Thermal hard turbulence in mercury (tentative)

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Abstract

We studied the scaling properties of turbulence in a Rayleigh-Bénard convection cell using Hg, a low Prandtl number fluid (Pr=0.024). The length scales of thermal and viscous layers for Rayleigh numbers (Ra) ranging from 10^6 to 10^8 in an aspect ratio 1 cell, are estimated more precisely than previous analyses. The viscous boundary layer is thinner than the thermal one over the entire Rayleigh numbers range, unlike in He, water, and other fluids. We investigated the scaling of the Reynolds number (Re) and the Nusselt number (Nu) using different aspect ratio cells ($0.5, 1$ and 2) for Ra ranging from 10^5 to 2×10^9. We did not observed the hypothetical ultimate regime ($Nu \sim Ra^{1/2}$) even though the two boundary layers are inverted. We also analyzed temperature time series recorded in different positions across the boundary layers in the aspect ratio 1 cell and found that several non-dimensional quantities have a unique, Ra-invariant profile, if distance is normalized by the thermal boundary layer thickness. This may indicate an asymptotic regime of thermal

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turbulence in which the two boundary layers couple. The skewness of thermal fluctuations and its time derivative reveal that temperature fluctuations are not buoyancy driven but passively swept by the mean circulation in the bulk flow, outside the velocity boundary layer.

I. INTRODUCTION

Many recent experimental studies of the statistical properties of thermal turbulence in various fluids show a new type of developed turbulent state [**?**,**?**,**?**,**?**,**?**], called the hard turbulence, distinct from the earlier concept of classical thermal turbulence. Hard turbulence has the following characters, (1) the histogram of the temperature fluctuations has a long exponential tail at the center of the convection cell, while it is Gaussian in soft turbulence. (2) a large scale flow stably circulates near the wall of the cell. (3) the scaling exponents with dependence on the Rayleigh number, Ra, of the Nusselt number, Nu, Reynolds number, Re, and the mean square temperature fluctuation differ from the classical two field model. (4) the temperature power spectrum decays as a power law with $P(\omega) \sim \omega^{-1.4}$ in the inertial range for $Ra < 10^{11}$ and has two slopes for $Ra > 10^{11}$.

Classical theory for the scaling of thermal turbulence, hypothesized the existence of two regions, the thermal boundary layer and the center region [**?**,**?**]. The temperature gradient in the center is very weak because of strong mixing by turbulent flow, and steeper in the thin area near the top and bottom plate where flow is suppressed so heat is transported only by thermal diffusion. Therefore the entire temperature drop occurs within the this surface of the two regions, called 'the thermal boundary layer'. Thermal boundary layer is assumed to be marginally stable for the convective instability. $Nu \sim Ra^{1/3}$, $v \sim Ra^{3/7}$ and $\lambda_T \sim Ra^{-1/3}$ are derived from those assumptions, where v is the characteristic velocity of flow and λ_T the length scale of the thermal boundary layer.

On the other hand, the theory for the scaling exponent of the Nusselt number as a function of the Rayleigh number in the hard turbulence supposes a viscous sublayer between above center and thermal boundary layers, in which means velocity reaches its maximum because of the the large scale circulation [**?**]. Theory does not explain the Reynolds number sacaling or the meaning of the spontaneously organized large scale flow. (see **section** I B).

Is hard turbulence the ultimate regime of thermal turbulence? Some theoretical and experimental predictions suggest the existence of a new turbulent state. Kraichnan's theory for the ultimate regime of thermal turbulence predicts the scaling, $Nu \sim Ra^{1/2}$. Theories to explain it make one of two assumptions. The first one is that boundary layers effectively disappear at very high Ra, where heat can be transported by thermal structures, like buoyancy force driven plumes advected at the free fall velocity which gives $Nu \sim Ra^{1/2}$. The other argument is that the viscous boundary layer becomes thinner than the thermal one at high Ra (around $Ra = 10^{14}$ for He gas [**?**]), since the viscous length scale varies as $Ra^{-1/2}$ and the thermal length scale as $Ra^{-2/7}$. Again $Ra^{1/2}$ (see **section** I C).

A. Non-dimensional numbers

Four common non-dimensional numbers describe flow. First, the Rayleigh number is the non-dimensional temperature difference defined as:

$$Ra = \frac{\alpha g \Delta T L^3}{\kappa \nu}, \tag{1}$$

where g is the earth's gravitational acceleration, ΔT the temperature difference supplied to the cell, L the height of the cell, α the thermal expansion coefficient, ν the kinematic viscosity and κ is the thermal diffusivity of fluid. The Nusselt number is the non-dimensional heat flux given by:

$$Nu = \frac{Q}{\chi \frac{A}{L} \Delta T}, \tag{2}$$

where Q is the heat flux, χ the thermal conductivity, and A the cross sectional area of the cell. The Prandtl number, Pr, is defined as:

$$Pr = \frac{\nu}{\kappa} = \frac{l^2/\kappa}{l^2/\nu}. \tag{3}$$

The Pr is the ratio of two characteristic diffusive time scales in a thermal convection, the thermal diffusion time scale ($\tau_t \equiv l^2/\kappa$) and the viscous diffusion time series ($\tau_v \equiv l^2/\nu$). For large Pr fluids, thermal structures, for example plumes and thermal boundary layers, dominatethe flow in the system because $\tau_t \gg \tau_v$. On the other hand, thermal structures are less important for low Pr fluids because $\tau_t \ll \tau_v$, which show strong non-linearities because of inertial forces. The Prandtl number of Hg is much smaller ($Pr = 0.024$) at $20°C$ than that of water ($Pr = 2 \sim 7$) or He gas ($Pr = 0.7$). Finally, the Reynolds number is defined as:

$$Re = \frac{LU}{\nu}, \tag{4}$$

where L is the height of the convection cell and U the mean velocity of the large scale flow in the fluid. Re characterizes the ratio of inertial force and viscous force.

