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Gauge-fixing degeneracies and confinement in non-Abelian gauge theories

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Following several suggestions of Gribov we have examined the problem of gauge-fixing degeneracies in non-Abelian gauge theories. First we modify the usual Faddeev-Popov prescription to take gauge-fixing degeneracies into account. We obtain a formal expression for the generating functional which is invariant under finite gauge transformations and which counts gauge-equivalent orbits only once. Next we examine the instantaneous Coulomb interaction in the canonical formalism with the Coulomb-gauge condition. We find that the spectrum of the Coulomb Green's function in an external monopole-like field configuration has an accumulation of negative-energy bound states at $E=0$. Using semiclassical methods we show that this accumulation phenomenon, which is closely linked with gauge-fixing degeneracies, modifies the usual Coulomb propagator from $|\vec{k}|^{-2}$ to $|\vec{k}|^{-4}$ for small $|\vec{k}|$. This confinement behavior depends only on the long-range behavior of the field configuration. We thereby demonstrate the conjectured confinement property of non-Abelian gauge theories in the Coulomb gauge.

I. INTRODUCTION

It has recently been observed by Gribov[1] that in non-Abelian gauge theories, in contrast with Abelian theories, standard gauge-fixing conditions of the form

$$F^a[A_\mu^b(x)]=0\,, \tag{1.1}$$

where $A_\mu^b(x)$ is the gauge field, fail to uniquely specify the gauge. For example, in the Coulomb or Landau gauge one can find a continuous multiplicity of fields ${}^{(g)}A_\mu^b$ all related by finite gauge transformations g which satisfy $F^a[{}^{(g)}A_\mu^b(x)]=0$. In the quantum theory this gauge-fixing degeneracy is dangerous because the gauge degrees of freedom are not to be quantized and must be completely removed. Usually, the gauge-fixing condition is thought to specify uniquely the field for canonical quantization and to ensure that gauge-equivalent field configurations are not counted separately in the path-integral formalism. The observation of a gauge-fixing degeneracy implies that the naive canonical procedure fails and that the usual Faddeev-Popov[2] prescription for the path integral is incomplete at least in the Coulomb and Landau gauges.

The Feynman rules for the perturbative expansion of the amplitudes are insensitive to the behavior of the theory under finite gauge transformations. Moreover, it is known that non-Abelian gauge theories are not Borel summable[3] and that the perturbative expansion is not the complete solution to the theory. The failure of the Borel sum to define the theory and gauge-fixing degeneracy are correlated phenomena.

In this article we examine the gauge-fixing degeneracy further. First, we reexamine the usual Faddeev-Popov prescription, show how it can fail, and then give a modified prescription that takes gauge-fixing degeneracies into account. In particular, while the Faddeev-Popov prescription works for infinitesimal gauge transformations, it fails for finite ones if the degeneracy is present. The generating functional must be invariant under finite as well as infinitesimal gauge transformations. We give a formal prescription that satisfies this and which counts each gauge-equivalent orbit only once in the sum over paths.

Second, we examine, in the canonical procedure, the instantaneous Coulomb interaction. In this case we impose the Coulomb-gauge condition

$$\partial_i A_i^a(x)=0\,. \tag{1.2}$$

The gauge-fixing degeneracies associated with the Coulomb gauge have been studied by Gribov[1] in terms of the "pendulum equation" which is also discussed by Wadia and Yoneya.[4] We review their work here.

For the SU(2) gauge theory the field is

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+g\epsilon^{abc}A_\mu^b A_\nu^c \tag{1.3}$$

and we set $g=1$ in what follows (it is easily recovered). In the canonical procedure[5] the transverse components of A_i^a and F_{0i}^a are conjugate variables. Here

$$F^a_{0i} = {}^T F^a_{0i} + {}^L F^a_{0i} , \tag{1.4}$$

with

$$\partial_i F^a_{0i} = \partial_i {}^L F^a_{0i} \text{ and } \epsilon^{ijk}\partial_j {}^L F^a_{0k} = 0 . \tag{1.5}$$

Hence

$${}^L F^a_{0i} = -\partial_i \rho^a , \quad {}^T F^a_{0k} \equiv E^a_k ,$$

and (1.6)

$$B^a_i \equiv \tfrac{1}{2}\epsilon_{ijk} F^a_{jk} .$$

The Hamiltonian is

$$H = \tfrac{1}{2}\int d^3x\,[(E^a_i)^2 + (B^a_i)^2 + (\nabla_i \rho^a)^2] , \tag{1.7}$$

where the last term is the instantaneous Coulomb interaction. This can be written in terms of the independent variables A^a_i and E^a_i using

$$\rho^a(x,t) = \int d^3y\, D^{ab}(x,y;A)\epsilon^{bcd} A^c_k(y,t)\, E^d_k(y,t) , \tag{1.8}$$

where the Green's function $D^{ab}(x,y;A)$ is the solution to

$$\begin{aligned}\partial_i D_i(A) D^{ad} &= (\partial_i\partial_i \delta^{ab} + \epsilon^{acb} A^c_k \partial_k) D^{bd}(x,y;A)\\ &= \delta^{ad}\delta^3(x-y) .\end{aligned}$$

In contrast with the Abelian gauge theory, this Green's function depends on the field strength A^a_i. We may represent the Green's function as

$$D^{ab}(x,y;A) = \sum_n \frac{V^a_n(x;A)V^{b*}_n(y;A)}{-E_n} , \tag{1.9}$$

where $V^a_n(x;A)$ are the eigenfunctions of the operator $\partial_i D_i(A)$:

$$\partial_i D_i(A) V^a_n(x;A) = -E_n V^a_n(x;A) . \tag{1.10}$$

The functions $V^a_n(x;A)$ are complete and normalized according to

$$\sum_n V^a_n(x;A)V^{b*}_n(y;A) = \delta^{ab}\delta^3(x-y)$$

and

$$\int d^3x\, V^a_n(x;A) V^{a*}_m(x;A) = \delta_{nm} . \tag{1.11}$$

We have examined the Coulomb propagator Green's function D^{ab} in the presence of external monopole-like configurations of the gauge field of the form

$$\begin{aligned} &A^a_i(x) = \epsilon^a_{ij} x_j f(r)/r^2 , \quad r^2 = \vec{x}^2 \\ &\partial_i A^a_i(x) = 0 . \end{aligned} \tag{1.12}$$

A sufficient condition for finite energy is that $f(r)$ approaches a constant as $r \to \infty$ and that $f(0)=0$ or $f(0)=2$. Our major result is that for $f(\infty) > \frac{9}{8}$ the field A^a_i is strong enough to produce an infinite number of negative-energy bound states in the propagator D^{ab}. Semiclassical estimates imply that the bound-state energy levels accumulate at $E_\infty = 0$ like $E_n \sim -|E_0| e^{-\beta n}$, where β is a constant. This accumulation phenomenon is independent of the details of the field configuration $A^a_i(x)$ for finite $|x| < \infty$. It depends only on the long-range infrared tail of the field configuration.

