Partial-Waves Calculation Code for Coupled-Channel $\bar{K}N$ Scattering Release Notes

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(for the Joint Physics Analysis Center)

These notes constitute a reprint of what can be found at the JPAC Webpage and cover the usage of the Fortran code for the calculation of the $\bar{K}N$ scattering partial waves. You are free to redistribute the code but we encourage to enclose both the `README.tex` and `README.pdf` files with these notes. If you use this code, please remember to cite [1] in any associated publication. We also encourage you to contact the author with questions and comments.

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I. SUMMARY OF THE FORMALISM

The full model, fitting procedure, and results are detailed in [1]. We report here only the main features of the model.

A. Observables

The differential cross section and polarization observable for the processes $\bar{K}N, \pi\Sigma, \ldots \to \bar{K}N, \pi\Sigma, \ldots$ are given by

$$\frac{d\sigma}{d\Omega}(s,\theta) = \frac{1}{q^2}\left[|f(s,\theta)|^2 + |g(s,\theta)|^2\right], \tag{1}$$

$$P(s,\theta) = \frac{2\,\mathrm{Im}\left[f(s,\theta)\,g^*(s,\theta)\right]}{|f(s,\theta)|^2 + |g(s,\theta)|^2}, \tag{2}$$

where q is the center of mass momentum of the incoming kaon, θ is the scattering angle in the center of mass frame. The amplitudes $f(s,\theta)$ and $g(s,\theta)$ give the contribution from no spin-flip and spin-flip, respectively.

Specifically, in this work we consider the following cases which have been measured (dropping the s and θ dependence)

$$f^{K^-p\to K^-p} = \frac{1}{2}f^1_{\bar{K}N\to\bar{K}N} + \frac{1}{2}f^0_{\bar{K}N\to\bar{K}N}, \tag{3}$$

$$f^{K^-p\to\bar{K}^0n} = \frac{1}{2}f^1_{\bar{K}N\to\bar{K}N} - \frac{1}{2}f^0_{\bar{K}N\to\bar{K}N}, \tag{4}$$

$$f^{K^-p\to\pi^-\Sigma^+} = -\frac{1}{2}f^1_{\bar{K}N\to\pi\Sigma} - \frac{1}{\sqrt{6}}f^0_{\bar{K}N\to\pi\Sigma}, \tag{5}$$

$$f^{K^-p\to\pi^+\Sigma^-} = \frac{1}{2}f^1_{\bar{K}N\to\pi\Sigma} - \frac{1}{\sqrt{6}}f^0_{\bar{K}N\to\pi\Sigma}, \tag{6}$$

$$f^{K^-p\to\pi^0\Sigma^0} = \frac{1}{\sqrt{6}}f^0_{\bar{K}N\to\pi\Sigma}, \tag{7}$$

$$f^{K^-p\to\pi^0\Lambda} = \frac{1}{\sqrt{2}}f^1_{\bar{K}N\to\pi\Lambda}, \tag{8}$$

and similarly for $g(s,\theta)$.

These amplitudes are related to the s-channel isospin $I=0$ and $I=1$ amplitudes through a general relation

$$f(s,\theta) = \alpha^0\, f^0_{kj}(s,\theta) + \alpha^1\, f^1_{kj}(s,\theta), \tag{9}$$

$$g(s,\theta) = \alpha^0\, g^0_{kj}(s,\theta) + \alpha^1\, g^1_{kj}(s,\theta), \tag{10}$$

where $f^I_{kj}(s,\theta)$ and $g^I_{kj}(s,\theta)$ are the isospin amplitudes. Here α^0 and α^1 are the corresponding Clebsch-Gordan coefficients for isospin zero and one, respectively, and kj label the initial (k) and final (j) state, respectively.

