RESTRICTIONS ON SU(3) MIXING IMPLIED BY EXCHANGE DEGENERACY

C. B. CHIU and J. FINKELSTEIN *
CERN, Geneva

Received 7 August 1968

It is shown that the hypothesis that the imaginary parts of exchanged Regge contributions must cancel in channels which contain no resonances implies restrictions on the way in which SU(3) may be mixted and broken in the vector and tensor meson nonets; for example, the mixing angle should satisfy $\tan^2\theta = \frac{1}{2}$.

There is an intriguing possibility that S matrix principles, such as analyticity and crossing, may imply restrictions on the possible symmetry groups of strong interactions (e.g. ref. 1). In this note, we explore some consequences of a consistency condition recently proposed by Harari (ref. 2), and also implied by work of Van Hove (ref. 3) and Henzi (ref. 4), which is an extension of the idea of exchange degeneracy (ref. 5). In particular, we calculate the mixing angles for the vector and tensor meson nonets.

It has been suggested by Harari (ref. 2) that the contributions of Regge trajectories, excluding the Pomeron, in the t channel may be built up, in the sense of finite-energy sum rules, by resonances in the s channel. In this note we are interested in a special case of this suggestion: when there are no resonances in the s channel, then the sum of the imaginary parts of the exchanged trajectories (again excluding the Pomeron) should vanish. This requirement is a generalization of the hypothesis of exchange degeneracy proposed by Arnold (ref. 5), and may be considered to be a consistency condition which s channel information can impose on the t channel trajectories.

This condition can also be motivated, without any reference to finite-energy sum rules, from ideas discussed by Van Hove (ref. 3) and Henzi (ref. 4). These authors compare elastic scattering amplitudes, such as the pp (or Kp) elastic amplitudes, with the amplitudes for the u channel reactions $\bar{p}p$ (or $\bar{K}p$). They argue that the part of the overlap function which survives at infinite energy comes from those inelastic channels which can be produced by exchange of vacuum quantum numbers, and so is the same for pp and $\bar{p}p$; in addition, there is a part of the $\bar{p}p$ (or $\bar{K}p$) overlap function coming from baryon (or strangeness) annihilation channels which has no counterpart in pp (or Kp), and which must disappear as the energy increases. Thus they predict that the overlap functions, and hence the total cross-sections in channels such as pp or Kp have a much smaller energy-dependent part than they do in channels such as $\bar{p}p$ or $\bar{K}p$. When this behaviour is described in Regge language, it implies that the imaginary parts of the exchanged trajectories (excluding the Pomeron) cancel for those channels which do not permit annihilation. Since (at least in the examples we shall consider), the channels which do not permit annihilation are those which do not contain resonances, we are led back to the consistency condition proposed by Harari (ref. 2). We shall now show that, in the approximation that this condition is satisfied exactly, there are restrictions on the mixing and breaking of SU(3) for the vector and tensor nonets.

In a reaction in which, for example, only the ρ and the A_2 trajectories may be exchanged, we have, at high energies

$$A(s,t) = \beta_\rho(t)\,\{1-\exp(-i\pi\alpha_\rho(t))\}s^{\alpha_\rho(t)} + \beta_{A_2}(t)\,\{1+\exp(i\pi\alpha_{A_2}(t))\}s^{\alpha_{A_2}(t)}\,. \tag{1}$$

The condition $\operatorname{Im} A(s,t)=0$ then requires ‡ $\alpha_\rho(t) = \alpha_{A_2}(t)$, and $\beta_\rho(t) = \beta_{A_2}(t)$. In the following, we do not specify the value of the variable t, since our results will be valid for any t (including for

* NATO Postdoctoral Fellow.

‡ We note that the inclusion of Regge cuts does not change these results, at least if the cuts are calculated according to the hybrid model we have proposed earlier (ref. 6). To first order in inelastic transitions, the amplitude is real if and only if the Regge pole term is real.

example $t = m_\rho^2$) at which the consistency condition is valid†. When we apply this requirement to baryon-baryon scattering, and assume the absence of states with baryon number 2 ††, we obtain the exchange degeneracy proposed by Arnold (ref. 5): the trajectories (ρ, A_2), (K^*K^{**}), (ω, f^o) and $(\varphi, f^{o\prime})$ are pairwise equal, and their couplings to the baryons are also pairwise equal.

