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PHOTON AS A GOLDSTONE BOSON IN (2 + 1)-DIMENSIONAL ABELIAN GAUGE THEORIES

A. KOVNER[1,*], B. ROSENSTEIN[1,2,**] and D. ELIEZER[1,***]

[1] *Physics Department, University of British Columbia, Vancouver, B.C., Canada, V6T 2A6*
[2] *Institute of Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*

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We examine the relationship between the masslessness of a photon and the realization of global symmetries in abelian gauge theories in 2 + 1 dimensions: scalar and spinor QED and their immediate generalizations. We find that this masslessness (in the Coulomb phase) directly follows from the spontaneous breakdown of a symmetry generated by the magnetic flux. In spinor QED with two flavours the chiral symmetry is broken as well but a linear combination of the flux and chiral symmetries remains unbroken. A similar symmetry breaking pattern $U(1) \otimes U(1) \to U(1)$ is realized also in Chern–Simons electrodynamics for a particular value of the Chern–Simons coefficient at which the photon becomes massless (anyon superconductor). The pertinent order parameter for the Higgs–Coulomb phase transition in scalar QED is identified with the vev of the magnetic vortex creation operator $V(x)$. We calculate, using weak coupling perturbation theory, the vev and the correlator of $V(x)$ in both phases. This turns out to be equivalent to evaluation of the euclidean QED partition function in the presence of the external current which produces a magnetic monopole (with the contribution of the Dirac string subtracted). In the Higgs phase this vev vanishes in accord with the Wigner–Weyl realization of the flux symmetry. In the Coulomb phase of scalar QED we obtain a nonzero value of the order parameter whereas in the spinor QED it vanishes. This indicates that in the scalar QED the symmetry breaking is of the usual Nambu–Goldstone type while in the spinor QED it is of Kosterlitz–Thouless type.

1. Introduction

Scalar massless particles have long been associated with the spontaneous breaking of underlying continuous symmetries of a quantum field theory. There are numerous examples of Goldstone bosons in $d > 2$. Physical realization of this phenomenon can be found in QCD where pseudoscalar mesons are approximately Goldstone bosons associated with the spontaneous breaking of chiral symmetry. In $d = 2$, although the vev of any order parameter vanishes [1], this phenomenon

* KOVNER@PHYSICS.UBC.CA
** BARUCH@PHYSICS.UBC.CA
*** ELIEZER@PHYSICS.UBC.CA

continues to manifest itself in the guise of massless Kosterlitz–Thouless bosons [2, 3]. For example, the massless mode in the Gross–Neveu model appears as a result of the Kosterlitz–Thouless realization of the chiral U(1) symmetry [3].

Generally the Goldstone theorem can also lead to the appearance of nonscalar massless particles. This, for example, is the case of Goldstinos which appear as a result of SUSY breaking.

There is another important class of theories which contain massless particles: gauge theories. Masslessness of gauge bosons is usually not thought to be related to a symmetry among the physical states of the theory. It is sometimes attributed to gauge invariance. In general, however, there is no connection between the two. Gauge bosons may acquire mass due to the Higgs phenomenon or related effects, such as topological mass generation in QED_3. The mass of gauge particles is a dynamical question, yet massless gauge particles often appear. In these theories one would like to understand the exact masslessness of gauge bosons as resulting from spontaneous breaking of a continuous symmetry*.

Closely related to this question is the phase structure of these theories. Theories exhibiting a conventional Goldstone phenomenon usually have two phases, distinguished by different realizations of a continuous symmetry. In the unbroken phase the relevant symmetry is realized in the Wigner–Weyl (WW) mode, and the spectrum has an energy gap. As one approaches the critical point, the energy gap vanishes, indicating the appearance of large-scale fluctuations. At the critical point, objects carrying the symmetry charge condense, and the symmetry breaks spontaneously beyond this point.

Abelian gauge theories such as the Higgs model (scalar QED), both in 2 + 1 and 3 + 1 dimensions, have a very similar phase structure. In the Coulomb phase there is a massless mode – the photon. In the Higgs phase, on the other hand, there are no massless modes. Usually this phase transition is not associated with the Nambu–Goldstone (NG) phenomenon. The common jargon is that in the Higgs phase "the local gauge symmetry is spontaneously broken". This should not be taken literally since local symmetries are never broken (by the Elitzur theorem [5]). Moreover, even the global electric charge U(1) symmetry is not broken in the Higgs phase [6]. In fact, the appropriate local order parameter for this phase transition has not been found [7].

One can take a radically different point of view on this question**. If the photon were a massless mode associated with the spontaneous breakdown of some global symmetry, then the Higgs–Coulomb phase transition is to be understood as another example of a symmetry breaking phenomenon.

* There were attempts to understand photons as "Goldstone bosons" [4]. In this case, however, the proposed global transformation was a part of the local gauge group. These symmetries are not symmetries between the physical states, and so are not of the type considered here.

** A short account of some ideas presented in this paper appeared in ref. [8].

In this paper we pursue this line of reasoning in the framework of abelian gauge theories in 2 + 1 dimensions. In 2 + 1 dimensions the analysis is greatly simplified compared to realistic (3 + 1)-dimensional theories since the photon in the Coulomb phase is in fact a scalar particle. The main idea, however, can be generalized to 3 + 1 dimensions.

We identify the symmetries responsible for masslessness of the photon in both scalar and spinor QED_3. In scalar QED it is the flux symmetry defined by the conserved current

$$\tilde{F}_\mu = \tfrac{1}{2}\epsilon_{\mu\nu\lambda}F^{\nu\lambda}. \tag{1.1}$$

The corresponding charge is the magnetic flux $\Phi = \int B(x)\,d^2x$. In the Coulomb phase this symmetry is spontaneously broken, causing the masslessness of the photon. In the Higgs phase it is realized in the WW mode, and the theory contains excitations carrying nonzero flux (magnetic vortices) [9]. We construct the pertinent order parameter – the vortex creation operator – and calculate its vacuum expectation value and correlator in weak coupling perturbation theory.

We also consider spinor QED with two flavours,

$$\mathscr{L} = \bar{\psi}(i\not{\partial} - \mu - m\tau_3 - e\not{A})\psi - \tfrac{1}{4}F^2, \tag{1.2}$$

where ψ^a, $a = 1,2$ is a doublet of two complex (Dirac) fermions, μ and m are parity-violating and parity-conserving fermion masses, respectively, and $\tau_3 = \mathrm{diag}(1, -1)$. The theory is symmetric under flux symmetry generated by Φ and chiral transformations generated by $Q^5 = \int d^2x\,\psi^\dagger\tau_3\psi$. For $|m| > |\mu|$, both the flux Φ and the chiral charge Q^5 are spontaneously broken and connect the vacuum to the one-photon state. The following combination, however, remains unbroken:

$$I_\mu = J^5_\mu - \frac{e}{2\pi}\,\mathrm{sign}(m)\tilde{F}_\mu. \tag{1.3}$$

The symmetry breaking pattern therefore is $U_\Phi(1) \otimes U_\chi(1) \to U_I(1)$. For $|m| < |\mu|$ both symmetries are unbroken. Precisely for these values of the parameters the Chern–Simons (CS) term is generated dynamically, rendering the photon massive (see fig. 1 for phase diagram).

We find that even in the broken phase the expectation value of the order parameter vanishes. This indicates that the symmetry is realized not in familiar NG mode but rather in the KT mode common in (1 + 1)-dimensional models [2,3] (we review definitions and some basic features of various modes of symmetry realizations in appendix A). Along similar lines we consider the CS electrodynamics [10].

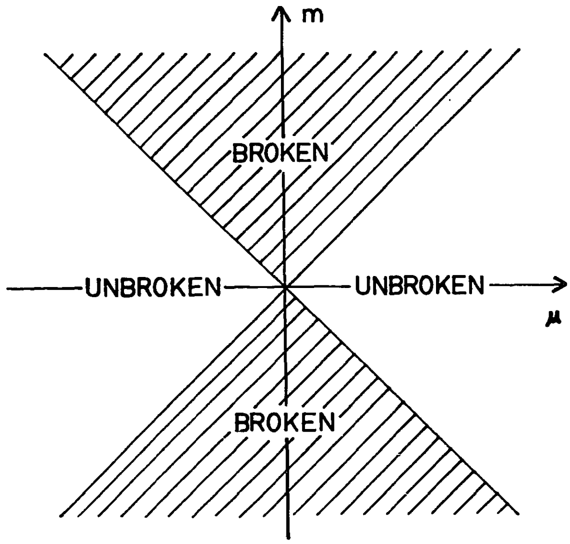

Fig. 1. Phase diagram of the spinor QED_3. The lines separate the broken phase from the unbroken phase.

This paper is organized as follows. In sect. 2 we consider symmetry breaking patterns and massless modes in several (2 + 1)-dimensional abelian gauge theories. In subsect. 2.1 we define the flux symmetry, first in the case of a free photon and then in scalar QED. In subsect. 2.2 we remark that the "chiral" lagrangian corresponding to the symmetry breaking pattern requires softness of interactions between photons. In subsect. 2.3 the phase structure and the symmetry breaking pattern of the spinor QED and in subsect. 2.4 of the CS electrodynamics are analyzed.

