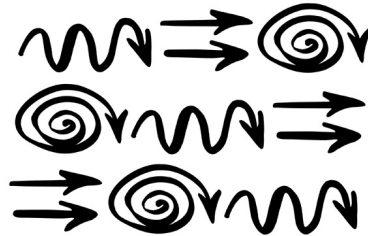arXiv:0907.3074v1 [hep-ph] 17 Jul 2009

LARGE EXTRA DIMENSIONS [*]
Becoming acquainted with an alternative paradigm

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This is a colloquium style pedagogical introduction to the paradigm of large extra dimensions.

1. Road Map (Instead of Introduction)

They say God does not exactly know how parts of his Creation work. When he sees a nice theory which he likes he says: "OK, let it be so..."

Today theoretical high energy physics deals basically with two options: (i) Grand Desert stretching from $\sim 10^2$ GeV to $\sim 10^{16}$ GeV, with no new physics inside; and (ii) Large Extra Dimensions paradigm various versions of which predict new physics at a much lower scale of energies. If the first option is realized, this would mean that high-energy physics in the future

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will face a serious menace of becoming a non-empirical science: experiments at energies in the ballpark of $\sim 10^{16}$ GeV are impossible in terrestrial conditions.

The LED paradigm was born from the desire "to have new physics around the corner," in an attempt to keep high-energy physics as an experiment-based discipline. One may hope that God will like it.

The topic of large extra dimensions (LED) experienced an explosive development since the mid-1990's. Since then thousands of works dedicated to this subject were published. The reason why the LED paradigm attracted so much attention is due to the fact that it brings the scale of new fundamental physics from inaccessible 10^{16} or 10^{19} GeV down to 10 or 100 TeV or so.

A comparison with a huge country the exploration of which is not yet completed is in order here. This lecture presents a bird's eye view of the territory, giving a brief and nontechnical introduction to basic ideas lying behind the large extra dimension paradigm and a particular braneworld model.

The task of describing the large extra dimensions paradigm is "multi-dimensional" in itself. First, there exist three main scenarios which sometimes compete and sometimes complement each other. Second, each scenario starts from a general design of a basic model, while phenomenological consequences come later. Moreover, some scenarios predict new macrophenomena, such as modifications of gravity at distances comparable with the observed Universe size.

Below we will focus on the simplest LED scenario – that of Arkani-Hamed–Dimopoulos–Dvali (ADD) – limiting our forays into alternative scenarios to a minimum. We start from a brief review of fundamental regularities of our world in the context of the paradigm that had existed before the advent of LED. The latter was based on the standard model and its supersymmetric version, supersymmetric grand unification and great desert. We will refer to this paradigm as to the great desert paradigm, or, sometimes, good old paradigm. (We hasten to add, though, that it was not particularly old or particularly good.) Then, after familiarizing ourselves with the history of the topic, we will discuss the very same regularities as they are interpreted from the standpoint of the LED paradigm.
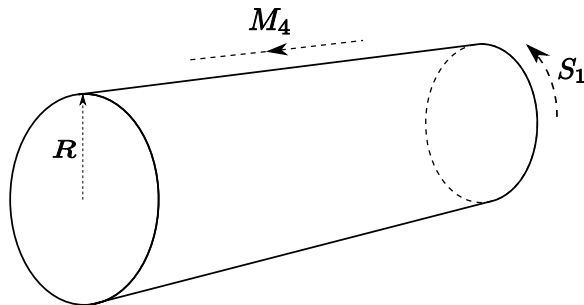
Fig. 1.   The Kaluza–Klein set-up.

2. Genesis/Glimpses of history

2.1. *Kaluza–Klein Theory*

The story starts in the 1920's. At that time time Theodor Kaluza and Oscar Klein, who were working on the unification of Einstein's gravity and electromagnetism, invented the Kaluza–Klein (KK) mechanism.[1,2] Its essence is as follows. Assume that our world, rather than being four-dimensional, is in fact $(4+n)$-dimensional, $n \geq 1$, but the extra dimensions are compact. An illustration is presented in Fig. 1, where $n = 1$, so that our world is a direct product of the four-dimensional Minkowski space M_4 and a circle S_1 with the radius R. All fields are defined on this "cylinder." For a scalar field the single-valuedness on the cylinder can be written as

$$\Phi(x_\mu,\ Z) = \Phi(x_\mu,\ Z + 2\pi R)\,, \qquad \mu = 0, 1, 2, 3\,. \tag{1}$$

