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Ref.TH. 2837-CERN

THE EFFECTS OF COLOURED GLUE IN THE QCD MOTIVATED BAG OF HEAVY QUARK - ANTIQUARK SYSTEMS

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ABSTRACT

The QCD motivated bag model is elaborated and applied to heavy quark-antiquark systems. The effect of coloured glue in the model is shown to explain the rapid cross-over of the static $Q\bar{Q}$ potential from the asymptotically free Coulomb region into the linear confinement regime. The spin-dependent force between static quarks is derived in Coulomb gauge from the exchange of a confined transverse gluon. The dimensional bag parameter Λ_B = 235 MeV and the quark-gluon coupling constant α = 0.38 (as defined at $r_{Q\bar{Q}} \sim 0.2$ fermi) are determined from a good fit of the $c\bar{c}$ and $b\bar{b}$ spectra. The fit is in serious disagreement with the widely accepted MIT parameters. As an important test of our model, we calculate the rich spectrum of ($Q\bar{Q}$ glue) states. In Υ particle spectroscopy we predict a narrow ($Q\bar{Q}$ glue) state with exotic quantum numbers $J^{PC} = 1^{-+}$ below the $B\bar{B}$ threshold. Its experimental confirmation would be the first direct evidence for coloured glue in the hadron spectrum.

Ref.TH.2837-CERN
27 March 1980

1. INTRODUCTION

The study of strong interaction physics in quantum chromodynamics (QCD) clearly indicates the existence of two distinct scales. At very short distances, below about 0.1 fermi, asymptotic freedom[1] operates while on the scale of 1 fermi the problem of quark confinement[2] appears.

Heavy particle spectroscopy may be regarded as the bridge between the two scale regimes. The purpose of our present investigation is to develop a QCD motivated bag picture for heavy particle spectroscopy. The model is meant to be an approximate phenomenological consequence of the QCD Lagrangian. Less ambitiously, it may serve to shape our intuition about the intimate connection between the asymptotically free scale and the confinement scale.

The physical vacuum (ether) is recognized now in QCD as a complicated and highly non-perturbative quantum state. Gauge fields and quarks cannot propagate freely in this medium. However, the insertion of a static point-like $Q\bar{Q}$-pair into the "ether" creates a bubble inside which perturbation theory is applicable provided that the $Q\bar{Q}$-separation $r_{Q\bar{Q}}$ is much smaller than the inverse of the scale parameter Λ.

This is best seen by observing that the bag picture for $r_{Q\bar{Q}} \ll \Lambda^{-1}$ is consistent with asymptotic freedom, since the bag radius becomes very large compared with $r_{Q\bar{Q}}$. The confinement radius R_c of perturbation theory is defined as a distance where the chromoelectric field strength E becomes comparable to Λ^2. Since we have an approximate dipole field,

$$E(R) \sim \alpha^{1/2}(r_{Q\bar{Q}}) \frac{r_{Q\bar{Q}}}{R^3} , \tag{1}$$

at a distance R from the $Q\bar{Q}$-pair, the ratio $r_{Q\bar{Q}}/R_c$ is determined as

$$\frac{r_{Q\bar{Q}}}{R_c} \sim (r_{Q\bar{Q}})^{2/3} \ln^{1/2}\left(\frac{1}{r_{Q\bar{Q}}\Lambda}\right) . \tag{2}$$

Equation (2) shows that the confinement radius is very large compared with $r_{Q\bar{Q}}$ [or the characteristic distances in quantum loops[3]] and asymptotic freedom becomes a self-consistent property of the large bubble. The way the perturbative vacuum crosses over into the outside medium at the confinement radius is a rather irrelevant matter in the asymptotically free regime.

It becomes a relevant problem at distances $r_{Q\bar{Q}} \sim \Lambda^{-1}$. The confinement radius R_c becomes then comparable to the $Q\bar{Q}$-separation and we depart from asymptotic freedom. Here we need an additional phenomenological assumption.

2. THE QCD BAG

For $r_{Q\bar{Q}} \geq \Lambda^{-1}$ we shall assume the bag picture[4,5] for the heavy $Q\bar{Q}$-system[6]. Accordingly, a bubble with well-defined surface is created around the heavy $Q\bar{Q}$-pair. Perturbation theory is valid inside with a chromoelectric permeability ε approximately one. Across the surface there is a rapid change of ε from one to zero. Correspondingly, the chromomagnetic permeability $\mu = 1/\varepsilon$ crosses over to infinity in the outside medium. The ether is a perfect diaelectric and paramagnetic medium against the gluon fields. On the basis of current Monte Carlo calculations[7] one may conjecture that the bag picture we advocate will be supported or disproved in the near future.

There is considerable activity now to describe the outside medium in terms of an instanton liquid[8], monopole condensate[9], or a condensate of chromomagnetic flux lines[10]. From those investigations we borrow the idea that the difference in energy density between the Fock vacuum and the outside medium is positive and calculable in terms of Λ. The energy required to create the bag phase is $\Lambda_B^4 V + \sigma A$ where V designates the volume of the bubble and A is the surface area.

In order to simplify the forthcoming discussions we ignore here the surface energy σA and only the volume energy is kept in the calculations. This should have very little influence on our numerical predictions in phenomenological applications. In heavy particle spectroscopy the presence of the surface term can be compensated by the readjustment of the volume energy density Λ_B which is often denoted by $B^{1/4}$ in the bag literature[4,5].