Sect. 3 is devoted to a perturbative evaluation of the order parameters. In subsect. 3.1 the order parameter is calculated in the simple free photon theory. In subsect. 3.2 we define the order field $V(x)$ (the vortex creation operator) in the interacting case. We calculate in subsect. 3.3 the vev and the correlator of $V(x)$ in the Coulomb phase of scalar QED and verify that $V(x)$ is a local scalar field. In subsect. 3.4 an analogous calculation is performed in the Higgs phase. The slope of the exponential of the correlator $\langle V(x)V^*(y)\rangle \sim \exp(-m|x-y|)$ determines the mass of the magnetic vortex. To leading order in the electromagnetic coupling it coincides with the energy of the classical solution [9, 11]. In subsect. 3.5 we define the order parameter in spinor QED [which slightly differs from $V(x)$].

Sect. 4 contains discussion of our results and generalization of this approach to nonabelian gauge theories and QED_4.

In appendix A we systematically classify and review various modes of realization of continuous symmetries, emphasizing the delicate distinction between the Kosterlitz–Thouless (KT) and NG modes.

2. Symmetry breaking and phase structure in QED_3

2.1. SCALAR QED AND THE FLUX SYMMETRY

We first consider scalar QED, defined by the lagrangian

$$\mathscr{L} = |D_\mu \phi|^2 + m^2|\phi|^2 - \tfrac{1}{4}F^2 + V(\phi^*\phi), \tag{2.1}$$

where $D_\mu = \partial_\mu - ieA_\mu$ and $V(\phi^*\phi)$ is a polynomial up to third order. This theory is renormalizable in 2 + 1 dimensions. It has two phases – a Higgs phase, in which the photon is massive, and a Coulomb phase, in which it is massless. Sometimes the phase transition is associated with "breakdown of a local gauge symmetry". However, it is clear that one cannot make a physical distinction between phases on the basis of realization of a gauge symmetry. All gauge group generators annihilate physical states due to the Coulomb constraint and therefore the states are invariant under the action of the gauge group in every phase.

The global $U_e(1)$ electric charge symmetry generated by Q [6],

$$Q = \int d^2(x)\, J_0(x) = \frac{1}{e}\int d^2x\, \partial_i E_i(x),$$

$$J_\mu = i\phi^* D_\mu \phi - \text{h.c.}, \tag{2.2}$$

is a symmetry between physical states. However, this symmetry is not broken in any one of the phases. In fact, in both phases all the finite-energy states are neutral under the action of this symmetry. This is evident in the Higgs phase. Since the electric charge is given by an integral of the electric field at spatial infinity and the electric field decays exponentially (the photon is massive), any finite energy configuration has zero charge. In the Coulomb phase the $U_e(1)$ symmetry is neutral for a different reason. The exchange of massless photon induces a logarithmic confinement of electric charges and therefore the energy of any charged state is logarithmically divergent. Consequently this symmetry does not distinguish between the two phases.

We shall show now that the symmetry broken at the Coulomb–Higgs phase transition is the flux symmetry. This symmetry is generated by the magnetic flux through the plane $\Phi = \int d^2x B(x)$. The corresponding identically conserved current is [12]

$$\tilde{F}_\mu(x) = \epsilon_{\mu\nu\lambda}\partial^\nu A^\lambda. \tag{2.3}$$

As usual for currents of this type the symmetry charge generates transformation on fields only at space infinity. In hamiltonian gauge one has

$$[A_i(x), \Phi] = 0, \qquad [E_i(x), \Phi] = i \int d^2y\, \epsilon_{ij} \partial_j^y \delta^{(2)}(x-y). \tag{2.4}$$

In the Higgs phase the symmetry is realized in the Wigner–Weyl mode while in the Coulomb phase it is broken. The breakdown of this symmetry in the Coulomb phase leads to the appearance of a corresponding massless mode – the photon.

We begin the discussion of the flux symmetry with the simple case of free photon. In 2 + 1 dimensions the photon has only one transversal polarization and is equivalent to a massless scalar particle*. Canonically (in hamiltonian gauge) the theory is described by the hamiltonian

$$H = \tfrac{1}{2} \int d^2x (E^2 + B^2) \tag{2.5}$$

with the constraint

$$\partial_i E_i = 0. \tag{2.6}$$

The linear constraint is easily solved in terms of a single scalar field $\chi(x)$,

$$E_i = \epsilon_{ij} \partial_j \chi. \tag{2.7}$$

The magnetic field $B(x)$ is just the momentum canonically conjugate to $\chi(x)$,

$$B = -\pi. \tag{2.8}$$

In these variables the hamiltonian becomes

$$H = \tfrac{1}{2} \int d^2x \left[\pi^2 + (\partial_i \chi)^2\right]. \tag{2.9}$$

The flux symmetry in these notations is a familiar shift transformation $\chi(x) \to \chi(x) + \text{const}$. This symmetry (in $d = 2+1$) is spontaneously broken [13] with χ itself (which interpolates the photon) as a Goldstone boson.

This statement remains true in the Coulomb phase of the interacting theory as well. The Coulomb constraint now is $\partial_i E_i = j_0$. It is no longer solved by the expression (2.7), but the matrix element of $\tilde{F}_\mu$ between the vacuum and one-photon state is easily calculated and still does not vanish. The correlator of the flux

* There is no helicity which distinguishes the massless photon from a massless scalar in 3 + 1 dimensions.

current and the photon field is

$$\int d^3x\, e^{ipx} \langle T\tilde{F}_\mu(x) A_\nu(0) \rangle = \epsilon_{\mu\nu\lambda} \frac{p^\lambda}{p^2[1-\Pi(p^2)]}, \tag{2.10}$$

where $\Pi(p^2)$ is the vacuum polarization contribution to the symmetric part of the inverse photon propagator. In the Coulomb phase $\Pi(p^2)$ is regular as $p^2 \to 0$ and the matrix element of the current $\tilde{F}_\mu(x)$ between the vacuum and the one-photon state is given by*

$$\langle 0|\tilde{F}_\mu(x=0)|1, \boldsymbol{p}\rangle = -\frac{i}{2\pi\sqrt{2p_0}} \epsilon_{\mu\nu\lambda} p^\nu \epsilon^\lambda \lim_{p^2 \to 0} \frac{1}{1-\Pi(p^2)}, \tag{2.11}$$

where ϵ^λ is the photon polarization vector. This quantity is gauge invariant, and using hamiltonian gauge,

$$\epsilon_0 = 0, \qquad \epsilon^i = \epsilon^{ij} \frac{p_j}{|\boldsymbol{p}|}, \tag{2.12}$$

we obtain

$$\langle 0|\tilde{F}_\mu(0)|1, \boldsymbol{p}\rangle = -\frac{i}{2\pi\sqrt{2p_0}} p_\mu \lim_{p^2 \to 0} \frac{1}{1-\Pi(p^2)}, \tag{2.13}$$

where p_μ is momentum of the one-photon state. This is the familiar form of the matrix element of spontaneously broken current between the vacuum and the state of one Goldstone boson [13]. Eq. (2.13) implies that in the Coulomb phase the vacuum is degenerate under the action of the flux symmetry and the photon is the corresponding massless mode.

In the Higgs phase, eq. (2.13) is not valid any more. The photon now is massive and has two polarizations. The vacuum polarization $\Pi(p^2)$ has a pole at zero momentum and the matrix element (2.11) vanishes. The magnetic flux annihilates the vacuum state and the symmetry is not broken. The Higgs particle and the massive photon are flux neutral. The spectrum also contains finite mass excitations carrying nonzero flux: magnetic vortices. This gives a simple and natural interpretation of the superconducting vacuum – it is a state, annihilated by magnetic flux. Since there is a finite gap in the spectrum of flux carrying excitations (vortices), a sufficiently small external magnetic field cannot penetrate the medium (the Meissner effect).

As one approaches the phase transition point the mass of the vortices vanishes and they condense. The Higgs–Coulomb phase transition can therefore be thought

* We use the canonical "nonrelativistic" normalization of states $\langle \boldsymbol{p}|\boldsymbol{k}\rangle = \delta^2(\boldsymbol{p}-\boldsymbol{k})$.

of as a condensation of vortices. We shall discuss this as well as the relevant order parameter in subsect. 2.2.

2.2. EFFECTIVE "CHIRAL" LAGRANGIANS

It is well known [14] that Goldstone bosons, unlike other scalars, must interact softly among themselves and sometimes with other particles. For any symmetry breaking pattern $G \to H$ one generally can write down an effective "chiral" lagrangian with fields interpolating Goldstone bosons living on the manifold G/H.

In the case of flux symmetry in $2+1$ dimensions discussed above, the breaking pattern is very simple: $U(1) \to 1$. In general, for abelian symmetry breaking the n-particle to m-particle S-matrix element is proportional to (at least) $(m+n)$ powers of momenta. This is indeed the case for effective interactions of photons due to virtual pair creation processes.

In principle, the low-energy ("chiral") effective lagrangian is derived by integrating out all of the massive degrees of freedom [15, 16]. The effective "chiral" lagrangian corresponding to the flux symmetry breaking coincides with the $(2+1)$-dimensional analog of the Schwinger effective lagrangian

$$\begin{aligned}\mathscr{L} &= -\tfrac{1}{4}F^2 - i\,\mathrm{Tr}\ln\left[(\partial_\mu - ieA_\mu)^2 + m^2\right] \\ &= \alpha(e^2,m)F^2 + \beta(e^2,m)F\partial^2F + \gamma(e^2,m)(F^2)^2 + \text{higher-derivative terms}.\end{aligned} \tag{2.14}$$

It depends on the gauge-invariant field strength $F_{\mu\nu}$ and its derivatives only, and therefore the effective photon–photon interaction is soft. The interaction of Goldstone bosons with other particles need not be soft [16].