B. Characteristic of hard turbulence and two boundary layers

The Chicago measured the temperature fluctuations in hard turbulence in low temperature Helium gas in the latter half of 1980's. In hard turbulence, some scaling laws which relate the dimensionless numbers, differ from those of classical theory. The new scaling exponents, γ and ϵ, for the dependence of the Nusselt number and the Reynolds number on the Rayleigh number were:

$$Nu \sim Ra^{\gamma} = Ra^{0.285 \pm 0.004} \tag{5}$$

and

$$Re \sim Ra^{\epsilon} = Ra^{0.485 \pm 0.005} \tag{6}$$

for $10^8 < Ra < 10^{12}$. For $10^6 < Ra < 10^8$, soft turbulent regime, γ equals to 1/3 agreeing with classical theories [**?**,**?**] The theory of soft turbulence supposes two regions. One is the zone within the thermal boundary layer in which only thermal diffusion transport the heat flux and the other is the center region in which temperature is constant because of strong mixing.

Castaing *et al.*'s hard turbulence theory also includes the mixing layer between the thermal boundary layer and the center and predicts $\gamma = 2/7$ close to the experimental γ for He gas for $10^8 < Ra < 10^{12}$. The theoretical Reynolds number exponent is $\epsilon = 3/7$, which slightly differ from the observation in some experimental $\epsilon = 1/2$.

Fig. 1 shows schematically the spatial structure of hard turbulence and temperature and velocity profiles in a convection. z is the distance from the bottom plate. Near the plate, temperature profile is linear because only thermal diffusion transports heat. On the other hand, in center region of the cell average temperature T equals to $\Delta T/2$ due to strong mixing. Thus the thickness of the thermal boundary layer, λ_T, is the distance between the plate and the position at which the extrapolation of the linear part of the temperature profile equals the central mean temperature. The fluid velocity is zero at the surface of the plates and increases gradually because of the stably circulating large scale flow in the convection cell. Near the plate, the fluid velocity reaches its maximum and decreases towards the center of the cell where the mean velocity is zero due to symmetry. The viscous boundary layer thickness, λ_v, is the distance between the plate and the position of the velocity maximum. λ_T and λ_v of the top plate should be same with those of the bottom plate becouse of symmetry.

Belmonte *et al.* measured directly the thickness of the two layers in compressed SF_6 gas was measured directly using movable detectors [?]. The boundary layers become thinner with increasing Ra as follows:

$$\lambda_T \sim Ra^{-0.29\pm0.01} \simeq Ra^{-\gamma} \tag{7}$$

and

$$\lambda_v \sim Ra^{-0.44\pm0.09} \simeq Ra^{-\epsilon}. \tag{8}$$

The thermal boundary layer is thinner than the viscous boundary layer over whole range they measured for $10^8 < Ra < 10^{12}$, but should become thicker around $Ra \sim 10^{14}$. A theory of hard turbulence predicts these relations [?]. The heat flux, Q, in the thermal boundary layer is:

$$Q = \chi \frac{\frac{\Delta T}{2}}{\lambda_T}. \tag{9}$$

By dimensional analysis, this equation implies that:

$$Nu = \frac{L}{2\lambda_T} \tag{10}$$

and

$$\frac{\lambda_T}{L} \sim \frac{1}{Nu} \sim Pr^{1/7} Ra^{-2/7} \sim Ra^{-\gamma}. \tag{11}$$

The viscous boundary layer thickness scales as:

$$\lambda_v \sim \frac{\nu}{U} \sim \frac{L}{Re} \tag{12}$$

and

$$\frac{\lambda_v}{L} \sim \frac{1}{Re} \sim Pr^{5/7} Ra^{-3/7}. \sim Ra^{-\epsilon} \tag{13}$$

Eqns. (11) and (13) produce crossing like the extrapolation of the experimental results for SF_6. Also from eqns. (11) and (13),

$$\frac{\lambda_v}{\lambda_T} \sim Pr^{4/7} Ra^{-1/7}. \tag{14}$$

Equation (14) implies that the lower the Prandtl number, the smaller the ratio of the boundary layer thicknesses, λ_v/λ_T. The critical Rayleigh number for crossing in liquid Hg(Pr $= 0.024$) should be $Ra \sim 10^5 - 10^8$ [?], which we covered in our previous work ??. Also $Ra \sim Ra^5$ for Hg is derived by more precious estimation [?].

C. Ultimate regime of thermal turbulence?

In previous section, the possibility of the inverted viscous and thermal boundary layers is shown by both of theories and experiments. What happens if those boundary layers are inverted? $Nu \sim Ra^{1/2}$ have been expected to be seen in the ultimate regime of thermal turbulence according to two ideas [?,?]. The first assumption is that the thickness of viscous boundary layer becomes negligible because it varies $\sim Ra^{-1/2}$. Thus heat is advected directly by buoyancy driven thermal structures, for example plumes, without thermal diffusive regions. In such case, viscous force can become negligible everywhere in the convection cell and the thermal structures move at the free fall velocity. Then the inertial force balances the buoyancy one,

$$\alpha g \Delta T \sim (v \sim \nabla) v \sim \frac{v^2}{L}. \tag{15}$$

And the typical velocity of the thermal structures scales as

$$v \sim (\alpha g \Delta T L)^{1/2}. \tag{16}$$

As heat flux Q equals $\rho c_p \Delta T \times v \times A$, where ρ is the density of fluid, c_p the heat capacity, and A the cross section area of the convection cell, and the Nusselt number scales as

$$Nu \sim Ra^{1/2}. \tag{17}$$

The other argument is that when the viscous boundary layer becomes thin at higher Ra, it cuts the thermal boundary layer and the thickness of the viscous boundary layer limits the heat flux, which also gives $Nu \sim Ra^{1/2}$.

We can test 'ultimate regime' in this work.

