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MONOPOLES IN A NON-ABELIAN GAUGE FIELD THEORY *

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ABSTRACT

The introduction, in a manner analogous to that employed for the abelian case, of point "magnetic" charges into an unbroken non-abelian gauge field theory is discussed.

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Magnetic point charges have been introduced into ordinary electrodynamics by considering them as the end-point of a magnetic dipole string whose other end is somewhere at infinity.[1)] Although magnetic charges have not been observed,[2)] such an approach has become of considerable interest recently, since an analysis of a massive gauge field theory in the presence of two "magnetic" charges of opposite sign leads to, in a suitable limit, an effective action for the open-ended string ("magnetic" dipole) which contains a term proportional to the area of the world sheet swept out by the string. Such a term can be identified with the usual dual string action.[3)]

In general, it is thought that for hadrons a non-abelian gauge theory, where the additional degrees of freedom are associated with colour and normal hadrons are colourless ("magnetically" neutral)[4)], should be more relevant. The purpose of this note is to attempt to introduce "magnetic" charges into an unbroken non-abelian gauge field theory by using a procedure similar to that employed by Dirac[1)] for the abelian case.

In the presence of an "electric" current j_μ the equation of motion for the gauge field B^a_μ is given by:

$$\partial_\mu F^a_{\mu\nu} - 2e f_{cba} F^c_{\nu\mu} B^b_\mu = - j^a_\nu \,, \qquad (1)$$

where $F^a_{\mu\nu}$ is the antisymmetric field tensor, e is a coupling constant and the f_{abc} are the usual SU(3) structure constants. The corresponding expression in the presence of a "magnetic" current k^a_ν is:

$$\partial_\mu \tilde{F}^a_{\mu\nu} - 2e f_{cba} \tilde{F}^c_{\nu\mu} B^b_\mu = - k^a_\nu \,, \qquad (2)$$

where the "dual" of an antisymmetric, e.g. $F_{\mu\nu}$, is defined by $\tilde{F}^a_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^a_{\alpha\beta}$. In order to satisfy both the above equations, we set[1)] $F^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu - 2ef_{abc} B^b_\mu B^c_\nu - \tilde{G}^a_{\mu\nu}$ where the antisymmetric tensor $G^a_{\mu\nu}$ will satisfy:

$$\partial_\mu G^a_{\mu\nu} - 2e f_{abc} B^b_\mu G^c_{\mu\nu} = - k^a_\nu \,. \qquad (3)$$

The total magnetic current, which will be conserved, is given by the sum of the right-hand side and the second term on the left-hand side of Eq.(3).

Let us then postulate the following expression for the total conserved current[5)]:

$$\partial_\mu G^a_{\mu\nu} = - g^a \int \frac{dz^a_\nu}{d\tau} d\tau\, \delta^4(x - z^a(\tau,0)), \tag{4}$$

where $z^a_\mu(\tau,0)$ describes the world line of the magnetic charge g^a. The corresponding expressions for $G^a_{\mu\nu}$ and the current k^a_ν are:

$$G^a_{\mu\nu} = g^a \iint d\tau\, d\sigma\, \delta^4(x - z^a(\tau,\sigma)) \left(\frac{\partial z^a_\mu}{\partial \tau} \frac{\partial z^a_\nu}{\partial \sigma} - \frac{\partial z^a_\nu}{\partial \tau} \frac{\partial z^a_\mu}{\partial \sigma} \right) \tag{5}$$

and

$$k^a_\nu = \iint d\tau\, d\sigma \left[g^a \frac{\partial}{\partial z^a_\nu} \delta^4(x - z^a) + 2e f_{abc} B^b_\nu g^c \delta^4(x - z^c) \right], \tag{6}$$

where $z^a_\mu(\tau,\sigma)$ describes the position of a point on the world sheet traced out by the string associated with the magnetic charge g^a.

The above equations of motion, in the absence of an "electric" current, may be obtained from the following Lagrangian $\mathcal{L}$:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - m^a \int d\tau\, \delta^4(x - z^a(0,\tau)) \left(\frac{dz^a_\mu}{d\tau} \frac{dz^a_\mu}{d\tau} \right)^{\frac{1}{2}}, \tag{7}$$

where the second term is associated with the mechanical mass m^a of the monopole.[5] Variation of the corresponding action with respect to B^a_μ leads to Eq.(1) (for $j^a_\nu = 0$). Variation with respect to the co-ordinates $z^a_\mu(\tau,0)$ of a magnetic pole yields

$$m^a \frac{d}{d\tau} \left[\frac{dz^a_\mu}{d\tau} \left(\frac{dz^a_\lambda}{d\tau} \frac{dz^a_\lambda}{d\tau} \right)^{-\frac{1}{2}} \right] = g^a \tilde{F}^a_{\mu\nu} \frac{dz^a_\nu}{d\tau} \tag{8}$$

and with respect to a point $z^a_\mu(\tau,\sigma)$ on the world sheet traced out by the string:

$$\varepsilon_{\mu\nu\alpha\beta} \left(\frac{\partial z^a_\alpha}{\partial \tau} \frac{\partial z^a_\beta}{\partial \sigma} - \frac{\partial z^a_\beta}{\partial \tau} \frac{\partial z^a_\alpha}{\partial \sigma} \right) \frac{\partial F^a_{\rho\nu}}{\partial z_\rho} = 0, \tag{9}$$

which implies that the "electric" current flow in a direction perpendicular to the sheet vanishes.[6]

