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NEW RELATIONS FOR TWO-BODY REACTIONS FROM INCLUSIVE FINITE-MASS SUM RULES

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Abstract: Finite-mass sum rule and duality considerations for the inclusive process $ab \to cX$ lead to a number of interesting relations amongst the quasi-two-body cross sections $ab \to cd_i$, where d_i denotes the prominent resonances in the missing-mass channel. The processes $\pi(\bar{K})N \to NX$ are studied in detail, and the main results are the following:

(i) A superconvergence relation for reggeon (ρ)-particle (π) scattering gives a sum rule connecting the ρ-exchange cross sections $(d\sigma/dt)$ $[\pi^- p \to (\pi^\circ, \omega, A_2^0)n]$.

(ii) Semi local duality considerations in reggeon-particle scattering relate the resonance production cross section $(d\sigma/dt)$ $[\pi(\bar{K})N \to N d_i]$ to the resonance mass in terms of the trajec tory $\alpha(t)$ of the exchanged reggeon. From this the two following corollaries follow:

(iii) The ratio of π to (f-ω) exchange contributions increases linearly with the resonance (mass)^2.

(iv) The resonance production cross sections get less peripheral with increasing resonance mass.

All the above predictions agree reasonably with the available data on mesonic resonance production.

1. Introduction

The ordinary finite-energy sum rules for two-body scattering have been recently extended to the case of inclusive reactions [1], where they are referred to as finite-mass sum rules (FMSR). For the inclusive process $ab \to cX$, the FMSR relate an integral over the low missing-mass (small M^2) region to the triple-Regge limit (large M^2). Several applications [2, 3] of FMSR have been made in the study of inclusive reactions, in particular to estimate the triple-Regge couplings, of which there are not yet reliable estimates from direct fits to the missing-mass distributions.

It has not been generally recognised, however, that FMSR have strong predictive power in the context of quasi-two-body reactions. The present work is exclusively devoted to applications of FMSR to processes of this kind. The crucial hypothesis is

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the Harari-Freund two-component duality [4] which associates the background and resonance parts of the missing mass integral with the pomeron and Regge components of the triple-Regge term, respectively. Thus one gets a relation between a sum of resonance production cross sections on one hand and a triple-Regge term(s) on the other (i.e. eqs. (10) and (11) of sect. 2). It is possible to derive from this many useful relations amongst the resonance production cross sections even without knowing the triple-Regge couplings.

The main results we derive follow from two considerations of the FMSR, (a) superconvergence and (b) semi-local duality.

(a) It is possible to construct a combination of inclusive cross sections, corresponding to exchange of exotic quantum numbers, for which the triple-Regge terms vanish. In this case the corresponding FMSR reduces to a sum-rule involving several resonance production cross sections.

(b) Assuming the FMSR to hold semi-locally in the missing mass, M^2, means that the resonance production cross section is related, on the average, to the resonance mass M_R by the relation

$$\left\langle \frac{\mathrm{d}\sigma_i^R}{\mathrm{d}t} \right\rangle \sim \left\langle M_R^2 \right\rangle^{\alpha_M(o)-2\alpha_i(t)} ,$$

at fixed s and t. There are two immediate corollaries from this:-

(i) The ratio of π to f-ω exchange in resonance production is predicted to increase linearly with the resonance mass squared.

(ii) The resonance production cross section $\mathrm{d}\sigma/\mathrm{d}t$ is predicted to show an antishrinkage with increasing resonance mass [5].

It is worth emphasising that we are interested only in the low missing-mass end of the inclusive reaction. This is described by a reggeon-particle amplitude for which the application of two-component duality is believed to be unambiguous. The only exception may be with respect to pomeron-particle scattering [6] which we shall not consider here [7].

In sect. 2 we discuss kinematics and outline the essential steps in the derivation of the FMSR. Several applications to quasi-two-body cross sections are categorised in sect. 3 and compared with data on production of meson resonances, $\pi(\bar{\mathrm{K}})\mathrm{N}\to\mathrm{NX}$. In sect. 4 we discuss some of the practical aspects involved in extracting the reggeon-particle forward amplitude from experimental data. We have, of course, exhausted only a very small part of the quasi-two-body data. Many more phenomenological analyses can be made in this direction, using the existing resonance production data. A few of these are indicated in the concluding section.

2. Formalism

Consider the inclusive process $\mathrm{ab}\to\mathrm{cX}$ in the kinematic region (fig. 1a)

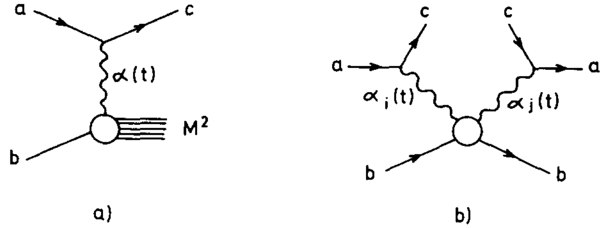

Fig. 1. (a) The inclusive process ab → cX. (b) The forward ab$\bar{c}$ → ab$\bar{c}$ amplitude for $s \gg t, s \gg M^2$.

$$s \gg t \ , \tag{1}$$

$$s \gg M^2 \ , \tag{2}$$

where s is the incident energy, t the momentum transfer between a and c and M is the missing mass. In this region the inclusive cross section is dominated by the leading Regge exchanges in the a$\bar{c}$ channel

$$\frac{d^2\sigma}{dM^2 dt} = \sum_{ij} \beta^i_{a\bar{c}} \beta^{j*}_{a\bar{c}} \xi_i \xi_j^* s^{\alpha_i(t)+\alpha_j(t)-2} \operatorname{Disc} f_{ij}(M^2, t) \ . \tag{3}$$

Here β stands for the reggeon coupling and ξ is its signature factor. According to the generalised optical theorem, the inclusive cross section is proportional to a discontinuity in M^2 of the forward ab$\bar{c}$ → ab$\bar{c}$ amplitude (fig. 1b). Hence $f_{ij}(M^2, t)$ represents the amplitude for reggeon-particle forward scattering.

The $M^2 \to \infty$ limit of $f_{ij}(M^2, t)$ is given by the triple-Regge formula:

$$\operatorname{Disc} f_{ij}(M^2, t) \xrightarrow[M^2 \to \infty]{} \sum_k g^k_{ij}(t) \beta^k_{b\bar{b}}(o) (M^2)^{\alpha_k(o)-\alpha_i(t)-\alpha_j(t)} \tag{4}$$

where $g^k_{ij}(t)$ is the triple-Regge coupling (fig. 2).

The analyticity properties in M^2 of $f_{ij}(M^2, t)$ have been investigated in perturbation theory [8] and in the dual resonance model [9]. Based on these results, one normally assumes that $f_{ij}(M^2, t)$ has the same analyticity structure as the elastic two-body amplitude, namely right- and left- hand cuts corresponding to the s-channel and the u-channel physical regions.