Similar conclusions are valid in spinor QED to which we now turn.

2.3. SPINOR QED

Let us consider spinor QED with two fermionic species,

$$\mathscr{L} = \bar{\psi}(i\partial\!\!\!/ - \mu - m\tau_3 - eA\!\!\!/)\psi - \tfrac{1}{4}F^2, \tag{2.15}$$

where ψ^a, $a=1,2$ is a doublet of two component complex (Dirac) fermions, $\tau_3 = \mathrm{diag}(1,-1)$, and m and μ are independent mass parameters*. In addition to the global electric charge $U_e(1)$ and the flux $U_\Phi(1)$ symmetries there is also a chiral $U_\chi(1)$ symmetry. The corresponding currents are the electromagnetic current

* The m- and μ-terms have different properties under the parity transformation defined as $\psi(x,y) \to \gamma_1\tau_1\psi(-x,y)$, where τ_1 is a Pauli matrix acting on flavour indices. The m-term preserves this symmetry, while the μ-term violates it.

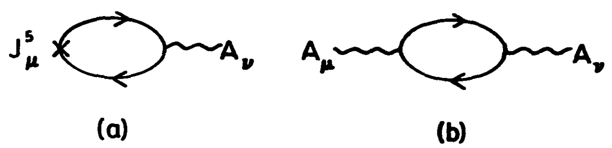

Fig. 2. Diagrams generating (a) matrix element of J^5_μ between the vacuum and one-photon state; (b) topological mass for the photon.

$J_\mu = \bar{\psi}\gamma_\mu\psi$, $\tilde{F}_\mu$ and the chiral current $J^5_\mu = \bar{\psi}\gamma_\mu\tau_3\psi$. To avoid confusion note that our chiral transformation is not the same as what is sometimes called a "chiral" rotation $\psi \to e^{i\theta\tau_2}\psi$ – this we refer to as an "axial" rotation*.

This theory was extensively studied for a variety of reasons. Deser et al. [10] found that in the $m = 0$ case a Chern–Simons (CS) term is dynamically generated at the one-loop level and the photon acquires a CS mass (usually referred to as a topological mass). In the $\mu = 0$ case the photon remains massless and induces a logarithmic confinement of electric charge. The dynamical breaking of the axial symmetry in the massless case was studied in ref. [17].

The theory defined by eq. (2.15) like scalar QED exhibits the phenomenon of flux symmetry breaking but has some new and interesting features.

The theory has two phases. When $|\mu| > |m|$ (the Chern–Simons phase) the photon is massive and all three symmetries are unbroken. When $|\mu| < |m|$ (the Coulomb phase) the photon is massless. The flux and chiral symmetries are both broken but the combination

$$I_\mu = J^5_\mu - \frac{e}{2\pi}\,\mathrm{sign}(m)\tilde{F}_\mu \tag{2.16}$$

(with charge denoted by I) remains unbroken.

Let us now consider the lagrangian (2.15) in two extreme cases: (i) $\mu = 0$, (ii) $m = 0$.

(i) $\mu = 0$. The CS term is not generated to all orders in perturbation theory [19]. Therefore the spectrum of the theory contains a massless photon. As in scalar QED, $\tilde{F}_\mu$ connects the vacuum to a one-photon state. The chiral charge $Q^5 = \int d^2x J^5_0(x)$ annihilates the vacuum in lowest order. However in the next-to-leading order one encounters the diagram of fig. 2a. It gives the following contribution to the correlation function:

$$\int d^3x\, e^{ixp}\langle 0|TJ^5_\mu(x)A_\nu(0)|0\rangle = -\frac{iem}{2\pi}\epsilon_{\mu\nu\lambda}p^\lambda\frac{1}{(-p^2)^{3/2}}\,\mathrm{arctg}\left(\frac{-p^2}{4m^2}\right)^{1/2}. \tag{2.17}$$

*This symmetry is sometimes called "chiral", since in terms of 4 × 4 Dirac matrices it is given by $\psi \to e^{i\theta\gamma_5}\psi$. We use two-dimensional Dirac matrices which are natural in 2 + 1 dimensions. The transformation which we call chiral in terms of 4 × 4 Dirac matrices takes form $\psi \to e^{i\theta\gamma_0\gamma_1\gamma_2}\psi$. It is the axial, rather than the chiral symmetry, that has recently been extensively studied analytically in ref. [17] and numerically in ref. [18].

This leads to a nonvanishing matrix element of the chiral current between the vacuum and the one-photon state,

$$\langle 0|J_\mu^5(0)|1,\boldsymbol{p}\rangle = -ie\,\mathrm{sign}(m)\frac{p_\mu}{4\pi^2\sqrt{2p_0}}\,. \tag{2.18}$$

In fact the diagram fig. 2a is the only contribution to eq. (2.17) to all orders in perturbation theory. This can be proved by making use of the arguments of ref. [19]. Therefore to all orders the only modification of the right-hand side of eq. (2.17) is to multiply it by $1/(1-\Pi(p^2))$. Eq. (2.18) shows that the vacuum is not invariant under chiral rotations.

The combination I_μ in eq. (2.16), however remains unbroken. Therefore the symmetry breaking pattern is $U_\phi(1)\otimes U_\chi(1)\to U_I(1)$.

(ii) $m=0$. The correlation function (2.17) in this case vanishes to all orders. The vacuum is invariant under chiral rotations and the symmetry is realized in the WW mode. This is correlated with the generation of a topological mass for the photon. Although on the classical level the photon is massless, quantum corrections induce a topological mass [10]. To all orders in perturbation theory this mass is given by the single one-loop diagram of fig. 2b [19]*.

The crossover between the two extreme cases is straightforward. The matrix element of the chiral current and the photon topological mass for arbitrary m and μ are given by**

$$\langle 0|J_\mu^5(0)|1,\boldsymbol{p}\rangle = -[\mathrm{sign}(m+\mu)+\mathrm{sign}(m-\mu)]\frac{ie}{4\pi\sqrt{2p_0}}p_\mu\,, \tag{2.19}$$

$$m_{\mathrm{ph}} = \frac{e^2}{4\pi}[\mathrm{sign}(m+\mu)-\mathrm{sign}(m-\mu)]\,. \tag{2.20}$$

* Note that the photon mass generation can be viewed as a kind of generalized Higgs mechanism [20]. Indeed, the diagram by fig. 2b also contributes to the antisymmetric part of the correlator of the electromagnetic current and the photon field,

$$G^A_{\mu\nu} = \int d^3x\, e^{ixp}\langle 0|TJ_\mu(x)A_\nu(0)|0\rangle = -ie\mu\epsilon_{\mu\nu\lambda}p^\lambda\frac{1}{\pi(-p^2)^{3/2}}\,\mathrm{arctg}\left(\frac{-p^2}{4\mu^2}\right)^{1/2}.$$

If the photon were massless, then as $p\to 0$ one would get an expression analogous to that of eq. (2.18). Similarly in this case the photon would have had to be interpreted as a NG (or KT) (see appendix A for definition) boson corresponding to the breaking of electric charge symmetry. However, because the CS term is generated, the electric charge is not broken. This can be viewed as a "self-Higgs" mechanism, when the photon plays the roles of both the "swallowed" NG (KT) boson and the "swallowing" gauge field [8]. The number of degrees of freedom in this case is evidently not changed.

** As a regulator, we use a pair of Pauli–Villars fermionic fields with masses of opposite sign. This regularization preserves parity, which is present in the theory for $\mu=0$.

Eq. (2.19) follows in full analogy to eq. (2.20) [10]. The two fermion species running in the loop of fig. 2 contribute to the matrix element independently.

We observe that the lines $|m| = |\mu|$ are critical (fig. 1). We conclude that when $|\mu| > |m|$ the photon is massive and both symmetries (flux and chiral) are unbroken. In the rest of parameter space the charge I remains unbroken while Q^5 and Φ are broken. Similar analysis holds in QED with any even number of flavours.

2.4. CHERN–SIMONS QED

The present considerations are easily extended to a theory of N fermions with a parity breaking mass and a Chern–Simons term,

$$\mathscr{L} = -\tfrac{1}{4}F^2 + \bar{\psi}^i(i\partial\!\!\!/ - \mu - eA\!\!\!/)\psi^i + \frac{\gamma}{8\pi}\epsilon_{\mu\nu\lambda}A_\mu F_{\nu\lambda}, \tag{2.21}$$

where $i = 1,\ldots, N$. This theory possesses two conserved currents: the flux current $\tilde{F}_\mu$ and the particle number current $J_\mu \equiv \bar{\psi}\gamma_\mu\psi$*.