Here and below we will use small Latin letters for "our" four coordinates reserving capital Latin letters for extra dimensions. The $2\pi R$ periodicity in Z means that one can present the field $\Phi(x_\mu,\ Z)$ as a Fourier series,

$$\Phi(x_\mu,\ Z) = \sum_{k=0,\pm 1,\dots} \phi_k(x_\mu) e^{ikZ/R}\,. \tag{2}$$

The expansion coefficients depend only on "our" coordinates x_μ, and are often referred to as *modes*. As we will see shortly, the zero mode corresponding to $k = 0$ will play a special role. Modes with $k \neq 0$ are shown in Fig. 2. Each zero mode is accompanied by non-zero modes $k \neq 0$ which are referred to as the KK excitations or the KK tower.

From the four-dimensional point of view, the modes $\phi_k(x_\mu)$ represent a tower of regular four-dimensional fields, the so-called Kaluza–Klein tower.
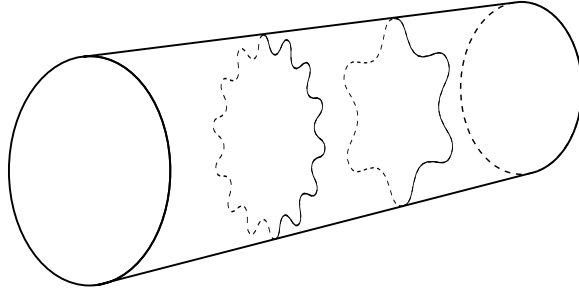
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Fig. 2.   The non-zero modes in the KK decomposition.

Let us start from the five-dimensional wave equation, assuming that the five-dimensional field $\Phi(x_\mu,\ Z)$ is massless,

$$\Box_5\, \Phi(x_\mu,\ Z) \equiv \left(\partial_\mu^2 - \frac{\partial^2}{\partial Z^2}\right)\Phi(x_\mu,\ Z) = 0\,. \tag{3}$$

Substituting the Fourier decomposition (2) we see that each mode ϕ_k satisfies the four-dimensional wave equation

$$\left(\Box_4 + \frac{k^2}{R^2}\right)\phi_k(x_\mu) \equiv \left(\partial_\mu^2 + \frac{k^2}{R^2}\right)\phi_k(x_\mu) = 0\,. \tag{4}$$

The zero mode ϕ_0 remains massless, while all other modes become *massive* four-dimensional fields, with $|k|/R$ playing the role of the mass term,

$$m_k = \frac{|k|}{R}\,. \tag{5}$$

Compactification of $(4+n)$-dimensional fields with spin exhibits another interesting phenomenon and leads to a richer KK tower, which includes a spectrum of four-dimensional spins. Consider for example the metric tensor G_{MN} in five dimensions. The vectorial indices corresponding to higher-dimensional world here and below are denoted by capital Latin letters. From the four-dimensional standpoint $G_{\mu\nu}$ is the four-dimensional metric tensor, $G_{5\mu} = G_{\mu 5}$ is a four-vector, while G_{55} is a scalar. So, the KK tower includes the zero and non-zero modes of spin-2, spin-1 and spin-0 fields.

In the KK picture one assumes that "R is small and $1/R$ is large" compared to some currently available energy scale. Moreover, all our four-dimensional world, in its entirety, including all experimental devices and all potential observers, is made from the zero modes. Then, given the energy limitation, the non-zero mode quanta cannot be produced, and we perceive our world as four-dimensional. Only when accessible energy becomes higher

than $1/R$ can we directly discover KK excitations, a signature of the extra dimension(s).

The infancy of the Kaluza–Klein scenario was eventful. Suffice it to mention that Schrödinger, Gordon and Fock worked on its development in the 1920's, while important contributions in the 1930's were due to Pauli,[3] and Einstein and Bergmann.[4] In particular, in considering compactification of two extra dimensions into sphere S_2 (see Fig. 3), Pauli discovered the Yang–Mills theory long before Yang and Mills. Since he did not know what to do with massless vector fields he never published anything on this discovery. However, gradually the interest to the Kaluza–Klein scenario languished, probably because of the absence of realistic applications in model-building of that time.
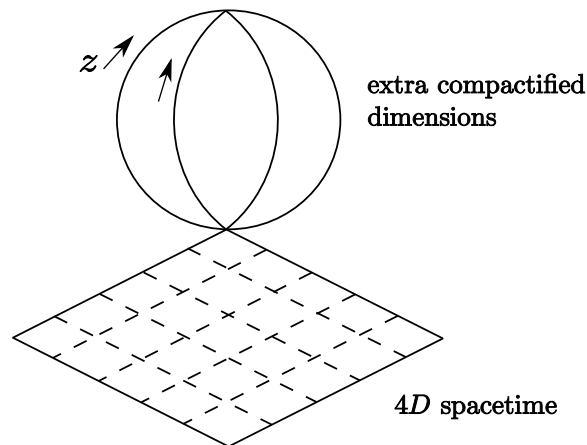

Fig. 3.   Pauli considered compactification of the six-dimension space onto $M_4 \times S_2$.

