Holcman, David

LETTER OF RECOMMENDATION FOR MR.DAVID HOLCMAN

I am writing this letter on behalf of Mr.David Holcman who is applying for a Position in your department .I have known him for about seven years, at the University of Paris VI, where he wrote his Ph.D. thesis. He defended it in 1998.All these years, I have followed his career very closely and have collaborated with him.

In my opinion Mr.Holcman is an exceptionally able mathematician who deserves a good position at an elite institution. A glance at his Vita shows that he has had a very diverse and unusual career, having been trained as an Engineer in telecommunications, prior to embarking on a career in Mathematics. He has quite impressive achievements to his credit and has already published 14 papers in journals having an excellent international reputation. Six others have been submitted for publication and four are in preparation. These papers cover a very wide range of topics such as geometric analysis on manifolds, differential geometry, dynamical system theory control theory and more recently neurobiology. Mr.Holcman has obtained deep and interesting results in these fields. In the mathematical modelling of neurobiological processes, he is one of the leaders. I think that Mr.Holcman has an exceptional mathematical talent. He has lots of initiative, a great creative power, lots of unusual and interesting mathematical ideas. On the technical side, he is very skilful at combining geometric ideas and analytic methods to generate interesting Mathematics. His great enthusiasm for mathematics and also other sciences makes it a delight to work with him. I feel he has a bright future in Mathematics if he can fully dedicate himself to it. In the following I am going to discuss in some detail his main results.

At the start of his career, he studied questions related to the Yamabe Problem. An important contribution was the solution of the prescribed scalar curvature problem for complete Riemannian manifolds. The compact case was solved earlier. But in the non-compact case new difficulties appear: the problem being framed in a variational setting, one has to prevent concentration phenomena at infinity of minimizing sequences. Moreover one has to show that the solution obtained, a Riemannian metric, is complete. For this problem he has proved several very interesting results for the existence of a solution under very precise metric assumptions on the manifold.

The type of equations studied by Mr.Holcman in the preceding problem led him to study two very important problems: the positive mass conjecture and the existence of nodal solutions for certain (Yamabe type) non-linear eigenvalue problems. The positive mass problem has its origin in the study of Gravitation. Mr.Holcman studies the mass of locally conformally flat compact manifolds with boundaries and shows that if their Yamabe invariant is positive and they are simply connected, their mass is positive and, by contrast, that conformally flat manifolds contain open sets with regular boundary whose mass is positive. These results complement very nicely the positive mass theorem of Shoen and Yau which states that, in the boundary-free case, the mass is positive if the scalar curvature is positive.

These studies of the mass led Mr.Holcman to results showing the influence of the mass to non-linear eigenvalue problems for equations of Yamabe's type and for Dirichlet boundary conditions. This led him to a deep study of the non-linear eigenvalue problem for compact manifolds without boundary. Contrary to the case where there is a boundary and Dirichlet boundary conditions are imposed, the boundary-free case has attracted very little attention. Under rather strong geometric conditions Shoen-Escobar proved the existence of positive solutions as does Moser in the special case of the sphere. Mr.Holcman shows the existence of nodal (i.e. sign changing) solutions under very simple and easily verifiable geometric conditions. He also finds upper and lower bounds for the eigenvalue terms of Cheeger's invariant and Sobolev's constant. As a fall-out, he obtains stationary solutions for certain types of non-linear wave and Schrödinger equations. Finally, let me mention that, in my opinion, the nodal case is the harder.

All the preceding work is related to the topics Mr.Holcman had started to study when writing his thesis. In his thesis he had studied operators such as Laplace-Beltrami and the Yamabe equation that were linked to the geometric problems mentioned above. Since then he has studied problems about subelliptic partial differential equations. With Professor Y.Kannai, he has solved the open problem of finding the elementary solution in closed form for the wave operator associated to a special sub-riemannian Laplacian , the Grushin operator. This is a very surprising result obtained an expression. In two papers submitted for publication he studies diffusion operators with a drift on compact manifolds when the diffusion coefficient (or ''viscosity'') tends to zero. This is the problem of the semi-classical approximation in quantum mechanics. For self adjoint operators (that is when there is no drift) the problem has been studied very extensively. But the drift case almost nothing has been done. Making general but simple assumptions on the dynamics of the drift (Morse-Smale),he proves new and very interesting results about, among others, the behavior of the first eigenvalue of the diffusion operator and more importantly on the weak limits as the diffusion coefficient tends to zero, of the measures having the normalized first eigenfunctions as densities. The results he gets,are very interesting.

Lately,Mr.Holcman has been working on problems related to neurophysiology. With a team of biologists at the Weizmann Institute he has constructed a mathematical model of the behaviour of calcium in dendrites spine. This a very important problem if one wants to understand how memory works. Mr.Holcman's model is one of the first to be constructed. With Professor Schuss, he has also studied the effect of absorbing sources on random walks. Again with Professor Schuss, he has studied the effect of the electrostatic field created by ions on the dielectric constant. I cannot say much more about this area of research being totally incompetent in the field but it appears that this is a topic with a great future.
Concluding it is my opinion that Mr.Holcman would be a valuable assets to a good mathematics department. I recommend him very strongly for a position at your department.

Ivan Kupka
Professor, University of Paris VI.