After integrating over all positions of the external source to restore translation invariance and integrating over isospin directions to restore SU(2) invariance, we find that the accumulation phenomenon produces a small-$|\vec{k}|$ singularity in the Fourier transform $D^{ab}(\vec{k})$ of the form $|\vec{k}|^{-4}$, which indicates a confining potential. Gribov[1] has speculated that this propagator becomes singular as $|\vec{k}| \to 0$ and our calculation fulfills this conjecture in detail.

This result is in sharp contrast with the usual $|\vec{k}|^{-2}$ Coulomb behavior which is obtained from configurations that do not give rise to level accumulation. We conclude that, at least in the Coulomb gauge, confinement is caused by field configurations that produce an infinite number of bound states in the Coulomb propagator. In this case the instantaneous Coulomb interaction term $(\nabla_i \rho^a)^2$ in the energy density grows at least linearly if one separates the nonsinglet sources. We conclude, with Gribov, that the spectrum of gauge theories cannot contain free gluons or quarks. One can only observe bound-state singlets.

Gauge-fixing degeneracy in the Coulomb gauge amounts to the observation that there are many field configurations related by finite gauge transformations, all satisfying $\partial_i A^a_i = 0$. For example, for the Ansatz (1.12), $f=0$ and $f=2$ are both pure gauges. We find that some of these gauge-equivalent configurations exhibit level accumulation and confinement while others do not. So it would appear superficially that gauge-equivalent configurations have different physics, an absurd conclusion.

The paradox is resolved by our generalization of the Faddeev-Popov prescription which informs us that we must sum over all configurations ${}^{(n)}A^a_\mu$ related by finite gauge transformations and divide by n to obtain a generating functional which is invariant under finite as well as under infinitesimal gauge transformations and which counts gauge-equivalent configurations only once. The physics can appear to be different for different gauge-equivalent configurations ${}^{(n)}A^a_\mu$ if one restricts oneself to infinitesimal gauge transformations. For example, the Faddeev-Popov determinants, $\det^{(n)}$, evaluated by diagonalization for the configuration ${}^{(n)}A^a_\alpha$, can be quite different since this evaluation takes into account only infinitesimal variations of the gauge field under a gauge transformation. Only by summing over all the gauge-equivalent configurations does one guarantee invariance under the full gauge group. Consequently those configurations exhibiting level accumulation are to be included. Since they are infinite in number they will survive the averaging; they do not comprise a set of measure zero in function space.

In this paper our entire discussion of confinement is carried out in the Coulomb gauge. However, confinement is a gauge-independent phenomenon. Unfortunately, it is not clear how to observe confinement in other gauges (such as the axial gauge[6] in which degeneracy is evidently absent). We have no progress to report on this difficult problem.

II. GAUGE-FIXING DEGENERACIES AND THE FADDEEV-POPOV PRESCRIPTION

The naive expression for the vacuum-vacuum transition amplitude in gauge theories is

$$Z = N^{-1}\int [dA^a_\mu]\exp\left(i\int \mathcal{L}\, d^4x\right), \tag{2.1}$$

where

$$\mathcal{L}(x) = -\tfrac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

and $F^a_{\mu\nu}$ is given by (1.3). The action $\mathcal{L}$ is invariant under local gauge transformations $U(g)$:

$$A_\mu \xrightarrow{g} (A_\mu)^g = U(g)A_\mu U^{-1}(g) + iU(g)\partial_\mu U^{-1}(g), \tag{2.2}$$

where

$$A_\mu = A^a_\mu \tau^a/2, \quad U(g)U^\dagger(g) = 1.$$

This gauge transformation acting on a particular field A_μ generates an orbit of fields A^g_μ all with the same action $S = \int \mathcal{L}\, d^4x$. This infinite variety of possible paths all with the same phase leads to an infinity in the generating functional Z. Faddeev and Popov[2] showed that this infinity is proportional to the volume $\int \prod_x dg(x)$ in group space and is independent of the field A_μ, and hence it can be lumped into the normalization factor N.

The starting point is to consider a gauge-fixing condition of the form

$$F^a[A_\mu(x)] = 0. \tag{2.3}$$

(This condition may, as we will discuss, fail to actually fix the gauge.) Equation (2.3) defines a "hypersurface" on the manifold of fields A_μ (see Fig. 1). Next consider the orbit of A_μ generated by the gauge transformation g. Then there are

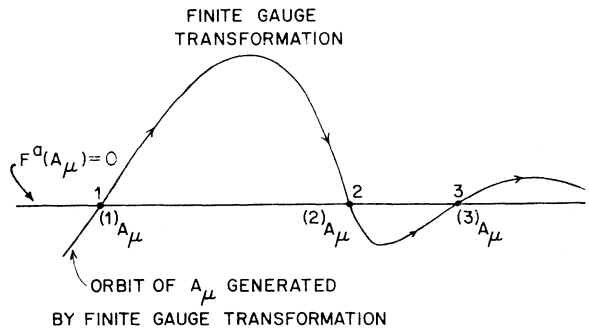

FIG. 1. Schematic representation of gauge-fixing degeneracies in configuration space.

various possibilities, the simplest being that the orbit intersects the hypersurface only once. Then the usual Faddeev-Popov prescription is valid. This is because the change in the gauge-fixing condition (2.3) under infinitesimal gauge transformations is all that need be considered to guarantee invariance of the generating functional under the full gauge group.

In what follows we assume that under *infinitesimal* gauge transformations the gauge is uniquely specified by the gauge-fixing condition (2.3). This is equivalent to assuming that there are no normalizable zero eigenvalues in the spectrum of the operator

$$O^{ab}(A) = \frac{\delta F^a(A)}{\delta A^b_\mu} D^{db}_\mu(A), \tag{2.4}$$

where

$$D^{db}_\mu = \partial_\mu \delta^{db} + \epsilon^{dcb} A^c_\mu .$$

Then the Faddeev-Popov determinant, evaluated by diagonalization, will be nonvanishing. A normalizable zero eigenvalue could only occur if there were a special (unknown) symmetry or very special field configurations and these would have no measure in the functional integral. So this will be a safe assumption.