A long period of a relative hibernation of the Kaluza–Klein theory gave place to a revival in the 1980's. The dawn of a "new era" was marked by Witten's no-go theorem.[5] Witten noted that fermions cannot be chiral if one starts from any eleven-dimensional manifold of the type $T = M_4 \times K$ where K is a compact manifold admitting the symmetry of the standard model (SM), namely, SU(3)×SU(2)×U(1). This was a sad conclusion since it meant that no realistic model could be based on the Kaluza–Klein theory since the fermions in our world are definitely chiral. Fortunately, it was negated, just a few years later, in the first superstring "revolution."

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2.2. *Strings*

Consistent superstring theory exists in ten dimensions. Nonsupersymmetric string is consistent in 26 dimensions. Our world is four-dimensional. In 1975 Joel Scherk and John Schwarz suggested[6] to consider superstrings in a product space of our conventional four-dimensional space-time and a six-dimensional compact manifold whose size is of the order of $M_{\rm Pl}^{-1}$.

The celebrated paper of Candelas *et al.*,[7] which opened the superstring revolution of 1985, demonstrates that if the six compact dimensions form the so-called Calabi–Yao manifold, then in the low-energy limit one recovers a $E(8) \times E(8)$ gauge theory which includes SM, with three generations of chiral fermions that are observed in nature. The Calabi–Yao compactification, conceptually, continues Pauli's line of reasoning. As we have already mentioned, in the 1930's Pauli observed that the KK model on $M_4 \times S_2$ produces three gauge bosons of SU(2) in the low-energy limit. The occurrence of these bosons is due to isometries of the sphere S_2. Of course, the six-dimensional Calabi–Yao manifold is much more contrived. Geometry of the Calabi–Yao manifold is so complicated that the explicit form of the metric is not known even now.

The typical sizes of the compact dimensions in the Calabi–Yao manifold are of the order of 10^{-33} cm. Needless to say, there is no way to observe such extra dimensions in a direct human-designed experiment.

Systematic searches for string-inspired realistic models of the SM type began in the 1990's.[8] Since from the "human" standpoint, extra dimensions in the Calabi–Yao scenario can be viewed as an auxiliary mathematical construction, it was suggested to replace the compactified coordinates by a more formal construction — internal free fermions propagating on the string world sheet. One can then completely abandon the geometrical description of the compactification and formulate it entirely in terms of free fermions on the string world sheet and their boundary conditions. One can then extract the physical spectrum, as well as the assignment of the quantum numbers under the four-dimensional gauge group.

Following this procedure realistic three-generation models were constructed.[8] They differed in their detailed phenomenological properties, but some elements were in common. In particular, an SO(10) grand unification, with an SO(6)³ flavor symmetries and a hidden E_8 gauge group were typical.

In the current millennium this topic — searches for string-inspired realistic models of the SM type — experienced a dramatic development based on D-brane engineering.[9] The advent of D branes[10] allowed one to find

string/D-brane models yielding just the SM massless fermion spectrum, with relative ease. One of the features predicted by the D-brane-based models is that the SM global symmetries — such as baryon and lepton numbers — are gauged symmetries whose anomalies are canceled (by a Green-Schwarz mechanism) only in the case of three quark-lepton generations. Proton stability and the Dirac nature of neutrino masses follow naturally.

This direction *per se* — string-inspired phenomenology — seems promising. What is important for our narrative is that typical sizes of the compact dimensions in the string/D-brane scenario are of the order of 10^{-33} cm; [†] hence, the masses of the excited states in the KK tower are of order of 10^{19} GeV. Such energies are (and will be) inaccessible to any terrestrial experiment. In other words, if physics were to be described by such scenarios, the KK tower would be unobservable, and the KK theory would have no practical implications.

