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Theoretical foundation for treating decays allowed by the Okubo-Zweig-Iizuka rule and related phenomena

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We derive an expression for the width of unstable mesons whose decays are allowed by the Okubo-Zweig-Iizuka rule. The starting point is an exact expression. In the lattice version a strong-coupling and hopping-parameter expansion of it yields the specific nonperturbative factor for particle-antiparticle creation in a strong field. A more general treatment leads to an expression for the width involving only one universal constant independent of flavor and state. The relation of our formulation to earlier phenomenological approaches is discussed.

I. INTRODUCTION

As is well known the description of decays depends strongly on the type of process involved. We have a good understanding of radiative and semileptonic weak decays (where we know the form of the current coupling to the quarks) as well as of inclusive decays forbidden by the Okuba-Zweig-Iizuka[1] (OZI) rule (where we can calculate the annihilation of the original quark-antiquark pair into gluons perturbatively). The decays allowed by the OZI rule, on the other hand, which, whenever possible, are the dominant ones, have resisted a deeper theoretical understanding up to now. The problem is to find an adequate description of the creation of the quark-antiquark pair within the hadronic color field, and the subsequent dynamical evolution of the system. Almost all phenomenological treatments have therefore directly resorted to simple models. The three essential types are the following.

(1) Elementary meson emission. Here one forgets about the quark-antiquark structure of one of the decay mesons and couples it directly to the quarks. This model has recently been applied successfully to a bulk of baryon decays by Isgur and Koniuk[2] and to meson decays by Godfrey and Isgur.[3]

(2) The quark-pair-creation (QPC) model of Le Yaouanc *et al.*[4] assumes that the $q\bar{q}$ pair is created with vacuum quantum numbers, i.e., in a $^3P_0^{++}$ state. This model successfully describes various polarization properties as well as subtle details due to nodes in wave functions.

(3) The ansatz of Eichten *et al.*[5] consists of an effective interaction operator which determines both the spectrum as well as decay amplitudes. It is of the form

$$H_I = -\tfrac{1}{2}\tfrac{3}{4}\int d^3x\, d^3y :\rho_a^\mu(x)V(x-y)\rho_{a\mu}(y): , \qquad (1.1)$$

where

$$\rho_a^\mu = \sum_A \bar{q}^{\,A}\frac{\lambda^a}{2}\gamma^\mu q^A$$

(a color, A flavor index). Unfortunately the computation can only be done numerically thus rendering a simple interpretation of the results impossible. Even if one is interested in spectrum calculations only, the possibility of pair creation will manifest itself through the phenomenon of screening.[6–8]

For an understanding of the decays allowed by the OZI rule a nonperturbative treatment of the creation of the quark-antiquark pair is a central point. In a fundamental paper, Schwinger[9] gave an exact expression for the creation of e^+e^- pairs in a constant uniform electric field E. His result for the creation probability per unit time and volume is

$$w = \frac{\alpha E^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-n\pi m^2/|eE|) . \qquad (1.2)$$

It clearly shows the nonperturbative character of the problem which is due to the fact that the process proceeds via tunneling: The virtual electron and positron created at the same point have to tunnel a distance $\sim m/|eE|$ in opposite directions in order to regain the minimum energy $2m$ necessary for the creation of a real pair.

Schwinger's formula has been applied to quark-antiquark creation in a hadronic decay or in a jet by Casher, Neuberger, and Nussinov.[10] They assumed essentially a homogeneous color-electric flux between the original $Q\bar{Q}$ pair and applied Eq. (1.2) in the flux tube, setting $w=0$ outside. For further work along these lines we refer to the literature.[11] It is obvious that essential features are missing in these treatments. The color field within the hadron is neither constant nor external, but has to be described as a quantum field in the hadron. The created quarks, furthermore, are not free but become bound into hadrons.

The treatment of all these quantum features is the intention of the present paper. We shall explore directly the time evolution of the decaying system. We recall that in ordinary quantum mechanics the persistence amplitude of an unstable state is simply evaluated by introducing a complete set of energy eigenstates yielding

$$\langle\phi(0)|\phi(T)\rangle = \int e^{-iET}\omega(E)dE, \quad \omega(E)\geq 0$$
$$\approx e^{-iE_0T}e^{-\Gamma T/2} . \qquad (1.3)$$

The exponential decay law holds under rather general assumptions on $\omega(E)$, except for extremely small times (this point has received some attention in connection with proton decay) and extremely large times [where the threshold behavior of $\omega(E)$ becomes important but where the system has practically completely decayed anyhow].

In Sec. II we present a general nonperturbative expression for the persistence amplitude. In Sec. III this expression is evaluated in a simple model using the strong-coupling and hopping-parameter expansion. Though this model involves drastic simplifications, it is very transparent and shows how Schwinger's nonperturbative factor in (1.2) naturally arises from a simple lattice expansion. In Sec. IV we calculate the persistance amplitude for an unstable meson. We reduce the general expression to a nonrelativistic quantum-mechanical path integral in the continuum. This has the advantage of being intimately connected to the simple classical picture of quark trajectories. The creation of the quark-antiquark pair deserves a special treatment. We end up with a formula for the width which, up to an obvious kinematical modification, is almost identical to the QPC model but going beyond this by explicitly specifying the dependence upon the mass of the created pair. In Sec. V we summarize our assumptions, compare with the model of Eichten *et al.* and given an outlook on processes which can be attacked within our approach.