II. EXPERIMENTAL SETUP

The experimental cell used to study the two boundary layers is vertical cylinder, aspect ratio 1, 10cm in height and 10cm diameter , as shown in Fig. 2. We also used other two cells (aspect ratio 1/2 and 2) with the same 10cm diameters but different heights, to measure the heat flux over wide range and check the Nusselt number scaling dependence on aspect ratio. The top and bottom plates are made of chromium plated copper 2cm thickness. The side wall cylinder is made of stainless steel 2mm thick to match the thermal conductivity of Hg. The high conductivity of mercury requires special heating and cooling. In the bottom plate, an insulated manganin wire heater (diamater 0.3mm) is embedded in copper in a spiral groove of semicircular section, with depth 0.4mm and horizontal spacing of 0.6mm.

The top plate forms the bottom of a large copper container with many copper fins embedded to promote good thermal exchange with the cooling water. Temperature controlled water enters the container through 5 inlets and exits from 4 outlets. The heater is supplied with constant power ranging from 1 watt to 1.2 K watts depending on the target Rayleigh number. The temperature of the cooling water is controlled precisely in stage: first by a refrigerator and an electric heater, then by a 60 liter mixing tank, and finally by Peltier elements for precise control. The worst temperature stability occurs for maximal heating and is 1% of the total temperature difference, ΔT, between the top and bottom plates. The best stability occurs for minimal heating and is of the order of $10^{-4\circ}K$. To prevent lateral heat leakage from the cell, thermal shield temperature regulated by water surrounds the heater. Setting the shield temperature of heater minimizes heat leak.

To measure the local temperature fluctuations, we use five thermistors (Thermometric B07PA) whose size is $300\mu m$ in diameter with insulation though the bare size is $200\mu m$. Their response time is sufficiently shorter than the flow's typical minimum timescale of 100-200 msec. As shown in Fig. 3, one[A] is placed at the center of the cell at midheight, two are vertically aligned at 1cm from the side wall, one at midheight[B], and the other 2mm above[C] to measure the meanm flow velocity. Two thermistors([D] and [E]) are fixed on the fine stainless tube which is tied to micro-translational stage, controlled by a stepping motor, and can be moved vertically along the center line of the cylinder. Each thermistor constitutes one arm of an AC capacitance bridge whose output is fed to a lock-in amplifier (PAR 124A). The output signal is first acquired by the digital spectrum analyzer (HP3563A). Temperature time series $T(z,t)$, $(t = 0, \Delta t, 2\Delta T, \cdots, n\Delta t)$ are measured at each height z with the sampling frequency ranging from 5 to 26 Hz, then analyzed by computer.

III. EXPERIMENTAL RESULTS

A. Statistical properties of thermal turbulence in mercury (aspect ratio 1 cell)

The histogram of the temperature fluctuations at the center of the aspect ratio 1 cell is nearly an exponential for Rayleigh numbers, $10^6 < Ra < 10^8$ [**?**]. Fig. 4 shows the histogram for $Ra = 7.54 \times 10^7$ measured at thermistor [A]. Moving thermistors: [D] and [E], along the center line of the cylinder, we analyzed the position dependence of the histograms of the temperature fluctuations (Fig. 5) which is Gaussian near the top plate and exponential near the center. The nondimensional fourth order moment of the temperature fluctuations, the 'flatness', is:

$$F = \frac{< (T(z,t) - < T(z,t) >)^4 >}{< (T(z,t) - < T(z,t) >)^2 >^2}, \tag{18}$$

where $< T(z,t) >$ is the time averaged temperature at height z, and $< (T(z,t) - < T(z,t) >)^2 >$ the variance. F characterizes large deviations from the probability distribution function of the temperature fluctuations. For instance, a Gaussian distribution corresponds to $F = 3$ and exponential distribution to $F = 6$. Fig. 6 plots F versus the distance z from the top plate.

We measured the mean flow velocity of the large scale circulation with the vertically aligned thermistors, [B] and [C], near the side wall. We estimated the group velocity of temperature fluctuations passing through the two detectors from the phase delay of the cross spectrum of the two signals [?]. The mean flow velocity was about 2 cm/sec at $Ra = 10^7$. Eqn. (4) gives the Reynolds and the peaked frequency of temperature power spectrum as a function of the Rayleigh number are shown in Fig. 6 and 7. The material constants of Hg used in these calculations was for $20°C$ and $1atm$ (Table 2). The large scale circulation rather than any secondary instability (*e.g.*, the oscillatory instability) causes a frequency peak in the temperature power spectrum, at f_p. Thus f_p also estimates the mean flow velocity, using the relation

$$V \sim \pi L f_p. \tag{19}$$

Fig. 7 shows the dimensionless parameter $Re' \equiv f_p \pi L^2/\nu$. Our direct and indirect measurements of the mean flow velocity coincide. The scaling relation,

$$Re \sim 6.24 Ra^{0.44 \pm 0.02}, \tag{20}$$

resembles hard turbulence in He gas but with a prefactor (6.24) about 20 times larger.

Fig. ??? shows the power spectrum of the temperature fluctuations at the center of the aspect ratio one cell for $Ra = 7.12 \times 10^7$. The spectra at the center of the cell fit to a form Wu:1990,

$$P(f) = \left(\frac{f}{f_0}\right)^{-\xi} \exp\left(-\frac{f}{f_c}\right) \tag{21}$$

In the aspect ratio 1 cell, ξ is $1.58 \pm 0.09 \sim 5/3$ for $10^6 < Ra < 10^8$. For Hg gas, ξ is 1.35 ± 0.05 ($\sim 7/5$). Fig. 9 plots the cutoff frequency, f_c, as a function of Ra. f_c scales as:

$$f_c \sim Ra^{\beta}, \tag{22}$$

with $\beta = 0.40 \pm 0.05$ in Hg, differing from that He hard turbulence where $\beta = 0.78$ by the same method [**?**].

In the mercury aspect ratio 1 cell for $10^6 < Ra < 10^8$:

$$Nu = 0.24 \times Ra^{0.25 \pm 0.02}, \tag{23}$$

was obtained. slightly smaller than for He gas. We did not obtain $Nu \sim Ra^{1/2}$.

