CHAPTER VIII

OTHER MEASURES OF DISORDER

We have previously discussed several ways to quantify the equilibration and disordering of a cellular pattern, including dynamical and distribution function measures. One natural way to look at disorder is to calculate an entropy. However, calculating the entropy of a pattern directly is sensitive to low amplitude noise and to lattice discretization. We therefore follow a suggestion made by Gollub for the analysis of the pattern of convective rolls in a large aspect ratio Rayleigh-Bénard cell and measure the entropy of the azimuthally averaged two dimensional Fourier transform of the pattern:[97]

$$
\begin{aligned}
\tilde{f}_0(\bar{\mathbf{k}}) &\equiv \int f(\bar{\mathbf{x}}) e^{i\bar{\mathbf{x}}\cdot\bar{\mathbf{k}}} d\bar{\mathbf{x}} \\
\tilde{f}(\bar{\mathbf{k}}) &\equiv \frac{\tilde{f}_0(\bar{\mathbf{k}})}{\int \tilde{f}_0(\bar{\mathbf{k}}) d\bar{\mathbf{k}}} \qquad \text{(VIII.1)} \\
S(f(\bar{\mathbf{x}})) &\equiv \int \tilde{f}(\bar{\mathbf{k}}) \log(\tilde{f}(\bar{\mathbf{k}})) d\bar{\mathbf{k}}
\end{aligned}
$$

This function depends in a simple way on the length scale, L, of the pattern $\tilde{f}(\bar{\mathbf{x}})$ as

$$S(f(L\bar{\mathbf{x}})) = S(f(\bar{\mathbf{x}})) + \delta \log(L), \qquad \text{(VIII.2)}$$

where δ is the dimension of the Fourier transform. For two patterns with the same basic length scale, a larger value of S indicates a more disordered pattern. In Fig. 49 (a) we plot $S(f(\bar{\mathbf{x}}))$ versus $\log(\langle a^{.5}\rangle)$ for Glazier *et al.*'s two dimensional air froth. For moderate length scales (late times) we observe (with some scatter due to poor statistics) the expected linear relationship.

Fig. 49 Spectral Entropy. (a) Spectral entropy versus time for a two dimensional air froth. (b) Spectral entropy versus the logarithm of the mean bubble radius for the same froth (From Glazier *et al.* 1989).[93]

