Effective Field Theory for Density Functional Theory I

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I. Overview of EFT, RG, DFT for fermion many-body systems

II. EFT/DFT for dilute Fermi systems

III. Refinements: Toward EFT/DFT for nuclei

IV. Loose ends and challenges, Cold atoms, RG/DFT

Outline

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Overview of Fermion Many-Body Systems

Density Functional Theory for Coulomb Systems

DFT for Nuclei? $\Longrightarrow$ EFT and RG

Summary I

Examples of Fermion Many-Body Systems

- Collections of “fundamental” fermions (electrons, quarks, . . .)
  - or of composites of *odd* number of fermions (e.g., protons)

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  - find charge distribution, binding energy, bond lengths, . . .
- Bulk solid-state materials
  - metals, insulators, semiconductors, superconductors, . . .
- Liquid ^{3}He (superfluid!)
- Cold fermionic atoms in (optical) traps [^{6}Li or ^{7}Li?]
- Atomic nuclei
- Neutron stars
  - neutron matter
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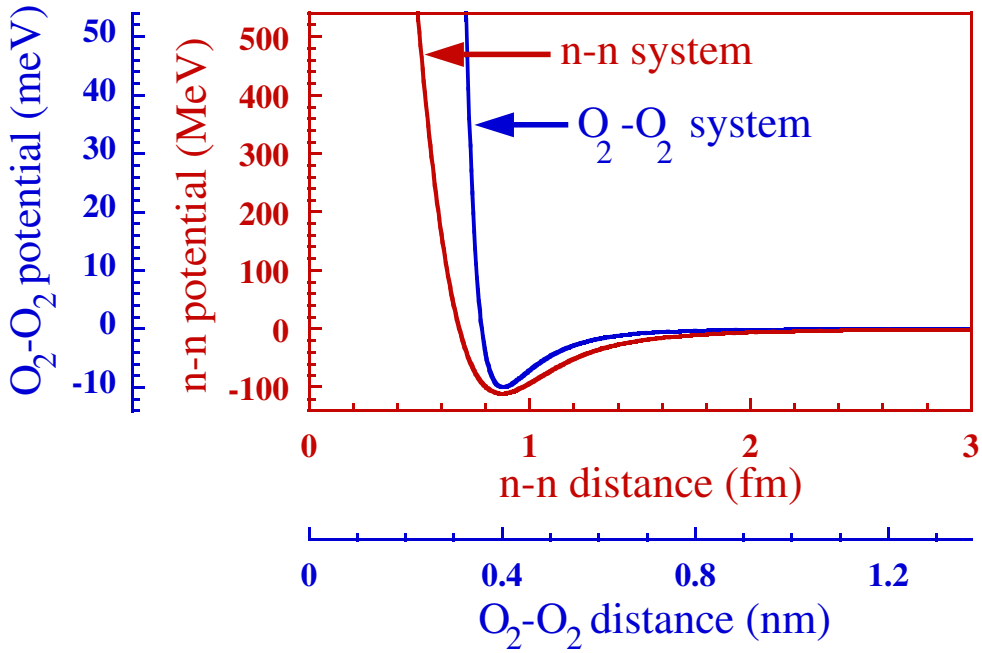
Nuclear and Cold Atom Many-Body Problems

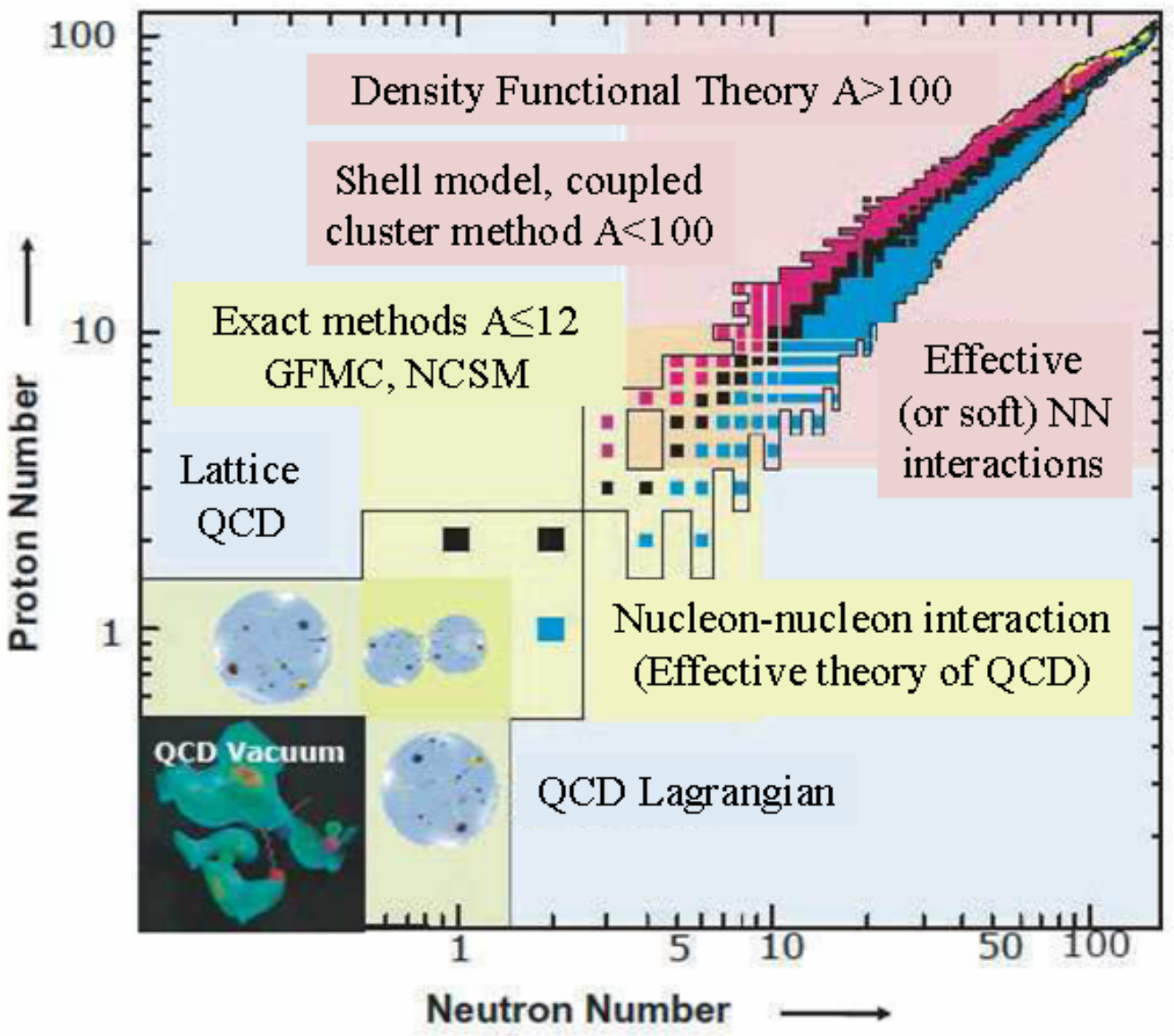
- Lennard-Jones and nucleon-nucleon potentials



[figure borrowed from J. Dobaczewski]

- Are there universal features of such many-body systems?
- How can we deal with “hard cores” in many-body systems?

The Big Picture (adapted from Richter @INPC2004)



The Many-Body Schrödinger Wave Function

[adapted from Joe Carlson]

- How to represent the wave function for an A-body nucleus?
- Consider ^{8}Be ($Z = 4$ protons, $N = 4$ neutrons)

$$|\Psi\rangle = \sum_{\sigma,\tau} \chi_\sigma \chi_\tau \phi(\mathbf{R}) \quad \text{where } \mathbf{R} \text{ are the } 3A \text{ spatial coordinates}$$

$$\chi_\sigma = \downarrow_1 \uparrow_2 \cdots \downarrow_A \ (2^A \text{ terms}) = 256 \text{ for } A = 8$$

$$\chi_\tau = n_1 n_2 \cdots p_A \ (\tfrac{A!}{N!Z!} \text{ terms}) = 70 \text{ for } ^8\text{Be}$$

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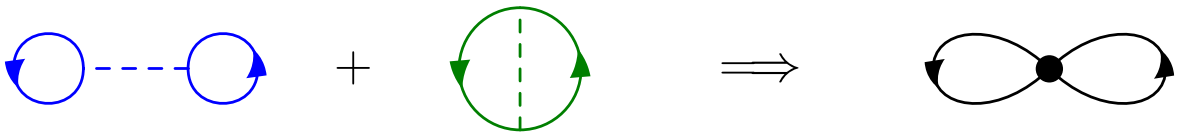
- So for ^{8}Be there are 17,920 complex functions in $3A - 3 = 21$ dimensions!
- Suppose you represent this for a nucleus of size 10 fm with a mesh spacing of 0.5 fm. You would need 10^{27} grid points!

Hartree-Fock Wave Function

- Best single Slater determinant in variational sense

$$|\Psi_{\rm HF}\rangle = \det\{\phi_i(\mathbf{x}), i = 1\cdots A\}\,, \quad \mathbf{x} = (\mathbf{r}, \sigma, \tau)$$

- Hartree-Fock energy:



$$\langle\Psi_{\rm HF}|\widehat{H}|\Psi_{\rm HF}\rangle = \sum_{i=1}^{A}\frac{\hbar^2}{2M}\int d\mathbf{x}\,\boldsymbol{\nabla}\phi_i^*\cdot\boldsymbol{\nabla}\phi_i + \frac{1}{2}\sum_{i,j=1}^{A}\int d\mathbf{x}\int d\mathbf{y}\,|\phi_i(\mathbf{x})|^2 v(\mathbf{x},\mathbf{y})|\phi_j(\mathbf{y})|^2$$

$$-\frac{1}{2}\sum_{i,j=1}^{A}\int d\mathbf{x}\int d\mathbf{y}\,\phi_i^*(\mathbf{x})\phi_i(\mathbf{y})v(\mathbf{x},\mathbf{y})\phi_j^*(\mathbf{y})\phi_j(\mathbf{x}) + \sum_{i=1}^{A}\int d\mathbf{y}\,v_{\rm ext}(\mathbf{y})|\phi_j(\mathbf{y})|^2$$

- Determine the ϕ_i by varying with fixed normalization:

$$\frac{\delta}{\delta\phi_i^*(\mathbf{x})}\left(\langle\Psi_{\rm HF}|\widehat{H}|\Psi_{\rm HF}\rangle - \sum_{j=1}^{A}\epsilon_j\int d\mathbf{y}\,|\phi_j(\mathbf{y})|^2\right) = 0$$

Hartree-Fock Wave Function

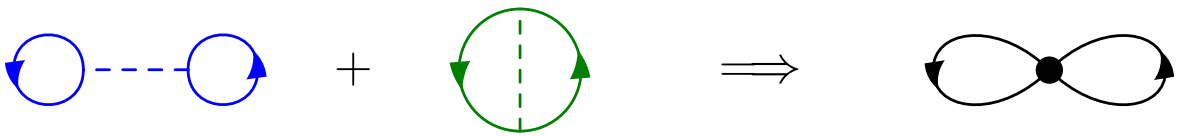
- Best single Slater determinant in variational sense

$$|\Psi_{\rm HF}\rangle = \det\{\phi_i(\mathbf{x}), i = 1\cdots A\}\,, \quad \mathbf{x} = (\mathbf{r}, \sigma, \tau)$$

- The $\phi_i(\mathbf{x})$ satisfy *non-local* Schrödinger equations:

$$-\frac{\boldsymbol{\nabla}^2}{2M}\phi_i(\mathbf{x}) + \Big(V_{\rm H}(\mathbf{x}) + v_{\rm ext}(\mathbf{x})\Big)\phi_i(\mathbf{x}) + \int d\mathbf{y}\, V_{\rm E}(\mathbf{x},\mathbf{y})\phi_i(\mathbf{y}) = \epsilon_i\phi_i(\mathbf{x})$$

with $V_{\rm H}(\mathbf{x}) = \int d\mathbf{y} \sum_{j=1}^{A} |\phi_j(\mathbf{y})|^2 \mathbf{v}(\mathbf{x},\mathbf{y})\,, \quad V_{\rm E}(\mathbf{x},\mathbf{y}) = -v(\mathbf{x},\mathbf{y}) \sum_{j=1}^{A} \phi_j(\mathbf{x})\phi_j^*(\mathbf{y})$

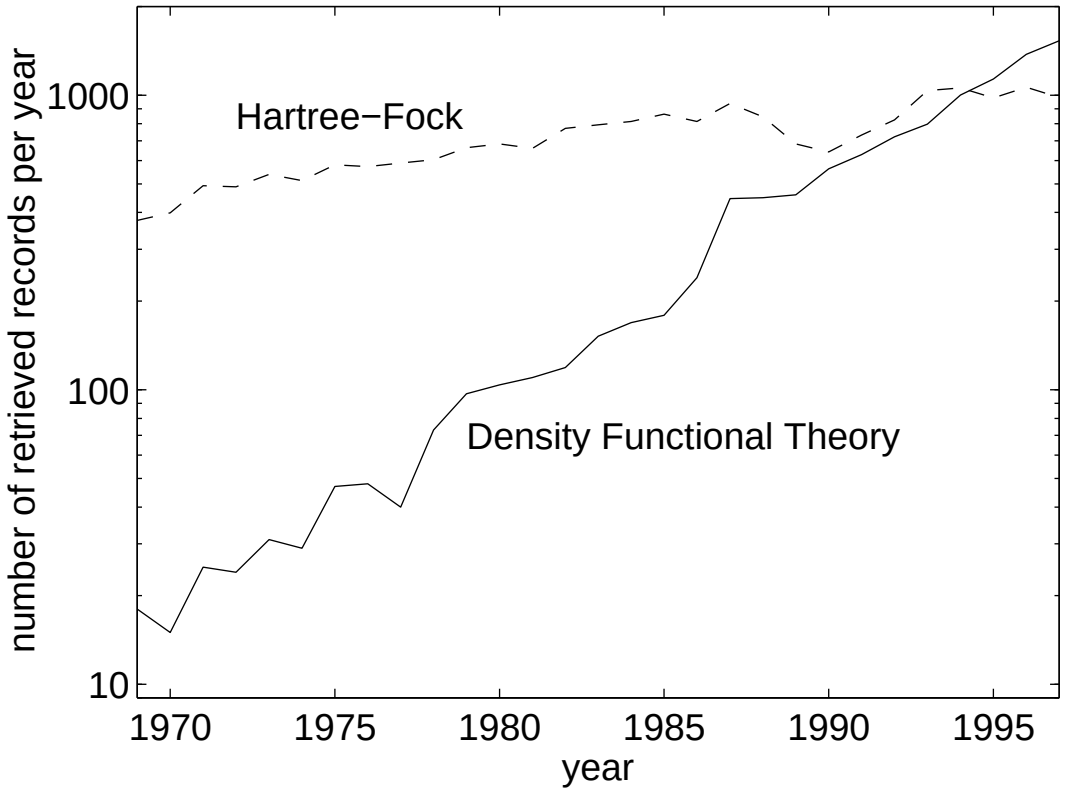


- Solve self-consistently; non-trivial because non-local

Outline

Density Functional Theory (DFT)

- Dominant application: inhomogeneous electron gas
- Interacting point electrons in static potential of atomic nuclei
- “Ab initio” calculations of atoms, molecules, crystals, surfaces, . . .

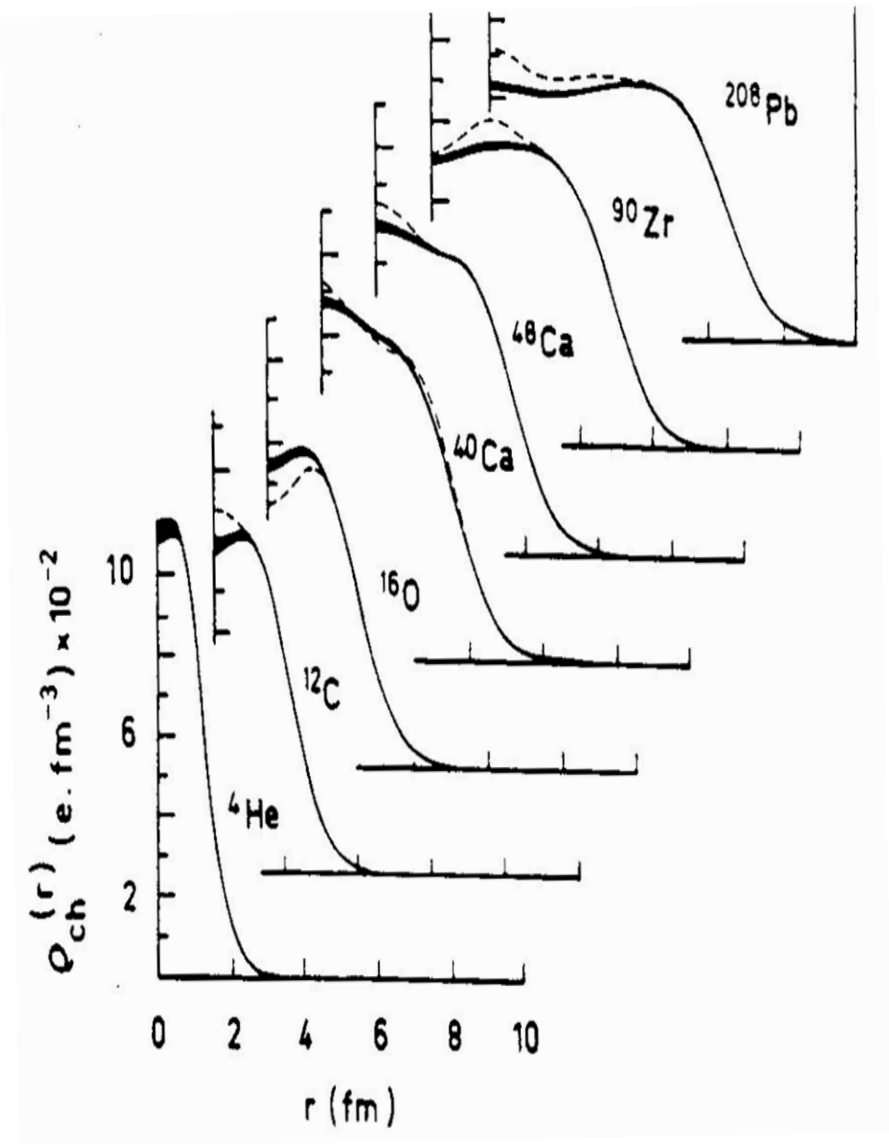


Density Functional Theory (DFT)

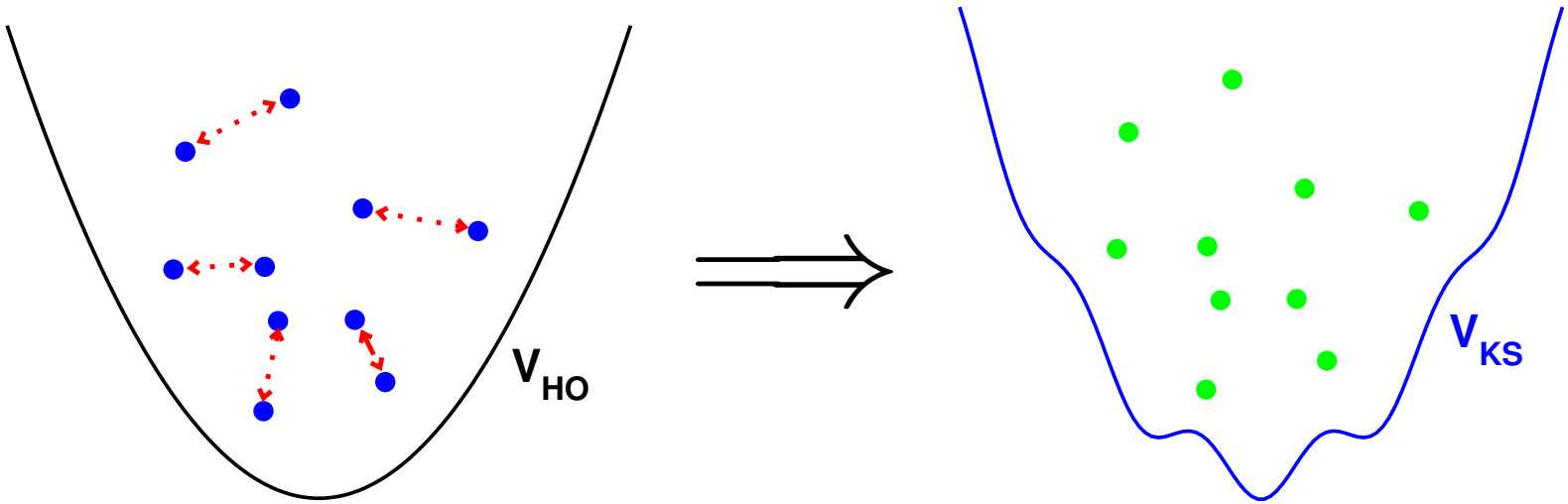
- Hohenberg-Kohn: There exists an energy functional $E_{v_{\text{ext}}}[\rho]$ ...

$$E_{v_{\text{ext}}}[\rho] = F_{\text{HK}}[\rho] + \int d^3x\, v_{\text{ext}}(\mathbf{x})\rho(\mathbf{x})$$

- F_{HK} is *universal* (same for any external v_{ext}) $\Longrightarrow$ H_2 to DNA!
- Useful if you can approximate the energy functional
- Introduce orbitals and minimize energy functional $\Longrightarrow$ E_{gs}, ρ_{gs}



Kohn-Sham DFT for $v_{\rm ext} = V_{\rm HO}$ Harmonic Trap



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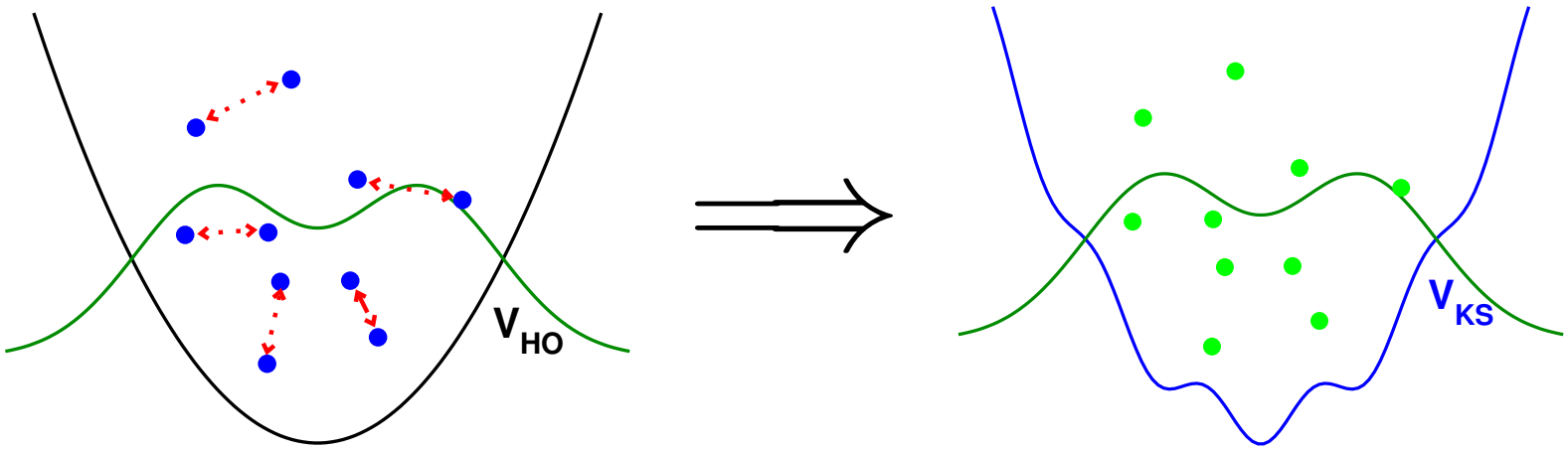


- Interacting density in $V_{\rm HO}$ $\equiv$ Non-interacting density in $V_{\rm KS}$
- Orbitals $\{\psi_i(\mathbf{x})\}$ in local potential $V_{\rm KS}([\rho], \mathbf{x})$

$$[-\boldsymbol{\nabla}^2/2m + V_{\rm KS}(\mathbf{x})]\psi_i = \varepsilon_i \psi_i \implies \rho(\mathbf{x}) = \sum_{i=1}^{A} |\psi_i(\mathbf{x})|^2$$

- Find Kohn-Sham potential $V_{\rm KS}([\rho], \mathbf{x})$ from $\delta E_{v_{\rm ext}}[\rho]/\delta\rho(\mathbf{x})$
- Solve self-consistently

DFT for Solid-State or Molecular Systems

- HK free energy for an inhomogeneous electron gas

$$F_{\rm HK}[\rho(\mathbf{x})] = T_{\rm KS}[\rho(\mathbf{x})] + \frac{e^2}{2}\int d^3x\, d^3x' \frac{\rho(\mathbf{x})\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + E_{\rm xc}[\rho(\mathbf{x})]$$

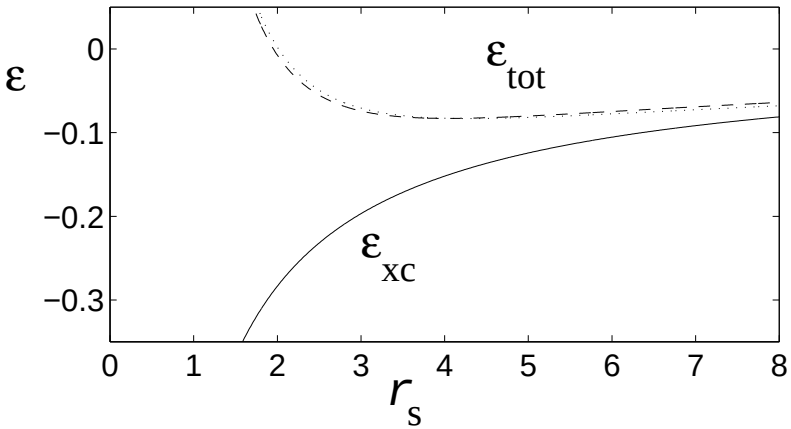
- Then $V_{\rm KS} = v_{\rm ext} - e\phi + v_{\rm xc}$ with $v_{\rm xc}(\mathbf{x}) = \delta E_{\rm xc}/\delta\rho(\mathbf{x})$
- Kohn-Sham $T_{\rm KS}[\rho(\mathbf{x})]$: find normalized $\{\psi_i, \epsilon_i\}$ from

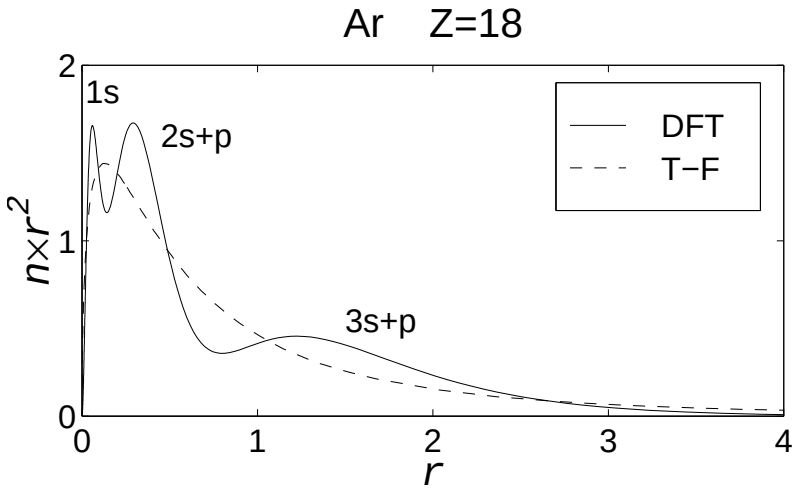
$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm KS}(\mathbf{x})\right)\psi_i(\mathbf{x}) = \epsilon_i\psi_i(\mathbf{x})$$

so that $\rho(\mathbf{x}) = \sum_{i=1}^{A} |\psi_i(\mathbf{x})|^2$ and

$$T_{\rm KS}[\rho(\mathbf{x})] = \sum_{i=1}^{A}\langle\psi_i| - \frac{\hbar^2}{2m}\nabla_i^2|\psi_i\rangle = \sum_{i=1}^{A}\epsilon_i - \int d^3x\, \rho(\mathbf{x})V_{\rm KS}(\mathbf{x})$$

- Local density approximation: $E_{\rm xc}[\rho(\mathbf{x})] \approx \int d^3x\, \mathcal{E}_{\rm xc}(\rho(\mathbf{x}))$
  - fit $\mathcal{E}_{\rm xc}(\rho)$ to Monte Carlo calculation of uniform electron gas





- in practice, use parametric formulas for energy density, e.g.,

$$\mathcal{E}_{\rm xc}(\rho)/\rho = -0.458/r_s - 0.0666 G(r_s/11.4)$$

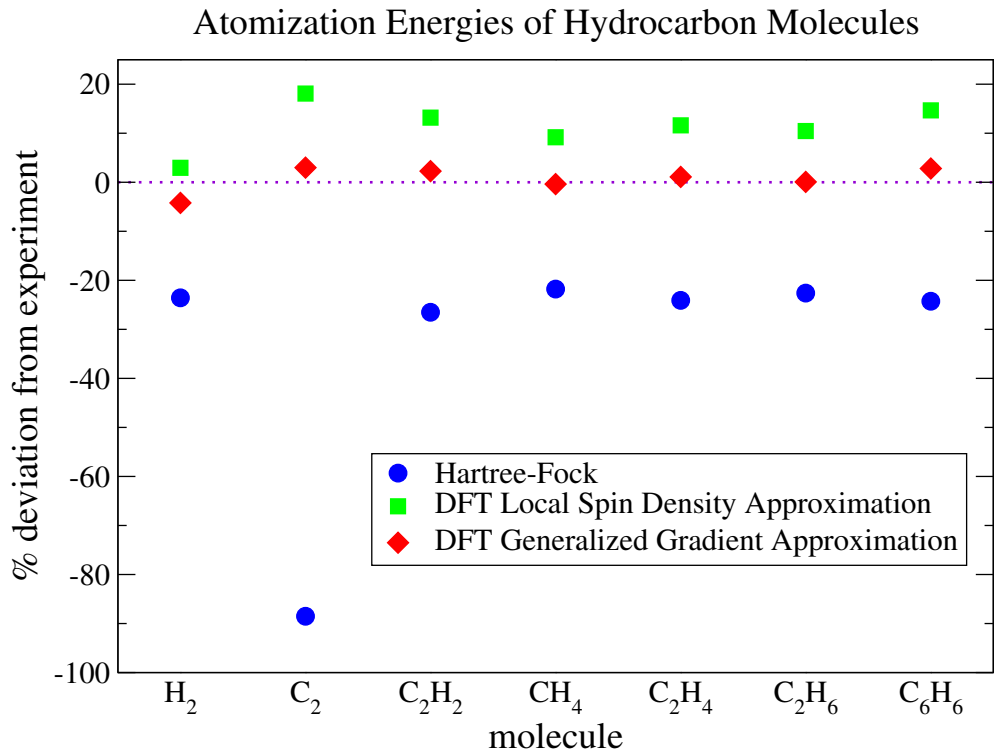
$$\text{with } G(x) = \frac{1}{2}\left\{(1+x)^3 \log(1+x^{-1}) - x^2 + \frac{1}{2}x - \frac{1}{3}\right\}$$

- just like “naive” Hartree approach with additional potential:

$$v_{\rm xc}(\mathbf{x}) = \left.\frac{d[\mathcal{E}_{\rm xc}(\rho)]}{d\rho}\right|_{\rho=\rho(\mathbf{x})}$$

Density Functional Theory (DFT)

- Dominant application: inhomogeneous electron gas
- Interacting point electrons in static potential of atomic nuclei
- “Ab initio” calculations of atoms, molecules, crystals, surfaces, . . .
- HF is good starting point, DFT/LDA is better, DFT/GGA is best



e.g., van Leeuwen–Baerends GGA

$$v_{\mathrm{xc}}(\mathbf{r}) = -\beta\rho^{1/3}(\mathbf{r})\frac{x^2(\mathbf{r})}{1+3\beta x(\mathbf{r})\sinh^{-1}(x(\mathbf{r}))}$$

with $x = |\nabla\rho|/\rho^{4/3}$

Quotes From the DFT Literature

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A Chemist's Guide to DFT (Koch & Holthausen, 2000)

"To many, the success of DFT appeared somewhat miraculous, and maybe even unjust and unjustified. Unjust in view of the easy achievement of accuracy that was so hard to come by in the wave function based methods. And unjustified it appeared to those who doubted the soundness of the theoretical foundations. "

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Meta-Generalized Gradient Approximation (Perdew et al., 1999)

"Some say that 'there is no systematic way to construct density functional approximations.' But there are more or less systematic ways, and the approach taken . . . here is one of the former."

Preview of DFT as Effective Action

- Recall ordinary thermodynamics with N particles at $T = 0$
- Use a chemical potential μ as source to change $\langle \widehat{N} \rangle$

$$\Omega(\mu) = -kT \ln Z(\mu) \qquad \text{and} \qquad N = -\left(\frac{\partial \Omega}{\partial \mu}\right)_{TV}$$

- *Invert* to find $\mu(N)$, Legendre transform to $F(N)$

$$F(N) = \Omega(\mu(N)) + \mu(N)N$$

$\Longrightarrow$ This is our energy function!

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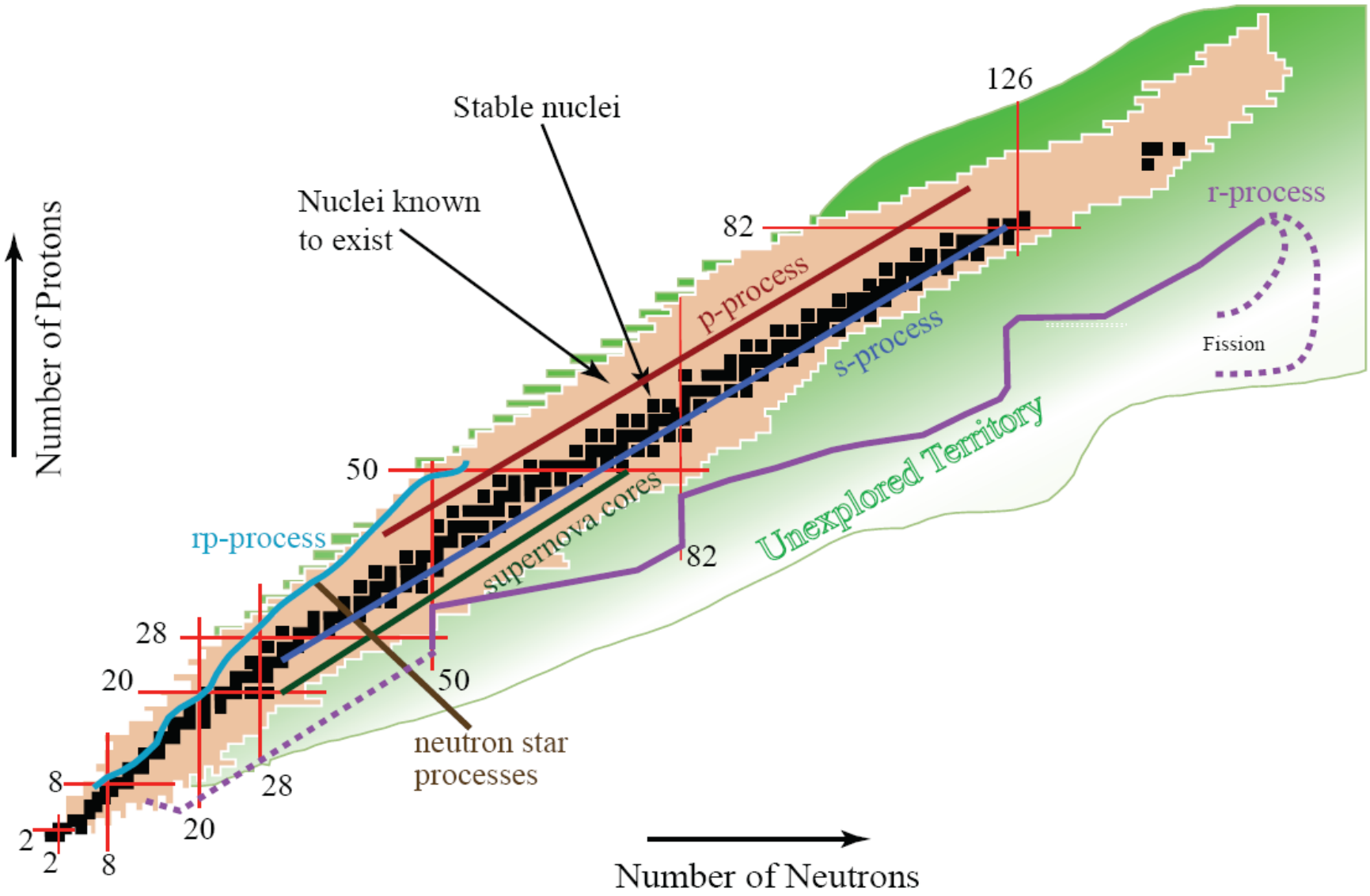
- Generalize to spatially dependent chemical potential $J(\mathbf{x})$

$$Z(\mu) \longrightarrow Z[J(\mathbf{x})] \qquad \text{and} \qquad \mu N = \mu \int \psi^\dagger \psi \longrightarrow \int J(\mathbf{x}) \psi^\dagger \psi(\mathbf{x})$$

- LT from $\ln Z[J(\mathbf{x})]$ to $\Gamma[\rho(\mathbf{x})]$, where $\rho = \langle \psi^\dagger \psi \rangle_J \Longrightarrow$ DFT!

