# CHAPTER I

# INTRODUCTION

## I.a Preview

Long before D'Arcy Thompson's *On Growth and Form*,[230] physicists studied the problem of pattern formation, but without much quantitative success. More recently, a great explosion of interest in non-linear dynamics has lead to new attempts to understand complex spatial patterns. In some areas these new techniques have lead to qualitatively new nonlinear approaches, also without too much quantitative success. In others, like those to be discussed here, the reexamination has resulted in a great increase in understanding by applying traditional techniques in new ways. The following pages contain almost no equations that are non-linear in the standard sense. The patterns to be analyzed are not active and dynamic like the reaction-diffusion equations of biological morphogenesis. Instead we find relaxation processes, gradual approaches to statistical equilibria, and almost universally linear equations. It is worth remembering, at a time when nonlinear phenomena receive so much attention, that linear problems involving energy minimization subject to constraints can result in notably complex patterns. Our success modeling and simulating simple large aspect ratio coarsening of this type is welcome given our general lack of understanding of large aspect ratio non-linear systems. Even so, our understanding is far from complete. As we discuss in later sections of this thesis, there are still many unsolved and interesting problems in linear pattern formation.

## I.b Overview

The study of the origin and development of granular materials is a large and well developed subject, as anyone who tries to survey the daunting quantity of literature presented in the bibliography will agree. Two schemes of dividing the field seem natural. We can consider the type of material under study, soap froths in one and two dimensions, fracture structures in basalt, grains in pure metals and in alloys (again in two or three dimensions) magnetic bubble systems, biological aggregates like cucumbers or human skin, ceramics, lipid monolayers, droplet condensation, etc., etc.. Alternatively we can characterize by philosophy of approach, mathematical studies of the topological and geometrical properties of random lattices, applied studies of real materials, modeling which attempts to mimic the detailed behavior of coarsening systems, modeling which considers only the basic underlying dynamical laws, engineering studies of practical applications, or philosophical musings on holistic patterns.

We will largely limit ourselves to a study of the coarsening process in cellular materials, paying particular attention to the way in which an initially ordered lattice can evolve into a disordered one. I will try to combine the two strategies mentioned above, to discuss three different systems and also a set of approaches to them. Only the case of the two dimensional soap froth is understood well enough at this point to discuss in detail. Its close cousin, the liquid-gas transition of the lipid monolayer is only now receiving adequate experimental attention. The realm of pattern formation in mag-

netic materials is better known, but the emphasis in the published literature is on device applications and the few studies of the coarsening problem are not notable for their quality.

## I.c Current Literature

Anyone trying to acquaint himself with the problems of coarsening and grain growth will want to refer to the many excellent review articles. Preeminence goes to Atkinson's invaluable and up to date "Theories of Normal Grain Growth in Pure Single Phase Systems,"[20] (1988) and to Weaire and Rivier's lively and thorough survey, "Soap, Cells and Statistics–Random patterns in Two Dimensions,"[247] (1984). Both are also notable for their excellent bibliographies. Another recent review is Nagai, Kawasaki and Nakamura, "Dynamics of two dimensional cell patterns,"[178] (1988). Cyril Smith's long series of reviews are older but full of interesting philosophical speculation and contain many helpful guides to the early literature on coarsening (1952-64).[206,207,208,209] Of the older papers on metal grains, the most complete is Beck's, "Annealing of Cold Worked Metals,"[24] (1954). A more recent overview of metallic grain growth may be found in Martin and Doherty's *Stability of Microstructure in Metallic Systems* (1976).[159] Several of the recent papers by Anderson *et al.* also contain reviews of the theory of grain growth.[12,16] Helpful reviews can also be found in several conference proceedings.[192] The popular literature has also given some attention to the coarsening of froths and metals, with a variety of brief summaries in various places.[21,44,91,153,238] Even the artistic world has chipped in.[221]