Partial wave expansion of isospin amplitudes is given by

$$f^I_{kj}(s,\theta) = \sum_{\ell=0}^{\infty}\left[(\ell+1)R^{I,kj}_{\ell+}(s) + \ell R^{I,kj}_{\ell-}(s)\right]P_\ell(\theta), \tag{11}$$

$$g^I_{kj}(s,\theta) = \sum_{\ell=1}^{\infty}\left[R^{I,kj}_{\ell+}(s) - R^{I,kj}_{\ell-}(s)\right]P^1_\ell(\theta), \tag{12}$$

where $P_\ell(\theta)$ is the Legendre polynomial with $P^1_\ell(\theta) = \sin\theta dP_\ell(\theta)/d\cos\theta$, $R^{I,kj}_{\ell\tau}(s)$ ($\tau=\pm$) are the partial waves which are to be considered as kj elements of the channel-space matrix $R_{\ell\tau}(s)$ as defined below, ℓ is the orbital angular momentum of the partial wave and $J=\ell+\tau/2$ is the total angular momentum for $R^{I,kj}_{\ell\tau}(s)$. The orbital angular momentum ℓ coincides with the orbital angular momentum of the initial $\bar{K}N$ state in $R^{I,kj}_{\ell\tau}(s)$ but it is not necessarily the orbital angular momentum of other possible initial states. For example, for the $I=1, \ell=0$ partial wave it is possible to have $\bar{K}\Delta(1232)$ in a D wave state ($L=2$) as initial (final) state.

Finally, the total cross section can be expressed in terms of the partial waves

$$\sigma(s) = \frac{4\pi}{q^2}\sum_{\ell=0}^{\infty}\left[(\ell+1)|R_{\ell+}(s)|^2 + \ell\,|R_{\ell-}(s)|^2\right], \tag{13}$$

where $R_{\ell\tau}(s) = \alpha^0 R^{0,kj}_{\ell\tau}(s) + \alpha^1 R^{1,kj}_{\ell\tau}(s)$.

B. Partial wave scattering matrix

For a given partial wave we write the scattering amplitude as a matrix in the channel-space

$$S_\ell = \mathbb{I} + 2iR_\ell(s) = \mathbb{I} + 2i\,[C_\ell(s)]^{1/2}\,T_\ell(s)\,[C_\ell(s)]^{1/2}, \tag{14}$$

where $\mathbb{I}$ is the identity matrix, $C_\ell(s)$ is a diagonal matrix which accounts for the phase space and $T_\ell(s)$ is the analytical partial wave amplitude matrix. We write $T_\ell(s)$ in terms of a K matrix to ensure unitarity

$$T_\ell(s) = \left[K(s)^{-1} - i\rho(s,\ell)\,\right]^{-1}. \tag{15}$$

For real s, $K(s)$ is a real symmetric matrix and $\rho(s,\ell)$ is a diagonal matrix. To ensure that $\rho(s,\ell)$ is free from kinematical cuts and has only the square-root branch point demanded by unitarity, we write it as a dispersive integral over the phase space matrix $C_\ell(s)$, ¡i¿a.k.a.¡/i¿ as the Chew-Mandelstam representation,

$$i\rho(s,\ell) = \frac{s-s_k}{\pi}\int_{s_k}^{\infty}\frac{C_\ell(s')}{s'-s}\frac{ds'}{s'-s_k}. \tag{16}$$

Here s_k is the threshold center of mass energy squared of the corresponding channel k and we define

$$C_\ell(s) = \frac{q_k(s)}{q_0}\left[\frac{r^2 q_k^2(s)}{1+r^2 q_k^2(s)}\right]^\ell. \tag{17}$$

The first factor on the r.h.s is related to the breakup momentum near threshold. For a meson-baryon pair with masses m_1 and m_2, respectively, $s_k = (m_1+m_2)^2$, and

$$q_k(s) = \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{2\sqrt{s}} \simeq \frac{\sqrt{m_1 m_2}}{(m_1+m_2)}\sqrt{s-s_k}. \tag{18}$$

The remaining factor ensures the threshold behavior and introduces the effective interaction radius, $r = 1$ fm. Finally, $q_0 = 2\,\text{GeV}$ is a normalization factor for the momentum in the resonance region. Evaluation of the dispersive integral can be found in [1].

