A MECHANISM FOR QUARK CONFINEMENT ★

Curtis G. ALLAN Jr.[1], Roger DASHEN
Institute for Advanced Study, Princeton N.J 08540, USA

and

David J. GROSS
Joseph Henry Laboratories, Princeton University, Princeton, N J 08540, USA

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Euclidean field configurations carrying half integral topological charge, merons, can, unlike ordinary instantons, confine quarks at moderate coupling g. Logarithmic interactions between merons prevent isolated ones from existing for small g. However, in four dimensional QCD a crude calculation indicates a phase transition to a quark confining plasma at an effective coupling $\bar{g}^2/4\pi^2 \approx \frac{1}{4}$.

Recently there has been much interest in finite action, non-trivial solutions of Euclidean non-abelian gauge theories. Polyakov [1] originally suggested that such field configurations might dominate the Euclidean path integral:

$$\int [\mathcal{D}A_\mu^a] \exp[-S(A)], \quad S(A) = \frac{1}{4g^2} \int \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} \mathrm{d}^4x,$$

in an asymptotically free theory where the effective coupling constant g grows large at large distances.

Until now attention has focused on the "instanton" solution of Euclidean Yang-Mills theory discovered by Belavin et al. [2], which minimizes the action in the sector with Pontryagin index one. This solution was understood to be an indication of vacuum tunneling [3–5] between an infinite number of classically degenerate stable vacua. Thus instanton configurations are of fundamental importance no matter how small the coupling is. One must take them into account in order to construct the correct vacuum, which is a coherent superposition of the classical vacua, labeled by a continuous parameter $|\theta| \leqslant \pi$ [4–6]. The main physical implications of this vacuum degeneracy has been the effect on fermionic symmetries. It was realized that in a θ vacuum, axial baryon number symmetry is broken without the generation of a Goldstone boson, thereby solving the notorious U(1) problem [3–5]. Other consequences of the θ-vacua are a possible mechanism for T-violation [3–5] and a possible source of the dynamical breaking of chiral SU(N) [5].

Do instantons play a role in quark confinement? We shall argue that their net effect on widely separated quarks is a finite mass renormalization. This argument is based on the "dilute gas approximation" [5], which is only valid for sufficiently weak coupling. Thus in an asymptotically free theory, such as QCD, one can only conclude with confidence that strong coupling (which is highly probable for large distances or large instantons) is required to confine quarks.

Once the effective coupling is not small other than minimal action field configurations become important. In this note we would like to focus on a particular class of configurations which might play a dominant role in quark confinement. These are configurations which have two important properties.

First they correspond to separated lumps of one-half topological charge, with an action (interaction energy) that increases only logarithmically with separation, and an independent entropy of position for each lump proportional to the logarithm of the volume. Thus while the contribution of such a pair separated by a distance R will be suppressed, for small effective coupling, by

$$R^4 \exp[-(\mathrm{const}/\bar{g}^2) \ln R],$$

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[1] On leave from Princeton University.

as $\bar{g}^2$ increases (in fact the effective coupling in the above action will be evaluated at a scale determined by R and will thus increase for increasing R) configurations with large separation become more probable. If one approximates the path integral by summing over such configurations, one obtains the partition function for a gas of merons ‡1 with attractive logarithmic interactions. Such a system is somewhat analogous to the two-dimensional Coulomb gas [7]. There one expects that at low temperature (weak coupling) the system is in a dielectric phase, composed of a weakly interacting gas of dipole pairs. However, since both the energy and entropy of a pair increase with the logarithm of the separation, at higher temperatures the entropy term in the free energy ($F = E - TS$) will take over and at some critical temperature (large coupling) isolated charges can appear. The pairs will then dissociate and the system will be in the plasma phase.