Outline

Table of the Nuclides



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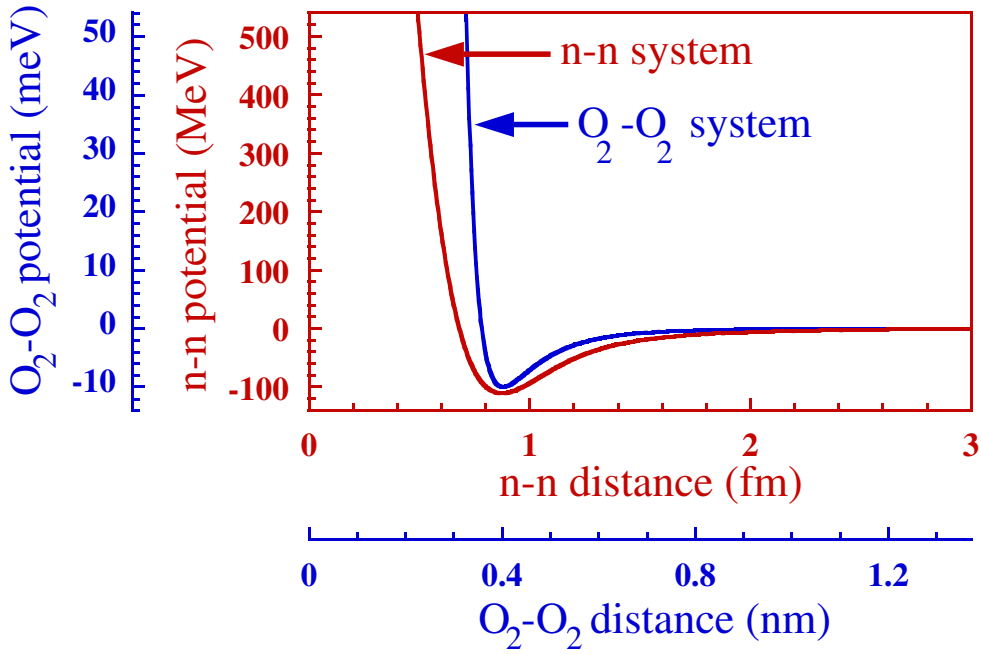

[figure borrowed from J. Dobaczewski]

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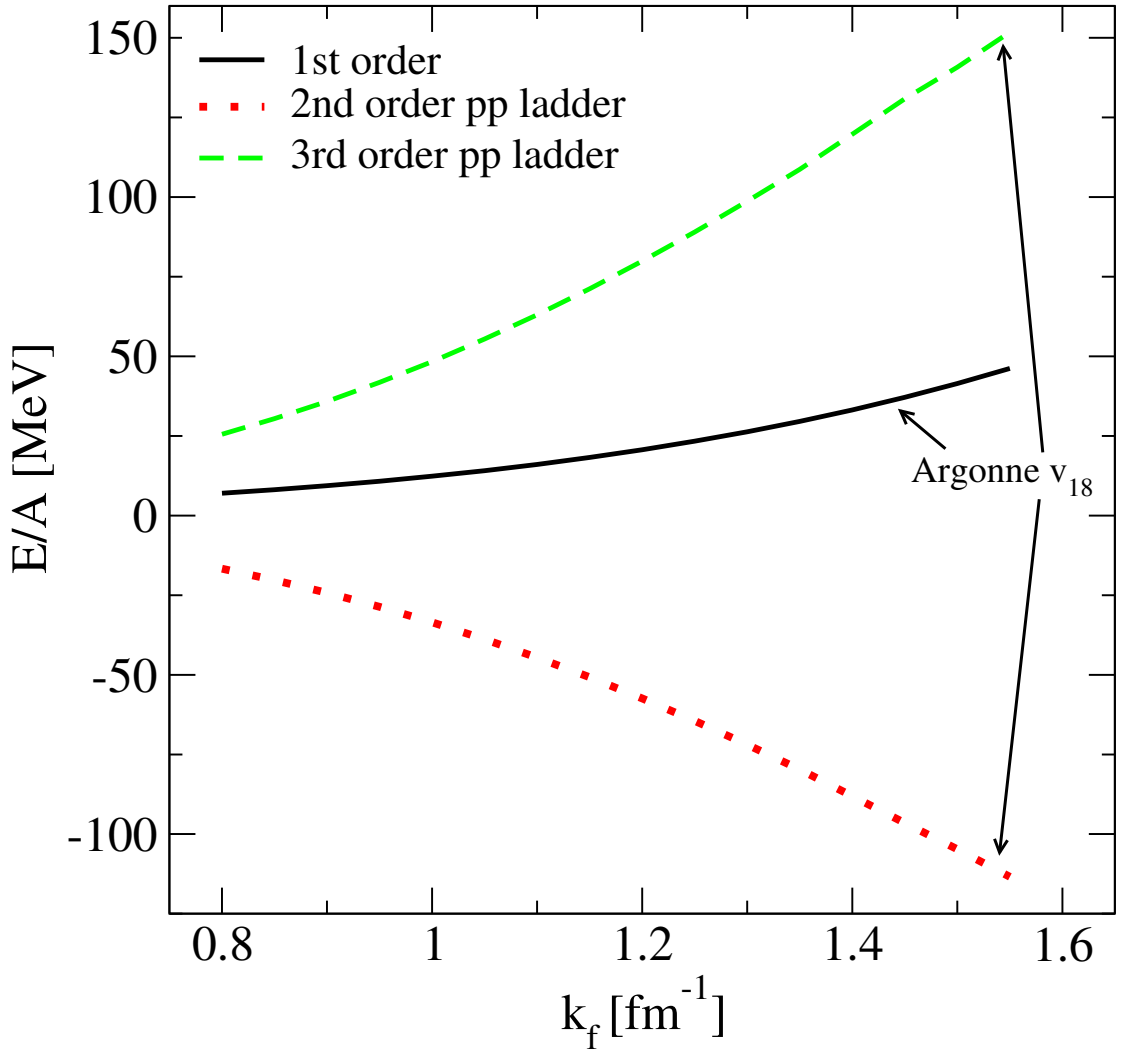
- How can we deal with “hard cores” in many-body systems?

Nuclear Matter in Low-Order Perturbation Theory

- Standard Argonne v_{18} potential
- Brueckner ladders order-by-order
- 1st order is Hartree-Fock $\Longrightarrow$ unbound!
- Repulsive core $\Longrightarrow$ series diverges



Hartree-Fock Wave Function

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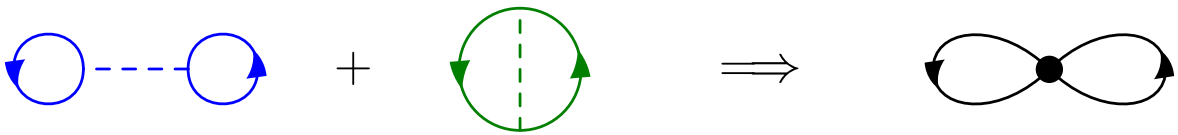
- The $\phi_i(\mathbf{x})$ satisfy *non-local* Schrödinger equations:

$$-\frac{\nabla^2}{2M}\phi_i(\mathbf{x}) + V_{\mathrm{H}}(\mathbf{x})\phi_i(\mathbf{x}) + \int d\mathbf{y}\, V_{\mathrm{ex}}(\mathbf{x}, \mathbf{y})\phi_i(\mathbf{y}) = \epsilon_i \phi_i(\mathbf{x})$$

with $V_{\mathrm{H}}(\mathbf{x}) = \int d\mathbf{y} \sum_{j=1}^{A} |\phi_j(\mathbf{y})|^2 v(\mathbf{x}, \mathbf{y})\,, \quad V_{\mathrm{ex}}(\mathbf{x}, \mathbf{y}) = -v(\mathbf{x}, \mathbf{y}) \sum_{j=1}^{A} \phi_j(\mathbf{x})\phi_j^*(\mathbf{y})$



- Solve self-consistently; somewhat tricky because non-local
  $\Longrightarrow$ much simpler if $v(\mathbf{x}, \mathbf{y}) \propto \delta(\mathbf{x} - \mathbf{y})$

Skyrme Hartree-Fock Energy Functionals

- Skyrme: Do Hartree-Fock with $V_2^{\mathrm{Skyrme}} + V_3^{\mathrm{Skyrme}}$, where $\langle \mathbf{k}|V_2^{\mathrm{Skyrme}}|\mathbf{k}'\rangle = t_0 + \frac{1}{2}t_1(\mathbf{k}^2 + \mathbf{k}'^2) + t_2\mathbf{k}\cdot\mathbf{k}' + iW_0(\sigma_1+\sigma_2)\cdot\mathbf{k}\times\mathbf{k}'$

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- Motivates Skyrme energy density functional (for $N = Z$):

$$\begin{aligned}\mathcal{E}[\rho,\tau,\mathbf{J}] \;=\; & \frac{1}{2M}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^{2+\alpha} + \frac{1}{16}(3t_1+5t_2)\rho\tau \\ & + \frac{1}{64}(9t_1-5t_2)(\nabla\rho)^2 - \frac{3}{4}W_0\rho\nabla\cdot\mathbf{J} + \frac{1}{32}(t_1-t_2)\mathbf{J}^2\end{aligned}$$

  - where $\rho(x) = \sum_i |\phi_i(x)|^2$ and $\tau(x) = \sum_i |\nabla\phi_i(x)|^2$ (and $\mathbf{J}$)

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  - where $\rho(x) = \sum_i |\phi_i(x)|^2$ and $\tau(x) = \sum_i |\nabla\phi_i(x)|^2$ (and $\mathbf{J}$)
- Minimize $E = \int d\mathbf{x}\,\mathcal{E}[\rho,\tau,\mathbf{J}]$ by varying the (normalized) ϕ_i's

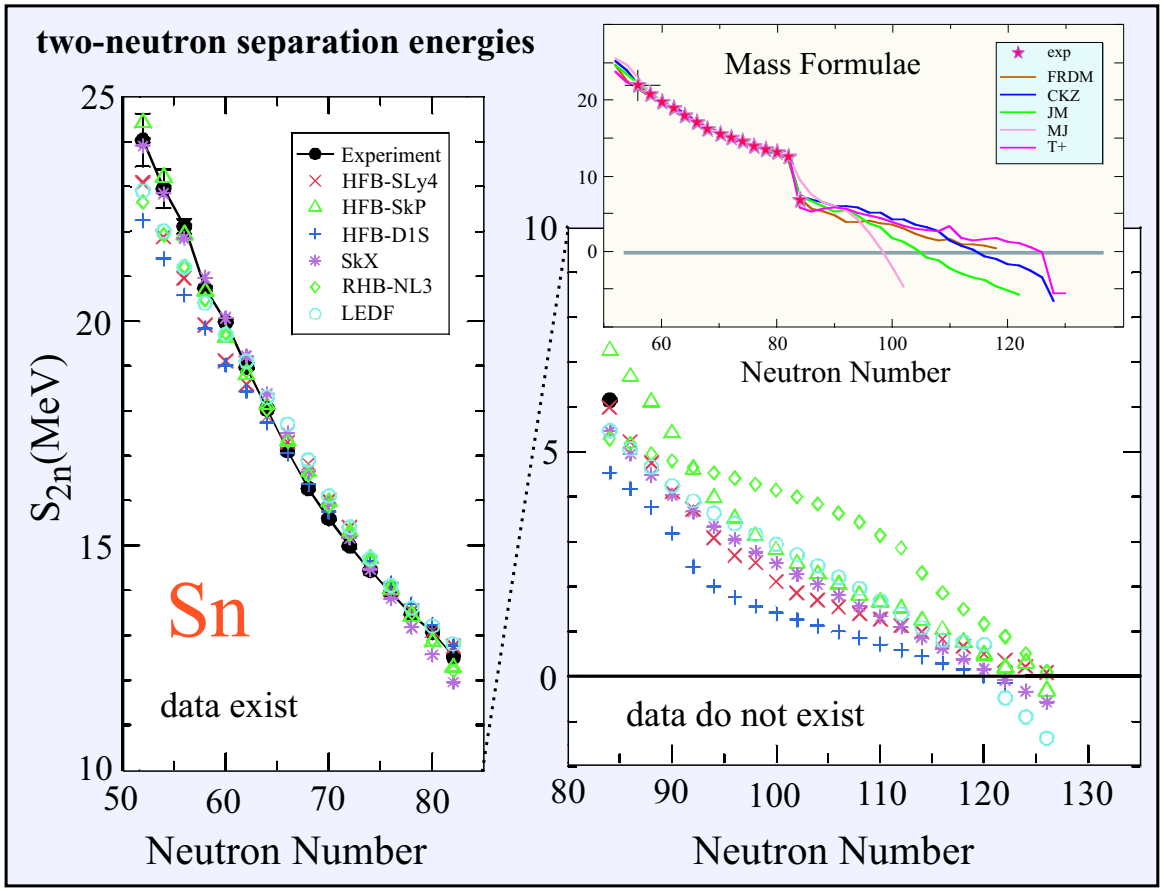
$$\left(-\nabla\frac{1}{2M^*(\mathbf{x})}\nabla + U(\mathbf{x}) + \frac{3}{4}W_0\nabla\rho\cdot\frac{1}{i}\nabla\times\sigma\right)\phi_\mathbf{i}(\mathbf{x}) = \epsilon_\mathbf{i}\,\phi_\mathbf{i}(\mathbf{x})\ ,$$

$U = \frac{3}{4}t_0\rho + (\frac{3}{16}t_1 + \frac{5}{16}t_2)\tau + \cdots$ and $\frac{1}{2M^*(\mathbf{x})} = \frac{1}{2M} + (\frac{3}{16}t_1 + \frac{5}{16}t_2)\rho$

- Iterate until ϕ_i's and ϵ_i's are self-consistent

Problems with Extrapolations

- Mass formulas and energy functionals do well where there is data, but elsewhere . . .



Questions and Criticisms of Skyrme HF

- Typical [e.g., SkIII] model parameters (in units of MeV-fm^n):
  $t_0 = -1129 \quad t_1 = 395 \quad t_2 = -95 \quad t_3 = 14000 \quad W_0 = 120$
  - These seem large; is there an expansion parameter?
  - Where does $\rho^{2+\alpha}$ come from? Why not $\rho^{2+\beta}$?
  - Parameter Fitting [von Neumann via Fermi via Dyson]:
    "With four parameters I can fit an elephant,
    and with five I can make him wiggle his trunk."

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    "Difficulty in a solution to a problem is always conserved
    regardless of the technique used in the solution."
  - How do we improve the approach? Is pairing treated correctly?

Questions and Criticisms of Skyrme HF

- Typical [e.g., SkIII] model parameters (in units of MeV-fm^n):
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- How does Skyrme HF relate to NN (and NNN) forces?

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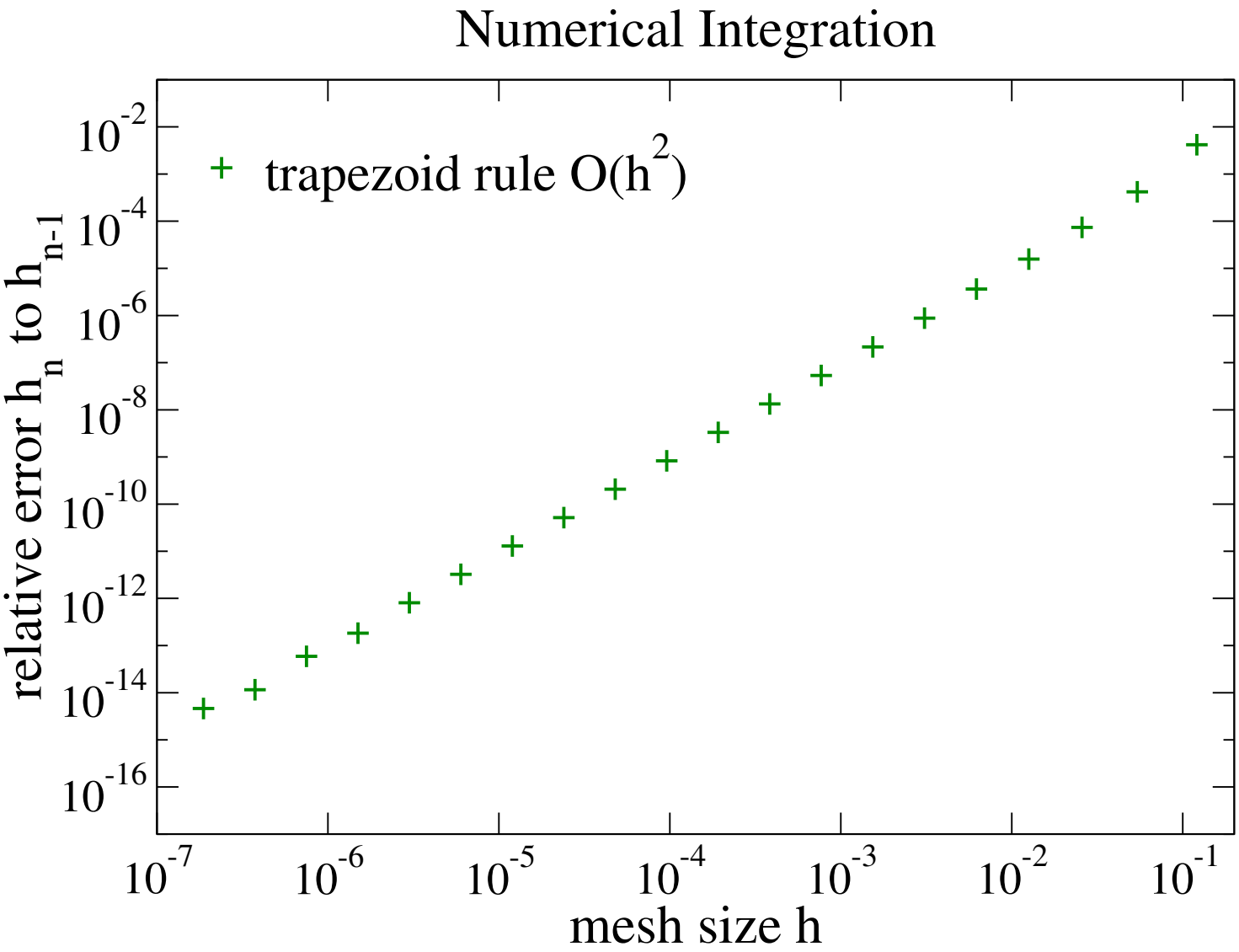
Choose approximations by "art"	Use physical arguments (often handwaving) to justify the subset of diagrams used

Why Use EFT For Many-Body Physics?

- Systematic calculations with error estimates
- Reliable, model independent extrapolation
- Analogy between EFT and basic numerical analysis
  - naive error analysis: pick a method and reduce the mesh size (e.g., increase grid points) until the error is “acceptable”
  - sophisticated error analysis: understand scaling and sources of error (e.g., algorithm vs. round-off errors)
    $\Longrightarrow$ Does it work as well as it should?
  - representation dependence $\Longrightarrow$ not all are equally effective!
  - extrapolation: completeness of an expansion basis
- Quantum mechanics makes EFT trickier!

Error Plots in Numerical Analysis



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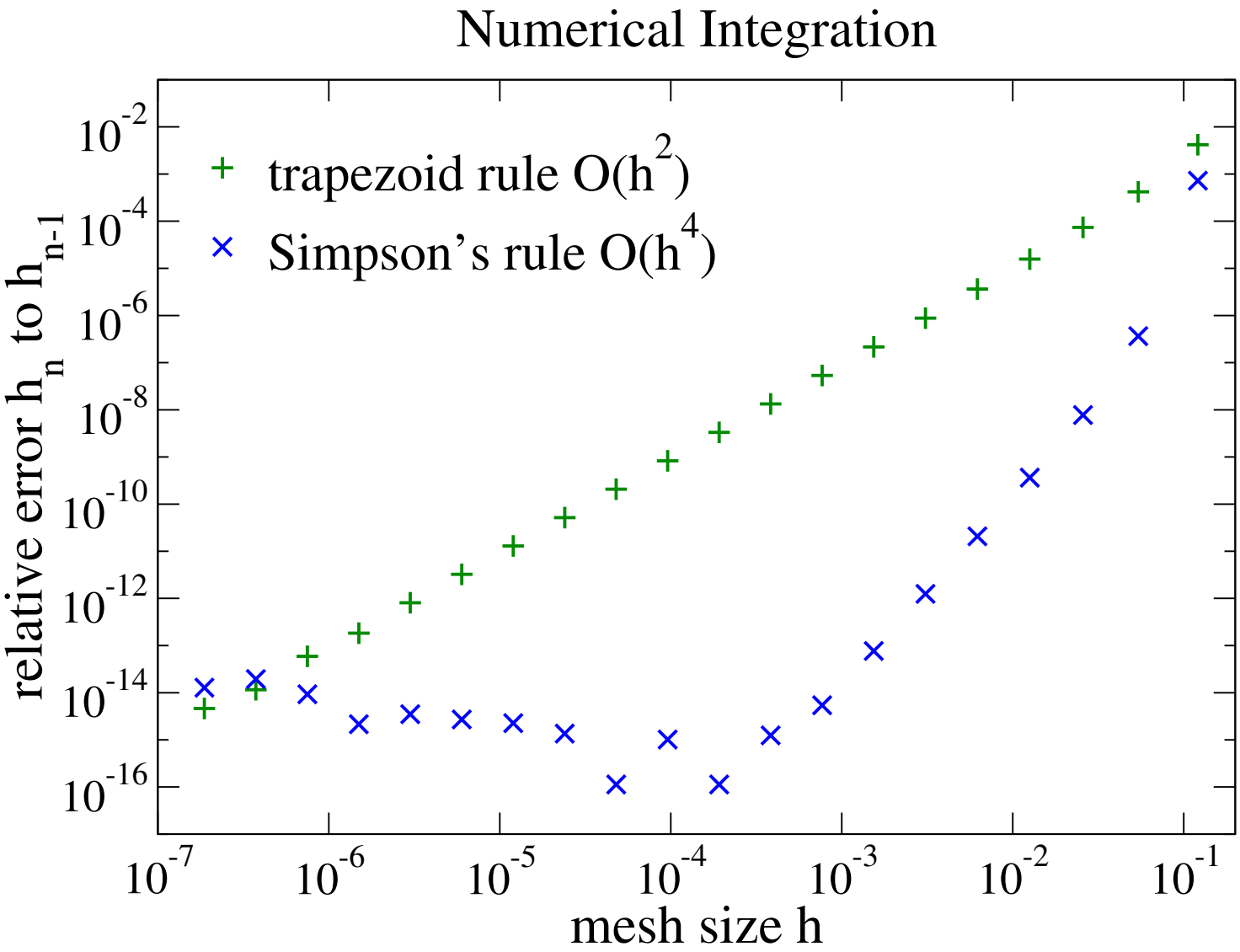

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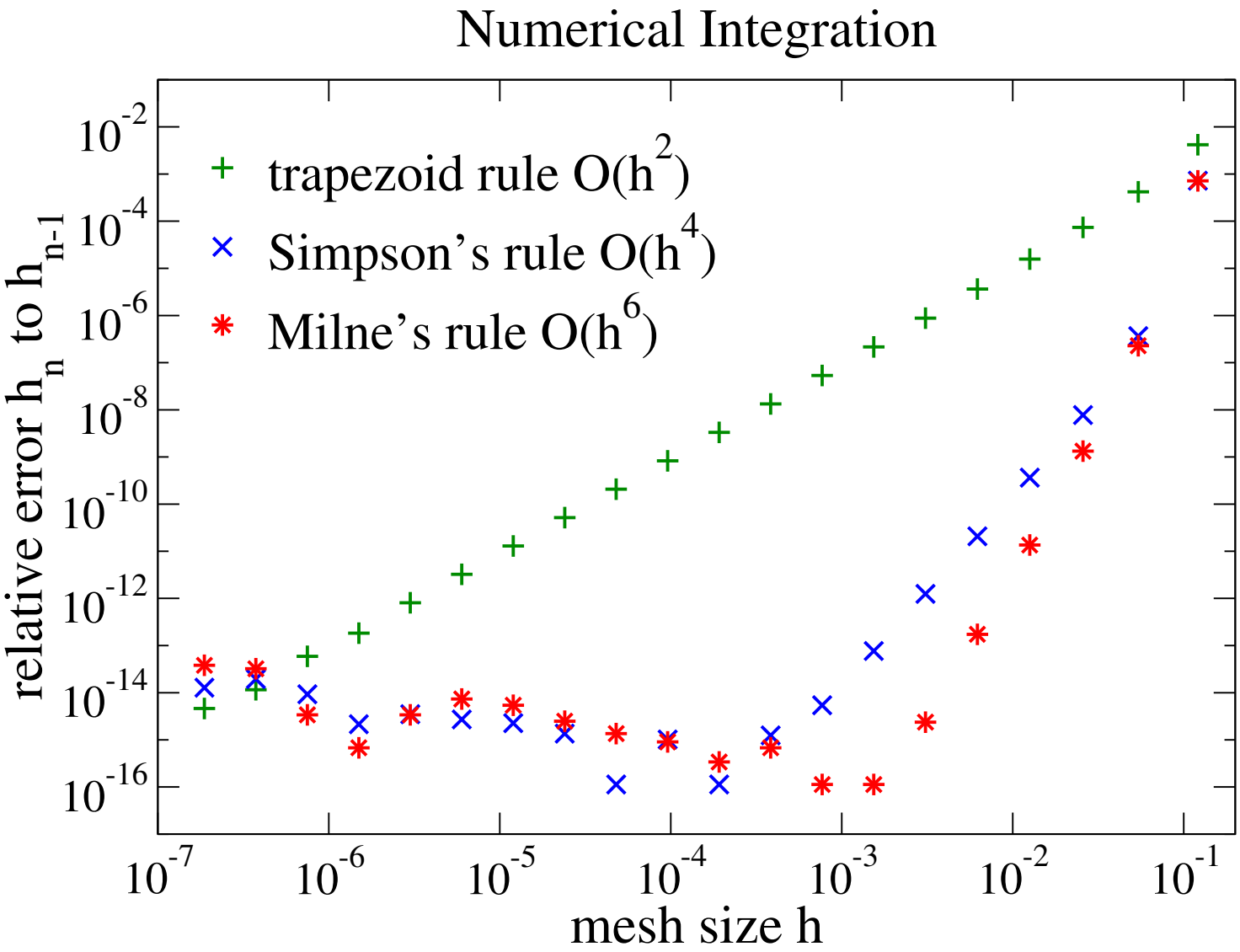

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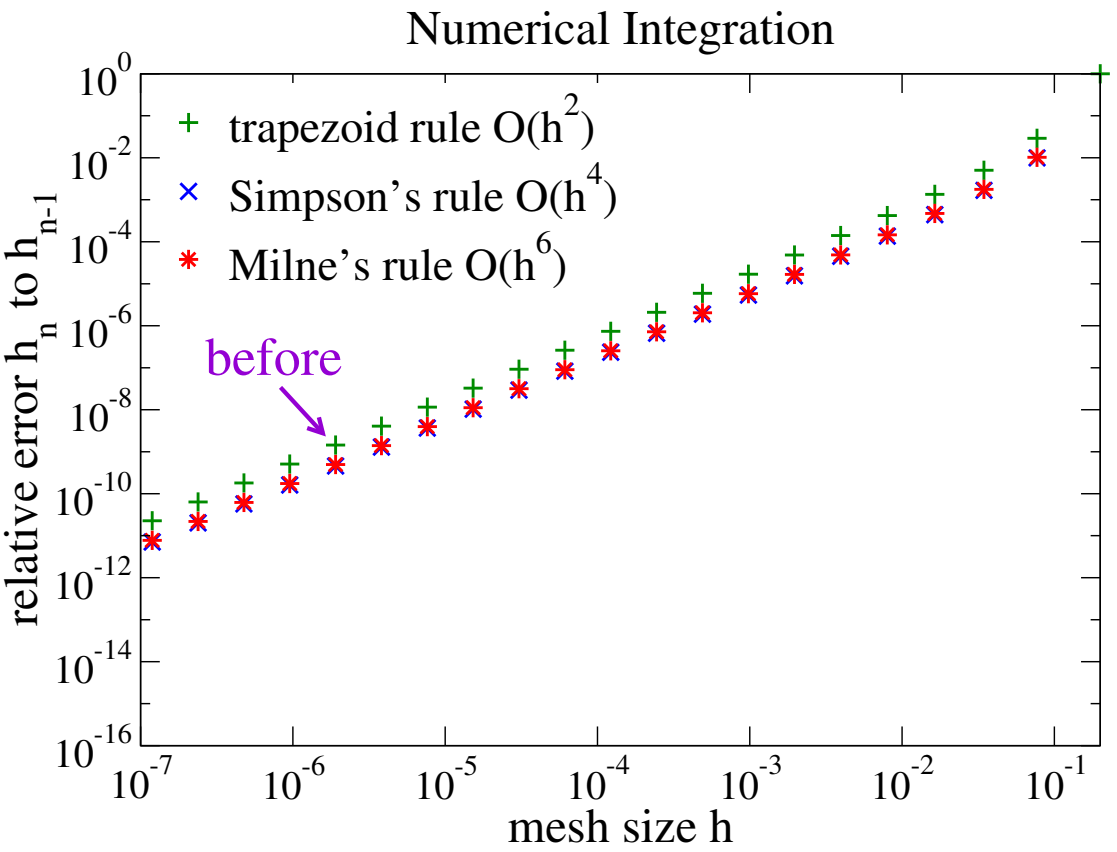

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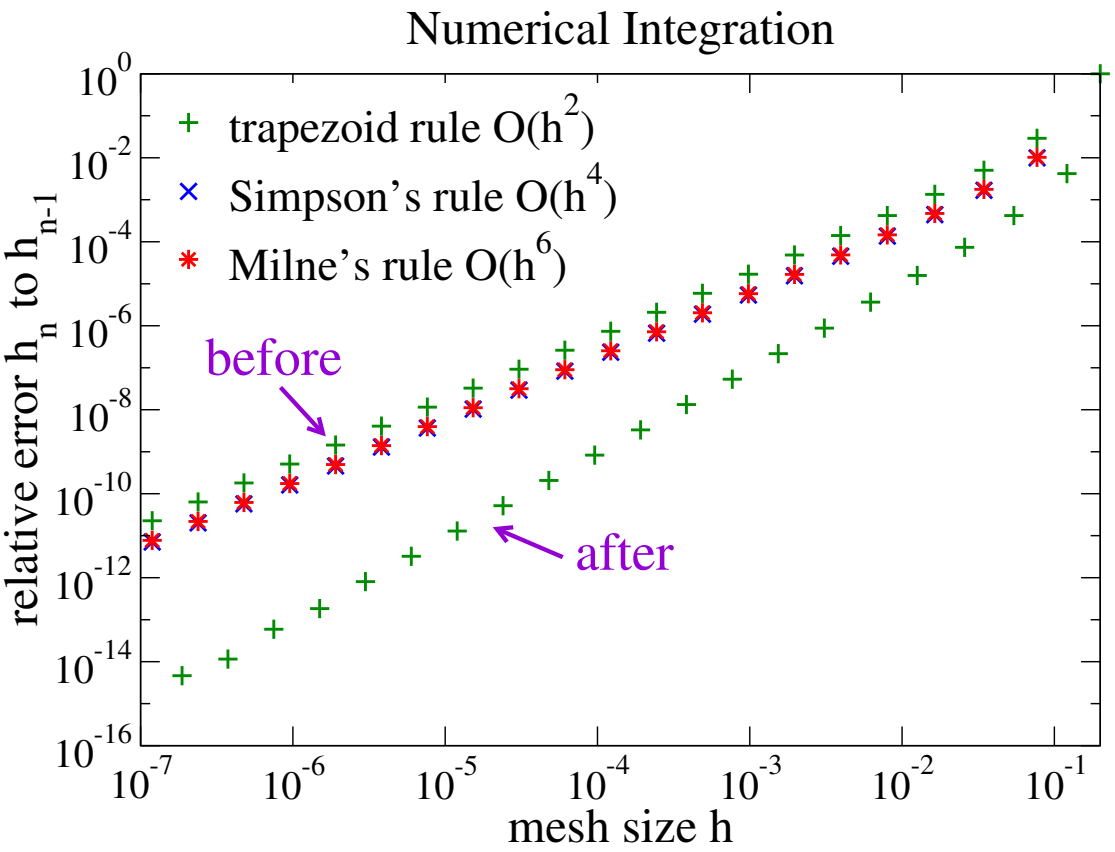

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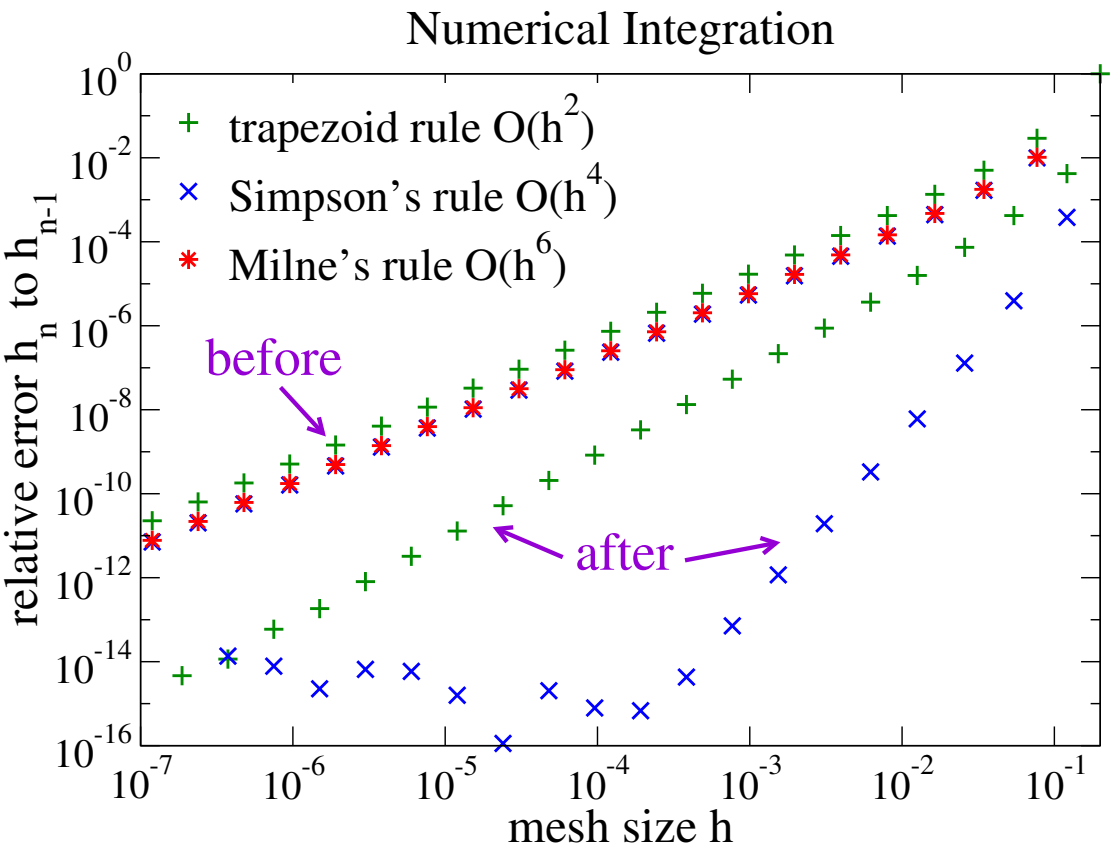

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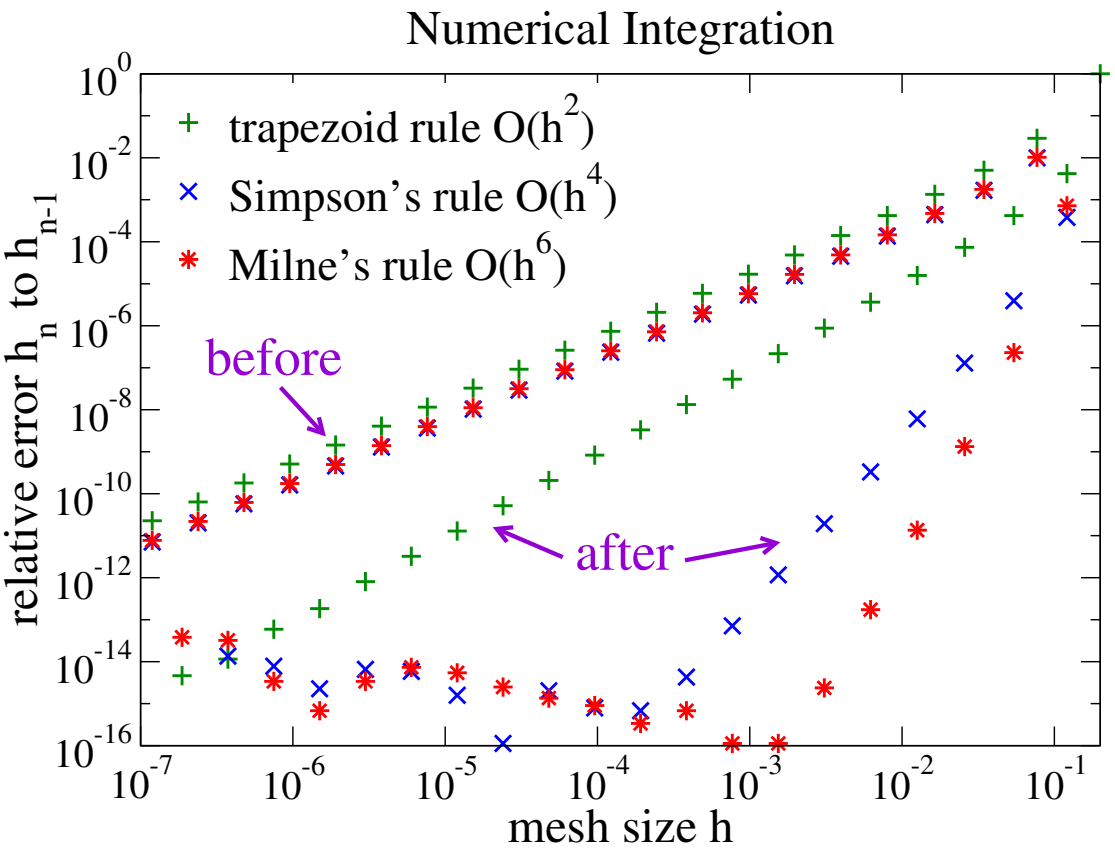
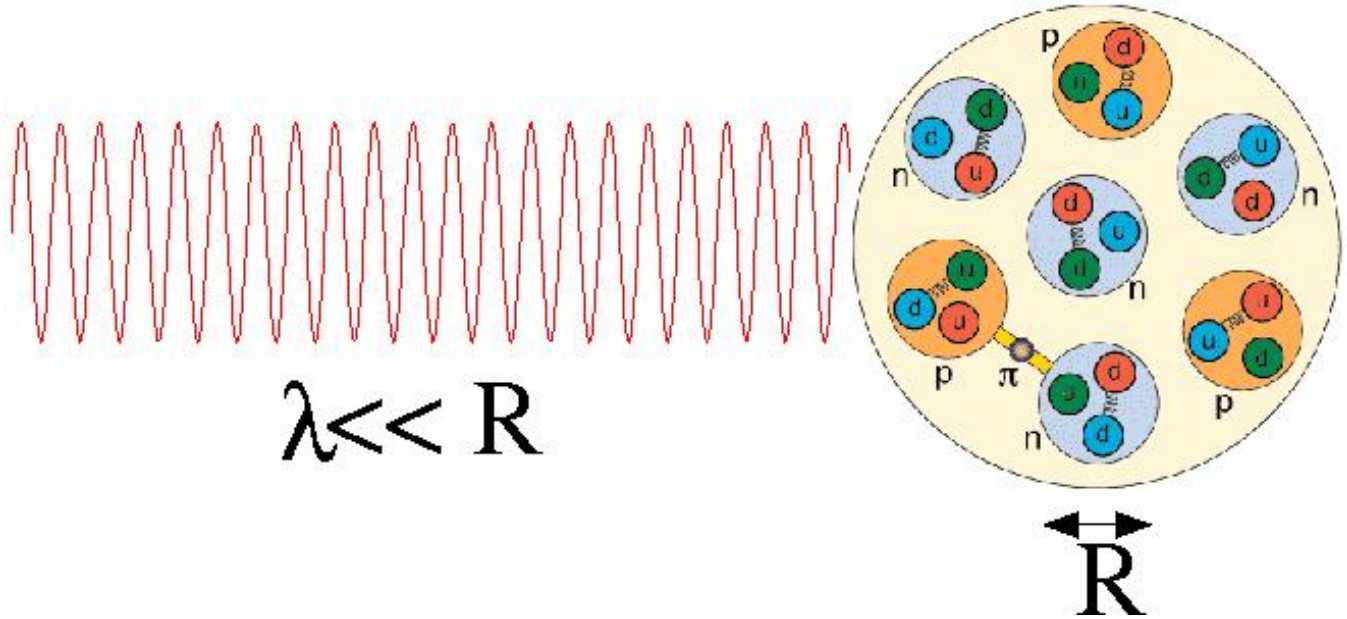