Quantum corrections generate an additional contribution to the photon mass**. When $\gamma = 2Ne^2$, the gauge field becomes massless. The matrix element of the current J_μ between the vacuum and a one-photon state has the form of eq. (2.18). Consequently for this value of the parameter γ both charges Φ and $Q = \int d^2x J_0(x)$ are broken. It is interesting that the combination of Φ and Q that remains unbroken coincides with the charge $I = \int d^2x\, \partial_i E_i$. This follows directly from the equation of motion

$$\partial_\nu F_{\mu\nu} = e\bar{\psi}\gamma_\mu\psi - \frac{\gamma}{4\pi}\epsilon_{\mu\nu\rho}F_{\nu\rho}. \tag{2.22}$$

Therefore the symmetry breaking pattern is $U_\Phi(1) \otimes U_e(1) \to U_I(1)$.

The charge I is logarithmically confined due to the exchange of massless photons. Physical excitations are therefore not interpolated by the field ψ^i, but rather by a field which commutes with I. This field, in addition to the charge Q, carries magnetic flux $\Phi = \gamma/4\pi e$. As a result, the excitations have fractional statistics under exchange, and spin $\frac{1}{2}(1 - 1/N)$ [21].

This model can be used to describe a parity-violating two-dimensional superconductor. The correlator of the current J_μ acquires a pole at zero momentum. When J_μ is minimally coupled to the real three-dimensional electromagnetic field [22], the pole in the propagator of the real photon is shifted from zero to some finite value, indicating superconductivity. The mechanism of superconductivity, i.e. cancellation of the renormalized CS coefficient is the same for the nonrelativistic

* For $N > 1$ there is, of course, the additional SU(N) flavour symmetry. As this symmetry is unbroken in the whole parameter space, it will be of no interest to us.

** We adopt Pauli–Villars regularization with sign $M_{PV} = -\text{sign}\,\mu$.

anyon gas and CS QED. In the nonrelativistic anyon gas with statistics $(1 - 1/N)$, the particles fill N Landau bands, each contributing $-e^2/4\pi$ to the induced CS coefficient, cancelling exactly the bare one. In the model (2.21), the role of the N Landau bands is played by the N fermion species, with the same effect on the renormalized CS coefficient. The number of fermion species N is equal to the number of filled Landau bands in the anyon gas [22]. The superconductivity of the anyon gas occurs at the point in parameter space at which the particle number symmetry is broken.

3. Order parameter

In this section, we define and calculate, in perturbation theory, the order parameter and the correlator of the order field in the theories we considered in sect. 2.

3.1. FREE PHOTON

Let us start with the simplest case of the free photon. As was discussed in sect. 2 the flux symmetry here is spontaneously broken. We shall now see this directly by calculating the order parameter. The analogous calculation in the interacting theories will coincide with this one in the leading order in the weak coupling expansion.

The relevant order parameter in this case is

$$\mathcal{O} = \langle V(x) \rangle , \tag{3.1}$$

where the order field $V(x)$ is defined by the requirement (see appendix A)

$$[V(x), B(y)] = -gV(x)\delta^2(x-y) . \tag{3.2}$$

Any operator of the form

$$V(x) = C\exp(ig\chi(x)) , \qquad \chi(x) = \int \mathrm{d}^2 y\, a_i(x-y) E_i(y) , \tag{3.3}$$

with C a constant, and the function $a_i(x)$ satisfying the condition

$$\epsilon_{ij}\partial_i a_j(x) = \delta^2(x) , \tag{3.4}$$

solves eq. (3.2). Functions $a_i(x)$ differing by the gradient of a scalar function define operators $V(x)$, which coincide on the Coulomb constraint (2.6). In what follows we take $a_i(x)$ to be

$$a_i(x) = \frac{1}{2\pi}\epsilon_{ij}\frac{x_j}{x^2}. \tag{3.5}$$

The constant C is necessary to absorb divergences.

The expectation value of the exponential of a linear operator is generally given by

$$\langle \exp(ig\chi) \rangle = \exp\left\{ \sum_{n=1}^{\infty} \frac{(ig)^n}{n!} \langle \chi^n \rangle_c \right\}, \tag{3.6}$$

where $\langle \chi^n \rangle_c$ is the connected n-point Green function. For free theory the only contribution comes from the connected two-point function,

$$\langle \chi(x)\chi(x) \rangle = \int \frac{d^3p}{(2\pi)^3} a_i(p) a_j(-p) \left[p_0^2 G_{ij}(p) - i\delta_{ij} \right]. \tag{3.7}$$

Here $G_{ij}(p)$ is the Feynman photon propagator in hamiltonian gauge*;

$$G_{ij}(p) = \frac{i}{p^2}\left(\delta_{ij} + \frac{p_i p_j}{p_0^2} \right). \tag{3.8}$$

The last term in eq. (3.7) originates from the time ordering in the definition of G**. Using the explicit form of a_i [see eq.(3.5)] one obtains

$$\langle \chi(x)\chi(x) \rangle = i \int \frac{d^3p}{(2\pi)^2} \frac{1}{p^2} = \frac{\Lambda}{\pi}, \tag{3.9}$$

where Λ is an ultraviolet momentum cutoff. As we shall see, this is the only ultraviolet divergence which will appear, even in the interacting theory. Absorbing this linear ultraviolet divergence in the constant C of eq. (3.3) renders all the Green functions of the vortex operator $V(x)$ finite.

With this definition, the renormalized order parameter is then finite, pointing to NG realization of the flux symmetry. In what follows, we choose C such that

* In the interacting case the hamiltonian gauge is known to have ambiguities at the two-loop level [23]. However this does not cause any difficulty in our work since we do not encounter loop integrals with internal photon lines. The advantage of the hamiltonian gauge in this setting is that it leads to the simplest canonical formalism.

** See e.g. ref. [24].

$\langle V(x)\rangle = 1$. The correlator of the order field is given by

$$\langle V(x)V^*(y)\rangle = C^2\langle \exp\{ig[\chi(x)-\chi(y)]\}\rangle = \exp[-g^2G(x-y)], \quad (3.10)$$

where G is the propagator of the massless scalar,

$$G(x-y) = \frac{1}{4\pi|x-y|}. \quad (3.11)$$

When $|x-y| \to \infty$ the correlator approaches a constant as usual in NG realization.

This calculation is conveniently represented in the euclidean path integral formalism. The vev of $V(x)$ can be naively rewritten as

$$\langle V(x)\rangle = C\int \mathcal{D}A \exp\left[-\tfrac{1}{4}F^2 - g\int \mathrm{d}^3y\, j_\mu(x-y)A_\mu(y)\right], \quad (3.12)$$

where

$$j_0 = 0, \qquad j_i(x) = \delta'(x_0)a_i(x). \quad (3.13)$$

This is obtained from eq. (3.3) using integration by parts. The integration by parts is naive since it disregards the equal-time singularity. In the canonical formalism this corresponds to dropping the δ_{ij}-term in eq. (3.7). The correct result is

$$\langle V(x)\rangle = C\int \mathcal{D}A \exp\left[-\tfrac{1}{4}F^2 - g\int j_\mu A_\mu + \tfrac{1}{2}g^2\iint j_\mu G^{\text{return}}_{\mu\nu} j_\nu\right], \quad (3.14)$$

with

$$G^{\text{return}}_{\mu 0}(p) = 0, \qquad G^{\text{return}}_{ij}(p) = -\frac{1}{p_0^2}\delta_{ij}. \quad (3.15)$$

The significance of the G^{return} term will become apparent in subsect. 3.3.

3.2. VORTEX OPERATOR IN INTERACTING THEORY

As we have seen in sect. 2 in the interacting theory $\tilde{F}_\mu$ remains a conserved current. The Coulomb constraint is, however, altered,

$$\partial_i E_i = eJ_0. \quad (3.16)$$

As a consequence, the operators $V(x)$ defined in eq. (3.3) for different coefficient functions $a_i(x)$ are no longer equivalent on the physical Hilbert space. We demand that the order field be a covariant scalar field, local with respect to the gauge-invariant operators $E_i(x)$ and $J_\mu(x)$. Since Φ is linear in $A_i(x)$ the order field in hamiltonian gauge should be exponential in $E_i(x)$,

$$V(x) = K[\phi, A_i] \exp ig \int d^2y\, a_i(x-y) E_i(y), \tag{3.17}$$

where $\epsilon_{ij}\partial_i a_j(x) = \delta^{(2)}(x)$. The dependence of K on ϕ and A_i can be restricted using locality of $V(x)$ (see ref. [25] for details). We will restrict ourselves to the form

$$V(x) = C \exp ig \int d^2y [a_i(x-y) E_i(y) + eb(x-y) J_0(y)], \tag{3.18}$$

with $b(x-y)$ a c-number function to be determined, and C a constant as before (in fact, the same constant).