The second important property of the configurations we propose is their effect on quarks. We shall argue that if the merons were in a plasma phase then they would confine quarks. Since the role of the temperature is played by the effective coupling, the system can be in different phases depending on the scale of distances being probed. At short distances the merons are tightly bound and have no effect on the quarks which are quasi-free. For widely separated quarks they behave as a plasma, and although the density of merons will be much less than that of instantons, they will control the long range properties of the system and confine the quarks.

To illustrate these ideas as it is useful to consider the two dimensional Abelian Higgs model. This model contains instantons which are simply the Nielson-Olessen vortices, whose topological charge is proportional to the magnetic flux. This theory is simple to analyze since it is superrenormalizable, and the instanton size is fixed (proportional to the inverse photon mass M). It is easy to see that, at least for weak coupling where the dilute gas approximation is valid, the model *will not* confine integrally charged quarks. As a measure of quark confinement one may consider the vacuum expectation value $C = \langle \exp \mathrm{i} e \oint_L A^\mu \mathrm{d}x_\mu \rangle$ where L is a Euclidean loop of spatial extent R and time extent T. For large T this is proportional to $\exp[-\epsilon(R)T]$, where $\epsilon(R)$ is the energy of massive charged quarks separated by a distance R ‡2. A linear potential between the quarks will result if $\ln C$ is proportional to the *area* of the loop $(R \cdot T)$ whereas if $\ln C$ is proportional to the circumference $(R + T)$ then this corresponds to a finite mass renormalization.

In the dilute gas approximation C is evaluated by considering the effect of one instanton. Since $e\oint A_\mu \mathrm{d}x^\mu$ equals $2\pi(0)$ if the instanton is totally within (without) the loop, $\epsilon(R)T$ will be proportional to the volume in which the instanton overlaps the loop, $(1/M)(R+T)$, times the density of instantons $M^2 \exp(-S_{\mathrm{cl}})$ and thus:

$$\epsilon(R) \underset{R \gg 1/M}{\sim} M \exp(-S_{\mathrm{cl}}) . \tag{1}$$

This R-independent interaction energy is simply a mass renormalization. An alternate derivation of this result can be obtained by considering $C_q = \langle \exp \mathrm{i}q \times \int A^\mu \mathrm{d}x_\mu \rangle$. In the dilute gas approximation this is given by $(V_0 \sim 1/M^2)$

$$C_q = I_q / I_0$$

$$I_q = \sum_{n_+ n_-} \int \prod_{i=1}^{n_+} \frac{\mathrm{d}^2 x_i^+}{V_0} \prod_{i=1}^{n_-} \frac{\mathrm{d}^2 x_i^-}{V_0} \times \exp[-(n_+ + n_-)S_{\mathrm{cl}} + \mathrm{i}(n_+^{\mathrm{L}} - n_-^{\mathrm{L}}) 2\pi q/e] . \tag{2}$$

where we sum over configurations with $n_+ (n_-)$ instantons (anti-instantons) and $n_\pm^{\mathrm{L}}$ is the number of such inside the quark loop L. Thus $\epsilon(R)$ is simply the change in the energy of the vacuum which occurs if θ is equal to zero outside the loop and $2\pi q/e$ inside the loop.

The vacuum energy is periodic in θ [5], so that for $q = e$ (integer charged quarks) one gets only a surface contribution to $\epsilon(R)$, leading to eq. (1). If, however,

‡1 Since these configurations have fractional topological charge we propose to call them merons, from the Greek root μεροσ meaning part or fraction. See Mero-, Shorter Oxford English Dictionary (Oxford, 1970). We thank A. Pais for suggesting this name.

‡2 The physical reason that the instantons confine in 2 dimensions is that they are responsible for restoring a discrete gauge invariance to the theory. This then wipes out the Higgs phenomenon ($\langle\phi\rangle = 0$) and although the vector meson acquires a mass there then exists a long range Coulomb interaction between charged sources. This then will confine fractional charges, but not integer charged sources which can bind to charged scalars.