Operationally the bag model of heavy particle spectroscopy is defined as follows. We write first the Hamiltonian in an external diaelectric medium as calculated in Coulomb gauge[11],

$$H = \int d^3x \left\{ \frac{1}{2} \vec{D}^a_{tr} \cdot \vec{E}^a_{tr} + \frac{1}{2} \vec{D}^a_{long} \cdot \vec{E}^a_{long} + \frac{1}{2} \vec{B}^a \cdot \vec{H}^a + \psi^+(-i\vec{\alpha}\vec{\nabla} + M)\psi - \vec{j}^a \vec{A}^a \right\} + \Lambda_B^4 \cdot V \quad (3)$$

In Eq. (3) V is the volume of the region around the heavy $Q\bar{Q}$-pair where the spatially varying dielectric permeability ε is one. The standard definitions $\vec{D}^a(\underline{x}) = \varepsilon(\underline{x})\vec{E}^a(\underline{x})$ and $\vec{B}^a(\underline{x}) = \mu(\underline{x})\vec{H}^a(\underline{x})$ are followed with the additional requirement $\varepsilon(x)\mu(x) = 1$. In Eq. (3) ψ designates the heavy quark fields and $\vec{j}^a(\underline{x})$ is their current. Color indices are not written out everywhere.

In the adiabatic Born-Oppenheimer approximation[5,6] we proceed in two steps. First, we solve the problem for fixed quark sources where the shape of the bag is determined by the minimum of the total energy with respect to the variation of the shape. In the second step the static energy of the $Q\bar{Q}$-system is used as the potential energy of the non-relativistic Schrödinger equation of the slowly-moving heavy quarks. With this method we generate the spectrum of $Q\bar{Q}$-states, the spin-dependent force between the $Q\bar{Q}$-pair, and a rich spectrum of $Q\bar{Q}$ glue states where the transverse gluon adiabatically occupies the bag around the slowly moving quarks.

The surface of the bag is not a classical notion in our model, since the outside medium is pictured as a complicated quantum state. Whether the boundary of this medium exhibits some collective excitations remains an open theoretical problem. The latent dynamical degrees of freedom associated with the geometric bag variables are frozen out in our calculation. Whenever there is an additional light particle inside the bag, we calculate first the total energy of the system for fixed $Q\bar{Q}$-position and some fixed bag shape. The effective potential energy of the fixed $Q\bar{Q}$-pair is derived by minimization of the energy with respect to the shape.

3. $Q\bar{Q}$ SPECTROSCOPY

Our previous experience[6] tells us that the numerical solution to the shape of the bag is nearly a sphere for $Q\bar{Q}$-separations $r < \Lambda_B^{-1}$. This simple geometry turns out to be an adequate approximation when compared with our complete numerical solution of the potential energy for the $c\bar{c}$ and $b\bar{b}$ systems. Based on our earlier work this was also pointed out recently by other authors[12].

The confined Coulomb Green's function for a sphere of radius R is given by

$$G(\underline{x},\underline{x}') = \frac{1}{4\pi}\left\{\frac{1}{\varepsilon R} - \frac{2}{R} + \frac{1}{|\underline{x}-\underline{x}'|} + \frac{R}{\left(R^4 + x^2x'^2 - 2R^2xx'\cos\theta\right)^{1/2}}\right. \tag{4}$$

$$\left. - \frac{1}{R}\ln\left[\frac{1}{2} - \frac{xx'\cos\theta}{2R^2} + \frac{1}{2R^2}\left(R^4 + x^2x'^2 - 2R^2xx'\cos\theta\right)^{1/2}\right]\right\},$$

where $\vec{x}$ and $\vec{x}'$ are measured from the centre of the sphere, and $\vec{x}\cdot\vec{x}' = xx' \cos\theta$. The outside permeability ε of the diaelectric vacuum is sent to zero at the end of the calculation.

The static chromoelectric energy of the $Q\bar{Q}$-pair is calculated in lowest order from Eq. (3). The longitudinal part of the Hamiltonian is evaluated by using the instantaneous Green's function $G^{ab}(\underline{x},\underline{x}') = \delta^{ab}G(\underline{x},\underline{x}')$. The static potential energy of the $Q\bar{Q}$-pair is shown in Fig. 1 after minimization with respect to R. The exact potential energy of a deformed bag and the $Q\bar{Q}$ ground state wave functions are also plotted.

At short distances for $r \ll \Lambda_B^{-1}$ the chromoelectric Coulomb energy dominates. For $r \sim \Lambda_B^{-1}$ we expect a rapid crossover from approximately spherical shape into cylindrical shape. The radius of the chromoelectric vortex for large $Q\bar{Q}$-separation is given by

$$R_{vort} = \left(\frac{8\alpha}{3\pi}\right)^{1/4} \frac{1}{\Lambda_B} \quad . \tag{5}$$

The energy per unit length (string tension) along the vortex tube is

$$\lambda = \left(\frac{32\pi}{3}\alpha\right)^{1/2} \Lambda_B^2 \quad . \tag{6}$$

The shape of the bag and the potential energy of the $Q\bar{Q}$-pair were also determined accurately in a computer calculation. The potential energy departs from the spherical approximation only for quark-antiquark separations larger than 0.7 fermi. The shape of the bag as a function of $r_{Q\bar{Q}}$ is shown in Fig. 1.

In our numerical fit to charmonium and T-spectroscopy $\alpha = 0.385$ and $\Lambda_B = 235$ MeV were chosen. The calculated tension with these parameters is $\lambda = 0.198$ (GeV)^2 and the radius of the asymptotic flux tube should be R = 0.64 fermi. In the Schrödinger equation $m_c = 1.35$ GeV and $m_b = 4.75$ GeV were taken for the masses of the c and b quarks, respectively. The Schrödinger equation was solved with our precise numerical potential which is Coulomb-like at short distances and linear at large distances.