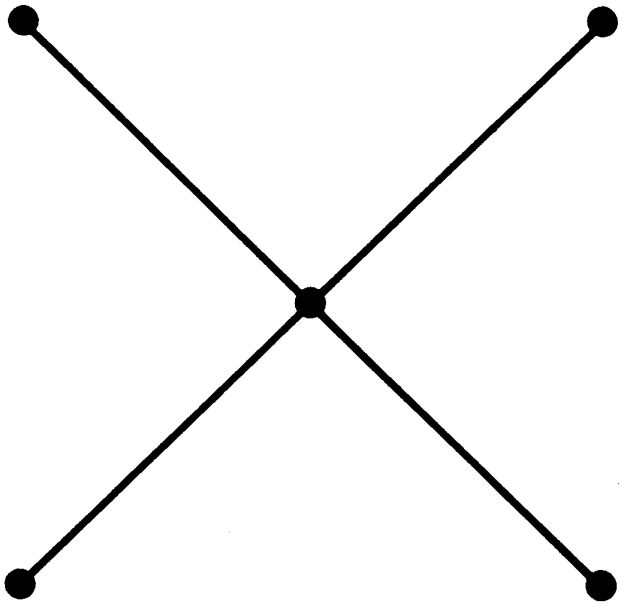
In the biological literature, the papers of Matzke and Lewis are erudite and contain helpful bibliographies.[140,146,161] For ceramics a good starting point is Brook, *Ceramic Fabrication Processes, Treatise on Materials Science and Technology* (1976).[39] For magnetic bubbles Eschenfelder, *Magnetic Bubble Technology* (1980), is the most useful, though it concentrates on device applications rather than disordered patterns.[63] There is no review available on the coarsening of lipid monolayers. The one published study is by Moore *et al.* (1986).[169]

## I.d What is a Two Dimensional Cellular Pattern?

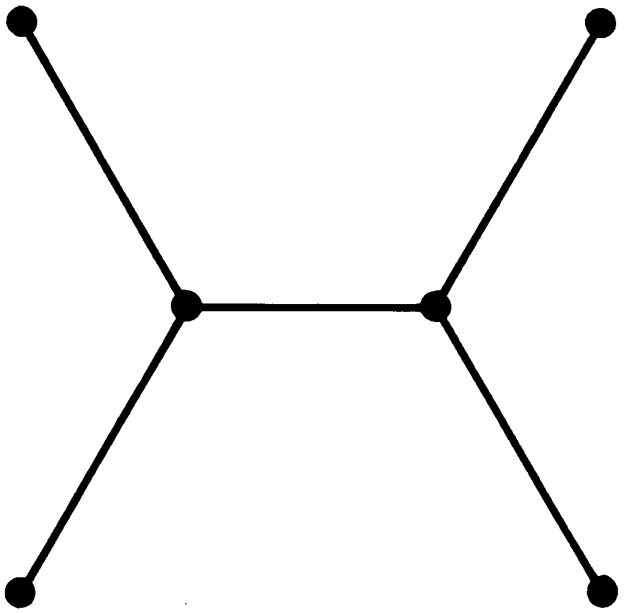
The bulk of this thesis will be devoted to the study of cellular patterns of a particular kind, two dimensional networks with coordination number three, and a dynamics driven by surface tension (or surface energy) forces. A pattern consists of a network of boundaries on a surface which has the property that three boundaries meet at every intersection or vertex. The topology and dynamics are related since the dominance of three-fold vertices results from considerations of structural stability in the presence of surface tension. Four-fold vertices tend to fall apart into pairs of three-fold vertices since the total length of four lines radiating from a single vertex to connect four points is almost always larger than the length of five lines radiating from two vertices (see Fig. 1).[87] The domains outlined by these boundaries

**Fig. 1 Length Minimization and Coördination Number.** The relative length of sides of four vertices connected by (A) four lines running to a single 90° vertex (length= 2.828) and (B) five lines running to two 120° vertices (length= 2.732). Since surface tension tends to minimize side length, (A) decays into (B).

(A)



(B)



are generally approximately polygonal in shape (though magnetic bubbles present an extreme case where this observation fails), with more or less curved boundaries. Curved walls result in energy differences across walls, and hence diffusion and wall motion.

In soap froths, magnetic bubbles and lipid monolayers, the equilibration time along the boundaries is short compared to the rate at which bubbles grow or shrink, so we may regard the pattern as fully relaxed, or equilibrated. In metal grains, the rates of diffusion along and across grain boundaries are comparable, but we will usually try to get by with assumptions of complete relaxation. We therefore tend to find Plateau's minimal surfaces experimentally, vertices of roughly 120°, walls that are nearly circular arcs, etc.[189,190] Another difference between these two classes of systems is that surface tension in lipids, soap bubbles and some magnetic materials is isotropic, while that in metal grains and other magnetic materials may be significantly anisotropic.

While the basic driving force is simply surface tension, it is the competition between surface minimization and conservation constraints that gives rise to patterns (we will usually neglect wall breakage in froths, grain coalescence in metals and mitosis in cells).

Similar rules hold in three dimensions, but vertices are four-fold connected with 109.47° internal angles, and walls take the form of sections of spheres. We will find however, that some of the geometrical rules that simplify our consideration of two dimensional networks are lost in three dimensions, considerably complicating the problem.