B. Statistical properties of thermal turbulence in mercury (aspect ratio 1/2 and 2 cells)

We also measured the heat flux and time series of temperature fluctuations at the center of the aspect ratio 1/2 (height: $h = 20cm$) and 2 ($h = 5cm$) cells for a wider range of Rayleigh numbers because Ra is proportional to h^3. As a result, we obtained the Nu - Ra relation for $9.0 \times 10^4 < Ra < 2.0 \times 10^9$. Fig. 16 shows the Nusselt number as a function of the Rayleigh number. $\gamma = 0.28 \pm 0.02$ and 0.26 ± 0.02 for aspect ratio 1/2 and 2 cells, respectively. $1/4 < \gamma < 2/7$ which is the same or slightly smaller than for He gas which is close to 2/7 [**?**,**?**]. The aspect ratio 2 cell has a small jump in the $Nu - Ra$ curve at $Ra \sim 2 \times 10^5$. $Nu \sim Ra^{1/2}$ did not occur.

We also measured the temperature fluctuations to characterize their aspect ratio dependence. For the aspect ratio 1/2 cell, $Ra = 2 \times 10^8$ was realized when the maximum heating

($1.3KW$) is supplied. Fig. 10 and Fig. 11 show the typical histogram and power spectrum of temperature fluctuations for $Ra = 3.0 \times 10^8$. The histogram is exponential. The low frequency peak of the spectrum, as shown in Fig. 15, is broad showing that large scale flow does not exist or is very weak.

For the aspect ratio 2 cell, Fig. 12 shows the histograms of temperature fluctuations at the center of the cell for various Ra. The abscissas are normalized by the mean temperature of the top plate (T_{top}) and the temperature difference. For $Ra < 2 \times 10^5$, except for (a): $Ra = 1.6 \times 10^5$, the histograms are symmetric with two peaks and not exponential. The power spectrum of the temperature fluctuations has striking low frequency peaks (f_p) for all Ra (Fig. 15), showing strong oscillating flow at the center. The scaling exponent ϵ ($f_p \sim Ra^\epsilon$) equals 0.40 ± 0.02. We calculated the mean temperature in the center of the cell, T_{center}, and found that $(T_{center} - T_{top})/\Delta T$ are fluctuated widely for $10^5 < Ra < 10^7$, suggesting the presence of two or more circulating rolls which frequently change directions. exist in the aspect ratio 2 cell and their directions may change frequently. Fig. 16 shows a transition around $Ra \sim 2 \times 10^5$. What happens at the transition? As shown in Fig. 13, the averaged temperature fluctuation in the center region of the cell varies widely. The normalized standard deviation ($STD/\Delta T$) of the temperature fluctuations (Fig. 14) is much smaller below the transition than for $Ra > 2 \times 10^5$, implying that the direction of convection is stable below the transition point. Above this point, the large scale flow may change its direction frequently.

C. The thermal boundary layer

Since the two thermistors ([D] and [E] in Fig. 3) move to various distances from the top plate. Their time series can reveal the inner structure of the two boundary layers. Fig. 17 plots the time average of the temperature, $T_{ave}(z)$, as a function of the distance from the top plate, z. Temperature profile fits the function:

$$\frac{T_{ave}(z) - T_{top}}{\Delta T} = m_1 tanh(m_2 z), \tag{24}$$

$$tanh(m_2 z) = \frac{e^{m_2 z} - e^{-m_2 z}}{e^{m_2 z} + e^{-m_2 z}}, \quad (25)$$

where $T_{ave}(z)$ is the mean temperature of the position at which a movable thermistor is placed and T_{top} is the mean temperature of the top plate. Eqn. (24) is linear near to and saturated far from the boundary. We define the thermal boundary layer thickness as $\lambda_T = 1/m_2$ from the fit. Thus λ_T is the distance at which the extrapolation of the linear part of the profile equals the center mean temperature.

The estimated thickness also agrees with the distance at which the root mean square of the temperature fluctuations, T_{rms}, reaches its maximum, as shown in Fig. 17. $\lambda_T = 4.0mm$ at $Ra = 3.5 \times 10^7$.

D. The viscous boundary layer

We estimated the thickness of the viscous boundary layer by two methods: from the highest frequency, f_h, of the temperature frequency spectrum and by fitting the power spectrum of the temperature fluctuations.

Measurements in water and SF_6 [**?**,**?**] have validated this method to evaluate the mean velocity, which is proportional to f_h,

$$v_{ave}(z) \sim f_h, \quad (26)$$

where $v_{ave}(z)$ is the mean velocity at the distance z from the top plate. Fig. 18 shows f_h as a function of a distance z. Taking the viscous boundary layer thickness as the distance at which f_h is maximal yields $\lambda_v = 2.7mm$ at $Ra = 3.5 \times 10^7$, so $\lambda_v \sim 0.7\lambda_T$, inverted form SF_6 where λ_v is much larger than λ_T.

We developed a more reliable method to evaluate the viscous boundary layer thickness. For each value of Ra, the power spectrum of temperature $P_z(f)$ at the distances z are shifted on the frequency axis to fit a reference spectrum $P_{z_0}(f)$, minimizing the quantity:

$$Er(\phi) = \frac{1}{f_c - f_H} \int_{f_H}^{f_c} \left[ln \frac{P_z(\phi f)}{P_{z_0}(f)}, \right]^2 df \quad (27)$$