II. GENERAL FORMULATION OF THE PROBLEM

We express the two-point function of a composite hadron as a functional integral. Since we are interested in nonperturbative contributions, we define this integral with the lattice regularization. In Sec. IV we shall go back to the continuum. We use Wilson's lattice action[12]

$$S=S_1+S_2\ ,$$
$$S_1=\kappa m\sum_{n,\mu}\bar\psi_n(1+\gamma^\mu)U_{n,\mu}\psi_{n+\mu}+\text{H.c.}-m\sum_n\bar\psi_n\psi_n\ , \tag{2.1}$$
$$S_2=3\beta\sum_{n,\mu,\nu}\text{Tr}U_{n,\mu}U^\dagger_{n,\nu}U^\dagger_{n+\nu,\mu}U_{n+\mu,\nu}\ .$$

ψ_n is a quark field at the lattice point n and $U_{n,\mu}$ is the gauge group element related to the link pointing from the lattice point n into direction μ. If $\mathbb{A}_\mu$ is the non-Abelian gauge field, then one has in the continuum limit

$$U_{n,\mu}=\exp(i\mathbb{A}_\mu dx^\mu)\ .$$

A meson can be constructed as a superposition of gauge-invariant expressions

$$\phi_n=\bar\psi_{n_1}U_p\psi_{n_2}$$
with
$$U_p=\prod U_{m,\mu}\ . \tag{2.2}$$

The lattice points n_1,n_2 are taken at the same Euclidean time. The product U_p is to be taken along a path connecting n_1 and n_2. The persistence amplitude is a double sum of the correlation functions

$$A_{n',n}=\langle\phi^*_{n'},\phi_n\rangle_S\ , \tag{2.3}$$

where $\phi_{n'}$ is constructed like ϕ_n, but at a later time. The expectation value $\langle\ \rangle_s$ denotes the functional integral over the quark and gauge field (gluon) degrees of freedom with the measure e^{-S}, S being the full action (2.1). In Sec. III we shall take the quarks as static sources, in Sec. IV the superposition will be made with a nonrelativistic wave function. As is well known, the fermion integration over the quark degrees of freedom can be performed, leading to[7,13]

$$A_{n,m}=\sum_\chi\langle Gc_\chi\text{Tr}U_\chi\rangle_{S_2}/\langle G\rangle_{S_2}\ . \tag{2.4}$$

Here χ are paths connecting n_1 to n'_1 and n_2 to n'_2, closed to a loop by the paths P_n, P_m connecting n_1 with n_2 and n'_1 with n'_2, respectively (Fig. 1). U_χ is the product of gauge group elements along the path χ. G is the fermion determinant, which results from the quark integration. The measure in (2.4) is given by the pure gauge Lagrangian S_2. The quantity c_χ does not depend on the gauge field.

The fermion determinant can be expressed as the exponential of a sum over all closed loops φ (see, e.g., Refs. 7 and 13):

$$G=\exp\left[\sum_\varphi c_\varphi\text{Tr}U_\varphi\right]\ . \tag{2.5}$$

We define the connected part of $\langle\text{Tr}\,U_\varphi\text{Tr}\,U_\chi\rangle$ by

$$\langle\text{Tr}U_\varphi\text{Tr}U_\chi\rangle_{\text{con}}=\langle\text{Tr}U_\varphi\text{Tr}U_\chi\rangle_{S_2}-\langle\text{Tr}U_\varphi\rangle_{S_2}\langle\text{Tr}U_\chi\rangle_{S_2}\ . \tag{2.6}$$

If the distance between the loops φ and χ increases, the connected part $\langle\text{Tr}\,U_\varphi\text{Tr}\,U_\chi\rangle_{\text{con}}$ goes to zero.

Many lattice calculations have been performed with fermion determinant fixed to 1. This quenched approximation leads already to satisfactory results for many quantities. It corresponds to an expansion where all hadrons are

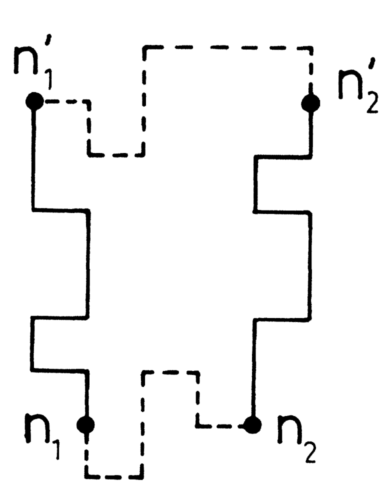

FIG. 1. Quark path χ in Eqs. (2.4) and (2.7). Solid lines: Quark paths. Dashed lines: Strings.

stable; therefore, an evaluation of the width has to take into account the deviation of the fermion determinant from 1. The first nontrivial contributions are one-loop terms:

$$G=1+\sum_{\varphi} c_\varphi \mathrm{Tr} U_\varphi \ .$$

In this approximation the persistence amplitude (2.3) takes the form

$$A_{n,m}=\sum_{\chi} c_\chi \left[\langle \mathrm{Tr} U_\chi \rangle_{S_2} + \sum_{\varphi} c_\varphi \langle \mathrm{Tr} U_\varphi \mathrm{Tr} U_\chi \rangle_{\mathrm{con}} \right] . \tag{2.7}$$

The first term on the right-hand side (RHS) is a sum over all paths of the original quarks. In the nonrelativistic limit it can be identified with a Feynman path integral. We recall that in the strong-coupling limit the integration over the gluon field can be performed explicitly yielding the area law and thus the linearly rising potential.

Equation (2.4) or (2.7) could be evaluated on a lattice by Monte Carlo methods and, in principle, the width of a meson could be extracted from such a numerical analysis. But, in practice, that would be extremely difficult: The Monte Carlo calculations have to be done in Euclidean space-time, whereas the extraction of the width has to be done from an expression in Minkowski space-time. The necessity for doing this is clearly seen from the general form (1.3) which refers to Minkowski time. If we introduce Euclidean time by continuing T to $-iT$; the Fourier transform becomes a Laplace transform which, in general, depends essentially on the mean energy E_0 only. It is only the oscillation of the exponential in (1.3) which makes the integral sensitive to the width. Thus for a simple and model-independent determination of Γ one has to continue (2.4) or (2.7) to Minkowski space. To our knowledge there is only one paper where the authors tried to attack the problem by Monte Carlo calculations.[14] The approach is quite different there and proceeds by extracting the $\rho\pi\pi$ coupling constant from a quark six-point function. Only preliminary results have been obtained there.