One may arrive at an effective action describing our system by first solving Eq.(1) (for $j^a_\mu = 0$) for B^a_ν

$$B^a_\nu(x) = \int d^4y\, D(x-y) \Big[2e f_{abc} F^c_{\mu\nu}(y) B^b_\mu(y) + \partial_\nu \partial_\mu B^a_\mu(y) + \partial_\mu \tilde{G}^a_{\mu\nu}(y) + 2e f_{abc} \partial_\mu \big(B^b_\mu(y) B^c_\nu(y) \big) \Big] \tag{10}$$

where D is a massless Green's function, and then substituting the above in the action discarding total divergences. The resulting effective action is

$$\begin{aligned}
\int d^4x\, \mathcal{L} = & -\frac{1}{2} \iint d^4x\, d^4y \Big[\frac{\partial}{\partial y_\mu} \tilde{G}^a_{\mu\nu}(y) + 4e^2 f_{cba} f_{cde} B^d_\mu(y) \cdot \\
& \cdot B^e_\nu B^b_\mu + \frac{\partial}{\partial y_\nu} \frac{\partial}{\partial y_\mu} B^a_\mu(y) + 2e f_{abc} B^b_\mu \big(\partial_\mu B^c_\nu - \partial_\nu B^c_\mu - \tilde{G}^c_{\mu\nu} \big) \Big] \cdot \\
& \Big[\frac{\partial}{\partial x_\rho} \tilde{G}^a_{\rho\nu}(x) - 2e f_{cba} f_{cde} B^b_\rho(x) B^d_\rho B^c_\nu - 2e f_{abc} \partial_\rho (B^b_\rho B^c_\nu) \Big] \\
& -\frac{1}{4} \int d^4x\, \tilde{G}^a_{\mu\nu} \tilde{G}^a_{\mu\nu} - m^a \int \left(\frac{dz^a_\mu}{d\tau} \frac{dz^a_\mu}{d\tau} \right)^{\frac{1}{2}} d\tau \\
= & -\frac{1}{2} \iint d^4x\, d^4y\, D(x-y) \Big[\frac{\partial}{\partial y_\mu} G^a_{\mu\nu}(y) \frac{\partial}{\partial x_\rho} G^a_{\rho\nu}(x) + 2e^2 f_{cba} \cdot \\
& \cdot f_{cde} B^a_\rho(y) B^c_\nu(y) B^b_\rho(y) \frac{\partial}{\partial x_\mu} \tilde{G}^a_{\mu\nu}(x) + \cdots \Big] - m^a \int \left(\frac{dz^a_\mu}{d\tau} \frac{dz^a_\mu}{d\tau} \right)^{\frac{1}{2}} d\tau,
\end{aligned} \tag{11}$$

where the first two terms have been exhibited previously.[7] The first term corresponds to the usual long interaction between "magnetic" charges, whereas the second term, on again substituting for B^a_μ according to Eq.(10), can lead to a positive mass-like contribution for the other gauge fields (for $g^c = 0$, for $c \neq a$). The omitted terms correspond to higher "order" interactions between the gauge fields and the magnetic charges or between different magnetic charges.[5]

Lastly, let us examine the conditions obtained on attempting a first quantization of the Lagrangian. We shall follow the procedure previously employed.[1] One finds that the conjugate variable $\beta^a_\mu(\sigma)$ to a string variable $z^a_\mu(\sigma, \tau = \text{constant})$ is given by

$$\beta^a_\mu = g^a \tilde{F}^a_{\mu\nu} \frac{dz^a_\nu}{d\sigma}. \tag{12}$$

In order to first quantize the above, we introduce the wave function of the system which is a function of particle, string and field variables and satisfies:

$$-i \frac{\partial \psi}{\partial z^a_\mu} = g^a \tilde{F}^a_{\mu\nu} \frac{d z^a_\nu}{d\sigma} \psi . \qquad (13)$$

Hence, from the above, if we loop the string around a point charge e^a and bring it back to its original position, the wave function acquires a phase

$$g^a \int \tilde{F}^a_{\mu\nu}\, d\sigma^{\mu\nu} = g^a \int dV^\nu \partial_\mu F^a_{\mu\nu} = 2\pi n , \qquad (14)$$

and on writing an expression for the total (conserved) current analogous to Eq.(4) we obtain the following relation between "electric" and "magnetic" charges:

$$e^a g^a = 2\pi n \quad , \qquad (15)$$

which, for the case of the usual SU(3) charge identification and quark triplet, leads to:

$$eg = 3\pi n \quad . \qquad (16)$$

As we can see, the introduction of "magnetic" charge into a non-abelian gauge theory can be done in a manner analogous to the abelian case, once a proper identification of the conserved current is made. It is of course possible to introduce magnetic charges of opposite sign, thus obtaining strings of finite length stretching from one monopole to the other. Also the gauge fields can be rendered massive through the introduction of coupled scalar fields and spontaneous symmetry breaking. There will then be in the action a term proportional to the area traced out by the string and, if we have more than one type of "magnetic" charge, there will also be different colour strings. Further we expect the area swept out by the strings to be quantized and Eq.(15) not to hold. We shall not discuss these results further, since they are analogous to those previously exhibited.[7),8)]

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REFERENCES

1) P.A.M. Dirac, Phys. Rev. 74, 817 (1948).

2) At the time this note was written.

3) Y. Nambu, Phys. Rev. D10, 4262 (1974).

4) Actually in such a context the words "electric" and "magnetic" are interchangeable. We shall use them rather freely without intending their actual identification with electric and magnetic charges.

5) In the following we may have either $g^c = 0$ for $c \neq a$, in which case there will just be one magnetic charge, or else, e.g. $g^c \neq 0$, in which case a sum over the relevant indices is intended, e.g. in Eq.(3).

6) A.O. Barut and G.L. Bornzin, Nucl. Phys. B81, 477 (1974).

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8) G. Venturi, "Monopoles and the Higgs model", University of Bologna preprint, 1975.