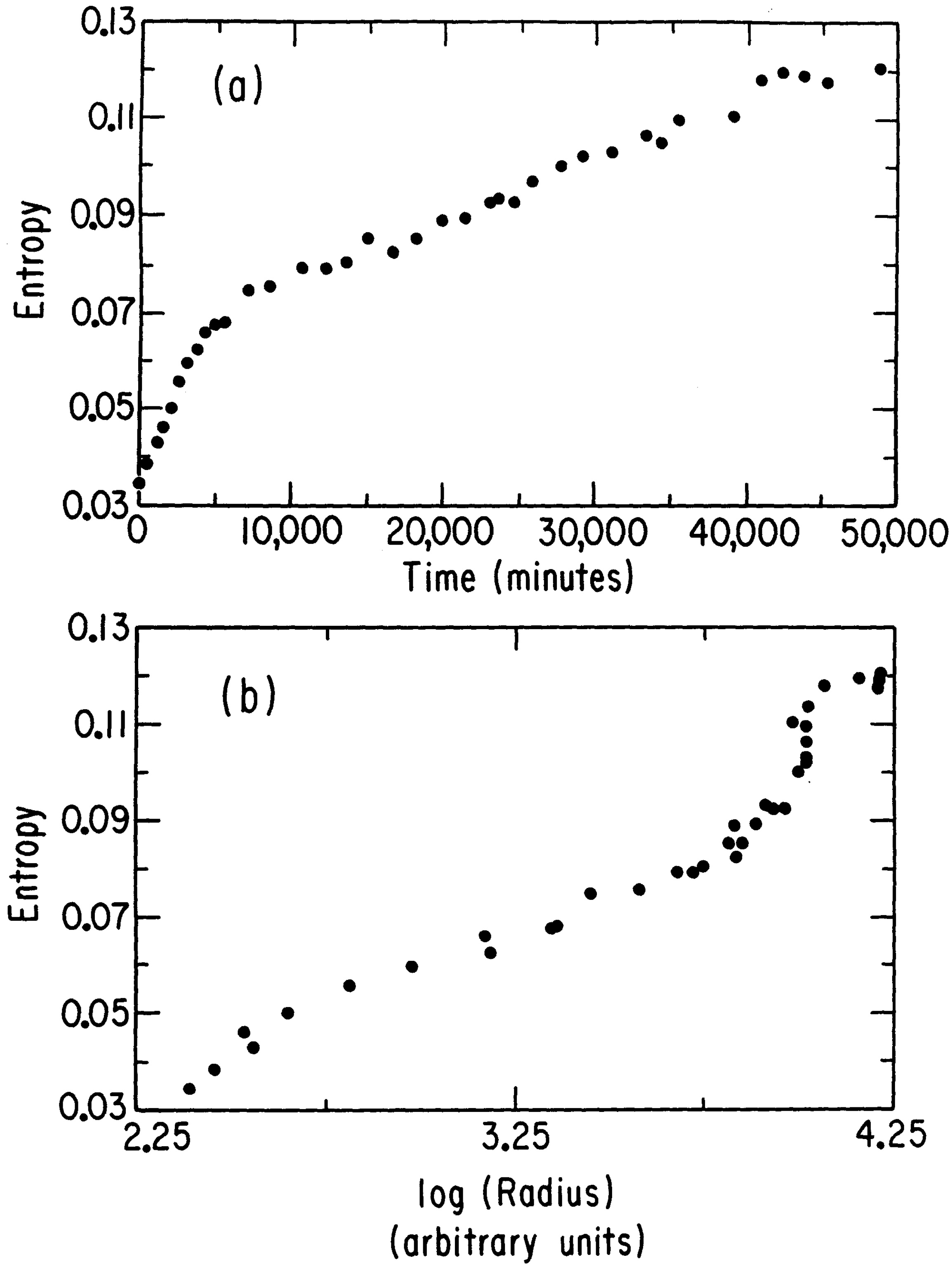

(a)
Entropy
0.03
0.05
0.07
0.09
0.11
0.13
0
10,000
20,000
30,000
40,000
50,000
Time (minutes)
(b)
Entropy
0.03
0.05
0.07
0.09
0.11
0.13
2.25
3.25
4.25
log (Radius)
(arbitrary units)

Fig. 50 Radius versus Time. Mean bubble radius versus time for the two dimensional air froth shown in Fig. 49. Note the nonmonotonic scatter at long times due to edge effects (From Glazier *et al.* 1989).[93]