Principles of Effective Low-Energy Theories



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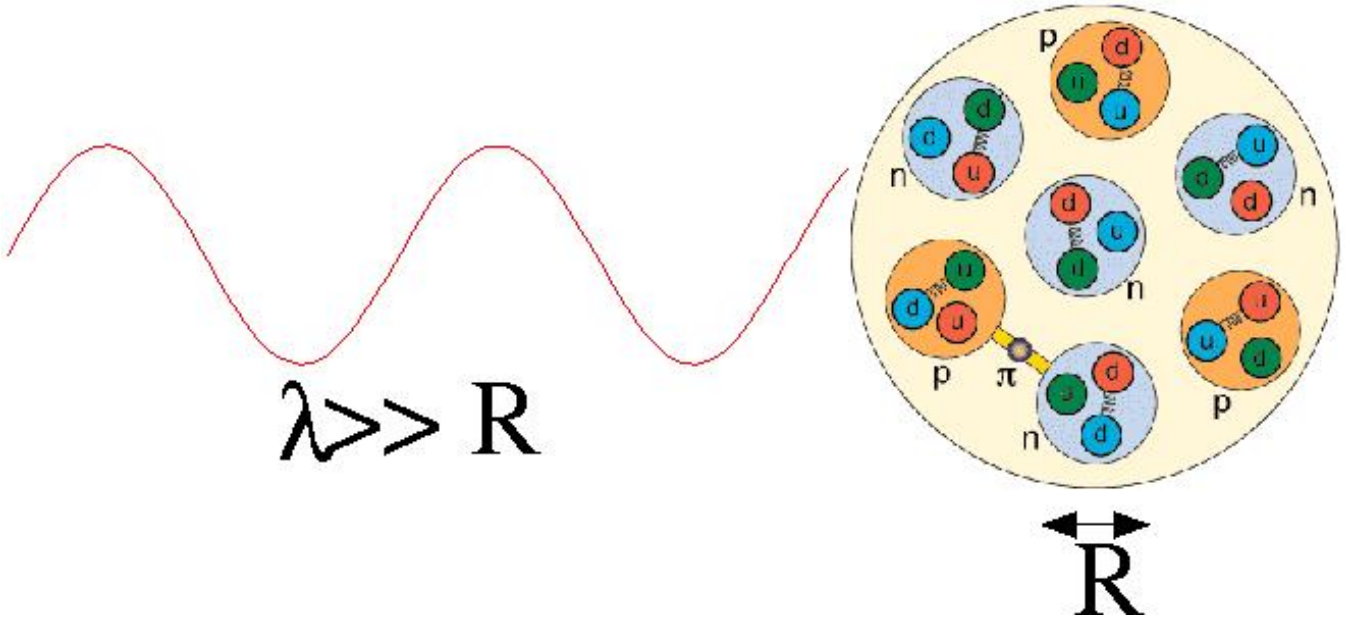

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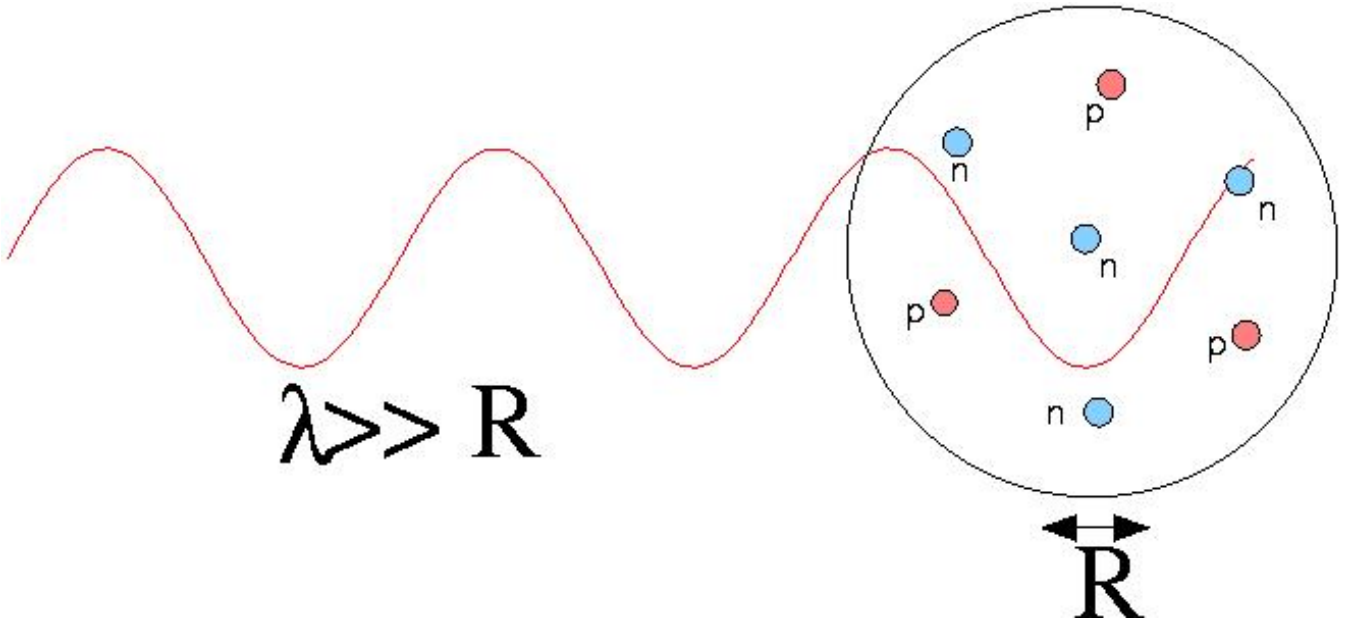
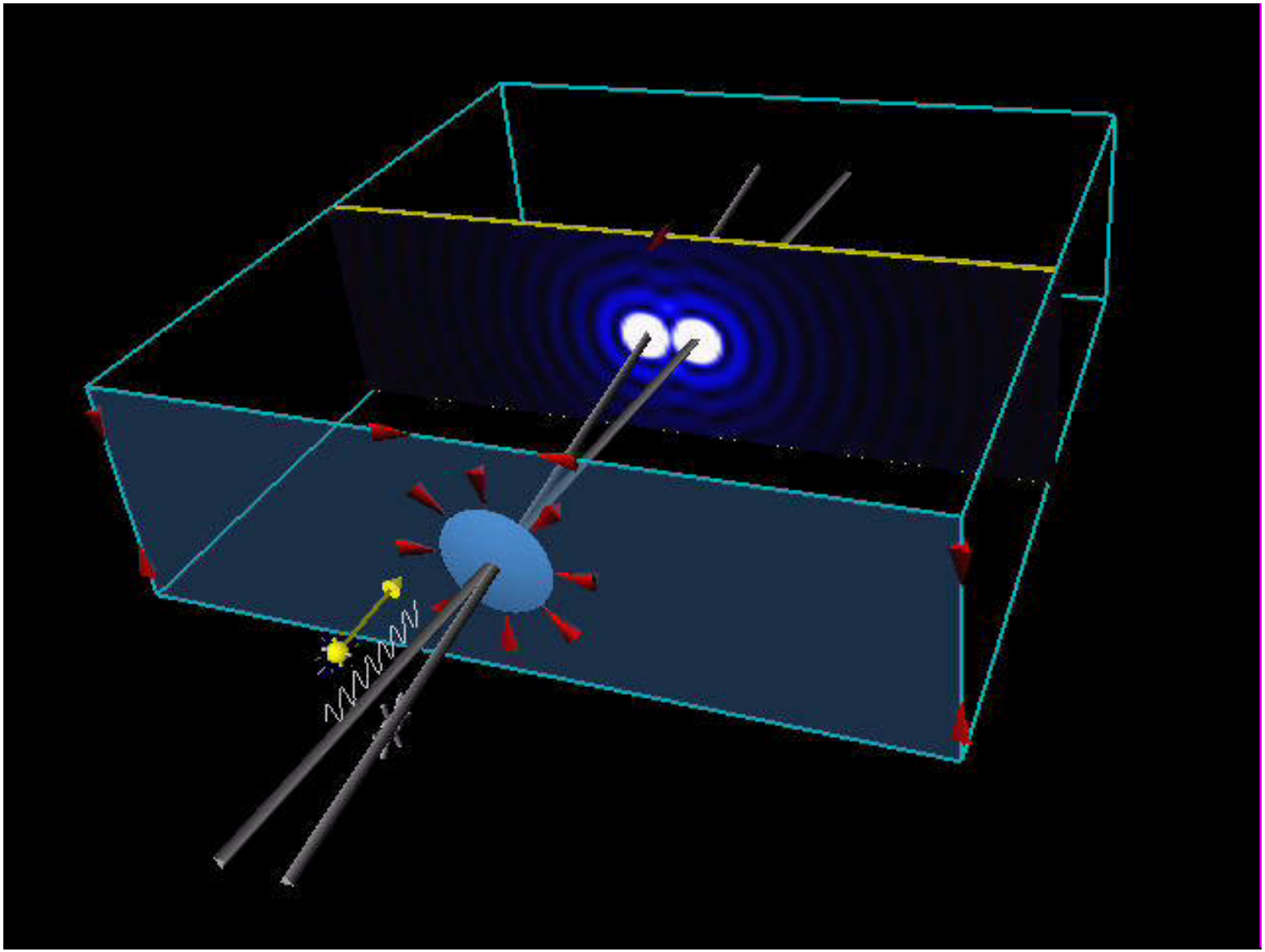


- If system is probed at low energies, fine details not resolved
  - use low-energy variables for low-energy processes
  - short-distance structure can be replaced by something simpler without distorting low-energy observables

Wavelength and Resolution



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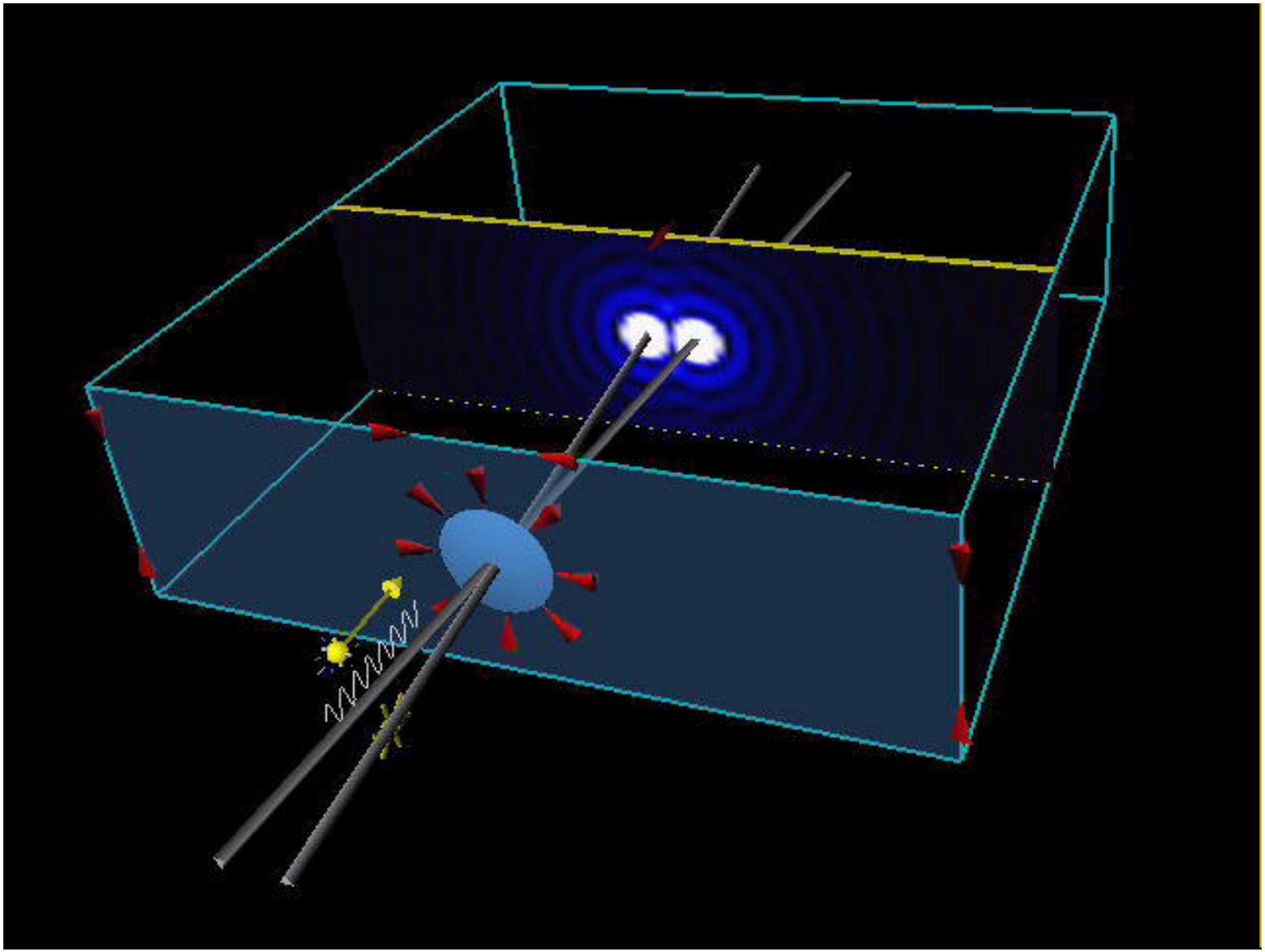

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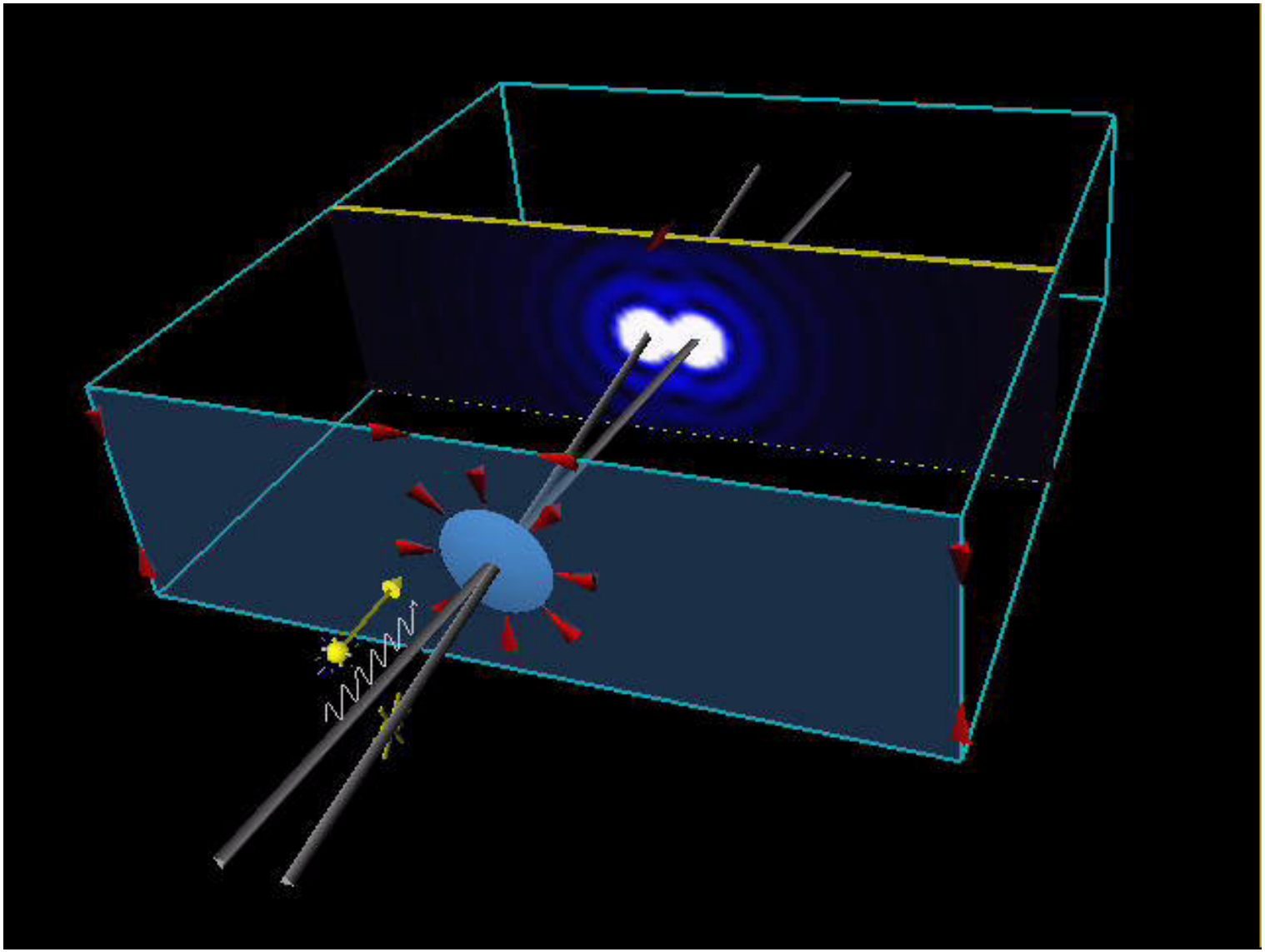

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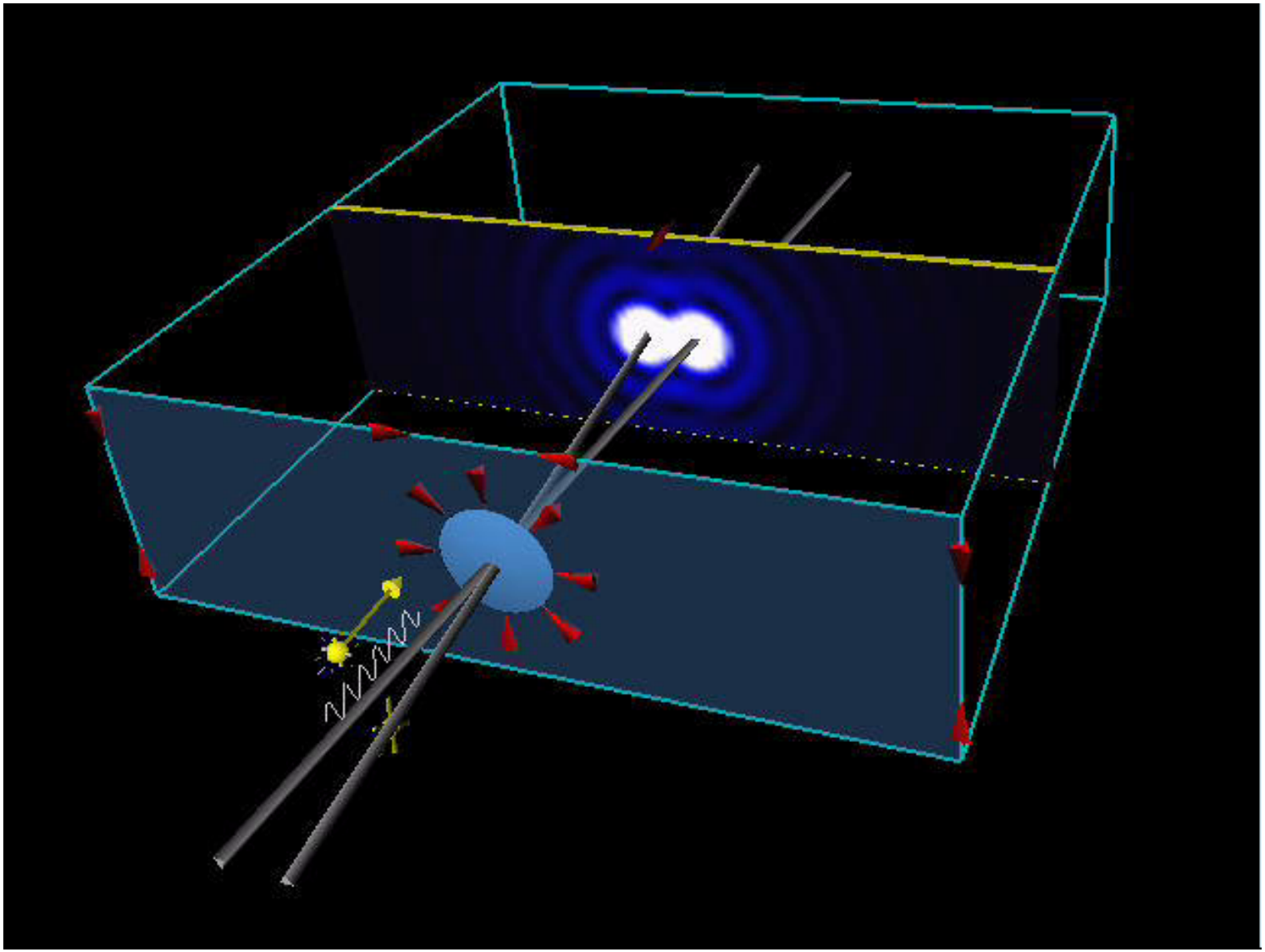

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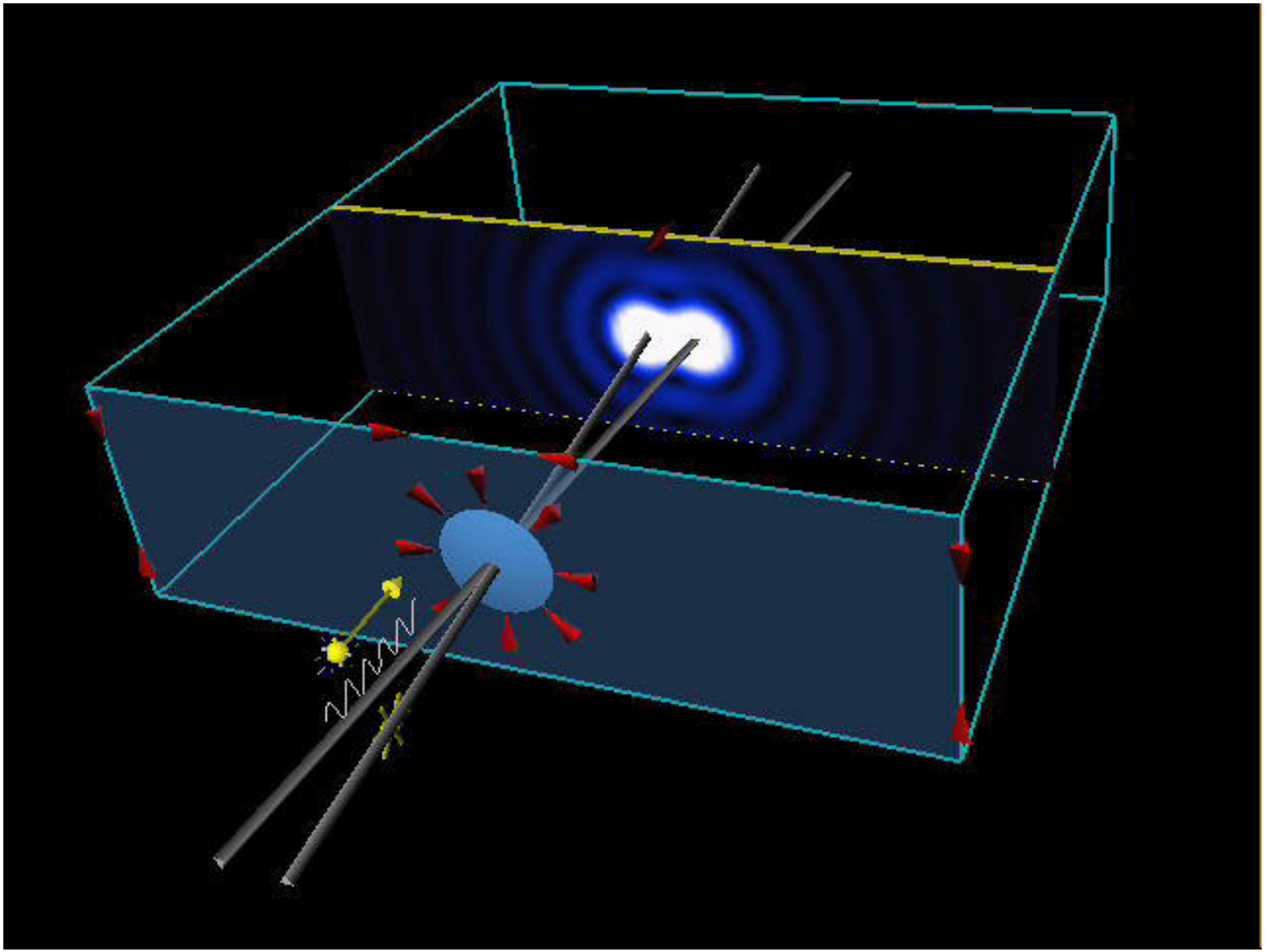

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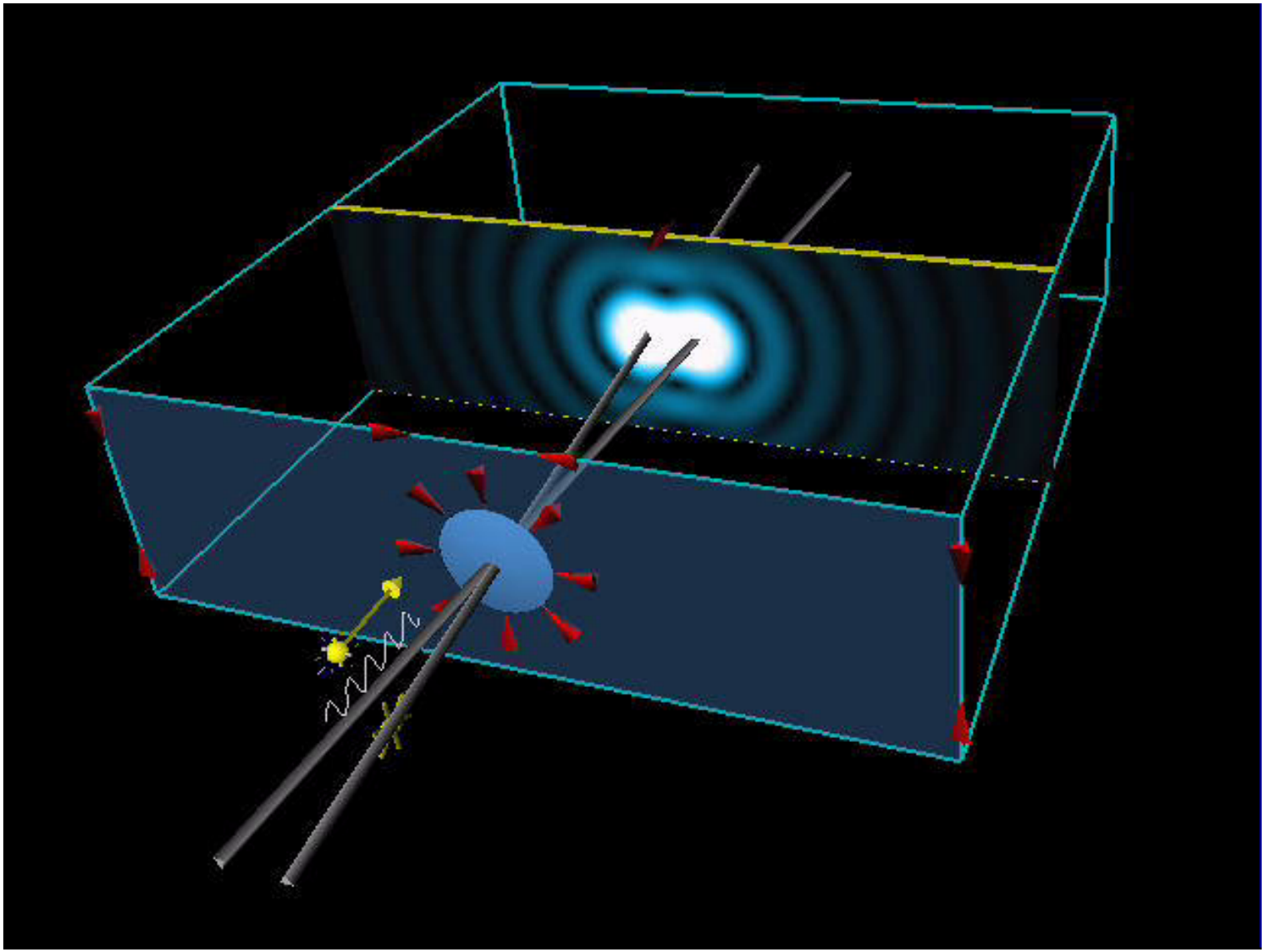

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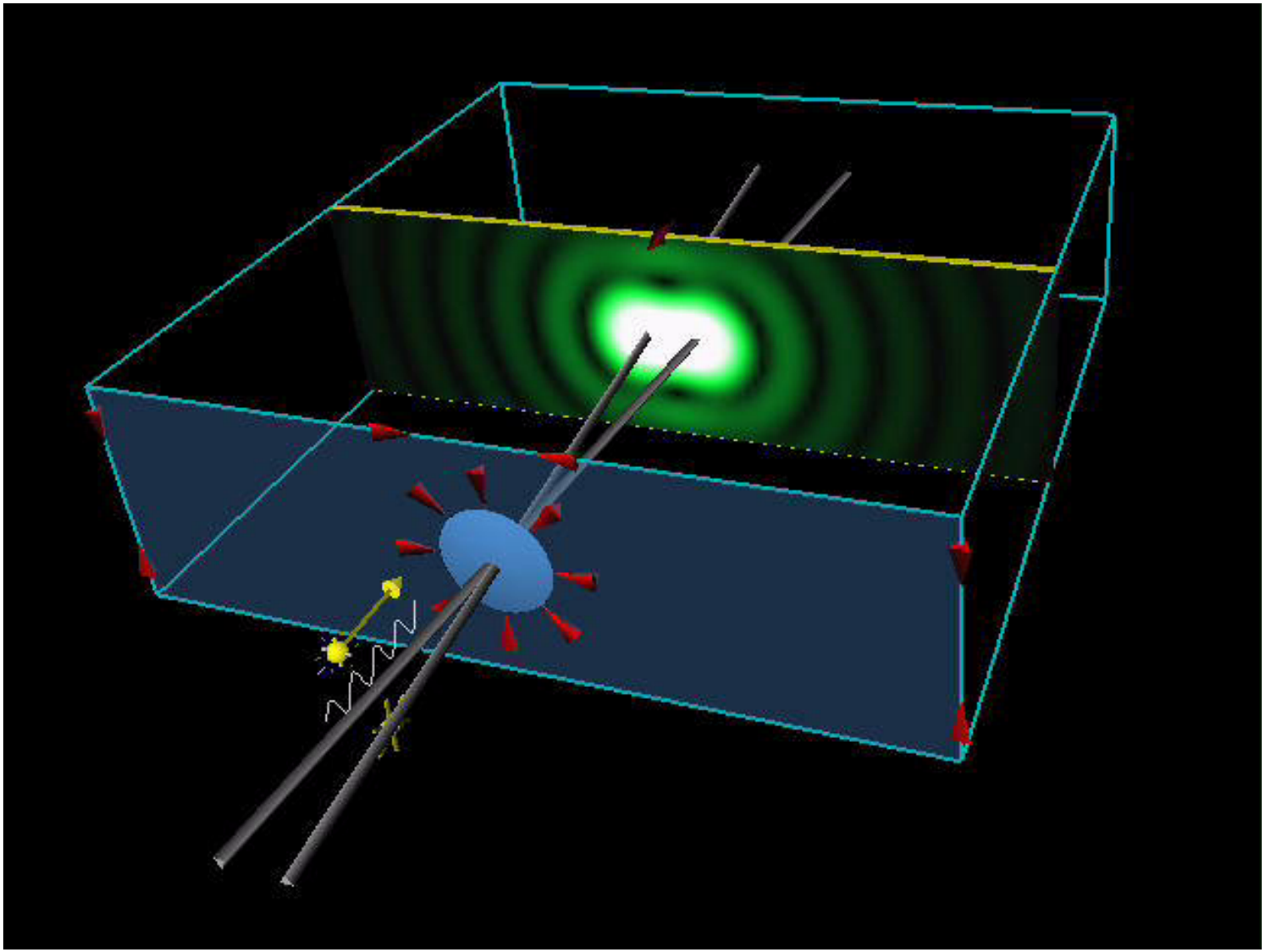

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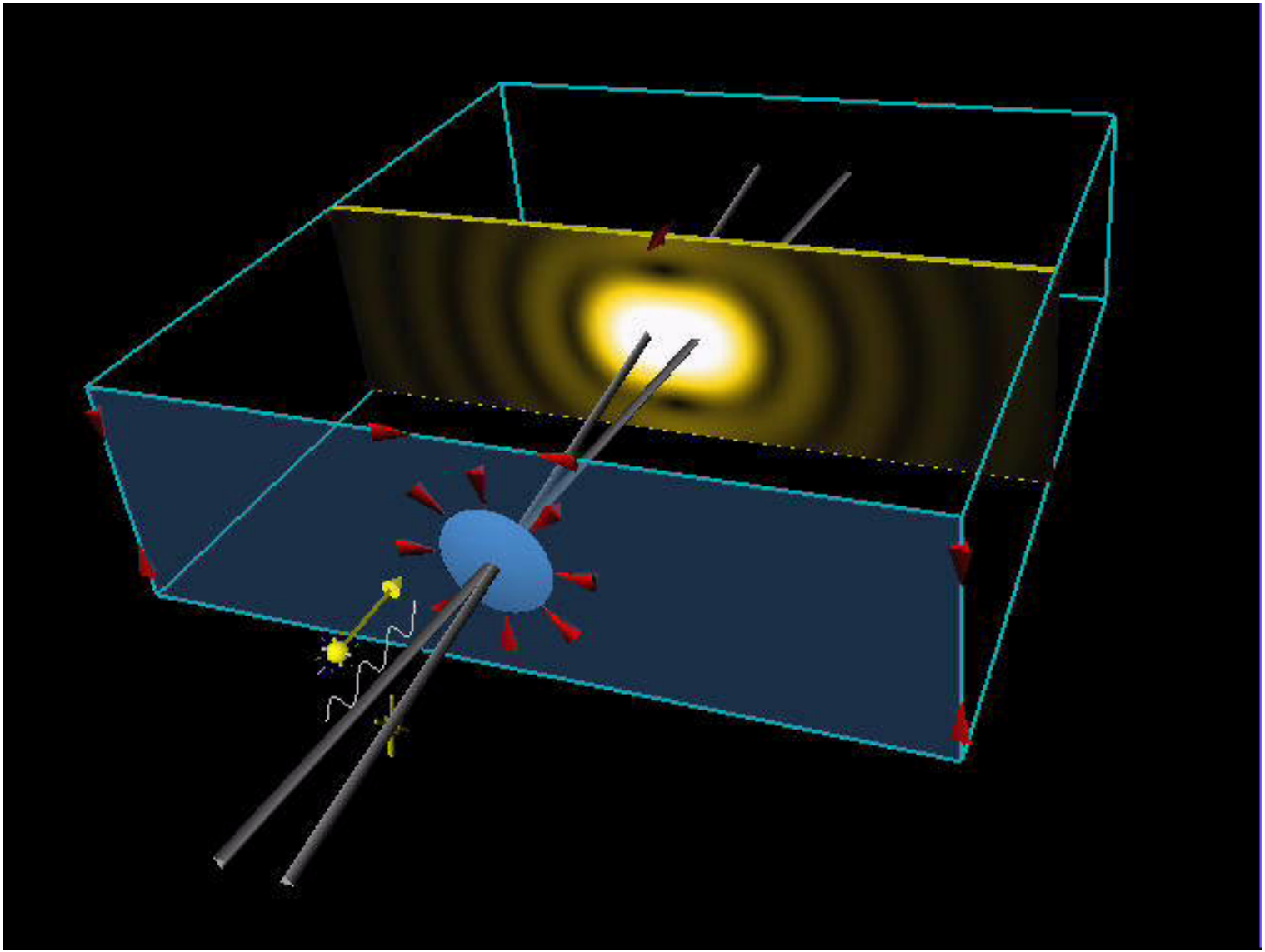