Knowing the analyticity properties and the high M^2 behaviour of $f_{ij}(M^2, t)$ the FMSR can be derived in the usual way [1]. It is convenient to use the variable

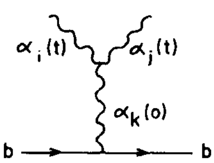

Fig. 2. Triple-Regge graph.

$$\nu = p_{\mathrm{b}} \cdot (p_{\mathrm{a}} - p_{\mathrm{c}}) = \tfrac{1}{2}(M^2 - t - m_{\mathrm{b}}^2) \ , \tag{5}$$

which is odd under $s \Leftrightarrow u$ crossing. Our analysis will be restricted to cases where there is no significant interference between different reggeons in the initial and final states – i.e. to amplitudes of the type $f_{ii}(M^2, t)$. If we write

$$A_{i\mathrm{b}}(\nu, t) \equiv \mathrm{Disc}\, f_{ii}(M^2, t) \ , \tag{6}$$

the u-channel discontinuity $A_{i\mathrm{b}}(-\nu, t)$ is given by

$$A_{\bar{i}\bar{\mathrm{b}}}(\nu, t) = A_{i\mathrm{b}}(-\nu, t) \ , \tag{7}$$

and can be obtained from the inclusive reaction $\mathrm{cb} \to \mathrm{aX}$ by factorisation as in (3). For a given exchange $i = j$ in (3) we now have the FMSR

$$\int_0^N \mathrm{d}\nu \nu^n \, [A_{i\mathrm{b}}(\nu, t) + (-1)^{n+1} A_{\bar{i}\bar{\mathrm{b}}}(\nu, t)] = \sum_\kappa (1 + (-1)^{n+1} \tau_\kappa) \times g_{ii}^\kappa(t) \beta_{\mathrm{b}\bar{\mathrm{b}}}^\kappa(0) \frac{N^{\alpha_\kappa(0) - 2\alpha_i(t) + n + 1}}{\alpha_\kappa(0) - 2\alpha_i(t) + n + 1} \ , \tag{8}$$

$$\int_0^N \mathrm{d}\nu \nu^n \, [A_{i\mathrm{b}}(\nu, t) + (-1)^n A_{\bar{i}\bar{\mathrm{b}}}(\nu, t)] = \sum_\kappa (1 + (-1)^n \tau_\kappa) \times g_{ii}^\kappa(t) \beta_{\mathrm{b}\bar{\mathrm{b}}}^\beta(0) \frac{N^{\alpha_\kappa(0) - 2\alpha_i(t) + n + 1}}{\alpha_\kappa(0) - 2\alpha_i(t) + n + 1} + R_{ii}^{(n)}(t) \ . \tag{9}$$

In (8) and (9), τ_κ is the signature of the trajectory κ and $R_{ii}^{(n)}(t)$ is the residue of the nonsense wrong signature fixed pole in the reggeon-particle amplitude. Following the two-component duality hypothesis [4], we shall assume (8) and (9) to be satisfied separately for the resonance (R) and background (B) components of $A_{i\mathrm{b}}$, with κ on the right corresponding to pomeron (P) and the leading meson Regge (M) exchanges, respectively. In terms of the inclusive cross sections of eq. (3) then, the FMSR are

$$\int_0^N \mathrm{d}\nu \nu^n \left[\frac{\mathrm{d}\sigma_i^{\mathrm{B,R}}}{\mathrm{d}M^2 \mathrm{d}t} + (-1)^{n+1} \frac{\mathrm{d}\bar{\sigma}_i^{\mathrm{B,R}}}{\mathrm{d}M^2 \mathrm{d}t} \right] = |\beta_{\mathrm{a}\bar{\mathrm{c}}}^i(t)|^2 |\xi_i(t)|^2 s^{2\alpha_i(t) - 2} \times [1 + (-1)^{n+1} \tau_{\mathrm{P,M}}] \, g_{ii}^{\mathrm{P,M}}(t) \beta_{\mathrm{b}\bar{\mathrm{b}}}^{\mathrm{P,M}}(0) \frac{N^{\alpha_{\mathrm{P,M}}(0) - 2\alpha_i(t) + n + 1}}{\alpha_{\mathrm{P,M}}(0) - 2\alpha_i(t) + n + 1} \ , \tag{10}$$

$$\int_0^N d\nu\nu^n \left[\frac{d\sigma_i^{B,R}}{dM^2dt} + (-1)^n \frac{d\bar{\sigma}_i^{B,R}}{dM^2dt}\right] = |\beta^i_{a\bar{c}}(t)|^2 |\xi_i(t)|^2 s^{2\alpha_i(t)-2}$$

$$\times \left[(1+(-1)^n \tau_{P,M}) g_{ii}^{P,M}(t) \beta_{b\bar{b}}^{P,M}(0) \frac{N^{\alpha_{P,M}(0)-2\alpha_i(t)+n+1}}{\alpha_{P,M}(0)-2\alpha_i(t)+n+1} + R_{ii}^{(n)B,R}(t)\right] , \tag{11}$$

where $(d\sigma_i/dM^2dt)$ $((d\bar{\sigma}_i/dM^2dt))$ denotes the i exchange contribution to the inclusive cross section for $ab \to cX$ $(cb \to aX)$. The M on the right corresponds to a summation over the exchange degenerate $\rho-\omega-f-A_2$ contributions.

3. Applications

We shall apply the resonance component of the FMSR (10) and (11) to the mesonic resonance production cross sections in $\pi N \to XN$ and $\bar{K}N \to XN$. The prominent resonances in the missing-mass channel X are $\pi, \rho, \omega, f, A_1, A_2, g$ for the π-beam and $\bar{K}$, $\bar{K}^*$, $\bar{Q}$, $\bar{K}^{**}$ for the $\bar{K}$ beam. The leading trajectories (i) in the $N\bar{N}$ channel are P, f, ω, ρ, A_2, π and presumably B.

There are certain problems involved in extracting the reggeon-particle amplitude $A_{ib}(M^2, t)$ from experimental data. The most important of these is to isolate the diffractive (P-exchange) part of the resonance production cross sections from the non-diffractive part, since we have assumed Harari-Freund duality only for the latter. Apart from the elastic cross sections, P-exchange can contribute to $\bar{K}^*$, $\bar{K}^{**}$, $\bar{Q}$, A_1 and A_2 production. The non-diffractive part of the elastic cross sections has been taken from the existing Regge fits, based on energy dependence, cross-over etc. As a first approximation we have assumed $\bar{K}^*$, $\bar{K}^{**}$ and A_2 to be purely non-diffractive on the basis of the Gribov-Morrison rule [10]. The recent Serpukhov data [11] on A_2 production seems to cast some doubt on this. However, we believe most of our results do not depend crucially on this assumption.

The A_1 and Q mesons are dominantly diffractively produced. We have estimated the amount of non-diffractive exchanges from the cross-over effect in the t-distributions [12].

The applications of FMSR to resonance production processes considered here fall into two broad categories: superconvergence relations and semi-local duality.

3.1. Superconvergence

It is sometimes possible to construct a linear combination of elastic reggeon-particle amplitudes which corresponds to exotic quantum number exchange in the $b\bar{b}$ channel. In this case the triple-Regge contribution on the right-hand side of eq. (10) vanishes. The background contribution to the missing-mass integral also vanishes by Harari-Freund duality. Thus we would get a sum rule involving the production cross sections for the resonances in the missing-mass channel, which is expected to hold over a significant t-interval.

One application of this kind was recently made [13] for a combination of $\rho \pm \mathrm{p}$ scattering amplitudes (in $\pi^{\pm}\mathrm{p} \to \pi^{0}\mathrm{X}$) corresponding to isospin 1 in the $\mathrm{p\bar{p}}$ channel. Evidently, this amplitude is not superconvergent, and one had to rely on the smallness of the non-flip $\rho\mathrm{p\bar{p}}$ coupling to ignore the triple-Regge term in eq. (10). The phenomenological success of the resulting sum rule was, nonetheless, very encouraging.

A genuine super convergent combination can be formed from the $\rho\pi$ amplitudes which corresponds to isospin 2 in the $\pi\pi$ channel. This is the combination

$$A_{\rho^-\pi^+} + A_{\rho^-\pi^+} - 2A_{\rho^0\pi^+} \ . \tag{12}$$

The prominent resonances in the $\rho\pi$ channels are π, ω and A_2. There is very little A_1 in the charge-exchange production mode.

Superconvergence relations can be written down for both the odd and even crossing sum rules and we discuss each in turn.

(i) *The odd crossing superconvergence relation.* Substituting $\mathrm{N\bar{N}}$ or $\mathrm{N\bar{\Delta}}$ for the $\mathrm{a\bar{c}}$ channel and using some isospin relations, eq. (10) reads for $n = 1$,

$$(-t)\ \frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \pi^0\mathrm{n}) - (m_\omega^2 - t - m_\pi^2)\ \frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \omega\mathrm{n})$$
$$+ (m_{\mathrm{A}_2}^2 - t - m_\pi^2)\ \frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \mathrm{A}_2^0\mathrm{n}) = 0 \ , \tag{13}$$

with an identical relation for the $\pi^+\mathrm{p} \to \pi^0\Delta^{++}$, $\pi^+\mathrm{p} \to \omega\Delta^{++}$ and $\pi^+\mathrm{p} \to \mathrm{A}_2^0\,\Delta^{++}$ cross sections.