C. Construction of the $K(s)$ matrix

We define the $K(s)$ matrix as the addition of $K_a(s)$ matrices

$$[K(s)]_{kj} = \sum_a x_k^a\,K_a(s)\,x_j^a, \tag{19}$$

where $K_a(s)$ can be of two kinds, pole and background:

$$[K_P(s)]_{kj} = x_k^P\,\frac{M_P}{M_P^2 - s}\,x_j^P, \tag{20}$$

$$[K_B(s)]_{kj} = x_k^B\,\frac{M_B}{M_B^2 + s}\,x_j^B, \tag{21}$$

Each partial wave employs a different amount of pole and background K matrices as well as a different amount of n_C channels. This information is summarized in Table I of Ref. [1].

The $K(s)$ and $T(s)$ matrices are connected through

$$[T(s)]_{kj} = \frac{1}{\mathcal{D}(s)}\sum_{a,b} x_k^a\,c_{ab}(s)\,x_j^b, \tag{22}$$

where $\mathcal{D}(s)$ and $c_{ab}(s)$ for the combination of up to six K matrices can be found in the Appendix in Ref. [1].

II. NOTATION

All the channels are treated as two-body (meson-baryon) states and are labeled as follows:

(i) if the state has the same orbital angular momentum (ℓ) as the partial wave the channel is identified by the names of the meson and the baryon, e.g. $\bar{K}N$ or $\pi\Sigma$;

(ii) if the baryon has spin 3/2, as it is the case of $\Sigma(1385)$, $\Delta(1232)$ and $\Lambda(1520)$ (in what follows Σ^*, Δ and Λ^* respectively), the orbital angular momentum of the initial state does not correspond to ℓ and a subindex L is added denoting the angular momentum of the initial (final) state. For example, in $\bar{K}N$ system the S_{01} denotes the isoscalar, $l = 0$ partial wave with total spin $J = 1$. It may couple to $\pi\Sigma^*$ with orbital angular momentum $L = 2$ (D wave) which we label as $[\pi\Sigma^*]_D$;

(iii) if the state contains a spin one $\bar{K}^*$ and a nucleon, they can couple to spin 1/2 which we name $\bar{K}^*_1N$ or to spin 3/2 which we name $\bar{K}^*_3N$. The $\bar{K}^*_1N$ state has the same orbital angular momentum as the $\bar{K}N$ and the partial wave but the $\bar{K}^*_3N$ does not, hence we add a L subindex to the last. For example, the S_{01} partial wave has as possible states $\bar{K}^*_1N$ and $\left[\bar{K}^*_3N\right]_D$.

For every partial wave we include an additional meson-hyperon channel that collectively accounts for any any missing inelasticity arising from channels not included explicitly. The kinematical variables for such a dummy channel is chosen arbitrarily as if it was a two-pion Λ or Σ state labeled as $\pi\pi\Lambda$ for $I = 0$ and $\pi\pi\Sigma$ for $I = 1$ partial waves. All the channels incorporated in the model have single-energy partial-wave data to fit except for the dummy channels $\pi\pi\Lambda$ and $\pi\pi\Sigma$ and the $\eta\Lambda$ and $\eta\Sigma$ channels in the S waves.

The full list of initial (final) states for each partial wave is:

S_{01} : $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_D$, $[\pi\Sigma^*]_D$, $\pi\pi\Lambda$;

P_{01} : $\bar{K}N$, $\pi\Sigma$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_P$, $[\pi\Sigma^*]_P$, $\pi\pi\Lambda$;

P_{03} : $\bar{K}N$, $\pi\Sigma$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_P$, $\left[\bar{K}^*_3N\right]_F$, $[\pi\Sigma^*]_P$, $[\pi\Sigma^*]_F$, $\pi\pi\Lambda$;¡/li¿

D_{03} : $\bar{K}N$, $\pi\Sigma$, $\bar{K}^*_1N$, $[\pi\Sigma^*]_S$, $[\pi\Sigma^*]_D$, $\pi\pi\Lambda$;

D_{05} : $\bar{K}N$, $\pi\Sigma$, $[\pi\Sigma^*]_D$, $[\pi\Sigma^*]_G$, $\pi\pi\Lambda$;¡/li¿