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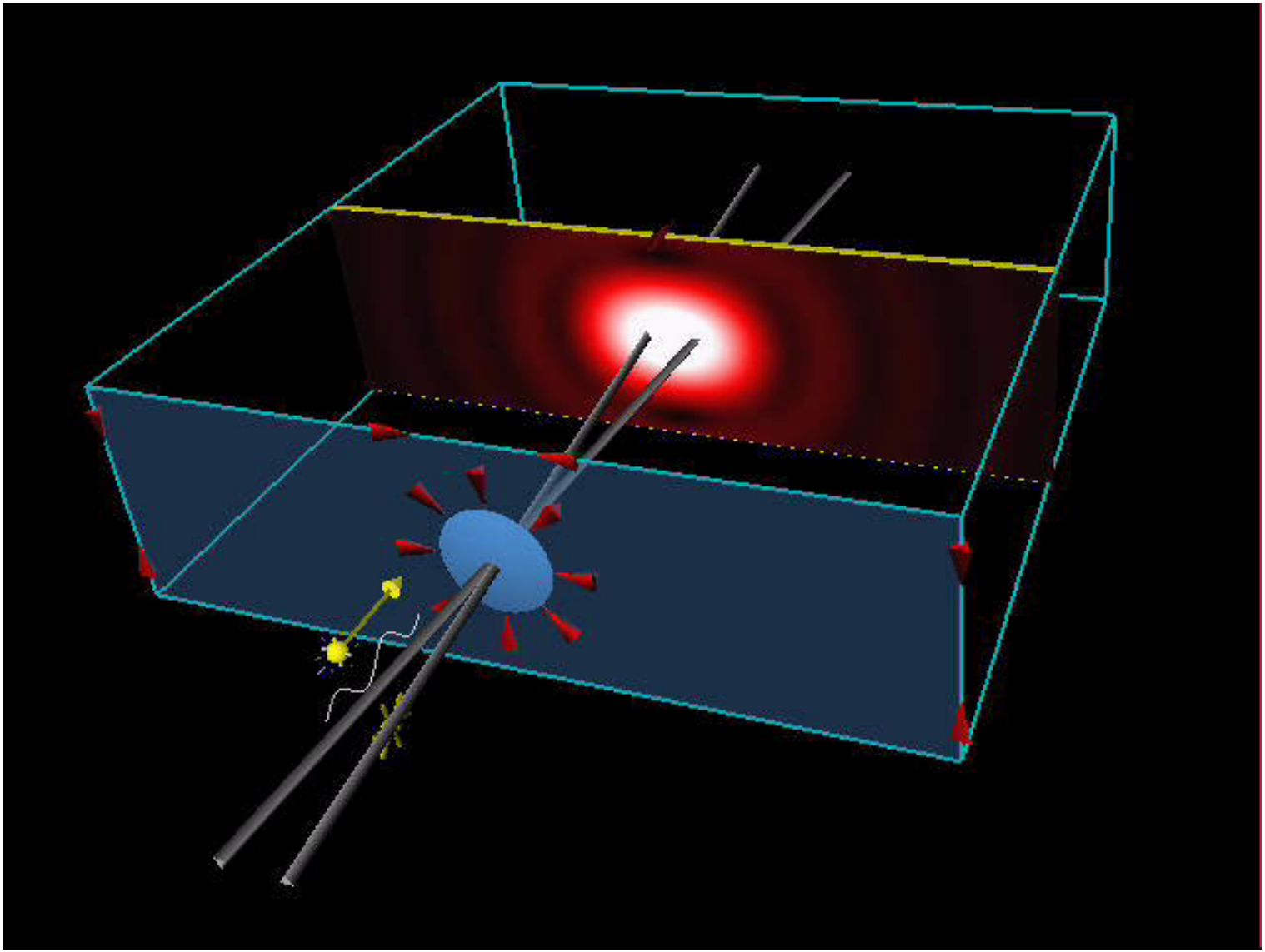
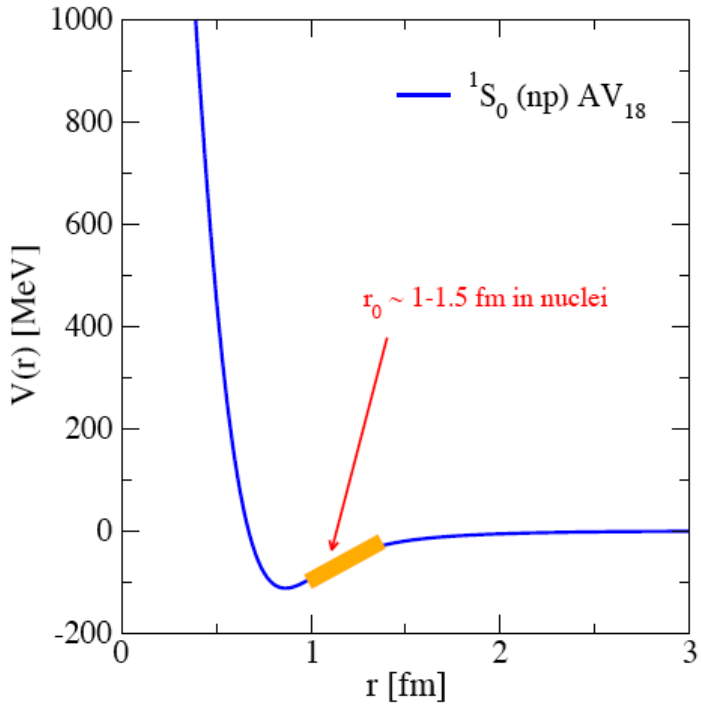


Sources of Nonperturbative Physics for NN

1. Strong short-range repulsion ("hard core")
2. Iterated tensor (S_{12}) interaction
3. Near zero-energy bound states



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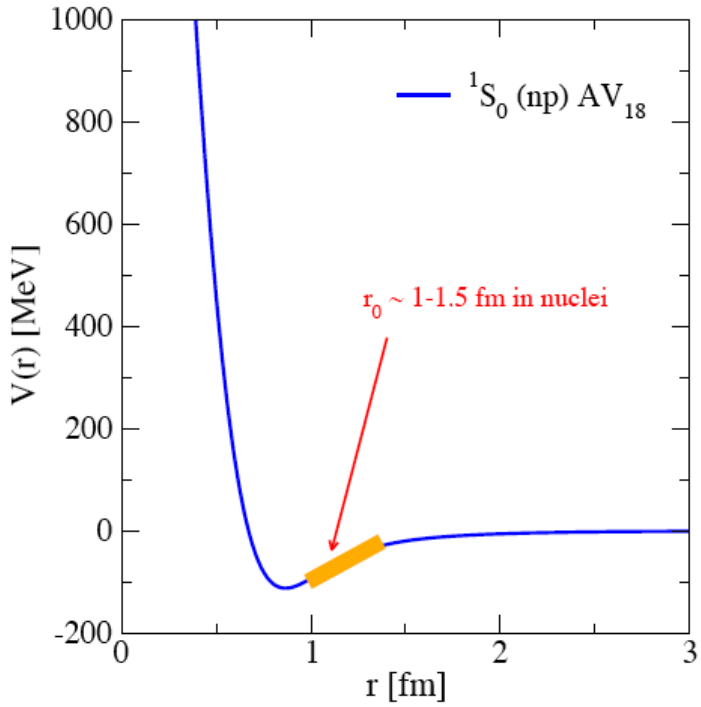

- Consequences:
  - In Coulomb DFT, Hartree-Fock gives dominate contribution $\Longrightarrow$ correlations are small corrections $\Longrightarrow$ DFT works!
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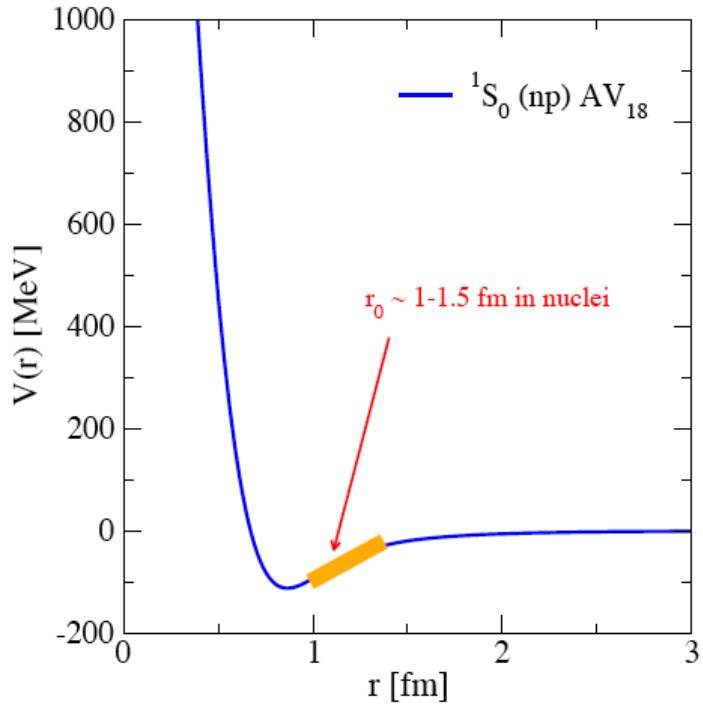

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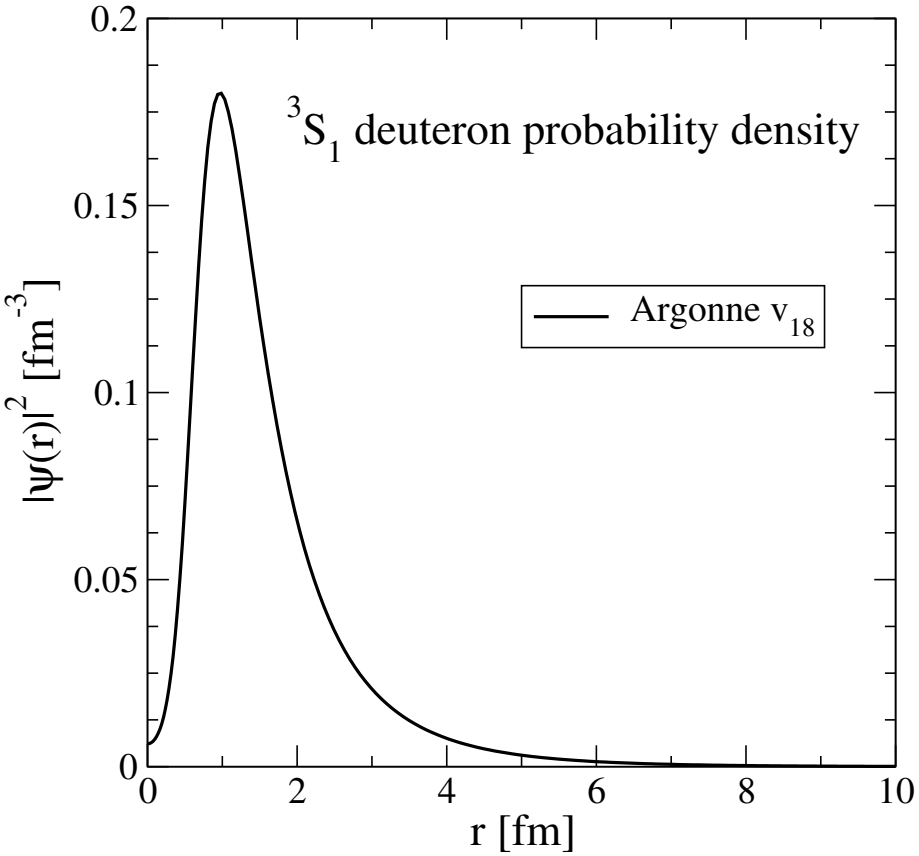
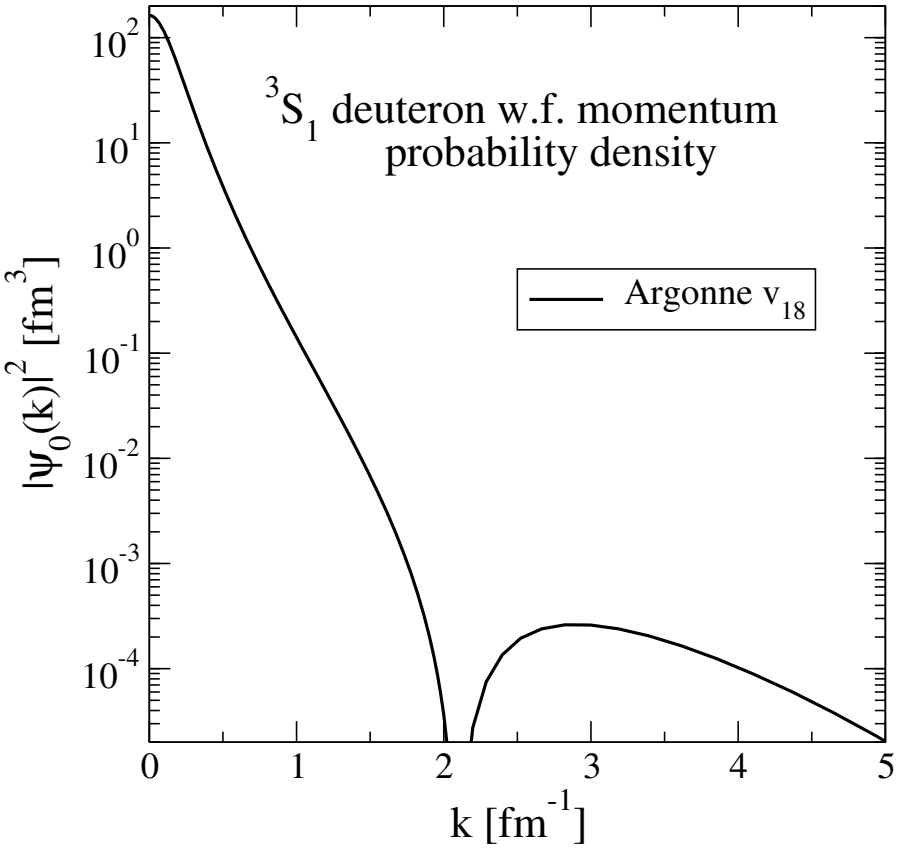
- However . . .
  - the first two depend on the *resolution* $\Longrightarrow$ different cutoffs
  - third one is affected by Pauli blocking

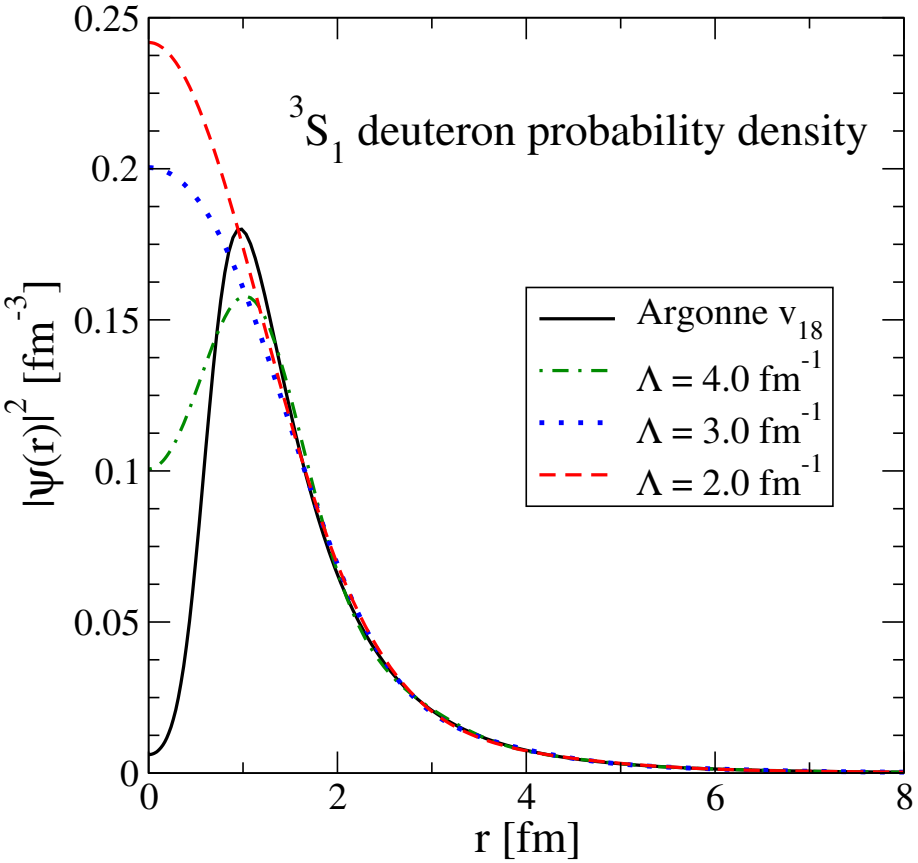
The Deuteron (Bound np) at High Resolution

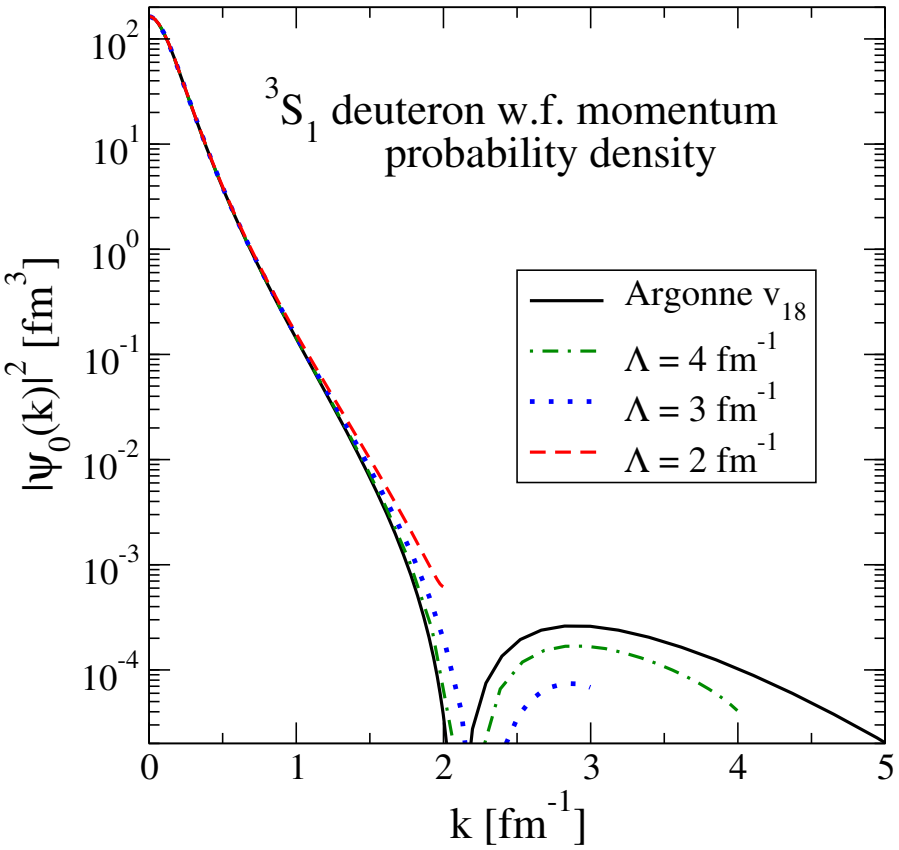




- Repulsive core $\Longrightarrow$ short-distance suppression
  $\Longrightarrow$ important high-momentum (small λ) components
- Makes the many-body problem complicated!

The Deuteron at Lower Resolutions

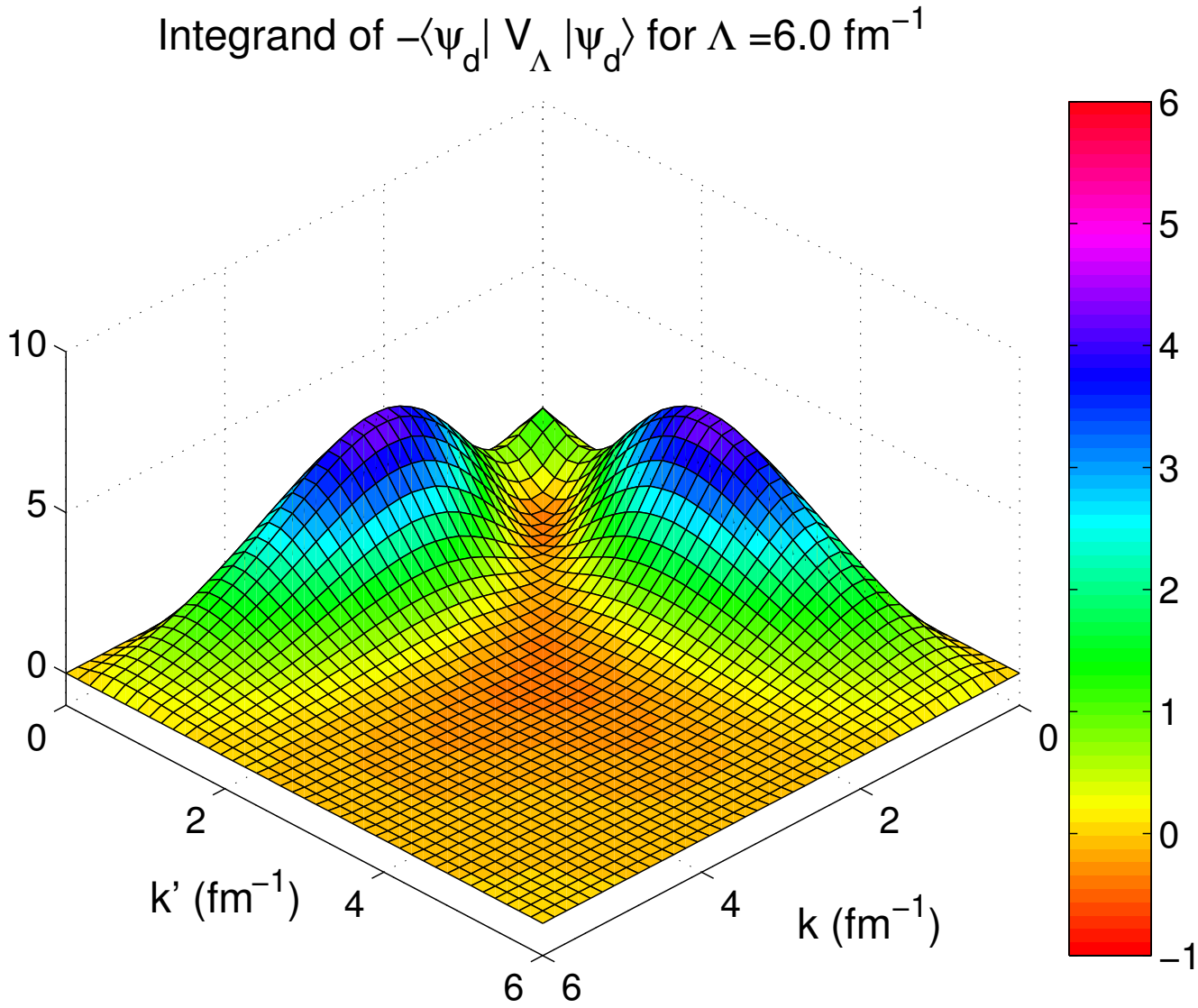




- Repulsive core $\Longrightarrow$ short-distance suppression / high-momentum components
- Low-momentum potential $\Longrightarrow$ much simpler wave function!

The Deuteron at Different Resolutions

more



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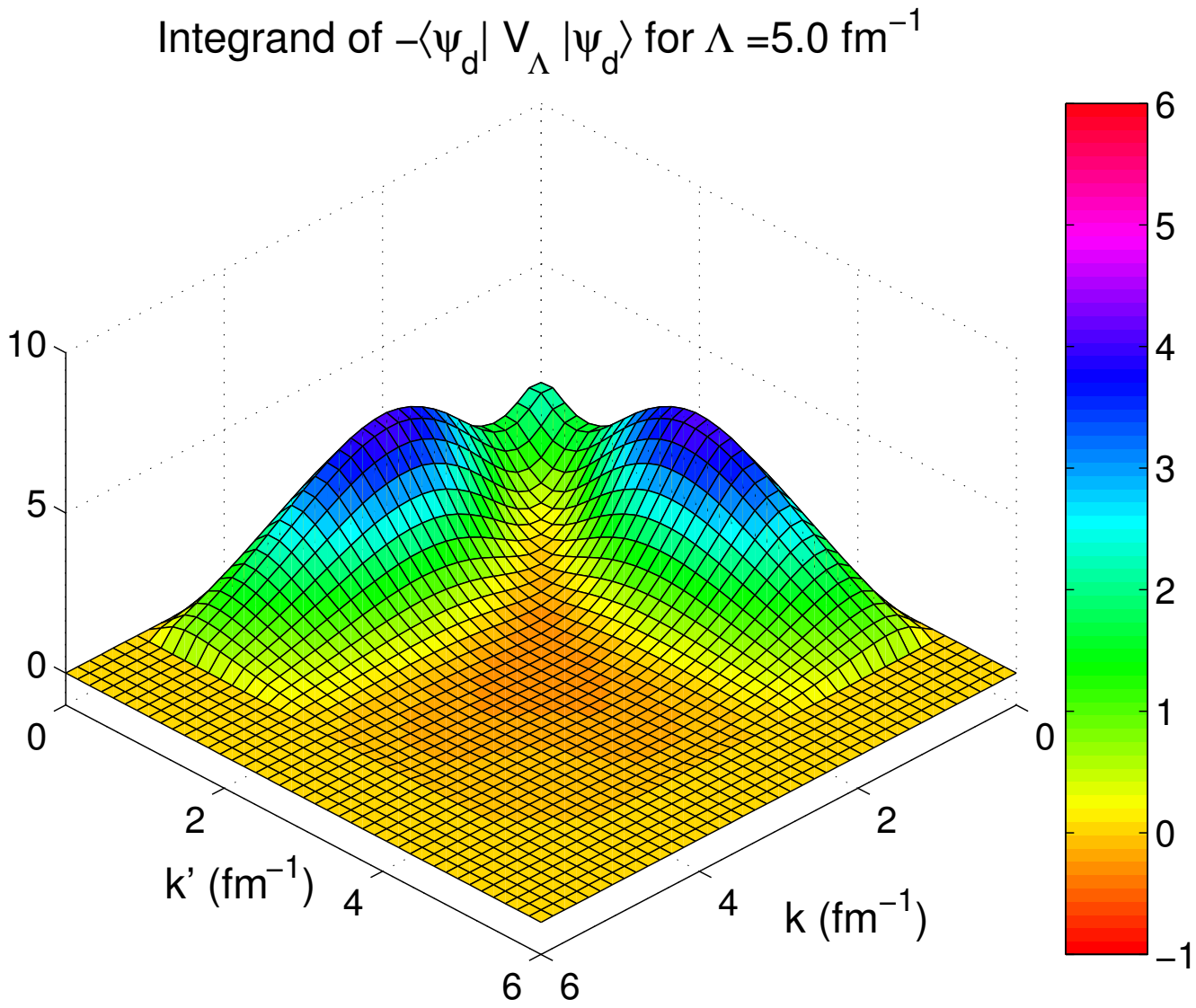

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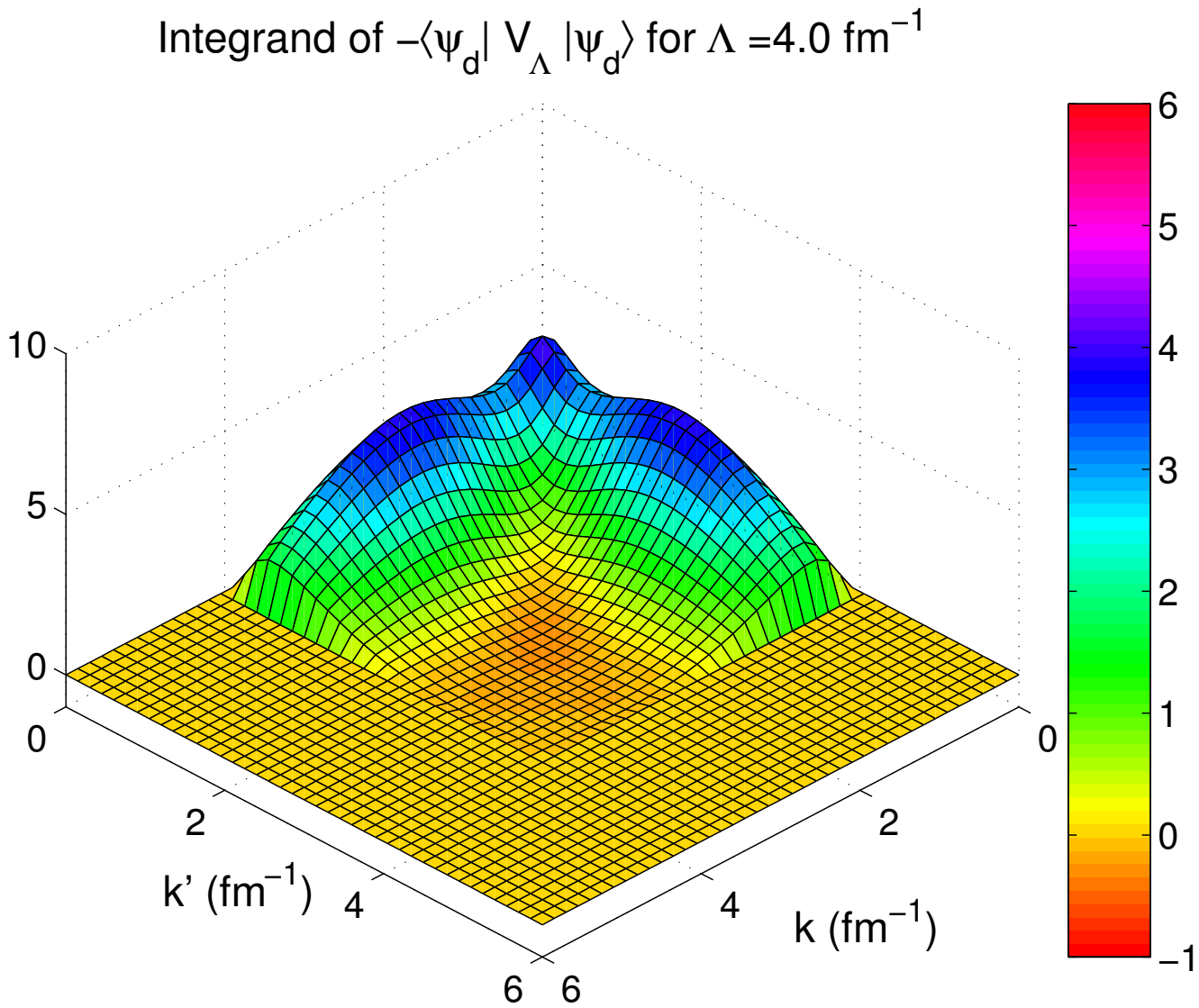

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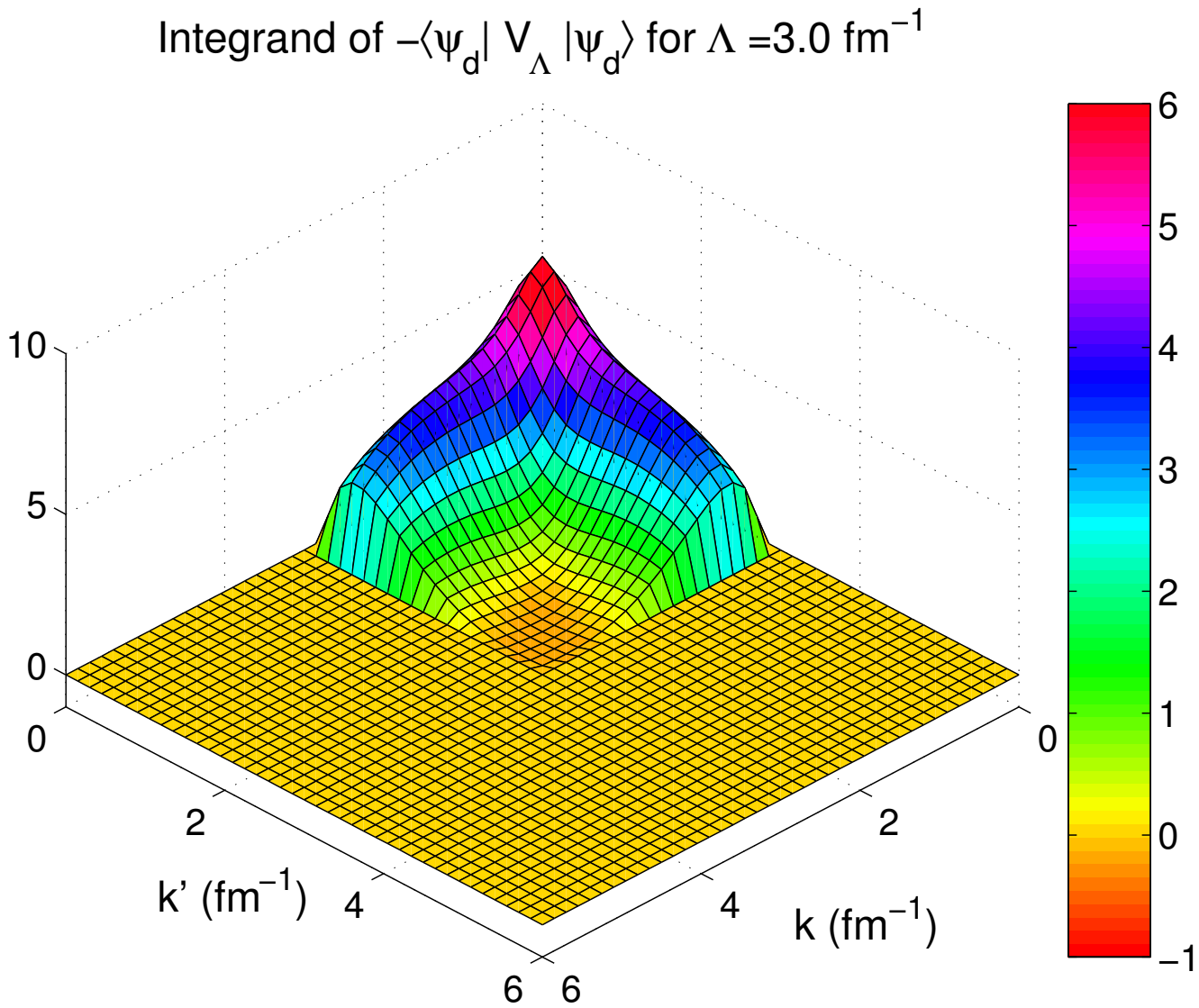