The locality of $V(x)$, in particular requires a zero commutator with $J_i(x)$ at space-like separations. Using eq. (3.18) one obtains

$$[J_i(x), V(y)] = 2i\{-ega_i(x-y) + \partial_i^x[egb(x-y)]_{\mathrm{mod}\, 2\pi}\}\phi^*(x)\phi(x)V(y). \tag{3.19}$$

There is no continuous function $b(x)$ for which the right-hand side of eq. (3.19) vanishes. There are, however discontinuous functions of this kind,

$$b(x) = \int_{C(x)} a_i \, dl_i, \tag{3.20}$$

where the contour $C(x)$ starts at the origin and ends at the point x. The function $b(x)$ has a branch cut which starts at the origin and depends on the choice of the contour $C(x)$. The simplest choice is

$$a_i(x) = \frac{1}{2\pi}\epsilon_{ij}\frac{x_j}{x^2}, \qquad b(x) = \frac{1}{2\pi}\Theta(x), \tag{3.21}$$

where $\Theta(x)$ is an angle between the vector x_i and the X-axis, $0 \leqslant \Theta < 2\pi$. In this case the discontinuity lies along the positive direction of the X-axis. In order that the discontinuity of $b(x)$ does not spoil the locality, the eigenvalue g must be quantized in units of $2\pi/e$,

$$g = 2\pi n/e. \tag{3.22}$$

The operator $V_n(x)$ defined by eq. (3.18) with this g will be called the vortex operator [26]. It creates the flux $2\pi n/e$ localized at a point x. If the field $V_n(x)$ is a scalar field its Green functions should not depend on the choice of the discontinuity line. We will show in subsect. 3.3 that this is indeed the case, at least in weak coupling perturbation theory.

Note that using the Coulomb constraint (3.16), we can rewrite the order field $V_n(x)$ in terms of the operator E_i only,

$$V_n(x) = C\exp\left\{i\frac{2\pi n}{e}\int \tilde{a}_i(x-y)E_i(y)\,\mathrm{d}^2y\right\}, \tag{3.23}$$

where

$$\tilde{a}_1(x) = 0, \qquad \tilde{a}_2(x) = \theta(x_1)\delta(x_2). \tag{3.24}$$

This form will be useful when we represent correlators of $V_n(x)$ in the path integral formalism.

3.3. SCALAR QED; COULOMB PHASE

We now calculate the expectation value of the order field $V_n(x)$ in the framework of weak coupling perturbation theory. The definition of $V_n(x)$, eq. (3.23), contains a nonperturbative factor $1/e$ in the exponent. The logarithm of $\langle V_n(x)\rangle$ is, however, expandable in e^2, although the series starts with the $1/e^2$ term. We will show that the expansion of $\ln\langle V_n(x)\rangle$ in powers of e^2 coincides with the loop expansion (or $\hbar$-expansion) of the ground-state energy of QED in an external field – the field of a magnetic monopole with the solenoid energy subtracted.

For this purpose it is convenient to use the euclidean path integral formulation. The expectation value of the order field (3.23) is

$$\langle V_n(x)\rangle = C\int \mathscr{D}\phi\,\mathscr{D}\phi^*\,\mathscr{D}A\exp\left\{-S(\phi,\phi^*,A) - \frac{2\pi n}{e}\int j_\mu A_\mu + \frac{1}{2}\left(\frac{2\pi n}{e}\right)^2\iint j_\mu G^{\text{return}}_{\mu\nu} j_\nu\right\}, \tag{3.25}$$

where S is the euclidean version of the action (2.1) and

$$j_0 = 0, \qquad j_i = \delta'(t)\tilde{a}_i(x) = \delta'(t)\theta(x_1)\delta(x_2)\delta_{i2}, \tag{3.26a}$$

or in momentum space,

$$j_0 = j_1 = 0, \qquad j_2 = p_0/p_1, \tag{3.26b}$$

and $G^{\text{return}}_{\mu\nu}(p)$ is given in eq. (3.15). The origin of the third term in the exponent is the same as in free theory (see subsect. 3.1). One can visualize j_μ (in the euclidean formulation) as a stationary three-dimensional current producing the magnetic field of a monopole with magnetic charge $2\pi n/e$. In the lowest order it produces the "external" potential (in hamiltonian gauge)

$$\mathcal{A}_\mu(p) = G_{\mu\nu}(p) j_\nu(p),$$

$$\mathcal{A}_0 = 0, \qquad \mathcal{A}_1 = -\frac{p_2}{p_0 p^2}, \qquad \mathcal{A}_2 = -\frac{1}{p_1 p_0} + \frac{p_1}{p_0 p^2}. \tag{3.27}$$

The corresponding three-dimensional magnetic field is

$$\mathcal{B}_\mu = \mathcal{B}_\mu^{\mathrm{M}} + \mathcal{B}_\mu^{\mathrm{string}}, \tag{3.28}$$

where $\mathcal{B}_\mu^{\mathrm{M}}$ is the field of a monopole,

$$\mathcal{B}_\mu^{\mathrm{M}} = i\frac{p_\mu}{p^2} \tag{3.29}$$

while $\mathcal{B}_\mu^{\mathrm{string}}$ describes the string supplying the flux to the monopole,

$$\mathcal{B}_0^{\mathrm{string}} = \mathcal{B}_2^{\mathrm{string}} = 0, \qquad \mathcal{B}_1^{\mathrm{string}} = -\frac{i}{p_1}, \tag{3.30a}$$

or, in configuration space,

$$\mathcal{B}_0^{\mathrm{string}} = \mathcal{B}_2^{\mathrm{string}} = 0, \qquad \mathcal{B}_1^{\mathrm{string}} = \theta(x_1)\delta(x_2)\delta(x_3). \tag{3.30b}$$

The appearance of the monopole is universal for any choice of the function $b(x)$ in eq. (3.20) consistent with locality requirements for the order field. The location of the solenoid coincides with the branch cut of the function $b(x)$. At first sight the appearance of the string breaks Lorentz covariance of the field $V_n(x)$. The third term in the exponent of eq. (3.25), as we shall see, completely cancels all the effects of the string (as in the free case), restoring Lorentz covariance.

It is straightforward now to develop a diagrammatic expansion for the weak coupling perturbation theory of expression (3.25). Without the term $j_\mu G_{\mu\nu}^{\mathrm{return}} j_\nu$ (which is independent of A_μ) one recognizes in expression (3.25) the generating functional Z of scalar QED in the presence of the external source j_μ. The logarithm of Z is the vacuum energy W, which is diagrammatically given by the sum of all connected vacuum diagrams. The current j_μ appears in the one-point vertex denoted by "x". The only contributions in leading order to W are those of free theory (fig. 3). The energy W is given by the classical energy of electromag-

Fig. 3. The diagram contributing to the order parameter in the Coulomb phase to leading order in e. "x" denotes an external current j_μ, eq. (3.26), creating a monopole.

netism in the presence of the external current creating a magnetic monopole, plus the $G_{\mu\nu}^{\text{return}}$ contribution,

$$-W=\int \tfrac{1}{2}\mathscr{B}^2+\frac{1}{2}\left(\frac{2\pi n}{e}\right)^2\iint\{j_\mu G_{\mu\nu}^{\text{return}} j_\nu\}$$

$$=\left(\frac{2\pi n}{e}\right)^2\frac{1}{2}\int \mathrm{d}^3p\left[\frac{1}{p_1^2}-\frac{1}{p^2}\right]+\frac{1}{2}\left(\frac{2\pi n}{e}\right)^2\int \mathrm{d}^3p\left(\frac{p_0}{p_1}\right)\left(-\frac{1}{p_0^2}\right)\left(\frac{p_0}{p_1}\right)$$

$$=-\tfrac{1}{2}\int(\mathscr{B}^{\mathrm{M}})^2. \tag{3.31a}$$

The second term is the $\mathscr{B}^{\text{string}}$-squared term, which cancels against the third term $(j_\mu G_{\mu\nu}^{\text{return}} j_\nu)$, as promised, so that the sum is just the magnetic energy of the monopole $\mathscr{B}_\mu^{\mathrm{M}}$ without the string,

$$-W=-\frac{1}{2}\left(\frac{2\pi n}{e}\right)^2\int\frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{p^2}. \tag{3.31b}$$

We recover the divergent expression of the free theory, see eqs. (3.6) and (3.9). At the next order in e^2 (i.e. order one) one encounters one-loop diagrams (see fig. 4). Other diagrams are of higher order in scalar–scalar couplings. The sum of diagrams in fig. 4 is the vacuum electromagnetic energy of the charged bosonic

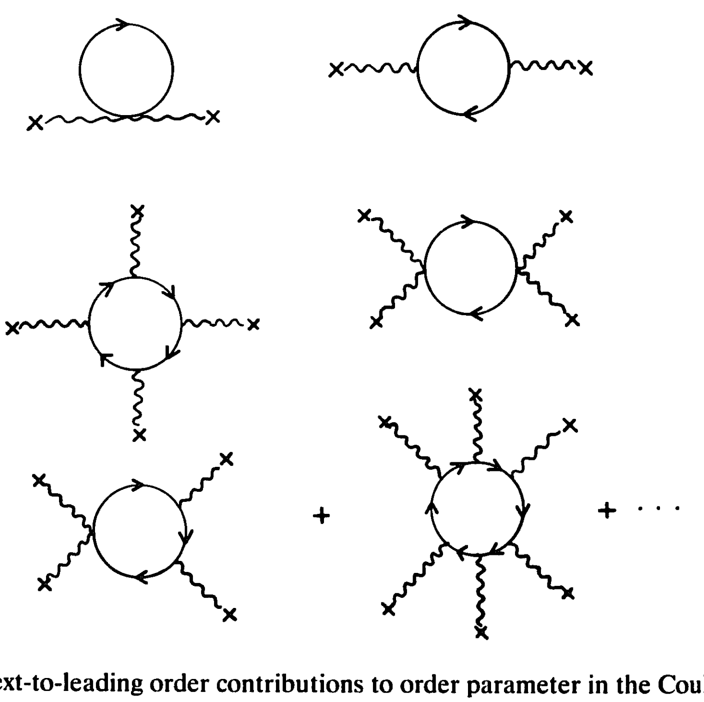

Fig. 4. Next-to-leading order contributions to order parameter in the Coulomb phase.

field in the presence of the external electromagnetic field $\mathcal{A}_\mu$, and hence is just the standard gaussian result

$$W = \operatorname{Tr}\ln\left[\frac{(\partial_\mu - ie\mathcal{A}_\mu)^2 + m^2}{\partial_\mu^2 + m^2}\right] = \ln\det\left[\frac{(\partial_\mu - ie\mathcal{A}_\mu)^2 + m^2}{\partial_\mu^2 + m^2}\right]. \qquad (3.32)$$

The calculation of W therefore reduces to the solution of a nonrelativistic quantum mechanical problem – a charged particle in the field of a magnetic monopole [27].

