RESEARCH STATEMENT

While completing the Ph.D. degree in Stony Brook, the direction of my research has been based on both my Mathematical preferences and also on a wider-perspective commitment to applying Dynamics in Neuroscience.

This has been a sustained personal goal along the years. Although in college I recognized the scientific promise in linking the two areas, Biology was at the time only a complementary study interest to my pure Mathematics major. The two disjoint halves came together after I arrived here.

I have been working on a thesis in Real Dynamics under the supervision of Prof. John Milnor, and my areas of mathematical interest include Ergodic Theory, Complex Dynamics and Kneading Theory. In 1999 I also started working on my first significant Neuroscience project with Prof. Paul Adams, faculty in the Neurobiology Department. Over the past few years, I continued learning about the physiology of the central nervous system, and neural networks (models of Hopfield, Oja, Kohonen). I have undergone a continuous effort to keep steady the balance between these two main fields. Last summer I completed a three-month internship in the Carlos Brody laboratory at Cold Spring Harbor Laboratories, where I plan to return and continue my research for several more months post graduation.

In the following paragraphs, I will describe in more detail my work in these two areas, the common themes being the complexity and plasticity of a dynamical system and/or a neural network. I will conclude with some of my plans for the future.

Description of Mathematical research

My dissertation illustrates how dynamic complexity of a system evolves under deformations. This evolution is in general only partly understood. Attempts to give a quantitative approach have considered simple examples of dynamical systems and have made use of the topological entropy $h(f)$ as a particularly useful measure of the complexity of the iterated map f. However, the only results so far have been obtained in the case of interval polynomials of degree 2 and 3.

The logistic family $Q = \{f_\mu(x) = \mu x(1-x)\, ,\, \mu \in [0,4]\}$ illustrates many of the important phenomena that occur in Dynamics. The theory in this case is the most complete ([D]): $\mu \to h(f_\mu)$ is continuous, monotonely increasing, and different values $h_0 = h(f_\mu)$ are realized for a single μ in some cases, but also for infinitely many in other cases. The cubic polynomials on the unit interval are organized as a 2-parameter family. In the compact parameter space of this family, the level sets of the entropy, called isentropes, were proved to be connected ([DGMT] and [MT]).

In general, families of degree d polynomials depend on $d-1$ parameters, so the same concepts are harder to inspect for higher degrees. It is most natural to research next a family of quartic polynomials that depends only on two parameters. I focused on showing the **Connected Isentropes Conjecture** for the family P^Q of compositions of logistic maps ([R]).

I briefly studied the more general combinatorics of $2n$-periodic orbits under alternate iterations of two $(+,-)$ unimodal interval maps.

I introduced a way to keep track of the succession of the orbit points along the unit interval I by defining the *order-data* as a pair of permutation $(\sigma,\tau)\in S_n^2$. If under alternate iterations of the two maps f and f' the two critical orbits are periodic, their order-data turns out to be strongly connected to the kneading-data of the composition $f'\circ f$.

For a given order-data (σ,τ), I defined the left/right bones in the parameter space P^Q to be the subsets for which either critical point has periodic orbit of order-data (σ,τ). The bones are algebraic curves , and by definition left bones can only intersect right bones. I called a crossing primary intersection, if it corresponds to a pair of maps with common periodic bicritical orbit and secondary intersection, if it corresponds to a pair of maps with disjoint critical orbits.

To obtain combinatorial properties of the bones, I compared the space P^Q with a model space of compositions of stunted tent maps. This technique is not accidental; the stunted sawtooth maps are generally useful models in kneading-theory, because they are rich enough to encode in a canonical way all possible kneading-data of m-modal maps. The combinatorial results made crucial use of Thurston's Uniqueness Theorem, and of an extension of it due to Poirier , interpreted by [MT].

In two following sections, I completed the description of the bones with two essential properties.

The bone-curves are $\mathcal{C}^1$-smooth and intersect transversally. Smoothness follows as in [M] at parameter points inside the hyperbolic components of P^Q. If the parameter point is outside these components, a quasiconformal surgery construction is necessary in order to perturb a map with a superattracting cycle to a map having an attracting cycle with small nonzero multiplier.

The bones are simple arcs in P^Q with two boundary points on ∂P^Q, in other words they contain no loops. [MT] proved the similar assertion in the case of cubic polynomials, either assuming true the well-known Fatou Conjecture or using a weaker theorem due to Heckman. I used instead a quite new and interesting rigidity result of [KSvS], that delivers density of hyperbolicity in my parameter space.

I defined the n-skeleton S_n^Q in P^Q to be the union of all bones of period at most $2n$, together with the boundary of the space. I put a dimension 2 topological cell structure on P^{ST} and P^Q as follows: the 0-cells are all intersections of bones in S_n and all boundary points of bones in S_n; the

1-cells are the 1-dimensional connected components obtained by deleting the 0-cells from the n-skeleton; the 2-cells are the 2-dimensional connected components of the complement of S_n.