The ρ-exchange is known to dominate the $\pi^-\mathrm{p} \to \pi^0\mathrm{n}$ and $\pi^+\mathrm{p} \to \pi^0\Delta^{++}$ cross sections. For the ω-production cases, we assume $\mathrm{d}\sigma_\rho/\mathrm{d}t$ to be given by $(\rho_{11} + \rho_{1-1})$ $\mathrm{d}\sigma/\mathrm{d}t$. This is supported partly by the energy dependence and partly by a sharp dip in the forward direction [14, 15], indicating small-cut effects*. For A_2^0 production we have no direct estimate of the ρ-exchange contribution. Evaluation of the density-matrix elements is complicated by the presence of s-wave background. There is strong indication of a large contribution to charge-exchange A_2 production from lower lying (e.g. B) singularities since the cross section falls like p_{lab}^{-2} in the 4–8 GeV/c range [16]. However, it is possible to estimate the ρ-exchange contribution in $\pi^-\mathrm{p} \to \mathrm{A}_2^0\mathrm{n}$ from the $\pi^-\mathrm{p} \to \mathrm{A}_2^-\mathrm{p}$ and elastic $\pi^-\mathrm{p}$ data, by using exchange degeneracy between the f-and ρ-coupling to $\pi\mathrm{A}_2$. Both the density matrix measurements and the energy dependence of $(\mathrm{d}\sigma/\mathrm{d}t)\,(\pi^-\mathrm{p} \to \mathrm{A}_2^-\mathrm{p})$ suggest this cross section to be dominated by f, ρ-exchange in the $5 \to 15$ GeV range.

Exchange degeneracy then gives

$$\frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \mathrm{A}_2^0\mathrm{n}) \Big/ \frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \pi^0\mathrm{n}) = \frac{\mathrm{d}\sigma_{\mathrm{f},\rho}}{\mathrm{d}t}(\pi^-\mathrm{p} \to \mathrm{A}_2^-\mathrm{p}) \Big/ \frac{\mathrm{d}\sigma_{\mathrm{f},\rho}}{\mathrm{d}t}(\pi^-\mathrm{p} \to \pi^-\mathrm{p}) \ . \tag{14}$$

* There is, however, no evidence for a wrong signature dip in $\pi^-\mathrm{p} \to \omega\mathrm{n}$ (ref. [14]) although there is some evidence for such a dip in $\pi^+\mathrm{p} \to \omega\Delta^{++}$ (ref. [15]).

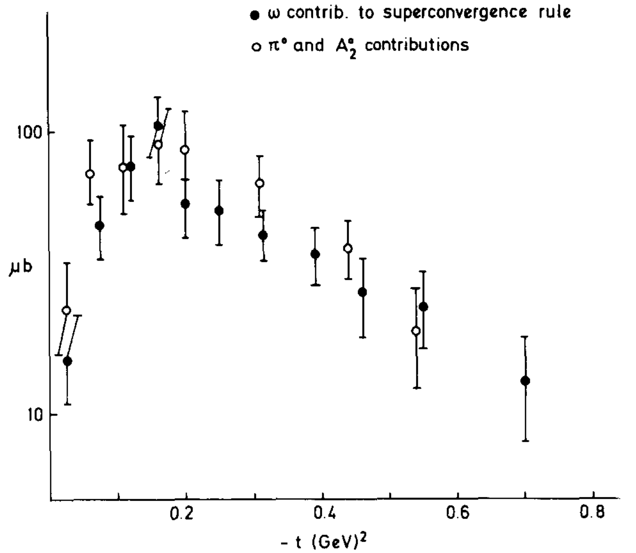

Fig. 3. Experimental values of contributions to the superconvergence relation (15), evaluated from the data of refs. [14, 17, 20] using the method described in subsect. 3.1.

We have estimated $(d\sigma_\rho/dt)$ $(\pi^- p \to A_2^0 n)$ from the data on the differential cross section and density-matrix elements of $\pi^- p \to A_2^- p$ at 7.5 GeV/c (ref. [17]) using the Barger-Phillips [18] estimate of f-and ρ-exchange in πp scattering.

The resulting contributions to the sum rule (13) are shown in fig. 3. The agreement is remarkable, considering the various approximations involved.

We should point out that the $(d\sigma_\rho/dt)$ $(\pi^- p \to A_2^0 n)$ calculated from the exchange degeneracy relation (14) or alternatively from our sum rule (13) turns out to be only $\sim$ 30% of the net $d\sigma/dt$ at 7.5 GeV/c. This ties in with the rapid fall off of $d\sigma/dt$ in this energy range. It may appear a very surprising result nonetheless, since ρ-exchange seems to account for $\sim$ 60% of the ω-production cross section [14] at this energy. However, we shall see from the semi-local duality considerations in subsect. 3.2. that the ratio of the contribution from B-exchange to ρ-exchange is, indeed, expected to increase by a factor ~ 3 $(= M_{A_2}^2/M_\omega^2)$ in going from ω to A_2^0 production. Hence we predict that the quantity $(\rho_{11} + \rho_{1-1})$ for A_2^0 production at 7 GeV/c will be much smaller than 1. Because of this dominance by non-leading Regge exchange at 7 GeV/c, the cross section at higher energies will continue to decrease faster than would be expected from pure ρ-exchange.

(ii) *The even crossing superconvergence relation – fixed poles and Regge cuts.* In contrast to the situation in particle-particle scattering, the fixed-pole residues for Reggeon-particle scattering have important physical significance. In the Gribov calculus [19] it is these quantities which govern the size of the Regge cuts for two-body scattering. From eq. (11) it can be seen that the evaluation of the even-crossing sum rule provides a method of estimating the fixed-pole residue, $R_{ii}^{(0)}(t)$. This is then related to the ii Regge-cut contribution for the forward bb scattering amplitude by

$$F^{\mathrm{cut}}(0) = \frac{i}{\pi^2 q} \int \mathrm{d}t \, [R_{ii}^{(0)}(t)\, s^{\alpha_i(t)-\frac{1}{2}}\, \xi_i(t)]^2 \ . \tag{15}$$

There have been some attempts to estimate the fixed-pole residues for ρ-p (ref. [3]) and pomeron-p (ref. [2]) scattering using the $n = 0$ sum rule with $I = 0$ in the $\mathrm{p\bar{p}}$ channel. These evaluations suffered from the large uncertainty in the relevant triple-Regge contributions, consequently no quantitative estimate could be given for the $\rho\rho$ or PP cuts in pp scattering.

The uncertainty of the triple-Regge couplings is, of course, removed if we choose a superconvergent sum rule – i.e. one where $\mathrm{b\bar{b}}$ is an exotic channel. Furthermore, it is of added phenomenological interest to calculate the size of the Regge-Regge cut for two-body scattering in the case where the t-channel is exotic.

We again consider the same combination (12) for ρ-π scattering, to ensure $I = 2$ in the $\pi\pi \to \rho\rho$ channel. The $n = 0$ sum rule then gives

$$\frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \pi^0\mathrm{n}) - \frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \omega\mathrm{n}) + \frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \mathrm{A}_2^0\mathrm{n})$$

$$= 2\, s^{2\alpha_\rho(t)-2} |\beta^\rho_{\mathrm{p\bar{n}}}(t)|^2 |\xi_\rho(t)|^2 R^{(0)}_{\rho\rho}(t) \ . \tag{16}$$

For the π^0 and ω production, we again use the 7 GeV/c data [14, 20] used in the superconvergent relation (13) while using that relation itself to estimate the A_2^0 cross section.

