Introduction to Network Science

Yong-Yeol "YY" Ahn

SCHOOL OF INFORMATICS AND COMPUTING

INDIANA UNIVERSITY

Bloomington

Q: Who are more popular, you or your friends (on average)?

1. You

2. Same

3. Your friends

1. You

2. Same

How can *everyone* feel that his/her friends are more popular?

"Friendship Paradox"

When are we living?

Google processes

20+ petabytes

per day

Most populated countries

1,300,000,000+

1,200,000,000+

300,000,000+

500,000,000+

900,000,000+

1,300,000,000+

1,200,000,000+

Most populated countries

Billions of people

recording their social life

in **Bits**.

40421551 40421561 40421571 40421581 40421591 40421601 40421611 40421621 40421631 40421641 40421651 40421661 40421671 40421681 4042 tgagcagacctatataagatggttatgaagattcacacagcggctcatgcctgtgatcccagcactttgggaggctgaggcaagtggagcacctgagatcatgagttcaagaccagcctggccaacatggtgaaaccccatctcta tgaacagacctatataagatggtt tgaagattcacacagtggctcatgcctgtgatcccagcac tgggaggctgagtcaagtggagcacctgagatcatgagtt ACCAGCCTGGCCAACATGGTGAAACCCCATCTA cagacctatataagatggtt aagatacacacagtggctcatgcctgtgatcccagcactt GGGAGGC GAGGCAAG GGAGCACC GAGA CATGAG TC cagcctggccaacatggtgaaaccccatctcta GACC A A AAGA GG A GAAGA CACACAG GGC C CC G GA CCCAGCAC I GGGAGGC GAGGCAAG GGAG ACC GAGA CA GAG CAAGACCAGCC GGACAACA GG AACCCCA C C A ATATAAGATGGTTATGAAGATTCACACAGTGGCTCATGCC tgatcccagcactttgggagg TGAGGCAAGTGGAGCACCTGAGATCATGAGTTCAAGACCA GCCAACATGGTGAAACCCCCATCTCTA
TCAGATGGTTATGAAGATTCACACAGTGGCTCATGCCTGTTATCCCAGCACTTTGGGAGGCTGAGGCAAGGGGAGCACCTG ATGAGTTC **GAACAG GAACAGAC** gaacagccc la La aagatggttatgaagattcacacagtggctcatgcctgtg TCCCAGCAC TTGGGAGCC GAGGCAAG GGAGCACC GA A GAG CAAGACCAGCC GGCCAACA GG GAAACCCCA TA A tgaacagacctatata gatggttatgaagattcacacagtagctcatgcctgtgat AGCACTTTGGGAGGCTGAGGCAAGGGGAGCACGTGA GAG I CAAGACCAGCC GGCCAACA GG GAAACCCCA C C A gacciatataagatggittatgaagattcacacagtggctc CIGTAATCCCATCACTTTGGGAGGCTGAGGCAAGTGGAGCCCTGAGATCATGAGTTCAAGA AGCCTGGCCAACATCGTGAAACCCCATATCTA GAACAGACC IA A AA 1GG ITA GAAGAT CACACAG GGC CATGCC G GATCC cactttgggatgctgaggcaagtggagcacctgagatcat CAAGACCAGCC GGCCAACA GG GAAACCCCA C C A ACC TATA TAAGA TGG TTA TGAAGA TTCACACAG TGGC TCA TTGTGA TCCCAGCAC TTTGGGAGGC TGAGGCAAG TGGAGA CACGAG TCAAGACCAGCC TGCCCAACA TGG C AACCCCCA C TC TA G GAACAGACCTATATAAGA GGTTACGAAGATTCACACAGTGGCTCATGCCTGTGATCCC cacattgggaggctgaggcaagtggagcacctgagatcat AAGACCAGCC GGCCAACA GG GAAACCCCA C C A tgaacagacctatataagat ttatgaagattcacacagtggctcatgcctgtgatcccag CTTTGGGAGGCTGAGGCAAGTGGAGCACCTGAGATCATGA agcc ggccaaca gg gaaacccca c c a aaga Licacacag ggc ca gccag ga cccagcac Lt GGGAGGC GAGGCAAG GGAGCACC GAGA AA GAG C GCC GGCCAACA GG GAAA CCCA C C A gaacagacc a a aaga gg GAACAGACC A A AAGA GG A aga t cacacagaggc ca gcc g ga cccagcac the AGGC GAGGCAAG GGAGCACC GAGA CA GAG CAAG CC GGCCAACA GG GAAACCCCA C C A CACACAG IGGC CA IGCC G GA CCCAGCACC