where f_H is the frequency at which the level of $P_{z_0}(f)$ becomes a fixed value P_0 and f_c is the cutoff frequency at which it sinks into the noise level (Fig. 20 shows a schematic drawing). The value ϕ^* for which Er reaches minimum is the ratio between the characteristic high frequencies of the two spectra as shown in Figure 21. The reference spectrum $P_{z_0}(f)$ is a power spectrum at an arbitrary distance z_0. We chose z_0 so the spectrum had maximum f_c. Therefore, the maximum value of the best fit parameter, ϕ^*, is 1 at $z \simeq \lambda_v$. Fig. 19 shows that ϕ^* did not change even if the lower cutoff, f_H, was varied by changing P_0 ($= P_{z0}(f_H)$) from $-45dB$ to $-80dB$. To evaluate the error bar ϕ^*_{err} of ϕ^*, $Er(\phi)$ can be fit by a quadratic:

$$\frac{1}{Er(\phi)}\left(\frac{d^2Er(\phi)}{d\phi^2}\right) \approx \frac{1}{\{\phi^*_{err}\}^2} \tag{28}$$

are derived. So the error bar is:

$$\phi^*_{err} \approx \left\{\frac{1}{Er(\phi)}\left(\frac{d^2Er(\phi)}{d\phi^2}\right)\right\}^{-1/2}. \tag{29}$$

This estimate of λ_v is easier and has less scatter than using the highest frequency of the power spectrum, f_h.

E. Thermal and viscous boundary layer thicknesses as a function of Rayleigh number

In two previous sections, we estimated the two length scales for $Ra = 6.1 \times 10^7$. The viscous boundary layer is thinner than the thermal. Figure 23 plots the thermal and viscous scales, estimated by the T_{rms} method and the fitted power spectrum method for various Ra respectively. Over the whole range of Ra, two boundary layers are inverted compared to those for compressed SF_6 gas. Fig. 24 shows that the ratio the two length scales is constant : $\lambda_v/\lambda_T = 0.63 \pm 0.05$, which indicates that the two layers couple. Thus the two inverted boundary layers both thin with increasing Ra, $\lambda_T, \lambda_v \sim -0.20 \pm 0.02$. The heat flux,$Q$, in the cell is:

$$Q = \chi\frac{\Delta T/2}{\lambda_T}, \tag{30}$$

if heat is transported in each horizontal plane of the cell without inclination and λ_T is constant at any positions of the plate. Eqns. (30) and (31) gives the Nusselt number from the thermal boundary layer thickness:

$$Nu = \frac{L}{2\lambda_T}. \tag{31}$$

Therefore from the inverse of eqn. (31),

$$\lambda_T = \frac{L}{2Nu}. \tag{32}$$

Fig.31 shows λ_T and $L/2Nu$ versus Ra in mercury. $L/2Nu$ is slightly smaller than λ_T.

IV. DISCUSSION

In mercury, the viscous boundary layer thikness was thinner than thermal, $\lambda_v < \lambda_T$, yet $Nu \sim Ra^{1/2}$ was not observed in the aspect ratio 1 cell. There is the mean flow in the center of the cell in aspect ratio 2 cell because some convection rolls exist. In aspect ratio 1/2 cell, λ_T and λ_v can be less than 1mm. Therefore we obtained the time series of temperature time series at various postions in only the aspect ratio 1 cell. The thermal and viscous bondary layers have not been measured in aspect ratio 2 and 1/2 cells. However, the peculiar scaling exponents, time series of temperature fluctuations and other histograms in the aspect ratio 1 cell reveal the inner structure and dynamics of the turbulence.

A. Advection from the boundary layer

The time series of temperature fluctuations show that plumes cannot be advected from the boundary layer. We estimate the Reynolds number of the shear flow based on the boundary layer thickness, to be $Re_\lambda \sim 500$, using $\lambda_v \sim 3mm$, so the boundary layer is turbulent. Considering that the turbulent viscous boundary layer is as thin as or thinner than the thermal boundary layer, turbulent shear stretches and mixes the plumes as soon as they detach from the thermal boundary layer and before they arrive at the center of the fluid. Simple plumes may not exist in low Prandtl number fluids.

The diffusion time for the smallest thermal structures, like plumes, of size τ_T is $\tau_T = \lambda_T^2/\kappa$, where κ is the thermal diffusivity of the fluid. The advection time is $\tau = L/V$. Therefore the ratio of the two time scales characterizes the importance of diffusion:

$$\frac{\tau_T}{\tau} = \frac{\lambda_T}{L}\frac{U\lambda_T}{\kappa} = \frac{\lambda_T}{L}Pe, \tag{33}$$

where Pe is the Peclet number for the thermal boundary layer. Using the estimates $\kappa = 4.3 \times 10^{-2} cm^2/sec$, $\lambda_T \sim 5mm$ and $V \sim 2cm/sec$, we obtain $\frac{\tau_T}{\tau} \sim 0.5$, so we must consider the effects of thermal diffusion. Turbulence may further increase the effective thermal diffusion.

We measured the temperature signal at different distances z from the top plate, for different values of Ra to check whether plumes exist (Fig. 26). To characterize the asymmetry of the histograms of the temperature fluctuations as shown in Fig. 5, we calculated the non-dimensional third-order moments: the skewness (S) defined as:

$$S = \frac{< (T(z,t) - < T(z,t) >)^3 >}{< (T(z,t) - < T(z,t) >)^2 >^{3/2}}. \tag{34}$$

Negative (positive) skewness corresponds to an asymmetry towards colder (warmer) temperatures.

Fig. 26 plots the skewness of the temperature time derivative, defined as:

$$S' = \frac{< \left(\frac{dT}{dt}\right)^3 >}{< \left(\frac{dT}{dt}\right)^2 >^{3/2}}, \tag{35}$$

where:

$$\frac{dT}{dt} = T_{i+1} - T_i. \tag{36}$$

S' characterizes the asymmetry of the derivatives of the signal. If buoyancy driven structures (plumes) exist near the cold boundary layer, a less rapid warming return follows excursion away from the mean (cooling). The cooling decreases S' while the warming increases. In total, cold plumes result in negative S'.

However, S' is positive from the cold top plate to the center in this experiment (see Fig. 26) so no cold plumes are present, unlike ordinary high Pr hard turbulence [**?**].