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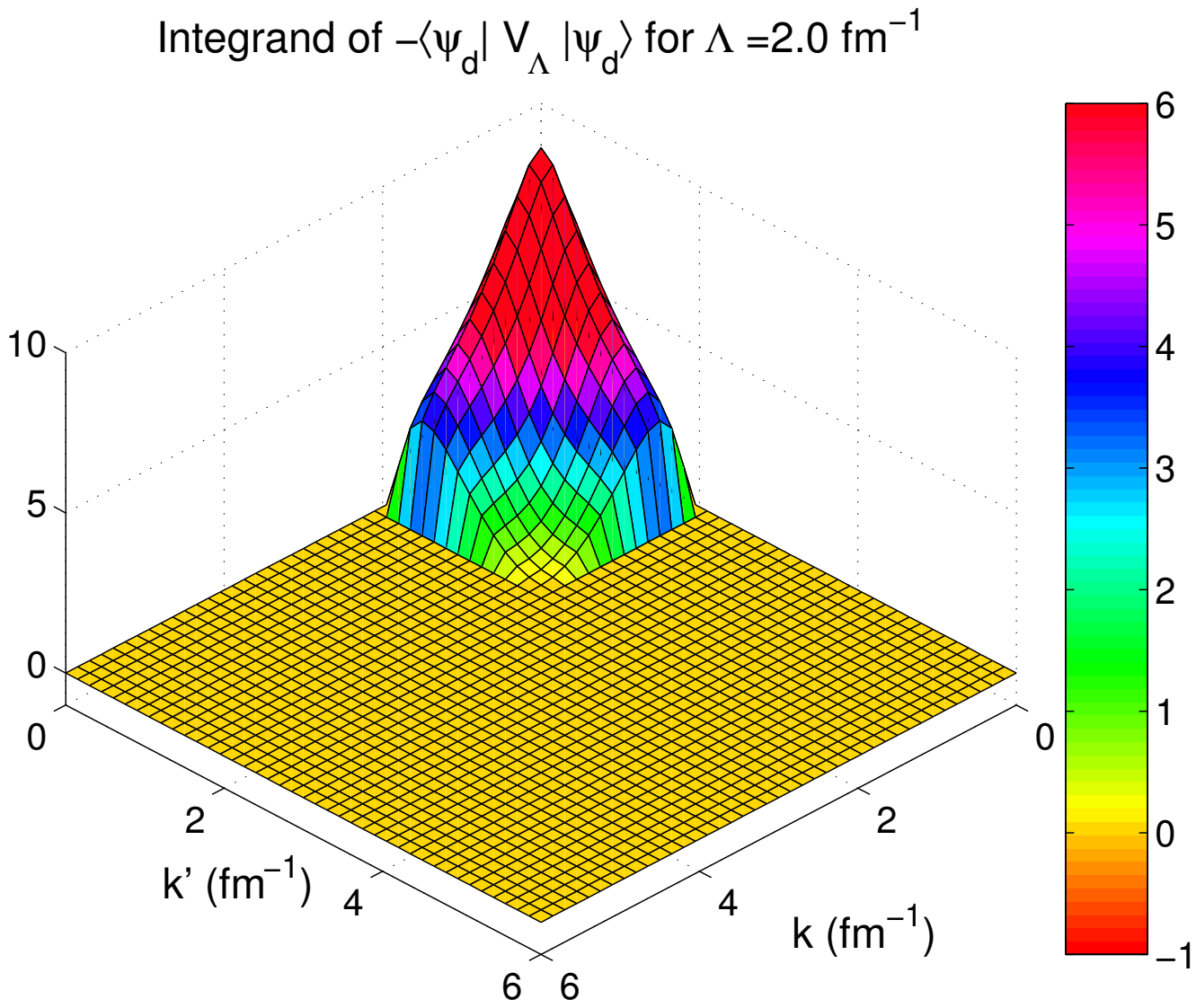
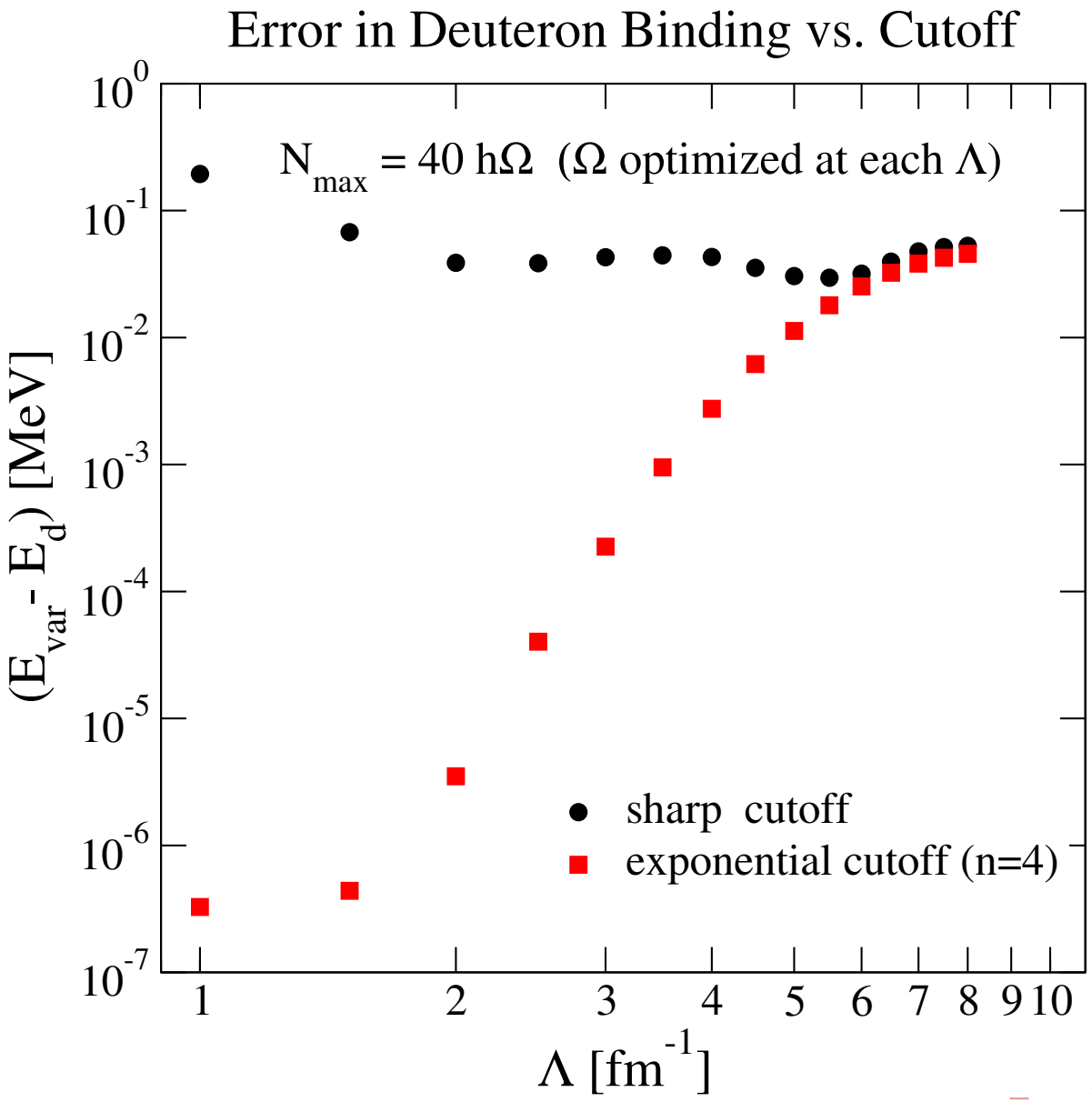


Consequence for Basis Expansions [nucl-th/0602017]



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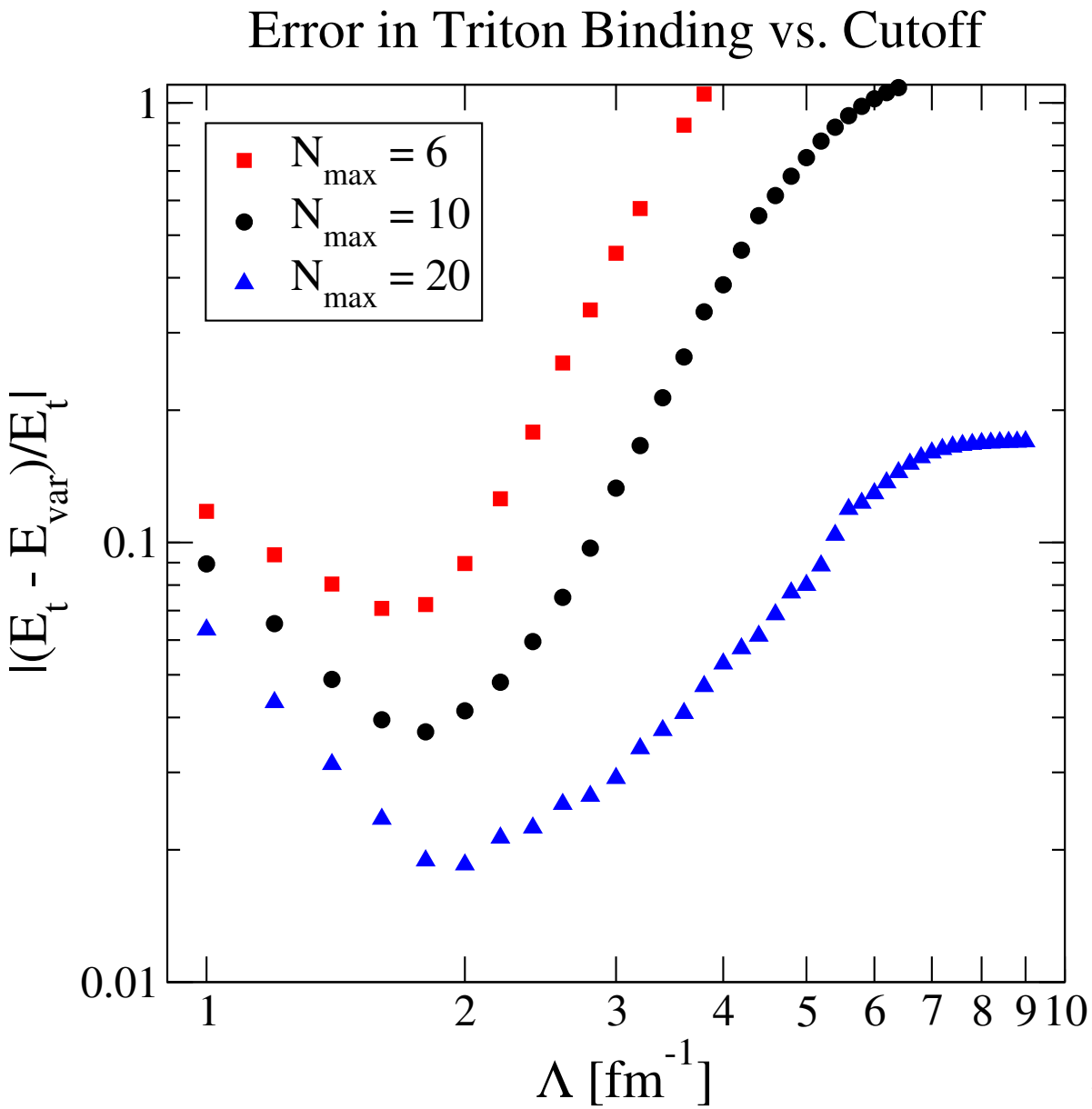

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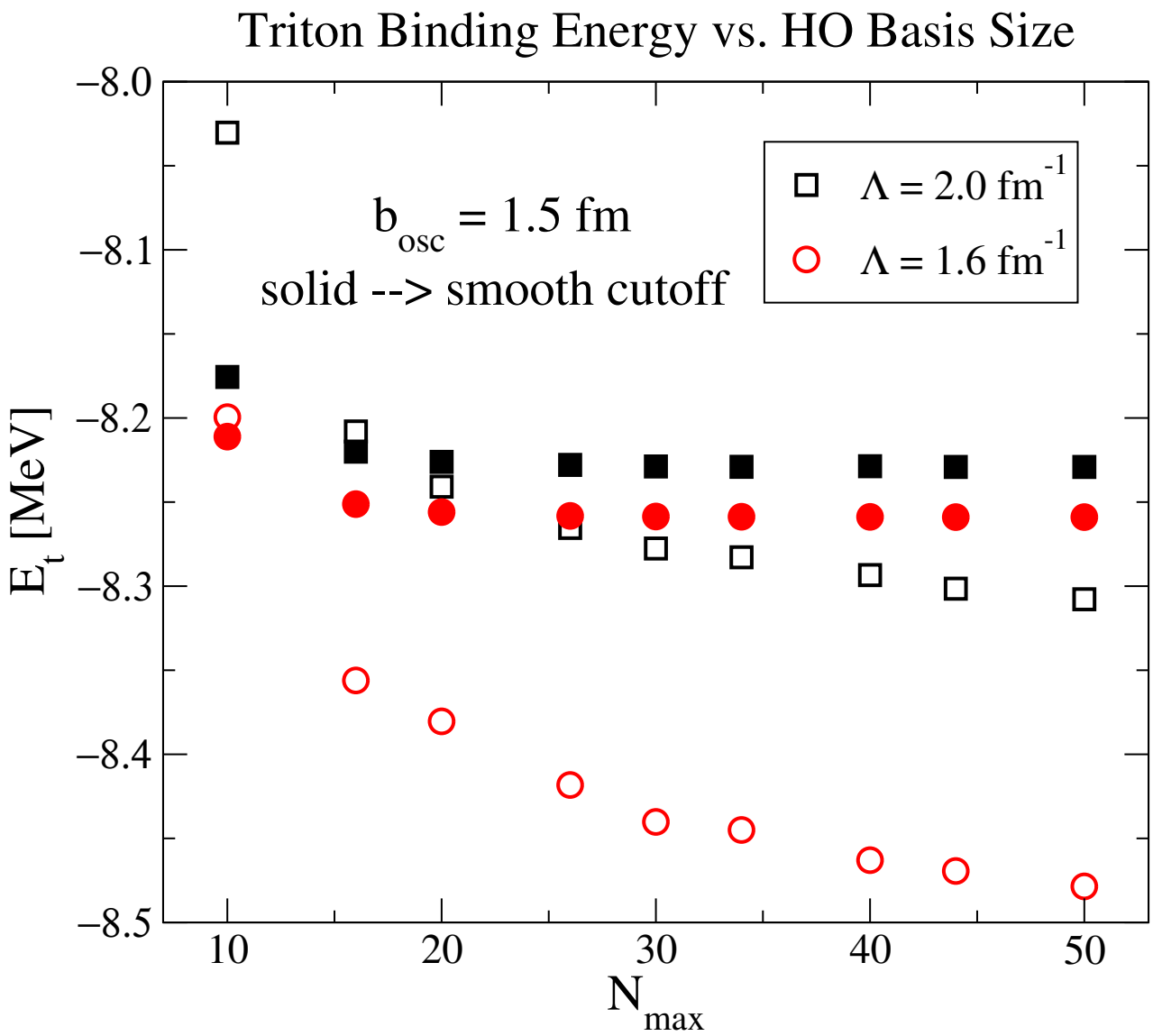
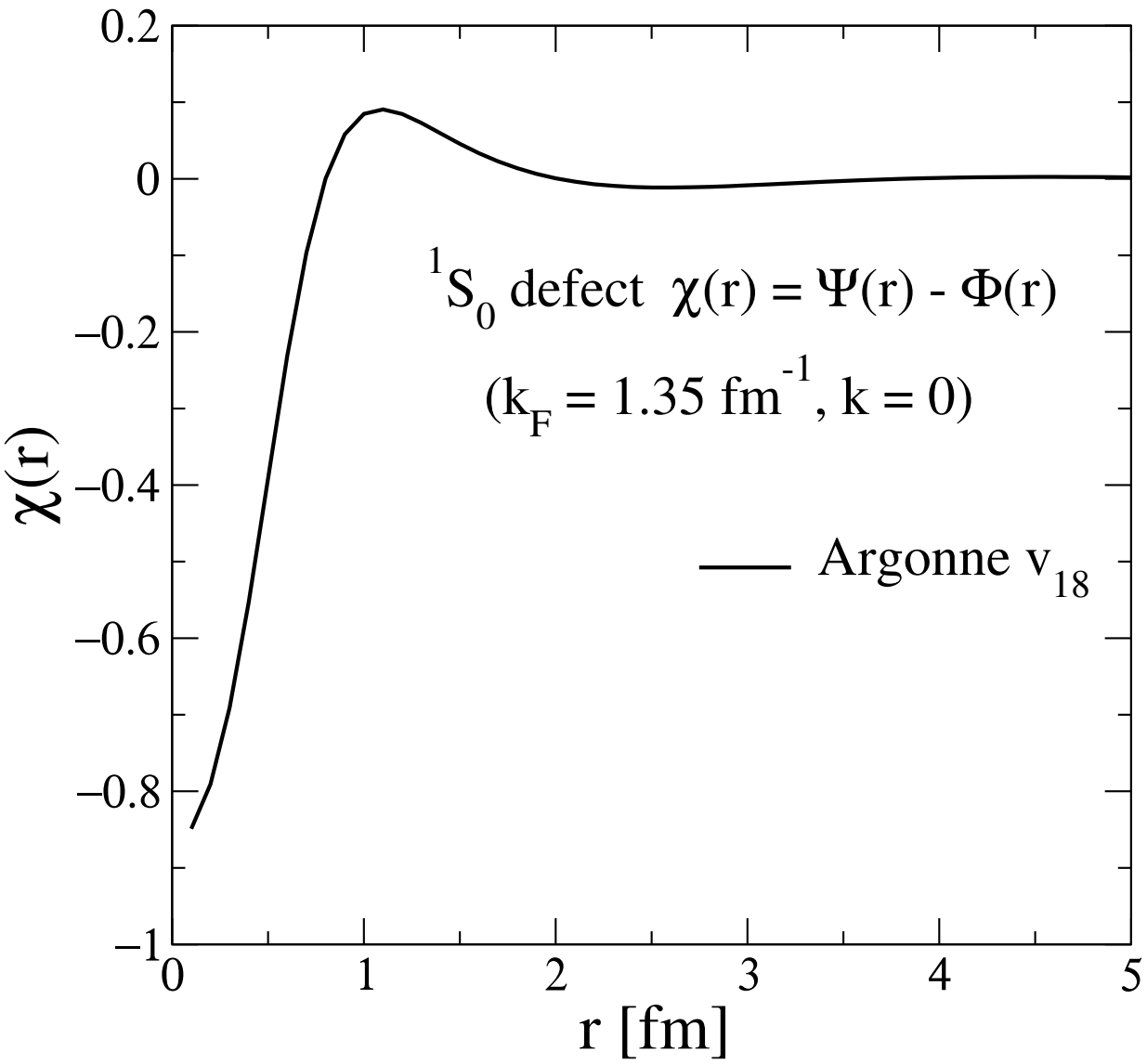


Two-Body Correlations in Nuclear Matter?



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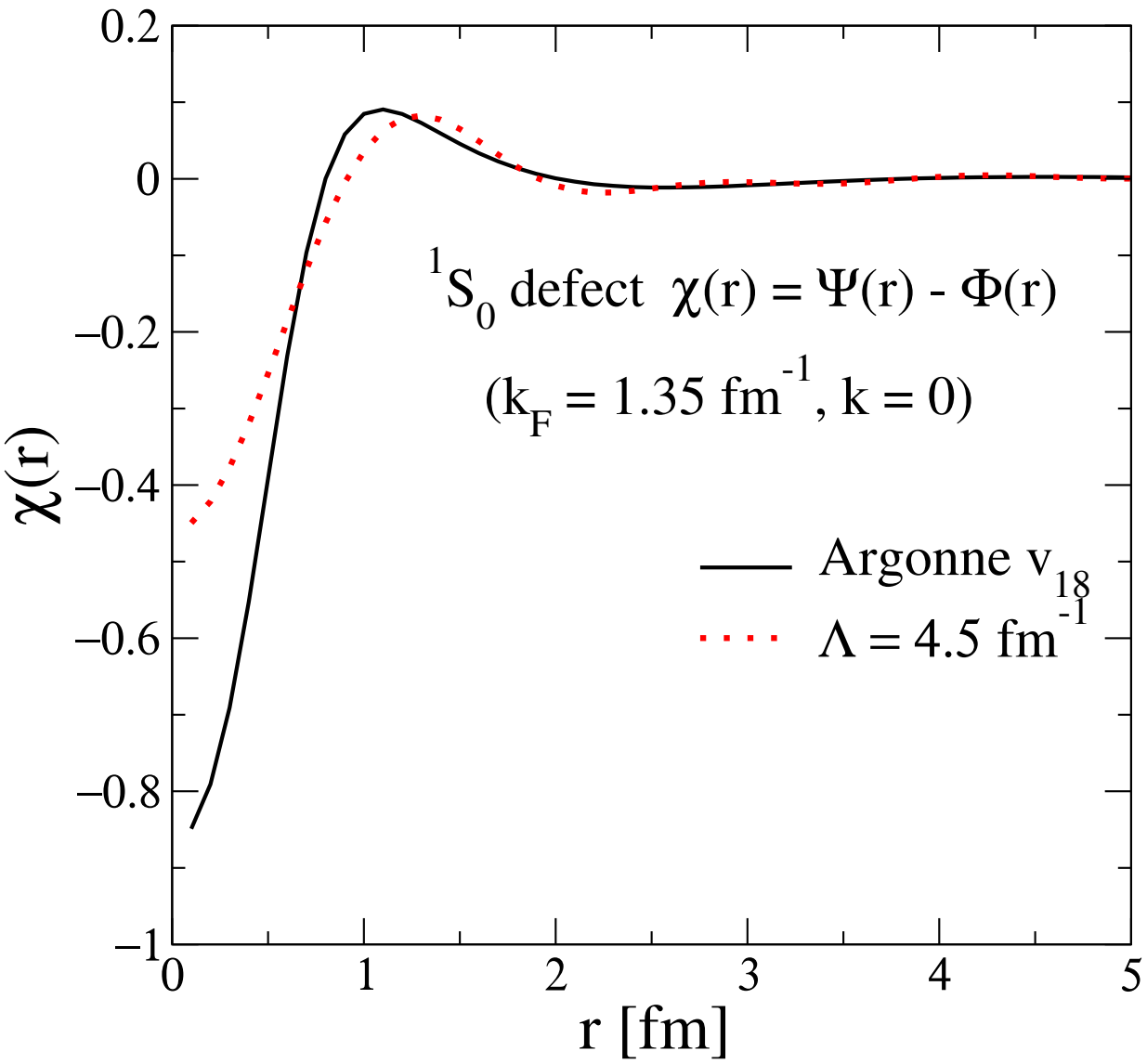

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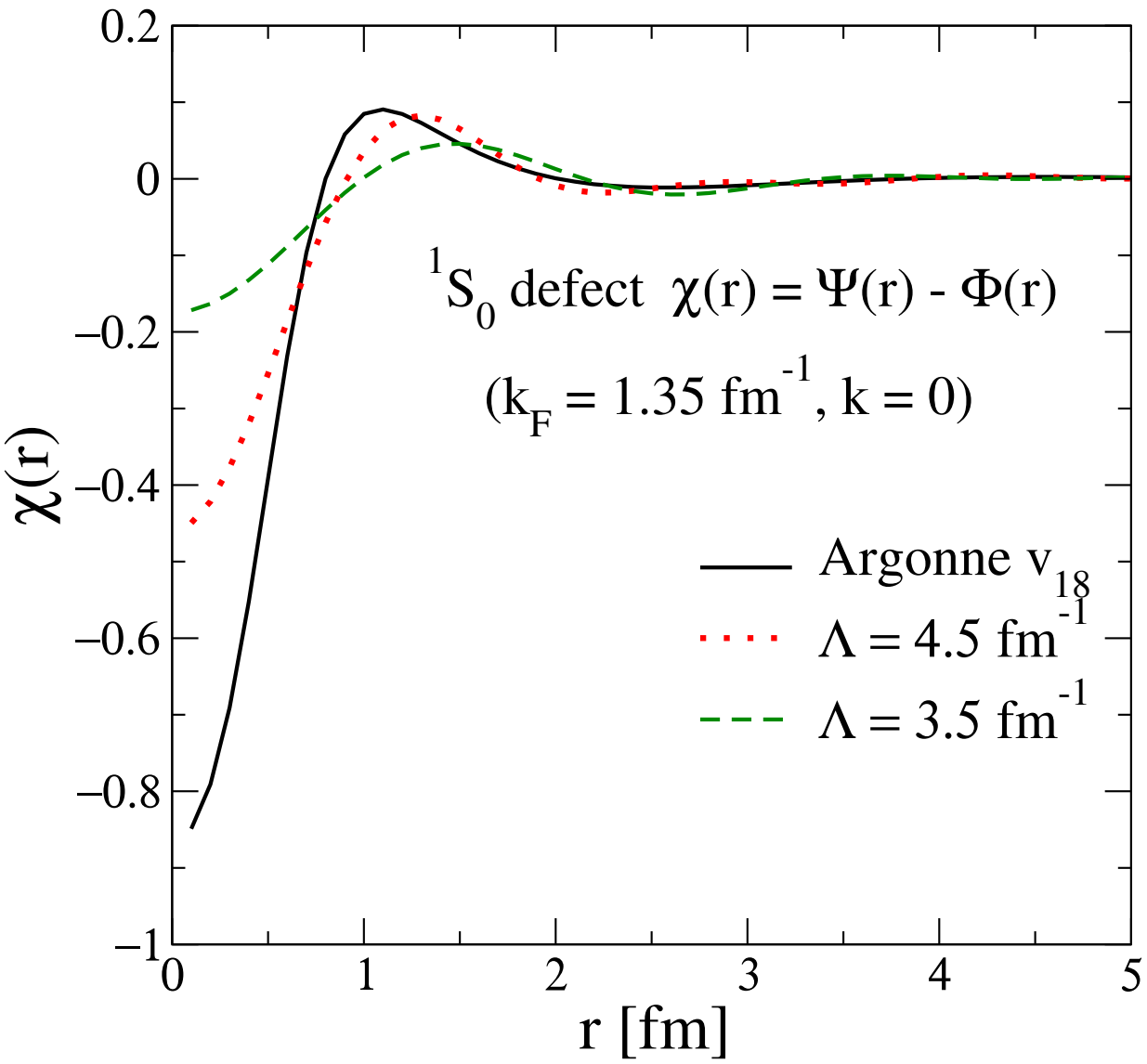

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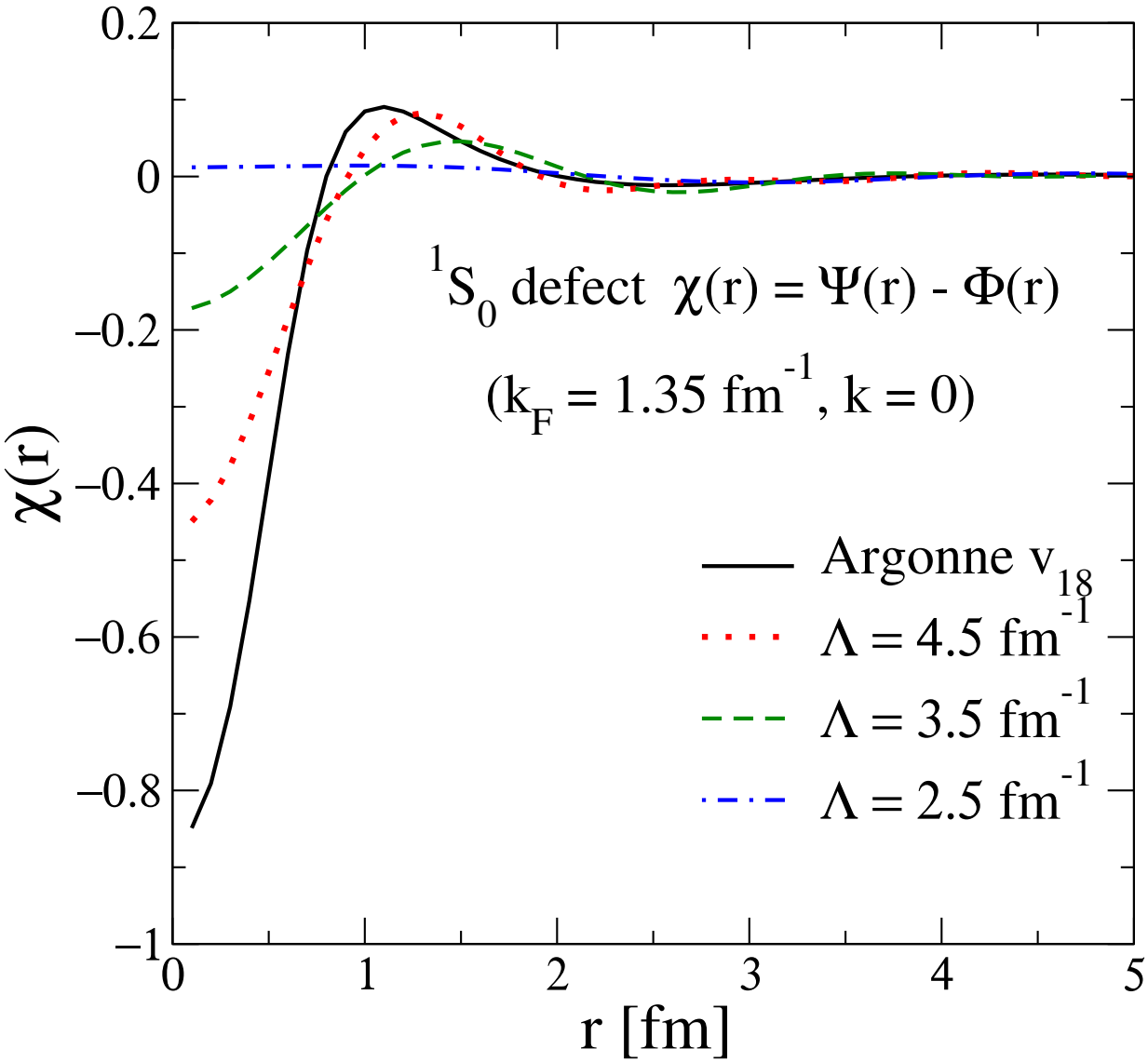

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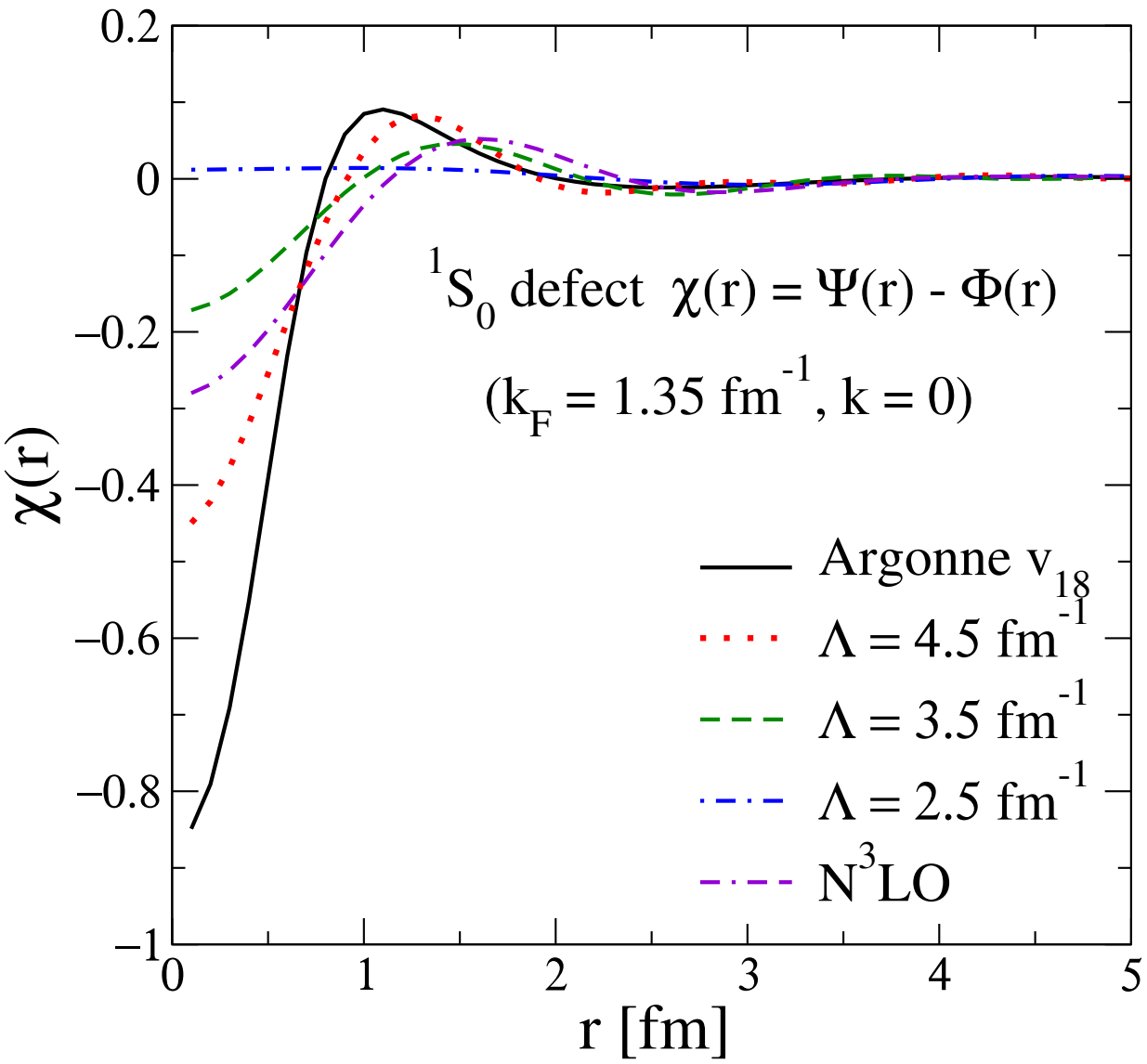


Why is In-Medium T Perturbative for $V_{\mathrm{low}\,k}$?

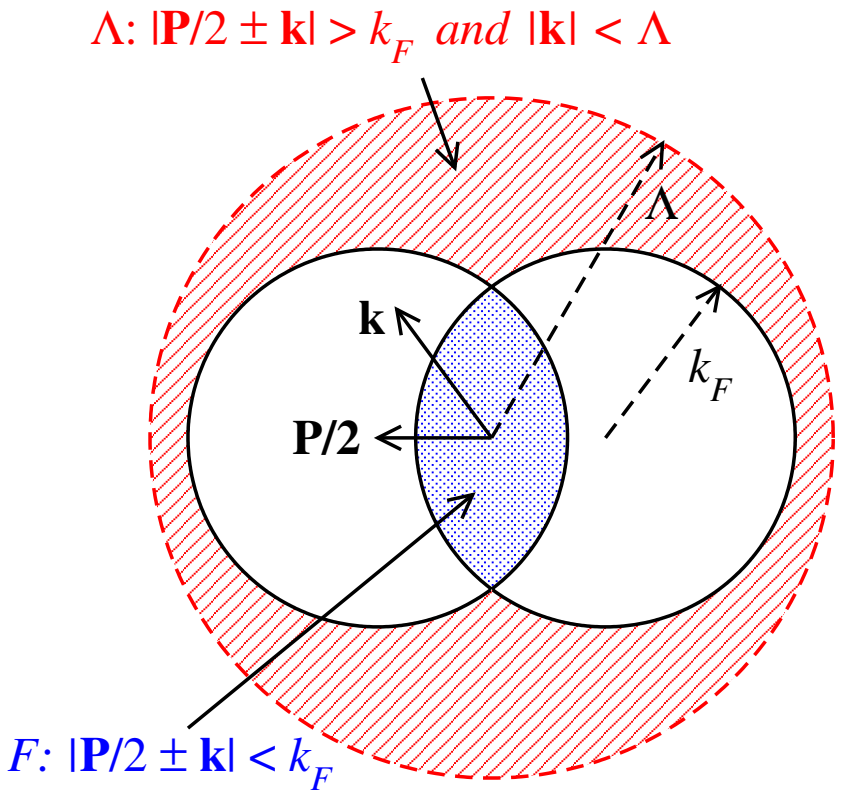
- Phase space in pp-channel strongly suppressed:

$$\int_{k_F}^{\infty} q^2\,dq\,\frac{V_{NN}(k',q)V_{NN}(q,k)}{k^2-q^2}$$

vs.