In fact we can eliminate $(\mathrm{d}\sigma_\rho/\mathrm{d}t)\,(\pi^-\mathrm{p} \to \mathrm{A}_2^0\,\mathrm{n})$ from (13) and (16) and express the fixed pole residue in terms of the $\rho\pi\pi$ vertex, $\beta^\rho_{\pi^+\pi^0}(t)$, which is well determined

$$R^{I_t=2}_{\rho\rho}(t) = \tfrac{1}{2}\pi|\beta^\rho_{\pi^+\pi^0}(t)|^2 \left[(m^2_{\mathrm{A}_2} - m^2_\pi)\frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \pi^0\mathrm{n}) - (m^2_{\mathrm{A}_2} - m^2_\omega)\frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \omega\mathrm{n})\right]$$

$$\times \left[(m^2_{\mathrm{A}_2} - t - m^2_\pi)\frac{\mathrm{d}\sigma_\rho}{\mathrm{d}t}(\pi^-\mathrm{p} \to \pi^0\mathrm{n})\right]^{-1} \ . \tag{17}$$

An exactly similar expression holds for $\pi^+\mathrm{p} \to (\pi^0, \omega, \mathrm{A}_2^0)\,\Delta^{++}$ (refs. [15, 20, 21]). In fig. 4 we show the resulting values of $R^{I_t=2}_{\rho\rho}(t)$ determined from the two sets of reactions. Using these values we can then proceed to evaluate the $I_t = 2$ $\rho\rho$ Regge cut in forward $\pi\pi$ scattering, giving

$$\sigma^{I_t=2}_{\mathrm{TOT}} \sim 40\ \mu\mathrm{b} \ , \qquad \left.\frac{\mathrm{d}\sigma}{\mathrm{d}t}^{I_t=2}\right|_{t=0} \sim 1.7\ \mu\mathrm{b/GeV}^2 \ ,$$

at $s = 10\ \mathrm{GeV}^2$.

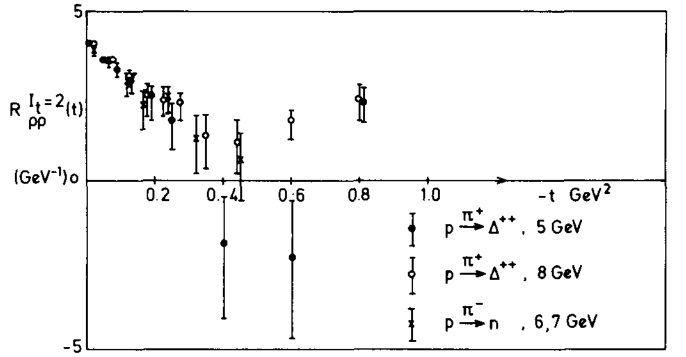

Fig. 4. Evaluation of the fixed-pole residue $R_{\rho\rho}^{I_t=2}(t)$ from the data of refs. [14, 15, 17, 20, 21].

It is of interest to compare the evaluation of this cut contribution with that derived from the eikonal model, where the fixed-pole residue is replaced by just the π^0 contribution. This corresponds to taking only the first term on the left-hand-side of (16). In the $n = 1$ sum rule, (13), the π^0 cross section is suppressed because of the damping factor t, which implies $\mathrm{d}\sigma_\rho\,(\mathrm{A}_2^0) \sim \frac{1}{3}\,\mathrm{d}\sigma_\rho\,(\omega)$. Experimentally, $\mathrm{d}\sigma_\rho\,(\pi^0)$ is roughly twice $\mathrm{d}\sigma_\rho\,(\omega)$ so that the three terms on the left-hand-side of (16) are roughly $2 : -1 : \frac{1}{3}$. Thus the sum $R_{\rho\rho}\,(t)$ is $\sim \frac{2}{3}\,\mathrm{d}\sigma_\rho\,(\pi^0)$ which means that the resulting amplitude F^{cut} is roughly half of the eikonal model estimate. Note that the rapid convergence of the series is reassuring both for the superconvergence hypothesis and for our practical calculation, where we have neglected the higher resonance contributions to the sum.

3.2. Semilocal duality

Perhaps the most interesting results of the scheme follow from semilocal duality considerations. Semilocal duality applied to reggeon-particle scattering suggests the leading Regge exchange (i.e. M in eq. (10)) to interpolate the resonance contributions in an average sense. In analogy with particle-particle scattering, this interpolation is expected to hold down to low-mass resonances, if the resonance contributions are dominantly additive and there are no fixed-pole contributions. The averaging is to be done on resonance contributions over a typical range of 1 GeV^2. In terms of the resonance production cross sections of eq. (10), this means that in an average sense

$$\left\langle \frac{\mathrm{d}\sigma_i^{\mathrm{R}}}{\mathrm{d}t} \right\rangle \sim \nu_{\mathrm{R}}^{\alpha_{\mathrm{M}}(0) - 2\alpha_i(t)} \; . \tag{18}$$

Therefore at fixed s and t, we know how the average resonance production cross section behaves as a function of the resonance mass, once we know the exchanged trajectory α_i.

Eq. (18) has two important, though rather obvious, corrollaries. Firstly the ratio of π, B to vector, tensor exchange contributions is expected to increase linearly with the resonance mass squared, i.e. $\sim \nu^{2(\alpha_{\mathrm{M}}(t) - \alpha_\pi(t))}$. Secondly, any given Regge-exchange contribution $d\sigma_i^R/dt$ is expected to show a logarithmic anti-shrinkage with increasing resonance mass squared, at fixed energy.

We present below a detailed comparison of eq. (18), and the two corrollaries mentioned above, for mesonic resonance production data involving π- and f, ω exchange. The trajectory parameters were taken to be

$$\alpha_{f,\omega}(t) = 0.5 + t\,, \qquad \alpha_\pi(t) = -0.02 + t\,. \tag{19}$$

(i) *The π-exchange.* Consider the π-exchange contribution to the reactions $K^-p \to nX$ and $\pi^-p \to nX$.

The relevant resonances in the missing-mass channel are (K^*, K^{**}) and (ρ, f, g). Moreover, π-exchange is known to dominate these production cross sections at small $|t|$ both from energy dependence and density-matrix measurements (dominance of ρ_{00} suggests small cut effects). The $K^-\pi^-$ and $\pi^-\pi^-$ channels being exotic, the fixed poles vanish, i.e. eq. (10) can be written for both odd and even n as

$$\int_0^N d\nu\nu^n \frac{d\sigma_\pi^R}{dM^2dt}\,[\pi^-(K^-)p \to nX] = s^{2\alpha_\pi(t)-2} f_\pi(t) N^{0.5-2\alpha_\pi(t)+n+1}\,. \tag{20}$$

Semilocal duality (i.e. eq. (18)), requires the Regge term on the right to average resonance contributions over bins of 1 GeV^2 in M^2. Since the resonances here are 1 GeV^2 apart, a single resonance is expected to average the Regge contribution in a given bin. The experimental data for $(d\sigma/dt)$ ($K^-p \to (K^*, K^{**})n$), multiplied by the density-matrix element ρ_{00}, are compared with the Regge contribution in fig. 5, over two bins defined by the mid points of the K^*-K^{**} and K^{**}-K^{***} mass squares. This is done separately for $n = 0$ and 1 in (a) and (b). Similarly the experimental values of ρ_{00} $(d\sigma/dt)$ ($\pi^-p \to (\rho, f, g)n$) are compared with the Regge contribution in figs. 6a and b over three bins, defined in an analogous fashion. The above figures suggest that semilocal duality works very well here even down to quite low masses. Moreover, the size of the integral ($\sim f_\pi(t)$) in the two cases agrees well with SU (3).

(ii) *The f, ω exchange.* Consider the f-ω exchange contributions to the reactions $K^-p \to X^-p$ and $\pi^-p \to X^-p$. The resonances in the missing mass are (K, K^*, Q, K^{**}) and (π, ρ, A_1, A_2, g), respectively.