Our ψ and T spectra are shown in Fig. 2 where the shape of the bag is also plotted for a separation of the $Q\bar{Q}$-pair at the mean square radius of the ground state wave function. Since we have a good over-all fit to the spectra, we expected T''' to be found at 10.6 GeV in agreement with preliminary data from Cornell[13]. The location of this level was also predicted on the basis of general arguments[14]. We expect little shift in the location of our T''' level due to spin-dependent forces.

We have to point out that the potential energy of the $Q\bar{Q}$-pair and the shape of the bag were determined right after the discovery of the ψ-particle[6]. What is new here is a general Green's function method to evaluate the static chromo-electric energy of an arbitrary colour charge distribution inside the bag. We can now generate on the computer the equivalent of Eq. (4) for very general shapes and multiquark configurations. The careful analysis of the ψ and T-particle spectra in our model is also new.

In the Coulomb region, for $r_{Q\bar{Q}} \ll \Lambda^{-1}$, the bag radius is large and perturbation theory is applicable. We find the relation in Eq. (2) to be valid self-consistently. The first loop correction makes the quark-gluon coupling constant run as a function of quark-antiquark separation. We would have no free parameter at short distances, if the scale parameter Λ in Coulomb gauge was determined somewhere. Phenomenologically our choice of α ignores the first loop correction, but we expect $\alpha = 0.38$ to be the strength of the Coulomb force at about $r \sim 0.2$ fermi.

At large $Q\bar{Q}$-separation perturbation theory remains valid in the model, since the bag becomes a cylindrical tube and the transverse gluon propagating along the vortex develops an effective mass of the order of R^{-1}. Radiative corrections to the "bare" linear Coulomb law are conjectured to remain infrared stable along the longitudinal direction of the $Q\bar{Q}$-axis. This should be seen by explicitly calculating the first loop correction to the Coulomb energy for $r_{Q\bar{Q}} \geq \Lambda^{-1}$. Operationally the coupling constant should freeze at some distance where the rapid crossover occurs from the asymptotically free scale into the linear confinement regime.

It is interesting to note that the shape of the bag is almost spherical when the precocious confinement force sets in at a separation of $r_{Q\bar{Q}} \sim 0.6$ fermi. This observation may suggest a new approach to the study of the space-time picture around a Wilson loop in Monte Carlo calculations[10]) or in some new more elaborate calculational scheme.

4. SPIN-DEPENDENT FORCE

We shall discuss now the exchange of a confined transverse gluon between the heavy $Q\bar{Q}$-pair. The corresponding part of the Hamiltonian in Eq. (3) can be written as

$$H_{spin} = -\frac{g}{2M}\left[\frac{1}{2}\lambda_1^a \vec{\sigma}_1 \vec{B}^a(\vec{r}_1) + \frac{1}{2}\lambda_2^a \vec{\sigma}_2 \vec{B}^a(\vec{r}_2)\right] , \tag{7}$$

where $\vec{B}^a(\vec{r}_1)$ is the chromomagnetic field evaluated at the position of the heavy quark. The second term describes the gluon emission of the antiquark.

In the second order of Schrödinger perturbation theory the level shift due to the exchange of the transverse gluon can be described as

$$\Delta E_{spin} = \frac{g^2}{4M^2}\; \frac{1}{2}\lambda_1^a \sigma_1 \cdot G_{mag}^{ab}(\vec{r}_1,\vec{r}_2)\cdot \sigma_2 \frac{1}{2}\lambda_2^a \quad , \tag{8}$$

where $G_{mag}^{ab} = \delta^{ab}\cdot G_m(\vec{r}_1,\vec{r}_2)$ is the static limit of the transverse chromomagnetic Green's function. In the spherical approximation, which is quite adequate for the ground state, G_{mag}^{ab} is calculable in closed form, similarly to our expression in Eq. (4).

The final formula for the energy shift is given by

$$\Delta E_{spin}(r) = -\frac{2}{3}\vec{\mu}_1\cdot\vec{\mu}_2\ \delta(\vec{r}) + \frac{1}{4\pi r^3}\left[\vec{\mu}_1\vec{\mu}_2 - 3\frac{(\vec{\mu}_1\cdot\vec{r})(\vec{\mu}_2\cdot\vec{r})}{r^2}\right] + \frac{R}{4\pi(R^2+\frac{1}{4}r^2)^3}\left[\frac{1}{4}(\vec{\mu}_1\cdot\vec{r})(\vec{\mu}_2\cdot\vec{r}) - R^2\,\vec{\mu}_1\cdot\vec{\mu}_2\right], \quad (9)$$

where $\vec{r} = \vec{r}_2 - \vec{r}_1$ is the quark-antiquark distance and $\vec{\mu} = \sqrt{4/3}\,(g/2M)\,\vec{\sigma}$ is the chromomagnetic moment of the quark. The second line of Eq. (9) displays the effect of confinement on the exchanged transverse gluon. When $\Delta E_{spin}(r)$ in Eq. (9) is sandwiched between charmonium ground state wave functions, the spin-dependent energy shift

$$\Delta E_{spin} = 180 \text{ MeV}$$

is found. It is larger than the experimental value mainly because our quark mass is rather low and confinement adds about 10 MeV to the contribution from free gluon exchange. We expect a decrease of ΔE_{spin} due to asymptotic freedom effects at short distances.

5. $Q\bar{Q}$ GLUE STATES

In the Born-Oppenheimer approximation we predict a new set of unconventional $Q\bar{Q}$ glue states[6] which bear some resemblance to diatomic molecules. The $Q\bar{Q}$ glue states are classified in Coulomb gauge where a transverse valence gluon is added to the heavy $Q\bar{Q}$-pair. Together they form a colour singlet, since the chromo-electric field of the colour octet $Q\bar{Q}$-pair is absorbed by the colour charge distribution of the transverse gluon.