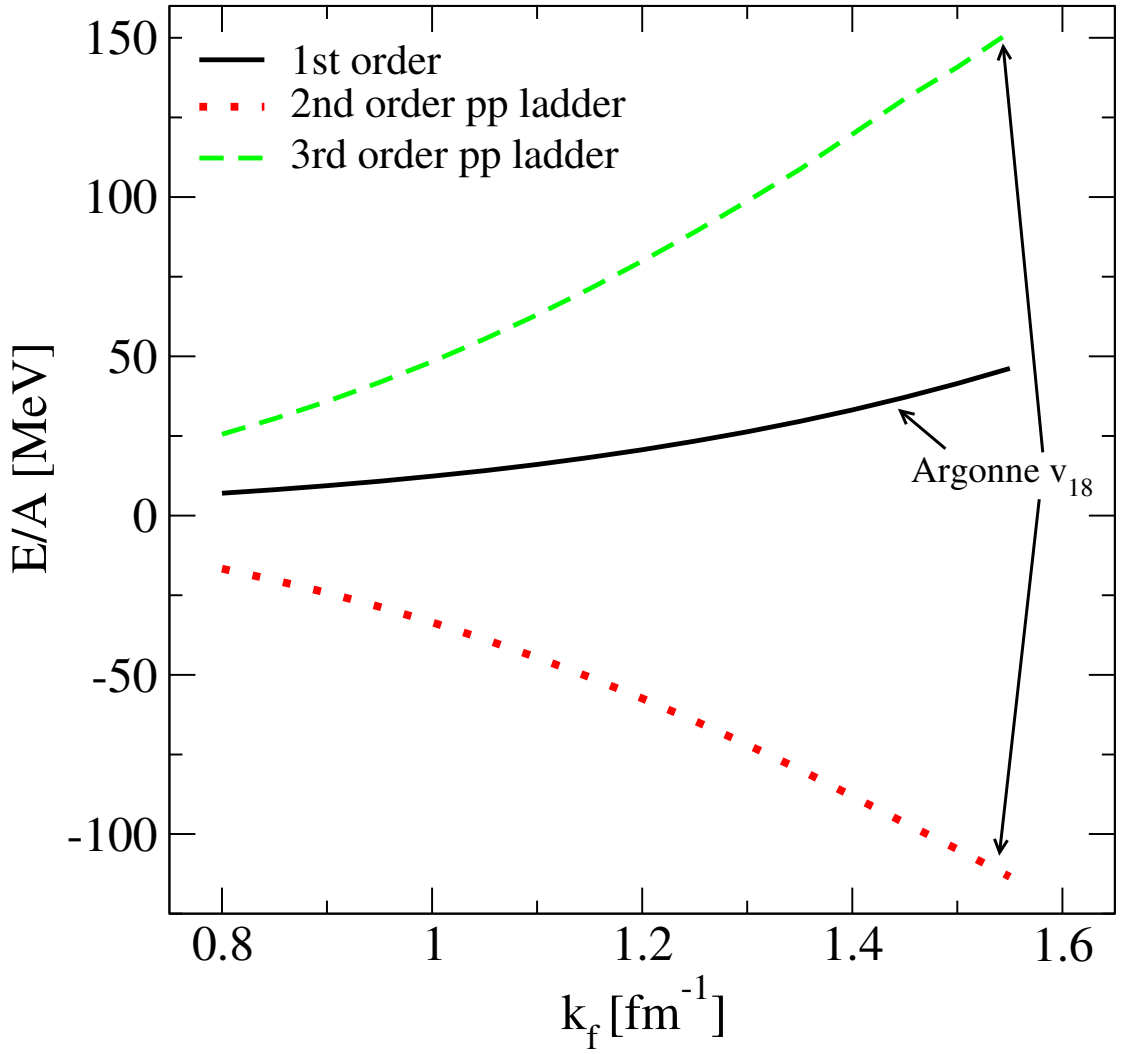
$$\int_{k_F}^{\Lambda} q^2\,dq\,\frac{V_{\mathrm{low}\,k}(k',q)V_{\mathrm{low}\,k}(q,k)}{k^2-q^2}$$

- Tames hard core, tensor, and bound state



Nuclear Matter with NN Ladders Only

- Brueckner ladders order-by-order
- Repulsive core $\Longrightarrow$ series diverges



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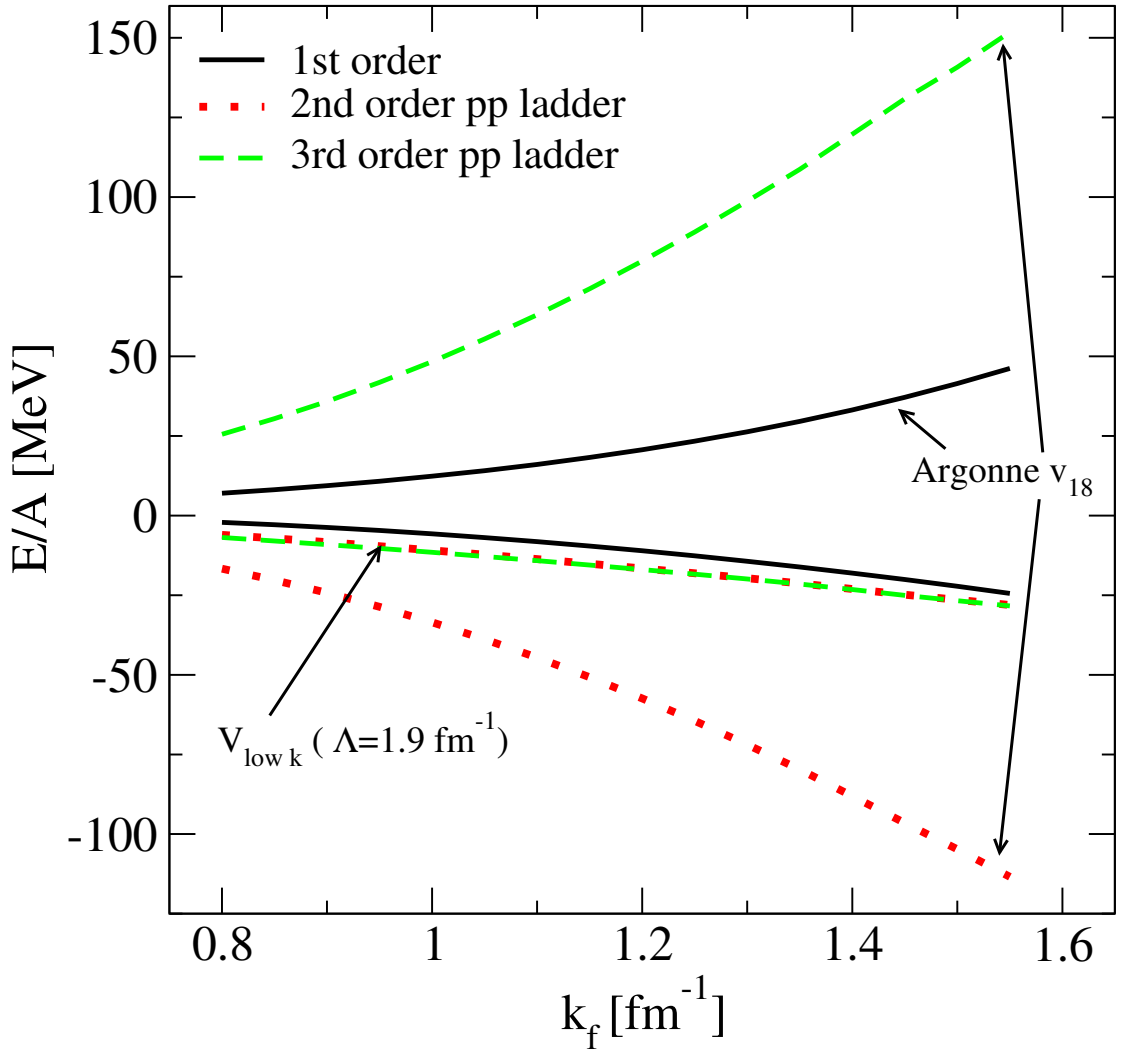

Nuclear Matter with NN Ladders Only

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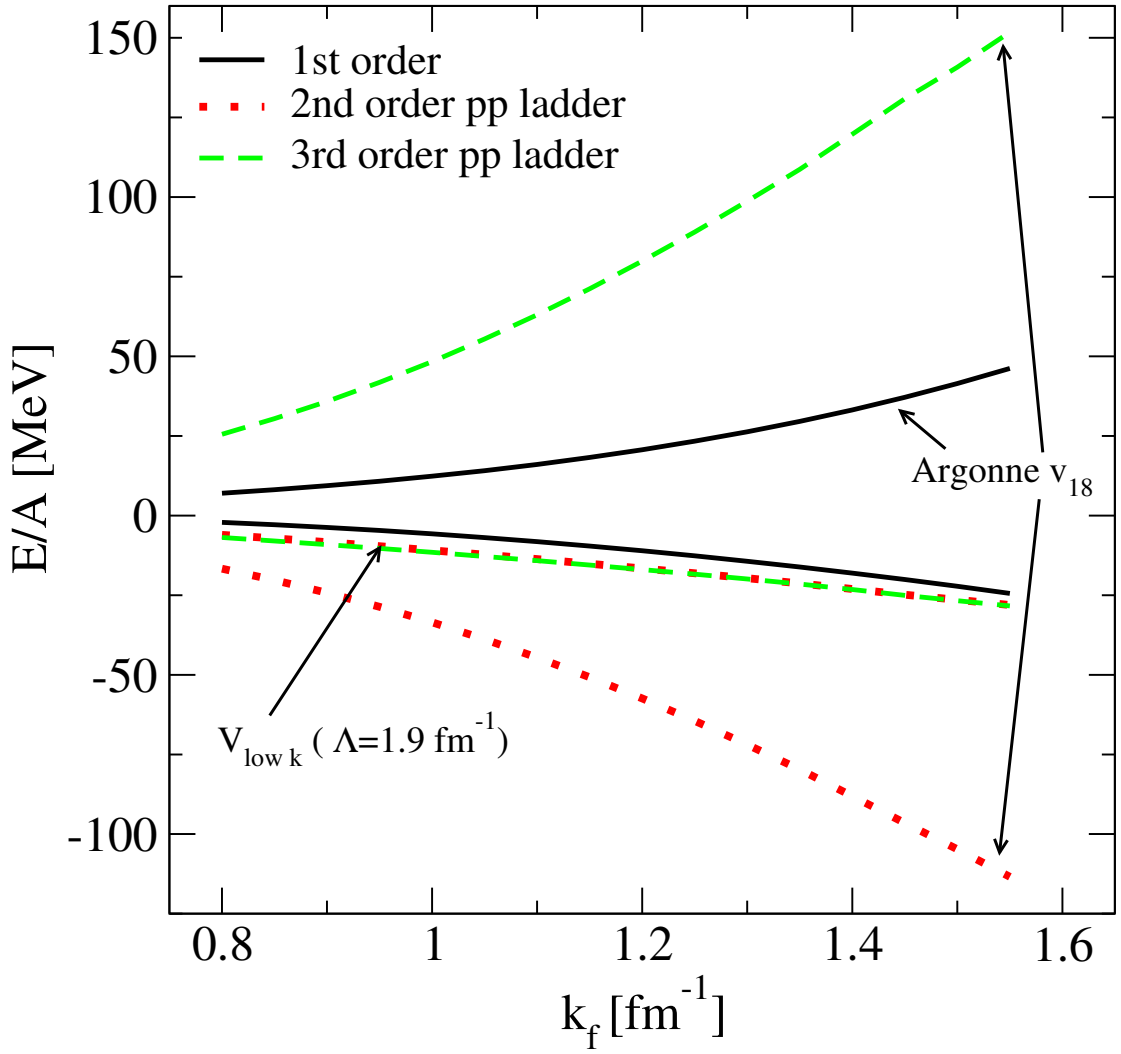
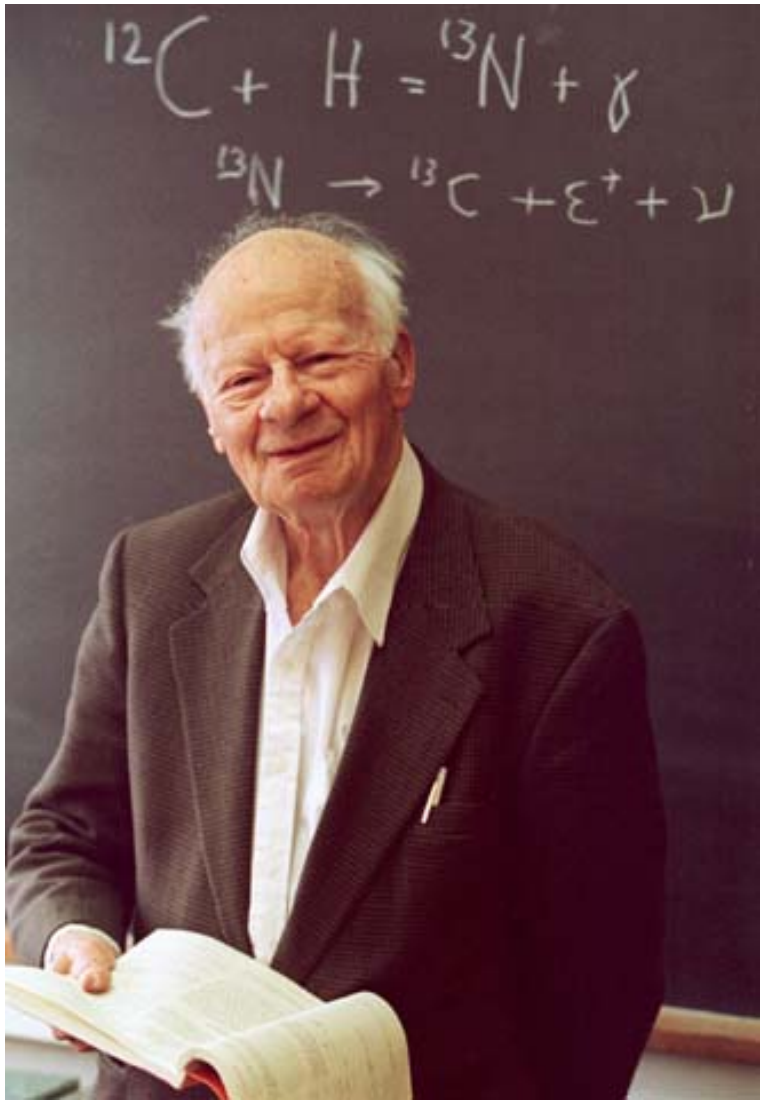
- Repulsive core $\Longrightarrow$ series diverges
- $V_{\text{low}\,k}$ converges
- No saturation in sight!



Deja Vu All Over Again?

- There were active attempts to transform away hard cores and soften the tensor interaction in the late sixties and early seventies.
- But the requiem for soft potentials was given by Bethe (1971):
  "Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."
- Next 30+ years trying to solve accurately with "hard" potential



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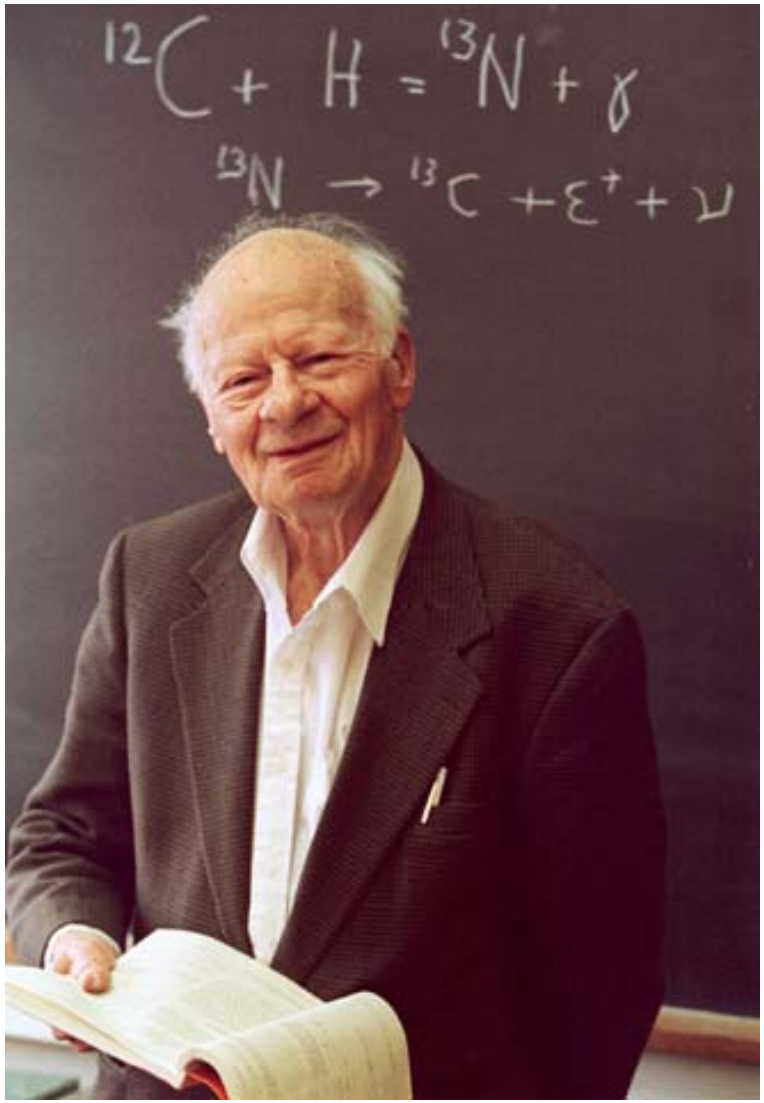
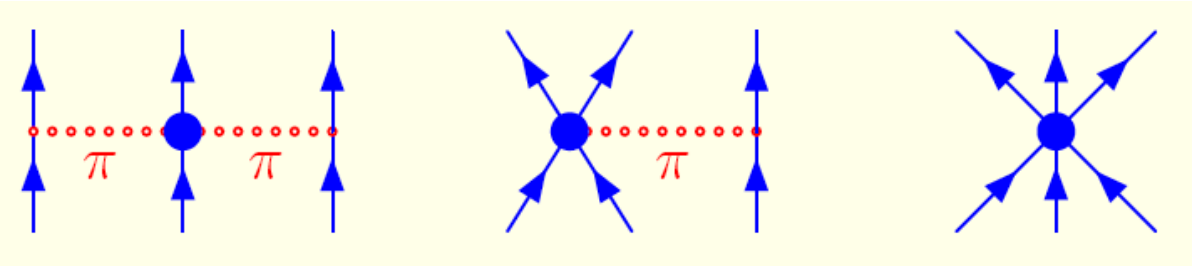
- But the story is not complete: three-nucleon forces (3NF)!



$V_{\mathrm{low}\,k}$ with Chiral 3NF

- Ideal: Start with chiral NN + 3NF EFT and run $\Lambda \downarrow$



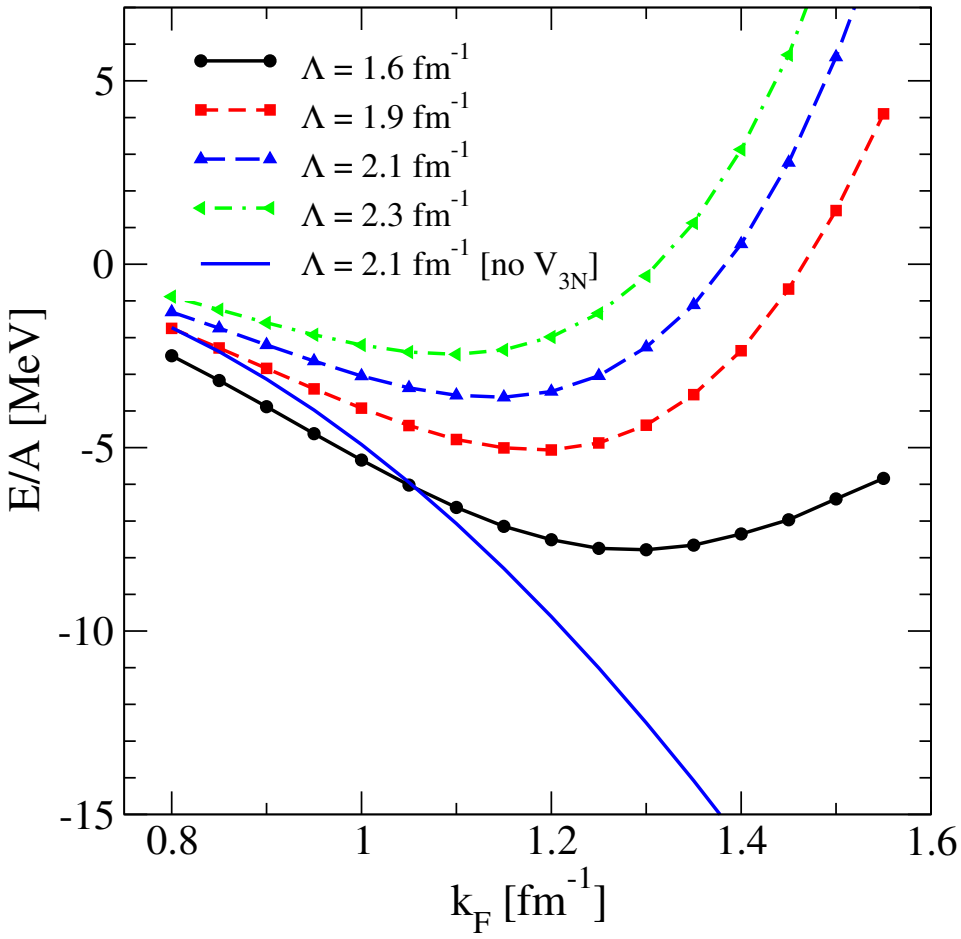
- Possible now: Run NN and fit 3NF EFT at each Λ

  Bogner, Nogga, Schwenk, Phys. Rev. C **70** (2004) 061002

  - two-pion-exchange c_i's from NN PSA fit
  - two free parameters fit to ^{3}H and ^{4}He binding energies
  - ratio 2NF/3NF consistent with chiral counting

(Approximate) Nuclear Matter with NN and NNN

Hartree-Fock



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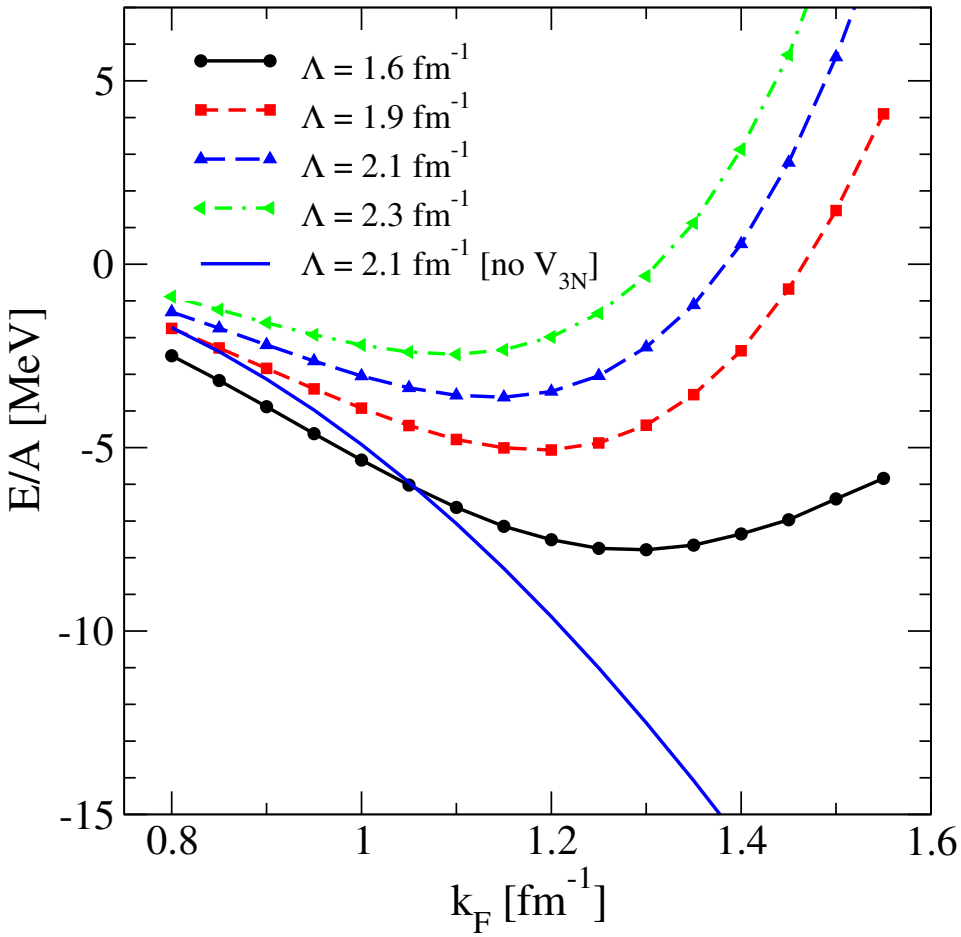
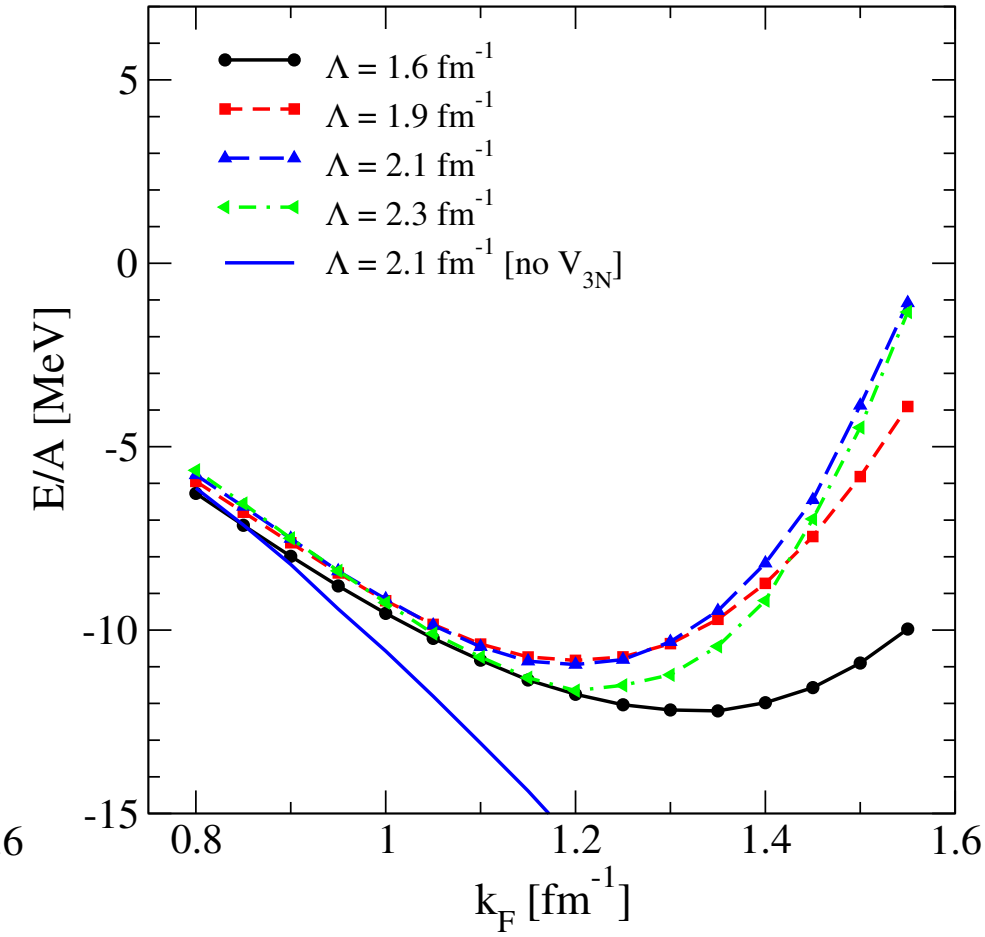


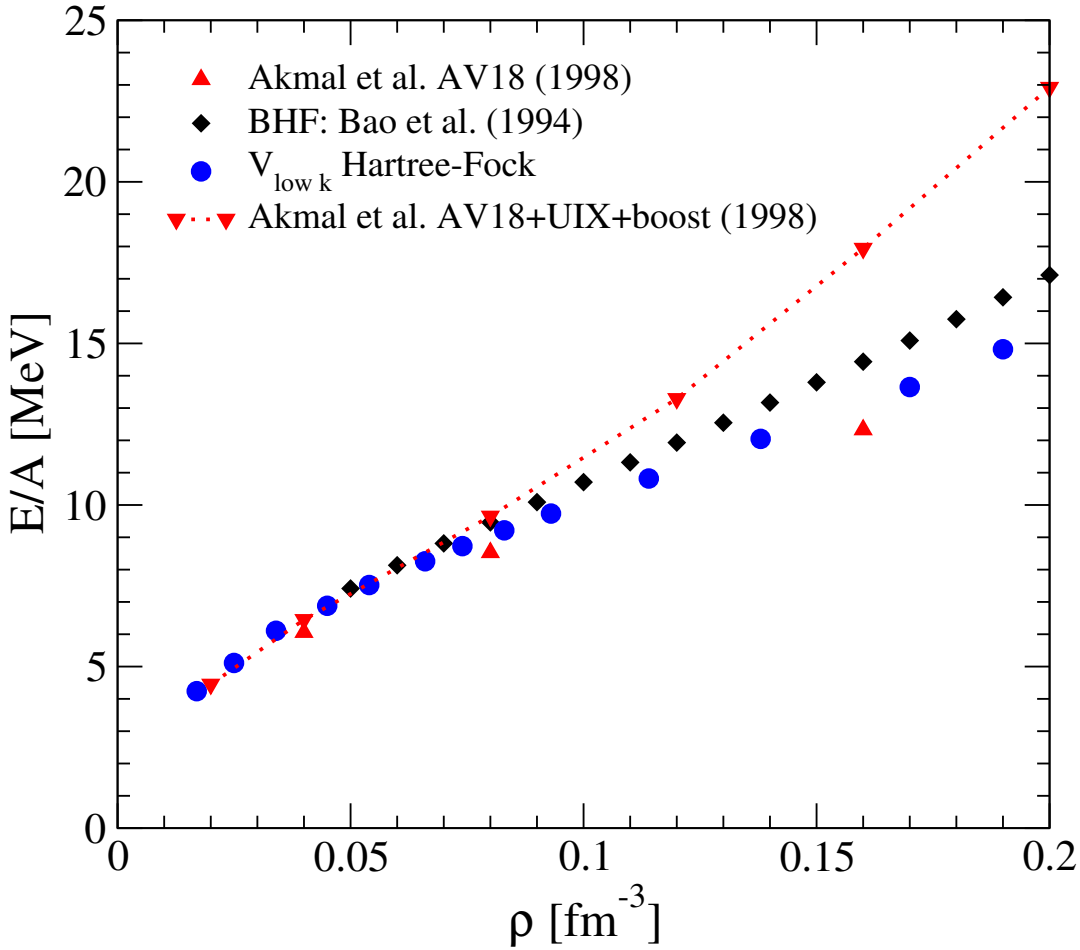
"≈ 2nd Order"



3-body out of control?

Low-Momentum Potential for Neutron Matter

- Removing hard core $\Longrightarrow$ simpler many-body starting point for neutron matter [Schwenk, Friman, Brown]



- Simple Hartree-Fock (circles) matches best calculations!

Outline

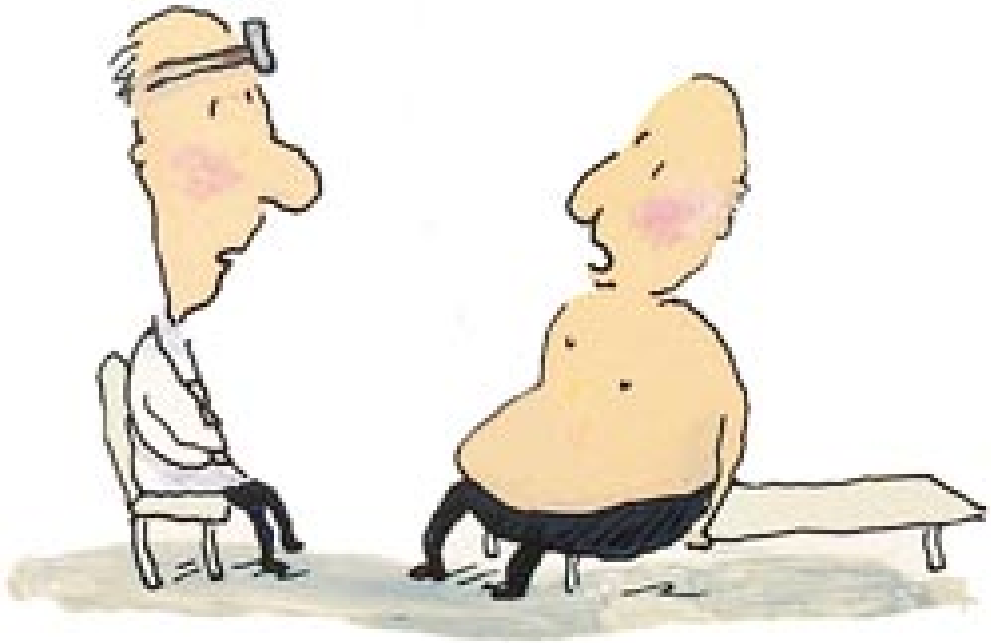
Bethe and Calculating Nuclear Matter

- Hans Bethe in review of nuclear matter (1971):

  "The theory must be such that it can deal with any nucleon-nucleon (NN) force, including hard or 'soft' core, tensor forces, and other complications. It ought not to be necessary to tailor the NN force for the sake of making the computation of nuclear matter (or finite nuclei) easier, but the force should be chosen on the basis of NN experiments (and possibly subsidiary experimental evidence, like the binding energy of H^3)."

EFT and RG Make Many-Body Physics Easier

- There's an old vaudeville joke about a doctor and patient . . .



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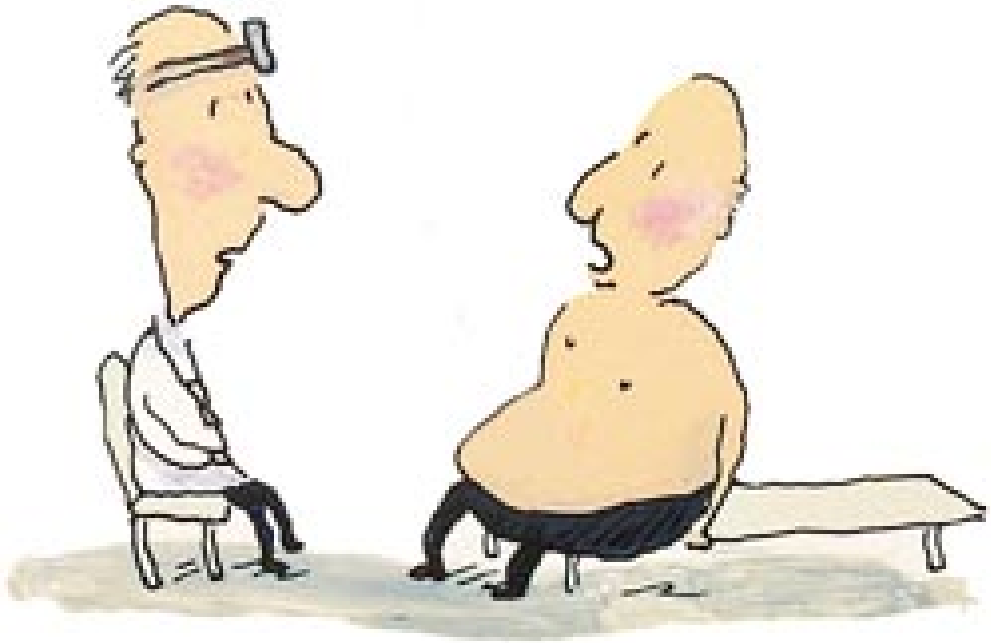

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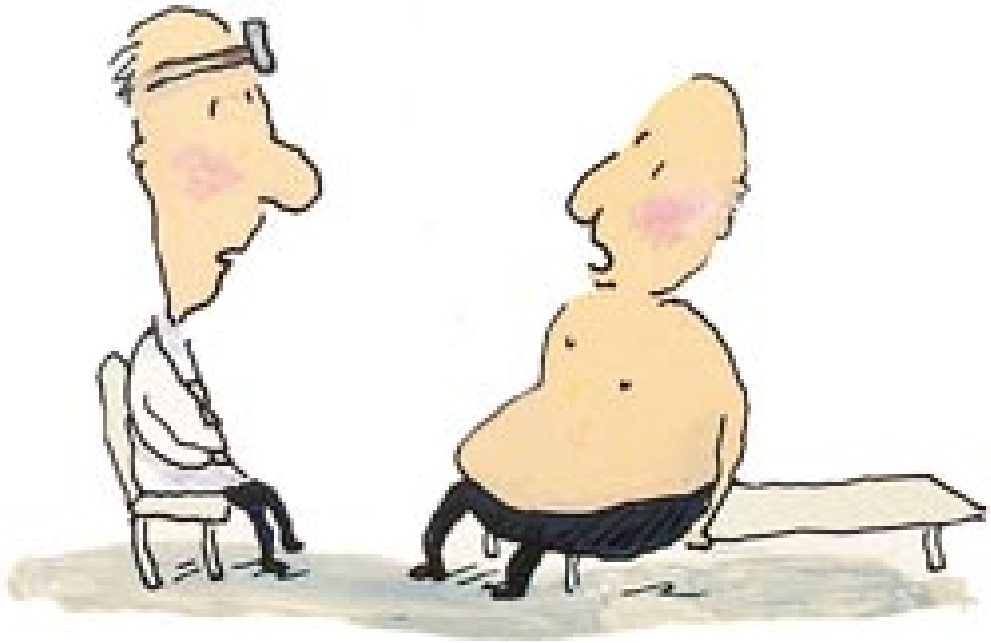

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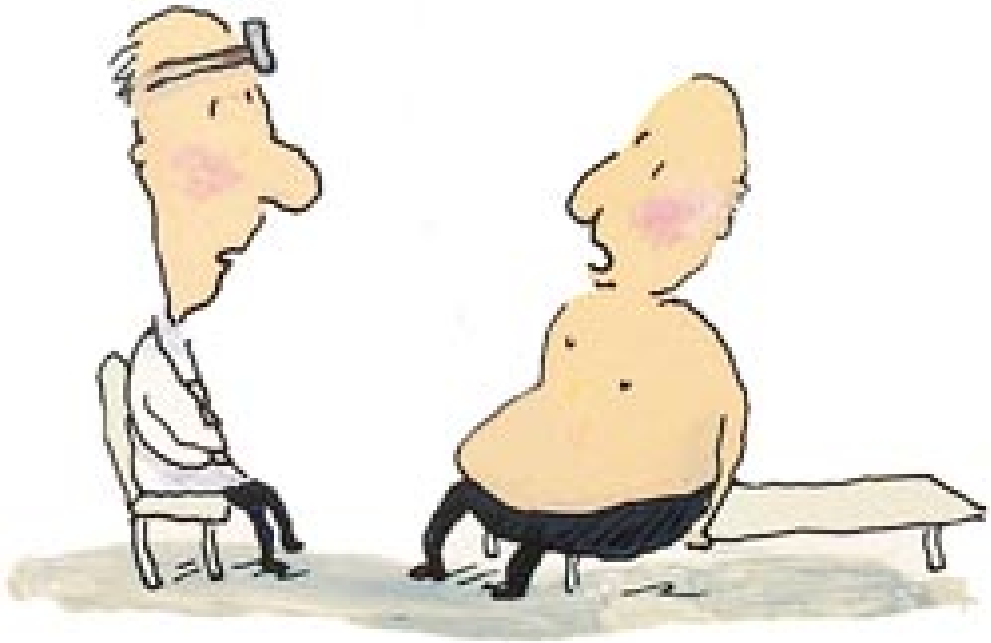

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- Weinberg's Third Law of Progress in Theoretical Physics:
  "You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you'll be sorry!"

(Nuclear) Many-Body Physics: "Old" vs. "New"

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One Hamiltonian for all problems and energy/length scales	Infinite # of low-energy potentials; different resolutions $\Longrightarrow$ different dof's and Hamiltonians

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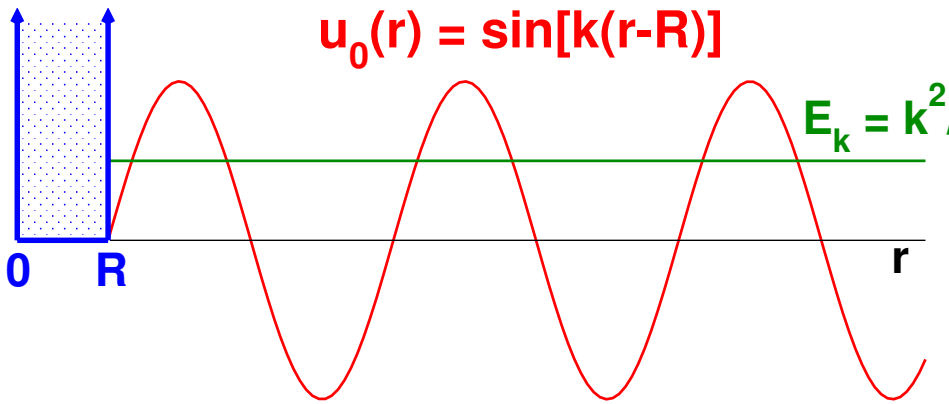
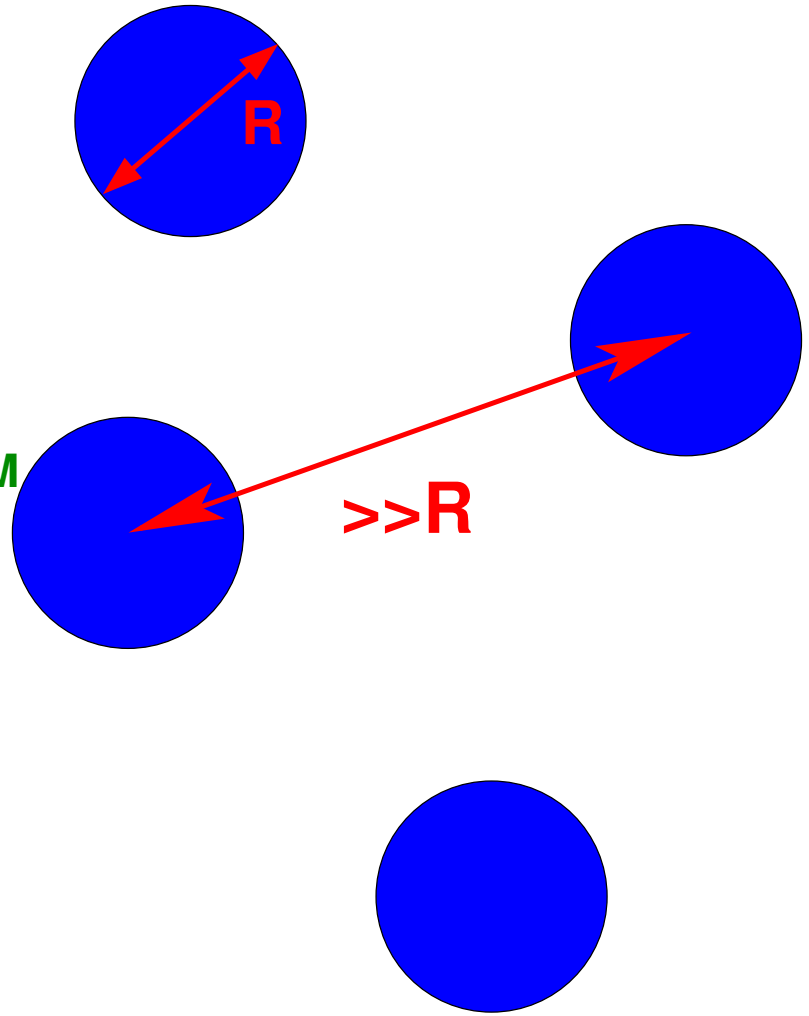
Choose diagrams by "art"	Power counting determines diagrams and truncation error

“Simple” Many-Body Problem: Hard Spheres

- Infinite potential at radius *R*
- Scattering solutions are simple:





- What is the energy / particle and density profile of the trapped many-body system?