In this paper we investigate the second term of the RHS of Eq. (2.7) which determines the decay width. This is done in the next section in the framework of a double strong-coupling and hopping-parameter expansion on the lattice. This treatment will serve as a guideline for a more realistic procedure applied in Sec. IV, where we derive a general expression for the decay amplitude of mesons into mesons.

III. A SIMPLE MODEL

In this section we stay on the lattice. The decaying state ϕ consists of a pair of static (i.e., infinitely heavy) quarks at positions $\mathbf{R}_1$ and $\mathbf{R}_2$, connected in a gauge-invariant way by the product of gauge group matrices U along the connecting straight line. The decay is possible through the production of light quarks. The persistence amplitude $A_{n,m}$ will depend on the spatial separation $\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2$ and the Euclidean temporal separation T. The path χ_c is thus a rectangle with the corners $(\mathbf{R}_1,0)$, $(\mathbf{R}_1,T)$, $(\mathbf{R}_2,T)$, $(\mathbf{R}_2,0)$. Equation (2.4) becomes in this static approximation

$$A(\mathbf{R},T)=c_\chi \langle \mathrm{Tr} U_{\chi_c} G \rangle_{S_2} / \langle G \rangle_{S_2} \ . \tag{3.1}$$

We shall make a double series expansion with respect to $\beta \sim 1/g^2$ (strong-coupling expansion) and the parameter κ (hopping-parameter expansion)[7,13] keeping only the leading terms of the double series. As mentioned in the previous section, the hopping-parameter expansion of the determinant G involves a sum over closed loops where each loop has a numerical factor (which is of no interest here) as well as a factor $(2\kappa)^n$ with n the circumference measured in lattice units. In our approximation there are only two types of contributions to be considered.

(a) No internal fermion loop, i.e., only the expectation value of the external Wilson loop χ_c. This is, of course, nothing else than the usual expression which has to be evaluated for obtaining the static potential in the framework of the pure gauge theory [Fig. 2(a)] [i.e., the first term of Eq. (2.7)].

(b) One rectangular fermion loop lying in the plane of the Wilson loop χ_c completely within the latter, with opposite orientation. This corresponds to the creation and subsequent annihilation of a light-quark pair [Fig. 2(b)] [second term of (2.7)].

In the strong-coupling expansion the leading term in β is obtained by considering only those plaquettes in the expansion of $\exp(-S)$ which form the minimal surface bounded by the loop (or loops). These minimal surfaces are shaded in Figs. 2(a) and 2(b). If their area in lattice units a^2 is denoted by A, the leading contribution to the expectation value is β^A. It is easily seen that all other types of geometrical configurations [some of them are shown in Fig. 2(c)] can be neglected in our approximation because, for any of them, one can find a graph of type (a) or (b) which has a lower order in β or κ.

Let us denote the lattice spacing by a, and the number of links of the internal fermion-loop rectangle in spatial (temporal) direction by n_r (n_t), and set $r=an_r$, $t=an_t$.

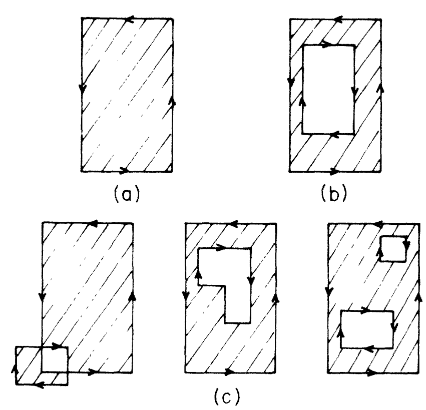

FIG. 2. (a) Wilson loop without internal loop. (b) Wilson loop with one rectangular internal loop in the interior. (c) Some examples of loops of higher order in β or κ which were neglected.

We then have

$$A(R,T)=\beta^{RT/a^2}(1+\eta)\ ,$$
$$\eta=8\sum_{n_r=1}^{R/a}\sum_{n_t=1}^{T/a}(R/a-n_r+1)(T/a-n_t+1)\times(2\kappa)^{2(n_r+n_t)}\beta^{-n_r n_t}\ . \tag{3.2}$$

The term proportional to η is the one due to the loops of the fermion determinant [type (b)]; the factors in parentheses count the number of rectangles with sides $r=an_r$ and $t=an_t$ fitting into the Wilson loop χ_c with sides R, T; the powers of 2κ and β are obvious from the hopping-parameter and strong-coupling expansion, respectively. The physical meaning of β and κ is easily extracted from (3.2). From the first term on the RHS we identify, as usual,

$$\beta^{RT/a^2}=e^{-\lambda RT}\ ,$$

i.e. ,

$$\beta=e^{-\lambda a^2}\ , \tag{3.3}$$

with λ the string tension. The meaning of κ becomes clear if we assume $\beta^{R/a}<(2\kappa)^2$ and take the limit of extremely large T. In this case the term with $r=R$ and $t=T$ dominates the sum in η and $\eta\gg1$. Physically, this means a pair creation of quarks at time zero, the quarks becoming directly attached to the sources. At time T they have to annihilate again, since we ask for the persistence amplitude of the original state. The amplitude becomes

$$A(R,T)\sim 8(2\kappa)^{2R/a}(2\kappa)^{2T/a}\ \text{for}\ T\to\infty \tag{3.4}$$

and

$$\beta^{R/A}<(2\kappa)^2\ .$$

On the other hand, we know that for very large T the time dependence of the system is determined by the ground state of mass $2m$, where $2m$ is the mass of the created quark pair. Thus, from

$$(2\kappa)^{2T/a}=e^{-2mT}$$

we obtain

$$2\kappa=e^{-ma}\ . \tag{3.5}$$

The inequality in (3.4) now reads $2m<\lambda R$ which is the condition that there is enough energy available in the string for the decay to become possible. The mass parameter in (3.5) has to be identified with the constituent mass here; to be more specific, it is the lowest eigenvalue of the system consisting of a quark bound to the external source.