$q/e \neq 1$ (mod 1), then there is a volume dependence of the vacuum energy, and

$$\epsilon(R)T \sim \exp(-S_{\mathrm{cl}})(1-\cos 2\pi q/e)RT/V_0. \quad (3)$$

Thus fractionally charged quarks are confined. Now note that confinement would also obtain if there existed isolated instanton configurations with half-integral flux. The insertion of a quark loop in a plasma of half-fluxons has the same effect of forcing θ to be equal to π inside the loop – thus providing an energy proportional to RT. Furthermore if the half-fluxons had an interaction (action) which only increased logarithmically say $S_{\mathrm{cl}}(R) \sim g^2 \ln R$, then for a pair of such half-fluxons we would get

$$\epsilon(R)T \sim \int_{|x^+|<R} d^2x^+ \int_{|x^-|>R} d^2x^- \exp[-g^{-2}\ln R] \sim TR^{3-g^{-2}} \quad (4)$$

which would confine if $g^2 \geqslant \frac{1}{3}$.

Now in point of fact it is impossible to find such configurations in the two-dimensional Higgs model. If one considers lumps of flux one half separated by a distance R they must be connected by a string-like region in which the phase of the Higgs fields changes from zero to π. This gives rise to an action that increases linearly with R, and thus the lumps never separate (alternatively in the dilute gas approximation the interaction energy of quarks due to such a configuration behaves as $Re^{-\mathrm{const}\,R}$). However, in four dimensional non-abelian gauge theories we shall show that configurations of this type do exist, with only logarithmic action and with equal power to confine quarks.

We shall now demonstrate that instantons by themselves do not confine quarks, at least as long as the effective coupling remains small enough to reliably replace the Euclidean functional integral by the partition function of a gas of instantons. In our analog gas an instanton of scale size $\rho = 1/\lambda$ has a chemical potential $\exp(-\beta\mu) = \exp(-8\pi^2/\bar{g}^2(\lambda/\mu))$ (μ = renormalization scale parameter) and an entropy of position [‡3] = $0.52\, d^4x\, d\rho/\rho^5(\bar{g}^2/8\pi^2)^4$. In the dilute gas approximation one can easily evaluate the (ordered) loop integral

$$C_{\mathrm{L}} \equiv \langle \mathrm{tr}\{T \exp i\oint_{\mathrm{L}} A^\mu dx_\mu\}\rangle/\langle 1\rangle$$

for a loop L in the z–t plane. In the $A_0 = 0$ gauge only the spacelike segments, sat at $t = 0$ and $t = T$, contribute to the integral and an instanton will only contribute if it is located at time t_{I} $0 \leqslant t_{\mathrm{I}} \leqslant T$ (otherwise it behaves like a pure gauge on the whole loop). For $T \gg R$, we can replace A_z by

$$A_z^{(x)} = U^{-1}(x-x_{\mathrm{I}})\partial_z U(x-x_{\mathrm{I}})$$

where

$$U(x) = \begin{cases} \exp[i\pi\boldsymbol{\tau}\cdot\boldsymbol{x}/\sqrt{\boldsymbol{x}^2+\rho^2}] & t=T \\ 1 & t=0 \end{cases}.$$

Since $U^{(r)}$ is equal to -1 when $|x| \gg \rho$, one gets a contribution to C_{L} only from instantons whose spatial extent overlaps with just one of the charged sources. Thus the interaction energy between two charged sources at distance R is

$$\epsilon(R) \sim \int^R \frac{d\rho}{\rho^5} \int_{|x|<\rho} d^3x \exp\left[-8\pi^2/\bar{g}^2\left(\frac{1}{\mu\rho}\right)\right]\left(\frac{8\pi^2}{\bar{g}^2}\right)^4. \quad (5)$$

Since this integral converges as $R \to \infty$, instantons do not confine quarks. One can, in principle, include the effects of instanton–anti-instanton interactions. However, these are rather short range [6] and we do not expect them to alter the conclusion.