The quantization of the magnetic flux g now appears in the guise of the Dirac quantization condition [28]. For nonquantized g the expression (3.32) is divergent due to the singular vector potential along the string. Moreover, as we will see shortly, the correlator $\langle V_n(x)V_n^*(y)\rangle$ would in this case be dependent on the location of the string, thus rendering $V_n(x)$ not a Lorentz scalar.

Since, in our case $(g = 2\pi n/e)$ the monopole satisfies the Dirac quantization condition, it is clear that any physical quantity is independent of the string. The solution of the quantum mechanical problem is well known [27]. The determinant of the Schrödinger operator (3.32) is both ultraviolet and infrared finite. Note that there is no additional ultraviolet renormalization of the composite operator $V_n(x)$ in this order and the constant C can be chosen the same as in the free theory. We expect this to hold to all orders at least when the scalar potential $V(\phi^*\phi)$ does not contain a $(\phi^*\phi)^3$ term, in analogy to 3 + 1 dimensions [25].

The appearance of the monopole configuration in this calculation is, in fact, natural. The vev of an order field $\mathcal{O}(x)$ generally is closely related to the tunneling amplitude between eigenstates of the (broken) charge with different eigenvalues. Consider the charge Q with integer eigenvalues, and an order field $\mathcal{O}(x)$,

$$[\mathcal{O}(x), Q] = \mathcal{O}(x). \qquad (3.33)$$

Let us choose in the Hilbert space the basis $|n(x)\rangle$ (for the sake of notational simplicity we work with discrete x), where

$$\mathcal{O}(x)|n(y)\rangle = \frac{1}{n^{1/2}(x)}|n(y) + \delta_{x,y}\rangle, \qquad (3.34)$$

$$Q|n(y)\rangle = q|n(y)\rangle, \qquad q = \sum_y n(y). \qquad (3.35)$$

Here the charge q is quantized and $n(x)$ is an integer valued function. The vacuum of the broken phase is a superposition of states with different charges q,

$$|\mathrm{vac}\rangle = \sum_{\{n(x)\}} \Psi[n(x)]|n(x)\rangle . \tag{3.36}$$

The vev of $\mathcal{O}(x)$ is

$$\langle \mathcal{O}(x)\rangle = \sum_{\{n(y)\}} \Psi^*[n(y)]\Psi\left[n(y)+\delta_{x,y}\right]n^{-1/2}(y) . \tag{3.37}$$

One recognizes in eq. (3.37) the tunneling amplitude over an infinite time between the states that differ by one unit of the charge,

$$\lim_{t\to\infty}\langle n(y)|\mathrm{e}^{-iHt}|n(y)+\delta_{x,y}\rangle = \Psi^*[n(y)]\Psi\left[n(y)+\delta_{x,y}\right] . \tag{3.38}$$

Therefore the order parameter is given by the sum of these tunneling amplitudes*. The monopoles appearing in eq. (3.29) represent semiclassically the tunneling processes between states with different flux.

The correlator of the order field is also easily calculated in the first two orders of perturbation theory. Quite generally, the correlator is given by the same expression as the vev in eq. (3.25) with $j_\mu(x)$ replaced by $j_\mu(x)-j_\mu(y)$. The leading order coincides with free theory of eq. (3.10), for which $W(x-y)=(2\pi n/e)^2G(x-y)$ [G is defined in eq. (3.11)] is the energy of the monopole–antimonopole pair without a string between them.

In the next-to-leading order one has to calculate the determinant of the Schrödinger operator of a particle in the field of a monopole–antimonopole pair. As a result, at large separations, only the coefficient of $1/|x-y|$ in the exponent is modified. Note that for the values of g in eq. (3.22) the correlator is independent of the shape of the string, and depends only on $(x-y)^2$. The same is true for any other Green function $V(x)$. Therefore the operator $V(x)$ is a covariant scalar field.

To conclude, to this order in perturbation theory, the order parameter is nonzero and the correlator of the order field approaches a finite value at large separations. This means that the flux symmetry is NG broken.

The validity of this result should not be taken for granted – higher orders or nonperturbative effects may be important. We shall discuss this in more detail in subsect. 3.6.

3.4. HIGGS PHASE

In the Higgs phase there are no broken symmetries, and therefore no massless particles. We expect that at the phase transition point the flux symmetry is

* If the order field has an eigenvalue m, this calculation will involve tunneling amplitudes between states with $\Delta q = m$.

restored. To see this explicitly, let us calculate the order parameter $\langle V_n(x)\rangle$ in the same approximation we used in the Coulomb phase.

In the path integral formalism,

$$\langle V_n(x)\rangle = C\int \mathscr{D}\phi\, \mathscr{D}\phi^*\, \mathscr{D}A \exp\left\{ -S(\phi,\phi^*,A) - \frac{2\pi n}{e}\int j_\mu A_\mu + \frac{1}{2}\left(\frac{2\pi n}{e}\right)^2 \iint j_\mu G_{\mu\nu}^{\text{return}} j_\nu \right\}. \tag{3.39}$$

To derive the e^2-expansion of W, let us rescale the fields and couplings of the theory, $A_\mu \to (1/e)A_\mu$, $\lambda \to e^2\lambda$, and $\phi \to (1/e)\phi$, where λ is the quartic coupling*. Then we can establish the factor $1/e^2$ in front of the lagrangian,

$$\langle V_n(x)\rangle = C\int \mathscr{D}\phi\, \mathscr{D}\phi^*\, \mathscr{D}A \exp\left\{ -\frac{1}{e^2}\left[S(\phi,\phi^*,A) - 2\pi n\int j_\mu A_\mu + 2\pi^2 n^2 \iint j_\mu G_{\mu\nu}^{\text{return}} j_\nu \right]\right\}. \tag{3.40}$$

Consequently the small-e expansion of W is equivalent to the $\hbar$-(or loop) expansion of the ground-state energy of the Higgs model.

In leading order W is just the sum of all tree vacuum diagrams, and is equal to the classical energy in the presence of external sources, corrected by the constant term $\sim \iint j_\mu G_{\mu\nu}^{\text{return}} j_\nu$ (which precisely subtracts the contribution of the string of the monopole).

Since in the Higgs phase the photon has nonzero mass μ the classical action of the three-dimensional monopole in the superconducting medium is linearly divergent in the infrared [11]. As a result, the order parameter vanishes.

Note that the ultraviolet divergences are the same as in the Coulomb phase, and therefore no additional renormalization is required.

The correlator of the order field is given in terms of the classical energy of a monopole–antimonopole pair in the superconductor. At large separation the magnetic flux is concentrated in a flux tube of thickness of order $\mu^{-1} = ev$, where v is the classical expectation value of the scalar field (see fig. 5a). The euclidean action of this configuration is proportional to the distance R between the monopole and antimonopole.

$$W = M_n R\,. \tag{3.41}$$

* We concentrate on a theory without $(\phi^*\phi)^3$ interaction term throughout this subsection.

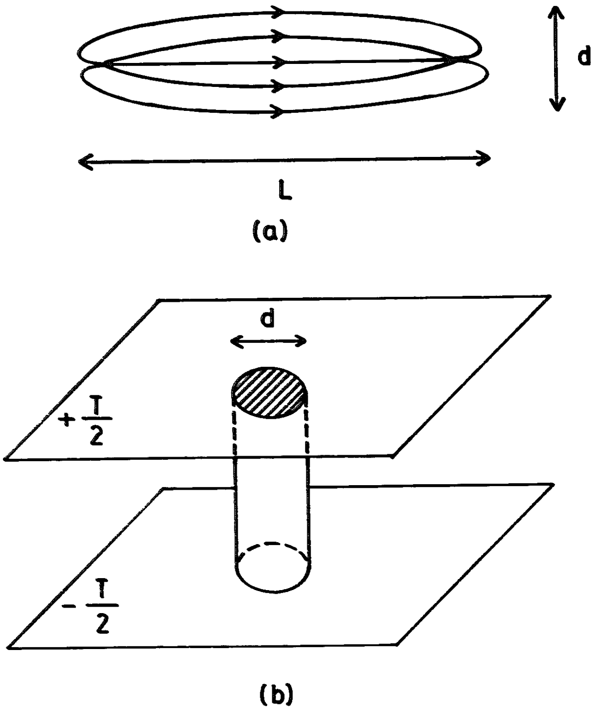

Fig. 5. (a) Flux tube produced by monopole–antimonopole pair in the Higgs vacuum. (b) Space-time diagram of a two-dimensional magnetic vortex.