We can perform the summation over n_t in (3.2) and obtain, using relations (3.3) and (3.5),

$$A(R,T)=e^{-\lambda RT}(1+\eta)$$

with

$$\eta=8\sum_{n_r=1}^{R/a}\left[(R/a+1-n_r)e^{-2mr}\frac{e^{aw}}{(1-e^{aw})^2}[e^{aw(T/a+1)}-(T/a+1)e^{aw}+T/a]\right] \tag{3.6}$$

and $w=\lambda r-2m$. For *very* large values of T the term $\exp(wT)$ dominates and we regain the result used earlier that for very large times the time dependence of the persistence amplitude is determined by the threshold $2m$. Equation (3.6) is an analytic interpolation function for the dependence on the Euclidean time T which can be continued to Minkowski time T by mapping $T\to iT$. For a small enough we can expand e^{aw} and replace the summation over n_r by an integration over r/a. This yields

$$\eta(iT)=\frac{8}{a^4}\int_0^R(R-r)e^{-2mr}\frac{e^{iwT}-iwT-1}{w^2}dr\ , \tag{3.7}$$

where T is now Minkowski time. The energy E_0 and the width Γ in equation (1.3) are most easily extracted by performing the time derivative of the logarithm of $A(R,T)$. As in ordinary time-dependent perturbation theory in Minkowski space we can treat η in the lowest order:

$$\frac{d}{dt}\ln A(R,i\Gamma)=-i\lambda R+\frac{8}{a^4}\int_0^R(R-r)e^{-2mr}i\frac{e^{iwT}-1}{w}dr=-iE_0-\Gamma/2\ . \tag{3.8}$$

For large T one has

$$i\frac{e^{iwT}-1}{w}=i\frac{\cos wT-1}{w}-\frac{\sin wT}{w}\to-\pi\delta(w) \tag{3.9}$$

which, as it should, yields energy conservation. Inserting it into Eq. (3.8) results in $E_0=\lambda R$ and

$$\Gamma=\frac{16\pi}{a^4\lambda^2}\theta(\lambda R-2m)(\lambda R-2m)e^{-4m^2/\lambda}\ . \tag{3.10}$$

We have thus obtained the essential nonperturbative factor $\exp(-4m^2/\lambda)$ which is also present in Schwinger's formula. The electrostatic force eE has now been replaced by the force λ of the string. The appearance of 4 instead of π is clearly a lattice artifact. The threshold condition $\lambda R>2m$ has come out automatically. The overall factor in front of Γ depends on the lattice spacing a. Since in the strong-coupling limit there is no unambiguous way to fix a, we have to leave this factor free. It is independent of the mass m.

IV. A GENERAL EXPRESSION FOR DECAYS ALLOWED BY THE OZI RULE

We shall now extend the treatment of the general formulas of Sec. II to cover the realistic case of meson decays, working from now on in continuum Minkowski space. The central idea is simple: Concerning the original quark and antiquark paths $\mathcal{X}$ in Eq. (2.7), instead of restricting them to the straight lines of the static sources, we have now to sum over *all* paths connecting $(\mathbf{R}_\alpha,0)$ with $(\mathbf{R}'_\alpha,T)$ $(\alpha=1,2)$. Our approximation will be to consider only those paths which correspond to a nonrelativistic motion. The expression (2.7) then goes over into the usual nonrelativistic path-integral representation for a Green's function. Since path integrals are dominated by the classical trajectories and very high values of the momenta lead to large oscillations in the weight factor e^{iS}, the assumption of dominance of nonrelativistic trajectories appears justified as well as, say, in spectrum calculations, which, as is well known, are very successful even for hadrons consisting of light quarks.[15]

As for the loops from the determinant describing the creation and subsequent annihilation of a quark-antiquark pair we shall still restrict ourselves to one loop; for the timelike parts we also restrict ourselves to nonrelativistic trajectories. The creation and annihilation of the quark pair needs a separate treatment. For the moment we only will assume that these processes take place rather instantaneously. Physically this means that the created quarks have to tunnel very fast to an appropriate distance in order to get back the energy which had to be borrowed for the creation. Later we will be more specific on this point.

The decaying meson will, as usual in spectrum and decay calculations, be represented by its quark-antiquark component:

$$|B(\mathbf{K}_B,\tau)\rangle=\int \underline{\bar{\psi}(\mathbf{R}_1,\tau)O\psi(\mathbf{R}_2,\tau)}\,|0\rangle \times\varphi(\mathbf{R})e^{-i\mathbf{K}_B\cdot\mathbf{X}}d^3R\,d^3X\ . \tag{4.1}$$

Here ψ, $\bar\psi$ are the quark operators, O describes spin and flavor, and $\varphi(\mathbf{R})$ is the spatial wave function. Quark and antiquark have position variables $\mathbf{R}_1,\mathbf{R}_2$ and momenta $\mathbf{Q}_1,\mathbf{Q}_2$, respectively. The state has been made gauge invariant by the insertion of the path-ordered exponential

$$P\exp\left[ig\int A_\mu dx^\mu\right]$$

along the string between quark and antiquark. Here and in the following this will be abbreviated by the horizontal bracket connecting the field operators. We have introduced relative and c.m. coordinates

$$\mathbf{R}=\mathbf{R}_1-\mathbf{R}_2,\quad \mathbf{X}=(M_1\mathbf{R}_1+M_2\mathbf{R}_2)/M\ ,\qquad M=M_1+M_2 \tag{4.2}$$

and the respective momenta

$$\mathbf{Q}=(M_2\mathbf{Q}_1-M_1\mathbf{Q}_2)/M,\quad \mathbf{K}_B=\mathbf{Q}_1+\mathbf{Q}_2\ . \tag{4.3}$$

Quantities with a prime refer to the time T. Since we are interested in the genuine decay but not in the trivial quantum-mechanical spreading of a localized state we have to use eigenstates of the c.m. momentum $\mathbf{K}_B$.