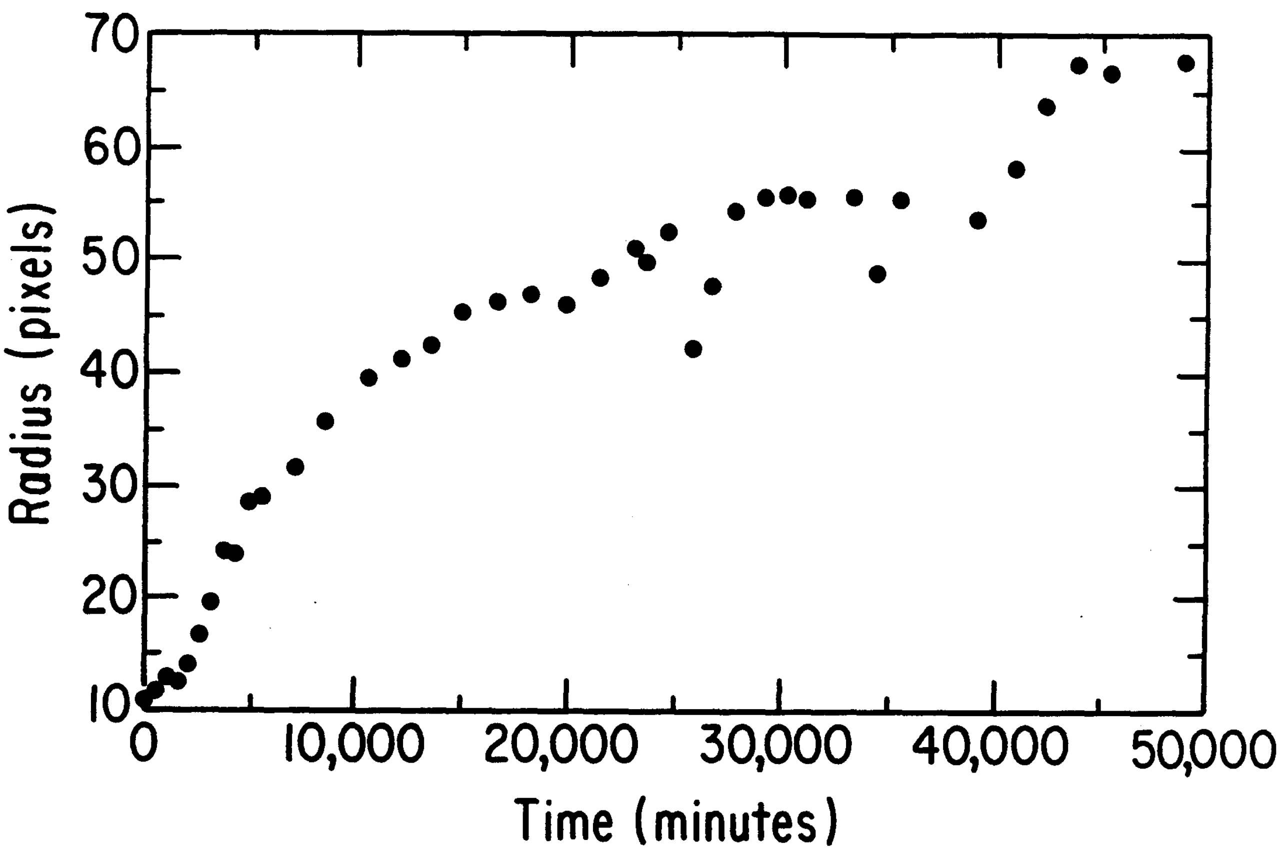
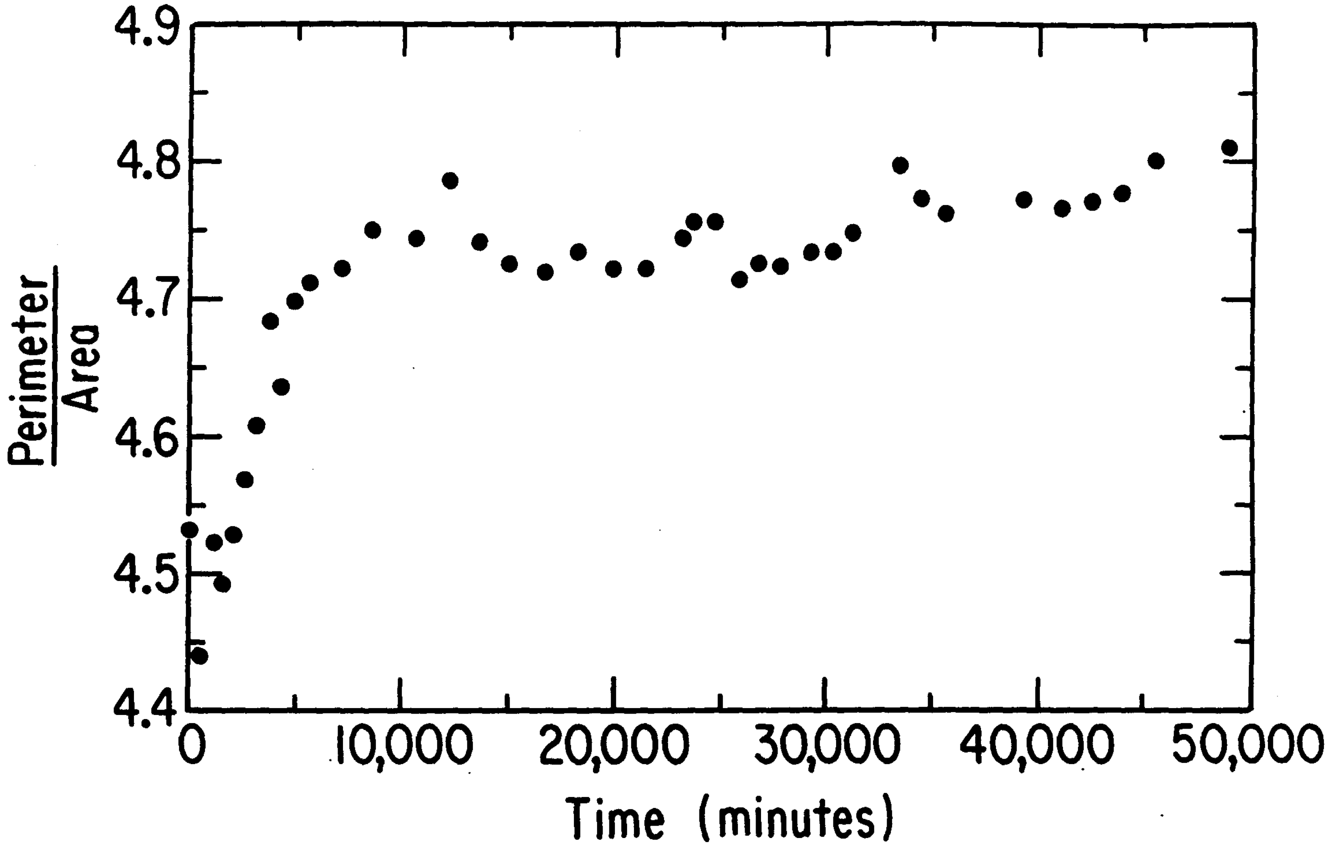