2.3. *Parallel development: localizing on topological defects*

Independent ideas which later formed one of the pillars of the LED paradigm emerged and were developed in the 1980's-90's. The idea of localizing matter on topological defects was formulated, in the most clear-cut form, [‡] by Rubakov and Shaposhnikov[14] in the paper entitled "Do We Live Inside A Domain Wall?".

It is convenient to explain the essence of this suggestion in a simplified setting where "our" world is assumed to be (1+2)-dimensional, while the coordinate z is treated as an "extra dimension." Assume that the underlying microscopic theory has several discrete degenerate vacua which are labeled by distinct values of an order parameter. Call two such vacua — they can be chosen arbitrarily — vacuum I and II. There exists a static field configuration, a domain wall, which divides the three-dimensional space in two parts, say, on the left hand-side our system is in the vacuum I while on the right-hand side in the vacuum II (Fig. 4).

The domain wall represents a transitional domain and is topologically stable. Once created, it cannot be destroyed. The thickness of the domain

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[†]It would be fair to add that some work on introducing larger extra dimensions in the string context had been done in the 1990's. For instance, Hořava and Witten pointed out[11] that a single extra dimension of the size $\sim 10^{-28}$ cm could eliminate the gap between the scale of grand unification and the Planck scale, see Sect. 3.4.

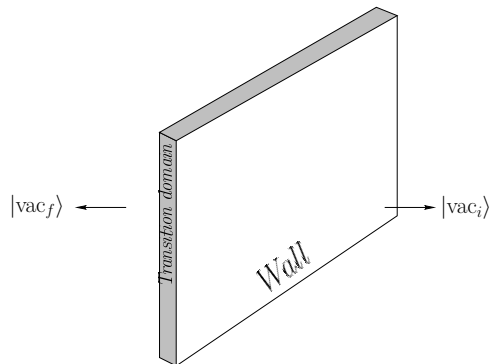[‡]Similar ideas were discussed around the same time in Refs. [12,13].

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$|\text{vac}_f\rangle$ *Transition domain*   *Wall* $|\text{vac}_i\rangle$

Fig. 4.   A domain wall separating two distinct degenerate vacua.

wall δ depends on details of the microscopic theory. At distances $\gg \delta$ the domain wall can be viewed as a two-dimensional surface.

One can excite the domain wall field configuration by pushing the wall at a certain point or by pumping in energy in any other way. All possible excitations naturally fall into two categories. Some of them are localized on the wall (their spatial extension in the perpendicular direction is of the order of δ). These are usually associated with *zero modes*. Being considered from the (1+2)-dimensional point of view, the zero modes represent massless particles which can propagate only along the wall surface.

Other excitations are delocalized and can escape in the bulk (i.e. in the perpendicular direction). They are represented by *nonzero* modes with typical energy eigenvalues of the order of $1/\delta$. From the (1+2)-dimensional standpoint each nonzero mode is a particle with mass $M_n \sim 1/\delta$.

Assume that all matter that we see around is made of the zero modes trapped on the domain wall surface. Then "our world" will be confined to the wall surface and will be effectively (1+2)-dimensional. To discover the third (perpendicular) spatial dimension an observer made of the zero modes will have to have access to energies larger than $1/\delta$.

An obvious distinction between the KK scenario and localization on the domain walls (or other topological defects) is the mass scale of the excited modes. In the KK model it is related to the inverse size of the extra dimension, while in the case of the domain walls the extra dimension is infinite, and the mass scale is set by the inverse thickness of the wall. This distinction turns out to be crucial in physical applications.

The existence of at least one zero mode is easy to demonstrate. In-

deed, the underlying microscopic theory has four-dimensional translational invariance. The domain wall breaks, spontaneously, the invariance with respect to translations in the z direction. Physics becomes dependent on the distance to the wall in the perpendicular direction. Correspondingly, in accordance with the Goldstone theorem, there emerges a Goldstone boson which is confined to the wall surface. If the profile of the order parameter describing the wall (we will call it $\phi(z)$) is known, then the profile of the translational zero mode is given by the derivative $d\phi/dz$, see Fig. 5.
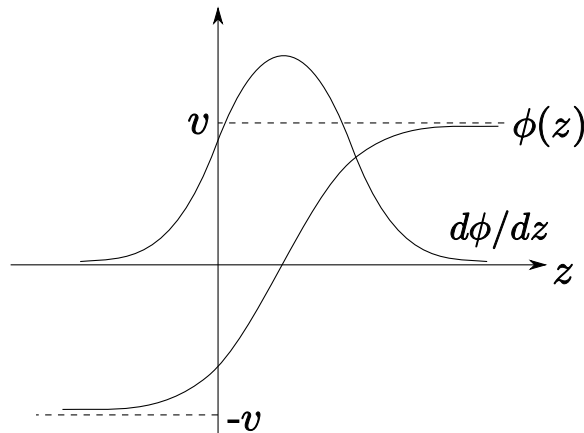

Fig. 5.   The profile function of the domain wall determines localization of the translational zero mode.