The ordinary Faddeev-Popov determinant is defined by

$$\det{}^{-1}[O^{ab}(A)] = \int_{\mathrm{I}} \prod_x dg(x) \prod_{x,a} \delta(F^a[A^g_\mu(x)]), \tag{2.5}$$

where the region of group integration I is limited to infinitesimal transformations $g \simeq 1$. One can show that this determinant is invariant under infinitesimal gauge transformations:

$$\det{}^{-1}[O^{ab}(A)] = \det{}^{-1}[O^{ab}(A^g)], \tag{2.6}$$

where $g \simeq 1$. However, it is not invariant under finite gauge transformations if there are gauge-fixing degeneracies. This corresponds to the orbit of A_μ intersecting the hypersurface many times (see Fig. 1), where the intersections are connected by finite gauge transformations. If one were to insert the factor

$$1=\det[O^{ab}(A)]\int_{\rm I}\prod_a dg(x)\prod_{x,a}\delta(F^a[A^g(x)]) \quad (2.7)$$

into the generating functional (2.1), the resulting expression would be invariant under infinitesimal gauge transformations only. Furthermore, one would count every intersection on the same orbit once rather than recognizing that each intersection lies on the same gauge-equivalent orbit. Consequently, the usual prescription fails if there is a gauge-fixing degeneracy under finite transformations.

For finite gauge transformations a modification is required. Suppose, in the case of gauge-fixing degeneracies, there are fields ${}^{(n)}A_\mu$, $n=2,3,4,\ldots$, related to ${}^{(1)}A_\mu$ by finite gauge transformations and satisfying

$$F^a[{}^{(n)}A_\mu]=0\,. \quad (2.8)$$

(We assume that the index n is discrete; in general it is a continuous variable, and sums have to be replaced with integrations.) Then the invariant expression over the full orbit is

$$\sum_n \det{}^{-1}\{O^{ab}[{}^{(n)}A]\}=\int_{\rm II}\prod_x dg(x)\prod_{x,a}\delta(F^a[A^g_\mu(x)]), \quad (2.9)$$

where the group integration region II covers all gauge transformations g. Each separate determinant at $n=1,2,\ldots$ may be quite different; only the sum (2.9) is invariant under all gauge transformations. The separate determinants can be evaluated for the configurations ${}^{(n)}A$ by the usual diagonalization.

Writing (2.9) as

$$1=\frac{1}{\sum_n \det^{-1}\{O^{ab}[{}^{(n)}A]\}}\int_{\rm II}\prod_x dg(x)\prod_{x,a}\delta(F^a[A^g_\mu(x)]) \quad (2.10)$$

and inserting this into (2.1) gives

$$\begin{aligned}Z&=N^{-1}\int[dA_\mu]\frac{1}{\sum_n\det^{-1}[O^{ab}({}^{(n)}A)]}\\&\quad\times\int_{\rm II}\prod_x dg(x)\prod_{x,a}\delta(F^a[A^g_\mu(x)])e^{iS}\\&=N^{-1}\int_{\rm II}\prod_x dg(x)\int[dA_\mu]\frac{1}{\sum_n\det^{-1}[O^{ab}({}^{(n)}A)]}\\&\quad\times\delta(F^a[A_\mu(x)])e^{iS}\\&=N'^{-1}\int[dA_\mu]\frac{1}{\sum_n\det^{-1}[O^{ab}({}^{(n)}A)]}\\&\quad\times\delta(F^a[A_\mu(x)])e^{iS}\,,\end{aligned} \quad (2.11)$$

which is the gauge-invariant generating functional in the case of gauge-fixing degeneracies. If the $\delta(F^a)$ function is satisfied more than once for gauge-equivalent configurations, this overcounting is compensated for by the sum over determinants in the denominator. So the formal expression (2.11) counts the gauge-equivalent orbits only once and is invariant under finite gauge transformations.

In the case of gauge-fixing degeneracies it is not possible to promote the determinant to ghost field terms in the Lagrangian in the usual way. Hence, the usual Feynman rules fail to specify the solution in the Coulomb and Landau gauges.

III. COULOMB-GAUGE DEGENERACIES AND THE INSTANTANEOUS COULOMB INTERACTION

In the canonical formalism[5] the SU(2) gauge field is fixed by the Coulomb-gauge condition

$$F^a(A)=\partial_i A^a_i=0\,. \quad (3.1)$$

The Hamiltonian is

$$H=\tfrac{1}{2}\int d^3x[(E^a_i)^2+(B^a_i)^2+(\nabla_i\rho^a)^2]\,, \quad (3.2)$$

with

$$\begin{aligned}&F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+\epsilon^{abc}A^b_\mu A^c_\nu\,,\\&F^a_{0i}={}^LF^a_{0i}+{}^TF^a_{0i}\,,\\&\partial_i F^a_{0i}=\partial_i\,{}^LF^a_{0i},\quad {}^LF^a_{0k}=-\partial_k\rho^a\,,\\&{}^TF^a_{0i}\equiv E^a_i,\quad B^a_i=\tfrac{1}{2}\epsilon_{ijk}F^a_{jk}\,,\end{aligned} \quad (3.3)$$

and E^a_i and A^a_i are the canonically conjugate independent variables. The operator ρ^a appearing in the instantaneous Coulomb interaction term is expressed in terms of the canonical variables as

$$\rho^a(x,t)=\int d^3y\,D^{ab}(x,y;A)\epsilon^{bcd}A^c_k(y,t)\,E^d_k(y,t)\,. \quad (3.4)$$

The Green's function $D^{ab}(x,y;A)$ is given by

$$D^{ab}(x,y;A)=\sum_n\frac{V^a_n(x;A)V^{*b}_n(y;A)}{-E_n}\,, \quad (3.5)$$

where V^a_n satisfies

$$\begin{aligned}\partial_i D_i(A)V^a_n(x,A)&\equiv(\partial_i\partial_i\delta^{ab}+\epsilon^{acb}A^c_k\partial_k)V^b_n(x,A)\\&=-E_nV^a_n(x,A)\end{aligned} \quad (3.6)$$

and the eigenfunctions are normalized and complete according to

$$\begin{aligned}&\int d^3x\,V^a_n(x)V^{a*}_m(x)=\delta_{nm};\\&\sum_n V^a_n(x)\,V^{b*}_n(y)=\delta^{ab}\delta^3(x-y)\,.\end{aligned} \quad (3.7)$$