Renormalization “Old” vs. “New”

More Deuteron Variational

Weinberg Eigenvalues

Renormalization: “Old” vs. “New”

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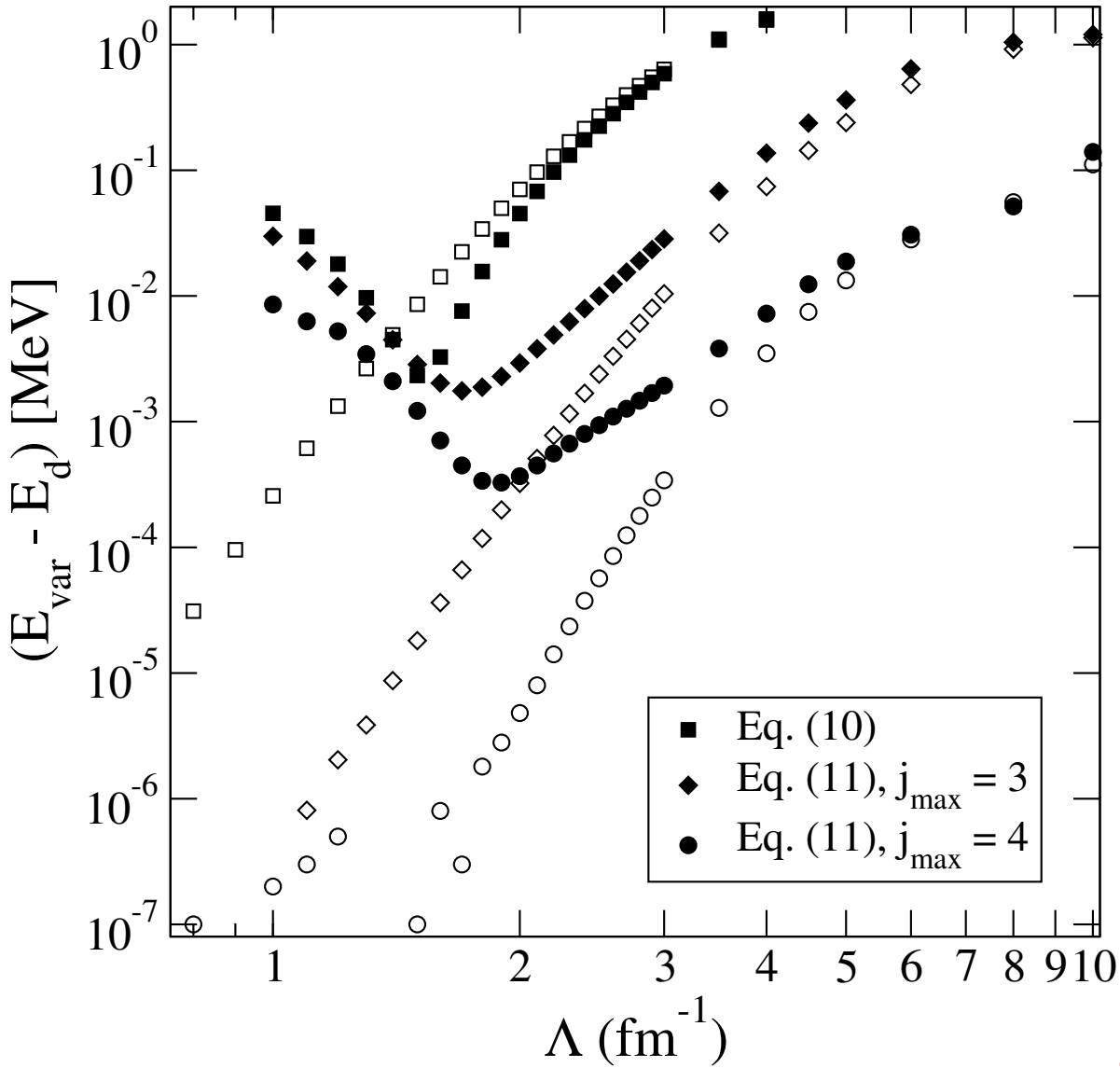
Non-renormalizable means no predictive power $\Longrightarrow$ renormalizable theories	Non-renormalizable $\Longrightarrow$ systematic expansion! Effective field theories

More Deuteron Variational Calculations



Convergence of the Born Series for Scattering

- Consider whether the Born series converges for given E

$$T(E) = V + V\frac{1}{E - H_0}V + V\frac{1}{E - H_0}V\frac{1}{E - H_0}V + \cdots$$

- For fixed E, find (complex) eigenvalues $\eta_\nu(E)$ [Weinberg]

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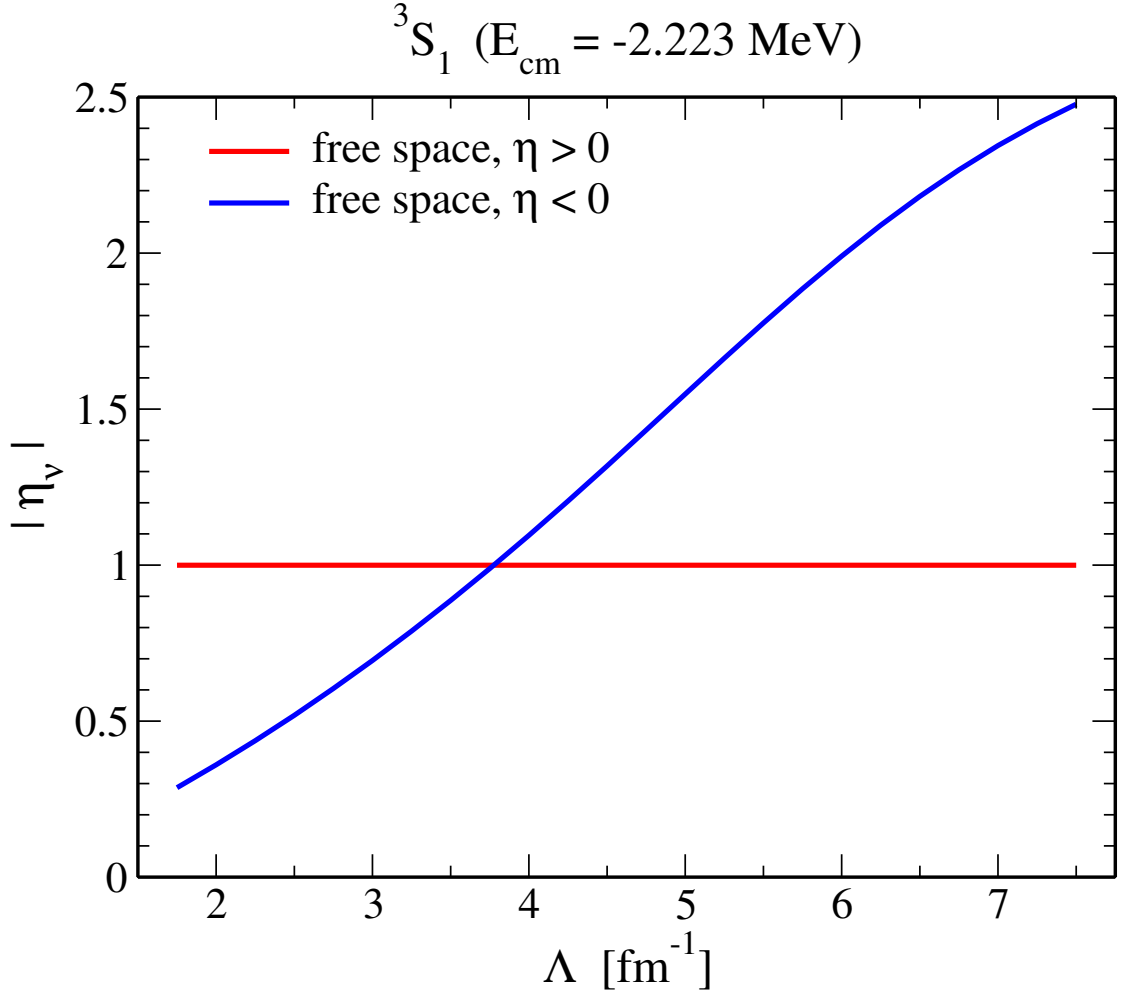
- For $E < 0$, same as finding η_ν where V/η_ν has bound state

$$(H_0 + V/\eta_\nu)|b\rangle = E|b\rangle \quad \text{with} \quad \eta_\nu > 0 \ \textit{or} \ \eta_\nu < 0$$

Weinberg Eigenvalues as Function of Cutoff

- Deuteron $\Longrightarrow$ attractive eigenvalue η_ν
  - $\Lambda \downarrow \Longrightarrow$ unchanged
- Hard core $\Longrightarrow$ repulsive eigenvalue η_ν
  - $\Lambda \downarrow \Longrightarrow$ reduced



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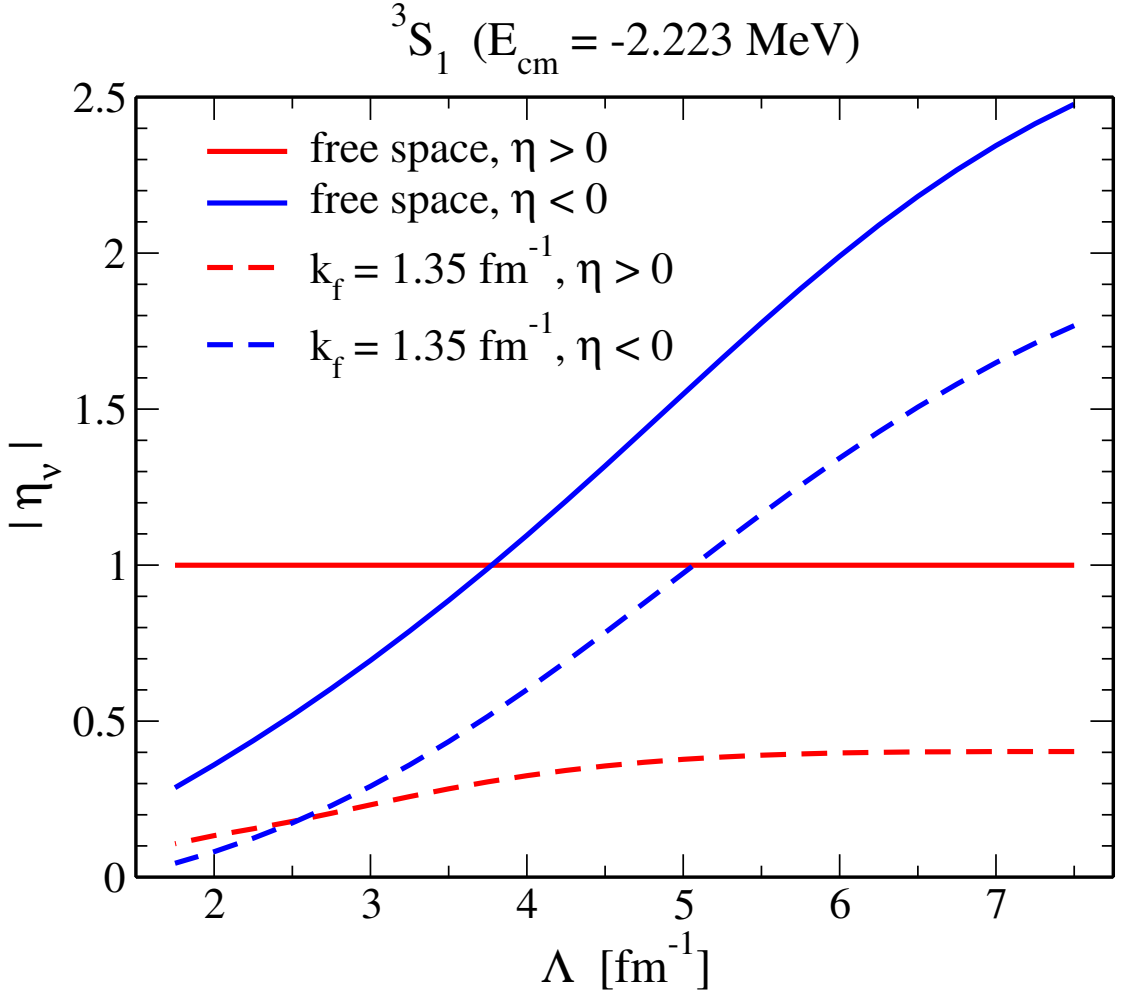
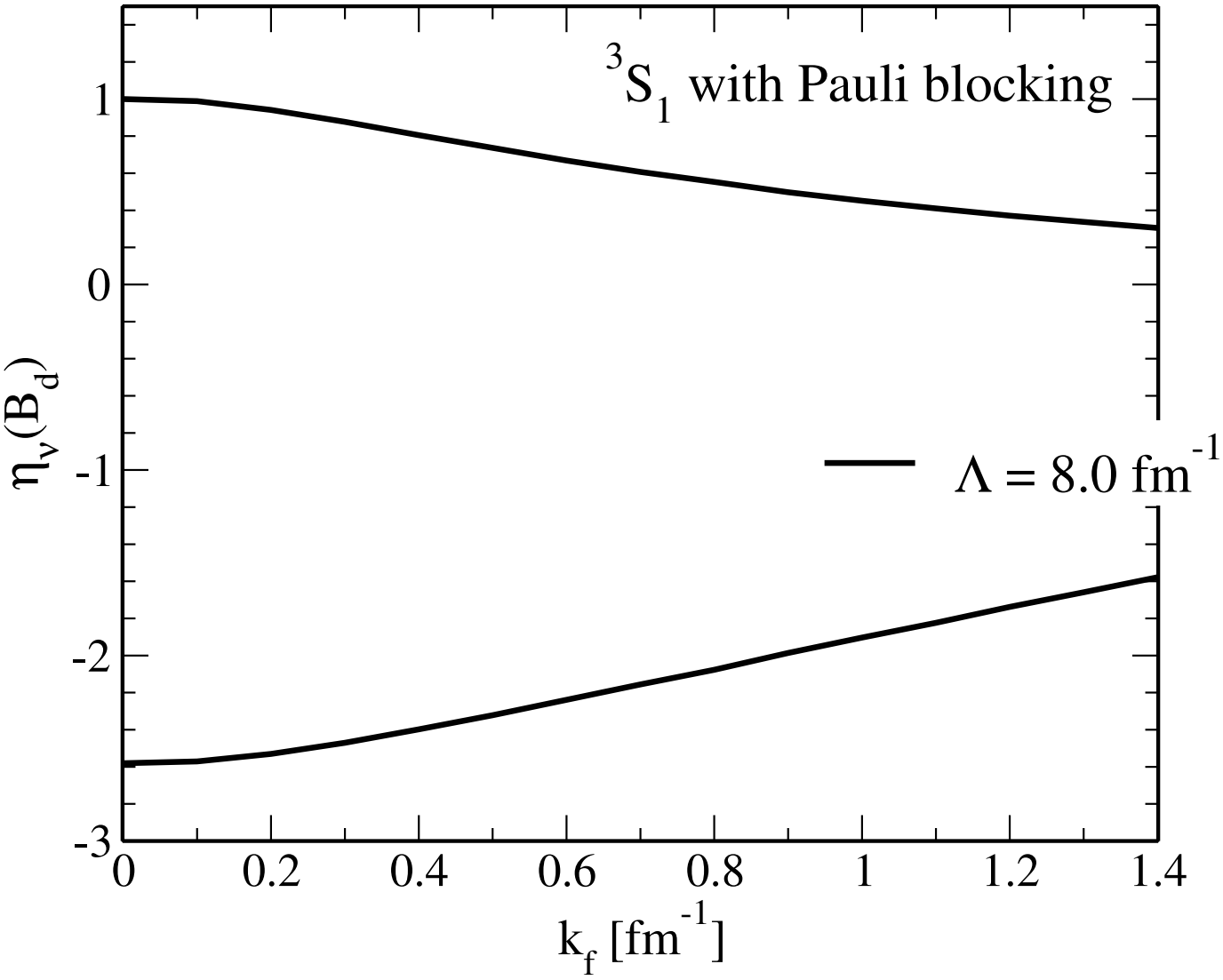
- In medium: both reduced
  - $\eta_\nu \ll 1$ for $\Lambda \approx 2\,\mathrm{fm}^{-1}$

$\Longrightarrow$ perturbative (in pp)



Weinberg Eigenvalues as Function of Density



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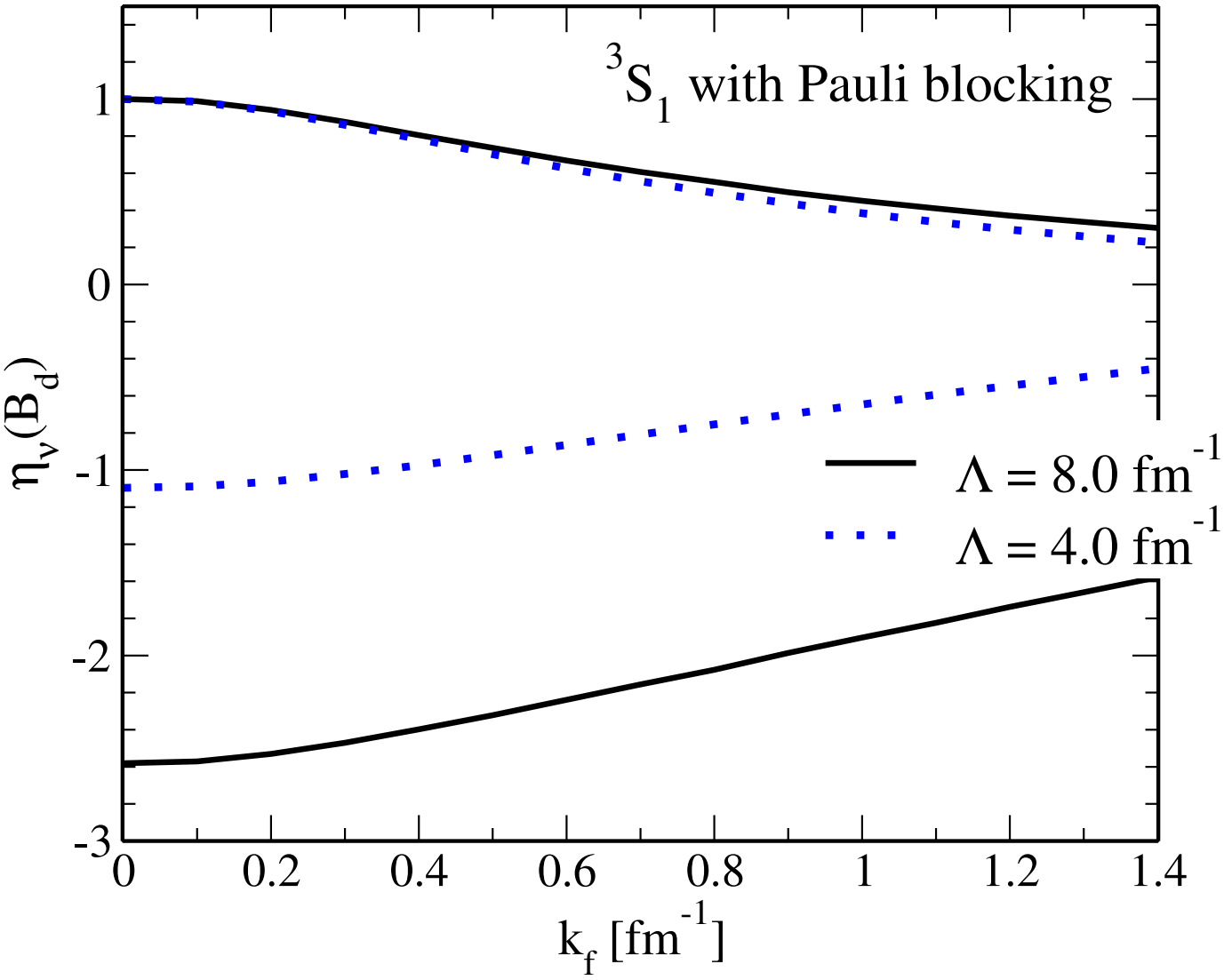

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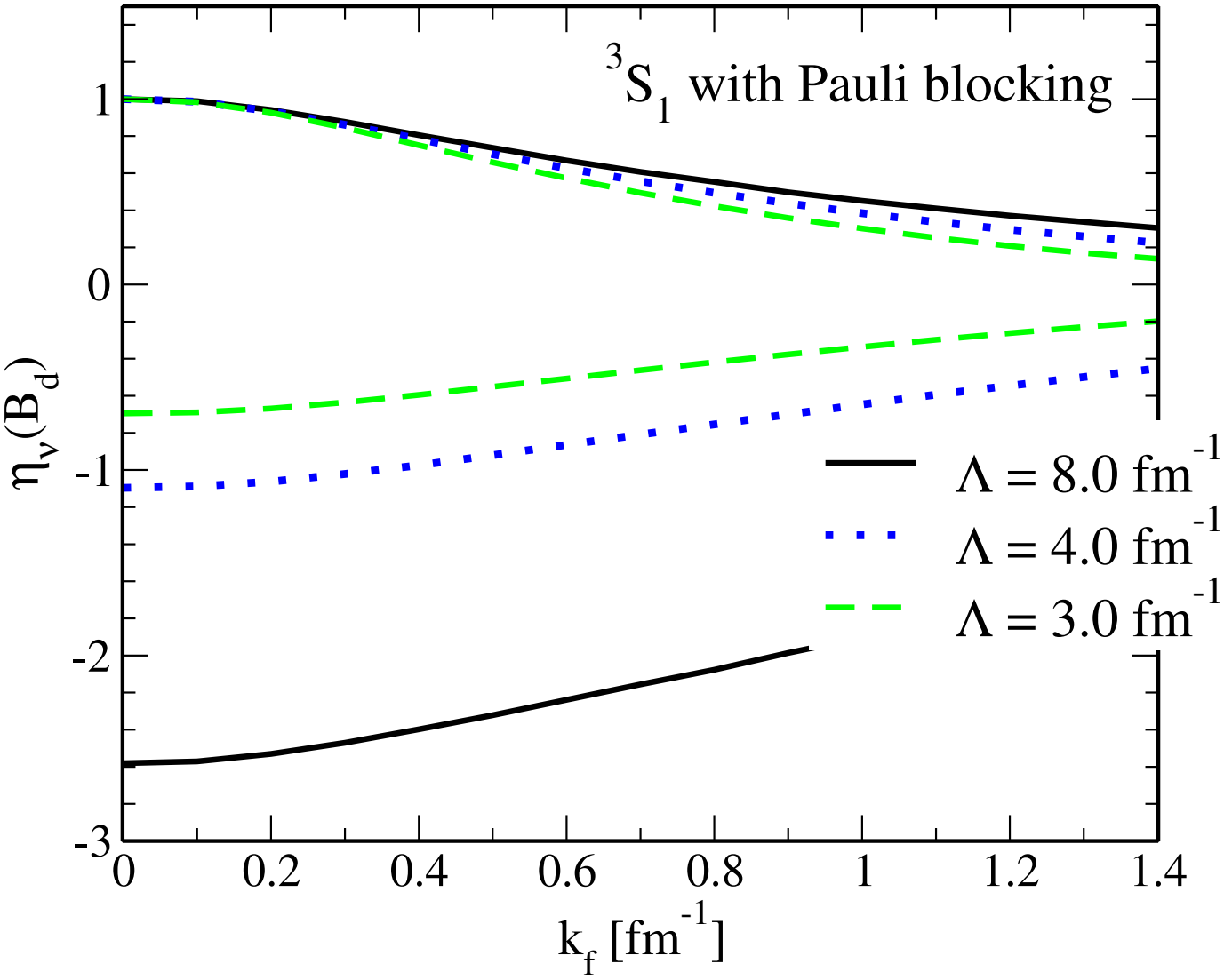

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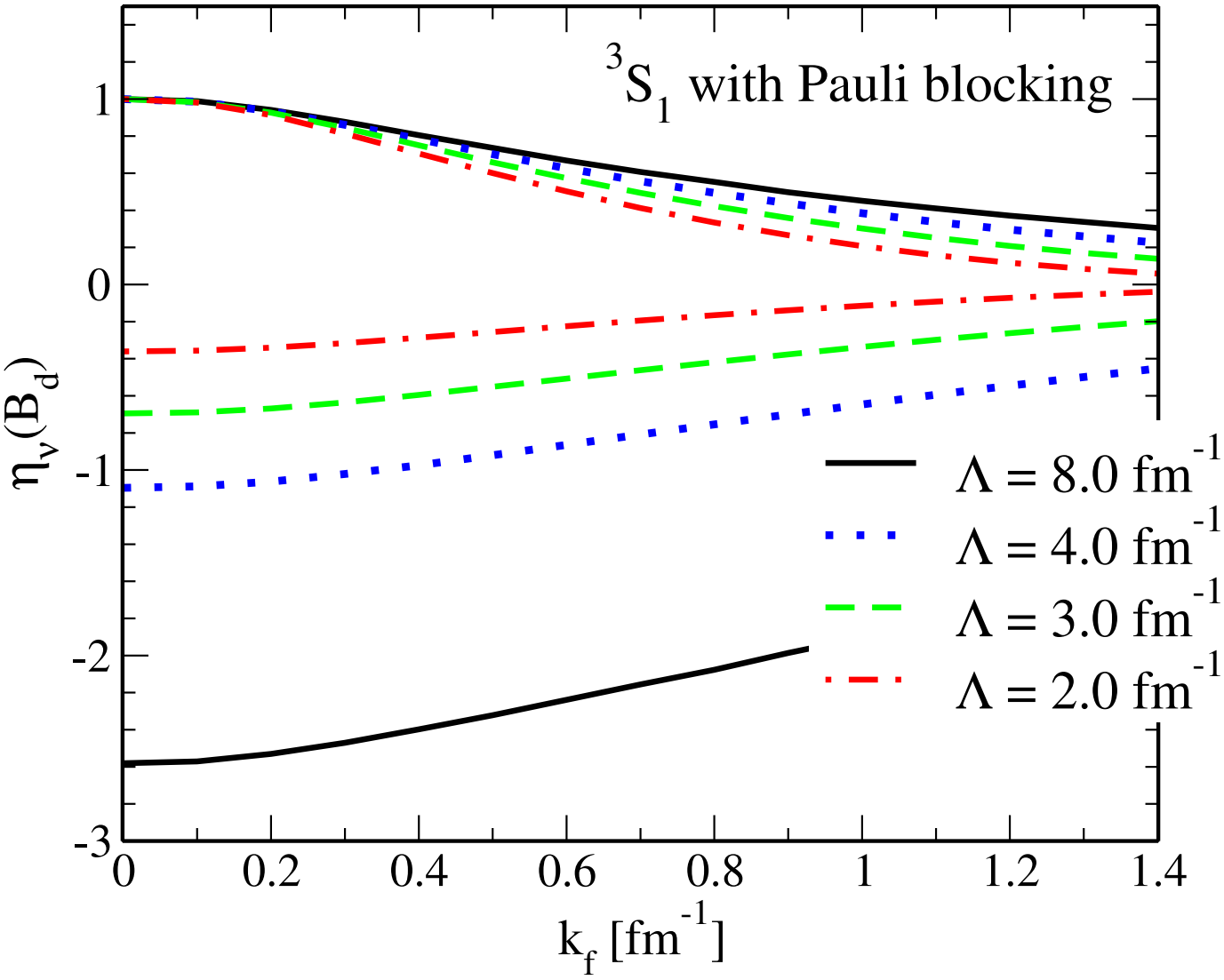
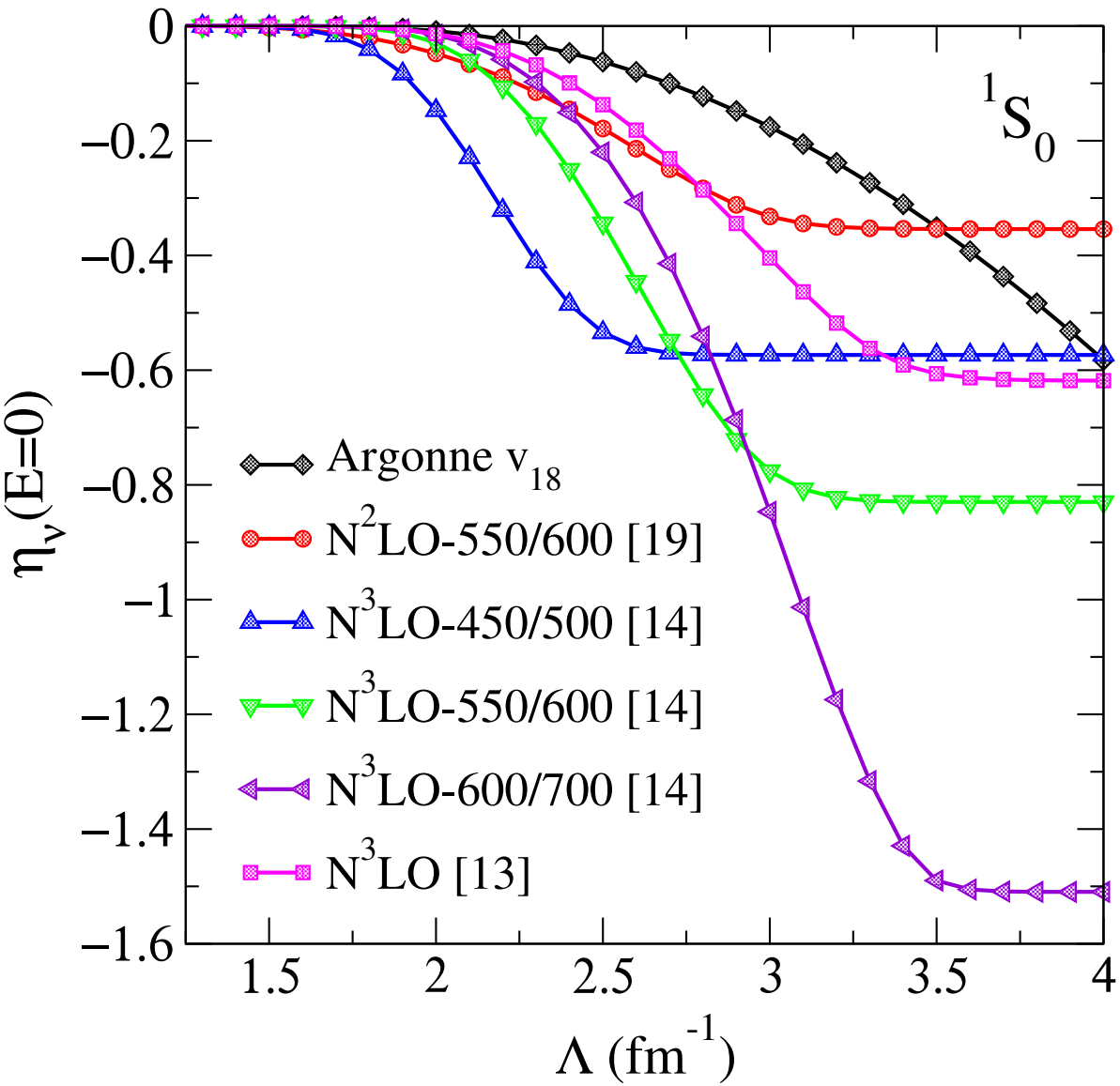


Collapse of Weinberg Eigenvalues



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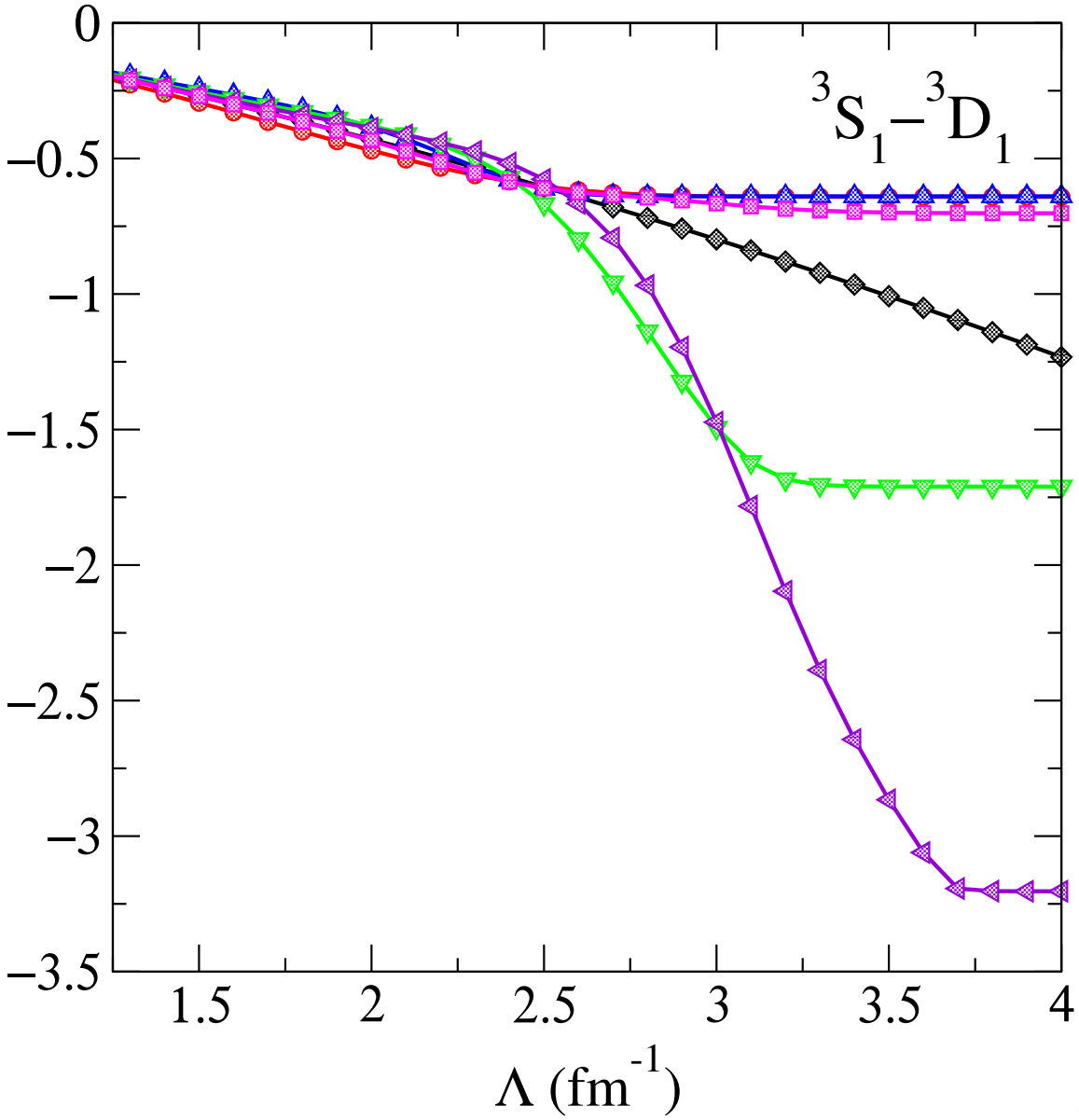
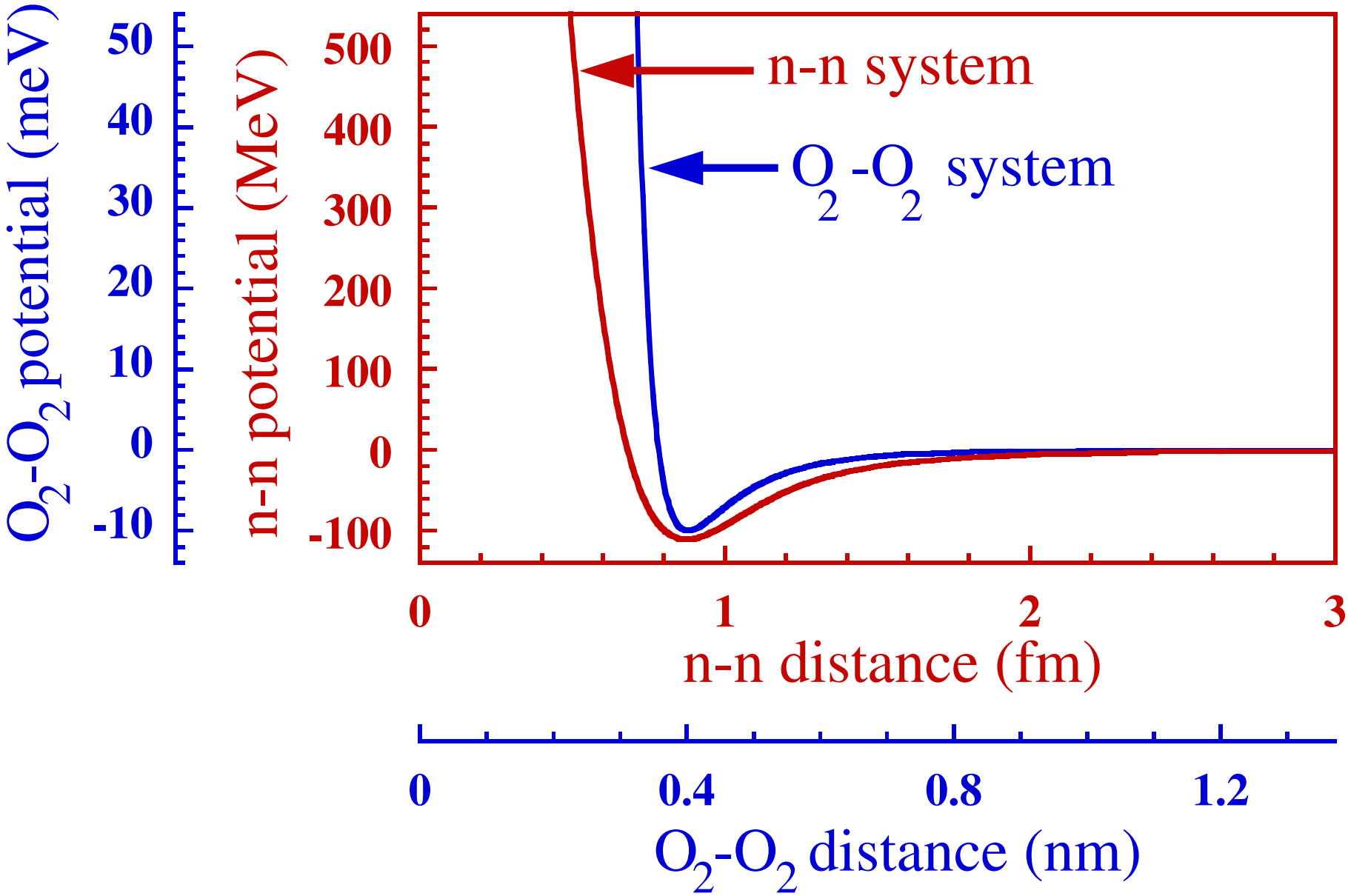
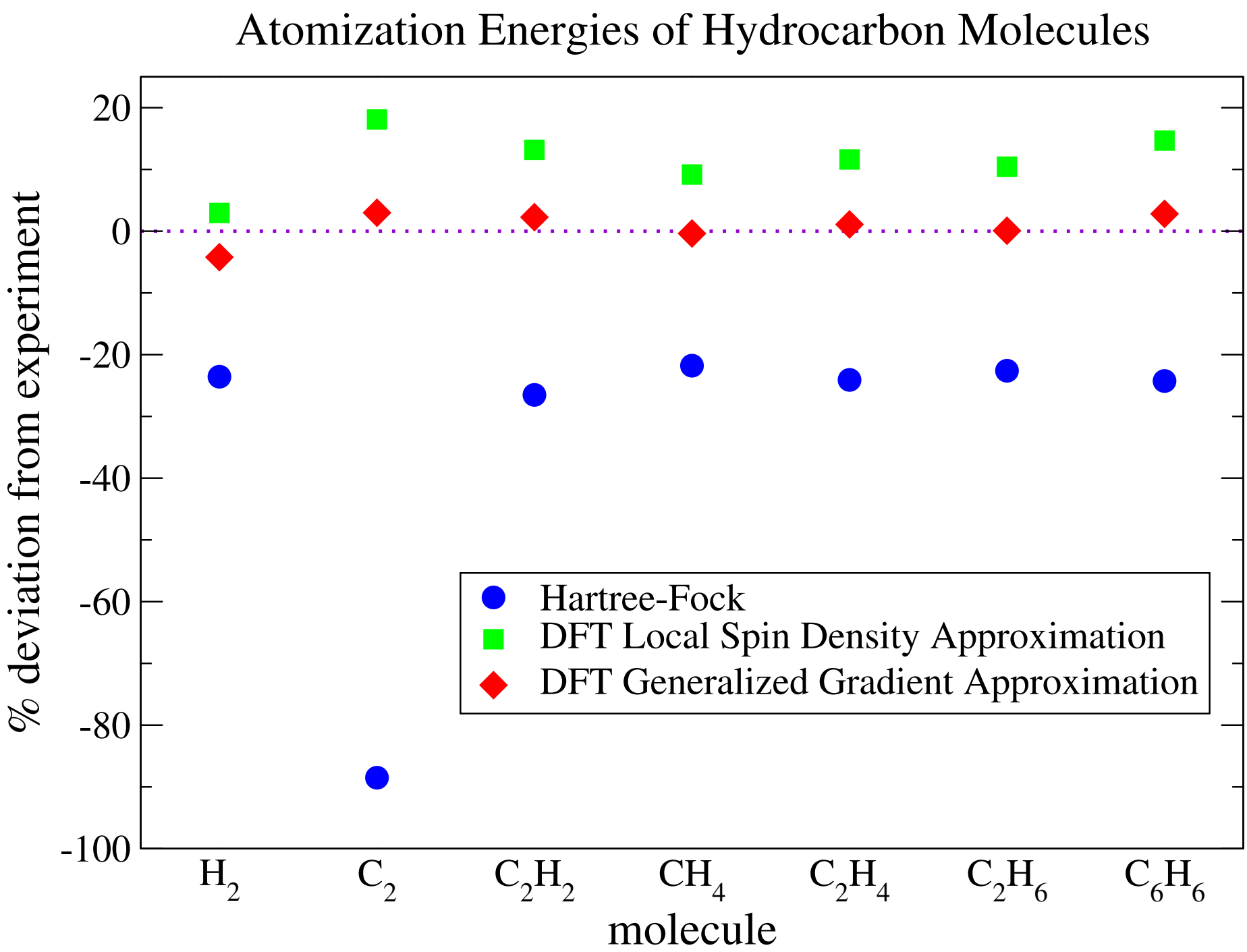