The relations between entropy and the sequence of cell complexes is emphasized in the last section of my paper. If two points in P^Q correspond to distinct values of the entropy, then any path connecting them crosses infinitely many bones. In more technical phrasing: for any $\epsilon > 0$, there is a large enough n for which the corresponding cell complex is fine enough to have variation of entropy less than ϵ on each of its closed cells. These considerations permitted me to transport some topological properties of the isentropes from the previously mentioned model space to similar properties of isentropes in P^Q. More precisely, contractibility of isentropes in the stunted tent maps model space translates as connectedness of isentropes in P^Q.

References

[D] A. Douady, *Topological Entropy of Unimodal Maps*, NATO ASI Series C: Mathematics and Physical Sciences, **464**

[DGMT] S.Dawson, R. Galeeva, J. Milnor, C. Tresser, *A Monotonicity Conjecture for Real Cubic Maps*, NATO ASI Series C: Mathematics and Physical Sciences, **464**, pp 165-184

[KSvS] O. Kozlovski, W. Shen and S. van Strien, *Rigidity of Real Polynomials*, Warwick preprint (2003)

[M] J. Milnor, *Hyperbolic Components in Spaces of Polynomial Maps*, IMS preprint, **3**, (1992)

[MT] J. Milnor, C. Tresser, *On Entropy and Monotonicity for Real Cubic Maps*, IMS preprint, **9**, (1998)

[R] A. Radulescu, *The Connected Isentropes Conjecture in a Space of Quartic Polynomials*, in perparation

Description of Neuroscience research

The area of neuroscience I am primarily interested in is the mechanisms of learning and memory in neural networks. However, I try to keep my interests as broad as possible and continue to understand how my particular models fit into a larger context. Most of my research is related to or inspired by theories and results in dynamical systems and PDE's, which in my opinion have been an important catalyst for some major breakthroughs in mathematical biology in recent times.

One of the early and yet still one of the most important learning principles is the postulate of Hebb: neurons that fire together wire together. Its phrasing leaves space for multiple levels of interpretation. The first models I studied considered a couple of these possibilities.

Oja's rule ([O]) mathematizes a linear network model which, through a process of unsupervised learning, adjusts the synaptic weights according to a normalized Hebbian rule. We introduced a modification that we called *the Adams' model*, based on the possible imperfections in the mechanical-chemical hardware that supports learning, which permit information "leaks" between neighboring cells. I made essential use of hyperbolic dynamics ([AR]) to prove the convergence of the weight vector to a stable state under iterations. We concluded that "chaos" in such a leaking network does not increase sensibly with its physical dimension. This constitutes just a starting point. We anticipate that the study will naturally enlarge to encompass a wider class of networks and become a piece of the puzzle in understanding information transfer.

A different interpretation of Hebb's postulate is offered by [SMA]. During the research program this summer, I worked on extending this model to mathematically back up in vitro results. Recent studies in the visual cortex suggest that intracortical inhibitory circuits are necessary to initiate and drive ocular dominance plasticity if an eye is stimuli deprived. The project proposed to integrate rapid GABA-ergic transmission with long-lasting spike timing-dependent plasticity mechanisms to explain how open-eye inputs achieve a competitive advantage and deprived inputs are selectively weakened over time. As expected, a direct analytic method seemed hopeless, so we developed a set of MATLAB simulations to illustrate the results. The idea of this competitive, correlation based plasticity model still seems absolutely amazing to me and I intend to delve further into it in the future.

Future plans

An apparently nonrelated topic I am attracted to is the role of attention in modulating brain processes, as described in [RC] and detailed in a few papers on gain modulation by Abbott and Salinas. An idea to which I plan to return for future research is to incorporate attention gain modulation into the plasticity model described before. I expect that the understanding of each of the mechanisms would be enhanced by seeing them at work simultaneously.

At present, I am learning about the kinetic representation described in [CTSM]. Part of my plan is to try to review the previous competition models putting them in the perspective of this approach.

Last year I presented some of these neuroscience projects in the mathematics graduate seminar, which was exciting, and I hope that I can keep doing this after I get my degree. I am planning to graduate in December, with the conviction that this combination is the kind of research that could fill my life. On a shorter and longer term, I would like my next professional positions to emphasize this and to help me gain more knowledge, more practice and more confidence in Mathematics and Neuroscience as separate fields and as a wide unitary science.

References

[O] E. Oja, *A simplified neural model as a principal component analyzer*, Journal of Mathematical Biology, vol 16, 1992

[RC] J. Reynolds, L. Chelazzi, *Attention Modulation of Visual Processing*, Annu. Rev. Neurosci. **27**, 2004, pp 611-647

[AB] P. Adams, A. Radulescu, *A Dynamical Analysis of Adams' model*, in preparation

[SMA] S. Song, K. Miller, L. Abbott, *Competitive Hebbian Learning through Spike-Timing-Dependent Synaptic Plasticity*, Nature Neuroscience, **9**, 2000

[CTSM] D.Cai, L. Tao, M. Shelley, D McLaughlin *An effective kinetic representation of fluctuation-driven neuronal networks with application to simple and complex cells in visual cortex*, PNAS, **101**, 2004

If any additional information may be needed, please don't hesitate to contact me.