We will examine in this section the behavior of the Fourier transform of $D^{ab}(x,y;A)$ in the presence of an external field A_i^a. In order to restore translation invariance, which requires that $D^{ab}(x,y;A)$ depend only on $|\vec{x}-\vec{y}|$, we will integrate over all possible positions $\vec{c}$ of the external field. If $V_n(x;A)$ is the eigenfunction for the external field $A_i^a(x)$ at position $\vec{c}=0$, then the translationally invariant Green's function can be written as

$$D^{ab}(x,y;A)=\int\frac{d^3c}{V}\sum_n\frac{V_n^a(x+c;A)V_n^{b*}(y+c;A)}{-E_n}\,,\tag{3.8}$$

where V is the volume of the d^3c integration. This integration over c is a device that guarantees that ρ in (1.8) is translation invariant as is required in the full exact theory. (Note that we introduce an infrared cutoff by this finite volume.) The Fourier transform of the propagator is then

$$\begin{aligned}G^{ab}(k)&=\delta^{ab}g(k^2)+k^ak^bf(k^2)\\&=\int d^3x\,e^{-i\vec{k}\cdot\vec{x}}D^{ab}(x,0;A)\,.\end{aligned}\tag{3.9}$$

Next we integrate over all directions of isospin to restore SU(2) invariance. The final result for the propagator is

$$\begin{aligned}D^{ab}(k^2)&=\delta^{ab}D(k^2)=\int\frac{d\Omega_k}{4\pi}G^{ab}(k)\,,\\D(k^2)&=g(k^2)+\tfrac{1}{3}k^2f(k^2)\,.\end{aligned}\tag{3.10}$$

Denoting

$$V_n^a(x+c;A)=\int\frac{d^3k}{(2\pi)^3}e^{ik\cdot(x+c)}V_n^a(k)\,,$$

substituting this into (3.5), and using (3.9) and (3.10), the d^3c integration can be done. We obtain

$$\begin{aligned}D^{ab}(k^2)&=\delta^{ab}D(k^2)\\&=\frac{\delta^{ab}}{3V}\sum_n\frac{V_n^c(k)V_n^{*c}(k)}{-E_n}\,.\end{aligned}\tag{3.11}$$

The result of our calculation will be to show that

$$D(k)\sim\frac{L}{V|k|^4}\quad(|k|\to0)\,,\tag{3.12}$$

which we interpret as confinement.

However, before we carry out this calculation we discuss the gauge-fixing degeneracies of the Coulomb gauge and review the work of Gribov[1] and the pendulum equation of Wadia and Yoneya.[4] Suppose we have a field $A_i=\tau^aA_i^a/2$ that satisfies the Coulomb-gauge condition

$$\partial_iA_i=0\,,\tag{3.13}$$

and consider a field A_i' on the same orbit as A_i:

$$A_i'=UA_iU^{-1}+iU\partial_iU^{-1}\,,\tag{3.14}$$

where U is the gauge transformation. Requiring that

$$\partial_iA_i'=0\tag{3.15}$$

implies that

$$D_i(A)(U^{-1}\partial_iU)=0\,.\tag{3.16}$$

Hence, if there are nontrivial solutions $U\neq1$ for field configurations A_i, we have a gauge-fixing degeneracy.

We restrict ourselves to field configurations of the form

$$A_i^a(\vec{x})=\epsilon_{aij}\frac{x_j}{r^2}f(r),\quad r^2=x_i^2\tag{3.17}$$

which satisfy $\partial_iA_i^a=0$. Some special cases are

$$\begin{aligned}&f(r)=0,2\ \text{ pure gauge}\,,\\&f(r)=1\ \text{ Wu-Yang monopole}\,.\end{aligned}\tag{3.18}$$

A sufficient condition for infrared-finite energy H is that

$$\lim_{r\to\infty}f(r)=\text{constant}\tag{3.19}$$

and we impose this restriction. Ultraviolet-finite energy can be obtained if

$$f(0)=0,2\,.\tag{3.20}$$

The gauge transformation is assumed to take the form

$$U(\alpha)=e^{i\alpha(r)\vec{x}\cdot\vec{\tau}/r}\,,\tag{3.21}$$

and regularity at $r=0$ requires that

$$\alpha(0)=n\pi\,.\tag{3.22}$$

Equations (3.19), (3.20), and (3.22) are suitable boundary conditions for our problem.

Substituting (3.17) and (3.21) into (3.16), we find that the Ansatz are consistent and we obtain

$$\nabla^2\alpha(r)-\frac{\sin[2\alpha(r)]}{r^2}[1-f(r)]=0\,.\tag{3.23}$$

The changed variable $r=e^t$ gives the pendulum equation[1,4] (with damping)

$$\ddot{\alpha}(t)+\dot{\alpha}(t)-\sin[2\alpha(t)][1-f(t)]=0\,,\tag{3.24}$$

satisfying the boundary conditions $\alpha(-\infty)=n\pi$ and (3.22). The potential energy of the pendulum is $V(\alpha)=-2\sin^2\alpha(t)[1-f(t)]$ (see Fig. 2).

For simplicity, we first consider the case $f(t)$ = constant. Note that if $f<1$ gravity points downward and if $f>1$ gravity points upward (see Fig. 2). For large negative values of t we can approximate the pendulum equation by

$$\ddot{a}(t)+\dot{a}(t)-2(1-f)a(t)=0\,,$$

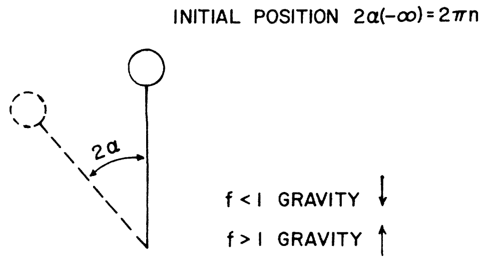

FIG. 2. Configuration of the pendulum for the pendulum equation (3.24).

where $a(t)=\alpha(t)-n\pi$ satisfies the boundary condition $a(-\infty)=0$. Thus, if $f<1$,

$$a(t)\sim\beta e^{\delta t}\quad(t\to-\infty),$$

where $\delta=[-1+(9-8f)^{1/2}]/2$. When t increases $a(t)$ becomes large and nonlinear effects become important. Then the pendulum eventually stops at the bottom $a(+\infty)=\pi/2$. Note that $|a(t)|<\pi$ because the initial kinetic energy is 0 $[\dot a(-\infty)=0]$.