For sufficiently large R the effective coupling, $\bar{g}^2(1/\mu R)/8\pi^2$, might grow too large for such a weak coupling approximation to make sense. However, we believe that before this occurs other field configurations will become important, and can lead to confinement even for effective couplings sufficiently small that semi-classical considerations are still reliable.

The configurations that we believe control the behavior of the quark loop consist of isolated lumps of one-half unit of topological charge, merons, whose positions are arbitrary and whose action only increases logarithmically with separation. To illustrate the effect of such configurations on the quark loop we shall exhibit a singular configuration of this sort. Consider the ansatz for the gauge field (for SU_2)

‡3 The evaluation of the determinant which give this expression is due to G. 't Hooft, ref. [3]. The reason that the entropy is proportional to $d^4x d\rho/(g^2\rho)^4$ is because it suffices to shift the orientation, scale size or location by g, $g\rho$ or $g\rho$ to obtain a physically distinguishable configuration.

$$A^a_\mu = (\epsilon_{0a\mu\nu} + \tfrac{1}{2}\epsilon_{abc}\epsilon_{bc\mu\nu})\partial_\nu \ln \rho(x). \qquad (6)$$

The Yang-Mills equations are then equivalent to $\Box\rho = C\rho^3$ (C is an arbitrary constant [8]. In particular

$$\rho(x) = [(x - x_1)^2 (x - x_2)^2]^{-1/2}, \qquad (7)$$

yields a solution of the field equations everywhere except the singular points x_i [12]. As noted in ref.[9], this solution has the property that the topological charge density is given by

$$Q(x) = (1/16\pi^2)\,\mathrm{tr}\,[F_{\mu\nu}\widetilde{F}_{\mu\nu}] = \tfrac{1}{2}[\delta^{(4)}(x - x_1) + \delta^{(4)}(x - x_a)], \qquad (8)$$

corresponding to two point merons. The action of this solution is infinite, due to the fact that the field is singular at x_i. However, this singularity can easily be removed by smearing the topological charge over a small sphere. It is then easy to see that the action will be proportional to $(1/g^2)\ln(x_1 - x_2)^2$, for large separation. These point merons are then analogous to point one-half fluxons of the type which we discussed above. If one were to neglect their logarithmic interaction they would confine quarks. This is easily seen once again by evaluating C_L in the $A_0 = 0$ gauge. An individual meron will give a non-trivial phase now if it is located at $0 \leqslant t \leqslant T$ and is no further than R from the loop leading to an energy between fixed quark sources which increases rapidly ($\sim R^3$) with increasing R. Now, of course, the meron interaction cannot be neglected. However, we shall argue below that its coefficient is proportional to $(\bar{g})^{-2}(1/\mu R)$, and for large enough R such that $\bar{g}^2/8\pi^2 \geqslant \frac{3}{28}$ the energy will increase, thus leading to confinement for an effective coupling sufficiently small that our semi-classical arguments might still be valid.

Having seen that merons can confine quarks let us now discuss how they could be treated in a systematic way. For small coupling one is used to approximating the Euclidean functional integral by a saddle point approximation. However, in general, the dominant contributions do not come from strict minima of the action, even in the case of instantons alone, since we must include multiple instanton–anti-instanton configurations which are not solutions of the equations of motion. In an infinite volume system the most important configurations are those for which the action is not too far from a minimum and for which the entropy is large. Such configurations occupy a large volume in function space, thus compensating for the fact that the action is not minimal.

The entropy associated with instantons, and with merons, comes from the fact that they are localized lumps of topological charge whose position is arbitrary. From the point of view of entropy the important variable is therefore the topological charge density $Q(x)$. We propose to define an effective field theory for $Q(x)$ by integrating over the gauge field and holding $Q(x)$ fixed, and only then integrating over $Q(x)$. The vacuum to vacuum amplitude would be $\int[\mathrm{d}Q]\,\mathrm{e}^{-W(Q)}$ where