F_{05} : $\bar{K}N$, $\pi\Sigma$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_P$, $\left[\bar{K}^*_3N\right]_F$, $[\pi\Sigma^*]_P$, $[\pi\Sigma^*]_F$, $\pi\pi\Lambda$;

F_{07} : $\bar{K}N$, $\pi\Sigma$, $\bar{K}^*_1N$, $\pi\pi\Lambda$;

G_{07} : $\bar{K}N$, $\pi\Sigma$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_D$, $\left[\bar{K}^*_3N\right]_G$, $\pi\pi\Lambda$;

S_{11} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\eta\Sigma$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_D$, $[\pi\Sigma^*]_D$, $[\pi\Lambda^*]_P$, $\left[\bar{K}\Delta\right]_D$, $\pi\pi\Sigma$;

P_{11} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_P$, $[\pi\Sigma^*]_P$, $[\pi\Lambda^*]_D$, $\left[\bar{K}\Delta\right]_P$, $\pi\pi\Sigma$;

P_{13} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_P$, $[\pi\Sigma^*]_P$, $[\pi\Lambda^*]_D$, $[\pi\Sigma^*]_F$, $[\pi\Lambda^*]_S$, $\left[\bar{K}\Delta\right]_P$, $\pi\pi\Sigma$;

D_{13} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_S$, $\left[\bar{K}^*_3N\right]_D$, $[\pi\Sigma^*]_S$, $[\pi\Sigma^*]_D$, $[\pi\Lambda^*]_P$, $[\pi\Lambda^*]_F$, $\left[\bar{K}\Delta\right]_S$, $\left[\bar{K}\Delta\right]_D$, $\pi\pi\Sigma$;

D_{15} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_D$, $[\pi\Sigma^*]_D$, $[\pi\Sigma^*]_G$, $[\pi\Lambda^*]_P$, $[\pi\Lambda^*]_F$, $\left[\bar{K}\Delta\right]_D$, $\pi\pi\Sigma$;

F_{15} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_P$, $\left[\bar{K}^*_3N\right]_F$, $[\pi\Sigma^*]_P$, $[\pi\Sigma^*]_F$, $[\pi\Lambda^*]_D$, $[\pi\Lambda^*]_G$, $\left[\bar{K}\Delta\right]_P$, $\pi\pi\Sigma$;

F_{17} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_F$, $\left[\bar{K}^*_3N\right]_H$, $[\pi\Sigma^*]_F$, $[\pi\Lambda^*]_D$, $[\pi\Lambda^*]_G$, $\left[\bar{K}\Delta\right]_F$, $\pi\pi\Sigma$;

G_{17} : $\bar{K}N$, $\pi\Sigma$, $\pi\Lambda$, $\bar{K}^*_1N$, $\left[\bar{K}^*_3N\right]_G$, $[\pi\Lambda^*]_F$, $[\pi\Lambda^*]_H$, $\left[\bar{K}\Delta\right]_D$, $\left[\bar{K}\Delta\right]_G$, $\pi\pi\Sigma$.

III. FORTRAN CODE

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- Last update: September 2015

A. Zip File Content

- README file: `README.tex` and `README.pdf`
- Fortran Source File: `partialwaves.f`
- Input File: `file.inp`
- Parameter files (contain the parameters for each partial wave):
  - `parameters.s01.inp`
  - `parameters.p01.inp`
  - `parameters.p03.inp`
  - `parameters.d03.inp`
  - `parameters.d05.inp`
  - `parameters.f05.inp`
  - `parameters.f07.inp`
  - `parameters.g07.inp`
  - `parameters.s11.inp`
  - `parameters.p11.inp`
  - `parameters.p13.inp`
  - `parameters.d13.inp`
  - `parameters.d15.inp`
  - `parameters.f15.inp`
  - `parameters.f17.inp`
  - `parameters.g17.inp`