The motion of the heavy $Q\bar{Q}$-pair is followed adiabatically by the constituent gluon which occupies a definite quantum state in the bag. This quantum state is calculated from the solution of the Maxwell equation as a stationary TE or TM mode which belongs to a given shape. We calculate the total energy of the $Q\bar{Q}$ glue state which includes the instantaneous Coulomb interaction energy of the colour charges inside the bag. The $Q\bar{Q}$-pair represents two point-like charges in colour octet and the gluon wave function corresponds to a continuous colour charge distribution. The confined Coulomb Green's function in Eq. (4) is used throughout the numerical calculation. The effective potential energy of the $Q\bar{Q}$-pair is given in Eq. (10) for the spherical approximation. The minimization $\partial V/\partial R = 0$ of the expression

$$V(r) = \frac{4\pi}{3} R^3 \Lambda_B^4 + \frac{\alpha_{nj}^\lambda}{R} + \frac{1}{6}\frac{\alpha}{r} + \frac{\alpha}{r}\left[-6 + \frac{3r^2}{8R^2} + \frac{1}{1-\frac{r^4}{16R^4}}\left(\frac{7}{24}\frac{r^2}{R^2}+\frac{3}{2}\right) + \frac{7}{12}\ln\frac{R^2+\frac{r^2}{4}}{R^2-\frac{r^2}{4}} - \frac{3}{4}\ln\left(1-\frac{r^4}{16R^4}\right)\right] \quad (10)$$

is required at fixed quark-antiquark separation. In Eq. (10) the term α_{nj}^λ/R is the field energy of the gluon mode without interaction. The eigenvalues α_{nj}^λ are solutions of transcendental equations where λ is TE or TM mode, j designates the angular momentum of the gluon and n is a radial quantum number of the gluon orbital.

The classification of the $Q\bar{Q}$ glue states goes as follows. The total angular momentum of the system is given by

$$\vec{J} = \vec{j} + \vec{\ell} + \vec{s} \quad , \quad (11)$$

where $\vec{j}$ is the angular momentum of the gluon, $\vec{\ell}$ is the orbital angular momentum of the $Q\bar{Q}$-pair and $\vec{s}$ designates the sum of the quark spins. First, we construct the wave function with eigenvalues L, L_z for the "orbital part" $\vec{L} = \vec{j} + \vec{\ell}$ and eigenvalue Λ for the projection of the gluon's angular momentum along the quark-antiquark molecular axis. The wave function without the quark spin is then

$$\vec{\phi}_{LL_z\Lambda}(\vec{r},\vec{r}_g) = \psi_{j\Lambda}^L(r)\, D_{L_z\Lambda}^{L\,*}(\phi,\theta,0)\, \vec{A}_{j\Lambda}^{\lambda}(\vec{r}_g\,') \,. \quad (12)$$

In Eq. (12) the vector pointing from $\bar{Q}$ to Q is denoted by $\vec{r}$ and $\vec{r}_g\,'$ is the gluon coordinate vector in the body-fixed coordinate system whose z'-axis coincides with the $Q\bar{Q}$-axis. The vector $\vec{r}_g\,'$ is measured from the centre-of-mass of the $Q\bar{Q}$-pair and ϕ,θ are the polar angles of $\vec{r}$ in the lab frame. The vector potential $\vec{A}_{j\Lambda}^\lambda(\vec{r}_g\,')$ describes a gluon mode with angular momentum j and projection Λ along the z'-axis.

The radial wave function $\psi^{L}_{j\Lambda}(r)$ of the $Q\bar{Q}$-pair is calculated from the Schrödinger equation

$$-\frac{1}{M_Q}\frac{d^2}{dr^2}\psi^{L}_{j\Lambda}(r)+\frac{1}{M_Q}\frac{\langle(\vec{L}-\vec{j})^2\rangle}{r^2}\psi^{L}_{j\Lambda}(r)+V(r)\psi^{L}_{j\Lambda}(r)=E\,\psi^{L}_{j\Lambda}(r)\;. \qquad (13)$$

The second term in Eq. (13) is the centrifugal energy of the $Q\bar{Q}$-pair with an average over the square of their orbital angular momentum.

For the lowest gluon mode we have $\alpha^{TE}_{01} = 2.744$. The corresponding effective $Q\bar{Q}$-potential is shown in Fig. 3. The ground state wave function of the $c\bar{c}$ glue and $b\bar{b}$ glue states as calculated for the α^{TE}_{01} gluon mode is also shown in Fig. 3. In the calculation of V(r) we used Eq. (10) which assumes a uniform colour charge distribution of the gluon inside the sphere. An exact numerical evaluation of the confined Coulomb interaction energy of the $Q\bar{Q}$ glue system proves this approximation to be adequate within a few MeV accuracy.

The repulsive Coulomb energy $\alpha/6r$ in Eq. (10) arises from the colour octet representation of the $Q\bar{Q}$-pair. At large $Q\bar{Q}$-separation a chromoelectric vortex tube is expected when deformations of the spherical shape are allowed.

This is understood by observing that the colour structure of the $Q\bar{Q}$ glue states can be described by attributing to the quarks and gluon two different kinds of effective Abelian charges. The quark and antiquark attract each other with respect to the charge of first kind and the gluon carries none. The force between the quark and antiquark is repulsive with respect to the charge of second kind whereas the Q glue and $\bar{Q}$ glue interactions are attractive. At large $Q\bar{Q}$-separation the second charges of the quark and antiquark are screened by a split gluon cloud and a chromoelectric vortex tube supported by the charge of the first kind develops. The linear potential energy of this flux tube is $V \sim (\sqrt{7/4})\,\lambda r$ where λ is the tension in $Q\bar{Q}$ systems. From Fig. 3 a precocious approach to this force law is seen before the spherical approximation breaks down.