Others have measured the boundary layer of slightly heated or cooled surfaces of horizontal plate, where the turbulent flow passively mixes temperature [**?**,**?**]. Above a heated plate, $S' < 0$ and $S > 0$, whereas $S' > 0$ and $S < 0$ above a cooled plate. Therefore, $S' \times S < 0$ for a passive scalar. SF_6 gas hard turbulence convection has the opposite sign: $S' \times S > 0$ ($S < 0, S' < 0$), outside the thermal boundary layer of the top plate [**?**]. Active thermal plumes driven by buoyancy explain this feature in water [**?**]. In Hg, $S \times S' < 0$ outside the viscous boundary layer as shown in Fig. 26. In fact, the sign of $S' \times S$ defines three distinct regions, as seen on figure 26. Inside the viscous boundary layer, $S' \times S > 0$. ¿From $z/\lambda_T = \lambda_v/\lambda_T \simeq 0.6$ to about 6, $S' \times S < 0$. In the central region, of stable temperature stratification, $S' \times S > 0$. However, in this region where the mean velocity is small or null, $S' \times S < 0$ may not imply that buoyancy is dominant. Actually, there is several indications of the passive character of temperature fluctuations in the central region of the cell, in low Pr hard turbulence [**?**,**?**]. Buoyancy only drives the mean, large scale flow within a thin viscous boundary layer along the walls , where $S' \times S > 0$. Outside the viscous layer, plumes are inactive. The shear along the boundary layer feeds energy to drive the turbulence in the rest of the cell, mixing the temperature as a passive scalar.

B. Fluid motion at the center of the cell

In addition to the absence of plumes near the boundary, surface (boundaries and side walls) and bulk flows differ. The typical temperature fluctuation (rms) at the center, θ, is about 3% of the total temperature difference, ΔT; thus $\theta/\Delta T \sim 10^{-2}$ at $Ra = 10^7$. The typical velocity is about $2cm/sec$ at $Ra \sim 10^7$. The buoyancy force, $\alpha g\theta$, of the fluctuation at the center is much smaller than the inertial term, $|U\nabla U| \sim U^2/L$; $\alpha g\theta/(U^2/L) \sim 10^{-2}$. Therefore, in the centeral region, buoyancy is negligible and temperature fluctuations resemble a passive scalar, perhaps causing the scaling behavior of the spectrum. Balancing the buoyancy of the boundary layer, $\alpha g \Delta T$, with the inertial term, U^2/L correct estimates (2 cm/sec) the mean flow velocity; implying that most of the detached boundary layer flows along the side wall in the large scale circulation, which drives the turbulent flow in the center. At larger scales, inertia balances the buoyancy term, while at smaller scales inertia balances the energy transfer. The Bolgiano scale, L_B, at which buoyancy and energy transfer are of the same order, is given by $L_B = Nu^{1/2}L/(Ra \cdot Pr)^{1/4}$ [?,?]. In the experiment $L_B \sim 2cm$ at $Ra = 10^7$ which corresponds to 1 Hz in the spectrum, narrowing the cascade range.

In Fig. 25, the value of the mean temperature just below the cold upper plate boundary layer is larger than $\Delta T/2$, revealing a temperature inversion in the cell of about 10% of the total temperature difference, ΔT. Outside the boundary layer, the flow shows a stable stratification in temperature. The amplitude of the temperature is about 3% of ΔT, which is significantly smaller than the temperature inversion.

C. Scalings

At very high Rayleigh number, the transition to $Nu \sim Ra^{1/2}$ requires that the velocity boundary layer becomes turbulent and blow out the thermal boundary layer. However, horizontal mean flow which is driven by the buoyancy should determine the tickness of the viscous boundary layer. Without a thermal boundary layer, mean flow cannot exist. The ultimate state of the boundary region in high Rayleigh number flow may be two matching boundary layers. For $\lambda_T \sim \lambda_v$, the scaling, $\gamma = 1/4$, of the Nusselt number may be explained by the following argument:

Let us define a nondimensional parameter, G, the ratio between the buoyancy force and the viscous force in the boundary layer:

$$G = \frac{\alpha g \Delta T}{|\nu \nabla^2 U|}. \tag{37}$$

The viscous force of the horizontal mean flow mainly generates viscous force. Therefore, by dimensional analysis reads $|\nu \nabla^2 U| \sim \nu U/\lambda_v^2$,

$$G \sim \frac{\alpha g \Delta T}{\nu U/\lambda^2} = \frac{\alpha g \Delta T \lambda^3/\kappa\nu}{(U\lambda/\nu)(\nu/\kappa)} = \frac{Ra_\lambda}{Re_\lambda Pr}, \tag{38}$$

because $\lambda = \lambda_T = \lambda_v$.

We expect that G approaches a constant value since the buoyancy should balance the viscous force in the boundary and dissipate mostly in the viscous sublayer. If so, the Rayleigh and Reynolds number of the boundary layer must scale with the same exponent.

$$Ra_\lambda \sim Re_\lambda \sim Ra^\eta. \tag{39}$$

Using Eqn. (39), we obtain

$$Ra\lambda^2 \sim Re. \tag{40}$$

If we suppose $Re \sim Ra^{1/2}$, the boundary layer thickness should scale as,

$$\lambda \sim Ra^{1/4}, \tag{41}$$

which gives $Nu \sim Ra^{-1/4}$. This picture is close to the model of the model of rigid body rotation, proposed by Cioni *et al.* [**?**].