In the persistence amplitude

$$\langle B(\mathbf{K}_B,0)\,|\,B(\mathbf{K}'_B,T)\rangle$$

the integration over the variable $\mathbf{X}+\mathbf{X}'$ is trivial due to translational invariance and gives the δ function $\delta^{(3)}(\mathbf{K}_B-\mathbf{K}'_B)$ of conservation of c.m. momentum. From now on we shall therefore drop the $(\mathbf{X}+\mathbf{X}')$ integration, take $\mathbf{X}'=-\mathbf{X}$, and go to the rest system $\mathbf{K}_B=\mathbf{K}'_B=0$. The persistence amplitude to be evaluated thus becomes

$$A(T)=\int\langle 0\,|\,\underline{\bar\psi(\mathbf{R}_2,0)\bar{O}\psi(\mathbf{R}_1,0)}\,\underline{\bar\psi(\mathbf{R}'_1,T)O\psi(\mathbf{R}'_2,T)}\,|\,0\rangle \times\varphi^*(\mathbf{R})\varphi(\mathbf{R}')d^3R\,d^3R'd^3(2X)\ . \tag{4.4}$$

We shall consider again two types of contributions to (4.4). The first is the one without internal quark loops, i.e., the first term of Eq. (2.7); it corresponds to the contributions of type (a) in the preceding section. It simply gives the exponential time dependence

$$A^{(a)}(T)=\exp[-i(M+E_0)T]\ , \tag{4.5}$$

where $M=M_1+M_2$ is the sum of the rest masses and E_0 the binding energy. This result can be also derived along the lines presented in the following, which would, however, be unnecessarily complicated here.

We next come to the contributions containing the creation and subsequent annihilation of a quark-antiquark pair [type (b) contributions]. Coordinates and momenta of "internal quarks" will be denoted by small letters $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{q}_1$, $\mathbf{q}_2$, their mass by m. The creation takes place at time t_i, the annihilation at time t_f, the lifetime of the pair is thus $t=t_f-t_i$. In accordance with our nonrelativistic kinematics we consider only loops for which $0\le t_i\le t_f\le T$. As a first step the integration over the gluon fields is performed formally. The time evolution is now described as follows (Fig. 3). For times $0<\tau<t_i$ the system consists of the original quark Q_1 and antiquark $\bar{Q}_2$ only. The contribution to the Green's function in

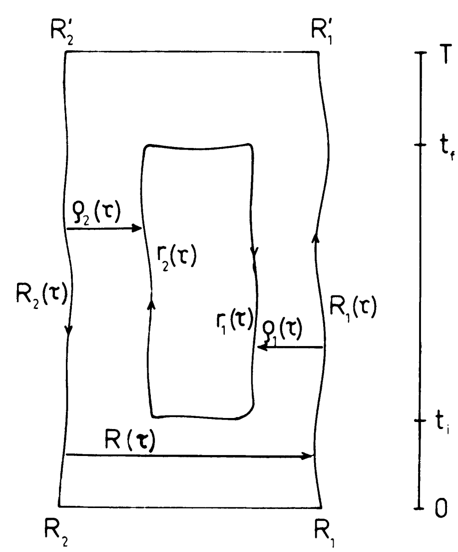

FIG. 3. Notations for the quark trajectories. The definitions of the relative coordinates and the notations for momenta are given in Eqs. (4.2), (4.3), and (4.7).

(4.4) is a path integral involving the integrations $D[\mathbf{R}_1(\tau)]D[\mathbf{R}_2(\tau)]$. Introduction of the relative and c.m. coordinates $\mathbf{R}(\tau)$ and $\mathbf{X}(\tau)$, defined analogously as in (4.2), factorizes the functional integral into the product of two ordinary Green's functions: The bound-state Green's function G_μ^V referring to $D[\mathbf{R}(\tau)]$ and the reduced mass

$$\mu=(M_1+M_2)/M_1M_2 \ ,$$

and the free Green's function G_M^0 of the c.m. motion.

At time t_i the pair $\bar{q}_1 q_2$ is created. For the moment we only assume that the creation can be described by a function F which may depend upon the three distances

$$\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2,\ \ \boldsymbol{\rho}_1=\mathbf{r}_1-\mathbf{R}_1,\ \ \boldsymbol{\rho}_2=\mathbf{r}_2-\mathbf{R}_2 \tag{4.6}$$

between the four quarks involved at time t_i, as well as upon the spins which we suppress for the moment. It is convenient to split off from F a factor i in order to make it a T-matrix element.