I GGG GC IGAGGCAAG IGGAGCACC GAGA CA IGAG I CAAGAC gaacagacclatataagatggtta **CCAACA GG GAAACCCCA C C A** ACTTIGGGAGGC GAGGCAAG GGAGCACC GAGA CA G **GAACAGACC A A AAGA GG A** CAG GGC CA GCC G GA CAACA GG GAAACCCCA C C A AC CGGGAGGC GAGGCAAG GGAGCACC GAGA CA G GAACAGACC A A AAGA GG A GAAG CAG GGC CA GCC G GA C AACA GG GAAACCCCA C C A GAACAGACC A A AAGA GG A GAAGA CAG GGC CA GCC G GA CC CC C GGGAGGC GAGGCAAG GGAGCACC GAGA CA G ACA IGG GAAACCCCA IC IC A GAACAGACC A C AAGA GG A GAAGA T GCGGC CA GCC G A C C TT GGGAGGC GAGGCAAG GGAGCACC GAGA CA GA ACA GG GAAACCCCA C A A CTCTTGCCTGTGATCCCAGCACTTTGGGAGGCTGACGCAATTGGAGCACCTGAGATCATGAGTTCAAGACCAGCCTGGCCATTGGTGAAACCCCATCTCTA GAACAGACC A A AAGA GG A GAAGA C CTCATGCCTGTGATCCCAGCACTTTGGGAGGCTGAGGCAATTGGAGCACCTGAGATCATGAGTTCAAGACCAGCCTGGCCA GAACAGACC A A AAGA GG A GAAGA C **GG GAAACCCCA CGC A** GIGA CCCAGCACITTIGGGAGGC GAGGCAAG GGAGCAC GA CA CA GAGT CAAGACCCGCC GGCCAACA GG GAAAC cca cicle GAACAGACC A A AAGA GG A GAAGA CA AGA GG TA GAAGA TCACACAG GGC CA GCC G GA CCAGCAC TTGGGAGGC GAGGCAAG GGAG ACC GAGA GAG I I CAAGACCAGCC I GGCCAACA I GG I GAAACCCCA I C I TA ACA IGG I I A I GAAGA I I CACACAG I GGC I CA I GCC I G I GA CITTI GGGAGGC I GAGGCAAG I GGAGCACC I GAGA I CA I GA CA GG G AACCCCA C C A GG HA GAAGA HCACACAG GGC CA GCC G GA CCC C C C GGGAGGC GAGGCAAG G agcacc gaga ca gag caagaccagcc g**caacat igaaacccca c c a ga <mark>teccagcae titigggagge gaggcaag iggagcaee</mark> time aan ag tilcaagaccagce iggccaaca igg igaaacceca te til Al A GAAGA CACACAG GGC CA A GAAGA CACACAG GGC CA GCC G GA CCCAGCA C GGGAGGC GAGGCAAG GGAGCACC GAGA CA GAG **CATGGTGAAACCCCATCTCTAC** CA GG GAAACCCCA C C C ga teccage ta tttgggagge tgaggaaag tggagcacet atcccagcactttgggaggctgaggcaagtggagcacctg CA GG GAAACCCCA C C A C GAGAGGC GAGGCAAG GGAGCACC GAGA CA GAG **GTGAAACCCCATCTCTAO** G GAAACCCCA C C A GGGA GC LAG CAA LG AGCACC GAGA CA GAG LC aggc gaggcaag ggagcacc gaga ca gag caag ді давасосса сісів iggggcaag iggagcacci gaga i ca i gag i i caagacca gigaaaccgigicicae gaggcaag tggagcacc tgaga tca tgag tilcaagacca GAAA CCCA C C A GAGGCAAG GGAGCACC GAGA CA GAG CAAGACCAG GAAACCCCA C C A AGGCAAG GGAGCACC GAGA CA GAG CAAGACCAGC **GAAACCCCA C C A** aggcaattigagciccigagatcatgagticaagaccagc дааасссса с с д **GCAAG GGAGCACC GAGA CA** AACCCCA C C A CAAG GGAGCACC GAGA CA GAG CAAGACCAGCC G AA CCCA C C A <u>caagiggagcaccigagalicaligaglii caagaccagccig</u> aaccccalicieiad AAG GGAGCACC GAGA CA GAG CAAGACCAGCC GG AACCCCA C C A AG GGAGCACC GAGA CA GAG CAAGACCAGCC GGC ACCCCG G A AG GCAGCACC GAGA CA GAG CAAGACCAGCC GGC acccca c c a **CCCCA C C AC** G GGAGCACC GAGA CA GAG CAAGACCAGCA GGCC GGAGCACC GAGA CA GAG CAAGACCAGCC GGCCAA **CA C C A** ggagcaccigaga i ga i gagitt caagaccaggg i ggccaa **CA C C A** ggagcacc gaga ca gag caagaccagcc ggccaa CG G G A GAGCACC GAGA CA GAG L CAAGACCAGCC GGCCAAC **CA G G A**