We consider now pseudoscalar meson-pseudoscalar meson scattering and initially assume the validity of SU(3). The exchanged trajectories are those of the vector and the tensor mesons. As observed by Harari (ref. 2), if we assume the absence of resonances in the [10] and the [27] representations, the consistency condition tells us that there must be a tensor SU(3) singlet degenerate with the tensor octet. Allowing then a tensor nonet (in addition to the Pomeron), the absence of [10] and [27] resonances allows us to compute the relative magnitude of the couplings of the vector and tensor octet and the tensor singlet to the pseudoscalar mesons (the vector singlet does not couple at all). This enables us to determine what linear combination of tensor singlet and octet does not couple to $(\pi\pi)$. The result is that if we define the state

$$|f'\rangle = \sin\theta\,|f_1\rangle + \cos\theta\,|f_8\rangle \ , \qquad (2)$$

then for $|f'\rangle$ not to couple to $(\pi\pi)$ requires $\tan^2\theta = \frac{1}{2}$. This result would also be expected in a model of the tensor mesons as l excited states of two quarks.

We can get further information on the couplings of the trajectories to baryons by considering pseudoscalar baryon scattering. Since we expect no resonances in the (KN) system in any charge state ††, the imaginary parts of the contributions of the $I = 0$ and the $I = 1$ trajectories must separately cancel. The possible $I = 0$ trajectories are the ω_8, the f_1 and the f_8. We have already determined the couplings of these trajectories to the mesons; from examining the baryon-baryon problem we see that the couplings of the baryons to ω_8 and to f_8 are equal. We can then use the absence of resonances (or annihilation) in the (KN) system to solve for the (relative magnitude of the) coupling of the f_1 to nucleons. This in turn determines the combination of f_1 and f_8 which does not couple to nucleons, which turns out to be the same combination [$\tan^2\theta = \frac{1}{2}$ in eq. (2)] which is decoupled from $(\pi\pi)$. Since we know from the baryon-baryon problem that the baryon couplings of the vector and of the tensor meson trajectories are the same, we can see that this same angle will decouple a linear combination of ω_1 and ω_8 from nucleons. Similarly, it can be shown, by letting the vector mesons be also external particles, that this same combination of ω_1 and ω_8 does not couple to $(\pi\rho)$. Again, these results would be expected in a quark model.

So far we have shown that linear combinations with $\tan^2\theta = \frac{1}{2}$ will decouple from $(\pi\pi)$, $(\pi\rho)$ and $(N\bar{N})$, but we have not yet shown that this should be the actual mixing angle. To do this, we go back to the example of meson-meson scattering. We consider the exchange of nonets of vector and tensor meson trajectories, but do not now assume SU(3) relations for the trajectory functions or residues. By applying the consistency condition in $\pi^+\pi^+$ scattering, we see that some *physical* f, say the f^o, has the same trajectory function as does the ρ. Combining this information with what we can learn from baryon-baryon scattering, we see that the trajectories fall into three groups - (ρ, A_2, ω, f^o), $(\varphi, f^{o\prime})$ and (K^*, KK^{**}) - with the trajectory functions within each group coinciding. If the particles of the first group had the same trajectory function as those of the second group, the mixing angles would be undefined. But in fact the $f^{o\prime}$ trajectory does not coincide with the f^o trajectory, and by remembering that the consistency condition implies a cancellation of the contributions of trajectories which do coincide, we shall now calculate the mixing angle by demanding cancellations among the trajectories of the first group.

In $\pi^+\pi^+$ and π^+K^+ scattering, we must have cancellations between the imaginary parts of the contributions of the ρ and the f^o, and in KK scattering with $I_t = 0$, between the imaginary parts of the contributions of the ω^o and the f^o. The required cancellations in these three reactions can only occur if

$$g^2_{\rho^o KK} = g^2_{\omega KK} \qquad (3)$$

where the g's are the couplings of the trajectories to the external mesons; if we choose $t = m_\rho^2 = m_\omega^2$, then the g's are the actual coupling constants. So far we have not used SU(3), but if we now write the physical ω as $|\omega\rangle = \cos\theta\,|\omega_1\rangle +$

† On the other hand, one might be most willing to believe in the validity of the consistency condition for t not too far from zero.

†† What about the deuteron ? It is quite possible that there are no channels which are absolutely free of resonances. A reasonable criterion for applying the the consistency condition is that there be many fewer strongly coupled low-mass states (or many fewer available inelastic channels) in the s than in the u channel. This criterion should be understood when we write, for brevity, that a channel contains no resonances.