The (1+2)-dimensional Goldstone boson appearing in this way has spin zero. In fact, the original work of Rubakov and Shaposhnikov was motivated by the desire to have a Higgs boson whose mass is protected by the Goldstone theorem from being dragged in the ultraviolet by quadratic divergences, typical of the scalar particle masses in field theory. The novelty of the idea and its potential were not recognized till mid-1990's since shortly after the Rubakov–Shaposhnikov publication supersymmetry gained the role of a universal saviour.

In general, localization of spin-zero bosons presents no problem, at least, at a conceptual level. Assume that the microscopic theory has a global symmetry group G which remains unbroken both in the vacuum I and II. Assume that on the given wall solution the symmetry G is (spontaneously) broken down to H. Then the Goldstone bosons corresponding to the broken generators will be confined to the wall; their interactions will be described

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by a coset G/H sigma model.

Localization of the spin-1/2 particles is also a long-known phenomenon. Fermion fields coupled to the wall can have zero modes too. The number of such zero modes is regulated by the Jackiw–Rebbi index theorem.[15] For fermion fields one must consider the mass matrix as a function of z. If in piercing the wall (i.e. in passing from $z = -\infty$ to $z = \infty$) k eigenvalues of the fermion mass matrix change sign, then one will have k fermion zero modes. The thickness of the profile of the fermion zero modes is of the order of the inverse fermion mass in the bulk. With all bulk fermions massive, localization of the fermion zero modes on the wall clearly presents no problems.

Localizing non-Abelian gauge fields on the wall (in the framework of field theory) is a more complicated task. A working mechanism was found in 1996.[16] The basic idea is as follows. Assume that in the bulk, outside the wall, we have a gauge theory (with the gauge group G) in the confining phase. The bulk dynamical scale parameter is Λ_{bulk}. Assume that inside the wall the gauge group is G' where $G' \in G$, the dynamical scale is Λ', and $\Lambda' \ll \Lambda_{\text{bulk}}$. Then the gauge fields of the G' theory will be localized on the wall. Indeed, at energies $\Lambda' \ll E \ll \Lambda_{\text{bulk}}$ the chromoelectric fields of the G' theory cannot escape in the bulk since, due to confinement, the lightest states in the bulk (glueballs) have masses $\sim \Lambda_{\text{bulk}}$. One can dualize this picture. Assume that the bulk G theory is Higgsed. Then the probe magnetic charges are confined in the bulk through formation of chromomagnetic flux tubes. If appropriate condensates vanish inside the wall, then the chromomagnetic flux can spread freely inside the wall.

In string theory localization of the gauge fields is achieved with no special effort, provided one identifies domain walls with D branes. The gauge bosons are then represented by open strings with the end points attached to the branes (Fig. 6). Thus, they are naturally confined to the brane surface. (Let us parenthetically note that the closed strings representing gravitons can freely propagate in the bulk in this picture. We will return later to the discussion of this feature.)

As was mentioned, the idea of localization on topological defects was in a rather dormant state till mid-1990's. In 1996 it was revived in the supersymmetric context.[17]

Why supersymmetry?

The reason for invoking supersymmetry is two-fold. First, in nonsupersymmetric theories the existence of degenerate discrete vacua requires spontaneous breaking of some discrete symmetry. On the other hand, in super-
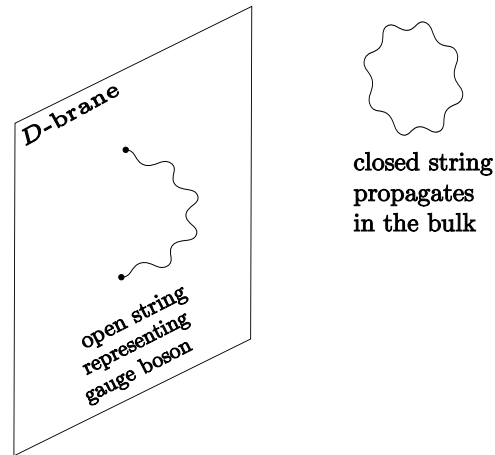
Fig. 6.   The gauge bosons are represented by open strings and are attached to the D-brane, while the gravitons represented by the closed strings "live" in the bulk.

symmetric theories the vacuum degeneracy is hard to avoid, it is typical. Indeed, all supersymmetric vacua must have the vanishing energy density and are thus degenerate. Therefore, domain walls are more abundant.