In the time interval $t_i<\tau<t_f$ the system contains two pairs of quarks and antiquarks which may be viewed as a virtual-meson pair, i.e., the would-be decay products. In analogy with the previous model we assume that we have to consider only the interaction between $Q_1\bar{q}_1$ and between $\bar{Q}_2 q_2$, i.e., within the virtual mesons, while there is no longer a sizable interaction between the two virtual mesons. We introduce the new coordinates

$$\boldsymbol{\rho}_\alpha=\mathbf{r}_\alpha-\mathbf{R}_\alpha$$

as in (4.6) and

$$\mathbf{X}_\alpha=(m\mathbf{r}_\alpha+M_\alpha\mathbf{R}_\alpha)/(M_\alpha+m),\ \ \alpha=1,2 \ . \tag{4.7}$$

The integrals over the four paths $D[\mathbf{X}_\alpha(\tau)]$, $D[\boldsymbol{\rho}_\alpha(\tau)]$ then factorize into four Green's functions, two of them describing the free motion of the centers of masses and two the internal motion in the would-be decay mesons. The further time development is clear. The function F responsible for the annihilation at time t_f fulfills $F_f=F_i^*$ from time-reversal invariance, while in the interval $t_f<\tau<T$ the situation is analogous to that in the first interval. We thus arrive at the following form for the contributions of type (b):

$$\begin{aligned}A^{(b)}(T)=\int & e^{-iMT}e^{-2imt}G_M^0(\mathbf{X}^{(i)}-\mathbf{X},t_i)G_\mu^V(\mathbf{R}^{(i)},\mathbf{R},t_i)iF(\mathbf{r}^{(i)},\boldsymbol{\rho}_1^{(i)},\boldsymbol{\rho}_2^{(i)})\\ &\times G^0_{M_1+m}(\mathbf{X}_1^{(f)}-\mathbf{X}_1^{(i)},t)G^0_{M_2+m}(\mathbf{X}_2^{(f)}-\mathbf{X}_2^{(i)},t)G^{V_1}_{\mu_1}(\boldsymbol{\rho}_1^{(f)},\boldsymbol{\rho}_1^{(i)},t)G^{V_2}_{\mu_2}(\boldsymbol{\rho}_2^{(f)},\boldsymbol{\rho}_2^{(i)},t)\\ &\times iF^*(\mathbf{r}^{(f)},\boldsymbol{\rho}_1^{(f)},\boldsymbol{\rho}_2^{(f)})G_M^0(-\mathbf{X}-\mathbf{X}^{(f)},T-t_f)G_\mu^V(\mathbf{R}',\mathbf{R}^{(f)},T-t_f)\\ &\times\varphi^*(\mathbf{R})\varphi(\mathbf{R}')d^3R\,d^3R'd^3(2X)dt\,dt_i d(i)d(f) \ .\end{aligned} \tag{4.8}$$

The rest masses M (which is present in the whole time T) and $2m$ (present only for the interval $t=t_f-t_i$) are exhibited explicitly. To obtain all loops of type (b) we have to integrate over the four positions of the quarks at time t_i and t_f [symbolically written as $d(i)d(f)$] and over t and t_i ($0\le t_i\le T-t$, $0\le t\le T$). The interpretation of the rest of the integrand should be clear from the preceding discussion.

To evaluate (4.8) we introduce the eigenfunction expansions for the Green's functions, i.e.,

$$G_M^0(\mathbf{X},t)=(2\pi)^{-3}\int e^{i\mathbf{K}\cdot\mathbf{X}}e^{-(i\mathbf{K}^2/2M)t}d^3K \tag{4.9}$$

for the free functions and

$$G_\mu^V(\boldsymbol{\rho}',\boldsymbol{\rho},t)=\sum_n \psi_n(\boldsymbol{\rho})\psi_n^*(\boldsymbol{\rho}')e^{-iE_nt} \tag{4.10}$$

for the bound-state functions. The following steps are elementary and will therefore only be sketched.

The integrations over $\mathbf{R}$ and $\mathbf{R}'$ project out one term, respectively, from the series for $G_\mu^V(R^{(i)},\mathbf{R},t_i)$ and $G_\mu^V(\mathbf{R}',\mathbf{R}^{(f)},T-t_f)$, namely, that containing the wave function φ. This introduces a factor

$$\varphi(R^{(i)})\varphi^*(R^{(f)})e^{-iE_0(T-t)} \ .$$

The X integration gives $(2\pi)^3\delta^{(3)}(\mathbf{K}+\mathbf{K}')$; the variable t_i drops out from the integrand; therefore, the t_i integration just gives a factor $(T-t)$. Instead of the eight integration variables $R_\alpha^{(j)},r_\alpha^{(j)}$ with $j=i,f$: $\alpha=1,2$, we now introduce the new variables

$$\mathbf{r}^{(j)},\boldsymbol{\rho}_\alpha^{(j)}$$

with

$$j=i,f;\alpha=1,2$$

and

$$\mathbf{z}=\tfrac{1}{2}(\mathbf{X}^{(i)}+\mathbf{X}^{(f)}),\ \ \mathbf{y}=\tfrac{1}{2}(\mathbf{X}_1^{(f)}-\mathbf{X}_1^{(i)}+\mathbf{X}_2^{(f)}-\mathbf{X}_2^{(i)}) \ . \tag{4.11}$$

This has the advantage that one can perform two more integrations over variables appearing in the exponential only: The z integration gives $\pi^3\delta^{(3)}(\mathbf{K})$; the y integration gives

$$(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2)$$

($\mathbf{k}_\alpha$ are the conjugate momenta of $\mathbf{X}_\alpha$). Thus from now on we set $\mathbf{k}_1=-\mathbf{k}_2=\mathbf{k}$. The integrals involving $\mathbf{r}^{(f)}$, $\boldsymbol{\rho}_1^{(f)}$, $\boldsymbol{\rho}_2^{(f)}$ have become the complex conjugate of those involving $\mathbf{r}^{(i)}$, $\boldsymbol{\rho}_1^{(i)}$, $\boldsymbol{\rho}_2^{(i)}$. We can combine them into an absolute square, dropping the indices i and f from now on. As an intermediate result we then arrive at