The f-ω contributions to the elastic cross sections are taken from standard Regge fits [18]. Those for K^*, K^{**} and ρ, A_2, g production are obtained using an isospin separation

$$\begin{aligned} \frac{d\sigma_{\omega,f}}{dt} &= \frac{d\sigma}{dt}(K^-p \to (K^{*-}, K^{**-})p) - \tfrac{1}{4}\frac{d\sigma}{dt}(K^-p \to (K^{*0}, K^{**0})n)\,, \\ \frac{d\sigma_{\omega,f}}{dt} &= \frac{d\sigma}{dt}(\pi^-p \to (\bar\rho, A_2^-, g^-)p) - \tfrac{1}{2}\frac{d\sigma}{dt}(\pi^-p \to (\rho^0, A_2^0, g^0)n)\,. \end{aligned} \tag{21}$$

This assumes that the isospin 0 exchange is dominantly f, ω and that there is negligible interference between the isospin 0 and 1 exchanges. The latter assumption has only been checked for ρ-production, but it is certainly a reasonable assumption for small

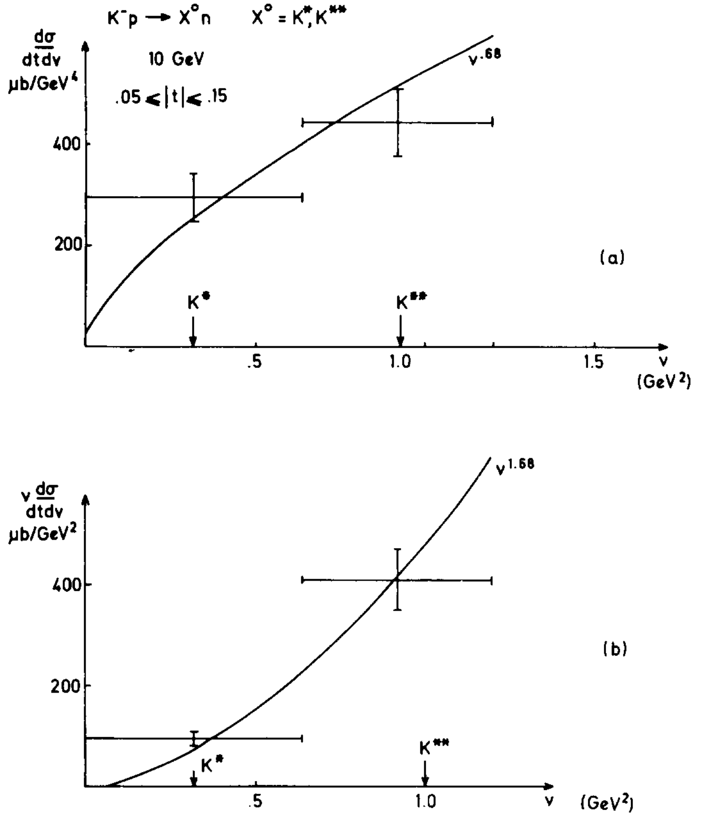

Fig. 5. Experimental ν^n $(\mathrm{d}\sigma_\pi/\mathrm{d}t)$ $(\mathrm{K}^-\mathrm{p}\rightarrow \mathrm{K}^*, \mathrm{K}^{**})\mathrm{n})$ at 10 GeV/c for $0.05 \leqslant |t| \leqslant 0.15$ compared with the π-exchange Regge contribution. (a) $n = 0$, (b) $n = 1$. Data from ref. [22].

t-values. The f- and ω-residues on the right of eq. (10) are equal by exchange degeneracy. The signature factors will also be equal at small-t values. Thus for small-t values one can treat them together as a single exchange, giving

$$\int_0^N \mathrm{d}\nu\nu^n \frac{\mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{f},\omega}}{\mathrm{d}M^2\mathrm{d}t}(\pi^-(\mathrm{K}^-)\mathrm{p}\rightarrow \mathrm{Xp}) = s^{2\alpha_{\mathrm{f},\omega}(t)-2} f_{f,\omega}(t)$$

$$\times N^{0.5-2\alpha_{\mathrm{f},\omega}(t)+n+1} \,. \tag{22}$$

In contrast to (20), eq. (22) is valid only for odd n, since σ and $\bar{\sigma}$ here are identical. The resonance contributions in the $\mathrm{K}^-\mathrm{p}$ reaction are compared with the Regge contribution in fig. 7 over the two mass bins defined earlier. The K^* bin contains the non-diffractive K-contribution as well. The cross-over observed in Q, $\bar{\mathrm{Q}}$ production [12] allows one to estimate the non-diffractive component which turns out to be

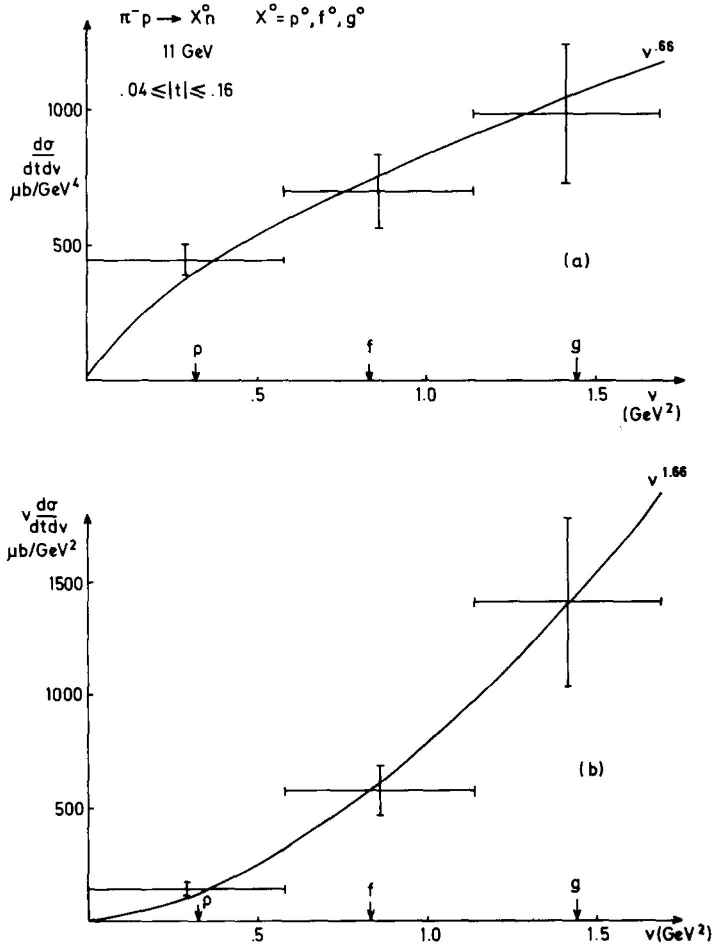

Fig. 6. Experimental ν^n $(d\sigma_\pi/dt)$ $(\pi^- \to (\rho^0, f^0, g^0)n)$ at 11 GeV/c for $0.04 \leq |t| \leq 0.16$ compared with the π-exchange Regge contribution. (a) $n = 0$, (b) $n = 1$. Data from ref. [24].

roughly 10% of all Q-production. In fig. 7, this contribution has been added to the K^{**} bin. Similarly the resonance contributions in the π^-p reaction are compared with the Regge interpolation in fig. 8 over the three mass bins defined earlier. A 10% non-diffractive component in the A_1 cross section was found to make only a small difference.

The agreement is quantitative in the K^-p reaction. However, for the π^-p case, the g-contribution falls short of the Regge interpolation, indicating an effective $\alpha_M(0) \sim 0.2$. We do not know, if this is a genuine effect of the lower-lying trajectories in the $b\bar{b}$ channel (which often tend to lower the effective α in the low-energy region), or is simply due to the uncertainty in evaluating $(d\sigma_\omega/dt)$ $(\pi^-p \to g^-p)$, as a difference of

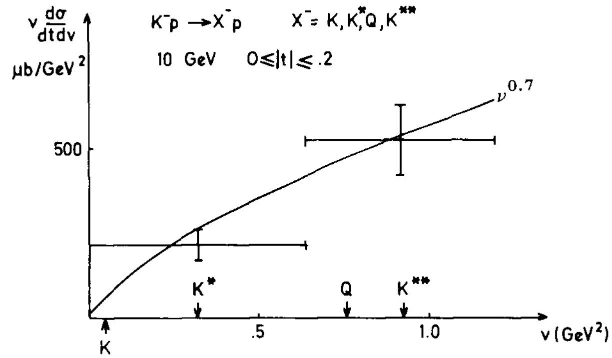

Fig. 7. f-ω exchange contributions to $\nu(\mathrm{d}\sigma/\mathrm{d}t)$ $(\mathrm{K}^-\mathrm{p} \to (\mathrm{K}, \mathrm{K}^*, \mathrm{Q}, \mathrm{K}^{**})\mathrm{p})$ at 10 GeV/c for $0 \leqslant |t| \leqslant 0.2$ compared with the f-ω exchange Regge contribution. Data from ref. [22].

two large experimental quantities in eq. (21). Moreover, an incident momentum of 8 GeV/c, above which we do not have simultaneous g^- and g^0 production data, may not be asymptotic for g-production. In fact, the energy dependence of the g^0 cross section between 7 and 11 GeV/c is slower than expected from π-exchange.

(iii) *Ratio of π to f-ω exchange.* From eq. (18), (20) and (22), one immediately gets, for fixed s and t

$$\left\langle \nu \frac{\mathrm{d}\sigma_\pi^\mathrm{R}}{\mathrm{d}t} (\pi^- (\mathrm{K}^-)\mathrm{p} \to \mathrm{nX}) \Big/ \frac{\nu \,\mathrm{d}\sigma_{\mathrm{f},\omega}^\mathrm{R}}{\mathrm{d}t} (\pi^- (\mathrm{K}^-)\mathrm{p} \to \mathrm{pX}) \right\rangle \sim (\nu_\mathrm{R})^{2\alpha_{\mathrm{f},\omega}(t) - 2\alpha_\pi(t)} , \tag{23}$$

i.e. the ratio of π to $\mathrm{f} - \omega$ exchange cross sections, averaged over the same resonance mass interval, should increase linearly with the resonance mass squared.

Fig. 9 shows a test of the above prediction for the 10 GeV/c $\mathrm{K}^-\mathrm{p}$ reaction. The first bin represents the ratio

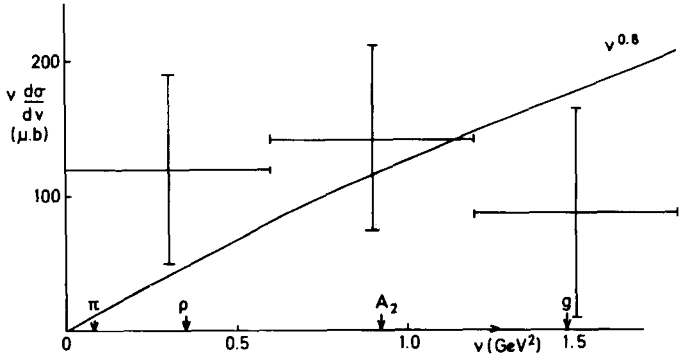

Fig. 8. f-ω exchange contributions to $\nu(\mathrm{d}\sigma/\mathrm{d}t)$ $(\pi^-\mathrm{p} \to (\pi, \rho, \mathrm{A}_2, \mathrm{g})\mathrm{p})$ at 8 GeV/c for $0.05 \leqslant |t| \leqslant 0.25$ compared with the f-ω exchange Regge contribution. Data from refs. [21, 25–27

$$\frac{d\sigma_\pi}{dt}(K^-p\to K^{*o}n)\Big/\left[\frac{d\sigma_{f,\omega}}{dt}(K^-p\to K^-p)+\frac{d\sigma_{f,\omega}}{dt}(K^-p\to K^{*-}p)\right] ,$$

and the second bin

$$\frac{d\sigma_\pi}{dt}(K^-p\to K^{**o}n)\Big/\left[\frac{d\sigma_{f,\omega}}{dt}(K^-p\to Q^-p)+\frac{d\sigma_{f,\omega}}{dt}(K^-p\to K^{**-}p)\right] .$$

In fig 10 we show a test of this ratio for the 8 GeV/c π^-p reaction. The three bins contain the ratios

$$\frac{d\sigma_\pi}{dt}(\pi^-p\to\rho^on)\Big/\left[\frac{d\sigma_f}{dt}(\pi^-p\to\pi^-p)+\frac{d\sigma_\omega}{dt}(\pi^-p\to\rho^-p)\right] ,$$

$$\frac{d\sigma_\pi}{dt}(\pi^-p\to fn)\Big/\frac{d\sigma_f}{dt}(\pi^-p\to A_2^-p) ,\quad \frac{d\sigma_\pi}{dt}(\pi^-p\to g^on)\Big/\frac{d\sigma_\pi}{dt}(\pi^-p\to g^-p) .$$

In each case the ratio does increase, and the rate is consistent with a linear dependence on the resonance mass squared.

Another example of this phenomenon is the increase of the ratio B/ρ exchange as we go from ω to A_2^0 production, as mentioned earlier. The increase of the π/f-ω exchange ratio, has been noted earlier as an experimental feature at least for the K$^-$p reaction [22]. It is clear that semilocal duality in the reggeon-particle amplitude provides a natural explanation of this phenomenon.

(iv) *Antishrinkage.* The semi-local duality relation (18) implies a logarithmic anti-

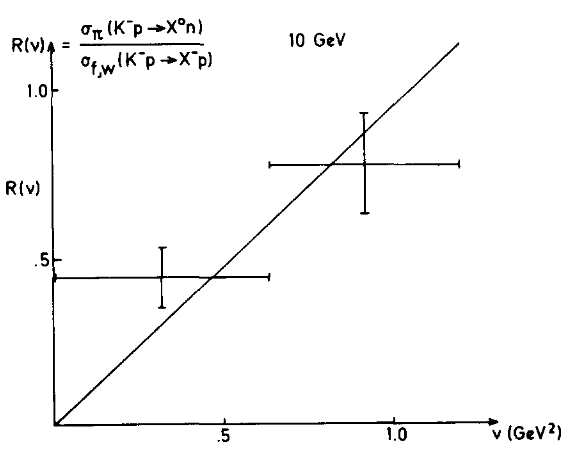

Fig. 9. Ratio of π/f-ω exchange to the reaction $K^-N\to XN$ at 10 GeV/c for $0\leq|t|\leq 0.2$. Data from ref. [22].

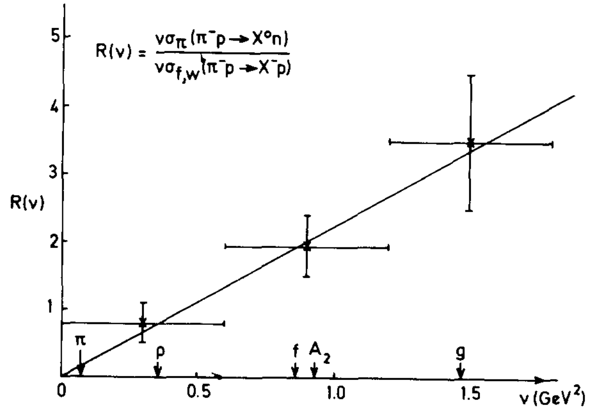

Fig. 10. Ratio of π/f-ω exchange to the reaction $\pi^- N \to XN$ at 8 GeV/c for $0.05 \leqslant |t| \leqslant 0.25$. Data from refs. [21, 25–27].

shrinkage of $d\sigma/dt$ as we go higher in the resonance mass at fixed incident momentum [15].