It follows that the correlator of the order field at large separations behaves as

$$\langle V_n^*(x) V_n(y) \rangle \sim e^{-M_n|x-y|}. \tag{3.42}$$

The operator $V_n(x)$ ($V_n^*(x)$) interpolates a physical particle of mass M_n and flux $2\pi n/e$ $(-2\pi n/e)$ – the vortex (antivortex). The mass M_n is just the classical energy per unit length of a Nielsen–Olesen vortex line [9]. This is seen as follows. Let us consider the solution of the classical field equations of the $(2+1)$-dimensional Higgs model with the following boundary conditions: the vortex of strength $2\pi n/e$ is located at the origin between times $-T/2$ and $T/2$ (see fig. 5b). On one hand it represents the vortex at rest which propagates in time and for $T \to \infty$ the classical action is $S = M_n T$. On the other hand, the same solution of the euclidean theory gives the energy of the NO vortex line in the $(3+1)$-dimensional Higgs model. Therefore the mass of the vortex is given by [11]

$$M_n = \pi v^2 \ln \frac{e}{\sqrt{\lambda}}, \tag{3.43}$$

where λ is the quartic self-coupling of the scalar field.

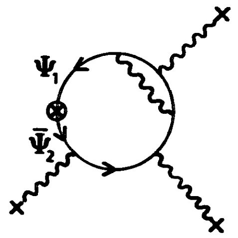

Fig. 6. Typical diagram contributing to the order parameter in spinor QED.

3.5. ORDER PARAMETER IN SPINOR QED

We turn now to the calculation of the order parameter in spinor QED. The symmetry breaking pattern is different: $U_\chi(1) \otimes U_\Phi(1) \to U_I(1)$. We should choose an operator which commutes with the unbroken generator I_μ and is an eigenoperator of a broken generator (J^5_μ or $\tilde{F}_\mu$)*. The simplest choice is

$$\mathscr{O}(x) = \bar{\psi}_1(x)\psi_2(x)V_2(x), \tag{3.44}$$

where $V_2(x)$ is the vortex operator defined in eq. (3.23) with $n = 2$, and the index on ψ refers to flavour. Its vev is given by a path integral,

$$\langle \mathscr{O}(x)\rangle = C\int \mathscr{D}\psi\,\mathscr{D}\bar{\psi}\,\mathscr{D}A\,\bar{\psi}_1\psi_2 \exp\left\{-S(\psi,\bar{\psi},A) - \frac{4\pi}{e}\int j_\mu A_\mu + 2\left(\frac{2\pi}{e}\right)^2 \iint j_\mu G^{\text{return}}_{\mu\nu} j_\nu\right\}. \tag{3.45}$$

Just as in the calculation of the vev of the vortex operator in subsect. 3.4, this can be interpreted diagrammatically. The typical diagram is shown in fig. 6. Since the electromagnetic interaction does not change flavour, any diagram of this type vanishes. Therefore, to all orders in perturbation theory the order parameter vanishes, and according to the definition in appendix A the symmetry breaking is of Kosterlitz–Thouless type.

3.6. VALIDITY OF PERTURBATION THEORY

Up to now, in calculating the order parameter, we have relied on weak coupling perturbation theory, to next-to-leading order. The validity of the approximation, however, can be questioned.

First, the real spectrum of the theory is qualitatively different from the perturbative one. The charged fermions (and also bosons in scalar QED) are logarithmically confined. Therefore, one expects important nonperturbative effects.

* If the operator carries nonzero quantum numbers of an unbroken symmetry its vev will trivially vanish.

As was shown recently by Sen [29], confinement of elementary charges does show up in order e^2, as an on-shell infrared divergence in the electron's propagator. However, other relevant effects, like the appearance of bound states, are not seen in perturbation theory.

Our calculations in both scalar and spinor QED are, up to now, at zeroth order in coupling (e^0), and do not incorporate the effects of confinement. The fact that corrections to the calculation we performed are indeed important can be seen from the following consideration. Let us consider the vev of the vortex operator $V_1(x)$, eq. (3.23), in spinor QED. This is not a good order parameter for the symmetry breakdown pattern $U_\Phi(1) \otimes U_\chi(1) \to U_I(1)$, since $V_1(x)$ is an eigenoperator of the unbroken charge,

$$[V_1(x), I] = V_1(x). \tag{3.46}$$

Since I is not broken, the expectation value of $V_1(x)$ must vanish in the vacuum. However, performing the calculation to order one, we obtain

$$\langle V_1(x) \rangle = \frac{\det|\partial\!\!\!/ - m - \mathcal{A}\!\!\!/|}{\det|\partial\!\!\!/ - m|}, \tag{3.47}$$

which is manifestly finite. We conclude therefore, that other contributions are important and should eventually lead to the vanishing of $\langle V_1(x) \rangle$.

This calculation is very similar to the one we performed in the Coulomb phase of scalar QED, subsect. 3.3. Therefore, one might expect that in that theory also higher-order and/or nonperturbative effects may lead to the vanishing of the order parameter, changing the realization of flux symmetry from NG to KT. We will address this question in a future publication. The interesting question of whether the symmetry breaking in both scalar and spinor QED is of NG- or KT-type beyond perturbation theory must be settled by nonperturbative methods, such as lattice simulation.

4. Discussion and conclusion

We studied realization of the flux and chiral symmetries in various (2 + 1)-dimensional abelian gauge theories. All these models have two phases. In the Coulomb phase the photon is massless. We show that the masslessness of the photon is a result of the spontaneous symmetry breaking of the flux symmetry. In the second phase the flux symmetry is unbroken and as a result the photon is massive. In scalar QED this is the Higgs phase. In spinor theories (QED and Chern–Simons electrodynamics) this is the phase where the photon acquires a topological mass. The vacuum in this phase is annihilated by the flux Φ which provides a natural interpretation of the superconducting state (the Meissner effect). There is a finite gap for flux carrying excitations (vortices) and as a result, small external magnetic fields are expelled.

We determined the symmetry breaking pattern and found a corresponding order parameter: the vortex operator. In the massive phases this order parameter vanishes, whereas in the Coulomb phase the situation is more delicate. If $\langle \mathscr{V} \rangle \neq 0$ it is another example of the familiar NG phenomenon, while if $\langle \mathscr{V} \rangle = 0$ the flux symmetry is KT broken. We performed a perturbative calculation of the order parameter. At next-to-leading order in e^2 in scalar QED, $\langle \mathscr{V} \rangle \neq 0$, while in spinor QED $\langle \mathscr{V} \rangle = 0$. This calculation is essentially equivalent to evaluating the partition function of QED in the presence of an external current creating a magnetic monopole (without the Dirac string). In this representation the mode of flux symmetry realization is determined by the long-distance asymptotics of the interaction between monopoles. If the renormalized interaction decays faster than ln r we get a NG mode, whereas for a slower decay KT mode is realized. In order to settle this question one should go beyond perturbation theory, for example to lattice simulation.

One should be aware, however, that the correct model on the lattice is *noncompact* QED. It is well known that in compact QED the flux current is not conserved. Instantons break the flux symmetry, and as a result the photon has a finite mass [30]. The continuum limit of compact QED is not the continuum theory we considered. Noncompact QED in $2+1$ dimensions unfortunately has not been thoroughly studied on the lattice but there are strong indications that it contains a massless photon and provides a proper regularization of continuum QED [31].

Let us now discuss the possible consequences of our general approach to other gauge theories. The first natural class of theories are nonabelian gauge theories in $2+1$ dimensions.

In pure Yang–Mills theory with a semi-simple gauge group G there is no direct generalization of the conserved (gauge invariant) current analogous to $\tilde{F}_\mu$. The natural candidates $\tilde{F}^a_\mu$ are only *covariantly* conserved, $D^{ab}_\mu \tilde{F}^b_\mu = 0$. Therefore one should not expect to find massless gluons, in agreement with general belief and lattice simulations. In more complicated cases one *perturbatively* encounters massless gauge bosons. This happens when the gauge group G is broken down to H, which contains an invariant abelian subgroup.

In this case one can define a quantity $\tilde{F}_\mu$, which is conserved within perturbation theory [32]. However one still cannot define a current which is conserved nonperturbatively. Semiclassically, this is apparent since instantons which break this "conservation law" explicitly have finite action. Indeed, generally, gauge bosons nonperturbatively acquire mass. One can expect that they can remain massless beyond perturbation theory only when a theory possesses an additional global symmetry which is spontaneously broken. We can find support for this point of view from the following examples taken from ref. [33]. Affleck et al. [33] considered an SO(3) gauge theory in $2+1$ dimensions, with gauge symmetry breaking pattern SO(3) → U(1),

$$\mathscr{L} = -\tfrac{1}{4}F^2 + \tfrac{1}{2}(D_\mu\phi)^2 + V(\phi) + \bar{\psi}\cdot(i\not{D} + g\phi\times)\psi\,, \tag{4.1}$$

where ψ^a and ϕ^a transform as adjoint representations of the gauge group. The system has a global U(1) symmetry, $\psi \to e^{i\theta}\psi$, $\bar{\psi} \to e^{-i\theta}\bar{\psi}$.

Affleck et al. [33] showed that this symmetry transformation connects the vacuum to a one-photon state. The symmetry remains broken nonperturbatively and the photon remains massless even when the non-perturbative instanton contributions are taken into account.