$$A^{(b)}(T)=-(2\pi)^{-3}e^{-i(M+E_0)T}\sum_{n_1,n_2}\int(T-t)\left|\int\varphi(\mathbf{r}-\rho_1+\rho_2)\psi^*_{n_1}(\rho_1)\psi^*_{n_2}(\rho_2)F(\mathbf{r},\rho_1,\rho_2)\right.$$
$$\times\exp\left[-i\mathbf{k}\cdot\left[\mathbf{r}-\frac{M_1}{m+M_1}\rho_1+\frac{M_2}{m+M_2}\rho_2\right]\right]d^3r\,d^3\rho_1 d^3\rho_2\Bigg|^2$$
$$\times\exp\left[-i\left[2m-E_0+E_{n_1}+E_{n_2}+\frac{k^2}{2(M_1+m)}+\frac{k^2}{2(M_2+m)}\right]t\right]d^3k\,dt. \quad (4.12)$$

The factor $e^{-i(M+E_0)T}$ is also present in the term $A^{(a)}$ in (4.5). The t integration can now be performed. As in Sec. III we next calculate $(d/dT)\ln A(T)$, choose T large, which again introduces the energy-conservation δ function, and determine the width Γ from comparison with (1.3). This leads to

$$\Gamma=(2\pi)^{-2}\sum_{n_1,n_2}\int\delta\left[E_0-2m-E_{n_1}-E_{n_2}-\frac{\mathbf{k}^2}{2(M_1+m)}-\frac{\mathbf{k}^2}{2(M_2+m)}\right]$$
$$\times\left|\int\varphi(\mathbf{r}-\rho_1+\rho_2)\psi^*_{n_1}(\rho_1)\psi^*_{n_2}(\rho_2)F(\mathbf{r},\rho_1,\rho_2)\right.$$
$$\times\exp\left[-i\mathbf{k}\cdot\left[\mathbf{r}-\frac{M_1}{m+M_1}\rho_1+\frac{M_2}{m+M_2}\rho_2\right]\right]d^3r\,d^3\rho_1 d^3\rho_2\Bigg|^2 d^3k\ . \quad (4.13)$$

We have next to specify the creation function F. One can give two alternative arguments, both leading to the same result. For the first we go back to the hopping-parameter expansion on the lattice, transformed into Minkowski space. For a spatial straight line of length $r=an$ there is a factor proportional to

$$\left[\frac{1+i\gamma\cdot\mathbf{r}/r}{2}\right]^n(2\kappa)^n=\frac{1+i\gamma\cdot\mathbf{r}/r}{2}e^{-mr}\ .$$

We made use of the fact that the first factor is a projector and used (3.5). The lattice spacing a has dropped out which gives some confidence in the formula. We therefore will use

$$F(\mathbf{r},\rho_1,\rho_2)=F(\mathbf{r})=Ce^{-mr}(1+i\gamma\cdot\mathbf{r}/r)/2\ , \quad (4.14)$$

where C is a universal constant of dimension (mass)^4. We can also derive Eq. (4.14) (and generalize it to arbitrary curves) in a different way where we can stay in the continuum. Let us go to Euclidean space for simplicity and consider the equation for the quark propagator $S(x,y;A)$ in a given external gauge field A. It satisfies the equation

$$\{\gamma_\mu[\partial^\mu_{(x)}-igA^\mu(x)]+m\}S(x,y;A^\mu)=\delta^4(x-y)\ . \quad (4.15)$$

Consider a curve $z^\mu(s)$ with the tangent vector $t^\mu(s)=dz^\mu(s)/ds$, normalized to $t^2=1$. Let $x^\mu=z^\mu(\sigma)$, $y^\mu=z^\mu(\tau)$ be points on this curve. We then look for an approximated propagator $\hat{S}$ obtained from (4.15) by only keeping the covariant derivative in the direction of the curve (the special case that the curve is a straight line $\mathbf{R}$=const of a particle at rest has been used as a starting point of a $1/m^2$ expansion by Eichten and Feinberg[16]. With $\hat{\partial}=t^\mu\partial_\mu$, etc., we obtain the equation

$$[\hat{\gamma}(\hat{\partial}_{(x)}-ig\hat{A})+m]\hat{S}(\sigma,\tau;A^\mu)=\delta(\sigma-\tau)\delta^3(\mathbf{x}_\perp-\mathbf{y}_\perp) \quad (4.16)$$

which has the solution

$$\hat{S}(\sigma,\tau;A^\mu)=\left[\theta(\sigma-\tau)\frac{1+\hat{\gamma}(\sigma)}{2}+\theta(\tau-\sigma)\frac{1-\hat{\gamma}(\sigma)}{2}\right]\times e^{-m|\sigma-\tau|}P(\tau,\sigma)\delta^3(\mathbf{x}_\perp-\mathbf{y}_\perp)\ . \quad (4.17)$$

Here

$$P(\tau,\sigma)=P\exp\left[ig\int_y^x A^\mu(z)dz_\mu\right]=P\exp\left[ig\int_\tau^\sigma\hat{A}(s)ds\right]$$

is the string operator along the curve. Again we have found the spin factor and the exponential suppression as in (4.14).