On the other hand, if $f>1$, then when $1<f<\frac{9}{8}$, δ is negative and when $f>\frac{9}{8}$, Re δ is negative. In either case there are no nontrivial solutions obeying our boundary condition.

It is clear from this example that for the zero-field configuration, $A_i^a=0$, $f=0$, there exist fields $A_i'^a$ also satisfying the Coulomb-gauge condition and characterized by at least four independent parameters β and $\vec{x}_0$, the displacement of the gauge transformation $U(x)\to U(x+x_0)$. (So the Coulomb vacuum is at least four-fold degenerate.) If one considers a t-dependent $f(t)$, the situation is even more complicated.

Next we consider the infinitesimal version of the gauge transformations $g\simeq 1$,

$$U(g)=1+iV^a(x)\tau^a\,, \tag{3.25}$$

where $V^a(x)$ parametrizes the transformation. The requirement (3.16) that there exist other fields on the orbit of A_i satisfying the same Coulomb-gauge condition for infinitesimal transformations is

$$\partial_i D_i(A)V^a(x)=0\,. \tag{3.26}$$

Let us examine the eigenvalue equation

$$\partial_i D_i(A)V^a(x)=-EV^a(x)\,. \tag{3.27}$$

With the Ansatz

$$A_i^a=\epsilon_{ij}^a\frac{x_j}{r^2}f(r),\quad V^a(x)=\frac{x_a}{r^2}y(r)\,, \tag{3.28}$$

we obtain from (3.27) the one-dimensional Schrödinger equation

$$y''(r)+[E-V(r)]\,y(r)=0\,, \tag{3.29}$$

where we have suppressed the small parameter $\hbar$ and where

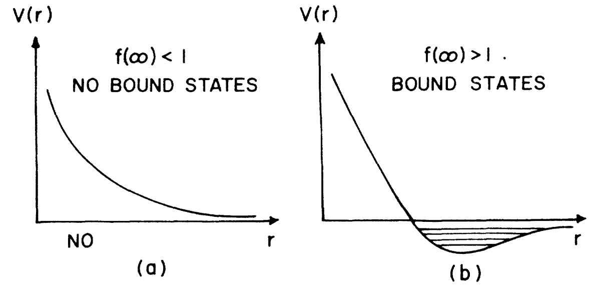

FIG. 3. The potential $V(r)$ for the Schrödinger equation (3.29) for the two cases $f(\infty)<1$ (no bound states) and $f(\infty)>1$ (bound states).

$$V(r)=\frac{2}{r^2}[1-f(r)]\,.$$

We assume, in what follows, that $f(r)$ satisfies the boundary conditions (3.19) and (3.20). If $f(0)=2$ the spectrum is unbounded from below so we assume that

$$f(0)=0,\quad f(\infty)=\text{constant}\,, \tag{3.30}$$

and for simplicity that $f(r)$ is monotonic. Then two cases may be distinguished (as in the pendulum problem): If $f(\infty)<1$ there are no bound-state solutions to the Schrödinger equation (3.29) [see Fig. 3(a)]; if $f(\infty)>1$ there are bound states [see Fig. 3(b)]. This latter case contains interesting physics.

It is not difficult to show that there is an accumulation of bound-state energies if $f(\infty)>\frac{9}{8}$. The accumulation is dependent only on the large-r tail of the potential. In general, level accumulation occurs if $V(r)\to-\gamma/r^\alpha$, providing that $\alpha\leqslant 2$. To show that in addition $f(\infty)$ must be $>\frac{9}{8}$ for accumulation to occur we set $E=0$ and the relevant potential problem is

$$y''(r)+\frac{\gamma}{r^2}\,y(r)=0\,, \tag{3.31}$$

where we define

$$\gamma=2[f(\infty)-1]\,. \tag{3.32}$$

Let $y(r)=r^s$. Then (3.31) implies that $s=\frac{1}{2}\pm(\frac{1}{4}-\gamma)^{1/2}$. Hence, if $\gamma<\frac{1}{4}$, $y(r)$ does not oscillate, and if $\gamma>\frac{1}{4}$, $y(r)$ oscillates. It is not difficult to see that for $\gamma>\frac{1}{4}$, $y(r)$ has an infinite number of nodes. For such one-dimensional potential problems, an infinite number of nodes implies that there are infinitely many levels below $E=0$.[7] Therefore, the requirement that $\gamma>\frac{1}{4}$ or $f(\infty)>\frac{9}{8}$ gives level accumulation.

We also note that the Faddeev-Popov determinant,

$$\det(\partial_i D_i)=\prod_n(-E_n)\,, \tag{3.33}$$

has an infinite set of negative eigenvalues as a consequence of the above remarks. This is simply a special case of the general observation of Gribov[1] that negative eigenvalues reflect gauge-fixing degeneracies.

Finally, we proceed to the calculation of the Coulomb Green's function $D^{ab}(k^2)=\delta^{ab}D(k^2)$ given by (3.11). The eigenfunctions $V_n^a(x;A)=x^a y_n(r)/r^2$ and bound-state energies E_n are just those associated with the Schrödinger equation (3.29),

$$y_n''(r)+[E_n-V(r)]y_n(r)=0\,, \tag{3.34}$$

with normalization (3.7)

$$\delta_{nm}=\int d^3x\, V_n^a(x)V_m^a(x)=4\pi\int_0^\infty dr\, y_n(r)y_m(r)\,. \tag{3.35}$$

For $V(r)=2[1-f(r)]/r^2$, $f(\infty)>\frac{9}{8}$ we have an accumulation of levels. Because there are an infinite number of loosely bound states, the semiclassical WKG approximation is appropriate to compute the sum over levels. Furthermore, the semiclassical analysis is insensitive to the precise details of the potential $V(r)$ for small r. Consequently, we may examine simultaneously all configurations that exhibit the accumulation phenomenon.

The potential $V(r)=2[1-f(r)]/r^2$, $f(\infty)>\frac{9}{8}$ is plotted in Fig. 4 and the accumulation of levels is indicated schematically. To simplify the subsequent analysis we will replace the actual potential by one that has a steep wall at $r=0$; this will eliminate the turning point near the origin without altering any of our conclusions which depend only on the large-r behavior.

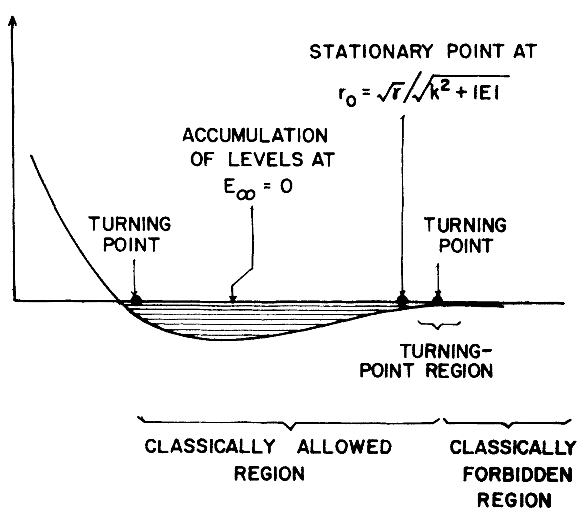

FIG. 4. The potential $V(r)$ in (3.29) with $f(\infty)>\frac{9}{8}$, showing the features of the WKB analysis: the accumulation of levels at $E_\infty=0$, the classically allowed and forbidden regions, the turning points, and the stationary-phase point.

For values of E near the accumulation point at $E_\infty=0$ the turning point A is given approximately by

$$A=(-\gamma/E)^{1/2}\,. \tag{3.36}$$

Thus, $A\to\infty$ as $E\to 0$.

The wave function y satisfying (3.34) may be approximated by three formulas which hold in each of three regions. In the classically forbidden region III the wave function $y(r)$ decays exponentially:

$$y_{\rm III}=C[V(r)-E]^{-1/4}\exp\left\{-\int_A^r[V(r')-E]^{1/2}dr'\right\}, \tag{3.37}$$

where C is a normalization constant to be determined. In the classically allowed region I, y oscillates:

$$y_{\rm I}=2C[E-V(r)]^{-1/4}\times\sin\left\{\int_r^A[E-V(r')]^{1/2}dr'+\pi/4\right\}. \tag{3.38}$$

In the turning-point region II about A $y(x)$ is given in terms of an Airy function that smoothly interpolates between $y_{\rm I}$ and $y_{\rm III}$:

$$y_{\rm II}=C\sqrt{\pi}\,2^{5/6}\gamma^{1/12}(-E)^{-1/4}\times{\rm Ai}[2^{1/3}\gamma^{-1/6}(-E)^{1/2}(r-A)]\,. \tag{3.39}$$

The eigenvalues are determined by the boundary condition that $y(r)$ vanish at $r=0$. Setting $y_{\rm I}(0)=0$ gives

$$\int_0^A[E-V(r')]^{1/2}dr'=(n+\tfrac{3}{4})\pi\quad(n=0,1,2,\ldots)\,. \tag{3.40}$$

To observe the accumulation phenomenon we replace the lower limit of integration by $B>0$, where to a good approximation

$$V(r)\sim-\gamma/r^2\quad(B\leqslant r\leqslant A)\,.$$

Then for large n we have

$$n\pi\sim\sqrt{\gamma}\int_B^A dr(r^{-2}-A^{-2})^{1/2}=\sqrt{\gamma}\int_{B/A}^1\frac{(1-x^2)^{1/2}}{x}dx\,.$$

This integral is logarithmically divergent as $A\to\infty$, so

$$-\sqrt{\gamma}\ln(B\sqrt{-E})\sim n\pi\,. \tag{3.41}$$

The density of states is

$$\rho(E)=\left|\frac{dn}{dE}\right|=\frac{\gamma^{1/2}}{2\pi|E|}\ . \tag{3.42}$$

Consequently, the sum over levels near $E=0$ in (3.11) (that is to say, the sum over large n) can be replaced by the integral

$$\sum_n \rightarrow \frac{\gamma^{1/2}}{2\pi}\int_0 \frac{dE}{E}\ , \tag{3.43}$$

a result we will shortly use.

Next we compute the normalization constant C in (3.37)–(3.39). The constant C is determined by the condition (3.34)

$$1=4\pi\int_0^\infty dr\, y^2(r)\ .$$

The only significant contribution to this integral comes from r in region I because outside this region the wave function is exponentially small. Using (3.38) we have

$$1=16\pi C^2\int_0^A \frac{dr}{[E-V(r)]^{1/2}}\sin^2\left[\int_r^A [E-V(r')]^{1/2}dr'+\pi/4\right].$$

The $\sin^2$ factor oscillates rapidly in the integration interval when n is large so we may approximate it by its average value $\frac{1}{2}$. Also, we may insert a lower limit of integration B above which $V(r) \sim -\gamma/r^2$:

$$1=8\pi C^2\int_B^A \frac{dr}{(\gamma/r^2-|E|)^{1/2}}$$
$$=\frac{8\pi C^2}{\sqrt{\gamma}}\int_B^A \frac{dr}{(1/r^2-1/A^2)^{1/2}}$$
$$=\frac{8\pi A^2C^2}{\sqrt{\gamma}}\,[1-(B/A)^2]^{1/2}.$$

Now if we let $E\rightarrow 0$ ($A\rightarrow\infty$), we obtain

$$C^2=\frac{|E|}{8\pi\sqrt{\gamma}}\ . \tag{3.44}$$

Next we must compute the Fourier transform of the wave function $V^a(x)=x^a y(r)/r^2$ [see (3.11)]:

$$k^aV(k)=V^a(k)=\int d^3x\, e^{i\vec{k}\cdot\vec{x}/\hbar}\,\frac{x^a}{r^2}\,y(r)\ .$$

Performing the angular integrations we obtain

$$k^2V(k)=\int d^3x\, e^{i\vec{k}\cdot\vec{x}/\hbar}\,\frac{\vec{k}\cdot\vec{x}}{r^2}\,y(r)$$
$$=4\pi i\hbar\int_0^\infty dr\, y(r)\left[\frac{\hbar}{kr}\sin\left(\frac{kr}{\hbar}\right)-\cos\left(\frac{kr}{\hbar}\right)\right]\ . \tag{3.45}$$

