CHAPTER XI

CONCLUSION

After our long ramble among various two dimensional coarsening phenomena it would be nice to be able to write something definitive. We are a long way from that achievement. We do understand a great deal about two dimensional coarsening in soap froths and pure metals. We understand how we go from an ordered pattern to a disordered scaling state and we have developed several methods to measure pattern disorder all of which give compatible results. We have developed a large variety of models, many of which seem accurately to duplicate the evolution of a froth. In the specific points where our models fail, we have some insight into why: anisotropy in the Potts model, failure to consider side shedding anticorrelations in our mean field theories, etc. Having identified the source of our difficulties we still need to study in detail how calculated distributions depend on anisotropy and correlations. We understand, at least qualitatively, the physical reasons why different systems (biological, geological, etc.) give different sorts of patterns, but we have not made any concerted effort to explain those patterns. Lewis' program for biology is still far from completion. In this sense we understand how the interaction of local energy minimization and topology produces complex patterns.

From an experimental point of view, repeating the soap froth experiment with larger cells and more and better controlled initial conditions would be desirable, but we would not expect any real surprises. Repeating the basic area versus time measurement in a drained cell with constant plateau border width would be ideal. If a drained cell still came up with a growth exponent different from one we would have to rethink our entire theory from the beginning. We also need to measure the anticorrelation between side shedding and side number. In the area of metal coarsening we need more information on anisotropy and temperature effects.

We have skirted or merely touched on a number of interesting points in two dimensional coarsening. We have discussed in a handwaving fashion how the zone refining of impurities and the presence of macroscopic defects leads to anomalous exponents in metal grain growth. It is a simple matter to drill holes and thread pins through a soap bubble cell to duplicate the effect. Glazier has made some preliminary measurements with a regular lattice of pins and found that coarsening slowed and eventually stopped. However, his observations were purely qualitative. A large cell with randomly placed pins would be a worthwhile experiment. It is less clear how we could duplicate anomalous grain growth experimentally. Several of Glazier *et al.*'s experiments suffered from undesired wall breakage. Perhaps, short duration intense heating could be applied intentionally to mimic random grain coalescence. More careful comparisons between the small-Q Potts model and metal systems exhibiting anomalous grain growth would also be interesting.

Theory and experiment are equally deficient when we come to three dimensional froths. Even the Potts model gives relatively poor results, for reasons that are not entirely clear. That we possess one adequate theory for two dimensional sections of three dimensional materials is no guarantee that we understand the full three dimensional problem. We are currently investigating the possibility of extending some of the existing models to three dimensions. But it will be some time before we have a developed theory. As a first step it would be nice to have a three dimensional von Neumann's law, or at least to know that it did not exist. The experimental picture is bleak. The best hope would be some sort of optical tomography on a rotating drum of froth. It would be an expensive and elaborate, but not impossible experiment. Data analysis and reduction might well prove to be an even worse problem than data collection. However, the good experimental data in three dimensions would be the most important new result we could obtain.

Finally, we have looked at two systems in which long range forces play a role. In the case of lipid monolayers, certain configurations look startlingly like ideal two dimensional grain growth while others look like mean field theory Ostwald ripening. In both cases we observe classical scaling states and familiar behavior. Indeed in the grain growth limit we expect the lipid monolayers to be closer to the ideal than any other system. We have suggested one way in which the von Neumann's law and Ostwald ripening theories could be combined, but lack the experimental data to test it. Additional experiment seems likely to lead to an understanding as complete as that we have for normal coarsening.

In the case of magnetic bubbles, long range interactions are fundamental. Though we have tried to draw analogies with more familiar patterns of behavior in metals and biology, many of the phenomena and patterns seem bizarre. Treating these patterns with an interaction mean field theory may work for "ordinary" bubble growth, but seems unlikely to explain behavior under field reversals, and the whole zoology of non-standard coarsening. A combination mean field theory plus boundary dynamic approach or mean field theory plus Potts model approach seems most promising.

Experimentally there is an enormous amount to do. All of the different coarsening scenarios need to be quantified. It seems clear that any future experiments should be done with an alternating bias field to smooth out bubble growth. The hysteretic patch reorganization produced by DC fields adds greatly to the complexity of interpreting a set of data already fearsomely complex. For both the lipid monolayers and the magnetic bubbles the fundamental problem is the lack of a basic dynamical equation. Top priority needs to go to an experimental measurement of von Neumann's law. For the lipids it probably exists. For the magnetic bubbles it may well not exist, in which case magnetic bubbles may prove a classic hard problem, like large aspect ratio Rayleigh-Bénard convection.

Spin-glasses may hold an important lesson. Statistics do not always tell the whole story. We ought to be surprised if magnetic bubble patterns could be described simply. It is nice to know that underneath these frightening non-linear problems, lies a simple linear problem that we have managed to solve.