$$\exp[-W(Q)] = \int[\mathcal{D}A^a_\mu] \times \prod_x \delta\left[Q(x) - \frac{1}{16\pi^2}\,\mathrm{tr}\,F_{\mu\nu}\widetilde{F}_{\mu\nu}\right] \exp[-S(A)],$$

with suitable gauge fixing and ghost terms implied. We can now imagine performing the A^a_μ integral by a straight forward saddle point approximation and then integrating over $Q(x)$ by more sophisticated methods. In first approximation W would be the minimum action for fields that satisfy the constraint and can be calculated by adding to the action a Lagrange multiplier $(1/4g^2)\lambda(x)\,\mathrm{tr}\,F\widetilde{F}$, solving the modified field equations $D^\mu F_{\mu\nu} + \partial^\mu\lambda\widetilde{F}_{\mu\nu} = 0$ and adjusting $\lambda(x)$ to obtain the desired $Q(x)$.

Of course, one would still have to perform the functional integral of $\mathrm{e}^{-W(Q)}$. Our purpose here is to suggest a program of studying this approximate field theory for $Q(x)$ and show how it could lead to confinement. The idea is to replace the Q-field theory by an analog model consisting of a gas of instantons and merons. Such a replacement can only be approximate, but our hope is that the model retains those degrees of freedom which are responsible for confinement.

The merons are most easily found by searching for minima of S subject to the constraint that the integral of $Q(x)$ over a given volume is equal to $\frac{1}{2}$. This can be achieved by taking Lagrange multipliers which are piecewise constant. Spherically symmetric configurations of this type are easy to construct. In the ansatz of eq. 6 let $\rho = \rho\,(\tau = \ln\sqrt{x^2}/r)$, with arbitrary scale r, and define $\phi(\tau) = -(\rho'/\rho + 1)$ where prime denotes differentiation with respect to τ. In terms of Φ the action is

$$S = \frac{3\pi^2}{g^2} \int_{-\infty}^{+\infty} d\tau [\phi'^2 + (\phi^2 - 1)^2],$$

and the topological charge density is $(6\pi^2/g^2)\phi'(1-\phi^2)$. The Yang-Mills equations are then equivalent to a particle in the potential $-(1-\phi^2)^2$ whose velocity can change discontinuously at any τ where the Lagrange multiplier $\lambda(\tau)$ is discontinuous. The function $\phi(\tau)$ must be continuous and approach ± 1 at $\tau = \pm\infty$ to yield a finite action. The standard instanton is the solution which proceeds from -1 at $\tau = -\infty$ to $+1$ at $\tau = +\infty$$\cdot$ $\phi(\tau) = \text{th}\,\tau$. Now consider a trajectory that leaves $\phi = -1$ at $\tau = -\infty$ and arrives at the stable minimum $\phi = 0$ at $\tau = 0$ at which point its velocity is discontinuously brought to zero. Then at $\tau = \ln R/r$ it is given a velocity kick ($\Delta\phi' = 1$) to make it arrive at $\phi = +1$ at $\tau = \infty$. Such a configuration consists of one-half of an instanton with half a unit of topological charge in the region $x^2 \leqslant r^2$, another half unit in the region $x^2 \geqslant R^2$ and zero charge density for $r^2 \leqslant x^2 \leqslant R^2$. Thus we have constructed a meron at $x = 0$ and another at $x = \infty$. A more physical configuration is obtained by performing an inversion about a point a_μ, with $a^2 = rR$. Taking $(x-a)^\mu \to rR\{(x-a)^\mu/(x-a)^2\}$ yields for $R \gg ra$ meron of size r located at the origin and another one of the same size at $D = \sqrt{rR}$ away. The topological charge is confined to two small spheres, each of size r, containing half a unit. In a similar fashion one can construct configurations with $\frac{1}{2}$ and $-\frac{1}{2}$ topological charges in the small spheres.