Here we mention only a few interesting features of the $Q\bar{Q}$ gluon spectrum and details will be given elsewhere[15]. The lowest lying $Q\bar{Q}$ glue states correspond to the TE gluon mode with $\lambda^{TE}_{01} = 2.744$ which is a nodeless state with $j = 1$. To construct a parity eigenstate from the wave function in Eq. (12) we have to introduce the linear combination

$$\vec{\phi}_{LL_z,\Lambda}(\vec{r},\vec{r}_g) + \eta\, \vec{\phi}_{LL_z,-\Lambda}(\vec{r},\vec{r}_g)\,, \tag{13}$$

where the parameter $\eta = \pm 1$ leads to the doubling of states. The parity and charge conjugation quantum numbers of the states is given after the inclusion of the total quark spin S as

$$P = \eta\,(-1)^{L+\Lambda+\lambda}\,,$$

$$C = \eta\,(-1)^{1+\Lambda+j+s+L}\,,$$

with $\lambda = 0$ for TM modes and $\lambda = 1$ for TE gluon modes, respectively.

Among the lowest lying states we have two particularly interesting ones summarized in the table below for $b\bar{b}$ glue states

J^{PC}	Mass (GeV)	Mode	j	Λ	s	L	η
1^{-+}	10.49	TE	1	+1	0	1	+
1^{--}	10.49	TE	1	+1	1	1	+

The mass 10.49 GeV ignores the self mass of the gluon mode and the level shift due to the exchange of a transverse gluon in the $Q\bar{Q}$ glue system. That leads to an estimated ±200 MeV ambiguity in our prediction.

The 1^{-+} state is exotic and can be seen in M1 photon transitions from narrow states produced in e^+e^- annihilation. If our exotic and narrow $b\bar{b}$ glue state stays below $B\bar{B}$ threshold after the above mentioned corrections, its experimental confirmation would be the first direct evidence for gluon degrees of freedom in the hadron spectrum. It would also point, for the first time, to the predictive power of the QCD motivated bag model.

The exotic 1^{-+} state can mix, in principle, with $(Q\bar{Q})_8(q\bar{q})_8$ states where $(q\bar{q})_8$ is a light $q\bar{q}$-pair in colour octet representation. In the same approximation which led to the prediction of 10.45 GeV for the mass of the $b\bar{b}$ glue state, we calculated the mass of the lowest $(Q\bar{Q})_8(q\bar{q})_8$ state to be about 10.9 GeV. With such a large energy denominator and our small coupling constant we expect little mixing of the 1^{-+} $b\bar{b}$ glue state with multiquark objects.

Our 1^{--} $b\bar{b}$ glue state may mix with ordinary $b\bar{b}$ states in a much more pronounced manner making the $B\bar{B}$ theshold region quite complicated.

We were pleased to observe the three bands of a molecular spectrum in our $b\bar{b}$ glue system. The gluon excitations above the ground state are of the order of 500 MeV. For fixed centrifugal energy in Eq. (13), the solutions of the Schrödinger equation correspond to the vibrational spectrum with 200 MeV excitations. Finally, the change in the centrifugal energy leads to rotational levels with 60 MeV separation. If this picture has some element of truth in it, there may be a band of $b\bar{b}$ glue states below $B\bar{B}$ threshold.

Apart from our first qualitative prediction[6)] of the $Q\bar{Q}$ glue states in the bag model, we have found another paper[16)] in the literature where the $Q\bar{Q}$ glue states are treated in a different fashion. The gluon is treated there as a point-like particle interacting with the $Q\bar{Q}$-pair through an *ad hoc* linear potential. There exists also an attempt to use a specific string model[17)].

6. CONCLUSION

We have developed a QCD motivated bag model to describe heavy particle spectroscopy in Born-Oppenheimer approximation. We pointed out that the strength of the Coulomb force at short distances should be ultimately a parameter-free potential thanks to asymptotic freedom. Our value of α and Λ_B is in serious disagreement with the widely accepted MIT parameters as determined from the so-called cavity approximation.

We find a precocious confinement force as the function of $Q\bar{Q}$ separation. The bag is almost spherical when the linear potential sets in after a rapid cross-over from the Coulomb regime.

Our calculation of the spin-dependent force seriously disagrees with the results of a recent paper[12] in which the authors used the old MIT parameters. Adapting their values of $\alpha = 2.2$ and $\Lambda_B = 145$ MeV, we would find the splitting between the ψ and η_c to be greater than 1 GeV. The value of 313 MeV was found in Ref. 12 for the same splitting in clear contradiction with our result. Our explanation of the discrepancy is their incorrect treatment of the exchange of a confined transverse gluon between heavy quarks.

In the near future we would like to apply our method to the spectroscopy of naked charm and beauty.

Acknowledgements

We benefited from the hospitality of the Theory Division at CERN and from very useful discussions with our colleagues, especially A. Martin and Chan Hong-Mo.