$- \sin\theta|\omega_8\rangle$, then since $|\omega_1\rangle$ does not couple to $(\mathrm{K\bar{K}})$, eq. (3) allows us to determine the value of θ. The result is $\tan^2\theta = \frac{1}{2}$; this angle should therefore be constant along the trajectories.

Similarly, one can show that the actual mixing angle for the tensor meson trajectories is given by $\tan^2\theta = \frac{1}{2}$. We now know that the physical f' and φ decouple from $(\pi\pi)$, $(\pi\rho)$ and (NN). Also, assuming the usual relation between mixing angle and masses and the Gell-Mann-Okubo formula, we can turn our "coupling-constant mixing angle" into a mass formula:

$$M^2_{\mathrm{K}^*} = \tfrac{1}{2}\,(M^2_\rho + M^2_\varphi)\ , \qquad (4)$$

and of course we have

$$M^2_\omega = M^2_\rho\ ; \qquad (5)$$

similarly for the tensor mesons. These relations are well satisfied experimentally†.

Clearly one can obtain other interesting results. For example, the consistency condition imposes constraints, such as eq. (3), on the Regge residue functions which could be used to simplify high-energy analysis. Also, eq. (4) and eq. (5), which should be true along the trajectories at any fixed (not necessarily integral) spin, can impose restrictions on possible parametrizations of the meson trajectories. We record here a linear parametrization of the trajectory functions, which satisfies eqs. (4) and (5) and closely reproduces the observed particle masses:

$$\alpha_{\mathrm{A}_2} = \alpha_\rho = \alpha_{\mathrm{f}^0} = \alpha_\omega = \left(\frac{0.48}{1.08}\right) + \frac{t}{1.08} \approx 0.44 + 0.93\ t,$$

$$\alpha_{\mathrm{K}^{**}} = \alpha_{\mathrm{K}^*} = \left(\frac{0.33}{1.15}\right) + \frac{t}{1.15} \approx 0.29 \pm 0.87\,t, \qquad (6)$$

and
$$\alpha_{\mathrm{f}^{0\prime}} = \alpha_\varphi = \left(\frac{0.18}{1.22}\right) + \frac{t}{1.12} \approx 0.15 + 0.82\ t.$$

We have shown that dynamics can restrict the way in which SU(3) can be mixed and broken. Of course, SU(3) might be violated slightly and violate the consistency conditions slightly. But if the violation of SU(3) is sufficiently great so that it must respect the consistency conditions, we then obtain the restrictions (4) and (5).

In summary, we have shown that the consistency conditions, which is an extension of the hypothesis of exchange degeneracy, has as consequences:

1) The vector and tensor trajectories occur in these groups: $(\rho, \omega, \mathrm{f}^0, \mathrm{A}_2)$ $(\mathrm{K}^*, \mathrm{K}^{**})$ and $(\varphi, \mathrm{f}^{0\prime})$. Within each group the trajectories are degenerate, but the groups need not be. The trajectories associated with different groups must be equally spaced for fixed α [eq. (4)].

2) Under the assumption of SU(3) symmetry for the couplings, the physical $\mathrm{f}^{0\prime}$ and φ are decoupled from $(\pi\pi)$, $(\pi\rho)$ and $(\mathrm{N\bar{N}})$. The result $\mathrm{f}^{0\prime} \not\to \pi\pi$ is independent of SU(3).

We have benefitted from discussions with Professor D. Amati, Dr. R. Henzi, Dr. J. J. J. Kokkedee and Dr. R. Van Royen. We thank Professor J. Prentiki and Professor L. Van Hove for the hospitality of the Theoretical Study Division of CERN.

† Using the masses given by A. H. Rosenfeld et al. (Jan. 1968), eq. (4) reads (in GeV^2), 0.80 = 0.81 for the vector mesons. and 2.01 = 1.99 for the tensors, while eq. (5) reads 0.61 = 0.59 for the vectors and 1.57 = = 1.70 for the tensors.

References

1. G. F. Chew, The Analytic *S* Matrix, (New York, 1966).
2. H. Harari, Phys. Rev. Letters 20 (1968) 1395.
3. L. Van Hove, in Particle interactions at high energy (1966) p. 63.
4. R. Henzi, Nuovo Cimento 52A (1967) 772.
5. R. C. Arnold, Phys. Rev. Letters 14 (1965) 657.
6. C. B. Chiu and J. Finkelstein, to be published.

* * * * *