BIG DATA

BIG DATA

INFORMATION

Pulse of the Nation: U.S. Mood Throughout the Day inferred from Twitter

http://www.ccs.neu.edu/home/amislove/twittermood

BIG DATA

BIG DATA

SOCIETY LIFE ECONOMY

BIG DATA

BIG DATA SOCIETY LIFE ECONOMY COMPLEX SYSTEMS

COMPLEX SYSTEMS

COMPLEX SYSTEMS

MANY parts,

INTERACTING with each other

in **NON-TRIVIAL WAYS**

NETWORKS

Nodes

Links (edges) between nodes

Degree: # of neighbors

Links (edges) between nodes

Y.-Y. Ahn, S. Ahnert, J. P. Bagrow, A.-L. Barabási, *Sci. Rep.* 2011

So what?

Pagerank = Random walk problem on a network

Linked in

~02010 LinkedIn - Get your network map at inmaps.linkedinlabs.com

H1N1 Pandemic prediction

Real Prediction

Reaction-diffusion system with transportation networks

Can we understand a **complex system**

without knowing the **structure** of it?

NETWORKS

Graph Theory

Leonhard Euler

"Sociogram"

EMOTIONS MAPPED BY NEW GEOGRAPHY

Charts Seek to Portray the Psychological Currents of -Human Relationships.

> **New York Times** April 3, 1933

What's the structure of networks?

 $p = 0.5$

Alfréd Rényi

Paul Erd

Clustering

Small-world

Heterogeneity

It's not random! We form clusters.

"Small world experiment"

Stanley Milgram

[http://oracleofbacon.org/index.php](http://livepage.apple.com/)

We're **clustered**, but at the same time we are **well-connected**.

Duncan J. Watts

 (a)

Steven H. Strogatz

Watts and Strogatz model Watts & Strogatz, Nature 1998

Networks are heterogeneous!

Albert-László Barabási **Réka Albert Hawoong Jeong**

Poisson distribution

Degree: # of neighbors

Liljeros et al., Nature 2001

GROWTH:

add a new node with m links

PREFERENTIAL ATTACHMENT: the probability that a node connects to a node with k links is proportional to k.

Error

Error

Attack

Attack

"We can't block epidemic spreading on scale-free networks"

Alessandro Vespignani

Epidemic spreading: "following links"

Epidemic spreading: "following links"

"Friendship Paradox"

Epidemic spreading: "following links"

"Friendship Paradox"

The disease quickly get to the **hubs**

How to effectively **detect** & **prevent** the disease spreading?

"Hubs"

Random person -> immunize a random friend of the person (but **not the original one!**)

Communities

Networks are not just **clustered**, but form **communities**

where $\mathcal{L}_{\mathbf{A}}$ is paper we show this can be achieved. In this can be achieved. In this can be achieved. The study of community structure in networks has a long history. It is closely related to the ideas of graph interconnected nodes" "a group of densely

in understanding and visualizing the structure of net-

time minimizing the number of edges that run between

processors, so that the amount of interprocessor commu-

nication (which is normally slow) is minimized. In gen-

eral, finding an exact solution to a partitioning task of

this kind is believed to be an NP-complete problem, mak-

ing it probibitively different to solve for large graphs, but for l

a wide variety of heuristic algorithms have been devel-

oped that give acceptably good solutions in many cases,

the best known being perhaps the Kernighan–Lin algo-

ever not particularly helpful for analyzing and under-

standing networks in general. If we merely want to find

if and how a given network breaks down into commu-

nities, we probably don't know how many such com-

munities there are going to be, and there is no reason

why they should be roughly the same size. Furthermore,

the number of inter-community edges needn't be strictly

minimized either, since more such edges are admissible

between large communities than between small ones.

rithm [20], which runs in time O(n3) on sparse graphs.

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Hierarchy

Hierarchy implies **communities**.

Hierarchical Random Graph model

A. Clauset, C. Moore, and M. E. J. Newman, *Nature* (2008)

Hierarchical community structure

Hierarchy - Communities

Sunday, October 7, 12

BUT,

large network we introduce the distributions of these four basic

G. Palla, I. Derényi, I. Farkas & T. Vicsek, *Nature* (2005)

existence of the their structural subunits (communities) associated (co

with more highly interconnected parts. Identifying these a priori

unknown building blocks (such as functionally related proteins5,6

industrial sectors7 and groups of people8,9) is crucial to the

understanding of the structural and functional properties of

networks. The existing deterministic methods used for large net-

 $w = \frac{1}{\sqrt{2}}$

networks are made of highly overlapping cohesive groups of

nodes. Here we introduce an approach to analysing the main

statistical features of the interwoven sets of overlapping commu-

nities that makes a step towards uncovering the modular structure

of complex systems. After defining a set of new characteristic

 $\frac{1}{\sqrt{2}}$

technique for exploring overlapping communities on a large scale.

