PATTERNS OF EXCHANGE DEGENERACIES[1] 5507

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CONTENTS

INTRODUCTION

Exchange degeneracies are relations between Regge trajectories associated with the absence of exchange forces. They arise naturally in potential scattering. In general, the potentials for positive and negative parity states are different and the Regge trajectories of the even and odd spin states are unrelated. However, if the exchange force vanishes, then the same potential acts on both parity states and the two sets of trajectories are equal, i.e., exchange degenerate.

The relativistic analogues of direct and exchange forces are the forces produced by the exchange of particles in the t and u channels. Exchange

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degeneracies between trajectories in relativistic theories are the result of the absence of forces in one of these crossed channels.

Internal symmetries introduce a new degree of freedom into exchange degeneracies. The complete absence of exchange forces results in exchange degeneracies between Regge trajectories with the same internal quantum numbers. However, if exchange forces are lacking only in selected representations, then trajectories with different internal quantum numbers will be exchange degenerate. These groupings are interesting because they correlate trajectories and particles in different multiplets.

This classificatory aspect of exchange degeneracy is the principal focus of our paper. The results we obtain are reminiscent of the predictions of higher symmetry schemes. Particles, and trajectories, in different multiplets are classified together, f/d and other coupling ratios are fixed, and representation mixing angles are obtained.

In actuality observations will of course always deviate from these exact results, and one is faced with the problem of describing these deviations, just as one must describe the perturbations about a higher symmetry limit. We devote an extensive discussion to the structure of these deviations.

We assume that the exchange forces are primarily due to single-particle exchange. The empirical absence of particles in certain representations gives the selective absence of exchange forces which underlies our exchange degeneracy patterns. In this context, these assumptions are called *duality*.

Duality was an extension of the finite-energy sum rules (1–4). Earliest applications of duality were primarily to mesonic representation mixing and exchange degeneracies of strange baryons (5–7). Difficulties in the application of duality to baryon-antibaryon scattering were pointed out by Rosner (8), who suggested the existence of mesons with exotic quantum numbers in order to resolve them. A convenient diagramatic visualization of duality was subsequently introduced (9, 10). The exchange degeneracies of meson and baryon trajectories were first displayed systematically by Capps, Rebbi, Slansky, and the authors (11–15).

It may be useful to point out that the view of duality taken here is quite different from that illustrated by the Veneziano model. There, zero-width resonance saturation is exact at all energies, and at any fixed momentum transfer, the leading Regge trajectory dominates the scattering at sufficiently high energies. With this stronger form of duality, exchange degeneracies are exact and each trajectory is accompanied by an infinite family of daughters. By contrast, we will emphasize the ranges of validity of nonexotic resonance saturation and Regge pole dominance in order to understand the deviations from exact exchange degeneracies.

The second section is a discussion of the duality concept and its applications. We give a precise definition of duality which incorporates the intrinsic limitations of its validity. The equations which express exchange degeneracies are derived and a model for estimating the deviations from exchange degeneracy is presented.

In this third section the exchange degeneracies are discussed. The patterns of meson and baryon trajectory multiplets are given along with the relations among their coupling constants. Relations between total cross sections which depend on the exchange degeneracies are derived and the difficulties associated with the application of duality to baryon-antibaryon channels are extensively discussed.

DISCUSSION

Duality is a hypothetical relation between direct-channel resonances and crossed-channel Regge poles. It is the assertion that the imaginary part of a scattering amplitude can be approximately calculated in two alternative ways: by summing all direct channel resonances, or by using the Regge parametrization. The contribution of a Regge trajectory to the imaginary part of the scattering amplitude is (16, 17):

$$\operatorname{Im} A(s, t) = \beta(t) s^{\alpha(t)} \qquad \textbf{1.}$$

where s is the center-of-mass energy squared and t the invariant momentum transfer squared; $\alpha(t)$ is the trajectory function, which passes through an integer when $t=m^2$, the mass squared of a *crossed-channel* resonance; and $\beta(t)$ is the residue function, which reduces to the coupling constant squared of the resonance at $t=m^2$.

More precisely, duality is the assumption that there exist amplitudes with the properties:

1. Their imaginary parts can be approximated by direct-channel resonance contributions, at least in the low-energy region.

2. At least at high energies, Regge pole exchange also describes their imaginary parts.

3. There is a region of energies and angles, called the *overlap region*, over which both these approximations may be simultaneously employed. Obviously, since resonant contributions are bumpy while a Regge pole gives a smooth energy dependence, these two descriptions can be equivalent only in some average sense.[5]
shown in Figure 1.

All amplitudes which do not have the quantum numbers of the vacuum in their crossed channels are candidates for amplitudes satisfying duality. Diffraction scattering precludes the possibility that amplitudes with no quantum numbers exchanged in their crossed channels could be dual. However, Freund & Harari (18, 19) have conjectured that even for these processes amplitudes lacking diffractive contributions can be constructed, and for these amplitudes there is a region over which the average of the resonances equals the Regge pole contribution.

Even in principle, scattering amplitudes can satisfy duality only approxi-

[5] The concept of average equivalence can be made more precise through the finite-energy sum rules (1–4).

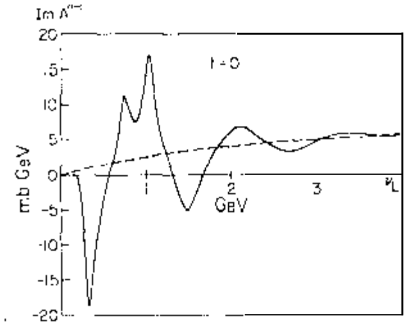

FIGURE 1. A dual amplitude—the $I_{(t)}=1$ spin nonflip amplitude for forward πN scattering. The solid curve is the measured amplitude; the dashed curve is the ρ trajectory fit of Arbab & Chiu (31). (See Dolen, Horn & Schmid 4)

mately: in the direct channel there are always nonresonant contributions and in the crossed channel the leading Regge trajectory never exactly describes the scattering.[6]

The duality between resonances and Regge poles is our primary dynamical assumption which we will use to relate different $SU(2)$ or $SU(3)$ multiplets of Regge trajectories. The general structure of the equations which express duality is

$$\left\langle \sum \begin{Bmatrix} \text{all resonances in } SU(2) \text{ or} \\ SU(3) \text{ representation } \mathbf{a} \end{Bmatrix} \right\rangle_{\text{average}} = \sum_b X^{ab} \sum \begin{Bmatrix} \text{Regge trajectories in } SU(2) \\ \text{or } SU(3) \text{ representation } \mathbf{b} \end{Bmatrix} \qquad 2.$$

where X is the crossing matrix which describes the direct-channel amplitudes resulting from the exchange of quantum numbers $\mathbf{b}$ in the crossed channel. This equality is valid only in the overlap region. The simplest way to use these equations would be to know all the contributions on one side of the equation, and in terms of them compute the other side. There are many channels, sometimes called *exotic*, which appear to contain no resonances at low energies. All known mesons seem to occur in either $SU(3)$ octets or singlets while baryons are found in singlets, octets, and decuplets.[7] (In the language of the quark model, all nonexotic mesons are in $q\bar{q}$ and all

[6] Since in reactions involving particles with spin, one can construct amplitudes which receive contributions from trajectories with only one natural parity ($P(-1)^J$) and PC, *by leading trajectory one means the highest trajectory with each natural parity and* PC.

[7] There are indications that strangeness $+1$ double-charged baryons may exist. If they exist, they are weakly coupled and quite massive so our results should be insensitive to their presence.

TABLE 1. Nonexotic resonant representations

$SU(3)$ representations	Strangeness	$SU(2)$ representations
Mesons: **1**, **8**	0 ±1	0, 1 1/2
Baryons: **1**, **8**, **10**	0 −1 −2 −3	1/2, 3/2 0, 1 1/2 0

nonexotic baryons in qqq.) Baryon number two channels are often considered exotic. There is very little experimental evidence about $B=2$ resonances however, and we will not use that assumption.[8] Exotic $SU(2)$ representations are those which occur *only* in exotic $SU(3)$ representations (see Table 1).

An illustration of the difference between exotic and nonexotic channels is provided by the contrast between K^+p and K^-p elastic scattering (Figure 2). In the nonexotic reaction K^-p, the total cross section is very bumpy, and on the average falls rapidly with increasing energy. On the other hand, in the exotic reaction K^+p, the total cross section is quite smooth and, except near threshold, shows very little energy variation. (The small ripples in K^+p scattering may be associated with the opening of new channels or

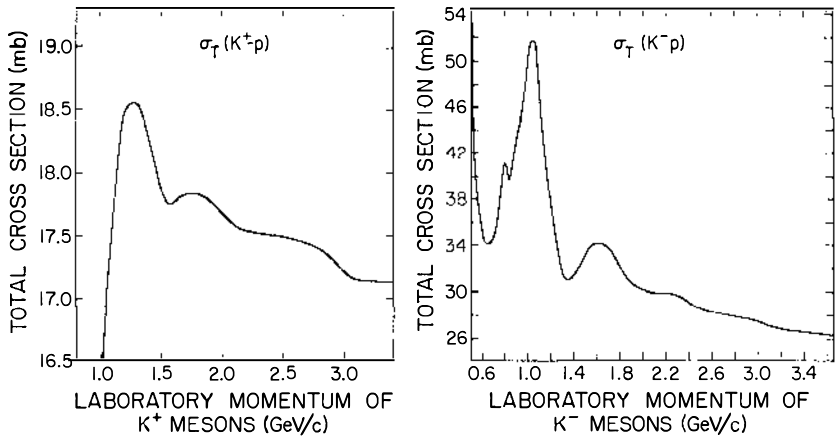

FIGURE 2. Comparison of exotic and nonexotic channels. K^+p is exotic while K^-p is not.

[8] The quark model might suggest that baryon number two resonances occur in all representations contained in 6q.

perhaps with weakly coupled resonances in exotic representations.) From the point of view of duality, the bulk of the K^+p total cross section results from diffraction scattering and is free from significant Regge pole terms, while in K^-p scattering there are substantial resonant contributions which on the average are equal to the crossed-channel Regge poles. Although we may describe the difference between K^+p and K^-p cross sections in terms of duality, this contrast cannot be taken as conclusive evidence for duality because diffraction scattering is poorly understood.[9]

The duality Equations 2 simplify dramatically when restricted to exotic direct channels. Then the resonance side vanishes leaving equations relating only Regge parameters.

By way of example, let us consider $\pi^+\pi^+$ elastic scattering. The direct channel is exotic, so by assumption the imaginary part of the Regge amplitude is zero. Since leading Regge trajectories with $I=0$ and 1 can contribute in the crossed channels, the duality equation will express a cancellation between these trajectories in the imaginary part of the direct-channel amplitude. Noting the form of a Regge pole contribution (Equation 1), we find that the duality equation is

$$X^{20}\beta^0(t)s^{\alpha_0(t)} + X^{21}\beta^1(t)s^{\alpha_1(t)} = 0 \qquad 3.$$

Since the cancellation must occur over a range of s (the overlap region), Equation 3 requires the two trajectories to be degenerate

$$\alpha_0(t) = \alpha_1(t) \qquad 4.$$

and gives a linear relation between the Regge residue functions

$$X^{20}\beta^0(t) + X^{21}\beta^1(t) = 0 \qquad 5.$$

An $I=0$ trajectory which couples to $\pi\pi$ will have particles with $J^{PC}=0^{++}$, 2^{++}, $\cdots$; an $I=1$ trajectory has particles with $J^{PC}=1^{--}$, 3^{--}, $\cdots$. Thus the trajectories related by duality are *exchange degenerate*, that is, their trajectory functions are equal but they have opposite signature (one trajectory has even spin particles, the other has odd spin particles). The leading $I=1$ and 0 trajectories which couple to $\pi\pi$ are the ρ and f trajectories and the predicted exchange degeneracy between them is well satisfied experimentally (Figure 3).

This is the general character of the solutions to our null duality equations. *Trajectories occur in exchange degenerate sets and linear relations obtain for their residue functions.*

Even in the resonance approximation, the real parts of scattering amplitudes for exotic (s) channels need not vanish since via the (fixed t) dispersion relations they receive contributions from crossed (u) channel

[9] For example, in K^+p it could be that at moderately low energies diffraction scattering slowly rises with increasing energy while a substantial Regge contribution falls, resulting in a total cross section which is essentially energy independent.

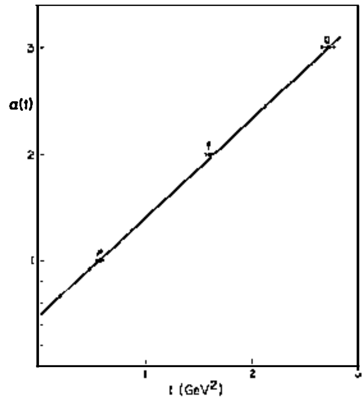

FIGURE 3. $\rho-f_0$ exchange degeneracy.

resonances. Hence, the simplification of the duality equations for the imaginary part that resulted by restricting them to exotic channels will not occur for the real part of the amplitude.

The relations we will use in this paper follow from the assumption that Regge pole contributions to imaginary parts of scattering amplitudes cancel out in exotic channels. Schematically, we may express this as

$$\sum_{\mathbf{b}} X^{ab}\,\mathrm{Im}\left\{\begin{matrix}\text{Regge pole contribution with}\\ t\text{-channel quantum numbers } \mathbf{b}\end{matrix}\right\} = 0 \qquad 6.$$

a an exotic representation

where X is the crossing matrix which describes the linear combinations of t-channel amplitudes **b** that correspond to single s-channel representations **a**. The contribution resulting from the exchange of a Regge trajectory in the t channel is

$$R = \beta(t)\,\frac{(\tau + e^{i\pi\alpha(t)})}{\sin \pi\alpha(t)}\,P_{\alpha(t)}(\cos\theta_t) \qquad 7.$$

where τ, the signature factor, is $+1$ when even spin particles occur on the trajectory and -1 when the particles have odd spin.[10]

For fixed t, the behavior of $P_{\alpha(t)}(\cos\theta_t)$ at large $|s|$ is

$$\begin{aligned} P_{\alpha(t)}(\cos\theta_t) &\sim s^{\alpha(t)},\ s \to \infty \\ &\sim e^{-i\pi\alpha(t)}u^{\alpha(t)},\ u \to \infty \end{aligned} \qquad 8.$$

[10] Signature $+1$ baryon trajectories have particles with spin 1/2, 5/2, 9/2, $\cdots$, while signature -1 baryon trajectories have particles with spin 3/2, 7/2, $\cdots$.

Therefore, duality gives

$$\sum_{\mathbf{b}} (X_{st})_{\mathbf{ab}} \sum_{k} \beta_k{}^{\mathbf{b}}(t) s^{\alpha_k^{\mathbf{b}}(t)} = 0 \qquad 9.$$

where **a** is an exotic s channel and

$$\sum_{\mathbf{b}} (X_{ut})_{\mathbf{ab}} \sum_{k} \tau_k{}^{\mathbf{b}} \beta_k{}^{\mathbf{b}}(t) u^{\alpha_k^{\mathbf{b}}(t)} = 0 \qquad 10.$$

where **a** is an exotic u channel.

The validity of these equations depends upon the compatibility of the two different approximations, Regge pole dominance and nonexotic resonance saturation. Equivalently, the regions in s or u in which they are valid (for fixed t) may be regarded as the overlap between the regions in which Regge pole dominance and nonexotic resonance saturation are each separately valid. If these approximations are very accurate and if the overlap region is large, one would expect the exchange degeneracies to be more accurate than if the overlap region were small.

If duality is valid over a very large energy region, the trajectories which cancel against one another must have essentially the same energy dependences $s^{\alpha_k{}^{b}(t)}$, which is to say the same trajectory functions $\alpha_k{}^{\mathbf{b}}(t)$. Removing the common factor s^{α} from the high-energy behavior of the imaginary part of the amplitudes leaves the linear relations among the residue functions of the exchange-degenerate trajectories:

$$\sum_{\mathbf{b}} (X_{st})_{\mathbf{ab}} \sum_{k} \beta_k{}^{\mathbf{b}}(t) = 0 \qquad 11.$$

for **a** an exotic s-channel representation, and

$$\sum_{\mathbf{b}} (X_{ut})_{\mathbf{ab}} \sum_{k} \tau_k{}^{\mathbf{b}} \beta_k{}^{\mathbf{b}}(t) = 0 \qquad 12.$$

for **a** an exotic u-channel representation. The solutions to these equations determine the representations and signatures of the trajectories which are exchange degenerate with each other (or at least minimal patterns of trajectories), since a nonvanishing $\beta_k{}^{\mathbf{b}}(t)$ requires the presence of a trajectory with its quantum numbers. The solutions also give relations between the couplings of those trajectories.

If the region over which duality is valid is less extensive, the energy dependences coming from the trajectories need not match as well, and so the trajectories can be less closely degenerate. Similarly, if the equations need not be satisfied so accurately, the couplings may differ from their expected values.

Even though we are concerning ourselves with exotic direct channels, the region of nonexotic resonance domination is crucial because our equations assert that Regge trajectories make a small contribution to exotic channels *relative* to their contributions to nonexotic channels. *The existence of high-mass exotic mesons would have exactly the same effect as the failure* *of resonance saturation, since in either case, above some energy exotic channels would cease to be depressed relative to ordinary ones. Such high-mass mesons could couple to ordinary mesons without seriously disturbing the mesonic exchange degeneracies.*

The accuracy of the exchange degeneracies also depends, obviously, on the size of the individual Regge pole contributions. The reason again is that duality is only approximate, so greater cancellations will be required when the individual terms are large. Since it appears experimentally that lower trajectories contribute less than leading ones, we might expect that their exchange degeneracies are less accurate.

From the assumptions that underlie the duality equations 9 and 10, we would expect that they are valid only in a fairly narrow range of t. One limitation on their validity is that the simple form of the imaginary part of a Regge contribution given in Equation 1 is correct only when the imaginary part of the trajectory function α is zero. When Im α is nonzero, the Regge contribution is more complicated, and so reliable information can be inferred from the equations only for small or vanishing Im α, i.e., the region between t slightly negative and t slightly above the crossed-channel threshold. [Of course, were duality an *exact* principle, we could infer that exchange-degenerate trajectories had identical trajectory functions—both real and imaginary parts. It is not exact however, and over a finite range of s differences in the real parts of $\alpha(t)$ can be masked by differences in the imaginary parts.] Another source of difficulty is that at negative t the cuts resulting from simultaneous Regge pole and Pomeranchuk exchange lie above the leading pole (20–22). Even though these cuts may be weaker than the poles, at sufficiently negative t they will distort the Regge contribution.

A natural boundary to the region in momentum transfer where resonance saturation is valid might be the place where the partial wave series diverges—the boundary of the larger Lehman ellipse. One fears that resonance saturation may become quite inaccurate should one stray too near the crossed-channel threshold.

The import of these remarks is that *the duality equations can only be considered to have been derived in a finite range of momentum transfers*. The region extends from slightly negative t to the positive value of t at which either the crossed-channel reaction becomes significantly inelastic or the imaginary part of the trajectory function becomes large. Since a large portion of this region may be absolutely inaccessible experimentally, one may ask how the exchange degeneracies are to be tested. Much of the phenomenological utility of these results rests on our ability to extrapolate trajectory and residue functions. We will test our results in the particle region by assuming
approximately linear in m^2 and that the *ratios* of residue functions vary smoothly.

There appears to be great variation in the accuracies of the predicted exchange degeneracies. For mesonic trajectories, this variation seems correlated with the threshold of the reaction from which the exchange degeneracy is deduced. One can make hierarchies of resonating channels, ordering them according to increasing threshold, the mesonic channels being pseudoscalar-pseudoscalar meson (PP), pseudoscalar-vector (PV), vector-vector (VV), baryon-antibaryon ($B\bar{B}$), decuplet-antibaryon ($\Delta\bar{B}$), and decuplet-antidecuplet ($\Delta\bar{\Delta}$), and the baryonic channels being PB, $P\Delta$, VB, and $V\Delta$. This hierarchy is reflected in the precisions of the exchange degeneracies derived from these reactions. The precision of an exchange degeneracy is determined by the best reaction from which it can be deduced.[11] The ones coming from PP are the best, those from PV not as good, those from VV fairly bad, and some of the trajectories predicted from $B\bar{B}$, $B\bar{\Delta}$, and $\Delta\bar{\Delta}$ have not been observed. Such a correlation with thresholds would be expected if the region of validity of nonexotic resonance saturation were essentially the same for all direct channels with the same quantum numbers while the Regge region depended strongly on the threshold.

The exchange degeneracies that follow from the application of duality to $B\bar{B}$, $B\bar{\Delta}$, or $\Delta\bar{\Delta}$ direct channels seem to be very badly violated. These channels give both mesonic and baryonic exchange degeneracies, through reactions like $B\bar{B}\rightarrow\bar{B}B$ and $B\bar{B}\rightarrow PP$. When $B\bar{\Delta}$ and $\Delta\bar{\Delta}$ channels are considered, it is impossible to satisfy the exchange degeneracy relations with only singlets and octets of mesons (8). Those reactions require that mesonic trajectories in $SU(2)$ and $SU(3)$ representations usually considered exotic lie not too far from (but certainly not degenerate with) ordinary mesonic trajectories. No such mesons have been observed. Although ordinary baryonic trajectories are sufficient to satisfy the exchange degeneracy constraints coming from annihilation channels, these constraints require many unobserved baryons and baryonic trajectories. The most spectacular prediction is that there be an $I=1/2$ nonstrange baryon resonance with $J^P=3/2^+$ approximately degenerate with the $\Delta(1238)$. No such state has been seen up to the present limit of phase-shift analyses.

The predictions coming from $B\bar{B}$, $B\bar{\Delta}$, and $\Delta\bar{\Delta}$ channels are of such a different quality from the other exchange degeneracies that we are prompted to segregate these into a separate subsection (page 312) and to consider only meson-meson and meson-baryon direct channels in the principal calculations of this paper. The poor quality of the predictions coming from baryon-antibaryon channels may be related to their very high thresholds (for channels with mesonic quantum numbers).

The failure of these exchange degeneracies presumably implies either that resonance saturation will be a bad approximation to reactions with baryon-antibaryon channels or that in these reactions exotic mesons will appear near threshold. If reactions like $B\bar{B}\rightarrow$mesons are not dominated by single resonance intermediate states, the Argand plots for these reactions will not show the

[11] For example, although the exchange degeneracy between the ρ and f trajectories can be inferred from $\pi\pi\rightarrow\pi\pi$, $\pi\pi\rightarrow\rho\rho$, $\rho\rho\rightarrow\rho\rho$, etc, the first reaction will determine the precision of the exchange degeneracy.

rapidly varying circular structures characteristic of resonance-dominated processes (23). On the other hand, if resonance saturation is a good approximation in all reactions at all energies, then resonances will occur in all $SU(3)$ representations. The mass at which a given $SU(3)$ representation first occurs will increase with the number of $q\bar{q}$ excitations in its quark wavefunction.

EXCHANGE DEGENERACY EQUATIONS

Meson Trajectories and Their Couplings

Trajectories.—First we consider the elastic scattering of octets of mesons, $M_1+M_1'\rightarrow M_2+M_2'$ (Figure 4). The t channel is taken to contain factorized Regge trajectories while the absence of resonances with exotic quantum

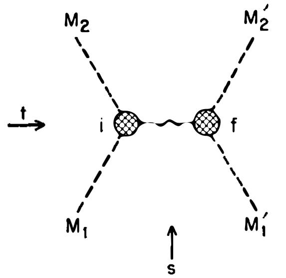

FIGURE 4. Meson scattering with factorized Regge trajectory exchange in the t channel.

numbers is imposed in the s and u channels. This leads to the following exchange degeneracy equations:

$$\sum_{\mathbf{b}} (X_{st})^{\mathbf{ab}} \sum_{k} \beta_k{}^{\mathbf{b}}(t) s^{\alpha_k{}^{\mathbf{b}}(t)} = 0 \qquad 9.$$

$$\mathbf{b} = \mathbf{10}, \overline{\mathbf{10}}, \mathbf{27}$$

$$\sum_{\mathbf{b}} (X_{ut})^{\mathbf{ab}} \sum_{k} \tau_k{}^{\mathbf{b}}\beta_k{}^{\mathbf{b}}(t) u^{\alpha_k{}^{\mathbf{b}}(t)} = 0 \qquad 10.$$

Since these equations hold for a range of s or u, trajectories must occur in exchange-degenerate sets $\{k,\mathbf{b}\}$ with their couplings constrained by

$$\sum_{\mathbf{b}} (X_{st})^{\mathbf{ab}} \sum_{k} \beta_k{}^{\mathbf{b}}(t) = 0 \qquad 11.$$

$$\sum_{\mathbf{b}} (X_{ut})^{\mathbf{ab}} \sum_{k} \tau_k{}^{\mathbf{b}}\beta_k{}^{\mathbf{b}}(t) = 0 \qquad 12.$$

We solve for the simplest possible sets $\{k,\mathbf{b}\}$ of trajectories whose couplings permit a solution to these equations. Factorization allows us to write the β as products of two Reggeon couplings. These couplings are written as

$s^{\pm}$, $d^{\pm}$, and $f^{\pm}$ corresponding for each signature ($\pm$) to $SU(3)$ singlets, octets coupling symmetrically, and antisymmetrically.[12]

Each type of coupling may arise through the exchange of a trajectory of either positive or negative signature. We therefore introduce for each type of coupling a two-dimensional vector $\mathbf{v}=(v^+, v^-)$ whose components are proportional to the coupling strengths of the positive or negative signatured trajectories. In terms of these vectors, the s-channel equations (11) become

$$\begin{aligned} \mathbf{s}_f\cdot\mathbf{s}_i - 3\mathbf{f}_f\cdot\mathbf{d}_i + 3\mathbf{d}_f\cdot\mathbf{f}_i - 2\mathbf{d}_f\cdot\mathbf{d}_i &= 0 \\ \mathbf{s}_f\cdot\mathbf{s}_i + 3\mathbf{f}_f\cdot\mathbf{d}_i - 3\mathbf{d}_f\cdot\mathbf{f}_i - 2\mathbf{d}_f\cdot\mathbf{d}_i &= 0 \\ \mathbf{s}_f\cdot\mathbf{s}_i + \mathbf{d}_f\cdot\mathbf{d}_i - 3\mathbf{f}_f\cdot\mathbf{f}_i &= 0 \end{aligned} \tag{13}$$

where the normalizations are defined in Appendix 1.

Since in the u-channel, trajectories of opposite signature contribute with opposite sign, it is convenient to define a new coupling vector $\mathbf{v}'$ derived from $\mathbf{v}$ by

$$\mathbf{v}' = (\tau_+ v^+, \tau_- v^-) = (v^+, -v^-) \tag{14}$$

The u-channel equations (12) are then written as

$$\begin{aligned} \mathbf{s}_f\cdot\mathbf{s}_i' + 3\mathbf{f}_f\cdot\mathbf{d}_i' + 3\mathbf{d}_f\cdot\mathbf{f}_i' - 2\mathbf{d}_f\cdot\mathbf{d}_i' &= 0 \\ \mathbf{s}_f\cdot\mathbf{s}_i' - 3\mathbf{f}_f\cdot\mathbf{d}_i' - 3\mathbf{d}_f\cdot\mathbf{f}_i' - 2\mathbf{d}_f\cdot\mathbf{d}_i' &= 0 \\ \mathbf{s}_f\cdot\mathbf{s}_i' + \mathbf{d}_f\cdot\mathbf{d}_i' + 3\mathbf{f}_f\cdot\mathbf{f}_i' &= 0 \end{aligned} \tag{15}$$

Solving these equations first for $i=f$, we find

$$\begin{aligned} \mathbf{s}^2 &= 2\mathbf{d}^2 \\ \mathbf{f}^2 &= \mathbf{d}^2 \end{aligned} \tag{16}$$

from the s-channel equations and because of charge conjugation either

$$f_+ = d_- = s_- = 0 \tag{17}$$

or

$$f_- = d_+ = s_+ = 0 \tag{18}$$

from the u channel. The general character of both solutions is an octet exchange degenerate with a nonet. If the charge conjugations of the external octets in the t channel are C_1 and C_2, the nonet trajectory will have $C=+C_1C_2$ (since it couples symmetrically, i.e., d type) while the octet will have $C=-C_1C_2$ (since it couples antisymmetrically, i.e., f type) (24). The argument holds for each natural parity. In particular, $PP\rightarrow PP$ gives **8** vector trajectories ($J^{PC}{}_\alpha=1^{--}, 3^{--}, \cdots$) exchange degenerate with **9** tensor trajectories ($2^{++}, 4^{++}, \cdots$). Isolating positive natural parity exchanges in

[12] The factored residue function of a singlet trajectory of positive signature would, for example, be written as $\beta_+{}^1 \sim s_f{}^+ s_i{}^+$.

$PV \to VP$,[13] however, leads to 9 vectors exchange degenerate with 8 tensors. Consideration of the two reactions forces nonets of both vector and tensor trajectories. Similarly, for negative natural parity exchanges, $PV \to VP$ and $VV \to VV$ yields for each charge conjugation exchange-degenerate nonets. For either natural parity, nonets of exchange-degenerate trajectories will also satisfy the constraints coming from the inelastic reactions $PP \to PV$ and $PV \to VV$. *The fact that mesons occur in nonets, at least on all leading trajectories, confirms the structure predicted by duality.*

The overlap region where both Regge pole dominance and nonexotic resonance saturation hold is expected to be much larger for $PP \to PP$ than for $VV \to VV$. Therefore, the predictions derived from $PP \to PP$ should be more accurate than those from $VV \to VV$. A measure of the accuracy of these predictions is the amount of singlet-octet mixing. Since exact duality requires singlet-octet degeneracy, strong mixing will occur when $SU(3)$ symmetry breaking is introduced. If duality is badly broken, however, the singlet and octet will be widely split and should not mix appreciably.

The large amount of mixing observed in the vector and tensor nonets indicates that duality is well satisfied in the reactions $PP \to PP$ and $PV \to VP$. The weak mixing present in the pseudoscalar nonet suggests that duality is badly broken in $VV \to VV$. This leads us to predict strong mixing for the 1^{+-} and 2^{--} multiplets since their nonet structure is required by $PV \to VP$. *Weak mixing should occur in the* 1^{++} *multiplet* since the singlet-octet degeneracy that would be required here comes from the reaction $VV \to VV$. Specifically, the 1^{++} octet must satisfy the Gell-Mann–Okubo mass formula (without mixing). Consequently, *the $D(1285)$ together with the $A_1(1070)$ and the $K^*(1240)$ forms an unmixed 1^{++} octet.* Indeed,

$$m^2(D) = \frac{4m^2(K^*) - m^2(A_1)}{3} \qquad 19.$$

$$1.65 \approx 1.67$$

Furthermore, assigning the D to a weakly mixed octet allows for its production in πN reactions as well as its decay into $\delta(960) + \pi$. Both features are observed experimentally (25).

All $SU(3)$ symmetric results for meson trajectories are summarized in Table 2.

Couplings.—In this section we first discuss the coupling patterns predicted by exchange degeneracy for meson vertices. Factorization of the leading vector-tensor trajectory and the absence of exotic resonances in meson-baryon scattering then leads to predictions for meson-meson, meson-baryon, baryon-baryon, and baryon-antibaryon total cross-section differences.

[13] The direct channel $PP \to VV$ is used only to fix the signatures of the trajectories which couple to PV.

TABLE 2 Summary of $SU(3)$-symmetric results for leading trajectories

A. Degeneracy patterns

Mesons: Exchange-degenerate nonets of opposite P and C.

Baryons: (a) Negative natural parity: **10** exchange degenerate with **8**.
(b) Positive natural parity: **8** exchange degenerate with $\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{10}$.

B. Specific degeneracies and the reactions giving them

Reaction	Meson trajectories related: $C=+C_1C_2$ (d coupling)		Meson trajectories related: $C=-C_1C_2$ (f coupling)
$PP\to PP$	**9** (2^{++})	$\Leftrightarrow$	**8** (1^{--})
$PV\to VP$	**9** (1^{--})	$\Leftrightarrow$	**8** (2^{++})
	9 (1^{+-})	$\Leftrightarrow$	**8** (0^{-+})
	9 (2^{--})	$\Leftrightarrow$	**8** (1^{++})
$VV\to VV$	**9** (0^{-+})	$\Leftrightarrow$	**8** (1^{+-})
	9 (1^{++})	$\Leftrightarrow$	**8** (2^{--})

Reaction	Baryon trajectories related
$PB\to BP$	**10** $(3/2^+)\Leftrightarrow$**8** $(5/2^-)$
	8 $(1/2^+)\Leftrightarrow(\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{10})(3/2^-)$
$P\Delta\to\Delta P$	**10** $(3/2^+)\Leftrightarrow$**8** $(5/2^-)$
	8 $(1/2^+)\Leftrightarrow(\mathbf{8}\oplus\mathbf{10})(3/2^-)$

Equation 16 implies that the linear combination

$$|Q\rangle = \sqrt{\frac{1}{3}}\,|1,\,Y=0,\,I=0\rangle - \sqrt{\frac{2}{3}}\,|8,\,Y=0,\,I=0\rangle \qquad 20.$$

of 2^{++} singlet and octet decouples from $\pi\pi$ since

$$|\pi\pi\rangle \propto \frac{1}{2\sqrt{2}}\,|1\rangle - \frac{1}{\sqrt{5}}\,|8_s\rangle$$

and[14]

$$s = -\sqrt{2}d \qquad 16.$$

Since $f'(1550)$ does not couple strongly to $\pi\pi$, we identify $|Q\rangle$ with

[14] The phase in Equation 16 is chosen so that the conventional relative phases in the expression for $|Q\rangle$ are obtained.

f'. The linear combination of singlet and octet is precisely that obtained in the quark model version of $SU(3)$ breaking. There, the f' transforms like a pure $\lambda\bar{\lambda}$ state. A graphical representation (26) of the selection rule $f' \not\leftrightarrow \pi\pi$ is given in Figure 5. The rules for drawing quark graphs are summarized in Appendix 2.

The reaction $PV \rightarrow VP$ requires additional 1^{--}, 1^{+-}, and 2^{--} nonets. Since on the $SU(3)$ level the overlap region for $PV \rightarrow VP$ is expected to be smaller than for $PP \rightarrow PP$, mixing in these nonets is a priori somewhat weaker than in the tensor nonet. The overlap region, however, depends on the masses of the external particles, and an examination on the $SU(2)$ level of the reactions governing the exchange degeneracies indicates that the

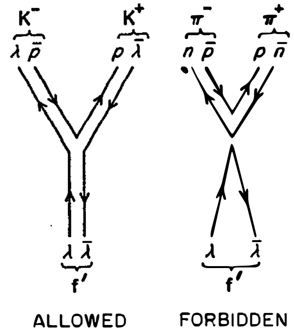

FIGURE 5. Quark model connectedness rule; $f' \not\rightarrow \pi\pi$.

1^{--} and 2^{++} mixing are essentially on the same footing, and distinguished from that of the 1^{+-} and 2^{--} nonets (see Table 3).

Specifically, A_2-f degeneracy follows from $\pi\pi \rightarrow \pi\pi$ and $K\bar{K} \rightarrow \bar{K}K$ while ρ-ω degeneracy comes from $\pi\rho \rightarrow \rho\pi$ and $K\bar{K} \rightarrow \bar{K}K$. Both $\pi\pi$ and $\pi\rho$ are the lowest-threshold reactions for the respective G parities. In the quark model these degeneracies are equivalent to the selection rules $f' \not\leftrightarrow \pi\pi$ and $\phi \not\leftrightarrow \rho\pi$.

The $I=1$, $I=0$ degeneracy in the 1^{+-} and 2^{--} nonets, on the other hand, would only follow from the higher-threshold reaction $K\bar{K}^* \rightarrow \bar{K}^*K$. We therefore expect these degeneracies to be less accurate than those of the vectors or tensors. Presently, there is no experimental evidence bearing on this point.

The mixing in the pseudoscalar 0^{-+} and axial 1^{++} nonets is expected to be even weaker. The π-η and A_1-D degeneracy would follow only from the reactions $K\bar{K}^* \rightarrow \bar{K}^*K$ and $\rho\rho \rightarrow \rho\rho$. Indeed, the π-η degeneracy is badly broken and the π, K, and η form a relatively pure octet. We predict the same behavior for the 1^{++} states: $A_1(1070)$, $K^*(1240)$, and $D(1285)$. Our predictions for mixing are summarized in Table 3.

TABLE 3. Meson mixing

Strength of meson mixing	Degenerate states in exact duality	Reactions giving degeneracies	Prototype "quark model" M^2 formula[a]
Strong	A_2-f $\rho-\omega$	$\pi\pi\to\pi\pi$ $K\overline{K}\to\overline{K}K$ $\pi\rho\to\rho\pi$ $K\overline{K}\to\overline{K}K$	$\omega=\rho$ $\phi=2K^*-\rho$
Inter-mediate	$B-(I=0,\ 1^{+-})$ $(I=1,\ 2^{--})-(I=0,\ 2^{--})$	$\pi\rho\to\rho\pi$ $K^*\overline{K}\to\overline{K}K^*$	$(H-B)(H'-B)$ $=\frac{4}{3}(K_A-B)(H'+H-2K_A)$
Weak	$\pi-(I=0,\ 0^{-+})$ $A_1-(I=0,\ 1^{++})$	$\rho\rho\to\rho\rho$ $K^*\overline{K}\to\overline{K}K^*$	$K=\dfrac{3\eta+\pi}{4}$

[a] If $SU(3)$ is broken as in the quark model, then our broken duality predictions for the amount of mixing may also be tested with these mass formulas (29).

A leading factorizable exchange-degenerate vector-tensor trajectory with the coupling pattern dictated by Equations 13 and 15 predicts the total cross-section relations

$$\begin{aligned}\sigma_t(\pi^+\pi^-,\text{ nondiffractive}) &= 2\ \sigma_t(\pi^+\pi^0,\text{ nondiffractive})\\ &= 2\ \sigma_t(\pi^+K^-,\text{ nondiffractive})\end{aligned} \qquad 21.$$

or, since diffraction corresponds predominantly to $I=0$ exchange[15]

$$\begin{aligned}\sigma_t(\pi^+\pi^-) - \sigma_t(\pi^+\pi^+) &= 2[\sigma_t(\pi^+\pi^0) - \sigma_t(\pi^+\pi^+)]\\ &= 2[\sigma_t(\pi^+K^-) - \sigma_t(\pi^+K^+)]\end{aligned} \qquad 22.$$

The graphical representation of Equation 21 is given in Figure 6.

Next we consider the scattering reactions $PB\to PB$. Factorization and the absence of $\overline{\mathbf{10}}$ and $\mathbf{27}$ in the direct channel lead to the following constraints on the couplings of the vector (V) and tensor (T) trajectories to $B\overline{B}$:

$$\begin{aligned}s_{B\overline{B}}^{(T)} &= d_{B\overline{B}}^{(T)} \pm 3\,f_{B\overline{B}}^{(T)}\\ d_{B\overline{B}}^{(T)} - f_{B\overline{B}}^{(T)} &= \pm\,(d_{B\overline{B}}^{(V)} - f_{B\overline{B}}^{(V)})\end{aligned} \qquad 23.$$

We then obtain the total cross-section relations[16]

[15] The total cross-section data from Serpukhov might suggest the presence of a small $I=1$ component to diffraction.

[16] Equation 24 can be derived from various assumptions. See H. J. Lipkin, *Proc. Heidelberg Int. Conf. Elementary Particles*, ed. H. Filthuth.

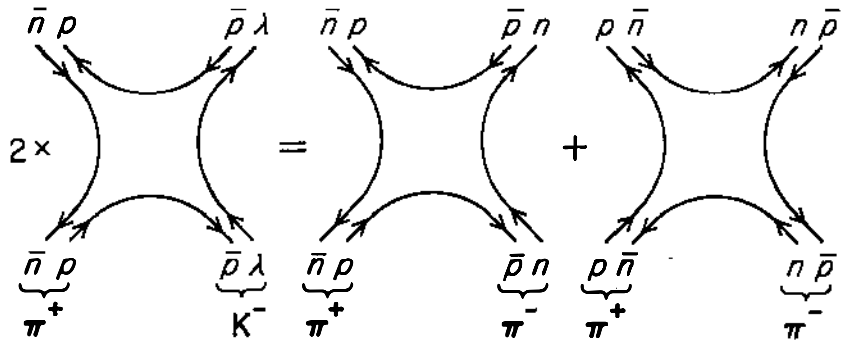

FIGURE 6. Quark graph derivation of the meson scattering total cross-section relations, Equation 21.

$$\begin{aligned}\sigma_t(\pi^-\mathrm{p}) - \sigma_t(\pi^+\mathrm{p}) &= \sigma_t(K^-\mathrm{p}) - \sigma_t(K^-\mathrm{n})\\ \sigma_t(K^+\mathrm{p}) &= \sigma_t(K^+\mathrm{n})\end{aligned} \qquad 24.$$

This last relation follows directly from the absence of a nondiffractive imaginary part in the $\overline{\mathbf{10}}$ and **27** representations.

If, in addition, we assume the f/d ratios of the vector and tensor couplings to $B\bar{B}$ to be the same, the f' decouples from $N\bar{N}$ and

$$\begin{aligned}\sigma_t(\pi^-\mathrm{p}, \text{nondiffractive}) &= \sigma_t(K^-\mathrm{p}, \text{nondiffractive}) \propto 2\, f_{B\bar{B}}\\ \sigma_t(\pi^+\mathrm{p}, \text{nondiffractive}) &= \sigma_t(K^-\mathrm{n}, \text{nondiffractive}) \propto f_{B\bar{B}} - d_{B\bar{B}}\end{aligned} \qquad 25.$$

These relations are represented by the quark graphs in Figure 7.

Similarly, again assuming equal f/d ratios for the vector and tensor couplings to $B\bar{B}$, consideration of $PB \rightarrow VB$ implies $\phi \not\rightarrow N\bar{N}$.[17] This is also suggested by quark graphs and appears to be confirmed experimentally.

Equal f/d for the vector and tensor trajectories, together with factorization, also implies that (27)

$$\sigma_t(BB, \text{nondiffractive}) = 0 \qquad 26.$$

Our results for total cross sections, together with some tests of factorization, are summarized in Table 4.

Baryon Trajectories and Their Couplings

Trajectories.—To determine the baryon exchange degeneracy patterns, we consider the scattering of octets of mesons off octets or decuplets of baryons. Baryon trajectories are exchanged in the s channel, and we impose

[17] Although the ϕ is not observed in the backward reaction $\pi N \rightarrow N\phi$, this suppression is not a definitive test of the selection rule, since the exchanged nucleon is off the mass shell.

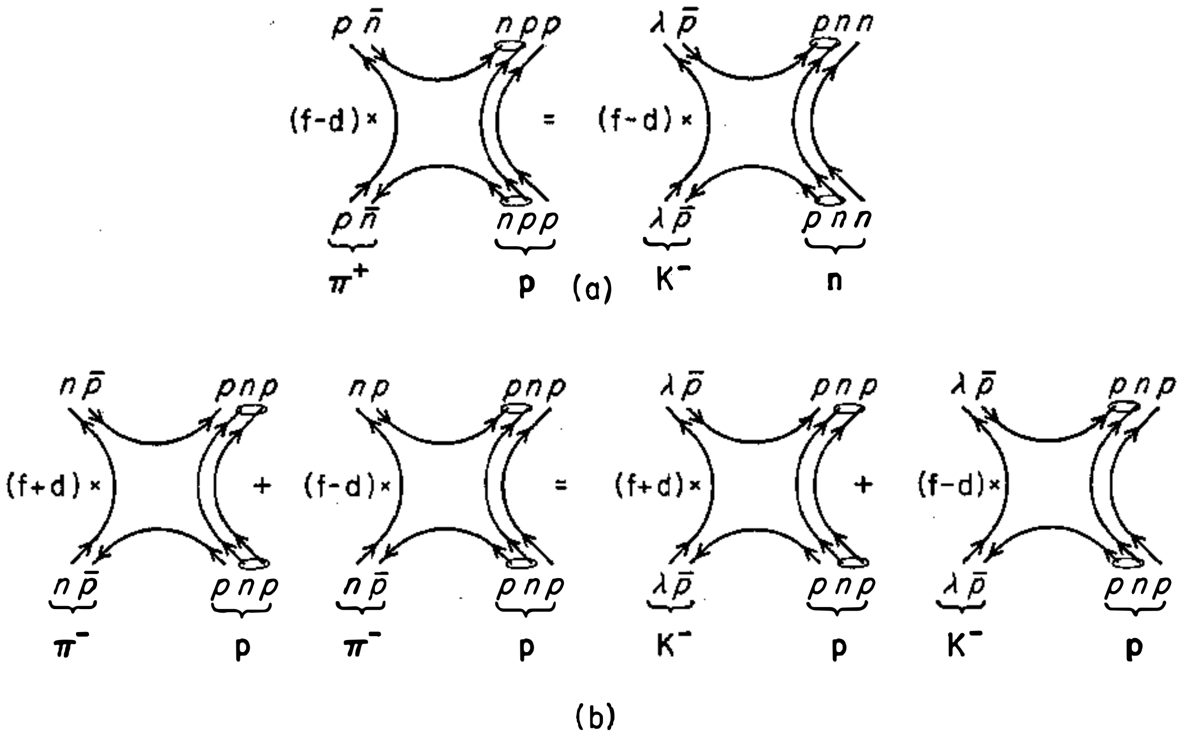

FIGURE 7. Quark graph derivation of the meson-baryon scattering total cross-section relations, Equation 25. (See Appendix 2 for rules.)

TABLE 4. Summary of total cross-section results

A. Factorization tests

$$[\sigma_t(\pi^-\mathrm{p}) - \sigma_t(\pi^+\mathrm{p})]^2 = \tfrac{1}{2}[\sigma_t(\pi^+\pi^-) - \sigma_t(\pi^+\pi^+)][\sigma_t(\bar{\mathrm{p}}\mathrm{p}) - \sigma_t(\bar{\mathrm{p}}\mathrm{n})]$$

$$[\sigma_t(\mathrm{pp}) - \sigma_t(\bar{\mathrm{p}}\mathrm{n})]\left[\frac{\sigma_t(K^-\mathrm{p}) - \sigma_t(K^+\mathrm{p}) + \sigma_t(K^-\mathrm{n}) - \sigma_t(K^+\mathrm{n})}{\sigma_t(K^-\mathrm{p}) - \sigma_t(K^-\mathrm{n})}\right]^2$$

$$= \sigma_t(\bar{\mathrm{p}}\mathrm{p}) - \sigma_t(\mathrm{pp}) + \sigma_t(\bar{\mathrm{p}}\mathrm{n}) - \sigma_t(\mathrm{pn})$$

B. Antisymmetric sum rule

$$\sigma_t(\pi^-\mathrm{p}) - \sigma_t(\pi^+\mathrm{p}) = \sigma_t(K^-\mathrm{p}) - \sigma_t(K^+\mathrm{p}) + \sigma_t(K^+\mathrm{n}) - \sigma_t(K^-\mathrm{n})$$

C. Cross sections for exotic reactions

$$\sigma_t(K^+\mathrm{p}) = \sigma_t(K^+\mathrm{n})$$

$$\sigma_t(\mathrm{pp, nondiffractive}) = \sigma_t(\mathrm{pn, nondiffractive}) = 0$$

D. Explicit formulas for MM → MM *and* MB → MB *nondiffractive cross sections*

Reaction	Relative cross sections calculated from quark graphs
$\pi^+\pi^- \to \pi^+\pi^-$	2
$\pi^+K^- \to \pi^+K^-$	1
$\pi^-\mathrm{p} \to \pi^-\mathrm{p}$	$2f$
$\pi^+\mathrm{p} \to \pi^+\mathrm{p}$	$f-d$
$K^-\mathrm{p} \to K^-\mathrm{p}$	$2f$
$K^-\mathrm{n} \to K^-\mathrm{n}$	$f-d$

the absence of imaginary parts in exotic u channels ($M\bar{B}$). Since nonexotic resonance saturation is not expected to hold in $B\bar{B}\to MM$ (resonance saturation may fail or exotic mesons may become important in these high-threshold reactions), sizable imaginary parts may be present in exotic t channels. We therefore do not include the constraints coming from the absence of exotic t-channel representations.

We are then left with the following sets of equations:

$$\sum_{\mathbf{b}} (X_{us})^{\mathbf{ab}} \sum_k \tau_k{}^{\mathbf{b}}\beta_k{}^{\mathbf{b}}(s)u^{\alpha_k\mathbf{b}(s)} = 0 \qquad 27.$$

where the exotic u-channel representations **a** are, respectively, **10** and **27** for $M_1+B\to M_2+B$, **27** for $M_1+B\to M_2+\Delta$, and **27** and **35** for $M_1+\Delta\to M_2+\Delta$. The mesons M_i are either pseudoscalar or vector octets.

A solution to these equations over a range in u requires sets $\{k,\mathbf{b}\}$ of exchange-degenerate baryon trajectories whose couplings satisfy

$$\sum_{\mathbf{b}} (X_{us})^{\mathbf{ab}} \sum_{k} \tau_k{}^{\mathbf{b}}\beta_k{}^{\mathbf{b}}(s) = 0 \qquad 28.$$

We introduce two-dimensional coupling vectors for the baryon trajectories, **s**, **f**, **d**, **t** at the meson-baryon vertex corresponding to the exchange of a singlet, an antisymmetric octet, a symmetric octet, and a decuplet; **E**, **T** at the meson-decuplet vertex corresponding to the exchange of octet and decuplet trajectories (see Figure 8). Once again the first and second

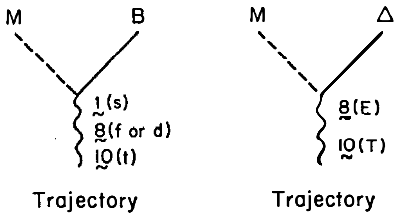

FIGURE 8. Factorized couplings of baryon trajectories to MB and $M\Delta$.

components of each vector refer to the exchange of trajectories with positive and negative signature; primed vectors are also defined as for the mesons (Equation 14). With the normalizations of Appendix I, the baryon exchange degeneracy equations read:

$M_1 + B \rightarrow M_2 + B$

no $\mathbf{10}_u$ $\quad \mathbf{s}_f\cdot\mathbf{s}_i' + 3\mathbf{f}_f\cdot\mathbf{d}_i' + 3\mathbf{d}_f\cdot\mathbf{f}_i' - 2\mathbf{d}_f\cdot\mathbf{d}_i' - 3\mathbf{t}_f\cdot\mathbf{t}_i' = 0$

29.

no $\mathbf{27}_u$ $\quad \mathbf{s}_f\cdot\mathbf{s}_i' + \mathbf{d}_f\cdot\mathbf{d}_i' + 3\mathbf{f}_f\cdot\mathbf{f}_i' + \mathbf{t}_f\cdot\mathbf{t}_i' = 0$

$M_1 + B \rightarrow M_2 + \Delta$

30.

no $\mathbf{27}_u$ $\quad \mathbf{E}_f\cdot\mathbf{d}_i' - \mathbf{E}_f\cdot\mathbf{f}_i' + \mathbf{T}_f\cdot\mathbf{t}_i' = 0$

$M_1 + \Delta \rightarrow M_2 + \Delta$

31.

no $\mathbf{27}_u$

or

no $\mathbf{35}_u$ $\quad 4\mathbf{E}_f\cdot\mathbf{E}_i' + 3\mathbf{T}_f\cdot\mathbf{T}_i' = 0$

Starting with the scattering of pseudoscalar mesons ($M_1 = M_2 = P$) off baryons, we look for sets of leading factorizable trajectories which satisfy these equations.

Excluding the trivial cases where all backward channels vanish in a reaction, the solutions with four or fewer trajectories are:

(i) $\mathbf{8} \Leftrightarrow \mathbf{10}$	(vi) $\mathbf{8} \Leftrightarrow \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{10}$
(ii) $\mathbf{8} \Leftrightarrow \mathbf{8}$	(vii) $\mathbf{1} \oplus \mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$
(iii) $\mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$	(viii) $\mathbf{1} \oplus \mathbf{8} \Leftrightarrow \mathbf{1} \oplus \mathbf{8}$
(iv) $\mathbf{10} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$	(ix) $\mathbf{8} \oplus \mathbf{10} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$
(v) $\mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{1}$	

Since no other multiplet has the same spin and parity as the $\Delta(1238)$ near its mass, the Δ, the leading negative natural parity trajectory, must be associated with solutions (i) or (iv). Noting that the only $5/2^-$ multiplet seen is the 1690 octet, we must choose the first solution ($\mathbf{8} \Leftrightarrow \mathbf{10}$) for the Δ. Similarly, the nucleon, the leading positive natural parity trajectory, must be associated with solutions (i), (ii), (iii), (v), or (vi). At around the mass expected for the exchange-degenerate recurrence of the nucleon octet, a $3/2^-$ octet and singlet are observed. This would prompt the choice of the fifth solution ($\mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{1}$) for the nucleon. We could also include the heavier $3/2^-$ decuplet at 1690 MeV (as would be expected in the quark model), giving the sixth solution ($\mathbf{8} \Leftrightarrow \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{10}$). Both solutions will be considered in more detail.

If we choose the sixth solution for the nucleon trajectory, the asymmetry between the exchange degeneracy patterns for positive and negative natural parities can be easily understood in the language of $SU(6) \times O(3)$: we then have the exchange degeneracy of the **56** $L=0$ with the **70** $L=1$, *the doublet and quartet states being separately exchange degenerate.*[18] In fact, the solutions show that *the* **56** $\mathrm{L}=0$ *must be accompanied by at least an exchange-degenerate* **70** $\mathrm{L}=1$.