We shall compare the prediction of eq. (18) with f-ω exchange cross sections. There is no clear way to estimate the π-exchange contribution at larger $|t|$ values. For the f-ω exchange cross sections, we should have

$$\left\langle \nu \frac{d\sigma^R_{f,\omega}}{dt} (\pi^- (K^-)p \to pX) \right\rangle_{t_1} \Big/ \left\langle \nu \frac{d\sigma^R_{f,\omega}}{dt} (\pi^- (K^-)p \to pX) \right\rangle_{t_2}$$

$$\sim (\nu_R)^{-2\alpha'_{f,\omega}(t_1 - t_2)} \tag{24}$$

The above prediction is compared with the K^-p data in fig. 11 with $t_1 = -0.2$ and $t_2 = -0.6$ GeV^2. The two bins contain K^{*-} + non-diffractive K^- and K^{**-} + non diffractive Q^-, as before.

In the π^-p case the ω- and f-exchange resonances should be treated separately. since the two signature factors are not equal at larger $|t|$ values. The ω-exchange resonances are ρ and g while the f-exchange resonances are π and A_2. For the first case, data is restricted to the $|t| < 0.3$ GeV^2 region, due to the ambiguity in extracting $(d\sigma_\omega/dt)\,(\pi^- p \to g^- p)$. In the second case one has the cross sections over a wider t-range, but the π-mass is too small to test a semilocal duality relation quantitatively. Nonetheless, we have compared the resonance production data for these two cases with eq. (24) in figs. 12 and 13. The agreement with the antishrinkage prediction for both the K^-p and π^-p reactions seems very encouraging.

It should be stressed that the quantitative prediction (24) is expected to hold only for individual Regge exchanges. If there are several Regge exchanges in the $a\bar{c}$ channel, then the net resonance production cross section may show an anti-shrinkage pattern quantitatively different from (24).

Finally we note that the semilocal duality relation may work better for the ratios

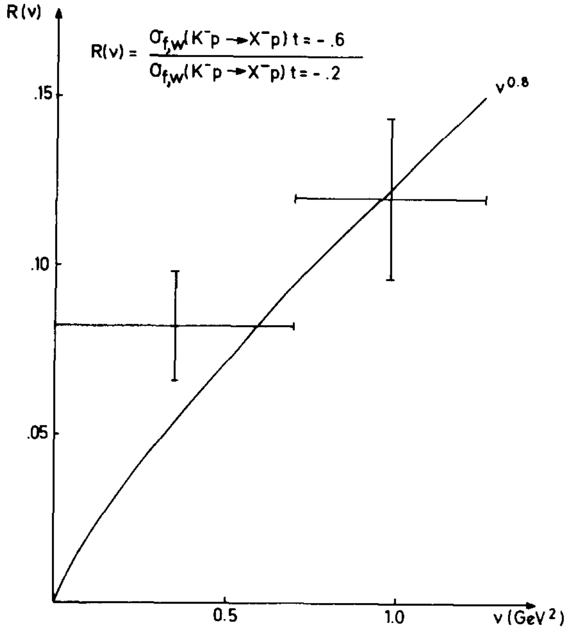

Fig. 11. Ratio of f-ω exchange contributions to the reaction $K^-p \to X^-p$ at 10 GeV/c at two values of t, -0.2 and -0.6 compared with the f-ω exchange Regge term. Data from ref. [22].

(i.e. eq. (23) and (24)), than for the individual exchanges of eq. (18). For instance, there could be a non-negligible contribution from a lower-lying trajectory in the

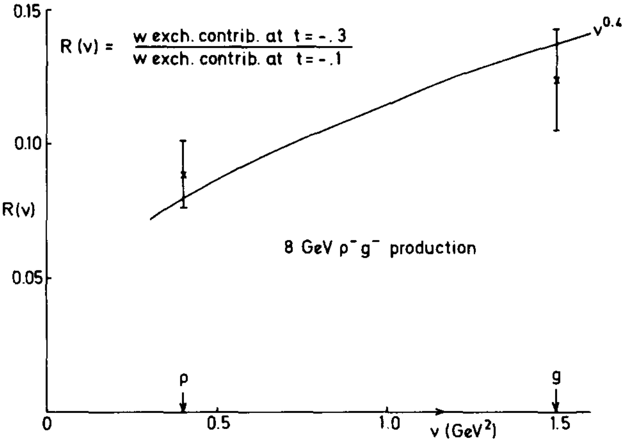

Fig. 12. Ratio of ω-exchange contributions at two values of t, -0.3 and -0.1 to the reactions $\pi N \to (\rho, g)N$. The ratio for ρ-production at 6 GeV of ref. [29] has been extrapolated to 8 GeV assuming Regge behaviour to compare with the g-production data of ref. [26].

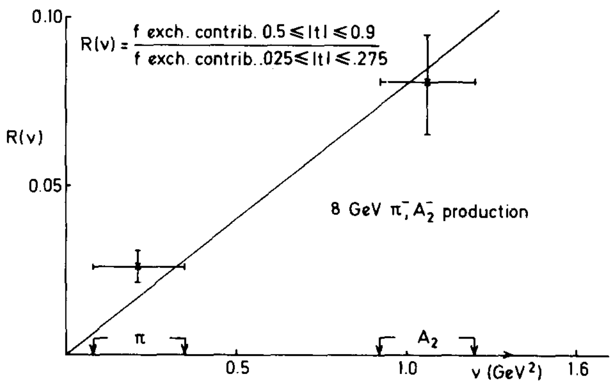

Fig. 13. Ratio of f-exchange contributions in two ranges of $|t|$ around 0.75 and 0.25 to the reactions $\pi N \rightarrow (\pi, A_2)N$ at 8 GeV/c. Data from ref. [21].

$b\bar{b}$ channel. As long as this contribution is additive, as in the intermediate energy region of two-particle reactions, its contributions to the numerator and the denominator cross sections of eq. (23) and (24) should be partly self-compensatory. In particular, if the effective $\alpha_M(0)$ for the two cross sections are lowered by the same amount, then eqs. (23) and (24) would not be affected.

4. Discussion

As we have seen, the FMSR provide several interesting relations between quasi-two-body reactions of the type $ab \rightarrow cd_i$. The experimental verification of these relations, on the other hand, lends support for the validity of the concept of duality for reggeon-particle amplitudes.

The use of quasi-two-body reactions for investigating duality in reggeon-particle scattering has several practical advantages, compared to a more inclusive approach using the entire reaction $ab \rightarrow cX$. We shall list some of them here.

(i) For most reactions $ab \rightarrow cd_i$, the production mechanism of the resonance d_i has already been extensively studied. From the energy dependence and the decay distributions we can separate out the specific trajectory whose contribution we want to investigate. We did not have to consider any interference terms. For the pairs π-B and ρ, ω-f, A_2 this follows from exchange degeneracy, whereas spin-parity constraints ensure no interference between the natural-and unnatural-parity exchanges in the unpolarized cross section.

Moreover, since we used the Harari-Freund two-component duality throughout for the reggeon-particle amplitude, we need never consider pomeron exchange in the $b\bar{b}$ channel. Altogether, this means that far fewer triple-Regge couplings are introduced into the FMSR than would be the case for the general inclusive reaction.

(ii) In many cases the reactions $ab \rightarrow cd_i$ can be studied experimentally, although the inclusive reaction ab → cX is difficult to observe for technical reasons. For example, there is no data on the reactions $\pi(\bar{K})p \to Xn$ for small-momentum transfers between the nucleons. However, the quasi-two-body reactions $\pi(\bar{K})\,p \to (\rho, f, g)$ $(K^*, K^{**})n$ have been studied in detail. The availability of a large number of different reactions makes a systematic study simpler for the quasi-two-body processes than for true inclusive reactions.

(iii) In all applications of the FMSR the condition $s \gg M^2$ must be satisfied in order to ensure that the cross section is proportional to a reggeon-particle amplitude. In our analysis M^2, being the $(\text{mass})^2$ of the first few resonances d_i, is generally rather small. Thus $s/M^2 \gg 1$ although the available data is at relatively low energies $(s \lessapprox 20\ \text{GeV}^2)$.