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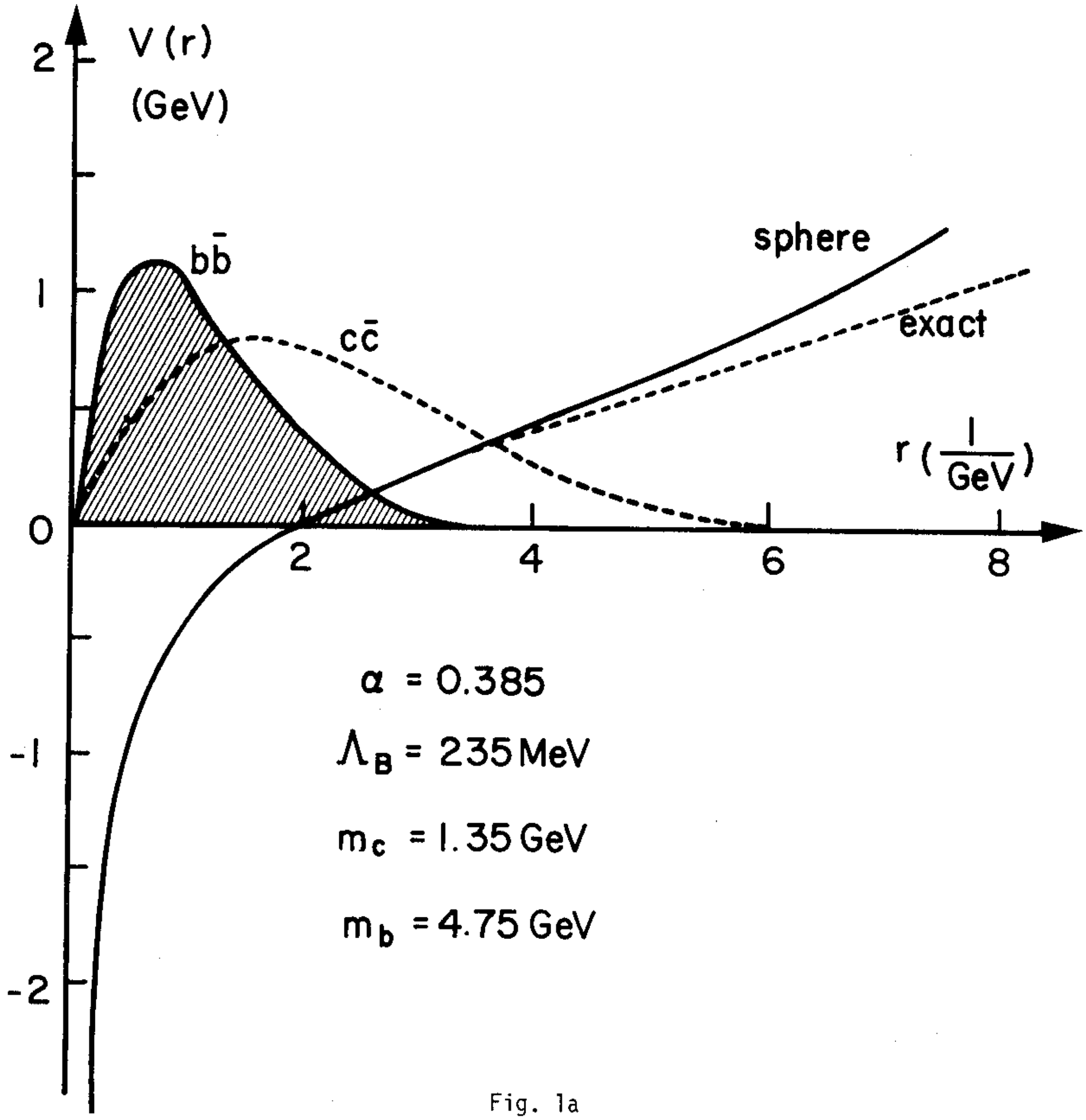
Figure captions

Fig. 1 : In Fig. 1a the potential energy of the $Q\bar{Q}$-pair is shown in the exact calculation and spherical approximation. The wave functions of the $c\bar{c}$ and $b\bar{b}$ ground states are also shown. Figure 1b displays the shape of the bag for different $Q\bar{Q}$-separations. The transverse size of the bag is depicted in Fig. 1c where the rapid convergence to the ideal vortex configuration is seen.

Fig. 2 : The point-like quarks are smeared according to their Compton wavelength to indicate the improvement of the static approximation moving to heavier quarkonia. The dashed line contour is the numerical bag shape as compared with the solid circle of the spherical approximation at the mean $Q\bar{Q}$-separation. The $Q\bar{Q}$ glue states are discussed later on.

Fig. 3 : The effective $Q\bar{Q}$-potential in the $Q\bar{Q}$ glue system and the ground state wave functions are shown. The potential in the spherical approximation is close to the conjectured form.



V(r)
(GeV)
2
1
0
-1
-2
$b\bar{b}$
$c\bar{c}$
sphere
exact
r (1/GeV)
2
4
6
8
α = 0.385
Λ_B = 235 MeV
m_c = 1.35 GeV
m_b = 4.75 GeV