Because we measure the thermal boundary layer thickness only along the center line of the convection cell, we cannot determine if λ_T depends on position. However Fig. 31 shows that the relation of the total heat flux through the cross section of the cell (Nu_{total}) to the heat flux through the center of the plate (Nu_{center}):

$$Nu_{total} > Nu_{center} = \frac{L}{2Nu}, \tag{42}$$

so the heat flux near the side wall is comparable with that at the center. Here we suppose that, in the cental region of thermal turbulence, heat flows in narrow area (width $= \lambda_T$) along the side wall because the strong large scale flow passively sweeps away plumes from the thermal boundary layer. So the area of the lateral boundary layer (A') is:

$$A' \simeq 2\pi r \times \lambda_T. \tag{43}$$

When a thermal structure of velocity v and $T = \Delta/2$ transports heat, the heat flux Q' through the area A' per second is:

$$Q' \simeq 2\pi r \times \lambda_T v \times \rho c_p \frac{\Delta T}{2}. \tag{44}$$

The heat flux through the plate Q is:

$$Q = A \times \chi \frac{\Delta T/2}{\lambda_T}, \tag{45}$$

and should equal Q'. Substituting $\kappa = \chi/(\rho C_p)$ and supposing $Q = Q'$,

$$\frac{\lambda_T^2}{r^2} = \frac{\kappa}{2vr} \sim \frac{1}{Re}. \tag{46}$$

Using the scaling relation, $Re \sim Ra^{0.44\pm0.04}$, for experiment,

$$\lambda_T \sim Ra^{-0.22}. \tag{47}$$

This scaling exponent is close to our experimental result.

D. Unique Ra-invariant profiles

In hard turbulence, the profiles of temperature, skweness, and so on, all depend on the distance from the plate z and the Ra. We define $\Theta(z)$,

$$\Theta(z) \equiv F(z, Ra), \tag{48}$$

where $\Theta(z)$ is a generalized function. Even z is normalized by any length scale, their profiles are not invariant for different Ra, as they are in SF_6 [**?**].

Figs. 27, 28, 29, and 30 plot nondimensional $< T >$, S and S', F, and the nondimensional characteristic frequency ϕ_* versus the distance normalized by the thermal boundary layer thickness z/λ_T respectively. Amazingly they have unique Ra-invariant profiles if distance z is normalized by λ_T. Therefore they can be expressed in terms of $z^* = z/\lambda_T$ as:

$$\Theta(z^*) = F(z^*). \tag{49}$$

Thus only one length scale, λ_T, exists.

V. SUMMARY

Our main results of the statistical and scaling properties of thermal turbulence in Hg in cells of aspect ratio 1, 1/2, and 2, are: 1) The histograms of the temperature fluctuations at the center of the aspect ratio 1 cell are exponential. 2) The viscous boundary layer is thinner than the thermal one for $8 \times 10^5 < Ra < 8 \times 10^7$ in the aspect ratio 1 cell. 3)The scaling exponent γ of the Nusselt number with the Rayleigh number is close to 1/4 which is slightly smaller than for He gas. In the aspect ratio 1/2 and 2 cells, $1/4 < \gamma < 2/7$. $Nu \sim Ra^{1/2}$ is not observed even though the two boundary layers are inverted. 4) The exponent ξ of the temperature fluctuations indicates that temperature behaves a like passive scalar in the central region of the cell. $S \times S' < 0$ outside the boundary layer also implies that plumes are not advected and passively swept away from the boundary layer. 5) Several nondimensional quantities have unique Ra-invariant profiles if the distance from the top plate is normalized by λ_T. So only one scale λ_T exists in our experiments.

Putting these results together, requires the construction of a consistent and global theoretical picture of thermal hard turbulence.

VI. ACKNOWLEDGEMENT

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TABLES

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TABLE 1. Material constants of Hg at $20°C$, $1atm$.

FIGURES

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FIG. 1. Schematic drawing of the spatial structure of hard turbulence and the temperature and velocity profiles in a cell. Temperature changes linearly near walls of a cell. It is really constant in the center region. The flow velocity is zero at the wall and at the center of the cell and maximum at a distance λ_v from the walls. Plumes(size $\sim$ d) detach from the thermal boundary layer and circulate around the cell. In He gas, the cut off frequency, f_c, of the temperature power spectrum gives $d \simeq \lambda_T$.

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FIG. 2. Schematic drawing of the convection cell. The cell has five thermistor. Three are fixed at stainless steel wires. Two are fixed on a fine stainless steel tube coneected to a micro-translation stage which can move move vertically along the center line of the cylinder.

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FIG. 3. The positions of the five thermistors in the convection cell. Five thermistors in the convection cell. [A] is at the center of the cell at midheight. [B] and [C] measure the mean velocity of the large scale circulation. [D] and [E] can move vertically along the center line of the cylinder to determine the two boundary layers.

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FIG. 4. A typical histogram of temperature fluctuations at the center of the cell. It is nearly exponential distribution at the center of aspect ratio 1 cell for $Ra = 7.54 \times 10^7$.

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FIG. 5. Histograms of the temperature distribution at various positions in the cell. They are normalized by ΔT and the mean temperature of the top plate T_{top}, at various heights for $Ra = 3.5 \times 10^7$. The heights are [a] z = 1.1mm, [b] 5.4mm, [c] 16.6mm, [d] 36mm and [e] 50mm. The arrow indicates $\Delta T/2$.

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FIG. 6. The flatness of the temperature fluctuation distribution as a function of the distance, z, from the top plate.

FIG. 7. Scaling of the Reynolds number of the box, $Re = UL/\nu$, as a function of the Rayleigh number.

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FIG. 8. f_p versus Ra in the aspect ratio 1 cell. The scaling exponent of f_p with the Rayleigh number is $\epsilon = 0.46 \pm 0.02$, close to that for He gas.

FIG. 9. Scaling of the cutoff frequency, f_c, as a function of Ra. The dotted line is the best fit, $f_c \sim Ra^{0.40 \pm 0.05}$

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FIG. 10. Histogram of the temperature fluctuations at the center of the cell in the aspect ratio 1/2 cell at $Ra = 3.0 \times 10^8$. The distribution is not exponential.

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FIG. 11. The power spectrum of the temperature fluctuations at the center of the cell for aspect ratio 1/2 cell. for $Ra = 3.0 \times 10^8$.

