CERN 72-18
Laboratory I
Theoretical Studies Division
18 October 1972

ORGANISATION EUROPÉENNE POUR LA RECHERCHE NUCLÉAIRE
CERN EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

HIGH-ENERGY PHYSICS 25 YEARS AFTER THE DISCOVERY OF THE π MESON

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*) Also at the Max-Planck-Institute for Physics and Astrophysics, Munich, Germany.

CERN - Service d'Information scientifique - H/520 - 4000 - octobre 1972

FOREWORD

This text is based on lectures given at the Physical Societies of Zurich, Belgium and the Netherlands, and on a lecture series at the Max-Planck-Institute for Physics and Astrophysics, Munich. It compares the state of high-energy physics in 1947 and 1972, reviews some of the most important developments in this period, and ends by listing a few current and future problems.

CONTENTS

		Page
1.	INTRODUCTION	1
2.	THE STATE OF HIGH-ENERGY PHYSICS IN 1947 AND NOW	1
	2.1 Particles	3
	2.2 Interactions	3
	2.3 Approximate symmetries of strong interactions	3
	2.4 Discrete symmetries and weak interactions	3
	2.5 Theoretical schemes	4
3.	HADRON CLASSIFICATION	5
4.	HADRONIC CURRENTS AND HIGHER HADRONIC SYMMETRIES	10
	4.1 Electromagnetic and weak currents	10
	4.2 Symmetry groups and currents	12
	4.3 Currents associated with SU(3) symmetry	13
	4.4 Axial currents and the chiral symmetry group	15
	4.5 More about chiral invariance	17
5.	HIGH-ENERGY HADRON COLLISIONS	18
	5.1 Diffractive collisions	19
	5.2 Exchange collisions and Regge theory	22
	5.3 Duality	24
	5.4 A remark on high-energy asymptotics	25
6.	CURRENT PROBLEMS AND FUTURE OUTLOOK	26
	REFERENCES	28

1. INTRODUCTION

A quarter of a century ago, two fundamental discoveries were made in high-energy and elementary particle physics, each of which marked the beginning of a series of very important developments.

The first one was the *discovery of the pion* in cosmic rays[1], followed within a year by the detection of abundant pion production at the Berkeley cyclotron[2]. It was rapidly recognized that the pion exists in three charge states π^+, π^0, π^- corresponding to isospin one, and that its strong interactions with nucleons (protons and neutrons) obey the law of isospin conservation. Soon after the pion the strange particles were discovered, their remarkable property of "associated production" was found, a property which was accounted for in 1953 by the introduction of the new quantum number of strangeness. The fifties were also marked by the successive discovery of many hadron resonances[3]. The rich spectrum of hadrons (hadron = strongly interacting particle) came to be recognized as a dominant although puzzling feature of elementary particle physics. Their classification became a central task of strong interaction theory.

The second discovery made in 1947 was the measurement of the *Lamb shift of hydrogen*[4] and its explanation in the framework of the *renormalization theory of quantum electrodynamics*[5]. Not only did quantum electrodynamics thereby become one of the most successful physical theories, it also provided the basis for great progress in general quantum field theory as the mathematical framework for the study of elementary particles and their interactions. One of the most important domains of application is modern S-matrix theory, more precisely the relativistic quantum formalism for decays and collisions based on a systematic exploitation of the properties of analyticity and unitarity of n-body amplitudes.

Since the glorious days of 1947, the field of elementary particle and high-energy physics has advanced at a high rate, important experimental discoveries alternating with new theoretical insights. This field has also changed in its instrumental nature and turned into a characteristic "big science" with a multitude of physicists working on and around a small number of giant accelerators. The present text attempts to summarize the scientific highlights of this development by contrasting the states of the field in 1947 and 1972, by describing some important domains of research, and by listing some outstanding problems. Such a presentation is, of course, influenced by the author's limited knowledge and personal judgement, and can therefore not be very objective. Its aim is only to take distance from the detailed research work of the day, and to survey the field and its progress from at least one general standpoint.

2. THE STATE OF HIGH-ENERGY PHYSICS IN 1947 AND NOW

We begin by comparing the state of knowledge in 1947 with that of the present day, in the form of two tables (Tables 1 and 2). They list the known particles (first column), the types of interactions (third column), some of their symmetry properties (fourth column), as well as the theoretical schemes most frequently used to describe the interactions of the particles (second column).

Particles	Theoretical schemes	Interactions	Approximate symmetries	
		Name	Isospin I	3d component I_3
Leptons neutrino ν electron e muon μ	Four fermion interaction	Weak interaction	nc	nc
Photon	QED	Electromagnetic interaction	nc	c
Hadrons proton p neutron n pion π	Yukawa theory	Strong interaction	c	c

Table 1

Survey of high-energy physics in 1947, as explained in the text

Particles	Theoretical schemes	Interactions	Approximate symmetries			
		Name	SU_3	I	I_3, S, P, C	PC
Leptons ν_e, ν_μ e, μ	V-A coupling	Very weak	nc	nc	nc	nc
		Weak	nc	nc	nc	c
Photon	QED	Electromagnetic	nc	nc	c	c
Hadrons baryons mesons	Hadronic currents	Strong	nc	c	c	c
	Diffraction Regge exch. Duality	Very strong	c	c	c	c

Table 2

Survey of high-energy physics in 1972, as explained in the text

2.1 Particles

In the *first column* of Table 1, the neutrino is mentioned, although in 1947 its existence was only a theoretical assumption. As shown in Table 2, we now know experimentally two types of neutrinos, ν_e and ν_μ, associated with the electron and muon, respectively. As to the hadrons, in contrast with 1947 many are now known, the baryons generalizing the nucleons and the mesons the pions. All leptons and baryons are theoretically expected to have distinct antiparticles (not mentioned in the tables), and many of these are known experimentally. The photons are their own antiparticles and the antiparticles of mesons are again mesons.

2.2 Interactions

The *third column* of Table 1 shows the well-known interactions of nuclear physics in order of increasing strength. They were sufficient for describing collisions and decays of the elementary particles known in 1947. This situation is essentially still unchanged in Table 2, although one has found it useful to distinguish two levels of weak and two levels of strong interactions. These distinctions are closely related to the question of approximate symmetries to be discussed presently.

2.3 Approximate symmetries of strong interactions

The *fourth column* of Table 1 mentions *isospin symmetry*, long known for protons and neutrons from nuclear physics and recognized to be valid also for the pion. This symmetry group and the associated quantum numbers I, I_3 are only defined for hadrons, and the symmetry is only obeyed by the strong interactions which therefore conserve I and I_3 (symbol "c" in the table). The electromagnetic and weak interactions violate it in specific ways, I_3 being non-conserved by the latter and I by both (symbol "nc" in the table).

The concept of approximate symmetries and partially conserved quantum numbers has occupied an increasingly central role in particle physics since the fifties. On the one hand, a far-reaching classification of hadrons became possible by introducing, in addition to isospin, the quantum number S of *strangeness*[6)], and by combining isospin and strangeness in the *unitary symmetry scheme* SU(3) [7)]. As shown in the fourth column of Table 2, S has the same conservation properties as those of I_3, while SU(3) symmetry is only approximately obeyed by strong interactions. This led to the convention of considering two levels of strong interactions, a stronger one which is by definition SU(3)-invariant and a weaker one embodying the SU(3)-violating parts. We have no precise mathematical description of these two parts of the strong interaction. What is known, however, is the group-theoretical SU(3) transformation property of the violation; it is revealed most clearly in the mass splittings in SU(3) multiplets[8)]. The size of the violation is considerable, of the order of 20%, but its group-theoretical transformation property implies relations between mass splittings which are experimentally well verified.

2.4 Discrete symmetries and weak interactions

On the other hand, the long-held dogma that *space inversion* P and *charge conjugation* C are exact symmetries of nature was shattered in 1956-57 [9)], when it was discovered that weak interactions involving leptons violate P and C maximally. For some years it was thought that these violations compensated each other exactly so as to ensure combined PC conservation, but in 1964 a very weak violation of PC was discovered in neutral kaon decay[10]. It led to the convention of considering two levels of weak interactions, a very weak one containing the PC-violating effects and another one for all remaining weak interaction effects. This is shown in the fourth column of Table 2. The very weak level is at present only known in a few decays of the neutral kaons. One should in fact question P, C and also T (time reversal) invariance at all levels of interactions, as well as the celebrated combined PCT invariance predicted by local quantum field theory. (There is recent evidence that T is violated and PCT conserved in the PC-violating neutral kaon decays.) As of now, looking at the whole fourth column of Table 2, one cannot help being struck by the systematic hierarchy of interactions in which greater strength always goes with higher symmetry.

2.5 Theoretical schemes

We come now to the discussion of the *second columns* of Tables 1 and 2. The lines crossing them from left to right simply summarize to which interactions the various types of particles are subject (strictly speaking, to the lowest order of interaction strength, because, for example, the higher-order electromagnetic properties of the muon are slightly affected by strong interaction effects). The square boxes in the second columns refer to the theoretical schemes used for describing these interactions.

Now, as in 1947, the greatest theoretical achievement in particle physics is beyond any doubt *quantum electrodynamics* supplemented by the *renormalization theory* to eliminate all divergences at all orders of perturbation. It is marked QED in the tables. Quantum electrodynamics can be regarded as a formally closed theory of the electromagnetic interactions of charged leptons ($e^{\pm}$ and $\mu^{\pm}$) and photons. It can also be applied to systems containing hadrons, such as atoms, but strong interaction effects are then present (e.g. the proton size in the hydrogen atom) which can only be estimated. More generally, strong and weak interaction effects are expected to modify somewhat all QED predictions, even those for charged leptons and hadrons. In many cases the expected modification is small, and agreement between experiment and QED is then impressive. As an example we quote the magnetic moment M of the muon, for which we give a few numbers[11]:

> Notation: $M = 2(1 + a)(e\hbar/2mc)$, m = muon mass
> Best experimental value: $a_{exp} = (116616 \pm 31) \times 10^{-8}$
> QED prediction: $a_{QED} = 116581.5 \times 10^{-8}$
> a_{QED} is calculated to third order in the fine structure constant α;
> the third-order term is
>
> $$21.2\,\alpha^3/\pi^3 = 26.7 \times 10^{-8} \;.$$
>
> The strong interaction contribution to a is estimated to be about 6×10^{-8}. It cannot be calculated exactly. The weak interaction contribution is much harder to estimate; some estimates give an order of magnitude of 10^{-8}.

