THE UNIVERSITY OF CHICAGO

DYNAMICS OF CELLULAR PATTERNS

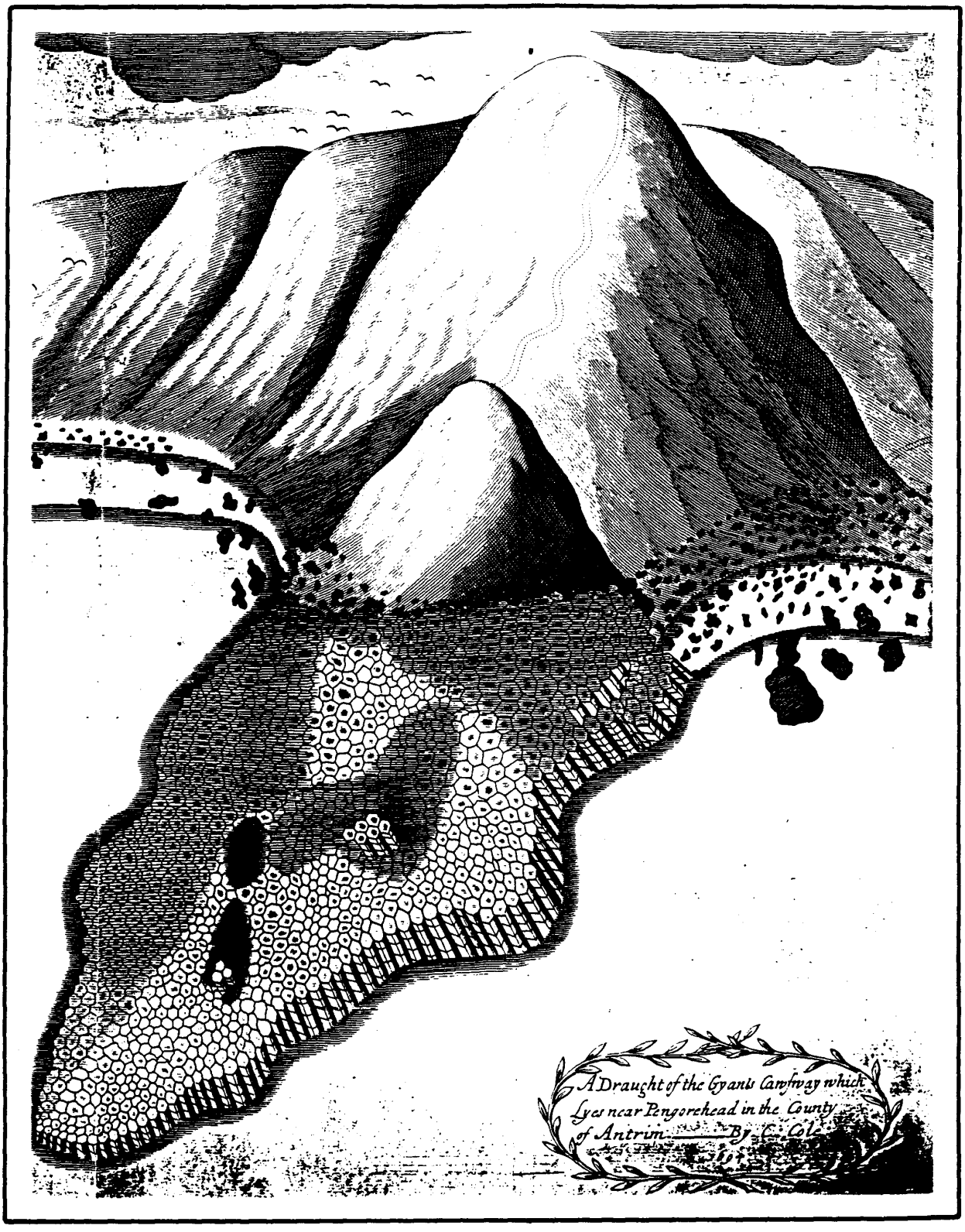
VOLUME ONE

A DISSERTATION SUBMITTED TO
THE FACULTY OF THE DIVISION OF THE PHYSICAL SCIENCES
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS

BY
JAMES ALEXANDER GLAZIER

CHICAGO, ILLINOIS
AUGUST, 1989


A Draught of the Gyants Cawsway which
Lyes near Pengorehead in the County
of Antrim ——— By C. Cole

Frontispiece: Giant's Causeway. Detail of a copperplate engraving of the Giant's Causeway, Northern Ireland, a basalt fracture pattern (From Sir. R. Bulkley 1693).[40]

Surface tension is of... paramount importance.– Sir D'Arcy Thompson

Like to bubbles that on water swim.– John Dowland

Double double toil and trouble, Fire Burn and Cauldron Bubble.– William Shakespeare

Astra ferar nomenque erat indelibile nostrum.– Ovid

PREFACE

The following discussion of coarsening is an eclectic summary. In most respects it makes no attempt at completeness. The discussion of grain growth in metals, in particular, is cursory and the selection of material arbitrary. The presentation of coarsening models, however, does attempt to present at least a few examples of each of the major types. The basic theme of the whole, for those who get lost among too many examples and too much detail, is how the interaction of diffusion and geometric constraints leads to complex patterns. Depending on context the terms grain, cell and bubble are used more or less interchangeably, as are film, soap film, wall boundary, and grain boundary; froth, pattern, network, array, lattice, and structure; and coarsening and grain growth.