B. Input File

Example of input file (`file.inp`):

```
s01
1 1
2.5
4.5
100
```

- The first line indicates the partial wave, the options are:
  - `all` $\rightarrow$ computes all the partial waves for a given channel
  - `s01` $\rightarrow S_{01}$
  - `p01` $\rightarrow P_{01}$
  - `p03` $\rightarrow P_{03}$
  - `d03` $\rightarrow D_{03}$
  - `d05` $\rightarrow D_{05}$
  - `f05` $\rightarrow F_{05}$
  - `f07` $\rightarrow F_{07}$
  - `g07` $\rightarrow G_{07}$
  - `s11` $\rightarrow S_{11}$
  - `p11` $\rightarrow P_{11}$
  - `p13` $\rightarrow P_{13}$

- `d13` $\to D_{13}$
- `d15` $\to D_{15}$
- `f15` $\to F_{15}$
- `f17` $\to F_{17}$
- `g17` $\to G_{17}$

- The second line indicates the process. It is made of two numbers where the first refers to the initial state and the last to the final. The options are:
  - `1` $\to \bar{K}N$,
  - `2` $\to \pi\Sigma$,
  - `3` $\to \pi\Lambda$,
  - `4` $\to \eta\Lambda$,
  - `5` $\to \eta\Sigma$,
  - `6` $\to \bar{K}_1 N$,
  - `7` $\to \left[\bar{K}_3 N\right]_-$,
  - `8` $\to \left[\bar{K}_3 N\right]_+$,
  - `9` $\to [\pi\Sigma^*]_-$,
  - `10` $\to [\pi\Sigma^*]_+$,
  - `11` $\to \left[\bar{K}\Delta\right]_-$,
  - `12` $\to \left[\bar{K}\Delta\right]_+$,
  - `13` $\to [\pi\Lambda(1520)]_-$,
  - `14` $\to [\pi\Lambda(1520)]_+$,
  - `15` $\to \pi\pi\Lambda$,
  - `16` $\to \pi\pi\Sigma$.

  The $+$ and $-$ subindices are short for the angular momentum of the state. The $+$ stands for the higher angular momentum and $-$ for the lower. If only one state of angular momentum is possible it corresponds to the $+$ state.

  Examples:

  Example 1: For any partial wave, `1 2` computes $\bar{K}N \to \pi\Sigma$.

  Example 2: For partial wave S_{01}, `15 10` computes $\pi\pi\Lambda \to [\pi\Sigma(1385)]_D$.

  Example 3: For partial wave F_{05}, `8 7` computes $\left[\bar{K}_3^* N\right]_F \to \left[\bar{K}_3^* N\right]_P$.

- The third line indicates the starting value of s in GeV^2.
- The fourth line indicates the final value of s in GeV^2.
- The fifth line indicates the the amount of points to calculate. There is a limit of 1000 points. It can be changed modifying variable `max_data_points=1000` in module `resonancesizes`.

IV. OUTPUT

The online and the downloadable versions produce an output file (`output.txt`) which contains six columns:

1. s (GeV^2),
2. $E_{\rm lab}$ (GeV),
3. $p_{\rm lab}$ (GeV),
4. the center-of-mass incoming momentum squared q^2 (GeV^2),

5. real part of the partial wave (adimensional), and
6. imaginary part of the partial wave (adimensional).

If the process is not kinematically allowed $p_{\text{lab}} = 0$ in the output file. If the selected partial is `all`, the partial waves are written in files `pw.s01.txt`, `pw.p01.txt`, and so on.

V. JPAC WEBPAGE

Further information and latest version of the code can be found at: JPAC Webpage. An online version of the code can also be run at the same webpage.

VI. DISCLAIMERS

- This code follows the *garbage in, garbage out* philosophy. If your parameters do not make sense, the output will not make sense either.
- You can use, share and modify this code under your own responsability.
- This code is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.
- No PhD students or postdocs were severely damaged during the development of this project.

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[1] C. Fernández-Ramírez, I. V. Danilkin, D. M. Manley, V. Mathieu, M. R. Pennington, and A. P. Szczepaniak, *Coupled-Channel Model for $\bar{K}N$ Scattering in the Resonant Region*, arXiv:1510.07065 [hep-ph].