In the $SU(3)$ limit, the deviations from exchange degeneracy are expected to be larger among the positive natural parity trajectories than among those of negative natural parity since for the same reactions more trajectories have to approximately cancel.

When several trajectories are coupled through an exchange degeneracy equation, the pattern of deviations from exchange degeneracy depends on their relative couplings. If two are more strongly coupled than the others, the two strongly coupled trajectories will be closely exchange degenerate, while the largest deviation is to be expected for the trajectory with weakest

[18] Decomposing $SU(6)$ representations into $SU(3) \times SU(2)$ multiplets, we have

$$\mathbf{56} \supset \mathbf{8}(S=1/2) \oplus \mathbf{10}(S=3/2)$$

$$\mathbf{70} \supset (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{10})(S=1/2) \oplus \mathbf{8}(S=3/2)$$

$$\mathbf{20} \supset \mathbf{8}(S=1/2) \oplus \mathbf{1}(S=3/2)$$

where we refer to the $S=1/2$ and $3/2$ states as doublets and quartets.

TABLE 5. Deviations from exchange degeneracy: Baryons[a]

Natural parity		Average $\Delta M^2 = \Delta\alpha/\alpha'$ (BeV^2)	Percent of decay to PB
Negative: $\mathbf{10}$ $(3/2^+) \Leftrightarrow \mathbf{8}$ $(5/2^-)$	$\mathbf{8}$ $(5/2^-)$	0.15	50%
Positive: $\mathbf{8}$ $(1/2^+) \Leftrightarrow \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{10}$ $(3/2^-)$	$\mathbf{1}$ $(3/2^-)$	0.1	90%
	$\mathbf{8}$ $(3/2^-)$	0.4	50%
	$\mathbf{10}$ $(3/2^-)$	0.9	15%

[a] The exchange degeneracy is more precise for the negative natural parity trajectories. The hierarchy of exchange degeneracies for positive natural parity trajectories reflects the spectrum of coupling strengths to PB. Linear trajectories have been assumed in the computation of ΔM^2.

coupling. *A spectrum in coupling strength will, therefore, lead to a hierarchy in the deviations from exchange degeneracy.*

This hierarchy is confirmed in the reaction $PB \rightarrow PB$. Experimentally, the **8** $(1/2^+)$ and **1** $(3/2^-)$ trajectories couple most strongly to PB, and their exchange degeneracy is the most accurate. The coupling of the **8** $(3/2^-)$ is weaker, and it is further from the nucleon trajectory. If one includes the **10** $(3/2^-)$ in the exchange degeneracy pattern, we would note that its coupling to PB is weakest, and it is the furthest from the nucleon trajectory.

Additional constraints on the **10** $(3/2^-)$ trajectory may be derived from the reaction $P\Delta \rightarrow P\Delta$. However, this reaction should lead to less accurate exchange degeneracies, since its threshold is higher. This again implies that the **10** $(3/2^-)$ will be far from the other exchange-degenerate trajectories as observed experimentally.

A summary of the exchange degeneracy patterns in the $SU(3)$ limit is given in Tables 2 and 5.

Couplings.—The exchange-degenerate set **10** $(3/2^+) \Leftrightarrow$ **8**$(5/2^-)$ will satisfy Equations 29, 30, 31 only if the f/d of the baryon octet is $-1/3$. This ratio holds for couplings either to pseudoscalar or vector mesons. It may be measured either through the decay modes of the particles lying on the trajectory, or in backward scattering. It implies, in particular, the suppression of the coupling $g_{\Lambda^* N\bar{K}}$ (and $g_{\Xi^*\Xi\eta}$) as appears to be the case for the $5/2^-$ octet. A detailed study (28) of all $5/2^-$ decay modes gives $f/d \approx -0.2$.

The explicit solution is given in Table 6 and expresses all six couplings in terms of two parameters.

In both of the exchange degeneracy patterns for the positive natural parity trajectories, all presently measurable coupling ratios are arbitrary except in singular cases. The couplings do not, unfortunately, pro-

TABLE 6. Baryon couplings

A. Coupling patterns

1. Negative natural parity trajectories = $10(3/2^+) \Leftrightarrow 8(5/2^-)$

$$\mathbf{E} = \left(\pm \frac{\sqrt{3}T}{2}, 0\right) \qquad \mathbf{T} = (0, T)$$

$$\mathbf{d} = (d, 0) \qquad \mathbf{f} = -\frac{1}{3}\mathbf{d} \qquad \mathbf{t} = \left(0, \pm \frac{2}{\sqrt{3}} d\right)$$

2. Positive natural parity trajectories = $8(1/2^+) \Leftrightarrow (1 \oplus 8)(3/2^-)$

$$\mathbf{E} = (E_1, \pm E_1)$$

$$\mathbf{d} = (d_1, d_2) \qquad \mathbf{f} = (f_1, d_2 \pm (f_1 - d_1))$$

$$\mathbf{s} = (0, \pm \sqrt{6f_1d_1 - 2d_1^2 - 4d_2^2 \mp 6d_2(f_1 - d_1)})$$

3. Positive natural parity trajectories = $8(1/2^+) \Leftrightarrow (1 \oplus 8 \oplus 10)(3/2^-)$

$$\mathbf{E} = \left(E_1 = \pm \sqrt{E_2^2 + \frac{3T^2}{4}}, E_2\right)^{a} \qquad \mathbf{T} = (0, T)$$

$$\mathbf{d} = (d_1, d_2) \qquad \mathbf{f} = \mathbf{d} - \frac{4t\mathbf{E}}{3T} \qquad \mathbf{t} = (0, t)$$

$$\mathbf{s} = \left(0, \pm 2\sqrt{\left(d_1 - \frac{tE_1}{T}\right)^2 - \left(d_2 - \frac{tE_2}{T}\right)^2}\right)$$

[a] We only give the solution with $T \neq 0$.

B. F/D ratios and suppressed decays

Natural parity	Octet	F/D theoretical[b]	F/D experimental	Suppressed decays
−	$8(5/2^-)$	$-\frac{1}{3}$	−0.2	$\Lambda^* \not\to N\bar{K}$ $\Xi^* \not\to \Xi\eta$
+	$8(1/2^+)$	~1	0.6 → 1	$\Sigma \not\to N\bar{K}$ $\Xi \not\to \Xi\pi$
	$8(3/2^-)$	~1	1.2	$\Sigma^* \not\to N\bar{K}$ $\Xi^* \not\to \Xi\pi$
	$8(5/2^+)$	~1	1.2	$\Sigma^* \not\to N\bar{K}$ $\Xi^* \not\to \Xi\pi$

[b] For the positive natural parity solution $8 \Leftrightarrow 1 \oplus 8 \oplus 10$.

vide us with the means of choosing between these two solutions. It is interesting, though, that there is a nontrivial relation between the couplings in the $(\mathbf{8})\Leftrightarrow(\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{10})$ pattern. The fact that the **10** $(3/2^-)$ couples weakly to PB even though it is very broad and so presumably strongly coupled to $P\Delta$ requires that the f/d ratios along both octet trajectories be approximately one.

Experimentally, the f/d ratios for the couplings of the $3/2^-$ and $5/2^+$ octets to PB are each $\sim$1.2 (28) while the f/d ratio of the nucleon octet is between 0.6 and 1. Of course, these f/d ratios are quite compatible with the smaller solution $(\mathbf{8})\Leftrightarrow(\mathbf{1}\oplus\mathbf{8})$ as well.

An f/d value near 1 means that the couplings $g_{\Sigma\bar{K}N}$ and $g_{\Xi\Xi\pi}$ are suppressed. For the **8** $(1/2^+)$, $g_{\Sigma\bar{K}N}/g_{\Lambda\bar{K}N}$ is small, while for the **8** $(3/2^-)$, $\Sigma^*(1660)\nleftrightarrow\bar{K}N$ and $\Xi^*(1820)\nleftrightarrow\Xi\pi$. The corresponding decays for the **8** $(5/2^+)$ are poorly determined.

All results for baryon couplings are summarized in Table 6.

Baryon-Antibaryon Channels

Additional baryon trajectories must be introduced when baryon-antibaryon channels are included along with meson-baryon channels. Some of these extra trajectories have not been seen. All may be chosen to be in **1**, **8**, or **10** $SU(3)$ representations. By contrast, the consideration of baryon-antibaryon channels for meson exchange degeneracies allows no solution with only **1** and **8** of mesons; exotic mesons are required.

One might think that since exotic mesons are needed to satisfy these equations, it is not reasonable to retain the information coming from the absence of exotic channels in our previous calculations. However, if these exotica are very massive, there will still be an extensive (overlap) region where the dual approximations of Regge pole and nonexotic resonance dominance will be valid.

It is important to point out that the predictions coming from baryon-antibaryon scattering, suitably interpreted, do not offend present observations. Certainly the evidence against a strongly coupled low-mass **70** $L=0^+$ baryon supermultiplet is strong, as is that against prominent low-mass exotic resonances. As exchange degeneracies, the predictions coming from these channels seem very badly violated. But if the intrinsically approximate nature of duality is taken seriously it would be quite natural for the additional resonances to be shifted to higher masses. It is entirely possible that the new states required to satisfy duality in baryon-antibaryon reactions (and even in exotic resonance scattering) are all present, although at higher mass. It may be that for classifying particles duality is applicable to all reactions.

Additional baryons.—A simple example of the fact that additional baryon trajectories are required when the annihilation channel $(B\bar{B}\to MM)$ equations are added to those of the meson-baryon channel is provided by pseudoscalar meson-decuplet scattering. There are only two independent conditions, one each from the reactions $\Delta\bar{\Delta}\to PP$ and $P\Delta\to\Delta P$.

$$2\mathbf{E}\cdot\mathbf{E} - 3\mathbf{T}\cdot\mathbf{T} = 0$$

$$4\mathbf{E}\cdot\mathbf{E}' + 3\mathbf{T}\cdot\mathbf{T}' = 0$$

It is apparent that these equations require that an octet of the same signature as the Δ be present, for if it is not ($E^- = 0$), elimination of the E^+ coupling between these equations gives (T^- is the $P\Delta\bar{\Delta}$ coupling)

$$3T^{+2} + T^{-2} = 0$$

which cannot be satisfied by nonzero real couplings. These conditions alone require an additional **8** ($3/2^+$), and if one accepts the $SU(6)$ particle classification, a **70** $L=0^+$.

To catalogue which additional baryon trajectories may be required, we must add to the conditions coming from backward meson-baryon scattering ($PB\to BP$) all those coming from the cancellation of contributions to the exotic meson (annihilation) channels in $B\bar{B}\to PP$, $B\bar{\Delta}\to PP$, and $\Delta\bar{\Delta}\to PP$. All the trajectories related by this combined set of relations will have approximately the same trajectory functions, and the relations between their residue functions,

$$\sum_{\mathrm{b}} (X_{s,t})^{\mathrm{ab}} \sum_{k} \beta_k{}^{\mathrm{b}}(t) = 0 \qquad 32.$$

are:

$M_1 + B \to M_2 + B$

no $\mathbf{10}_t$, $\overline{\mathbf{10}}_t$	$\mathbf{s}_f\cdot\mathbf{s}_i - 2\mathbf{d}_f\cdot\mathbf{d}_i + 3\mathbf{t}_f\cdot\mathbf{t}_i = 0$	33.
no $\mathbf{27}_t$	$\mathbf{s}_f\cdot\mathbf{s}_i + \mathbf{d}_f\cdot\mathbf{d}_i - 3\mathbf{f}_f\cdot\mathbf{f}_i - \mathbf{t}_f\cdot\mathbf{t}_i = 0$	

$M_1 + B \to M_2 + \Delta$

no $\overline{\mathbf{10}}_t$	$\mathbf{E}_f\cdot\mathbf{d}_i - 3\mathbf{E}_f\cdot\mathbf{f}_i - 3\mathbf{T}_f\cdot\mathbf{t}_i = 0$	34.
no $\mathbf{27}_t$	$\mathbf{E}_f\cdot\mathbf{d}_i + \mathbf{E}_f\cdot\mathbf{f}_i - \mathbf{T}_f\cdot\mathbf{t}_i = 0$	

$M_1 + \Delta \to M_2 + \Delta$

no $\mathbf{27}_t$	$2\mathbf{E}_f\cdot\mathbf{E}_i - 3\mathbf{T}_f\cdot\mathbf{T}_i = 0$	35.

We can solve these equations contemporaneously with the meson-baryon backward equations. The only solutions with four or fewer trajectories have the exchange degeneracy patterns

$$\mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$$

$$\mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{1}$$

$$\mathbf{8} \oplus \mathbf{1} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$$

$$\mathbf{8} \oplus \mathbf{10} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$$

$$\mathbf{8} \oplus \mathbf{1} \Leftrightarrow \mathbf{8} \oplus \mathbf{1}$$

There is no pattern in which the only $3/2^+$ low-mass resonance is the Δ, as was seen before. We can try to fit the Δ into the pattern $\mathbf{8} \oplus \mathbf{1} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$, but we must then explain why the $\mathbf{8}$ $(3/2^+)$ has not been seen. In fact, from the explicit solutions to the exchange degeneracy equations (see Appendix 3) one sees that the octet is coupled fairly weakly to PB as compared with the decuplet (15). This could account for our failure to observe this octet.[19]

There are two simple exchange degeneracy patterns in which the nucleon octet would be the only $1/2^+$ multiplet: $\mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{10}$ and $\mathbf{8} \Leftrightarrow \mathbf{8} \oplus \mathbf{1}$. In both of these patterns the f/d ratios of the octet couplings to PB are fixed. In the first pattern either all f/d ratios are $-1/3$, or the positive parity trajectory has pure d coupling while the negative parity trajectory has pure f, in complete disagreement (28) with the observed f/d ratios of the $1/2^+$, $3/2^-$, and $5/2^+$ octets. In the second pattern the f/d ratios are $+1$, but the octets, including the nucleon, decouple from $P\Delta$, which is hardly acceptable experimentally.

If one requires that particles be classified into $SU(6)$ multiplets, then the nucleon would be accompanied by the spin 1/2 members of the $\mathbf{70}$ $L=0^+$, a $\mathbf{1}$, $\mathbf{8}$, and $\mathbf{10}$. The decuplet, and specifically its $I=3/2$ component, has not been observed, though since the strength of its coupling to PB is not fixed by the exchange degeneracy equations, it might be very weakly coupled and thereby shifted considerably in mass.

Exotic mesons.—That the consideration of baryon-antibaryon channels for meson exchange degeneracies gives rise to exotic mesons (8) can be easily seen in $\Delta\bar{\Delta}$ forward scattering. Since each resonance contributes a positive imaginary part, the imaginary part of the Regge contribution cannot vanish in all isospin states. Yet the crossing relation (30)

$$\begin{bmatrix} A_0^{(s)} \\ A_1^{(s)} \\ A_2^{(s)} \\ A_3^{(s)} \end{bmatrix} = \begin{bmatrix} 1/4 & 3/4 & 5/4 & 7/4 \\ 1/4 & 11/20 & 1/4 & -21/20 \\ 1/4 & 3/20 & -3/4 & 7/20 \\ 1/4 & -9/20 & 1/4 & -1/20 \end{bmatrix} \begin{bmatrix} A_0^{(t)} \\ A_1^{(t)} \\ A_2^{(t)} \\ A_3^{(t)} \end{bmatrix} \qquad 36.$$

has no nontrivial solution with only $I=0$ and 1 contributions in both channels.

A diagramatic demonstration of the same result is that it is impossible to construct a duality diagram for baryon-antibaryon scattering with only $q\bar{q}$ intermediate states in each channel; one must have $q\bar{q}q\bar{q}$ (Figure 9). One's inability to draw such a diagram implies that there is no amplitude with nonzero imaginary parts in only the singlet and octet channels.

[19] Furthermore, since the f/d ratio for the coupling of this octet to PB is a free parameter, we can choose to make the nonstrange (nucleon-like) member couple as weakly as desired to πN. If we do choose $f/d=-1$, so that the decay $N^*(I=1/2)\rightarrow\pi N$ is forbidden, then the f/d ratio of the $\mathbf{8}(5/2^-)$ coupling to PB must be $-1/5$, which is certainly no worse experimentally than the value $-1/3$ selected on p. 321.

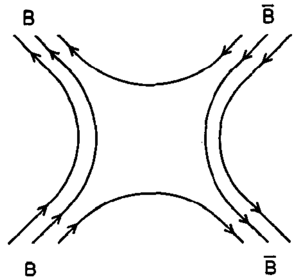

FIGURE 9. Nonexistence of baryon-antibaryon amplitudes with activity in only the **1** and **8** representations for both meson channels.

With our view of duality, the exotic meson trajectories cannot lie arbitrarily below the leading ordinary meson trajectories. For if exotics first appeared at a very high mass, far into the Regge region, there would be an extensive energy range over which Regge domination would be valid, yet in exotic representations there would be no imaginary parts. In this situation we would conclude that the exotic trajectories were very closely exchange degenerate with the ordinary ones, contradicting the assumption that the exotic mesons were extremely massive. The exotics cannot be too much heavier than ordinary mesons.[20,21]

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APPENDIX 1. Derivation of exchange degeneracy equations

In this Appendix we derive the exchange degeneracy equations 13 and 15 from the crossing matrices.

$(a)\quad M_1 + M_1' \rightarrow M_2 + M_2'.$

Requiring the absence of exotics in the s and u channels gives (from X_{st} and X_{ut}):

[20] Veneziano model amplitudes that have exotic trajectories which lie far below the leading ordinary ones have the difficulty that they do not exhibit Regge behavior until the energy reaches approximately the mass at which the exotics first appear.

[21] In exact duality, via the consideration of exotic meson scattering processes, it is required either that there be exotic mesons in the $\mathbf{10}+\overline{\mathbf{10}}$ and **27** representations degenerate with the leading ordinary meson trajectories (with exotic baryons being unnecessary) or that exotic mesons and baryons in ever more bizarre representations occur.

$$\text{No: } \mathbf{10}_s \quad \frac{1}{8}\left(\beta_+(1)+\beta_-(1)\right)+\frac{1}{\sqrt{5}}\left[\beta_+(8_{AS})+\beta_-(8_{AS})\right]$$

$$-\frac{1}{\sqrt{5}}\left[\beta_+(8_{SA})+\beta_-(8_{SA})\right]-\frac{2}{5}\left[\beta_+(8_{SS})+\beta_-(8_{SS})\right]=0$$

$$\overline{\mathbf{10}}_s \quad \frac{1}{8}\left[\beta_+(1)+\beta_-(1)\right]-\frac{1}{\sqrt{5}}\left[\beta_+(8_{AS})+\beta_-(8_{AS})\right]$$

$$+\frac{1}{\sqrt{5}}\left[\beta_+(8_{SA})+\beta_-(8_{SA})\right]-\frac{2}{5}\left[\beta_+(8_{SS})+\beta_-(8_{SS})\right]=0$$

$$\mathbf{27}_s \quad \frac{1}{8}\left[\beta_+(1)+\beta_-(1)\right]+\frac{1}{5}\left[\beta_+(8_{SS})+\beta_-(8_{SS})\right]$$

$$-\frac{1}{3}\left[\beta_+(8_{AA})+\beta_-(8_{AA})\right]=0$$

$$\mathbf{10}_u \quad \frac{1}{8}\left[\beta_+(1)-\beta_-(1)\right]-\frac{1}{\sqrt{5}}\left[\beta_+(8_{AS})-\beta_-(8_{AS})\right]$$

$$-\frac{1}{\sqrt{5}}\left[\beta_+(8_{SA})-\beta_-(8_{SA})\right]-\frac{2}{5}\left[\beta_+(8_{SS})-\beta_-(8_{SS})\right]=0$$

$$\overline{\mathbf{10}}_u \quad \frac{1}{8}\left[\beta_+(1)-\beta_-(1)\right]+\frac{1}{\sqrt{5}}\left[\beta_+(8_{AS})-\beta_-(8_{AS})\right]$$

$$+\frac{1}{\sqrt{5}}\left[\beta_+(8_{SA})-\beta_-(8_{SA})\right]-\frac{2}{5}\left[\beta_+(8_{SS})-\beta_-(8_{SS})\right]=0$$

$$\mathbf{27}_u \quad \frac{1}{8}\left[\beta_+(1)-\beta_-(1)\right]+\frac{1}{5}\left[\beta_+(8_{SS})-\beta_-(8_{SS})\right]$$

$$+\frac{1}{3}\left[\beta_+(8_{AA})-\beta_-(8_{AA})\right]=0$$

Factorization of the residues β allows us to write $\beta_+(1)=\bar{s}_f{}^+\bar{s}_i{}^+$, $\beta_+(8_{AS})=\bar{f}_f{}^+\bar{d}_i{}^+$, etc, with similar expressions for β_-. Finally, changing the normalizations of the couplings by

$$s^\pm=\bar{s}^\pm/2\sqrt{2}, \qquad f^\pm=-\bar{f}^\pm/3, \qquad d^\pm=\bar{d}^\pm/\sqrt{5} \qquad \text{A.1}$$

and writing $s=(s^+,s^-)$, $f=(f^+,f^-)$, $d=(d^+,d^-)$ leads to Equations 13 and 15.

(*b*) $M_1+B\rightarrow M_2+B.$

We first consider the equations coming from the absence of exotic antibaryons in the u channel. Proceeding as for $M_1+M_1'\to M_2+M_2'$ we introduce coupling vectors $\boldsymbol{s}$, $\boldsymbol{f}$, and $\boldsymbol{d}$ which we renormalize as in Equation A.1. In addition, we now have a decuplet trajectory whose coupling is given by $\boldsymbol{t}=(t^+,t^-)$,

$$t^\pm = \bar{t}^\pm/2\sqrt{3} \qquad \text{A.2}$$

with $\beta^\pm(10)=\bar{t}_f^\pm\bar{t}_i^\pm$. Equations 29 may then be obtained.

(*c*) $M_1+\Delta\to M_2+\Delta$.

No **27** or **35** in the u channel gives

$$\tfrac{2}{5}[\beta_+(8)-\beta_-(8)]+\tfrac{1}{4}[\beta_+(10)-\beta_-(10)]=0$$

with $\beta^\pm(8)=\bar{E}_f^\pm\bar{E}_i^\pm$ and $\beta^\pm(10)=\bar{T}_f^\pm\bar{T}_i^\pm$. Renormalizing

$$E^\pm=\bar{E}^\pm/\sqrt{5}, \qquad T^\pm=\bar{T}^\pm/\sqrt{6} \qquad \text{A.3}$$

we obtain Equation 31.

(*d*) $M_1+B\to M_2+\Delta$.

Factorization and Equations A.1–A.3 gives Equation 30.

APPENDIX 2. Rules for quark graphs

The rules for constructing quark graphs for vertices, which are well known (26), are that mesons are to be represented by a pair of lines pointed in opposite directions ($\bar{q}q$), baryons are to be represented by a triplet of lines pointed in the same direction (qqq), and *each line must begin and end in a different external particle.* Each diagram corresponds to a possible way of contracting $SU(3)$ indices to form an $SU(3)$ scalar.

For meson vertices, the coupling constant is proportional to the number of quark graphs, each graph being weighted by the product of the normalizations of the meson wavefunctions. For meson-baryon vertices, the graphs are weighted by an additional factor $(f+d)$ or $(f-d)$ depending upon whether or not a mesonic quark is contracted with a quark that has been symmetrized or antisymmetrized in the baryon wavefunction (d and f are the symmetric and antisymmetric couplings of the meson to baryon-antibaryon). Meson and baryon wavefunctions are given in Table 7. Examples of meson-baryon vertices are illustrated in Figure 10, where two antisymmetrized quarks are bound by a loop.

Diagrams for the imaginary part of forward scattering amplitudes may be assembled by connecting meson quark lines in vertex graphs (9, 10). Each diagram is a representation of a solution to the crossing matrix.

TABLE 7. Quark model states

Mesons			Baryons	
Weakly mixed octet	Strongly mixed nonet	Quark model wavefunction	Weakly mixed octet	Quark model wavefunction[a]
π^+	ρ^+	$\bar{n}p$	p	p[pn]
π^0	ρ^0	$(\bar{p}p-\bar{n}n)/\sqrt{2}$	n	n[pn]
π^-	ρ^-	$\bar{p}n$	Λ	$(p[n\lambda]+n[\lambda p]-2\lambda[pn])/\sqrt{6}$
K^+	K^{*+}	$\bar{\lambda}p$	Σ^+	$p[\lambda p]$
K^0	K^{*0}	$\bar{\lambda}n$	Σ^0	$(p[n\lambda]-n[\lambda p])/\sqrt{2}$
$\bar{K}^0$	$\bar{K}^{*0}$	$\bar{n}\lambda$	Σ^-	$n[n\lambda]$
K^-	K^{*-}	$\bar{p}\lambda$	Ξ^0	$\lambda[\lambda p]$
η	—	$(\bar{p}p+\bar{n}n-2\bar{\lambda}\lambda)/\sqrt{6}$	Ξ^-	$\lambda[n\lambda]$
—	ω	$(\bar{p}p+\bar{n}n)/\sqrt{2}$		
—	ϕ	$\bar{\lambda}\lambda$		

[a] The [ ] encloses the antisymmetrized quarks.

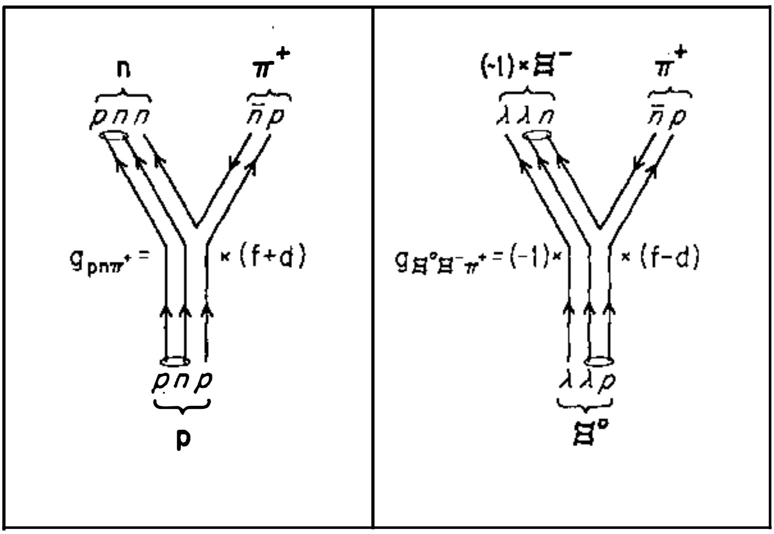

FIGURE 10. Quark graph rules for meson-baryon vertices.

APPENDIX 3. Baryon exchange degeneracy patterns

In this Appendix we give the three and four trajectory solutions to the complete set of baryon exchange degeneracy equations—those coming from the elimination of exotic direct channels in both meson-baryon backward scattering and baryon-antibaryon annihilation into two mesons. There are two solutions with three trajectories each: **8⇔8 ⊕1** and **8⇔8 ⊕10**. They are special cases of the solutions with four trajectories, which we discuss first.

8 ⊕ 10 ⇔ 8 ⊕ 1

In the exchange degeneracy pattern **8 ⊕10⇔8 ⊕1** there are nine coupling constants for each helicity state. We denote them as d_+, f_+, E_+; $t(=t_+)$, $T(=T_+)$; d_-, f_-, E_-; $s(=s_-)$. Three are independent, and we take them to be f_+, t, and T. In terms of these,

$$
\begin{aligned}
d_+ &= f_+ + \alpha_1 \sqrt{\frac{2}{3}}\, t \\
d_- &= -\alpha_1\alpha_2 \frac{1}{\sqrt{3}} f_+ + \alpha_2 \frac{2\sqrt{2}}{3} t \\
f_- &= -\alpha_1\alpha_2 \frac{1}{\sqrt{3}} f_+ - \alpha_2 \frac{\sqrt{2}}{3} t \\
s &= -\alpha_3 \frac{2\sqrt{2}}{\sqrt{3}} f_+ - \alpha_1\alpha_3 \frac{1}{3} t \\
E_+ &= \alpha_1 \frac{\sqrt{3}}{2\sqrt{2}} T \\
E_- &= \alpha_2 \frac{3}{2\sqrt{2}} T
\end{aligned}
$$

where the α_i are independent phases which may be ± 1. Note that

$$
f_- = -\frac{\alpha_1\alpha_2}{\sqrt{3}} d_+
$$

8 ⊕ 1 ⇔ 8 ⊕ 1

In this exchange degeneracy pattern none of the trajectories couples to $M\Delta$. There are six couplings to MB, denoted by $s_\pm$, $d_\pm$, $f_\pm$. Only two are independent, and we take them to be $s_\pm$. The solutions can be divided into two classes, which we call A and B.

Case A;

$$f_+ = d_+ = \alpha_1 \frac{1}{2\sqrt{2}} \sqrt{s_+^2 + 3s_-^2}$$

$$f_- = d_- = \alpha_2 \frac{1}{2\sqrt{2}} \sqrt{3s_+^2 + s_-^2}$$

Note that both f/d ratios are $+1$.

Case B:

$$d_+ = \alpha_1 \frac{1}{\sqrt{2}} s_+$$

$$d_- = \alpha_2 \frac{1}{\sqrt{2}} s_-$$

$$f_+ = \alpha_2\alpha_3 \frac{1}{\sqrt{2}} s_-$$

$$f_- = \alpha_1\alpha_3 \frac{1}{\sqrt{2}} s_+$$

$8 \oplus 10 \Leftrightarrow 8 \oplus 10$

There is only one more exchange degeneracy pattern with four trajectories, $\mathbf{8 \oplus 10 \Leftrightarrow 8 \oplus 10}$ (aside from those obtained by doubling a trajectory in one of the three trajectory patterns). There are ten couplings, which we denote $d_\pm$, $f_\pm$, $t_\pm$, $E_\pm$, $T_\pm$, of which three are independent. Two basically different coupling patterns satisfy the exchange degeneracy equations. We call them Cases A and B.

Case A:

$$E_+ = \alpha_1 \frac{\sqrt{3}}{2\sqrt{2}} \sqrt{T_+^2 + 3T_-^2}$$

$$E_- = \alpha_2 \frac{\sqrt{3}}{2\sqrt{2}} \sqrt{3T_+^2 + T_-^2}$$

$$d_+ = rE_+$$

$$d_- = rE_-$$

$$f_+ = -\tfrac{1}{3}rE_+$$

$$f_- = -\tfrac{1}{3}rE_-$$

$$t_+ = rT_+$$

$$t_- = rT_-$$

where T_+, T_-, and r take arbitrary real values, and the independent phases α_i are ± 1. Note that in this solution

$$f_+/d_+ = f_-/d_- = -\tfrac{1}{3}$$

Case B:

$$T_- = \alpha_1 T_+$$

$$E_- = \alpha_2 \sqrt{\frac{3}{2}}\, T_+$$

$$E_- = \alpha_3 \sqrt{\frac{3}{2}}\, T_-$$

$$d_+ = \alpha_1\alpha_2 \sqrt{\frac{3}{2}}\, t_-$$

$$d_- = \alpha_3 \sqrt{\frac{3}{2}}\, t_+$$

$$f_+ = -\alpha_2 \frac{1}{\sqrt{6}}\, t_+$$

$$f_- = -\alpha_1\alpha_2 \frac{1}{\sqrt{6}}\, t_-$$

The free parameters here are T_+, t_+, and t_-. The f/d ratios are not fixed, but satisfy

$$\left(\frac{f_+}{d_+}\right)\left(\frac{f_-}{d_-}\right) = \frac{1}{9}$$

8 ⇔ 8 ⊕ 1 and **8 ⇔ 8 ⊕ 10**

The three trajectory solutions are obtained as special cases of four trajectory solutions. The couplings in the **8⇔8⊕1** pattern are obtained by taking $s_+=0$ in the **8⊕1⇔8⊕1** solution. The Case B solution gives couplings reminiscent of the meson exchange degeneracy pattern. The coupling patterns in the **8⇔8⊕10** solution are obtained by taking $t_+=T_+=0$ in both cases of the **8+10⇔8+10** solution.

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