Some constrained systems are conveniently represented as abelian gauge theories. For example, the CP^{N-1} model can be represented in the gauge-invariant form [34]

$$\mathscr{L} = \frac{N}{f}|\mathscr{D}n|^2 + \alpha(n^\dagger n - 1)\,, \tag{4.2}$$

where the covariant derivative $D_\mu = \partial_\mu - iA_\mu$, with A_μ an auxiliary field $A_\mu = \frac{1}{2}i(n^*\partial_\mu n - \partial_\mu n^* n)$. On the classical level the theory possesses a conserved topological current $\tilde{F}_\mu$ [35]. To all orders in the $1/N$-expansion in the disordered phase (strong coupling) this symmetry is spontaneously broken [34]. As a result there appears a composite massless mode, "the photon". However, this theory resembles [36] compact QED_3 rather than the noncompact one. The hedgehog instanton's action in the continuum is finite [37] and recent lattice calculations [38] indicate a finite instanton density and an anomalous flux symmetry breaking in the disordered phase. The "photon" therefore becomes massive due to nonperturbative effects (in $1/N$).

In this paper we considered only the simple (2 + 1)-dimensional case. It would be more interesting to consider the realistic (3 + 1)-gauge theories with massless gauge bosons from this point of view. The natural generalization of the (2 + 1)-dimensional flux symmetry current $\tilde{F}_\mu$ is the dual field strength tensor $\tilde{F}_{\mu\nu}$. It satisfies the conservation law $\partial_\mu\tilde{F}_{\mu\nu} = 0$ and creates a photon from the vacuum. The corresponding charge $\Phi_\nu = \int\tilde{F}_{0\nu}\,d^3x$ is no longer a Lorentz scalar. $\Phi_0 = 0$ identically, while, of the three components of the magnetic flux Φ_i only two are independent. Breakdown of these two symmetries leads to the appearance of two massless photons. The photons are, however, not (Lorentz) scalars, due to the nontrivial commutators between the Lorentz group and the flux symmetry generators. It is interesting to further study these questions.

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Appendix A

MODES OF REALIZATION OF CONTINUOUS SYMMETRY

In this appendix we review the different modes in which a continuous symmetry may exhibit itself in quantum field theory. Let J_μ be a conserved current, $\partial_\mu J_\mu = 0$, and $Q = \int J_0$ a corresponding global charge. As is well known, in quantum field theory the physical spectrum does not always form multiplets under the action of Q.

The first important feature distinguishing between different modes of realization of the symmetry is the transformation properties of the vacuum under the action of Q. If $Q|0\rangle = 0$, the symmetry is said to be broken, while when $Q|0\rangle \neq 0$, the vacuum is degenerate and the symmetry is spontaneously broken. A hallmark of theories possessing a broken symmetry is an appearance of zero-mass excitations.

It is convenient to define finer subdivisions for the modes of symmetry realization. In the unbroken case one can distinguish between Wigner–Weyl (WW) mode, in which finite-energy excitations form multiplets carrying nonzero charge Q, and a neutral mode in which all finite-energy excitations are zero-charge singlets. Examples of WW mode are plentiful. Triality in QCD and electric charge $U_e(1)$ symmetry in the Higgs phase of QED are examples of neutral mode.

Among the broken symmetries, along with the familiar Nambu–Goldstone (NG) mode there exists a distinct mode of symmetry realization which we will call the Kosterlitz–Thouless (KT) mode. This is common in 1 + 1 dimensions: the O(2) symmetry in the low-temperature phase of the XY model [2] or chiral symmetry in various fermionic theories [3].

One distinguishes between NG- and KT-modes by the value of an order parameter pertinent to the symmetry. In order to make the distinction precise we define an order parameter $v_{\mathcal{O}}$ as the vev of an eigenoperator $\mathcal{O}$ of Q,

$$v_{\mathcal{O}} = \langle 0|\mathcal{O}(x)|0\rangle, \qquad [\mathcal{O}, Q] = \zeta\mathcal{O}. \tag{A.1}$$

When $v_{\mathcal{O}} \neq 0$ the symmetry is realized in NG mode. However, spontaneous breaking of a symmetry does not necessarily imply the nonvanishing of an order parameter. If $v_{\mathcal{O}} = 0$ for any eigenoperator O, we will call this the KT realization of symmetry*.

It is important to stress that the Goldstone theorem remains valid even in KT mode. The only assumption which is used in the general proof (see for example ref. [16]) is that there exists an operator χ such that $\langle 0|[\chi, Q]|0\rangle \neq 0$. The existence of such an operator is guaranteed by the fact that $Q|0\rangle \neq 0$. Consequently in the KT mode (as in NG) the existence of a massless excitation, for example, the spin wave in the XY model, is required by the Goldstone theorem.

* This definition is implicit in Witten's paper [3].

Alternatively one can distinguish between NG- and KT-modes by the behaviour of the correlator of the order field. It is convenient to use an eigenoperator which is a local field,

$$[J_0(x), \mathcal{O}(y)] = \zeta\mathcal{O}(x)\delta(x-y). \tag{A.2}$$

In the WW mode the order field $\mathcal{O}(x)$ interpolates a massive particle. It's correlator decays exponentially,

$$\langle 0|\mathcal{O}(x)\mathcal{O}(y)|0\rangle \sim e^{-m|x-y|}, \tag{A.3}$$

at large separations. The mass m is the lowest mass in the relevant channel. If the symmetry is in neutral mode m is infinite and the correlator vanishes.

The correlator falls slower in the broken symmetry case. For NG mode, since $\langle\mathcal{O}(x)\rangle = v_{\mathcal{O}} \neq 0$, it approaches a constant,

$$\langle 0|\mathcal{O}(x)\mathcal{O}(y)|0\rangle \to v_{\mathcal{O}}^2 \neq 0. \tag{A.4}$$

In the KT mode the correlator vanishes at infinite distance, but usually slower than an exponential, due to the existence of a zero mode. In the XY model, the Gross–Neveu model, and other (1 + 1)-dimensional models, it falls as a power [3],

$$\langle 0|\mathcal{O}(x)\mathcal{O}(y)|0\rangle \to \frac{1}{|x-y|^{\alpha}}. \tag{A.5}$$

The simplest and a rather generic example of the KT mode in $1+1$ is a free massless scalar,

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^2. \tag{A.6}$$

The "shift" symmetry $\phi \to \phi + \text{const.}$ is generated by $Q = \int\pi\,dx = \int\partial_0\phi\,dx$, the corresponding current being $J_\mu = \partial_\mu\phi$. The vacuum is degenerate: all coherent states of the form $\exp[ia\int dx\,\pi(x)]|0\rangle$, where π is the momentum conjugate to ϕ, have equal energy, and the shift symmetry is broken*. Yet for any choice of the vacuum state, the expectation value of the order field $\mathcal{O}(x) = \exp[i\zeta\phi(x)]$ vanishes. The particle ϕ is the corresponding massless mode. A generic order field is $\mathcal{O}(x) \sim C\exp i\zeta\phi(x)$,

$$[J_0(x), e^{i\zeta\phi(y)}] = i\zeta\, e^{i\zeta\phi(y)}\delta(x-y). \tag{A.7}$$

* Sometimes it is said that in these theories the symmetry is not broken. The reason is that the scalar product between different vacua is finite, unlike in higher dimensions where it vanishes exponentially with volume [13]. Therefore one is able to construct an invariant superposition. For finite volume (infrared cutoff) this state turns out to be a nondegenerate groundstate. However for infinite volume it becomes degenerate with other states and the symmetry is broken.

TABLE 1
Modes of symmetry realization. The table shows the characteristics that distinguish the various modes of symmetry realization

	Wigner–Weyl	Neutral	Nambu–Goldstone	Kosterlitz–Thouless
Degeneracy of vacuum	unbroken $Q\|0\rangle = 0$	unbroken $Q\|0\rangle = 0$	broken $Q\|0\rangle \neq 0$	broken $Q\|0\rangle \neq 0$
Corresponding massless excitations	none	none	NG bosons	KT bosons
Energy gap to charged excitations	finite	infinite	no charged eigenstates	no charged eigenstates
vev of order field	0	0	$\neq 0$	0
Correlator of order field as $r \to \infty$	e^{-rm}	0	const.	$r^{-\alpha}$

The expectation value of $\mathscr{O}(x)$ in any of the vacua can be easily calculated,

$$\langle \mathscr{O}(x) \rangle = C^2 e^{-\zeta^2 G(0)} \sim e^{(\zeta^2/4\pi)\ln \mu^2} = \mu^{\zeta^2/2\pi}, \tag{A.8}$$

where μ is an infrared cutoff. In the $\mu \to 0$ limit the order parameter vanishes. Therefore (in two dimensions only) the shift symmetry is realized in KT mode. The correlator of $\mathscr{O}(x)$ is

$$\langle 0|\mathscr{O}(x)\mathscr{O}(y)|0\rangle = e^{-\zeta^2 G(x-y)} = |x-y|^{-\zeta^2/2\pi}. \tag{A.9}$$

Although in all (1 + 1)-dimensional examples of KT realizations the correlator falls off as a power we are not aware of a general proof that this is always the case. Therefore as a definition of KT realization we choose the degeneracy of vacuum accompanied by the vanishing of the order parameter. The characteristic features of various symmetry realizations are summarized in table 1.

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