Radius (pixels)
70
60
50
40
30
20
10
0
10,000
20,000
30,000
40,000
50,000
Time (minutes)

Fig. 51 Perimeter/Area Ratio. Ratio of perimeter to area as a function of time for the two dimensional air froth shown in Fig. 49. The ratio reaches a constant value around 8000 minutes, when the froth reaches its scaling state (From Glazier *et al.* 1989).[93]


4.9
4.8
4.7
4.6
4.5
4.4
Perimeter
Area
0
10,000
20,000
30,000
40,000
50,000
Time (minutes)

At short length scales (early times), $S(f(\bar{x}))$ lay below the extrapolated line, indicating that the disorder of the pattern was increasing. As expected, the entropy reached equilibrium at approximately the same time as other measures of the disorder. At very long times the entropy rose abruptly above the extrapolated line. Most of this rise was due to non-monotonic fluctuations in the measured average bubble radius (See Fig. 50) at long times and does not represent a real change in the rate of entropy growth. If we plot instead, $S(f(\bar{x}))$ versus time (Fig. 49 (b)) we see a smooth increase in entropy, with a large slope during equilibration when the pattern was increasing in disorder and a smaller constant slope at long times when the pattern was merely increasing in length scale.

A disadvantage of the entropy method is its sensitivity to noise due to our difficulty in accurately measuring the average length scale of the pattern. However, its general applicability makes it a technique worth developing further.

Another technique to examine the stationarity of the pattern is to measure the ratio of the mean bubble circumference to the mean bubble radius as a function of time. We plot Glazier *et al.*'s values for the ratio versus time in Fig. 51. The ratio reached its equilibrium value of $4.73 \pm .4$ after approximately 8000 seconds, agreeing with the other measures of system equilibration.