Fig. 1a

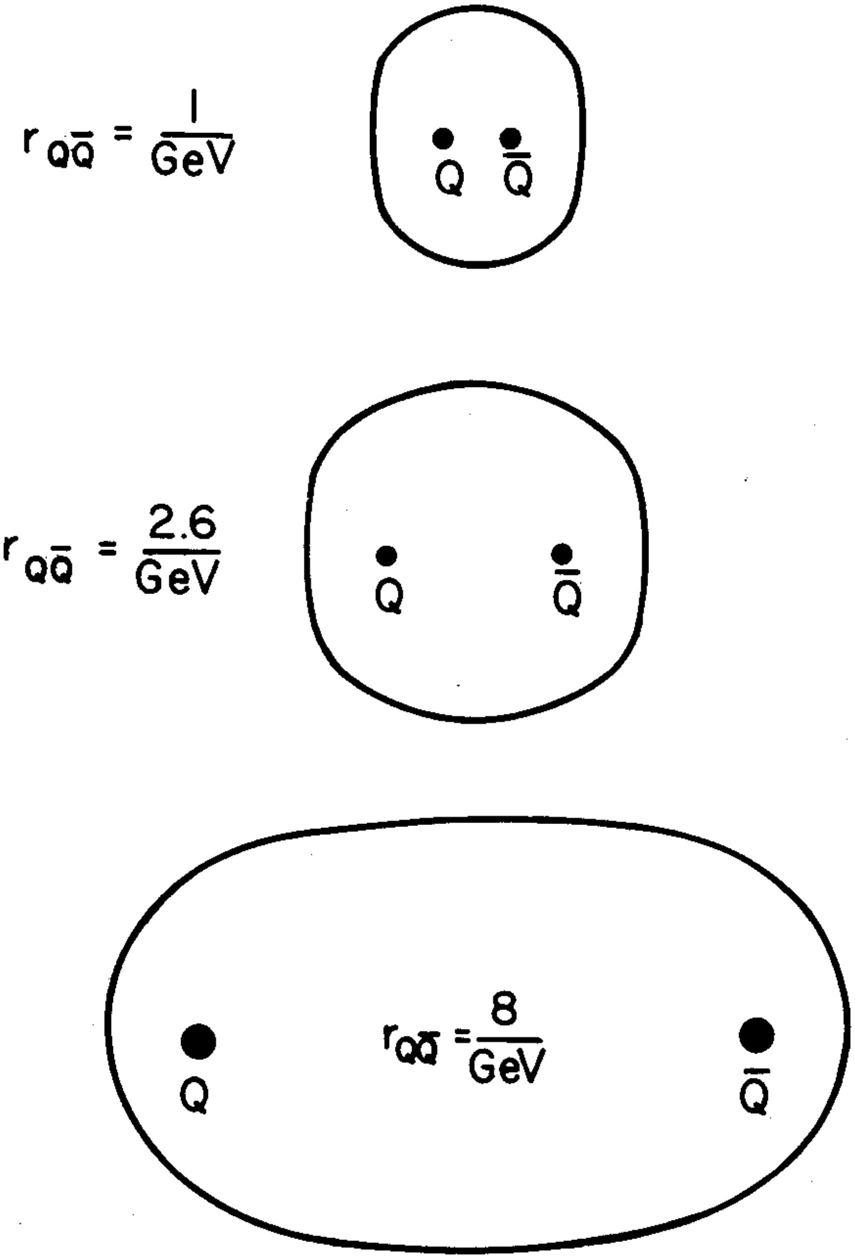


Fig. 1b

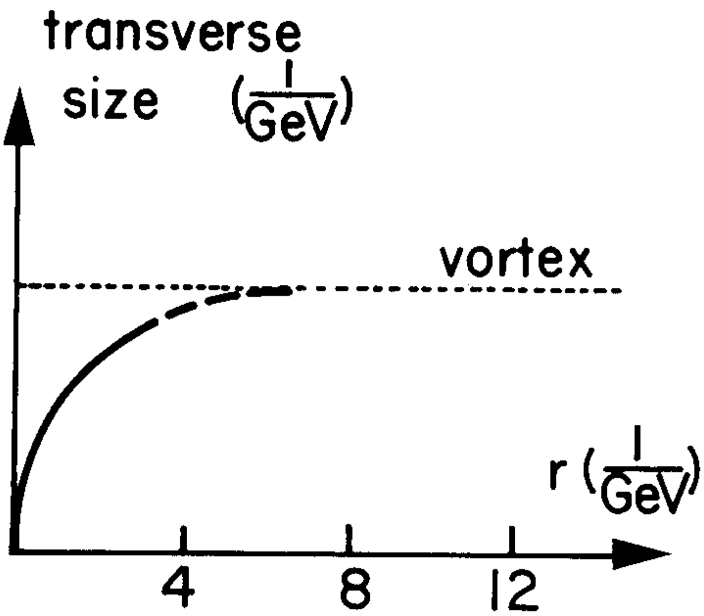


Fig. 1c

SPECTRA

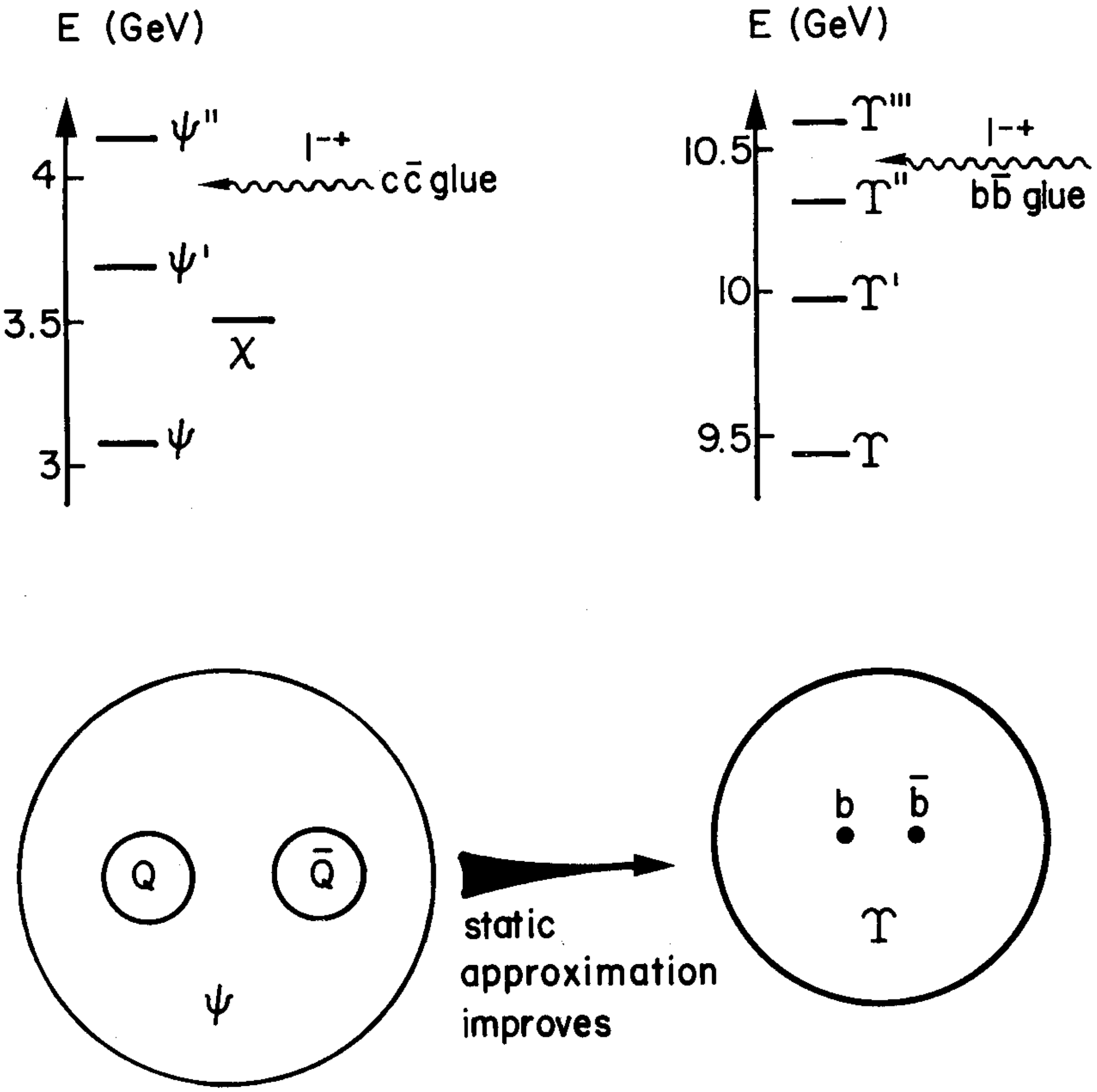