(iv) Finally the question of Regge cuts. The FMSR are valid only for factorizable pole exchanges. There is little point in trying to incorporate cuts into the formalism, as long as we have no reliable model for the cuts to start with. Thus the best thing one can do is to restrict oneself to cases where the cut contaminations are expected to be small from phenomenological analysis. The cuts in the $b\bar{b}$ channel are expected to be always small, in analogy with two-body forward elastic amplitudes. As to cuts in the $a\bar{c}$ channel, the situation is again best known for the quasi-two-body reactions. For the vector and tensor resonance productions there are, in fact, some phenomenological evidences to suggest that pole exchanges are dominating in the $a\bar{c}$ channel. The density matrix measurements show ρ_{00} and $(\rho_{11} + \rho_{1-1})$ dominance for the π-exchange and $f^-\ \omega$ exchange cross sections at small t.. All the cross sections show turnovers in the forward direction. Finally there is the well-known signature dip in $(d\sigma_\omega/dt)\ (\pi N \to \rho N)$ (ref. [29]).

5. Further possibilities

The aforesaid formalism is expected to be useful in a much wider context of two-body phenomenology. We wish to conclude by indicating some of these applications.

(a) *Resonance-background separation.* Like the resonance contributions, analysed here, the background contribution in the missing-mass channel is also constrained by eq. (10). The average background contribution $\langle d\sigma_i^B/dt \rangle$ is expected to grow like $(M^2)^{\alpha_P(0) - 2\alpha_i(t)}$. This constraint can be used in making a resonance-background separation in the large M^2 region, and also to study the duality properties of the broad daughter resonances, which are supposedly contained in the polynominal background.

We have looked at the background contributions in $K^-p \to Xn$ at 10 GeV/c and in $\pi^-p \to Xn$ at 11 GeV/c. The result is shown respectively in figs. 14 and 15. As a first approximation we have assumed that the background and the resonances have the same inelasticity and production mechanism. Figs. 14 and 15 show that the background behaving like $(M^2)^{1-2\alpha_\pi(t)}$ is consistent with the data. However, in the absence of data on the complete reaction $\pi(\bar{K})p \to Xn$ we are unable to draw any definite conclusions.

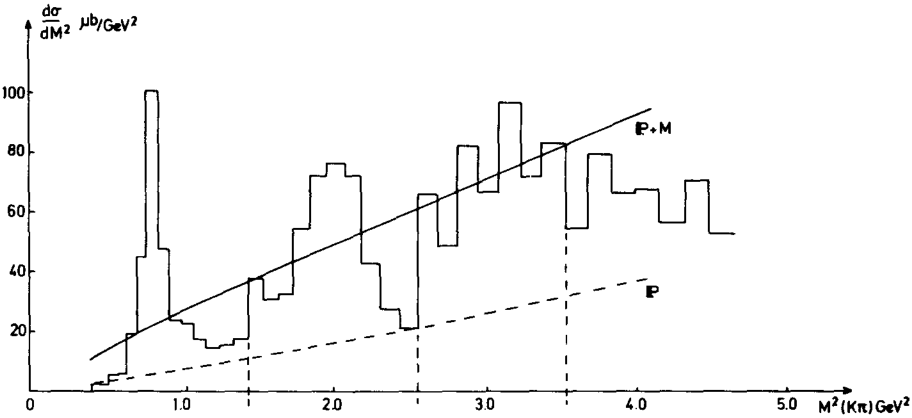

Fig. 14. M^2 distribution for $K^-p \to Xn$ at 10 GeV/c for $0.05 \leqslant |t| \leqslant 0.15$. The distribution has been obtained from the $M_{K^-\pi^+}$ distribution in $K^-p \to K^-\pi^+n$ (ref. [22]) by dividing the distribution into three bins (dashed vertical lines) and assuming the inelasticity in each bin to be that of the dominant resonance (K^*, K^{**} and K^{***}). The cross section has been multiplied by ρ_{00} to ensure π-exchange. The background fit, labelled by P, corresponds to a $(M^2)^{1-2\alpha_\pi(t)}$ behaviour.

This type of analysis should be very interesting for reactions like $\pi(\bar{K})p \to X\Lambda$, for which data on the inclusive M^2 distribution are available. The validity of the Harari-Freund hypothesis for reggeon-particle amplitudes could then be directly tested.

(b) *Mesons produced by baryon exchange.* The set of mesonic resonances considered occur also in the backward reactions $\pi(K)p \to pX$ and in the annihilation reactions $\bar{p}p \to \pi(K)X$. These reactions are dominated by baryon exchange, with $\alpha_B(0) \lessapprox -0.3$.

The resonance production cross section $\langle d\sigma^R/dt \rangle$ should then behave like $\nu_R^{\alpha_M(0)-2\alpha_B(t)}$.

We expect $\alpha_M(0) \simeq 0.5$ if these normal resonances are dual to normal Regge exchanges, and $\alpha_M(o) \ll 0.5$ if they are dual only to exotic exchanges [28]. The first alternative would suggest a much bigger g/ρ (or K^{**}/K^*) ratio in backward production or in annihilation, compared to the forward production case considered earlier. But this would not be the case if the second alternative were true. Thus a comparison of the forward and backward production cross sections of these meson resonances would help to clarify the duality situation in baryon-antibaryon scattering.

(c) *Baryon resonances.* The duality analysis should be extended to the case of baryon resonance production. On the one hand, one can study to what extent the non-diffractive resonances satisfy the semilocal duality constraint (20) and its two corollaries [30]. On the other hand, the diffractive resonances can presumably be isolated by considering $pp \to pX$ at ISR or Batavia energies. The behaviour of these diffractive resonances should help to answer the question of two-component duality for pomeron-particle scattering [7, 32].

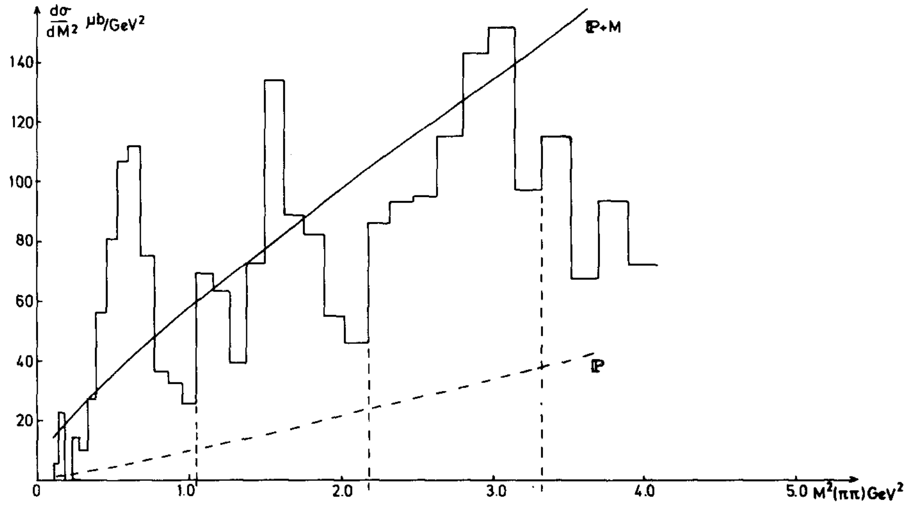

Fig. 15. M^2 distribution for $\pi^-p \to Xn$ at 11 GeV/c for $0.04 \leqslant |t| \leqslant 0.16$. The distribution has been obtained as for the K^-p reaction in fig. 14 from data on $\pi^-p \to \pi^-\pi^+n$ (ref. [24]). The curves labelled P (pomeron exchange and P + M (pomeron + meson exchange) have been drawn using the triple-Regge coupling constants determined from the fits of fig. 14 and values for $\beta_{\pi\pi}^{P,M}/\beta_{K\bar{K}}^{P,M}$ obtained from fits to total cross sections [31].

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