The classical action (unrenormalized) for such configuration is equal to $(8\pi^2/g_0^2)[1 + \frac{3}{4}\ln D/r]$. This consists of a term $4\pi^2/g_0^2$ from integrating over each sphere which represents the (bare) chemical potential of a meron and a term $(6\pi^2/g_0^2)\ln D/r$ which represents the (bare) interaction energy of a pair of merons of size r separated by a distance D. To determine the entropy of a meron at $x = \bar{x}$ as well as the renormalization of g_0 we must perform a Gaussian functional integral about a pair of merons. For fixed $Q(x)$ these are minima of S. However, there exist, upon relaxing the constraint, directions in function space in which S decreases and which do not correspond to varying the collective coordinates ($\bar{x}$, r, D). One can already see this instability developing in the radically symmetric ansatz. When D/r is big enough one can construct a configuration with smaller action by considering a trajectory which is given a very small kick at $\tau = 0$ ($\phi = 0$), so that it oscillates in the potential well with a period $\ln(D/r)/N$, and then after N oscillations is given another kick to enable it to approach $\phi = +1$ at $\tau = \infty$. In such a configuration $Q(x)$ oscillates between $\tau = 0$ and $\tau = \ln D/r$, corresponding to a nested set of ordinary instantons and anti-instantons between the merons. We claim that such modes will be automatically taken into account in a more systematic procedure by allowing for arbitrary superpositions of instantons and anti-instantons, and that therefore there is no need to introduce additional collective coordinates.

Examination of the functional integral about a pair of merons indicates that the renormalization length for the chemical potential is r, so that $\exp(-\beta\mu) = \exp(-4\pi^2/\bar{g}^2(1/r\mu))$, and for the interaction energy is D, so that the interaction energy of merons of sizes r and r' and separated by D (for $D \gg r, r'$) is

$$\exp(-\beta V) = \exp\left[-\frac{6\pi^2}{\bar{g}^2(1/\mu D)} \ln D/\sqrt{rr'}\right].$$

The entropy of merons we estimate to be $C(d^4x dr/r^5) \times (8\pi^2/\bar{g}^2)^4$ where $C \approx 1$. Note that the apparent divergence as $r \to 0$ is removed by asymptotic freedom, since $\exp(-\beta\mu)$ for a meron of size r (for the gauge group SU(2)) vanishes like $r^{11/2}$ as $r \to 0$.

Why do we restrict our attention to meron configurations and not consider isolated lumps of arbitrary fractional charge? It is clear that these would be as effective in producing confinement. The reason is that although one can easily construct such configurations with logarithmic interaction energy, it appears to be impossible to superimpose them in such a way as to obtain an independent entropy of volume for each lump. Consider an arbitrary field configuration which has an isolated lump of topological charge at the origin ($\text{tr}\,F\tilde{F} < 1/r^4$ for large r). The gauge field (in a non-singular gauge) must behave as $1/r$ at large distances. Now for an instanton the $1/r$ term is a pure gauge, and for a meron the $1/r$ term is a solution of the Yang-Mills equations. Lumps with other than one half charge require a $1/r$ term which does not satisfy the classical equations, thus leading to the problem described above.

We shall now discuss how the merons confine quarks and estimate the critical value of the effective coupling. If the gas of instantons and merons was sufficiently dilute one might perform a calculation of C_L as before. Now we would consider the effect on the quark loop of a pair of merons at x_1 and x_2. A non-trivial phase will obtain if one meron is inside the loop, the other outside. Integrating over the volume occupied by the merons outside the loop then yields (dropping irrelevant terms):

$$\epsilon(R) \sim \int_{|x|<R} d^3x\, dr/r \times \exp[-4\pi^2/\bar{g}^2(1/\mu r)]\, R^{4-6\pi^2/\bar{g}^2(1/\mu R)}. \tag{9}$$

Thus the interaction energy begins to grow with R when $\bar{g}^2/8\pi^2\,(1/R\mu) = \frac{3}{28}$. At this coupling the density of instantons is still reasonably small, i.e. $0.52\ \mathrm{ex}\,[-(8\pi^2/\bar{g}^2)]\,(8\pi^2/g^2)^4 = 0.35$. When $g^2/8\pi^2 = \frac{1}{8}$, $\epsilon(R)$ becomes linear; however, the density of instantons increases to 0.72. Thus confinement may be occurring for relatively weak coupling. However, one must improve on the above argument, taking into account instanton and meron interactions.