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FIG. 12. Histograms of the temperature fluctuations at the center of the cell for various Ra, in the aspect ratio 2 cell. (a)$Ra = 1.56 \times 10^5$, (b)$Ra = 1.71 \times 10^5$, (c)$Ra = 3.31 \times 10^5$, (d)$Ra = 5.51 \times 10^5$, (e)$Ra = 1.65 \times 10^6$, (f)$Ra = 3.52 \times 10^6$, (g)$Ra = 5.91 \times 10^6$, (h)$Ra = 8.90 \times 10^6$.

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FIG. 13. Average of the temperature fluctuations at the center of the aspect ratio 2 cell. The averaged temperature fluctuations normalized by the average of the top plate temperature T_{top} and the temperature difference ΔT. Fluctuations are larger than 10% for $10^5 < Ra < 10^7$.

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FIG. 14. The root mean square value of the temperature fluctuations, T_{rms}, at the center of the aspect ratio 2 cell. T_{rms} normalized by the temperature difference changes at $Ra = 2.0 \times 10^5$.

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FIG. 15. Power spectrum of the temperature fluctuations at the center of aspect ratio 2 cell for $Ra = 1.65 \times 10^6$. Low frequency peak correspond to the large scale circulation in the cell.

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FIG. 16. Nusselt number as a function of Rayleigh number; $\diamond$: aspect ratio 2, $\bullet$: 1, $\triangle$: 2. For the scaling exponent γ of Nu $1/4 \simeq \gamma \simeq 2/7$. In the aspect ratio 2 cell, a transition occurs at $Ra \sim 2 \times 10^5$ from oscillating to steady roll direction.

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FIG. 17. Mean, $T_{ave}(z)$ ($\circ$), and root mean square value , T_{rms} ($\bullet$), of the temperature fluctuations as a function of the distance z from the top plate. The solid line is the best fit to the function, $T_{ave} - T_{top} = m_1 tanh(m_2 z)$. The arrow shows the thermal boundary layer. The thickness of the thermal boundary layer (λ_T) is evaluated $\lambda_T = 3.8mm$ for $Ra = 7.8 \times 10^7$.

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FIG. 18. The highest frequency, f_h, as a function of the distance z from the top plate. The thickness of the viscous boundary layer (λ_T) is evaluated $\lambda_v = 2.0mm$ for $Ra = 7.8 \times 10^7$. Cleary $\lambda_v < \lambda_T$ in contrast to SF_6.

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FIG. 19. Error by shifting power spectrum as a function of $S(f_H)$. Error does not change when $S(f_H)$ is chosen ranging from 10^{-8} to 10^{-5}.

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FIG. 20. Schematic drawing of the shifting method used to estimate the viscous boundary layer thickness precisely. With shifting the higher frequency region of the frequency spectra of the temperature fluctuations overlap each other.

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FIG. 21. Error in comparison to the standard power spectrum as a function of the shifting parameter ϕ. ϕ^* is the minimum value of $Er(\phi)$ and the ratio of characteristics high frequencies of the two spectra. ϕ^*_{err} which is the error bar of ϕ^* is evaluated by eqn. (29).

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FIG. 22. The frequency shift ϕ^* ($\bullet$) as a function of distance z from the top plate for $Ra = 7.8 \times 10^7$. It has clear shape in comparison with f_h ($\diamond$). The position at which ϕ^* reaches its maximum is λ_v.

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FIG. 23. The thermal ($\circ$) and viscous ($\bullet$) boundary layer thickness as a function of Rayleigh number for aspect ratio 1. The two boundary layers scale in Ra with a slope -0.20 ± 0.02 (dashed line).

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FIG. 24. Ratio of the two boundary layer thicknesses as a function of Rayleigh number. The Ra-invariant ratio is about 0.6.

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FIG. 25. Temperature inversion in the center region. The temperature inversion is $\sim$ 10% of the total temperature difference ΔT around the center at $Ra = 3.5 \times 10^7$.

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FIG. 26. S and S' as a function of the distance from the top plate in the aspect ratio 1 cell for $Ra = 3.5 \times 10^7$. $S < 0$ ($\bullet$) and $S' > 0$ ($\circ$) outside the boundary layer near the top plate.

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FIG. 27. Temperature profiles normalized by T_{top} and ΔT near the top plate as a function of z/λ_T in the aspect ratio 1 cell. ($\bullet$) $Ra = 8.0 \times 10^7$, ($\circ$) $Ra = 6.1 \times 10^7$, ($\Box$) $Ra = 3.5 \times 10^7$, ($\triangle$) $Ra = 2.0 \times 10^7$, and ($+$) $Ra = 6.0 \times 10^6$.

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FIG. 28. S and S' versus z in the aspect ratio 1 cell. ($\Box$) $Ra = 8.0 \times 10^7$, ($\Diamond$) $Ra = 6.1 \times 10^7$, ($\circ$) $Ra = 3.5 \times 10^7$, ($\triangle$) $Ra = 2.0 \times 10^7$, and ($\bigtriangledown$) $Ra = 3.0 \times 10^6$.

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FIG. 29. The flatness of the temperature fluctuations, F, versus the distance z/λ_T in the aspect ratio 1 cell. The symbols are the same as in Fig. VI. Horizontal lines denote $F = 3$ and $F = 6$, corresponding to the values expected for Gaussian and exponential distributions, respectively.

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FIG. 30. The frequency shift ϕ^* as a function of z/λ_v in the aspect ratio 1 cell. Ra: (•) $Ra = 8.0 \times 10^7$, (○) $Ra = 6.1 \times 10^7$, (□) $Ra = 3.5 \times 10^7$, (◇) $Ra = 2.9 \times 10^7$, (△) $Ra = 2.0 \times 10^7$, (+) $Ra = 6.0 \times 10^6$, and (×) $Ra = 3.0 \times 10^6$.

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FIG. 31. λ_T and $L/2Nu$ as a function of Rayleigh number in the aspect ratio 1 cell. (□) λ_T, (•) $L/2Nu$.