We find that overlaps are significant, and the distributions we

introduce reveal universal features of networks. Our studies of

collaboration, word-association and protein interaction graphs

show that the web of communities has non-trivial correlations and

 \mathbb{R}^n are more highly connected to each other than to the rest of than to the rest of than to the rest of th

the network. The sets of such nodes are usually called clusters,

communities, cohesive groups or modules8,10,11–13; they have no

Most real networks typically contain parts in which the nodes

Arnold Perey, Social organization of *Oksapmin*, Papua New Guinea

Overlap is **pervasive.**
Overlap is **pervasive.**

Sunday, October 7, 12

[http://www.youtube.com/watch?v=SxuYdzs4SS8](http://livepage.apple.com/)

Hierarchical community structure

Hierarchy \longrightarrow Communities

Simple local structure

Complex global structure

Complex global structure

What the xxxx is this?

tests or validations of the efficiency of the efficiency of the efficiency of our algorithm. In particular, α **Word association network**: Network of "commonly associated English words"

of protein–protein–protein–protein–protein–protein interactions \mathcal{P}_c

represent the different meanings of this word. c, The communities of this word. c, The communities of the comm
The communities of the communities

above, because none of the others in the others in the others in the literature satisfy all the literature satisfy all the others in the literature satisfy all the others in the literature satisfy all the set of the liter

G. Palia, I. Derenyi, I. Farkas different networks. The communities are colour coded, the overlapping G. Palla, I. Derényi, I. Farkas & T. Vicsek, *Nature*, 2005 'bright' in the South Florida Free Association norms list (for which is the South Florida Free Association nor
The South Florida Free Association norms list (for which is the South Florida Free Association norms list (for

Here is the **PROBLEM.**

Communities exist.

Hierarchical structure exists.

 A

of clustering and community structure in networks [5, 6, 9,

 $10,$ 11]. Hierarchical structure goes beyond simple clustering, $\frac{1}{2}$

however, by explicitly including organization at all scales in

a network simultaneously. Conventionally, hierarchical struc-

ture is represented by a tree or *dendrogram* in which closely

related pairs of vertices have lowest common ancestors that

are lower in the tree than those of more distantly related

pairs—see Fig. 1. We expect the probability of a connec-

tion between two vertices to depend on their degree of relat-

edness. Structure of this type can be modelled mathematically

using a probabilistic approach in which we endow each inter-

nal node r of the dendrogram with a probability p^r and then

connect each pair of vertices for whom r is the lowest com-

This model, which we call a *hierarchical random graph*, is

similar in spirit (although different in realization) to the tree-

based models used in some studies of network search and nav-

Hierarchical community structure

Hierarchy \longrightarrow Communities

Hopeless?

Solution: Use **LINKS**

Solution: Use **LINKS**

"a group of densely interconnected nodes"

"a group of densely interconnected nuNKS"

LINK communities

Nodes: multiple membership

Links: unique membership

 A

of clustering and community structure in networks [5, 6, 9,

 $10,$ 11]. Hierarchical structure goes beyond simple clustering, $\frac{1}{2}$

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Hierarchy - Communities

So, How?
Similarity between links

Hierarchical Clustering

$$
e_{ik}
$$
\n
$$
e_{jk}
$$
\n
$$
i
$$
\n
$$
k
$$
\n
$$
j
$$
\n
$$
j
$$
\n
$$
n_{+}(i) \equiv \{x \mid d(i, x) \le 1\}
$$
\n
$$
S(e_{ik}, e_{jk}) = \frac{|n_{+}(i) \cap n_{+}(j)|}{|n_{+}(i) \cup n_{+}(j)|}
$$

$$
e_{ik}
$$
\n
$$
i
$$
\n
$$
i
$$
\n
$$
j
$$
\n
$$
n_{+}(i) \equiv \{x \mid d(i, x) \le 1\}
$$
\n
$$
S(e_{ik}, e_{jk}) = \frac{|n_{+}(i) \cap n_{+}(j)|}{|n_{+}(i) \cup n_{+}(j)|} \xrightarrow{4}
$$

Does it really work?

~600k nodes ~3M edges

Sunday, October 7, 12

Summary

- Networks matter.
- Particularly in the age of big data and social networks.
- Many interesting problems waiting for you!

Resources

- <http://yongyeol.com/courses/2012S-I590/>
- [http://yongyeol.com/w/index.php?](http://yongyeol.com/w/index.php?title=Network_science) [title=Network_science](http://yongyeol.com/w/index.php?title=Network_science)
- http://en.wikipedia.org/wiki/Network science

- <http://yongyeol.com>
- yyahn@indiana.edu