At low temperature = coupling constant, the merons will be tightly bound and indistinguishable from instanton configurations. To see whether a phase transition would take place one could carry out a mean field approximation by considering the screening effect of an instanton or a tightly bound meron–anti-meron pair on a well separated pair. In this highly non-linear gas the interactions are quite complicated, however, in a crude approximation the gas is analogous to a dielectric medium of quadrapolar objects. With this analogy we can estimate the transition temperature $= \bar{g}^2$, at which a transition to a plasma-like phase will occur, i.e. when pairs will dissociate and uncorrelated merons can be created. The mean field static dielectric constant $\epsilon\,(q^2=0)$ is proportional to $\langle D^4\rangle$, where D is the separation of the merons in a pair. The phase transition will occur when

$$\epsilon(q^2=0) \sim \langle D^4\rangle \sim \int d^4D D^4 \exp\left[-\frac{6\pi^2}{\bar{g}^2(1/\mu D)} \ln D\right]$$

diverges, which occurs at $\bar{g}^2/8\pi^2 = \frac{3}{32}$. (Note that for this g the density of instantons is so small, 0.17, that this estimate might be meaningful. Furthermore dimensional reasoning suggests that $\epsilon(q^2)$ develops a pole, $1/q^2$, when $\langle D^2\rangle$ diverges, thus leading to exponential screening. This occurs at $g^2/8\pi^2 = \frac{1}{8}$ which happens to coincide with our previous estimate, based on the quark loop, of when the quark-antiquark interaction energy becomes linear.

Much work clearly remains in developing all stages of our program. However, it might be useful even at this stage, to consider the phenomenological implications of our mechanism. The critical value of the coupling, $\bar{g}^2/8\pi^2 = \frac{1}{8}$, which leads to confinement, is essentially the value of the effective coupling at a distance corresponding to the size of the hadron, in units of the renormalization scale parameter. What is remarkable is that such a small coupling, where semi-classical arguments and low order perturbation theory are still valid, can confine quarks. We also note that the value $\bar{g}^2/4\pi^2 \approx \frac{1}{4}$ is consistent with the observed departures from asymptotic freedom.

We are obviously far from being able to calculate hadronic masses (the acid test of a theory of hadrons), however, one might speculate that our mechanism would lead to a phenomenological model similar to the MIT bag. In a system with more than one phase one can have localized regions in the "wrong" phase (e.g. bubbles in a liquid) at a cost of some energy per unit volume which may be compensated by other effects. In the presence of (separated) quarks our Euclidean gas may find it favorable to be in a dielectric phase (in a cylindrical region along the "time" axis) at a cost in action of $BV\cdot T$, where B is some constant energy per unit volume. The quarks would be almost free particles inside this region. Since for $\bar{g}^2/8\pi^2$ in the range $\frac{3}{32}$ to $\frac{1}{8}$ the density of instantons is increasing extremely rapidly one might expect the transition from the dielectric-asymptotic freedom phase to the plasma-confining phase to be rather abrupt, thus producing an effective "bag" with a sharp boundary. Elaboration of this idea might allow one to develop a bag whose parameters, range of validity and dynamics would be calculable.

Finally, we note that our confinement mechanism is special to non-abelian gauge theories. Not only do we rely heavily on the asymptotic freedom and infra-red slavery of QCD, but in addition we require meron configurations which satisfy the Euclidean equations of motion except in small regions and have $F_{\mu\nu} \sim 1/r^2$ at large distances, thus giving rise to long range correlations in $\langle A(x)A(y)\rangle$ which make the quark loop behave as $\exp(-RT)$. Such configurations do not exist in Abelian theories.

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