We next introduce (4.14) into (4.13), go to momentum space, and take F between spinors $\bar{u}(q_2)$ and $v(q_1)$, normalized to $\bar{u}u=-\bar{v}v=1$. The momenta in $\tilde{\varphi}$, $\tilde{\psi}^*_{n_1}$, $\tilde{\psi}^*_{n_2}$ are $\mathbf{Q}$, $\mathbf{p}_1$, $\mathbf{p}_2$, respectively, and $\mathbf{q}_1=-\mathbf{q}_2=\mathbf{q}=\mathbf{k}-\mathbf{Q}$. The ρ_1 integration gives

$$(2\pi)^3\delta^3(\mathbf{Q}+\mathbf{p}_1-M_1\mathbf{k}/(M_1+m))\ ,$$

the ρ_2 integration

$$(2\pi)^3\delta^3(\mathbf{Q}-\mathbf{p}_2-M_2\mathbf{k}/(M_2+m))\ .$$

In the lowest order of q/m one has

$$\bar{u}_2(-\mathbf{q})(1+\gamma\cdot\mathbf{q}/m)v_1(\mathbf{q})=2\chi_2^\dagger\sigma\cdot\mathbf{q}\chi_1^c/m \quad (4.18)$$

with $\chi^c=-i\sigma_2\chi$. The r integration therefore involves

$$\int(1+i\gamma\cdot\mathbf{r}/r)e^{-mr}e^{i\mathbf{q}\cdot\mathbf{r}}d^3r=\frac{8\pi m}{(m^2+q^2)^2}(1+\gamma\cdot\mathbf{q}/m)\ . \quad (4.19)$$

Putting all this together we arrive at our final formula for the decay width:

$$\Gamma=16(2\pi)^3|C|^2\sum_{n_1,n_2}\int\delta\left[E_0-2m-E_{n_1}-E_{n_2}-\frac{k^2}{2(M_1+m)}-\frac{k^2}{2(M_2+m)}\right]\times\left|\int\tilde{\varphi}(\mathbf{Q})\tilde{\psi}^*_{n_1}(\eta_1\mathbf{k}-\mathbf{Q})\tilde{\psi}^*_{n_2}(\mathbf{Q}-\eta_2\mathbf{k})\frac{\chi_2^\dagger\sigma(\mathbf{k}-\mathbf{Q})\chi_1^C}{[m^2+(\mathbf{k}-\mathbf{Q})^2]^2}d^3Q\right|^2 d^3k \tag{4.20}$$

with $\eta_j=M_j/(M_j+m)$. As mentioned, this result contains only one free parameter, the universal constant C.

V. DISCUSSION AND CONCLUSION

The main features of our final formula (4.20) can be easily understood. The δ function is due to energy conservation. In applications one would prefer the relativistic version

$$\delta(M_0-(M_{n_1}{}^2+k^2)^{1/2}-(M_{n_2}{}^2+k^2)^{1/2}) .$$

Usually only a few low-lying states labeled by n_1,n_2 can contribute kinematically and the respective terms in the sum are the corresponding partial widths. Our spin structure is identical to that of the QPC model[4] stating that the created pair has vacuum quantum numbers and is thus in a $^3P_0^{++}$ state. This is due to the creation function F which does not allow the exchange of quantum numbers between the created pair and the original quarks. Our kinematics is somewhat different from that in Ref. 4, which would become identical to ours in the limit m/M_1, $m/M_2\to 0$. The meaning of the momenta appearing in $\tilde{\psi}^*_{n_1},\tilde{\psi}^*_{n_2}$ is simply understood: Because $\mathbf{Q}$ is the momentum of the original quark (mass M_1), $\mathbf{q}$ that of the created antiquark (mass m), $\mathbf{k}$ that of the decay meson ($\mathbf{k}=\mathbf{Q}+\mathbf{q}$), these are just the correct relative momenta $\mathbf{p}_{1,2}$ in the decay mesons. [The authors of Ref. 4 also found this obvious modification of their formula (private communication).] The factor

$$|\mathbf{k}-\mathbf{Q}|[m^2+(\mathbf{k}-\mathbf{Q})^2]^{-2}$$

in (4.20) goes beyond the formulation in Ref. 4. It gives an additional momentum dependence which is of dynamical origin, together with the explicit dependence on the constituent-quark mass m. The mass dependence is, however, less dramatic as it might look because the decay momentum $\mathbf{k}$ is usually rather large (several hundred MeV). For a test of the dependence on the mass of the created quarks the $I=1$ resonances $A_2(1320)$, $\rho'(1600)$, $g(1690)$, which decay both into nonstrange and strange mesons, are especially suited. This test will be presented elsewhere.

For the understanding of decays allowed by the OZI rule it would be important to clarify the assumptions which lead to the QPC model[4] on one side or to the model of Ref. 5 on the other. In our approach we took into account the interaction within the decay mesons but did not consider any interaction between them. This picture would be exact in the strong-coupling approximation[17] and has been extended here to arbitrary potentials. It seems to us the appropriate approximation, since the forces inside a hadron are much stronger than the interhadronic forces.

While our assumptions which led to the $^3P_0$ QPC model were motivated from the strong-coupling expansion, the approach of Ref. 5 can be motivated from perturbation theory. Indeed from one-gluon exchange one would obtain Eq. (1.1) with $V(r)=-\alpha_s/r$ in the nonrelativistic limit. The creation function F would be the perturbative one with a gluon exchanged between an original quark and the created pair. It would no longer lead to $^3P_0^{++}$ quantum numbers only. In this model the extension from perturbation theory consists in admitting a more general potential than the Coulomb one. This modification is exactly true for all graphs which simply modify the gluon propagator. Such an approach, however, would imply a pure vector potential. This is in contrast with convincing arguments that the long-range part of the potential is scalar.[18,15] For these reasons we believe that our picture is relevant for decays. Nevertheless, it would be very interesting to pursue our approach within the ansatz of Eichten *et al.* Unfortunately, this cannot be done in a simple way because one then has interactions between all pairs of quarks thus preventing the factorization of the problem.

The approach presented here can be extended to a variety of processes where nonperturbative effects are supposed to play an important role. We think especially of baryon decays, jets, sequential pair production[19] and annihilation (like $p\bar{p}$). In all these cases an investigation of the influence of the flavor dependence and kinematics appears especially promising in our approach.

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