In other cases, strong interaction effects profoundly modify QED predictions, as in e^+e^- scattering when the energy is near the threshold for hadron pair production. This shows that in reality, despite its mathematically closed structure, QED is an incomplete and open theory which is only accurate for special systems under special conditions.

For *weak interactions*, the theoretical scheme prevalent in 1947 was the local interaction of four fermions (more exactly, four Dirac spinor fields), as proposed originally by Fermi[12]. One then believed that one had the choice between all couplings compatible with P, C, and T conservation. The discovery of C and P violation in 1956-57 led to the unique scheme of vector and axial couplings with equal strength, indicated by V-A in the second column of Table 2. This scheme gives unrenormalizable divergences whenever it is applied to second or higher orders of perturbation. Attempts are currently under way to make it renormalizable by means of hypothetical massive gauge fields which would mediate the weak interaction[13]. This work is quite remarkable by the fact that it proposes a unified theory of weak and electromagnetic interactions. If it finds experimental confirmation, it will constitute a very fundamental step forward.

The application of QED and V-A coupling theory to hadrons raises a new category of problems related to the fact that virtual effects of the strong interaction greatly modify the electromagnetic and weak interaction properties of the hadrons. While these problems can be avoided to a large degree in small momentum transfer processes (in particular in low-energy nuclear physics), they play a central role in all reactions and decays where transfers of the order of 0.5 GeV/c or larger are involved. They are tackled in the theory of *hadronic currents*, an important chapter of hadron physics which has developed in the last 15 years (see second column of Table 2).

Finally, the search for a systematic theory of *strong interactions* was marked by the fact that the Yukawa scheme of a meson exchange theory of nuclear forces (second column of Table 1), popular until the early fifties, had to be considerably modified. In the second column of Table 2 we have replaced "Yukawa theory" by the schemes which are now used in the analysis of high-energy hadron collisions (diffraction, Regge exchange, duality). With hadronic currents and higher hadronic symmetries, they form the bulk of present-day theoretical work on strong interactions. These topics are reviewed in the following sections.

3. HADRON CLASSIFICATION

The 1950's were marked by the experimental discovery of a great many hadrons, all but a few being so short-lived that they are detected through their decay products and therefore appear as resonance states. Once a sufficient number of them were discovered and their main quantum numbers known, the problem of finding an over-all classification of hadrons became manageable. After various unsuccessful attempts, the solution was found in the *unitary symmetry* scheme of Gell-Mann and Ne'eman[7]. It classifies hadrons in multiplets described by linear representations of the special unitary group in three dimensions, SU(3), i.e. by tensors $T^{\gamma\cdots}_{\alpha\beta\cdots}$ with indices taking the values $\alpha, \beta, \gamma\ldots = 1, 2, 3$. The group SU(3) embodies the isospin group SU(2) through the rule that index values $\alpha = 1$ and 2 correspond to $I = \frac{1}{2}$ $I_3 = \frac{1}{2}$ and $-\frac{1}{2}$, respectively, while $\alpha = 3$ corresponds to $I = 0$. It also embodies the strangeness S, with the rule that $\alpha = 1, 2$ corresponds to $S = 0$, and $\alpha = 3$ to $S = -1$.

It is quite remarkable that all meson multiplets appear to be nonets, of tensor structure M^{β}_{α}. The ones which are completely known are listed in Table 3. It should be noted that the nonet is a reducible tensor, the reduction being in a traceless tensor N^{β}_{α} (an octet) and a unit tensor δ^{β}_{α} (a singlet)

$$M^{\beta}_{\alpha} = N^{\beta}_{\alpha} + \delta^{\beta}_{\alpha}\, N', \quad \sum_{\alpha} N^{\alpha}_{\alpha} = 0\,, \quad N' = \frac{1}{3}\sum_{\alpha} M^{\alpha}_{\alpha}\ .$$

Meson nonets

Name, spin-parity J^P	Pseudoscalar, 0^-	Vector, 1^-	Tensor, 2^+
Strangeness, isospin	Symbol (mass, Γ = width)	Symbol (mass, Γ = width)	Symbol (mass, Γ = width)
S = 0, I = 1	$\pi^+\pi^-$ (140 MeV) π^0 (135 MeV)	ρ^+, ρ^-, ρ^0 (765 MeV) (Γ = 125 MeV)	A_2^+, A_2^-, A_2^0 (1310 MeV) (Γ = 80 MeV)
S = 0, I = 0	η^0 (549 MeV) η'^0 (957 MeV) (Γ < 4 MeV)	ω^0 (784 MeV) (Γ = 11 MeV) ϕ^0 (1019 MeV) (Γ = 4 MeV)	f^0 (1269 MeV) (Γ = 154 MeV) f'^0 (1514 MeV) (Γ = 73 MeV)
S = 1 and -1, I = ½	K^+, K^- (494 MeV) K^0, $\bar{K}^0$ (498 MeV)	K^{*+}, K^{*-}, K^{*0}, $\bar{K}^{*0}$ (893 MeV, Γ = 50 MeV)	K_N^+, K_N^-, K_N^0, $\bar{K}_N^0$ (1408 MeV, Γ = 107 MeV)

Table 3

The nonets of best-known mesons. The width Γ is only indicated for mesons decaying by strong interactions. It is very small for $\pi^\pm$, $K^\pm$, K^0, $\bar{K}^0$ which decay by weak interactions, and for π^0, η^0 which decay electromagnetically.

Baryon octet

Spin, parity = $\frac{1}{2}^+$	
Strangeness, Isospin	Symbol (mass)
$S = 0, \quad I = \frac{1}{2}$	p^+ (938 MeV) n^0 (940 MeV)
$S = -1, \quad I = 0$	Λ^0 (1116 MeV)
$S = -1, \quad I = 1$	Σ^+ (1189 MeV) Σ^0 (1193 MeV) Σ^- (1197 MeV)
$S = -2, \quad I = \frac{1}{2}$	Ξ^0 (1315 MeV) Ξ^- (1321 MeV)

Baryon decuplet

Spin, parity = $\frac{3}{2}^+$	
Strangeness, Isospin	Symbol (mass, Γ = width)
$S = 0, \quad I = \frac{3}{2}$	$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$ (1236 MeV, Γ = 115 MeV)
$S = -1, \quad I = 1$	$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$ (1385 MeV, Γ = 36 MeV)
$S = -2, \quad I = \frac{1}{2}$	Ξ^{*0}, Ξ^{*-} (1531 MeV, Γ = 7 MeV)
$S = -3, \quad I = 0$	Ω^- (1672 MeV)

Table 4

The octet and decuplet of best-known baryons. The width Γ is only indicated for baryons decaying by strong interactions. It is very small for n, Λ, $\Sigma^\pm$, Ξ^{0-}, Ω^- which decay by weak interactions, and for Σ^0 which decays electromagnetically. The proton p is stable.

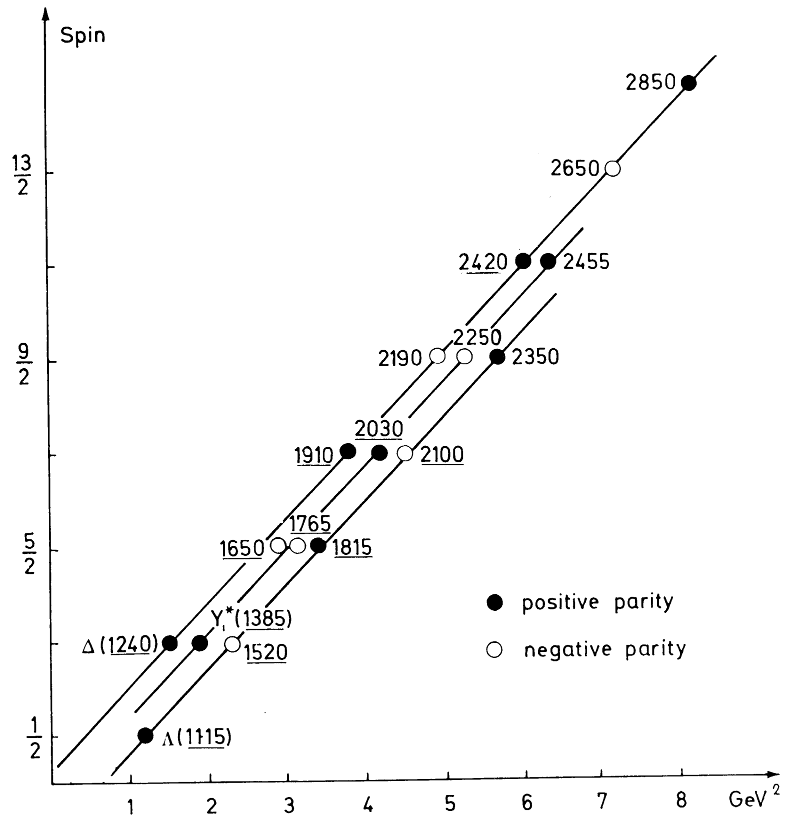

Fig. 1 The plot of spin versus mass squared for a number of baryons. The numbers 1115, 1240, etc., give the mass in MeV (c = 1). They are underlined when spin and parity of the corresponding baryon are actually measured. This figure is taken from Ref. 14 with one modification: the Δ(2420) has been underlined because its spin and parity are now determined. Some small and recent improvements in the mass values have not been incorporated in the figure. The straight lines are the Regge trajectories mentioned in the text.

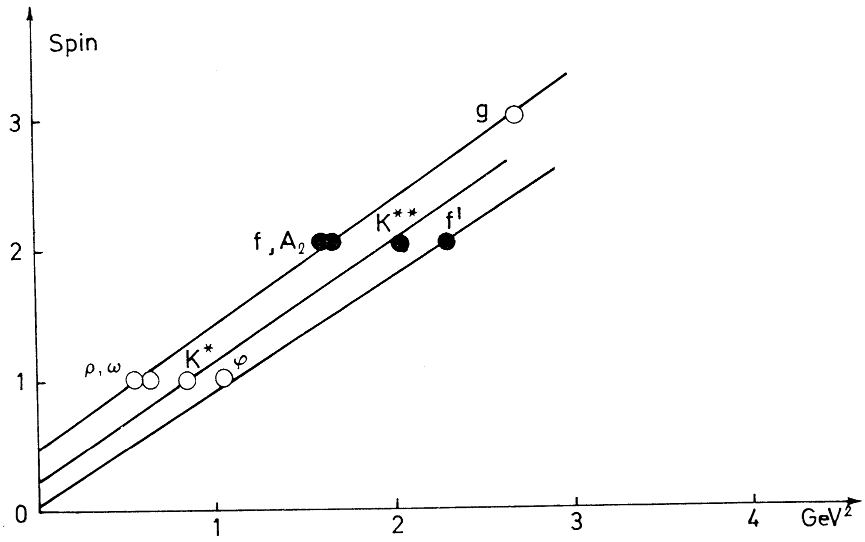

Fig. 2 The plot of spin versus mass squared for a number of mesons, taken from Ref. 14. Parities are indicated as in Fig. 1. All spins and parities are actually measured. The straight lines are the Regge trajectories mentioned in the text.

Except perhaps for the pseudoscalar mesons, the actual particles with I = S = 0 correspond to a mixture of the singlet and the I = S = 0 element of the octet.

All known baryon multiplets correspond to representations deduced from a three-index tensor $B_{\alpha\beta\gamma}$. The fully antisymmetric part is a singlet, an example of which is the hyperon Λ^*(1405 MeV) of width Γ = 40 MeV and of spin-parity $\frac{1}{2}^-$. The mixed symmetry part gives an octet and the fully symmetric one a decuplet (representation of dimension 10). Table 4 lists the best-known octet and decuplet of baryons. The masses given in Tables 3 and 4 show within each multiplet an amount of splitting which is due to SU(3) symmetry violation. With the exception of the pseudoscalar mesons (see next section), it is of the order of 20%.

The fact that all baryons can be represented by SU(3) tensors $B_{\alpha\beta\gamma}$ and all mesons by tensors M_α^β would become immediately understandable if there would exist SU(3) triplets Q_α which would be the building blocks of hadrons. The Q_α would be the celebrated quarks, baryons being composed of three quarks, and mesons of a quark and an antiquark:

$$B_{\alpha\beta\gamma} = Q_\alpha Q_\beta Q_\gamma , \quad M_\alpha^\beta = Q_\alpha \bar{Q}^\beta ,$$

with simple couplings for the quark spins which should have the value $\frac{1}{2}$. The Q_α should have baryon number $\frac{1}{3}$ and electric charges $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$ (for α = 1, 2, 3, respectively). Since no quarks have been found despite extensive searches, the nature of the SU(3) classification of hadrons remains puzzling. The universality of this classification is nevertheless striking, as well as the manifestation of SU(3) symmetry in the decays of resonances and the simple group-theoretical property of the SU(3) symmetry violation. In addition, as explained in the next section, SU(3) provides an important basis for the theory of hadronic currents.

Complementary to the SU(3) multiplet structure is another classification of hadrons referring to their spin and parity. In an SU(3) multiplet, the particles all have the same spin and parity, but they differ in isospin and strangeness. In the so-called *Regge classification*, a family is composed of particles all having the same isospin and strangeness but forming a series of increasing spin and alternating parity:

$$\text{spin} \quad J = J_0, \ J_0 + 1, \ J_0 + 2, \ldots$$

$$\text{parity} \quad P = P_0, \ -P_0, \ P_0, \ldots$$

The remarkable thing is that several such families have been found where the spin grows linearly with the mass squared:

$$J = \alpha' m^2 + \alpha_0 ,$$

the slope having always about the same value $\alpha' \doteq 1\ (\text{GeV}/c^2)^{-1}$. Striking examples are given in Fig. 1 for baryons and Fig. 2 for mesons. The lines interpolating between the (J,m^2) points are the *Regge trajectories*. It is very tempting to speculate that they continue to rise, implying the existence of a large, perhaps an *infinite* spectrum of hadrons with ever higher mass and spin values. As explained later, the exchange of such Regge trajectories is believed to be an important dynamical mechanism in high-energy collisions of hadrons.

4. HADRONIC CURRENTS AND HIGHER HADRONIC SYMMETRIES

One of the most successful developments over the last ten years has been the systematics of hadronic currents in relation to hadron symmetries, a programme largely pioneered by Gell-Mann[7,15]. In the picture which emerged, a central role is played by current and charge density operators which have a high degree of universality in the sense that many electromagnetic and weak processes of hadrons are described by taking the matrix elements of these operators between many different hadronic states. At the same time, the currents are directly associated to (approximate) symmetries of the strong interactions, and therefore obey (approximate) conservation laws.

4.1 Electromagnetic and weak currents

To lowest order, the *electromagnetic properties* of hadrons can all be described by the coupling of a single (real or virtual) photon to hadronic matter, as pictured in the vertex of Fig. 3. The lines a and b describe the hadronic states before and after the coupling with the photon γ. The electromagnetic current and charge density operator of hadrons, to be written $J_\mu(x)$, where x is a space-time point, describes this coupling through the amplitude (μ is summed over 0, 1, 2, 3)

$$e \int \langle b|J_\mu(x)|a\rangle \, \phi_\mu(x) \, d_4x \ ,$$

where e is the elementary electric charge and $\phi_\mu(x)$ the electromagnetic four-potential representing the virtual photon. The current J_μ is universal in the sense that, whatever the hadronic states a and b, $\langle b|J_\mu|a\rangle$ is a matrix element of the same operator. For a = b = proton, we obtain the coupling of a proton to a (virtual) photon or to an external electromagnetic field. For $a = \Sigma^0$ and $b = \Lambda$, we obtain the amplitude for the electromagnetic decay $\Sigma^0 \to \Lambda + \gamma$. For $a = \rho^0$ and b = vacuum, we obtain the amplitude for conversion of a ρ^0 meson into a virtual photon as occurs, for example, in the decay $\rho^0 \to e^+ + e^-$ (Fig. 4).

By determining experimentally for which hadronic states the matrix element $\langle b|J_\mu|a\rangle$ is non-zero, one can find out which quantum numbers are carried by the operator J_μ. The spin-parity is of course 1^-, and the "internal" quantum numbers are

baryon number	$\Delta B = 0$
electric charge	$\Delta Q = 0$
strangeness	$\Delta S = 0$
isospin	$\Delta I = 0$ and 1 .

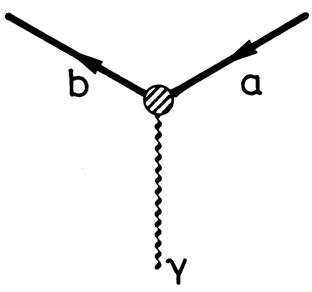

Fig. 3 The hadron-hadron-photon vertex.

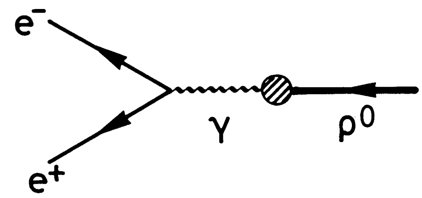

Fig. 4 Feynman diagram for $\rho^0 \to e^+ + e^-$ decay.

We have used the conventional notation ΔB, ΔQ, ..., which stems from the fact that these quantum numbers are equal to the differences between the corresponding quantum numbers of the hadronic states a and b; for example,

$$\Delta B = B_b - B_a \ ,$$

where B_a and B_b are the baryon numbers of a and b. Stating the quantum numbers carried by J_μ is another way of expressing the selection rules obeyed by electromagnetic transitions of hadrons. In the above list, the only item for which we still lack precise tests is the isospin, for which a small component of J_μ with $\Delta I > 1$ is not excluded. Experimentally, this question can be investigated in photoproduction experiments of pions on protons and neutrons:

$$\gamma + N \rightarrow N' + \text{one or more pions}; \quad N,N' = \text{nucleon states} \ .$$

The same procedure is used to describe *weak interactions* of hadrons with leptons. One has now two current density operators, $V_\mu(x)$ and $A_\mu(x)$, the first of vector type (spin-parity 1^-) and the second of axial type (spin-parity 1^+). The interaction of hadrons with an electron-neutrino pair or a muon-neutrino pair (Fig. 5) is represented by the amplitude

$$(G/\sqrt{2}) \int \langle b|V_\mu(x) + A_\mu(x)|a\rangle \, L_\mu(x) \, d_4x \ ,$$

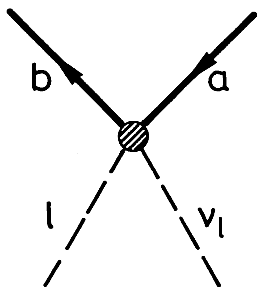

Fig. 5 The hadron-hadron-ℓ-ν_ℓ vertex, ℓ being an electron or muon and ν_ℓ the corresponding neutrino.

where G is the weak coupling constant ($G \simeq 10^{-5} \, m_p^{-2}$, m_p = proton mass). The states a, b are again the initial and final hadronic states. The function L_μ describes the leptonic contribution and is of the form

$$\langle d|\bar{\psi}_\ell(x)\gamma_\mu (1 + \gamma_5) \, \psi_{\nu_\ell}(x)|c\rangle \ ,$$

$$\ell = \text{electron or muon} \ .$$

where ψ_ℓ, ψ_{ν_ℓ} are the Dirac field operators for charged leptons ($\ell = e^-$ or μ^-) and the associated neutrinos (ν_e or ν_μ). The states c, d are the initial and final lepton states. We give a few examples:

Neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$:

$$a = n \ , \quad b = p \ , \quad c = \text{vacuum} \ , \quad d = e^- + \bar{\nu}_e \ .$$

Leptonic K^- decay $K^- \rightarrow \mu^- + \bar{\nu}_\mu$:

$$a = K^- , \quad b = \text{vacuum} \ , \quad c = \text{vacuum} \ , \quad d = \mu^- + \bar{\nu}_\mu \ .$$

Neutrino reaction $\nu_\mu + p \rightarrow \mu^- + p + \pi^+$:

$$a = p \ , \quad b = p + \pi^+ \ , \quad c = \nu_\mu \ , \quad d = \mu^- \ .$$

The internal quantum numbers carried by $V_\mu + A_\mu$ are, as far as is now known,

$$\Delta B = 0\,, \quad \Delta Q = 1\,, \quad \Delta S = 0 \text{ or } 1\ .$$

$$\Delta I = 1 \quad \text{if} \quad \Delta S = 0\,, \quad \Delta I = \tfrac{1}{2} \quad \text{if} \quad \Delta S = 1\ .$$

If one wants to describe transitions with $\Delta Q = Q_b - Q_a = -1$, one uses the Hermitian conjugate operators

$$G \int \langle b | V_\mu^\dagger + A_\mu^\dagger | a \rangle \, \langle d | \bar{\psi}_{\nu_\ell} \gamma_\mu (1 + \gamma_5) \psi_\ell | c \rangle\ .$$

Examples are

Leptonic K^+ decay $K^+ \to \mu^+ + \nu_\mu$:

$a = K^+$, b = vacuum , c = vacuum , $d = \mu^+ + \nu_\mu$.

Antineutrino reaction $\bar{\nu}_\mu + p \to \mu^+ + n$:

$a = p$, $b = n$, $c = \bar{\nu}_\mu$, $d = \mu^+$.

It should be noted that the above description does not cover those weak interactions of hadrons which do not involve leptons, e.g. the so-called non-leptonic decays $\Lambda^0 \to p + \pi^-$ or $K^0 \to \pi^+ + \pi^-$. Many attempts have been made to describe them by products $(V_\mu + A_\mu)(V_\mu^\dagger + A_\mu^\dagger)$, but no clear success has been encountered. A related question is the occurrence of weak interaction effects in hadronic transitions which conserve strangeness and involve no leptons. They are searched for in the form of parity-violating γ transitions of atomic nuclei[16].

4.2 Symmetry groups and currents

We now recall the well-known fact that, in a Lagrangian field theory, the knowledge of a continuous group of symmetry transformations leads to the definition of a corresponding current density. Let

$$\int L(\phi, \partial_\mu \phi)\, d_4 x$$

be the action integral, where the Lagrangian L depends on field operators denoted collectively by $\phi = \phi(x)$ and on their space-time derivatives $\partial_\mu \phi = \partial \phi / \partial x_\mu$. The field equations are

$$\partial_\mu [\partial L / \partial(\partial_\mu \phi)] = \partial L / \partial \phi\ .$$

Consider an infinitesimal transformation $\delta\phi$ acting on the field operators ϕ. The current density associated with it is

$$j_\mu(x) = [\partial L / \partial(\partial_\mu \phi)] \cdot (\delta\phi / \delta u)\ ,$$

where δu is the infinitesimal parameter of the transformation, and where a summation over fields is assumed. The divergence is

$$\partial_\mu j_\mu = \partial_\mu [\partial L / \partial(\partial_\mu \phi)] \cdot (\delta\phi/\delta u) + [\partial L / \partial(\partial_\mu \phi)] \cdot [\delta(\partial_\mu \phi)/\delta u]\ .$$

Using the equation of motion this becomes

$$\partial_\mu j_\mu = \delta L / \delta u\ .$$

If the transformation corresponds to a symmetry exactly obeyed by the system, we have $\delta L/\delta u = 0$ and the current j_μ is exactly conserved. In case the symmetry is only approximate, $\delta L/\delta u$ is small and the current is approximately conserved in the sense that it has a small divergence. Thus all symmetries which are relevant to the strong interactions provide us with currents, and we shall now see that these are directly related to the physically observable currents described in Section 4.1.

The spin parity of j_μ is 1^- if u is invariant for all space-time transformations, i.e. if u is a scalar parameter. We shall also encounter cases where u is pseudoscalar, corresponding to spin-parity 1^+ for the current j_μ (axial current).

4.3 Currents associated with SU(3) symmetry

The group SU(3) has eight continuous parameters and consequently eight linearly independent infinitesimal transformations. By the general method just outlined, one obtains eight currents $V_\mu^i(x)$, $i = 1, \ldots, 8$. It is reasonable to assume that SU(3) transformations commute with all space-time transformations; the SU(3) parameters are consequently scalars and the V_μ^i are vector currents.

In the now conventional notation due to Gell-Mann, the currents V_μ^i carry the following internal quantum numbers (in the sense defined in Section 4.1):

$$V_\mu^1 \pm i V_\mu^2 : \quad \Delta S = 0, \quad \Delta I = 1, \quad \Delta I_3 = \Delta Q = \pm 1$$

$$V_\mu^3 : \quad \Delta S = 0, \quad \Delta I = 1, \quad \Delta I_3 = \Delta Q = 0$$

$$V_\mu^4 \pm i V_\mu^5 : \quad \Delta S = \pm 1, \quad \Delta I = \tfrac{1}{2}, \quad \Delta I_3 = \pm\tfrac{1}{2}, \quad \Delta Q = \pm 1$$

$$V_\mu^6 \pm i V_\mu^7 : \quad \Delta S = \pm 1, \quad \Delta I = \tfrac{1}{2}, \quad \Delta I_3 = \mp\tfrac{1}{2}, \quad \Delta Q = 0$$

$$V_\mu^8 : \quad \Delta S = \Delta I = \Delta I_3 = \Delta Q = 0$$

All eight currents carry baryon number $\Delta B = 0$.

The currents $V_\mu^{1,2,3}$ belong to the isospin rotation group, for which the strong interactions are exactly invariant. From the general principle outlined in Section 4.2 we immediately conclude that, up to electromagnetic and weak corrections, one has

$$\partial_\mu V_\mu^i = 0 \quad \text{for} \quad i = 1, 2, 3 .$$

One can say a little more. The electromagnetic interaction leaves the third component of isospin invariant, hence

$$\partial_\mu V_\mu^3 = 0 \quad \text{up to weak interaction effects} .$$

Since the other components of isospin are not left invariant by the electromagnetic interaction, we have in contrast

$$\partial_\mu V_\mu^1 = 0 \quad \text{and} \quad \partial_\mu V_\mu^2 = 0 \quad \text{up to electromagnetic effects} .$$

The current V_μ^8 is associated with the one-dimensional group of transformations exp (i u Y), where Y is the so-called hypercharge which is defined by

$$Y = S + B, \qquad S = \text{strangeness},\ B = \text{baryon number}.$$

B is exactly conserved, conservation of S is only violated at the weak interaction level. Hence

$$\partial_\mu V^8_\mu = 0 \quad \text{up to weak interaction effects .}$$

The situation is different with the strangeness-carrying currents V^i_μ, i = 4 to 7. They correspond to SU(3) transformations which leave invariant only the very strong part of the strong interaction (see Table 2). As a consequence, the currents in question are conserved only at "very strong level". In other words, the divergences $\partial_\mu V^i_\mu$, i = 4 to 7, are given by the SU(3)-violating part of the strong interaction Lagrangian.

We end by giving the relations between the V^i_μ and the electromagnetic and weak currents described in Section 4.1, as they are now believed to hold on the basis of a number of successful predictions. The electromagnetic current is given by

$$J_\mu = V^3_\mu + (1/\sqrt{3})V^8_\mu$$

and the vector part of the weak current by

$$V_\mu = (V^1_\mu + iV^2_\mu)\cos\theta + (V^4_\mu + iV^5_\mu)\sin\theta \ ,$$

where θ, the so-called Cabibbo angle, is supposed to be a universal constant of about 15°. In these formulae, the same normalization has been used for all V^i_μ, so that the actual values of the coefficients are significant. It is clear that a precise understanding of these matters requires a more detailed explanation than can be given here. A very good approach to a better understanding is to consult a collection of articles and preprints published by Gell-Mann and Ne'eman[17)].

The above expression of V_μ in terms of the Cabibbo angle is the modern way of expressing the principle of universal strength of all weak interactions involving leptons, whether fully leptonic like muon decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$, or semi-leptonic like $n \to p + e^- + \nu_e$ or $\Lambda \to p + e^- + \nu_e$. Although some discrepancies still exist, the principle holds with reasonable precision for a great many reactions.

The vector part of the strangeness-conserving weak interactions is now described by $(V^1_\mu + i\,V^2_\mu)\cos\theta$, where $V^1_\mu + i\,V^2_\mu$ is simply the current for the appropriate isospin component. A particularly nice prediction of this scheme concerns the rare pion decay mode $\pi^- \to \pi^0 + e + \bar{\nu}_e$. Since the momentum transfer of π^- to π^0 is very small, the rate can be directly calculated from the vector part of the neutron beta-decay without any form factor correction. The branching ratio $(\pi^- \to \pi^0 + e + \bar{\nu}_e)/(\pi^- \to \bar{\nu}_\mu + \mu^-)$ is $(1.02 \pm 0.07) \times 10^{-8}$ and the experimental value is in excellent agreement with the theoretical prediction.

As an illustration of theoretical predictions resulting from the form $J_\mu = V^3_\mu + (V^8_\mu/\sqrt{3})$ of the electromagnetic current, we mention the magnetic moments of Λ and Σ hyperons which can be calculated from those of the proton and the neutron with an uncertainty of some 20%, characteristic of the strength of SU(3)-violation. The Λ moment is predicted to be half of the neutron moment, whereas experimentally $\mu_\Lambda/\mu_n = 0.37 \pm 0.04$.

4.4 Axial currents and the chiral symmetry group

The eight vector currents V_μ^i transform into each other under SU(3) and form the octet representation which is also the regular representation of the group. They obey the commutation relations

$$\left[V_\mu^j(x), Q^k\right] = i \sum_\ell f_{jk\ell} V_\mu^\ell(x) \ ,$$

where the $f_{jk\ell}$ are the structure constants of the group and the i Q^k its infinitesimal generators. The operators Q^k, which can also be called SU(3) charge operators, are related to the current densities by the relations

$$Q^i = \int V_0^i \, d_3\vec{x} \ ,$$

where one integrates over space at constant time $t = x^0$, x^0 being the time component of the four-vector x in the current densities $V_\mu^j(x)$. The $Q^{1,2,3}$ are the three components of the isospin operator $I_{1,2,3}$; $Q^3 + (Q^8/\sqrt{3})$ is the electric charge operator, and $2\ Q^8/\sqrt{3}$ the hypercharge operator. The degree of conservation of the eight charges Q^i is, of course, the same as for the corresponding currents V_μ^i.

It is very natural to assume that the weak axial current $A_\mu(x)$ has SU(3) properties entirely analogous to those of the weak vector current $V_\mu(x)$. This amounts to postulating the existence of eight axial currents $A_\mu^i(x)$ with the following properties:

i) they transform like the octet representation of SU(3), i.e.

$$\left[A_\mu^j(x), Q^k\right] = i \sum_\ell f_{jk\ell} A_\mu^\ell(x) \ ,$$

ii) their relation to the weak current A_μ is the same as the relation of the V_μ^i to the weak current V_μ, i.e.

$$A_\mu = \left(A_\mu^1 + iA_\mu^2\right) \cos\theta + \left(A_\mu^4 + iA_\mu^5\right) \sin\theta \ ,$$

where θ is again the Cabibbo angle.

Property (i) implies that the internal quantum numbers carried by each A_μ^j are the same as those of the V_μ^j of same index j. This knowledge of the quantum numbers combined with property (ii) gives a large number of experimental predictions concerning the axial parts of the weak hadronic decays involving leptons. The degree of success of these predictions is quite remarkable.

It is again natural in analogy with the V_μ^j to enquire whether the axial currents can be assumed to be conserved to some approximation:

$$\partial_\mu A_\mu^j(x) \simeq 0 \ .$$

This speculation also turned out to be very fruitful. The following formulation of this hypothesis of so-called *"partial conservation of axial currents"* (PCAC) seems at present to be the most successful: the divergence $\partial_\mu A_\mu^i$ can be assumed to be small to the degree that m_i^2 can be taken as a small quantity, where m_i is the mass of the pseudoscalar meson with same internal quantum numbers as A_μ^i. As seen in Tables 3 and 4, the π^+, π^-, and π^0 masses are appreciably smaller than all other hadron masses. This then suggests that the $\Delta S = 0$,

$\Delta I = 1$ currents $A_\mu^{1,2,3}$ are conserved to good approximation, whereas the remaining currents have larger divergences $\partial_\mu A_\mu^j$ corresponding to the larger masses of K and η mesons. The zero mass approximation for pions will be discussed further below.

With the vector currents V_μ^j we introduced the eight charge operators

$$Q^j = \int V_0^j \, d_3\vec{x} \ .$$

The i Q^j are the infinitesimal generators of the unitary symmetry group SU(3). Similarly, we can define axial charges

$$\tilde{Q}^j = \int A_0^j \, d_3\vec{x} \ ,$$

the space integrals again being taken at constant time, and we can consider the i $\tilde{Q}^j$ as infinitesimal generators of continuous families of transformations. The question which arises is whether they form a group in a natural way. The Q^i, of course, obey the SU(3) commutation rules

$$[Q^j, Q^k] = i \sum f_{jk\ell} \, Q^\ell \ .$$

From the SU(3) transformation properties of the A_μ^j one obtains furthermore:

$$[\tilde{Q}^j, Q^k] = i \sum f_{jk\ell} \, \tilde{Q}^\ell \ .$$

Gell-Mann[18] has suggested closing this system of equations by postulating

$$[\tilde{Q}^j, \tilde{Q}^k] = i \sum f_{jk\ell} \, Q^\ell \ .$$

In all these relations the Q^j and $\tilde{Q}^j$ are supposed to be defined by integrals of V_0^j and A_0^j over one and the same space-like hyperplane (equal time commutation rules). If one defines

$$Q_\pm^j = \tfrac{1}{2}(Q^j \pm \tilde{Q}^j) \ ,$$

the above commutation rules can be rewritten as

$$[Q_\pm^j, Q_\pm^k] = i \sum f_{jk\ell} \, Q_\pm^\ell, \quad [Q_+^j, Q_-^k] = 0 \ ,$$

which shows that the i Q^j, i $\tilde{Q}^j$ generate the direct product group SU(3) $\otimes$ SU(3). It is called the *chiral group* of strong interaction physics (the word *chiral transformation* applies, in particular, to the transformations generated by the axial charges $\tilde{Q}^j$).

To summarize these considerations, one can imagine an approximate theory of strong interactions which would be strictly invariant for the chiral group SU(3) $\otimes$ SU(3). It would imply exact unitary symmetry in the sense of Section 3 as well as vanishing masses for all pseudoscalar mesons. Such a theory needs various symmetry-breaking corrections in order to reproduce the actual facts:

i) Except for the pions, the pseudoscalar mesons have rather large masses. Hence a large breaking of SU(3) $\otimes$ SU(3) is expected, except for the subgroup SU(2) $\otimes$ SU(2) generated by the charges $Q^{1,2,3}$ and $\tilde{Q}^{1,2,3}$. The latter charges have the same internal quantum numbers as those of the pion ($\Delta S = 0$, $\Delta I = 1$).

ii) This SU(2) ⊗ SU(2) chiral subgroup is a better symmetry group, broken by a smaller correction term corresponding to the lower pion mass.

iii) The large mass differences between the pseudoscalar mesons manifest also a breaking of the unitary symmetry discussed in Section 3 [the corresponding SU(3) group is the one generated by the eight charges Q^j]. Tables 3 and 4 show that the pseudoscalar mass splittings are much larger than for the other SU(3) multiplets. This can be understood if breaking of SU(3) ⊗ SU(3) and breaking of SU(3) have a common origin; in the absence of these breakings, the masses of the π, η, and K mesons would vanish, while the masses in other hadron multiplets (baryons, vector mesons, etc.) would be equal without vanishing.

iv) The ordinary isospin group, which is the subgroup SU(2) generated by $Q^{1,2,3}$, is of course a very good symmetry of strong interactions. All known violations are sufficiently small to be attributed to electromagnetic and weak effects.

4.5 More about chiral invariance

In order to clarify how the size of the pseudoscalar meson masses is related to the approximate conservation of the axial currents, we consider the pion decay $\pi^- \to \mu^- + \bar{\nu}_\mu$. As explained in Section 4.1, the hadronic part of the decay amplitude is

$$\langle 0|V_\mu + A_\mu|\pi^-\rangle \ ,$$

where $|0\rangle$ is the vacuum. Since π^- is pseudoscalar, only A_μ contributes. As to internal quantum numbers, we need the $\Delta S = 0$, $\Delta I = 1$ part of A_μ. Consequently

$$\langle 0|V_\mu + A_\mu|\pi^-\rangle = \cos\theta \, \langle 0|A_\mu^{(+)}|\pi^-\rangle \ , \qquad A_\mu^{(+)} = A_\mu^1 + iA_\mu^2 \ .$$

By Lorentz invariance

$$\langle 0|A_\mu^{(+)}|\pi^-\rangle = f_\pi p_\mu \ ,$$

where p_μ is the four-momentum of the pion, and f_π a constant calculable from the experimental decay rate (its value is $\simeq$ 130 MeV). The divergence $\partial_\mu A_\mu^{(+)}$ has the matrix element

$$\langle 0|\partial_\mu A_\mu^{(+)}|\pi^-\rangle = f_\pi p_\mu p_\mu = f_\pi m_\pi^2 \ ,$$

m_π being the pion mass. Exact conservation would require this quantity to vanish. Since we obviously do not wish to decouple the current, we keep $f_\pi \neq 0$ and see that exact conservation would require a vanishing pion mass.

It is also of great interest to discuss the conservation of $A_\mu^{(+)}$ in the matrix element for neutron decay $n \to p + e + \bar{\nu}_e$ or neutrino reactions $\nu_\ell + n \to \ell + p$ (ℓ = e or μ). The matrix element has the form

$$\langle p|A_\mu^{(+)}|n\rangle = \frac{i}{2}\, \bar{u}_p(q_p)\, [G_1(k^2)\gamma_\mu\gamma_5 + G_2(k^2)k_\mu\gamma_5]\, u_n(q_n) \ ,$$

where q_n and q_p are the four-momenta of neutron and proton, u_n and u_p their Dirac spinors, $k = q_n - q_p$ the momentum transfer, and k^2 its relativistic square. One knows from beta-decay that $G_1(0) = g_A$ has the value $g_A \simeq 1.2$. One finds from the preceding formula

$$\langle p|\partial_\mu A_\mu^{(+)}|n\rangle = \frac{1}{2}\, [2m_N\, G_1(k^2) - k^2\, G_2(k^2)]\, (\bar{u}_p\gamma_5 u_n) \ ,$$

m_N being the nucleon mass, so that exact conservation requires

$$G_2(k^2) = 2m_N\, G_1(k^2)/k^2 \ .$$

Neither $G_1(0)$ nor the nucleon mass vanish. Hence G_2 has a pole at $k^2 = 0$ corresponding to exchange of a mass zero pseudoscalar meson. This is the pion with its mass vanishing under exact conservation of A_μ.

In the latter consideration we find that conservation of $A_\mu^{(+)}$ implies not only the vanishing of the pion mass but even the very existence of this zero mass particle. One can therefore speculate that the pion exists as a consequence of chiral SU(2) ⊗ SU(2) symmetry, and that similarly the K and η mesons exist because of the SU(3) ⊗ SU(3) symmetry. If these symmetries were strictly valid and the mesons strictly massless, the vacumm state would be degenerate with states containing any number of zero momentum mesons, just as the ground state of an isotropic ferromagnet is degenerate for simultaneous rotation of all spins. The pseudoscalar mesons would be analogous to the magnons of ferromagnetism, the two cases being examples of a general situation first recognized by Goldstone[19]. The chiral symmetry is broken, of course, and this corresponds to the pseudoscalar mesons having non-vanishing masses.

There are two good experimental confirmations of the approximate conservation of $A_\mu^{(+)}$, i.e. of approximate chiral SU(2) ⊗ SU(2) symmetry. One expresses the ratio g_A/f_π, which is a ratio of weak interaction quantities of nucleon and pion, respectively, in terms of nucleon mass m_N and pion-nucleon coupling constant g, which are pure strong interaction properties. It is the Goldberger-Treiman relation[20]

$$\sqrt{2}\, g_A/f_\pi = g/m_N \ .$$

The second expresses g_A in terms of the pion-nucleon scattering amplitude, again a pure strong interaction property. It is called the Adler-Weisberger relation[21]. Both relations are well satisfied.

5. HIGH-ENERGY HADRON COLLISIONS

Another domain of strong interaction physics where much activity has developed in the last ten years is the field of high-energy hadron collisions, i.e. of collisions between hadrons at centre-of-mass energies of a few GeV or more. In this energy range the collisions appear to follow a more unified pattern and to have a smoother energy dependence than at lower energy, where the effects of resonances and the threshold phenomena caused by production of additional particles unavoidably introduce rapid variations. All these can be expected to be smoothed out at higher energies, and it was therefore hoped that simple theoretical schemes could be constructed to describe the high-energy region. While extensive experimental work over the last ten years confirmed that the empirical data are indeed smooth and have a simple empirical behaviour at high energies, the success of theoretical interpretations has been more limited than, for example, in the domain of hadronic currents reviewed above. Many theoretical ideas were developed, some of them being quite new and interesting, but we are not yet able to single out a convincing theoretical scheme on the basis of truly successful experimental tests. Still, some theoretical concepts and models have done better than others and have a considerable power of qualitative description. We shall briefly describe three of them in this review: elastic and inelastic diffraction, Regge theory of exchange collisions, and duality. Before doing so, we mention a few empirical laws which dominate the whole field of high-energy hadron collisions.

i) *Law of constant cross-sections:* both the total cross-section σ_T and the elastic cross-section σ_{el} become approximately constant at high energy (elastic collisions are defined as collisions A + B → A + B where the outgoing hadrons, A, B are exactly the same as the incoming ones). The ratio σ_{el}/σ_T is of order 0.2.

ii) *Law of small transverse momenta:* the transverse momenta $\vec{p}_T$ of all outgoing hadrons (i.e. the projections of their momentum vectors on a plane perpendicular to the incident direction) have a distribution concentrated at small values, with averages $\langle p_T \rangle$ of order 0.3 to 0.4 GeV/c. This distribution is largely independent of the incident energy as well as of the nature of incoming and outgoing hadrons.

iii) *Law of low multiplicities:* the number of outgoing hadrons grows slowly with incident energy and is much smaller than would be the case if most of the available c.m. energy would be converted into particles (c.m. = centre of mass). The average multiplicity grows roughly as log E_{cm} or E_{cm}^a, $a \simeq 0.2$, where E_{cm} is the c.m. energy. For given E_{cm} the multiplicity of single collisions fluctuates widely around the average.

These laws hold as soon as $E_{cm} \gtrsim 5$ GeV. The third law means that most of the available c.m. energy is transformed into kinetic energy of the outgoing hadrons. Remembering the second law, we then see that the momenta of the outgoing hadrons obey on the average $|p_L| >> p_T$, where p_L is their longitudinal component (i.e. their projection on the incident direction). This means that the longitudinal direction plays a highly privileged role in h.e. hadron collisions (h.e. = high energy).

5.1 Diffractive collisions

Of all h.e. hadron collisions $A + B \to C_1 + \ldots C_n$ with specific hadrons $C_1, \ldots, C_n$ in the final state, the *elastic collisions* A + B → A + B have the largest cross-section $\sigma_{el} \simeq 0.2\ \sigma_T$. These elastic collisions are very peaked in the forward direction, corresponding to a small momentum transfer between A and B. A crude but universal parametrization of the forward elastic peak is given by an exponential

$$d\sigma/dt = a \exp(-bt) \ ,$$

t is the relativistic square of the four-momentum transfer defined by

$$t = -(p'_{A\mu} - p_{A\mu})\cdot(p'_{A\mu} - p_{A\mu}) = |\vec{p}\,'_A - \vec{p}_A|^2 - (E'_A - E_A)^2 \ ,$$

with $p_{A\mu}$, $p'_{A\mu}$ the four-momenta of incoming and outgoing particle A. In addition, the forward elastic amplitude is close to pure imaginary (and the constant a is therefore close to the value $\sigma_T^2/16\pi$ given by the optical theorem for a pure imaginary amplitude). The value of b is of order 8 to 12 $(\mathrm{GeV/c})^{-2}$ for all collisions; this approximate universality of b is a special case of the empirical law (ii) mentioned above. In contrast, the total cross-section σ_T and the parameter $a \propto \sigma_T^2$ depend strongly on the incident particles (extreme cases are K^+p and $\bar{p}p$ collisions, with σ_T = 18 and 44 mb at 50 GeV/c laboratory momentum p_{lab}. The latter is related to E_{cm} at high energy by the simple formula $p_{lab} \simeq E_{cm}^2/2m_T$, where m_T is the mass of the target particle, a proton in our examples). For given hadrons A and B, the quantities a and b vary only slightly for increasing p_{lab}.

In every respect the elastic scattering peak of hadrons at high energy looks like the manifestation of *diffraction* or *shadow scattering,* a familiar phenomenon in other branches of physics. Indeed, most of the h.e. hadron collisions are inelastic (the inelastic cross-section $\sigma_{in} = \sigma_T - \sigma_{el}$ is $\simeq 0.8\ \sigma_T$), and they are bound to produce shadow scattering in the elastic channel.

There is a class of inelastic hadron collisions which at high energy reveal three properties reminiscent of diffraction:

i) peaking in the forward direction, i.e. at small four-momentum transfer t,

ii) no exchange of internal quantum numbers,

iii) approximate energy-independence of cross-sections.

They are of the type

$$A + B \to A' + B' ,$$

where A' denotes either a single hadron or a set of several hadrons, and similarly for B'. The transfer t is defined by the relativistic square

$$t = -(p_{A'\mu} - p_{A\mu}) \cdot (p_{A'\mu} - p_{A\mu}) = -(p_{A'} - p_A)^2 ,$$

where $p_{A'\mu}$ is the total four-momentum carried by the hadronic system A', $p_{A\mu}$ being, as before, the four-momentum of the incoming particle A. Property (ii) means that

$$IQN(A') = IQN(A) , \quad IQN(B') = IQN(B) ,$$

where IQN(C) denotes the set of internal quantum numbers of the hadronic system C. (The internal quantum numbers are baryon number, electric charge, strangeness, isospin, and G-parity, the latter being defined only for non-strange mesons; if C is a system of several hadrons, IQN(C) denotes the total IQN of the system.) Note that the elastic scattering peak obeys the above properties with $A' = A$ and $B' = B$.

One speaks of *inelastic diffraction* when properties (i) to (iii) hold in cases where $A' \neq A$ *and/or* $B' \neq B$. This phenomenon is more commonly called *diffraction excitation* or *diffraction dissociation;* when $A' \neq A$ one considers A' as an excited or dissociated hadronic state produced by excitation of A in the course of the diffractive process, and similarly for B'.

Inelastic diffraction is probably a very common feature in h.e. hadron collisions. Some of the most striking experimental evidence for its occurrence is found in the detailed analysis of collisions of multiplicity 3 and 4. As an example we quote a thorough analysis of the four-body collisions at p_{lab} = 11 and 16 GeV/c,

$$\pi^- + p \to \pi^- + \pi^- + \pi^+ + p ,$$

carried out recently on the basis of a high statistics bubble chamber experiment[22]. The large majority of collisions fall into four groups, which are clearly separated in the sense that they are found in distinct regions of phase space. The four groups can be described by means of the above notation

$$A + B \to A' + B' \quad \text{with} \quad A = \pi^- , \quad B = p .$$

They correspond to the following composition of the hadronic systems A′, B′:

group I : $A' = \pi^-\pi^-\pi^+$, $B' = p$

group II : $A' = \pi^-$, $B' = \pi^-\pi^+ p$

group III : $A' = \pi^-\pi^-$, $B' = \pi^+ p$

group IV : $A' = \pi^-\pi^+$, $B' = \pi^- p$.

Property (i) above is found to be always satisfied in the sense that the momentum transfer $t = -(p_{A'\mu} - p_{A\mu})^2$ is small in each group ($p_{A\mu}$ is the momentum of the incident pion). Property (ii) can hold for groups I and II only; it is necessarily violated for groups III and IV (exchange of electric charge). Experiment shows that groups I and II have cross-sections which at high energy become approximately energy-independent, in accordance with property (iii). These groups are of diffraction dissociation type. In contrast, the charge-exchange processes in groups III and IV have cross-sections decreasing markedly with energy. At sufficiently high energy the diffractive processes of groups I and II will be the only ones to retain a sizeable cross-section.

Although its experimental study is still in an early phase, the phenomenon of inelastic diffraction is of great importance. Firstly, it is quite possible that this phenomenon will eventually account for the bulk of inelastic hadron collisions at very high energy, the dissociated systems A′, B′ having then a growing multiplicity as the energy increases. Secondly, despite the abundance of tentative models proposed in recent years, no good theoretical description is yet available. It must be noted in this context that, while the occurrence of elastic diffraction is a direct consequence of the existence of inelastic collisions (shadow scattering), this is not the case for inelastic diffraction whose occurrence and intensity depend on more specific dynamical properties of hadrons.

We end this section by explaining qualitatively how the separation of inelastic collisions in various groups, as illustrated above for the reaction $\pi^- + p \to \pi^- + \pi^- + \pi^+ + p$, looks in practice. Consider the c.m. system of the reaction. In each group of collisions $A + B \to A' + B'$, it turns out that the hadrons composing A′ have a c.m. momentum forming with the c.m. momentum of A an angle $\lesssim 90°$, and those composing B′ satisfy the same property with respect to B. The same fact can also be expressed in terms of longitudinal momenta p_L in the c.m. system. If our sign convention is such that $p_{LA} = -p_{LB} > 0$, the hadrons composing A′ have $p_L \gtrsim 0$ and those composing B′ have $p_L \lesssim 0$. In our example, group I is characterized by $p_L \gtrsim 0$ for all three outgoing pions, and one goes from group I to group III by changing the sign of $p_L(\pi^+)$. There is some overlap between groups when one or more p_L are small, but it turns out to be quite limited at high energy and low multiplicity (e.g. $n \lesssim 4$ at $p_{lab} \gtrsim 10$ GeV/c). The remarkable experimental fact at high energy is indeed that the size and the energy dependence of the collision amplitude are changing rapidly as one goes from one group of collisions to another by changing the sign of p_L for one outgoing particle, whereas they change much more smoothly inside each group. Because of the smallness of transverse momenta, the properties of the four-momentum transfer $t = -(p_{A'} - p_A)^2$ are also simply related to the sign of c.m. longitudinal momenta. Assume the energy to be very high and neglect all transverse momenta. One has then, on purely kinematical grounds, $t \simeq 0$ when all hadrons composing A′ have $p_L > 0$ and all those composing B′ have $p_L < 0$, whereas $t > 0$ for other signs of the p_L.

5.2 Exchange collisions and Regge theory

The general category of exchange collisions can be defined in terms that are similar to those of the diffractive case. They are of type

$$A + B \rightarrow A' + B'$$

with small momentum transfer $t = -(p_{A'} - p_A)^2$. As before, A′ can be a single hadron or a set of several hadrons, in which case $p_{A'}$ is their total four-momentum; B′ is similarly a single hadron or a set of hadrons. Exchange collisions are distinguished from diffractive ones by a non-vanishing exchange of internal quantum numbers, i.e. by the fact that at least one of the following inequalities holds:

$$\text{IQN}(A') \neq \text{IQN}(A), \quad \text{IQN}(B') \neq \text{IQN}(B) \ .$$

Taking again as example the collisions $\pi^- + p \rightarrow \pi^- + \pi^- + \pi^+ + p$ mentioned in Section 5.1, the groups III ($A' = \pi^-\pi^-$, $B' = \pi^+p$) and IV ($A' = \pi^-\pi^+$, $B' = \pi^-p$) are of exchange type, more precisely of charge-exchange type.

Whereas inelastic diffraction is occurring mainly in collisions of multiplicity $n \geq 3$, there are quite a few exchange collisions of multiplicity $n = 2$, and they have been the subject of many systematic experiments since some eight years. A list of such two-body exchange collisions is given in Table 5.

The dominant features of exchange collisions are the strong forward peaking of the differential cross-section $d\sigma/dt$ and the rapid decrease of $d\sigma/dt$ with increasing energy. The forward peaking is qualitatively similar to that of the diffractive case, and here the main interest concentrates on the decrease of $d\sigma/dt$ with energy. In the last ten years this problem has given rise to many interesting theoretical considerations, which will now be briefly reviewed.

The simplest dynamical model one can think of for the description of exchange collisions is the *one-particle exchange* model based on the Feynman diagram of Fig. 6. Here C denotes a single virtual hadron being exchanged between A and B. The third column of Table 5 lists hadrons C which can be exchanged in each case. For the two last collisions, no such hadron exists; for $\pi^- + p \rightarrow K^+ + \Sigma^-$ it should be a doubly-charged meson and for $p + K^- \rightarrow K^- + p$ a baryon with strangeness +1. No particles or resonances of this type have been found, and it should be noted that such hadrons would not fit in the SU(3) multiplets described in Section 3. It has become customary to refer to such hadronic combinations of internal quantum numbers as *"exotic"*. To realize them one needs a set of at least two hadrons. Similarly, for the two last collisions of Table 5 one talks about *exotic exchange*.

One still knows little about exotic exchange collisions. They have appreciably smaller cross-sections than those of the remaining single-hadron exchange collisions. This is perhaps not surprising since they require exchange of two hadrons, a process which is naturally expected to be rarer than single-hadron exchange. Experimental information on exotic exchange collisions is still too limited to test the very few theoretical ideas proposed so far.

The case of single-hadron exchange has advanced much more. The example of $\pi^- + p \rightarrow \pi^0 + n$ illustrates the interest of the problem very well. Simple considerations show that in this case the particle C exchanged should be a meson with isospin $I = 1$ coupling

Collision A + B → A′ + B′	Internal quantum numbers exchanged	Possible hadron exchanged
$\pi^- + p \rightarrow \pi^0 + n$	charge	ρ^-
$K^- + p \rightarrow \bar{K}^0 + n$	charge	ρ^-, A_2^-
$n + p \rightarrow p + n$	charge	π^+, ρ^+, A_2^+
$K^- + p \rightarrow \pi^- + \Sigma^+$	strangeness	$\bar{K}^0$
$\pi^- + p \rightarrow K^0 + \Lambda^0$	charge, strangeness	K^+
$p + \pi^+ \rightarrow \pi^+ + p$	baryon	n
$p + \pi^- \rightarrow \pi^- + p$	double charge, baryon	Δ^{++}
$\pi^- + p \rightarrow K^+ + \Sigma^-$	double charge, strangeness	"exotic"
$p + K^- \rightarrow K^- + p$	strangeness, baryon	"exotic"

Table 5

A list of simple exchange collisions, as explained in the text

to two pions. Such a meson must have spin-parity $J^P = 1^-, 3^- \ldots$. We know of two such mesons, the 1^- meson (see Table 3), and the heavier meson g of Fig. 2, which has $J^P = 3^-$. If particle C in the diagram of Fig. 6 has spin J, calculation of the corresponding Feynman amplitude gives a cross-section which at high energy has the form

$$\frac{d\sigma}{dt} = F(t)\, s^{2J-2} ,$$

where $s = E^2_{cm}$ is the square of the c.m. energy of the collision, and where F(t) is a function of t depending on the vertex contributions of the Feynman diagram. This gives dσ/dt constant in energy for ρ exchange (J = 1), and growing like s^4 for g exchange (J = 3), an obviously wrong result. The question was then how to construct a modified theory of single-particle exchange which would account for the experimental decrease of dσ/dt with energy, a decrease which for $\pi^- + p \rightarrow \pi^0 + n$ is measured to be approximately

$$\frac{d\sigma}{dt} \propto s^{-1} .$$

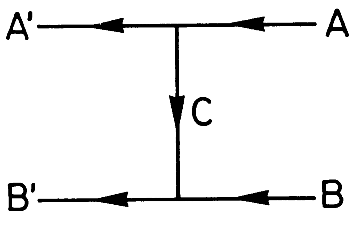

Fig. 6 Feynman diagram for a single-particle exchange collision.

An answer was found in *Regge theory*, an extension of scattering theory in which the angular momentum J is treated as continuous variable, and analytic continuation is performed in the *complex angular momentum plane*[23]. Just as the exchanged particle C of Fig. 6 carries a four-momentum squared $(p_{A'} - p_A)^2 = -t$ different from the value it would take if C were a real particle (this value would be $-t = m_C^2$ with m_C the mass of C), in the same way Regge theory attributes to the exchanged particle a spin α which is different from the value J_c that it takes for a real particle C. It states that α is an analytic function $\alpha(-t)$ of -t with the property of taking the real spin value J_c when the particle is real, i.e.

$$\alpha(m_c^2) = J_c \ .$$

The applicability of this idea to high-energy hadron collisions was independently recognized in 1961-62 by several people: Gribov and Pomeranchuk[24], Blankenbecler and Goldberger[25], and Chew, Frautschi and Mandelstam[26]. Although it was first thought that Regge theory would apply in the same way to both diffractive and exchange collisions, it is now realized that this is not so, and it is only for single-hadron exchange collisions that the scheme has had significant success, at least on one very important point.

This point is the following. In Regge theory the above formula $d\sigma/dt \propto s^{2J-2}$ is replaced by

$$d\sigma/dt \propto s^{2\alpha(-t)-2} \ .$$

But $\alpha(-t)$ is a function which extrapolates the spin values according to $\alpha(m_c^2) = J_c$ in exactly the way done by the Regge trajectories of Figs. 1 and 2. These figures show that a linear extrapolation holds. Let us apply it to $\pi^- + p \to \pi^0 + n$. The particle C can then be ρ and g, so that we take the upper trajectory of Fig. 2. We must extrapolate $\alpha(-t)$ to small and positive t values. This gives $\alpha(-t) \simeq 0.5$, which in turn implies

$$d\sigma/dt \propto s^{2\alpha(-t)-2} \simeq s^{-1} \ ,$$

in good agreement with experiment. Even the t-dependence of the exponent could be verified experimentally.

The above consideration can be summarized by saying that the energy dependence of single-hadron exchange collisions can indeed be described in terms of exchange, that the exchanged hadron must be given a *variable spin*, and that the variation follows the *Regge trajectories* discussed at the end of Section 3 and illustrated in Figs. 1 and 2. This is the crucial point of Regge theory. It is so far the only one to have qualitative simplicity, because the last years have revealed that many complications occur as soon as one tries to analyse the finer details of the exchange reactions.

5.3 Duality

Having discussed diffractive and exchange collisions, we now come to another interesting concept of hadron theory which emerged four years ago. It was introduced as an answer to a natural question. Consider once more a collision

$$A + B \to A' + B' \ .$$

Beyond diffraction and exchange, another collision mechanism is of course conceivable, namely *resonance formation* in which A and B form a hadronic resonance R which then decays in A′ and B′:

$$A + B \to R \to A' + B' \ .$$

Figures 1 and 2 show that hadrons of relatively high mass are already known, and extrapolation of the Regge trajectories to higher masses suggest that very heavy ones should exist. They would probably be broad resonances and could serve as intermediate states R in the above reactions when E_{cm} is equal to the resonance mass within the resonance width. The question to be faced concerns the *additivity of the contributions of the various collision mechanisms:* Can one write the complete collision amplitude T as a sum

$$T = T_d + T_e + T_r ,$$

where T_d describes a diffractive contribution, T_e a contribution of hadron exchange, and T_r a resonance formation term?

For exchange collisions as considered in Section 5.2 there is no diffractive term T_d, and one knows that the phenomena are qualitatively described by an exchange term T_e of Regge type. For diffractive collisions as considered in Section 5.1, the main term is T_d, but one knows that exchange effects of Regge type are also present in addition as energy-dependent corrections; they contribute the term T_e. In both cases the question is whether there is a further term T_r contributed by resonance formation.

At present the best answer to this question is a rather unexpected one: both terms T_e and T_r exist, their imaginary parts are equal and have to be counted only once in the imaginary part of the complete amplitude T. In formulae

$$\text{Im } T_e = \text{Im } T_r , \quad \text{Im } T = \text{Im } T_d + \text{Im } T_e = \text{Im } T_d + \text{Im } T_r .$$

This is called *duality;* more precisely, *Regge exchange and resonance formation are dual descriptions of one and the same basic collision mechanism.* Duality was first proposed by Dolen, Horn and Schmid[27]. Its role with respect to diffraction was formulated by Freund[28] and Harari[29]. From the theoretical standpoint, it is a very interesting concept with far-reaching consequences. It implies, in particular, that the Regge trajectories of Figs. 1 and 2 must continue to rise indefinitely, thereby predicting the *existence of hadrons of arbitrarily high mass.* Also from the practical standpoint the consequences are manifold. The most striking is that if resonance formation $A + B \to R$ is impossible (because of exotic quantum numbers, e.g. in $K^+ + n \to K^0 + p$ collisions), the various exchanges (here ρ, A_2) compensate each other to give a vanishing Im T_e. This puts remarkable constraints on Regge trajectories and couplings.

The explicit construction of collision amplitudes obeying duality is a difficult problem. Veneziano[30] has proposed such an amplitude for spinless particles, but it assumes that all particles, even the heavy ones, are stable (zero width resonances). Despite intensive efforts it has not yet been possible to construct physically more acceptable solutions. At a qualitative level, duality seems nevertheless to be an important new concept of hadron physics.

5.4 A remark on high-energy asymptotics

As indicated at the beginning of Section 5, all main properties of hadron collisions seem to develop a very smooth dependence on energy as the latter increases. This had created the hope that here we are dealing with a truly *asymptotic situation* and that the relevant terms of an asymptotic series would gradually be extracted from the experimental data. Recent developments have challenged this hope.

Consider, for example, pp elastic scattering, which is the most studied hadron reaction. It has now been measured up to the very high energies available at the CERN Intersecting Storage Rings (E_{cm} up to 55 GeV, equivalent to a proton beam of momentum p_{lab} up to 1600 GeV/c hitting a fixed target). The shape of the elastic peak changes slowly with energy, becoming steeper as the energy increases. It shows, in addition, some interesting structures. Unfortunately, many theoretical models are able to describe these features, each of them contains many free parameters, and it is therefore unlikely that more measurements of the same reaction will allow us to decide which model is the correct one.

This illustrates the double difficulty we are facing in this domain. Firstly, established theory puts only very few limitations on the freedom of the various models, each of which corresponds in fact to a different asymptotic expansion of the collision amplitude. Secondly, practice shows that it is much harder than expected to decide experimentally between the various models. But experiments at ever-increasing energy become extremely expensive and will only be justified if they can conclusively resolve some truly essential questions. What seems to be lacking is sufficient insight into what is essential as opposed to incidental in the field of strong interactions at very high energies. This insight will probably be gained more readily by the comparative study of a broad variety of reactions, without too much emphasis on every quantitative detail, than by the very accurate description of just a few reactions as a function of the incident energy.

6. CURRENT PROBLEMS AND FUTURE OUTLOOK

In this final section of our review we first mention two domains of research which are pursued very actively at the present time, and then list a few outstanding problems.

In the field of high-energy hadron collisions, the qualitative description of phenomena summarized in Section 5 has been developed in the last ten years for collisions which are mostly of the two-body type. The most common collisions have higher multiplicities (of order of magnitude 6 at $p_{lab} \simeq 20$ GeV/c increasing to about 15 at the CERN Intersecting Storage Rings), so that a detailed analysis is much more difficult. A large amount of experimental and theoretical work is currently devoted to these *many-body collisions*. It tries to extract from highly complex data a number of significant physical features which, hopefully, will be qualitatively simple and of general validity. This work goes in the direction mentioned at the end of the previous section. It should eventually lead to a description and understanding of all common hadron collisions in the high-energy region.

A fascinating development which is in full swing unites the main themes of hadronic currents and of high-energy hadron collisions reviewed in Sections 4 and 5. It concerns high-energy collisions between leptons and hadrons in the region of large momentum transfers, a type of process often referred to as *deep inelastic lepton-hadron scattering*. These collisions are of two types, illustrated by the electron-proton and neutrino-proton reactions

$$e + p \to e + N^*$$

$$\nu_\mu + p \to \mu + N^* ,$$

where N^* denotes a system of hadrons of high total mass. The first reaction proceeds through the electromagnetic interaction, the second is a weak interaction process.

Deep inelastic electron-proton and electron-neutron scattering has been extensively studied[31], and the results are of the greatest interest. They confirm a *scaling property* originally predicted by Bjorken[32]. We briefly point out its significance. Consider the process $e + p \to e + N^*$. It can be described by Fig. 6 with the identification $A = e$, $A' = e$, $B = p$, $B' = N^*$. The exchanged particle C is now a virtual photon. Deep inelastic scattering is characterized by the fact that both the momentum transfer t carried by the photon and the mass M of the final hadronic system N^* are large, the dimensionless ratio t/M^2 being finite. The remarkable fact is that the hadronic part of the process turns out to depend only on the ratio t/M^2 and not on t and M^2 separately, as one would normally expect. All attempts to interpret this experimental fact go back in one form or another to the basic feature of Bjorken's prediction, which is that the *electric charge distribution inside the proton has a granular structure composed of point-like elements*.

This property can also be formulated in more mathematical terms by means of the electromagnetic current operators J_μ (see Section 4.1). It then says that the commutation relations of current operators $J_\mu(x)$, $J_{\mu'}(x')$ involve in their leading singular terms no constants with dimension of a length when the four-vector $x - x'$ is near the light cone, i.e. when $(x - x')^2 \sim 0$. It thereby suggests generalizations of the algebraic properties of currents discussed in Section 4 in the direction of *approximate scale invariance*, i.e. invariance of large momentum transfer processes for changes of the scale of length.

In the case of neutrino-nucleon reactions, the simplest prediction of Bjorken's scaling property is that the total neutrino-nucleon cross-section increases linearly with the laboratory momentum of the neutrino. The data obtained in the CERN neutrino experiments confirm this prediction[33].

We can now ask the question, Could these properties of deep inelastic lepton-hadron scattering have a counterpart in purely hadronic processes? It is rather likely that they will not, in the region where the transverse momenta p_T are small, because one can show that deep inelastic scattering involves large p_T. This brings us to the first class of problems which are expected to play an increasing role in the future. It concerns the investigation of *high-energy hadron processes in the domain of large transverse momenta*, a domain where the strong interaction cross-sections are very small. If it is true (as indicated by deep inelastic ep scattering) that the proton has point-like properties of electromagnetic nature, one can even speculate that strong and electromagnetic effects may become of a comparable order of magnitude in the large p_T region, thereby modifying the hierarchy of interactions and symmetries summarized in Table 2. The speculations on approximate scale invariance belong to a second class of long-range problems. It concerns the elaboration of an *over-all scheme of broken symmetries for the strong interactions* and its possible interpretation in dynamical terms. This is a continuation of the line of work described in Section 4.

In addition, of course, one expects some of the more familiar *long-standing problems* of particle physics to remain in the forefront of future research. How far does the spectrum of hadrons extend into the high mass region? Are there more leptons? How does the weak interaction get modified as the energy increases? Is there a deep connection of particle physics with gravitation? And last but not least, Will also the very rich world of elementary particles one day become understandable as a complex and multifarious manifestation of a basically simple set of fundamental physical laws?

Acknowledgements

The author is indebted to Y. Goldschmidt-Clermont, W. Heisenberg, H. Schopper and I. Waller for valuable comments and critical reading of the manuscript.

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