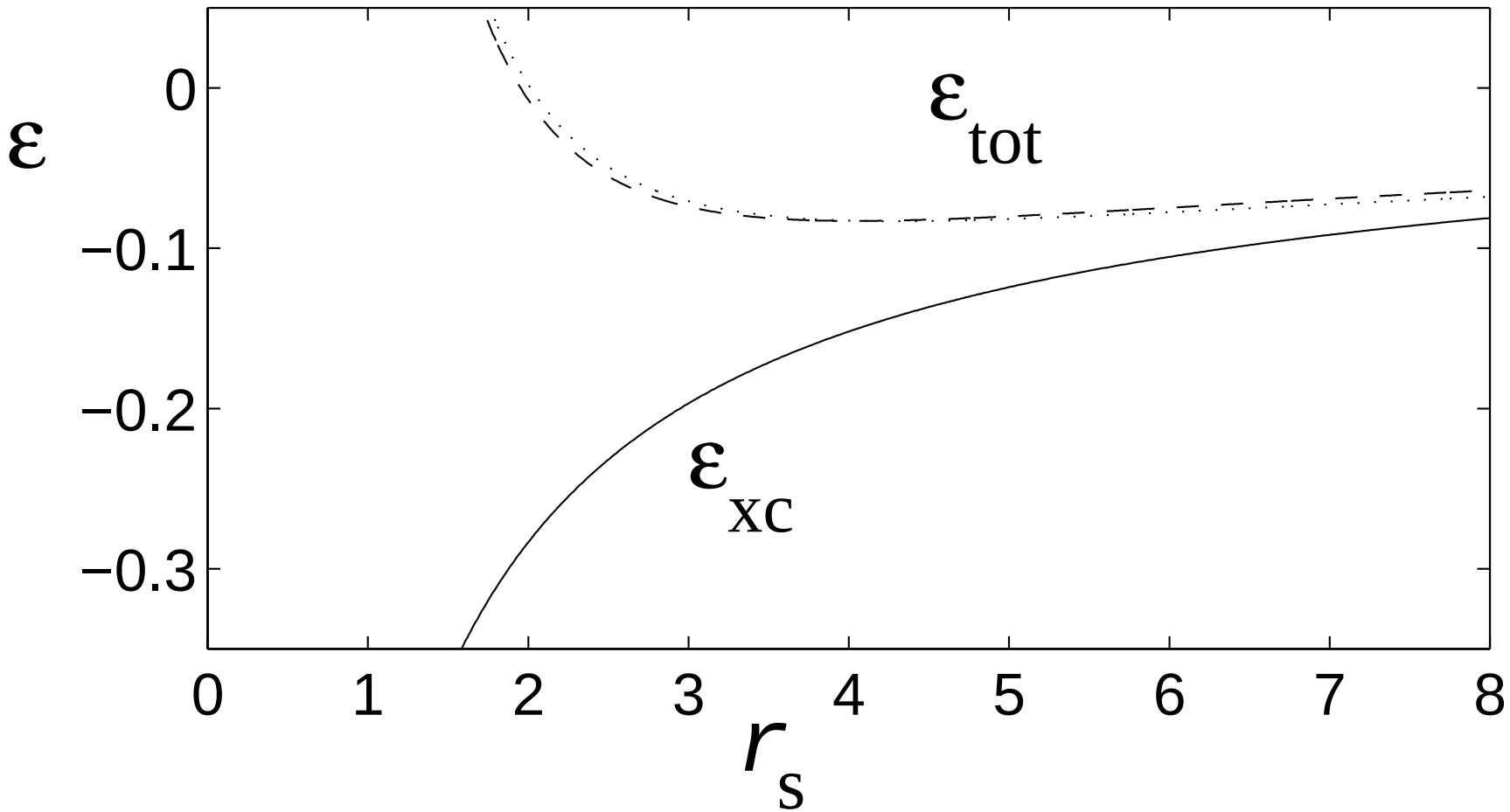


[figure borrowed from J. Dobaczewski]

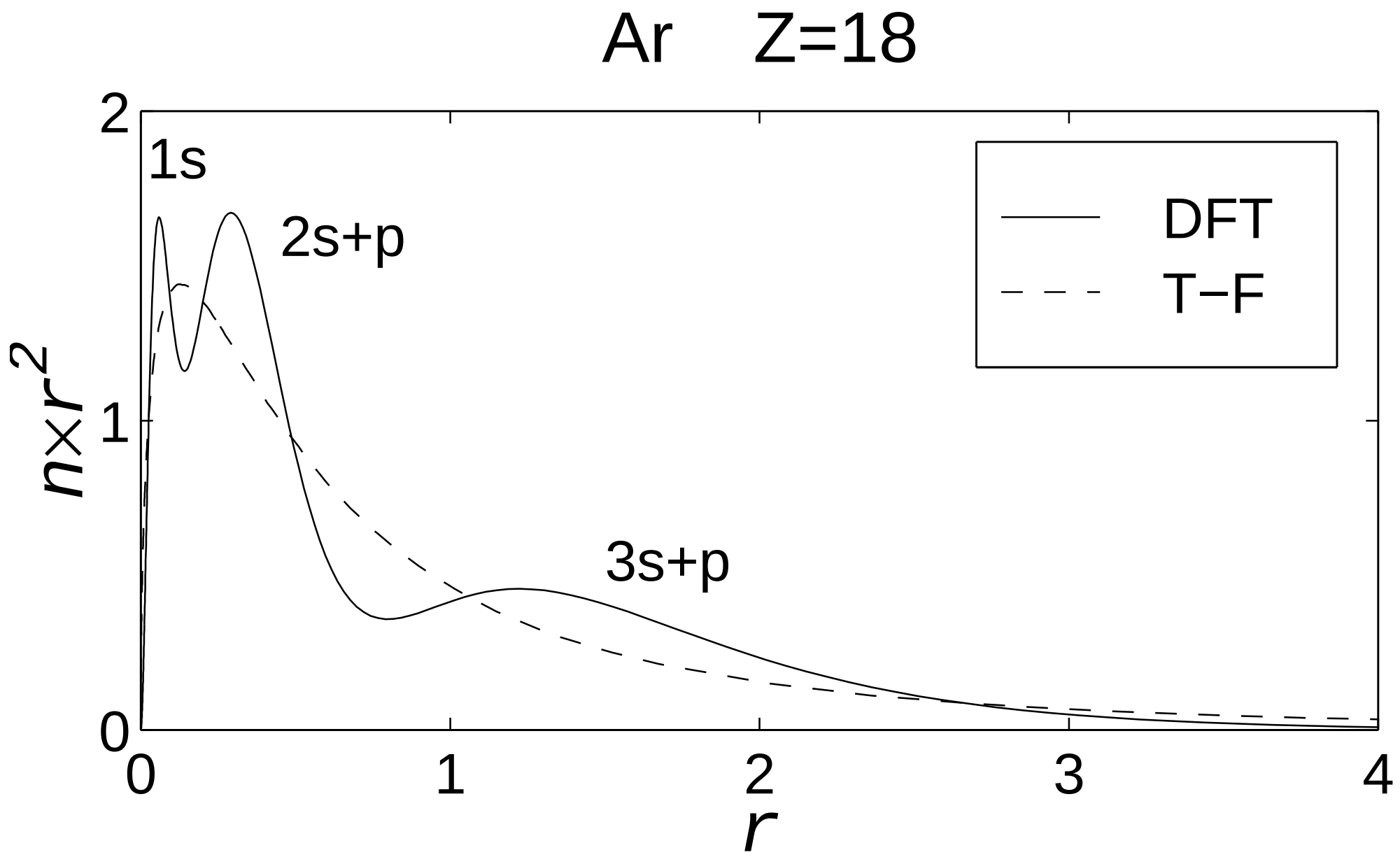


e.g. van Leeuwen–Baerends GGA

- Local density approximation
  - fit $\mathcal{E}_{\mathrm{xc}}(\rho)$ to Monte Carlo cal

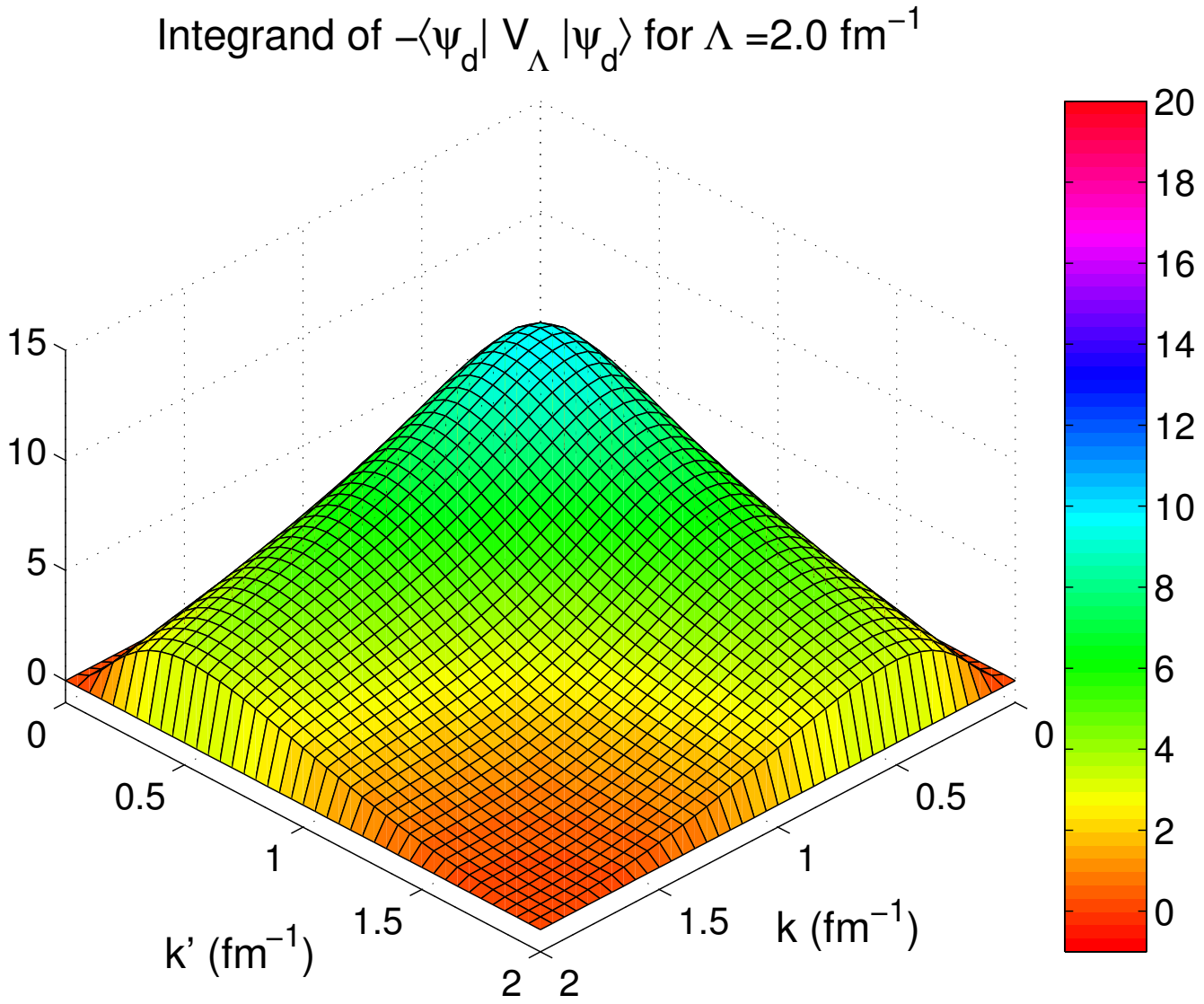


lculation of uniform electron gas



The Deuteron at Different Resolutions

back



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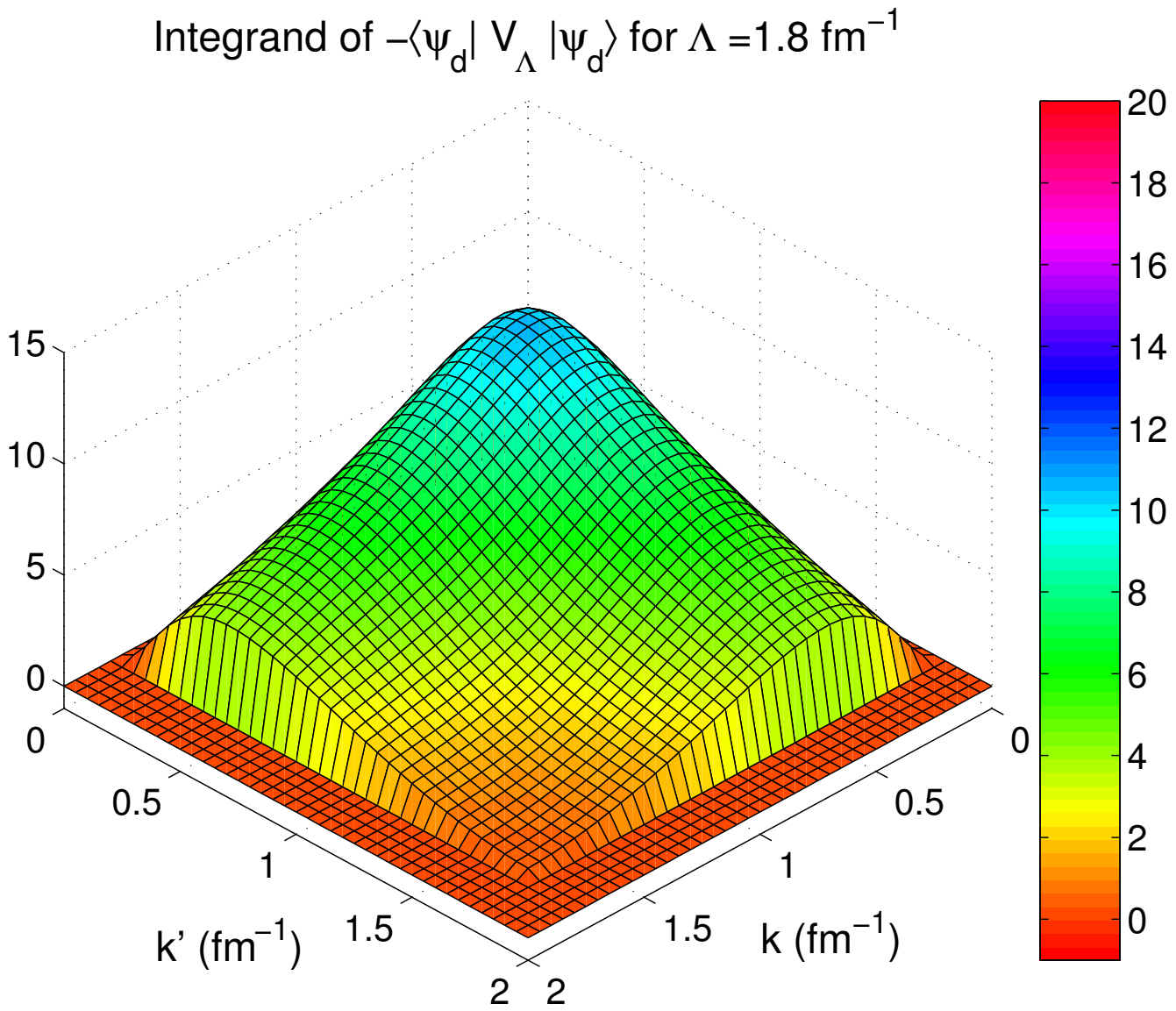

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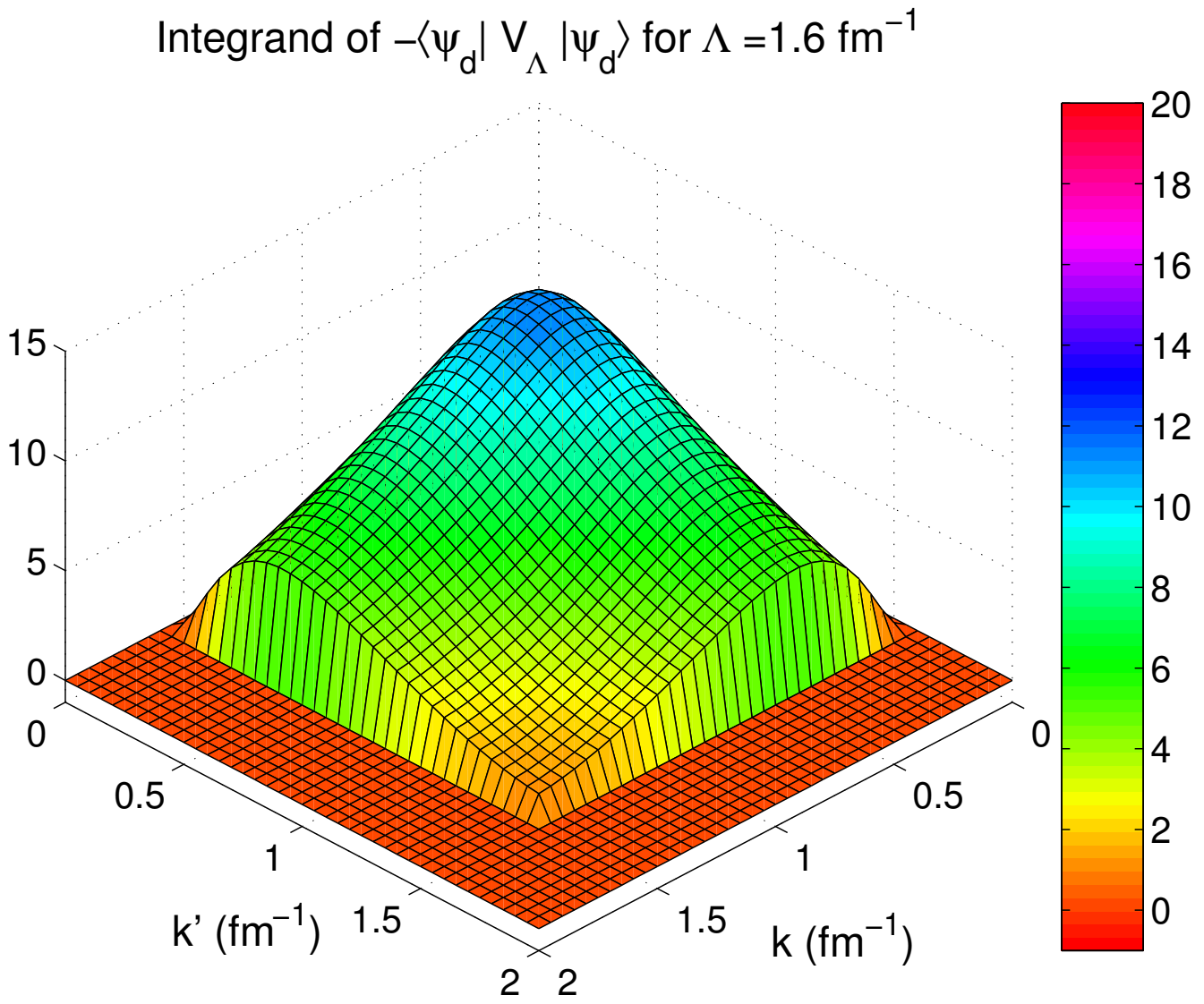

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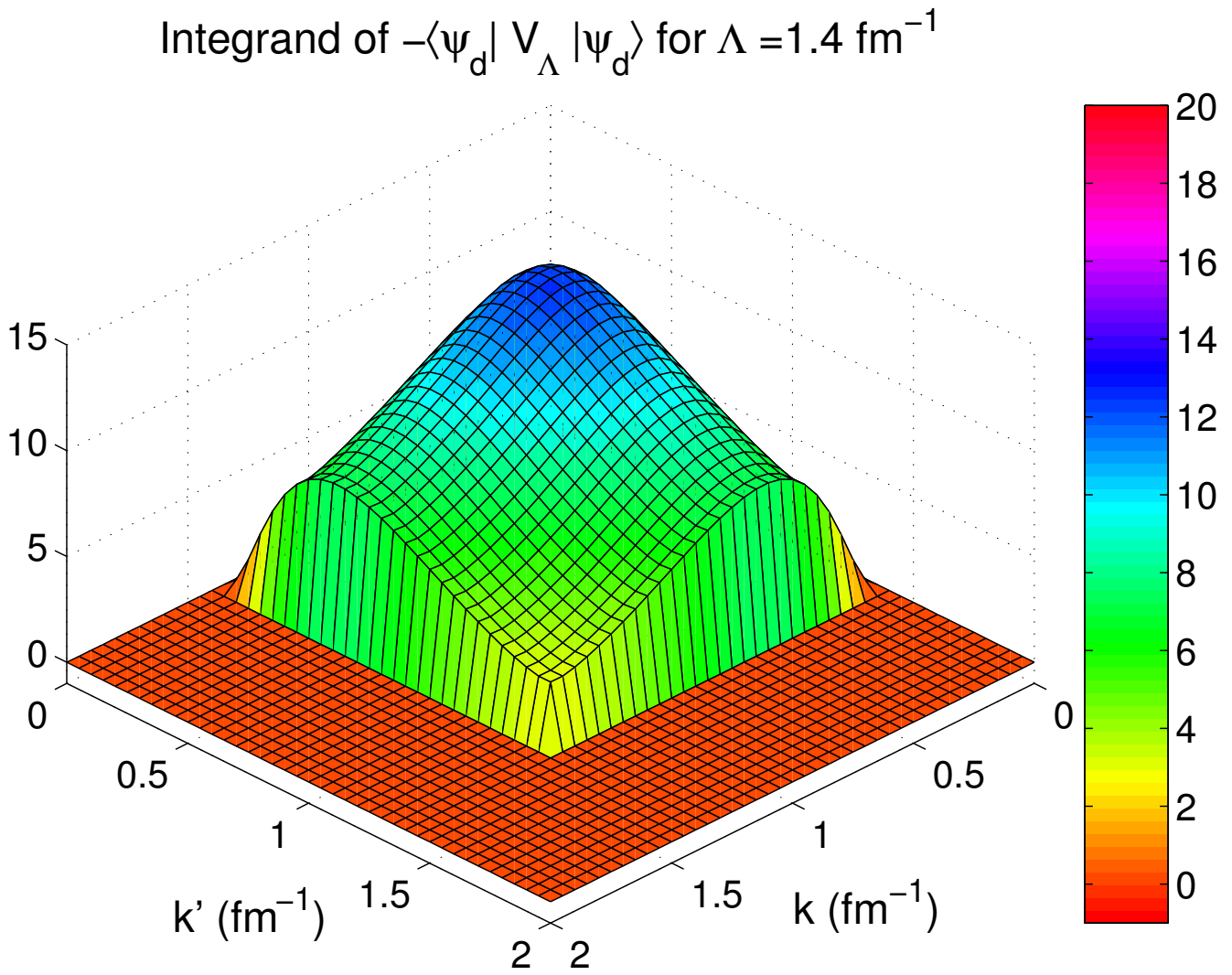

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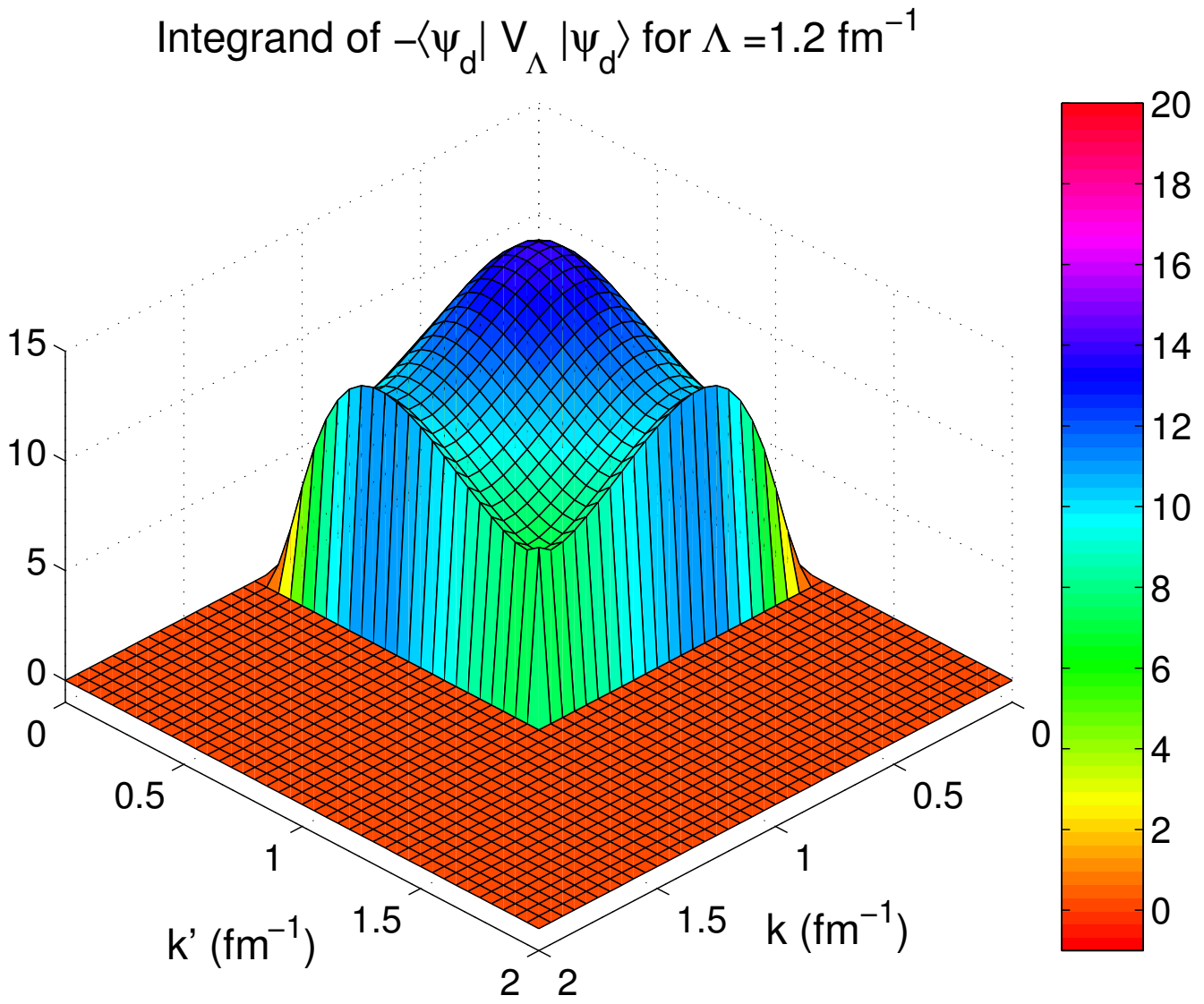

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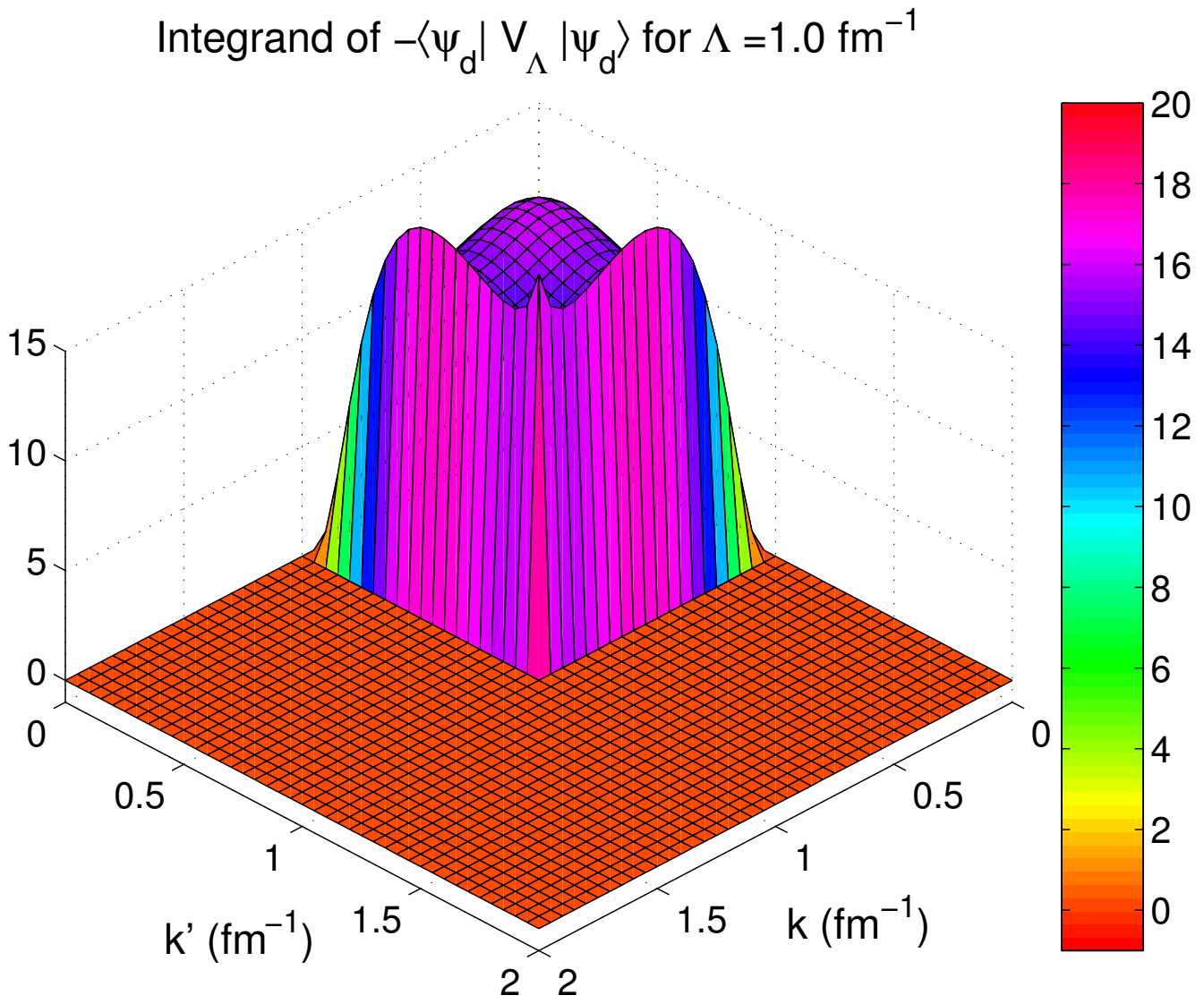
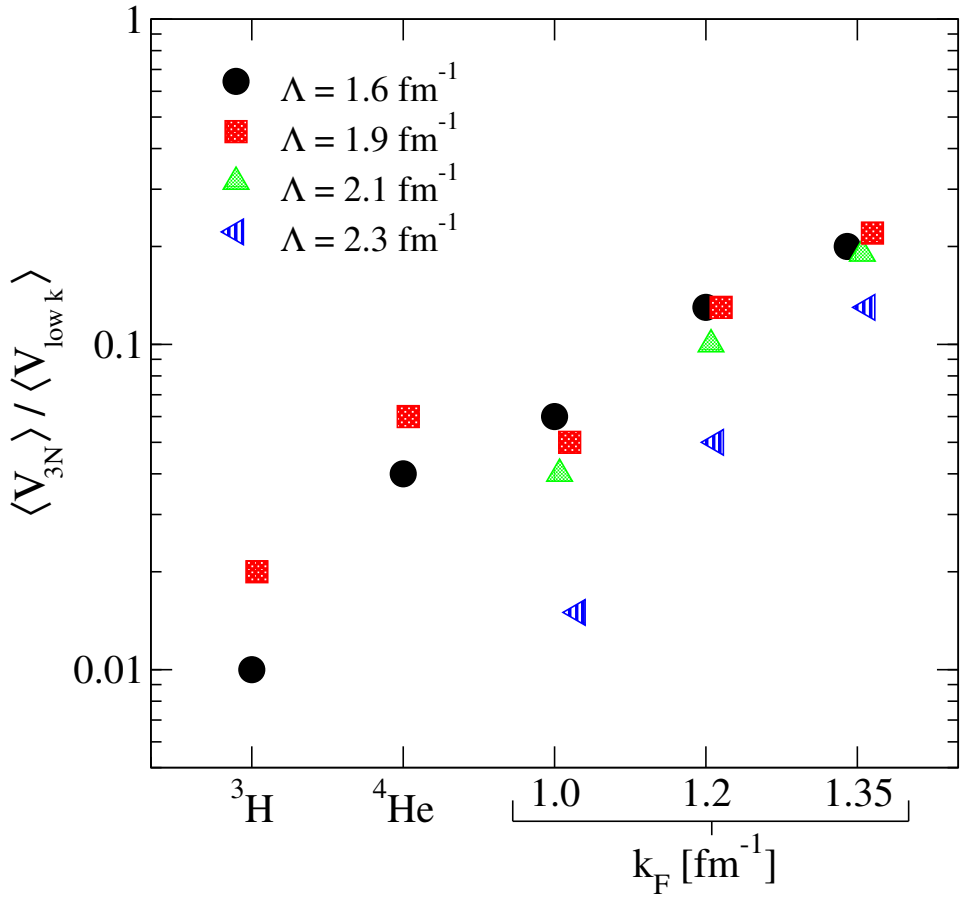


Is Three-Body Contribution Out of Control?

back

- Saturation driven by 3NF
- Unnaturally large?
- Chiral: $\langle V_{3N} \rangle \sim (Q/\Lambda)^3 \langle V_{NN} \rangle$
  $\Longrightarrow$ consistent

Check ratios:



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  $\Longrightarrow$ consistent
- Power counting still needed
  with NN + 3N HF at LO
- Four-body contributions?

Check ratios: