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UNITARY COUPLED CHANNEL DECK MODEL AND THE Q MESON RESONANCE REGION

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ABSTRACT

We construct a unitary Deck model with coupled $K^*\pi$ and $K\rho$ channels, including only one resonance in the Q region. Adjusting the resonance parameters, we achieve a satisfactory description of the experimental phase variations and structure in the mass spectra. The resonance is determined to belong to the $J^{PC} = 1^{+-}$ SU(3) octet, and is thus the Q_B. The relative coupling strength $K^*\pi/K\rho$ is $\sim 2/3$.

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Among the two octets of axial-vector meson resonances predicted by the quark model, only the B meson has been unambiguously identified. The apparent absence of the others is an outstanding difficulty[1]. Very recently, a SLAC group[2] reported evidence for the existence of two strange axial-vector mesons, Q_1 and Q_2. Their conclusion is based both on structure in the $K^*\pi$ and $K\rho$ mass distributions and on observed phase variations. In this article, we show that all these significant features of the data may be understood in terms of only one axial-vector resonance, and a non-resonant Deck background[3]. The resonance couples to both the $K\rho$ and $K^*\pi$ channels. For reasons we describe, it must have odd charge conjugation relative to the K. It is thus the Q_B, with $J^{PC} = 1^{+-}$. We find that its mass lies between 1.3 and 1.4 GeV, and its width is of order 150 MeV. Our description of the data without a Q_A $(J^{PC} = 1^{++})$ resonance is consistent with the apparent absence of a resonance signal in the $J^P = 1^+$ $\pi\rho$ A_1 system[4].

We begin with two assumptions. First, there are non-resonant Deck amplitudes, sketched in Fig.1, for both the $K^*\pi$ and $K\rho$ channels. We extract the $J^P = 1^+$ partial wave from each of these. Second, we assume that there is one resonance, with mass, width and branching ratios to be determined, which couples to both $K^*\pi$ and $K\rho$ channels. The rest is a classical two-channel problem of finding a properly analytic and unitary $J^P = 1^+$ partial wave amplitude which fulfills our assumptions. We then vary the parameters of the resonance to achieve an acceptable representation of the data. The structure in the data requires that the resonance have odd C.

The Deck model has been described at length elsewhere[3]. We quote here the analytic form valid near $t = 0$ for the $J^P = 1^+$ S wave $K\pi^*$ amplitude, with helicity zero in the Gottfried-Jackson frame.

$$A_{K^*\pi}(s,M^2,t) = \frac{2g_{K^*K^+\pi^-}|\vec{K}_{K^*}|\, s\sigma_{\pi p}}{(M^2 - m_\pi^2)} \exp(b_\pi t) \quad . \tag{1}$$

Kinematic variables are defined in Fig.1. Here, $|\vec{K}_{K^*}|$ is the incident kaon three-momentum as seen in the K^* rest frame; $\sigma_{\pi p}$ is the asymptotic πp total cross-section; b_π is the slope parameter for πp elastic scattering; and $g^2_{K^*K^+\pi^-} = 4\pi(1.66)$.

Likewise, for the kaon exchange Deck graph in Fig.1(b), the $J^P = 1^+$ S wave helicity zero $K\rho$ amplitude is

$$A_{\rho K}(s,M^2,t) = \frac{2g_{\rho KK}|\vec{K}_\rho|\, s\sigma_{Kp}}{(M^2 - m_K^2)} \exp(b_K t) \quad . \tag{2}$$

The symbols have the same meanings as above except that $|\vec{K}_\rho|$ is the incident kaon three-momentum evaluated in the ρ rest frame. For normalization, we adopt the SU(3) relationship $2g_{\rho^0K^+K^-} = g_{\rho^0\pi^+\pi^-}$, with $g^2_{\rho^+\pi^-\pi^-} = 4\pi(2.4)$. We evaluate

$|\vec{K}_{K^*}|$ and $|\vec{K}_\rho|$ at $t_2^{eff} = -0.2$ GeV^2 which we take to be independent of t and M. For the rest of this article, we specialize to $t = 0$.

We write the Deck contribution as a two component vector:

$$T_D = \begin{pmatrix} A_{K^*\pi} \\ A_{\rho K} \end{pmatrix} . \tag{3}$$

The Deck amplitudes provide a $J^P = 1^+$ cross-section $d\sigma/dM$ which rises from threshold and then decreases at large M^2 as M^{-3} . These cross-sections are shown as dashed lines in Fig. 2; they show no structure.

Our second assumption is that in addition to the Deck exchange contribution, there is one resonance in the $J^P = 1^+$ wave, which decays into both $K^*\pi$ and $K\rho$. A unitary S matrix representing the coupled channel $K^*\pi$ and $K\rho$ S wave scattering is easily constructed. We begin with the real symmetric K matrix

$$K = \begin{pmatrix} \frac{g^2}{s_1 - M^2} & \frac{gf}{s_1 - M^2} \\ \frac{gf}{s_1 - M^2} & \frac{f^2}{s_1 - M^2} \end{pmatrix} . \tag{4}$$

Here s_1 is related to the squared mass of the resonance and g and f to its coupling constants to $K^*\pi$ and $K\rho$, respectively; $\alpha^2 = g^2 + f^2$. The S matrix is then

$$S = \left[1 - K(M^2)C^+(M^2)\right]^{-1} \left[1 - K(M^2)C^-(M^2)\right] , \tag{5}$$

where the C matrix is diagonal $(C)_{ij} = \delta_{ij}C_i(M^2)$. The C_1 and C_2 are the usual unequal mass Chew-Mandelstam functions[5] for $K^*\pi$ and $K\rho$ respectively. E.g., $C_1(M^2)$ is cut from $M^2 = (m_{K^*}+m_\pi)^2$ to ∞ ; for $M \geq (m_{K^*}+m_\pi)$, it satisfies

$$\mathrm{Im}\, C_1(M^2) = \frac{2q}{M} \equiv \left[M^2-(m_{K^*}+m_\pi)^2\right]^{\frac{1}{2}}\left[M^2-(m_{K^*}-m_\pi)^2\right]^{\frac{1}{2}}\Big/M^2 \quad . \tag{6}$$

Each C_i is defined so that $C_i(0) = 0$. Equation (5) then provides a strong interaction S matrix with proper analyticity and unitarity properties. For a given function $F(M^2)$, the notation $F^{\pm}(M^2)$ is used to denote $F(M^2 \pm i\varepsilon)$, and $\Delta F(M^2) = (F^+ - F^-)/2i$.

We now face the standard problem of correcting a production mechanism, here the Deck amplitude, by final state interactions.[6,7] In our case we must use a coupled channel formulation.[8] The $J^P = 1^+$ S wave amplitude vector should be analytic in the complex M^2 plane cut from $-\infty$ to M^2_L (left-hand cut) and from M^2_R to $+\infty$ (right-hand cut). It should satisfy the following discontinuity relationships across these cuts:

a) $T^+(M^2) = S(M^2)T^-(M^2)$ $\quad M^2 \geq M^2_R$; Generalized Watson Theorem[9], where in the unitarity relation, we retain only the $K^*\pi$ and the $K\rho$ intermediate states.

b) The left-hand cut discontinuity is given by the Deck amplitude: $\Delta T(M^2) = \Delta T_D(M^2)$, for $M^2 \leq M^2_L$.

Furthermore, we require that $T(M^2)$ reduce to $T_D(M^2)$ when the S matrix, Eq. (5) is identically unity.[10]

The unique solution to this problem is the Cauchy integral[6,7,8]

$$T(M^2) = \frac{D(M^2)}{\pi} \int_{-\infty}^{M_L^2} \frac{D^{-1}(s')\Delta T_D(s')ds'}{(s'-s)} \quad , \tag{7}$$

where $D(M^2)$ is an invertible 2x2 matrix, whose elements are analytic in M^2 . $D(M^2)$ possesses only the right-hand cut, and satisfies $D^+(M^2) = S(M^2)D^-(M^2)$ across this cut, above the thresholds[11]. $D(M^2)$ is unique up to normalization[8] which cancels in Eq.(7). The factorized form chosen for $K(M^2)$ in Eq.(4) permits us to obtain an analytic expression[8] for $D(M^2)$

$$D(M^2) = \frac{1}{s_1-M^2-f^2C_2-g^2C_1} \begin{pmatrix} gs_1 & -f(s_1-M^2-\alpha^2C_2) \\ fs_1 & g(s_1-M^2-\alpha^2C_1) \end{pmatrix} \quad . \tag{8}$$

Because the S wave Deck amplitudes, Eq.(3), are simple poles in M^2, the integrals in Eq.(7) may be performed explicitly. For the final $J^P = 1^+$ partial wave amplitude, we derive

$$T(M^2) = \frac{\alpha^{-2}}{(s_1-M^2-f^2C_2-g^2C_1)} \begin{bmatrix} A_{K^*\pi}(M^2)\left\{g^2\left[s_1-m_\pi^2-\alpha^2C_1(m_\pi^2)\right]+f^2\left[s_1-M^2-\alpha^2C_2(M^2)\right]\right\} \\ +\ gfA_{K\rho}(M^2)\left\{M^2-m_K^2-\alpha^2C_2(m_K^2)+\alpha^2C_2(M^2)\right\} \\ \hline A_{K\rho}(M^2)\left\{f^2\left[s_1-m_K^2-\alpha^2C_2(m_K^2)\right]+g^2\left[s_1-M^2-\alpha^2C_1(M^2)\right]\right\} \\ +\ gfA_{K^*\pi}(M^2)\left\{M^2-m_\pi^2-\alpha^2C_1(m_\pi^2)+\alpha^2C_1(M^2)\right\} \end{bmatrix} \tag{9}$$

The upper element of the vector $T(M^2)$ is the $K^*\pi$ channel amplitude, whereas the lower is the $K\rho$ amplitude. The structure of each amplitude in Eq. (9) is that of a resonance term

$[s_1-M^2-f^2C_2-g^2C_1] \simeq (s_1-M^2-i\Gamma M)$ convoluted with a **sum** of the $K^*\pi$ and $K\rho$ Deck amplitudes. Moreover, multiplying each of the Deck amplitudes $A_{K^*\pi}$ and $A_{K\rho}$ is a complex function of M^2, with zeroes in the real part occuring at values of M^2 fixed by the resonance parameters s_1 and (f/g). These zeroes provide structure in the cross-section $d\sigma/dM$.

It is instructive to consider the equal mass SU(3) limit $(m_K = m_\pi\ ;\ \ m_\rho = m_{K^*})$. We define the ratio $r = g_\rho\sigma_{Kp}f/g_{K^*}\sigma_{\pi p}g$. Doing so, we find that $T(M^2) \to T_D(M^2)$ if $r = -1$. Thus, in the SU(3) limit, the production mechanism is left unaffected by the resonant final state interaction if the Deck and resonance amplitudes are orthogonal. By contrast, if $(g_{K^*}\sigma_{\pi p}/g_\rho\sigma_{Kp}) = (g/f)$, then $T(M^2) = T_D(M^2)\dfrac{s_1-m_\pi^2-\alpha^2C_1(m_\pi^2)}{s_1-M^2-\alpha^2C_1(M)}$.

In this latter case, with the Deck and resonance amplitudes proportional to each other, the final amplitude in each channel is the Deck amplitude multiplied directly by a resonance line shape. The resonance factor provides an enhancement in the amplitude of order $|s_1/\alpha^2C_1(M)| \simeq (\sqrt{s_1}/\Gamma)$ at the resonance location. With $\sqrt{s_1} \simeq 1.3$ GeV and $\Gamma = 0.1$ GeV, this enhancement would increase the amplitude by an order of magnitude.

In the physical situation with $m_K \neq m_\pi$ and $m_\rho \neq m_{K^*}$, the $K^*\pi$ and $K\rho$ thresholds are displaced from each other. Nevertheless, it remains true that the ratio r is the critical parameter in the problem. With normalization of the Deck amplitude fixed, we vary f/g and observe the resultant changes in the $K^*\pi$ and $K\rho$ amplitudes. Our optimal results are shown in Fig. 2 and are compared with the simple Deck results. The overall agreement with data[2] is qualitatively excellent and is even quantitative for the $K\rho$ mass distribution (Fig.2(b)), the $K\rho$ vs $K^*\pi$ relative phase (Fig.2(c)), and the relative cross-sections. The results shown are insensitive to reasonable modifications of the Deck amplitudes. For example, the use of form factors in the π and K exchange legs of Fig.1 may change the relative normalization of the two Deck terms. By making compensating changes in our (f/g) ratio, we can maintain the essential features of Fig.2.

Several points should be emphasized. To obtain a dip in the $K^*\pi$ mass distribution, it is necessary that (f/g) be negative relative to $(g_{K^*K\pi}/g_{\rho KK})$. Therefore, the resonance must have odd C relative to the kaon. It is thus the $Q_B, J^{PC} = 1^{+-}$. The relative heights of the two peaks in the $K^*\pi$ mass spectrum, the width of the threshold structure in the $K\rho$ mass distribution, and the $K^*\pi$ vs. $K\rho$ phase variation are controlled by the magnitude of (f/g). In our solution, we find $(f/g) \simeq -1.5$, with the $K\rho$ coupling favored. The SU(3) prediction is $(f/g) = -1$ for the Q_B. The position of the dip in the $K^*\pi$ spectrum is influenced by our choice of resonance position. We use $M_{res} = 1340$ MeV to obtain the results in Fig. 2. Lowering this value, we would displace the dip to lower mass. It is the resonance which generates the sharp structure near the $K\rho$ threshold. By contrast, as shown in Fig.2(b), the Deck amplitude alone rises slowly from threshold.

The variation of ϕ_{rel} shown in Fig.2(c) is in excellent agreement with the data, increasing from -30^o at $M \simeq 1.26$ GeV to $+40^o$ at $M \simeq 1.35$ GeV, and then falling again. The phase of our $K^*\pi$ amplitude varies very slowly in the region $M > 1.35$ GeV, with an average value of -5^o. Owing to the $K^*(1420)$ resonance, we expect the $2^+K^*\pi$ phase to vary relative to the $1^+K^*\pi$ phase as M is increased through 1420 GeV. However, the rise may be limited because the $K^*\pi$ branching fraction of the $K^*(1420)$ is only 30%. Both experimental groups[2] indeed observe a phase increase of roughly +50% in the neighborhood of the 1420, consistent with our expectations.

Two discrepancies may be noted between our theoretical curves, Fig.2, and the data. We obtain roughly 2/3 of the measured absolute cross-section. Second, our $K^*\pi$ mass distribution is somewhat too broad. It rises too quickly from threshold and falls off too slowly above 1.4 GeV. Narrowing of the mass distribution can be accomplished with form factors in t_2 and/or Reggeization of the π and K exchanges in the Deck amplitudes. Otherwise, the K matrix, Eq.(4), can be improved. The factorized form in Eq.(4) was dictated by our desire to obtain an analytic answer. At the expense of additional parameters, for example, a non-resonant background in K, we would have greater flexibility to describe the data in detail. The shortage of overall cross section opens the issue of whether there is room for a second Q resonance, a state with $J^{PC} = 1^{++}$. In a more complete Deck amplitude, one would include graphs with K^* and ρ exchanges, in addition to the π and K exchange graphs shown in Fig.1. Data suggest that these vector exchanges contribute with cross-sections roughly equal to the pseudoscalar terms,[3] thereby making up the cross-section shortage. Moreover, we argued above that a resonance with $C > 0$, i.e. $(f/g) > 0$ results in an enhancement of the Deck amplitudes, of order M_{res}/Γ at the resonance. Thus, only a very broad Q_A could be tolerated in our framework.

In this article we have discussed the dominant t-channel helicity $\lambda_t = 0$ amplitudes at small t . The relative size of the Deck amplitudes for $K^*\pi$ and $K\rho$ changes as t is increased. This alters the positions and relative heights of the two peaks in the $K^*\pi$ mass distribution, as well as the $K^*\pi/K\rho$ relative cross-section and phase. The $\lambda_t = 1$ Deck amplitudes have a more complicated dependence on M than Eqs.(1) and (2). In a future more detailed paper we expect to address the variation with t as well as the full spin problem.

In conclusion, we have constructed an explicit unitary $J^P = 1^+$ S wave coupled channel amplitude for the Q region, composed of one resonance and a Deck component. By adjusting the parameters of the resonance in interference with the Deck amplitude, we achieve a good representation of the data. We determine the mass $M \simeq 1340$ MeV, width $\Gamma \simeq 150$ MeV and relative $K^{*}\pi/K\rho$ coupling $f/g \simeq -1.5$ of this Q_B resonance, as well as its charge conjugation. The normalization is controlled by our Deck term. The overall agreement with the data leaves little room for a Q_A $J^{PC} = 1^{++}$ resonance in the same mass region unless its width is very large. We note that the interplay of Deck and the Q_B resonance is crucial for determining the $K\pi\pi$ mass distribution. In non-diffractive processes, e.g. $\bar{p}p$ or charge-exchange production, the shape of the $K\pi\pi$ mass spectrum in the Q_B region may be very different.

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FIGURE CAPTIONS

Fig. 1 a) Pion-exchange Deck graph for $K^- p \to K^{*o} \pi^- p$;

b) Kaon-exchange Deck graph for $K^- p \to \rho K^- p$. The kinematic variables are indicated; P stands for Pomeron exchange.

Fig. 2 Mass dependence at $t = 0$ of $d^2\sigma/dMdt$ in the $J^P\lambda_t = 1^+0$ partial wave for a) the $K^*\pi$ and b) the $K\rho$ channels. The dashed lines represent the pure Deck model. The solid lines are from our unitarized Deck model.

We determine the mass and width of the Q_B resonance (1.34, 0.15 GeV) from the position of the second sheet pole. These correspond to $\sqrt{s_1} = 1.43$ GeV, $g = -0.35$, and $f = 0.55$ in Eq. (4) .

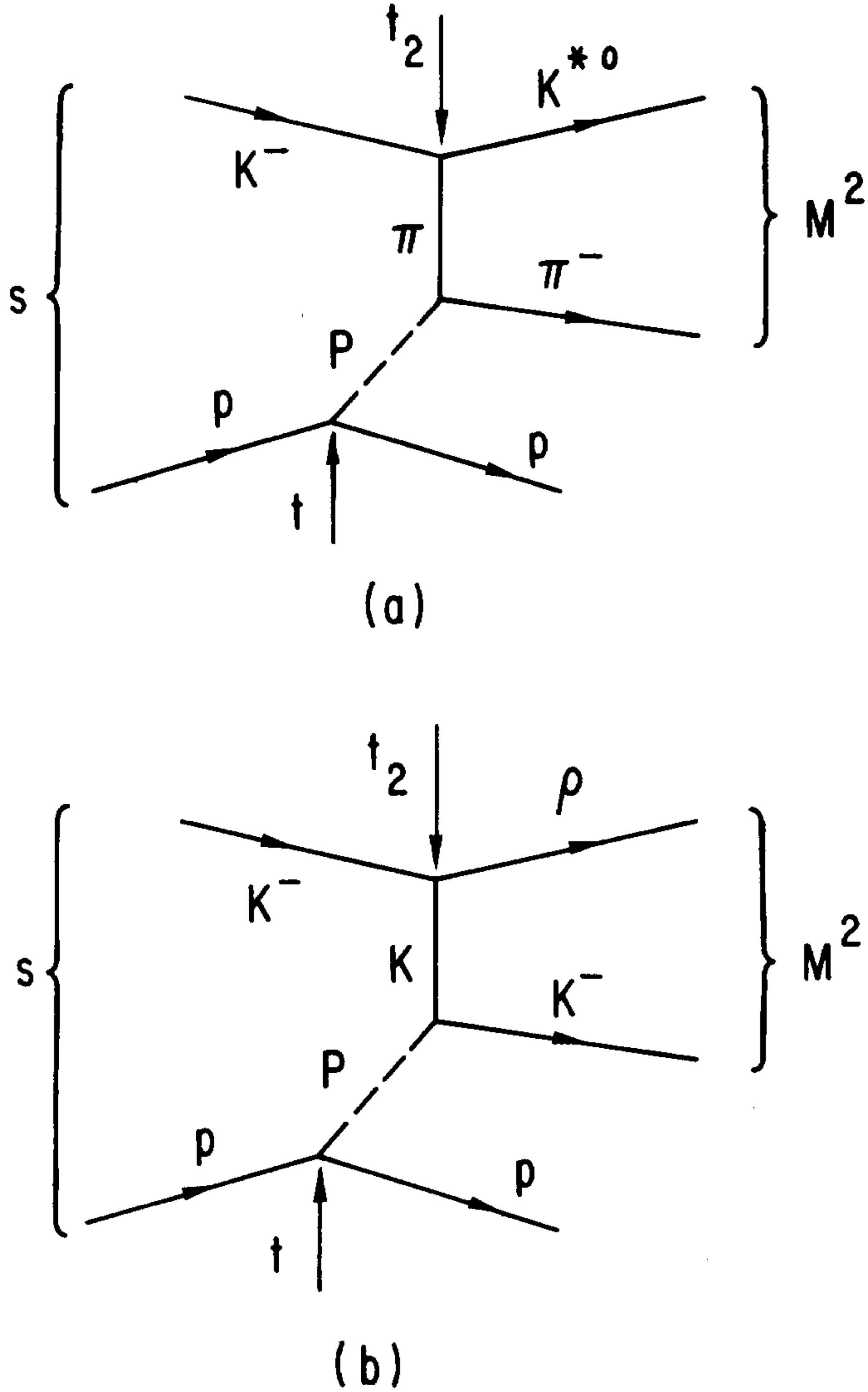

Figure 1

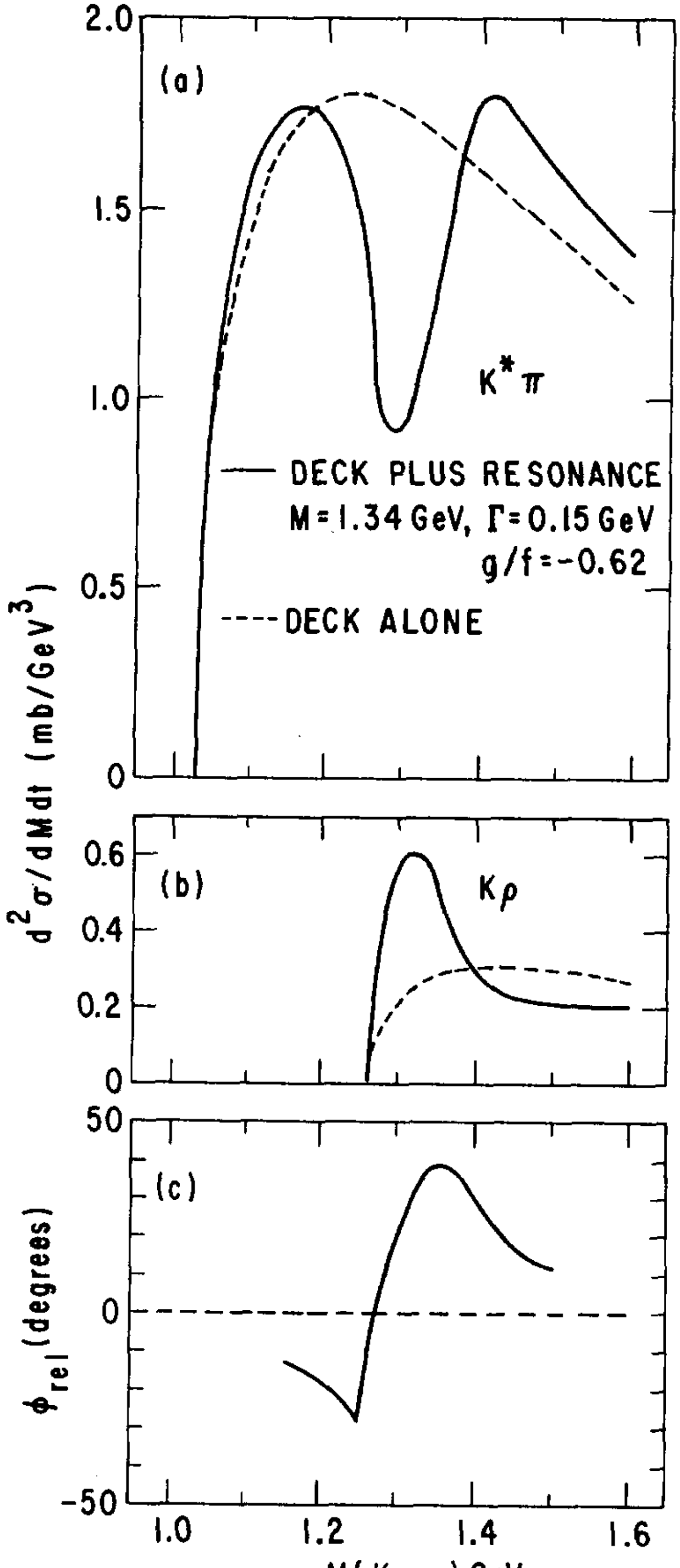
(a)
$K^*\pi$
DECK PLUS RESONANCE
M = 1.34 GeV, Γ = 0.15 GeV
g/f = -0.62
DECK ALONE
(b)
$K\rho$
(c)
$d^2\sigma/dMdt$ (mb/GeV^3)
ϕ_{rel} (degrees)
M(Kππ) GeV
2.0
1.5
1.0
0.5
0
0.6
0.4
0.2
0
50
0
-50
1.0
1.2
1.4
1.6

Figure 2