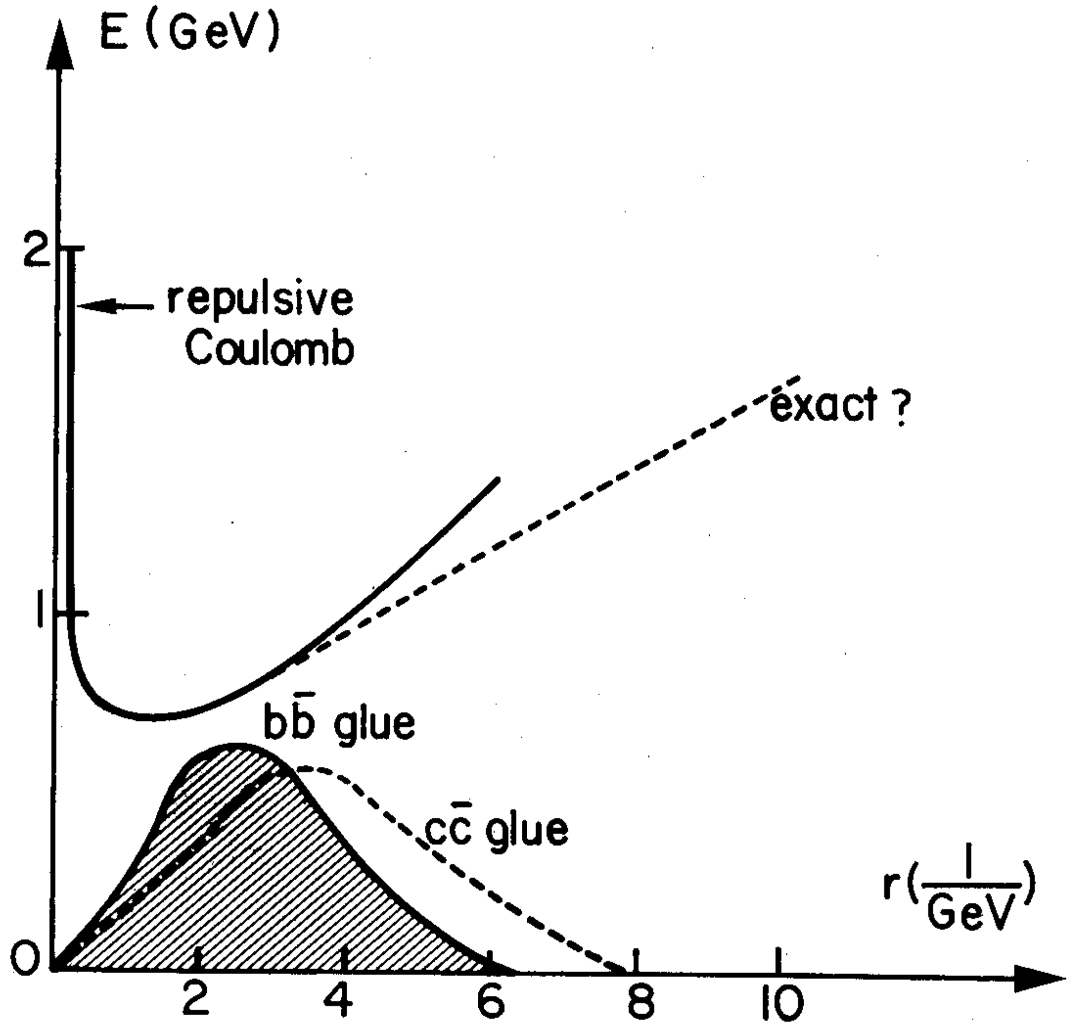


Fig. 2



E (GeV)
2
repulsive
Coulomb
exact ?
1
$b\bar{b}$ glue
$c\bar{c}$ glue
$r(\frac{1}{GeV})$
0
2
4
6
8
10

Fig. 3