I would like to thank Prof. Albert Libchaber for supervising and supporting this work, cheerfully and completely (even if, as occasionally happened, he disapproved of it). I would also like to thank Prof. Denis Weaire for many ideas, as well as hospitality at Trinity College Dublin, Prof. Nicholas Rivier, and Prof. Brian Kenny for useful criticism and discussions, and Prof. Vladimir Fradkov and Prof. Dmitri Udler for most stimulating correspondence. Dr. Pierre Molho, Dr. Bruno Berge and Marcello Magnasco generously provided data in advance of publication. Much of the work on soap froths discussed herein was done in collaboration with Dr. Joel Stavans, Dr. Gary Grest, Dr. Mike Anderson, Steve Gross and Andrew Belmonte.

Edward Pool did a beautiful job on the figures under tight time pressure. I am happy to acknowledge the support of a Grainger fellowship, which made much of this work possible.

I dedicate this thesis to Prof. Cyril Stanley Smith.

TABLE OF CONTENTS

TABLE OF CONTENTS, *continued*

TABLE OF CONTENTS, *continued*

LIST OF FIGURES

LIST OF FIGURES, *continued*

LIST OF FIGURES, *continued*

LIST OF FIGURES, *continued*

LIST OF TABLES

LIST OF SYMBOLS

Published models and experiments use a great variety of inconsistent notation. While we have tried to reduce our notation to some sort of order, some multiple usage remains. We give definitions of the most important symbols below.

Symbol	Meaning
α	Growth rate exponent. $< a > \propto t^{\alpha}$.
α, β, γ	Angles.
a_0	Initial average bubble area.
$< a >, < A >$	Average bubble area in a pattern.
A	Total area of experimental cell or pattern.
A_{ij}	Contact area between bubbles i and j.
A_n	Area of a single n-sided bubble.
$< a_n >, < A_n >$	Average area of an n-sided bubble.
β	Auxiliary growth exponent. $\alpha = 1/(\beta - 1)$
$c, c_1, c_2, d, \kappa_1, \kappa_2$	Fitting parameters.
δ	Spatial dimension.
$< \delta a >$	Width of area distribution.
ΔP	Pressure difference across bubble walls.
Δt	Time step in simulation.

LIST OF SYMBOLS, *continued*

d_{ij}	Distance between walls of bubble i and bubble j.
D	Diffusion constant for probability, Also the number of disordered bubbles in a pattern.
∂	Disorder parameter.
$< f >$	Average number of faces per polygon.
F	Force on vertex or boundary.
$\mathcal{F}$	Free energy of pattern.
H	Magnetic field.
$\mathcal{H}$	Hamiltonian.
i, j, k	Indices.
$j(r)$	Probability current.
J	Coupling or interaction strength.
κ	Diffusion constant for area. Units vary depending on context.
k	Boltzmann's constant.
λ_n	Relative area of an n-sided bubble $\lambda_n \equiv < a_n > / < a >$.
ℓ	Length of curved bubble wall.
L	Typical length scale.
μ	Boundary or vertex mobility. Units vary depending on context.

LIST OF SYMBOLS, *continued*

μ_2, μ_3, μ_4	Moments of side distribution.
$m(n)$	Average number of sides of neighbors to an n-sided bubble.
M	Magnetization.
n	Number of sides of a bubble.
$< n >$	Average number of sides per bubble.
$\hat{n}$	Local normal to a boundary.
$< n_f >$	Average number of edges per face in three dimensions.
N	Total number of bubbles in a pattern.
N_{edges}, $N_{vertices}$, etc.	Number of edges, vertices, etc.
O	The number of ordered bubbles in a pattern.
ψ	Angle mismatch between spins and lattice.
Q	Number of ground state spins in Potts model.
ρ	Radius of curvature in two dimensions.
ρ_i, ρ_j	Principle radii of curvature in three dimensions.
$\rho(n)$, $\rho(A)$, $\rho(n, A)$	Probability that a bubble has n-sides, area A, n-sides and area A.
$\mathcal{R}$	Ratio between $\rho(5)$ and $\rho(6)$. Also frictional damping force.
$< r_n >$	Average radius of an n-sided bubble.

LIST OF SYMBOLS, *continued*

σ	Surface tension. Units vary depending on context.
$\sigma_{(i,\ j)}$	Spin at lattice site $(i,\ j)$.
S	Entropy.
$\theta(n)$	Average internal angle for an n-sided bubble.
t	Time. Units vary depending on context.
T	Temperature.
$\mathcal{T}$	Topological charge. $\mathcal{T} \equiv n - 6$.
$\vec{v}$	Velocity of vertex or boundary.
W	Width of side distribution.
x, $\vec{x}$	Parameterized positions along bubble walls.