To evaluate this integral for small $\hbar$ we use the method of stationary phase, which we summarize as follows: Consider the integral

$$I=\int_a^b dx\, f(x)e^{i\phi(x)/\hbar}\ . \tag{3.46}$$

If $\phi'(x)$ is nonvanishing between a and b then

$$I\sim O(\hbar)\ .$$

However, if $\phi'(x_0)=0$, but $\phi''(x_0)\neq 0$, $a<x_0<b$ (x_0 is called a stationary point), then

$$I\sim O(\sqrt{\hbar})\ .$$

Explicitly, the contribution to I from the region surrounding the stationary point x_0 is

$$I\sim\begin{cases} f(x_0)\left(\dfrac{2\pi\hbar}{\phi''(x_0)}\right)^{1/2}\exp[i\phi(x_0)/\hbar+i\pi/4] & \text{if } \phi''(x_0)>0\ ,\\ f(x_0)\left(\dfrac{2\pi\hbar}{-\phi''(x_0)}\right)^{1/2}\exp[i\phi(x_0)/\hbar-i\pi/4] & \text{if } \phi''(x_0)<0\ .\end{cases} \tag{3.47}$$

Now we return to (3.45) and recall that we have suppressed a factor of $1/\hbar$ preceding the integral in (3.38) because we suppressed $\hbar$ in the Schrödinger equation (3.33). Since $y(r)$ is oscillatory in region I, a stationary-phase point exists. In regions II and III the wave function $y(r)$ is exponentially damped so no stationary-phase point exists and these regions make negligible contributions as $\hbar\rightarrow 0$. Therefore

$$k^2V(k)\sim 4\pi\hbar i\int_0^A dr\, y_{\rm I}(r)\left[\frac{\hbar}{kr}\sin\left(\frac{kr}{\hbar}\right)-\cos\left(\frac{kr}{\hbar}\right)\right]\ . \tag{3.48}$$

Using (3.38) we obtain

$$k^2V(k) \sim -2i\pi\hbar C\int_0^A \frac{dr}{[E-V(r)]^{1/4}}\left[\left(\frac{\hbar}{kr}-i\right)e^{i\pi/4+i\phi_+(r)/\hbar}+\left(-\frac{\hbar}{kr}-i\right)e^{i\pi/4+i\phi_-(r)/\hbar}+\text{c.c.}\right],$$

where

$$\phi_\pm(r)=\pm kr+\int_r^A [E-V(r')]^{1/2}dr'.$$

Stationary points r_0 must satisfy

$$\phi'_\pm(r_0)=\pm k-[E-V(r_0)]^{1/2}=0. \tag{3.49}$$

But $k>0$, so $\phi_-(r)$ has no stationary points and we need only consider $\phi_+(r)$ when k is small. There are in fact two real solutions r_0 to (3.49), one located near $r=0$ and the other given approximately by

$$r_0=\left(\frac{\gamma}{k^2-E}\right)^{1/2},$$

where E is near 0, which lies just to the left of the turning point at A (see Fig. 4). Note also that by virtue of (3.48),

$$\phi''(r_0)=\frac{V'(r_0)}{2k}.$$

Therefore, using (3.47) we have for each stationary point r_0, a contribution to (3.48) of the form

$$k^2V(k)=\frac{(4\pi\hbar)^{3/2}Ci}{[V'(r_0)]^{1/2}}\times\left\{\frac{\hbar}{kr_0}\sin[\phi_+(r_0)/\hbar]-\cos[\phi_+(r_0)/\hbar]\right\}. \tag{3.50}$$

For large r the behavior of the potential is $V(r)\sim-\gamma/r^2$. Therefore, $V'(r)\sim 2\gamma/r^3$ and

$$V'(r_0)\sim\frac{2}{\sqrt{\gamma}}(k^2-E)^{3/2}. \tag{3.51}$$

Observe that as E and k approach 0, $V'(r_0)$ becomes large and can produce a singularity in (3.50). At the other stationary point near $r=0$, $V'(r_0)$ does not vanish as E and k approach 0. Therefore, this point does not produce a singularity and we need not consider it further.

Combining (3.44), (3.50), and (3.51) gives

$$|V(k)|^2=\frac{-4\pi^2E\{\cos[\phi_+(r_0)]-(kr_0)^{-1}\sin[\phi_+(r_0)]\}^2}{k^4(k^2-E)^{3/2}}, \tag{3.52}$$

where we have set $\hbar=1$ because our semiclassical approximations are completed.

Besides the accumulation of bound states at $E=0$ there is a continuum of positive-energy states beginning at $E=0$ and these are also to be included in the sum on states. To treat the continuum we first discretize the spectrum by putting the system in a box of length $L=(3V/4\pi)^{1/3}$ corresponding to our infrared cutoff. Then the maximum positive energy is $E_{\max}=k^2-\gamma/L^2$. If one computes the density of states and the normalization constant $C(E)$ for the continuum they are separately L dependent; but the combination $C^2dn/dE=(16\pi^2)^{-1}$ that enters the calculation is L independent. The amplitude $|V(k)|^2$ for the continuum $E>0$ is the same as (3.52) with the phase factor given in terms of the variable $x=E/k^2$,

$$\gamma^{-1/2}\phi_+(r_0)=(1+xk^2L^2/\gamma)^{1/2}+\ln\left[\frac{(Lk\gamma^{-1/2})[1+(1-x)^{1/2}]}{1+(1+xk^2L^2/\gamma)^{1/2}}\right], \tag{3.53a}$$

in the continuum $E\gtrsim 0$, $x\geqslant-\gamma/L^2k^2$ and

$$\gamma^{-1/2}\phi_+(r_0)=\ln\left(\frac{1+(1-x)^{1/2}}{\sqrt{-x}}\right) \tag{3.53b}$$

for the bound-state region $E\lesssim 0$, $x\leqslant-\gamma/L^2k^2$. Since these expressions interpolate continuously in E from the bound states to the continuum region our calculation for the sum on levels can be given in terms of a single integral over E from some bound-state energy $-E_0$ to the maximum continuum energy $E_{\max}=k^2-\gamma/L^2$.

We now have all the ingredients required to compute the Green's function $D^{ab}(k)=\delta^{ab}D(k)$ given by (3.11). Using (3.52), (3.43), and the definition $V^a(k)=k^aV(k)$ we obtain

$$D(k^2)=\frac{2\pi\gamma^{1/2}}{3Vk^2}\,\mathrm{P}\int_{-E_0}^{E_{\max}}\frac{dE}{E(k^2-E)^{3/2}}\left[\cos\phi_+(r_0)-\left(\frac{1-E/k^2}{\gamma}\right)^{1/2}\sin\phi_+(r_0)\right]^2. \tag{3.54}$$

Let $E=xk^2$; then

$$D(k^2)=\frac{2\pi\gamma^{1/2}}{3Vk^5}\,\mathrm{P}\int_{-E_0/k^2}^{1-\gamma/L^2k^2}\frac{dx}{x(1-x)^{3/2}}\left[\cos\phi_+(r_0)-\left(\frac{1-x}{\gamma}\right)^{1/2}\sin\phi_+(r_0)\right]^2. \tag{3.55}$$

The final step is to let the infrared cutoff $L \to \infty$ with k held fixed. Then $x \to 1$ or $E_{\max} \to k^2$ as required. The result from (3.55) is

$$D(k^2) \to \frac{2\pi L}{3Vk^4} \qquad (3.56)$$

The variable k can be small with the restriction $k > L^{-1}$. In a pure gauge theory there is no length scale so L can be picked to set the scale $\sim(\mathrm{GeV})^{-1}$.

Had there been no accumulation of levels the integral in (3.54) would have been finite and the propagator $D(k)$ would have had the conventional behavior $D(k) \sim k^{-2}$ $(k \to 0)$. Our result in (3.56) corresponds to a confining potential.

The potential in configuration space corresponding to the propagator can be gotten from the expression for the Hamiltonian (1.7). The result is

$$V(r) \propto \int d^3k\, e^{i\vec{k}\cdot\vec{r}} |\vec{k}|^2 |D(\vec{k}^2)|^2 .$$

Regulating the singularity at $k=0$ in (3.55) according to

$$k^4 \to (k^2 + a^2)^2$$

and neglecting logarithms, this potential corresponds to an interaction of the form

$$\vec{T}_1 \cdot \vec{T}_2 |x_1 - x_2|^3$$

between the isotopic sources. Possibly Debye screening could reduce this r^3 potential to the phenomenologically successful linear potential, r.

IV. CONCLUSIONS

We have seen that for sufficiently "strong" potentials $A_i^a = \epsilon_{ij}^a (x_j/r^2) f(r)$ with $f(\infty) > \frac{9}{8}$, instantaneous Coulomb interactions become singular and give rise to a confining force between color-nonsinglet objects. On the other hand, in the case of "weak" potentials A with $f(\infty) < \frac{9}{8}$ there is no accumulation of levels and the ghost propagator remains nonsingular.

Apparently there exist finite gauge transformations which transform "small" and "large" potentials into each other while preserving the Coulomb-gauge condition. In fact the pendulum equation in Sec. III gives gauge transformations

$$U = \exp\left[i\alpha(r)\, \frac{\vec{\tau}\cdot\vec{x}}{r} \right], \qquad (4.1)$$

$$\alpha(0) = n\pi, \quad \alpha(\infty) = (\tfrac{1}{2} + n)\pi , \qquad (4.2)$$

which transform $A_i^a = \epsilon_{ij}^a (x_j/r^2) f$ with $f(\infty) < 1$ into $A_i'^a \sim \epsilon_{ij}^a (x_j/r^2)(2-f)$ $(r \to \infty)$. Thus, in the presence of gauge-fixing indeterminacy, strong and weak potentials are in general transformed into each other and we cannot make any distinction between them in a gauge-invariant way. In such a situation all the potentials ${}^{(n)}A$ $(n = 1, 2, \ldots)$ are to be treated on the same footing insofar as they obey the same gauge condition and are transformed to each other.

Thus, in general, large ${}^{(l)}A$'s and small ${}^{(m)}A$'s are lumped together to form a set $\{{}^{(n)}A\}$ corresponding to a given $F_{\mu\nu}$. Any breakup of such a set of potentials leads to results which are noninvariant under global gauge transformations. In particular, the ordinary Feynman perturbation theory is noninvariant because it singles out a small A out of each set of potentials $\{{}^{(n)}A\}$. In this way the requirement of gauge invariance under a full non-Abelian group necessarily forces us to a nonperturbative and global treatment of gauge fields.

Here the crucial fact is that the individual functional determinants, $\det[\partial_i D_i({}^{(n)}A)]$, strongly depend on ${}^{(n)}A$, although their sum,

$$\sum_n \det{}^{-1}[\partial_i D_i({}^{(n)}A)] , \qquad (4.3)$$

is globally gauge invariant. Similarly the instantaneous Coulomb force is dependent on ${}^{(n)}A$. For a given field $F_{\mu\nu}$ one may consider a "net" Coulomb potential,

$$\text{Coulomb force due to } F_{\mu\nu} \sim \left({\sum_n}' \text{ Coulomb force due to } {}^{(n)}A \right) \Big/ {\sum_n}' , \qquad (4.4)$$

where the curl of ${}^{(n)}A$ gives $F_{\mu\nu}$. Hence, if in this set of potentials there are sufficiently many ${}^{(n)}A$'s which give rise to a singular Coulomb force, we will obtain an averaged confining potential.

There is no known solution to the gauge field equations which gives a nonzero tunneling amplitude between the vacuums of integral and half-integral winding number. (Instantons interpolate between n and $n+1$.) If the tunneling amplitude is indeed zero between n and $n+\frac{1}{2}$ we have a superselection rule and confinement occurs only for the sector built from the vacuums of half-intergral winding number. Possibly widely separated pairs with half-integral winding number also give confinement.

The importance of half-integral charge for confinement has also been noticed by Callan, Dashen, and Gross.[8] Significantly the potential they find between static colored sources is r^3, the same as our result. So it is likely that these two approaches are closely related.

In this paper we have found that "large" transverse gauge fields give rise to confinement. Since we do not yet know in general what portion of the whole functional space of A is occupied by these configurations our argument for confinement is incomplete. It is indeed possible that there exist other configurations which alter or destroy this confinement effect. However, our purpose in this paper is to present a *plausible* mechanism for confinement. One might try to extend our argument by including other configurations such as multimonopoles, higher angular excitations, and so on. However, we do not expect that the qualitative results of this paper will be dramatically altered; the existence of gauge indeterminacy and level accumulation gives strong support to the view that the spectrum of non-Abelian gauge theories is entirely different from that of perturbation theory.

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