The second motivation is the ease with which one can localize simultaneously spin-0 and spin-1/2 fields on critical (or BPS saturated) domain walls.

2.4. *What to do with gravity?*

One cannot hope to successfully describe our world without including gravity. Unlike spin-0,1/2 and 1 fields, no field-theoretic mechanisms ensuring *bona fide* localization of gravity on domain walls are known. Moreover, if one approaches the problem from the string theory rather than field theory side, a drastic distinction between, say, gauge fields and gravity is obvious too. The gauge fields are represented by open strings with the endpoints attached to D branes. Thus, they are naturally localized on D branes. At the same time, gravity is represented by closed strings which can freely propagate in the bulk, see Fig. 6. In order for the LED paradigm to take off, a fresh idea regarding what to do with gravity was needed. It was not before long that it was put forward.

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3. Flat compact extra dimensions

Historically the first was the compact extra dimension model which goes under the name ADD, where ADD stands for Arkani-Hamed, Dimopoulos and Dvali, its inventors.[18] In the March 1998 paper, and a follow-up publication of the same authors with Antoniadis,[19] a marriage between the Kaluza–Klein scenario and localization on domain walls was suggested. Compactifying extra dimensions a lá Kaluza–Klein solves the problem of gravity, while localizing all other fields on the wall solves the hierarchy problem.

But let us begin from the very beginning.

3.1. *The Arkani-Hamed–Dimopoulos–Dvali (ADD) scenario*

The model starts from the assumption that the space-time is $(4 + n)$-dimensional, $n \geq 1$, while its geometry is factorized,

$$M_{\mathrm{world}} = M_4 \times K_n \,. \tag{6}$$

All SM particles are localized on a $(1 + 3)$-dimensional domain wall (3-brane) representing M_4 in the above expression (Fig. 7). At the same time, gravity spreads to all $4 + n$ dimensions (Fig. 8).
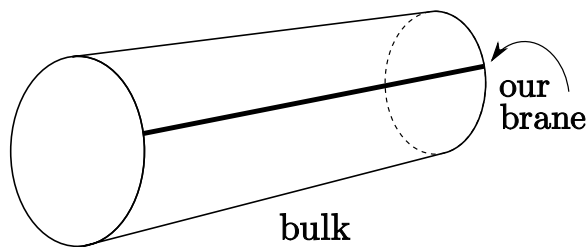

Fig. 7.   "Our world" is the 1+3 dimensional domain wall which is shown by a solid line in this figure. Perpendicular directions are compact.

If the thickness of the domain wall is chosen to be $\delta \lesssim (10 \text{ TeV})^{-1}$ then at energies $\lesssim 10$ TeV physics is effectively four-dimensional in all experiments except those with gravity.[§] Since gravity escapes in the bulk it be-

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[§]One may ask where does this mass scale, 10 TeV, come from? We will answer this question shortly.
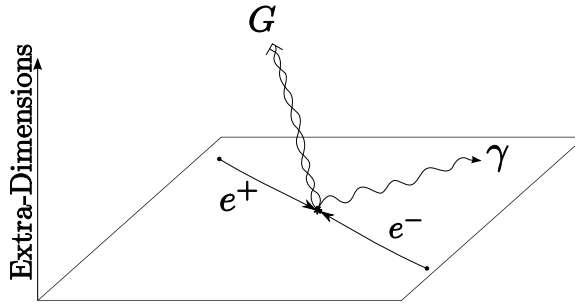
Fig. 8.   We, and everything around us, are made from the zero modes localized on the domain wall. Gravity escapes in the bulk.

comes four-dimensional only at distances $r \gg R$. At distances $r \lesssim R$ gravity is $4 + n$ dimensional.

We want $R \gg \delta$. The reason behind this requirement will become clear momentarily. Let us ask ourselves what values of the size of the compact dimensions R are compatible with what we know about our world today. Constraints on R are surprisingly lax. They follow from the fact that in the ADD scenario gravity becomes effectively four-dimensional at distances $\gtrsim R$. Experimentally, gravity is well-studied at large distances (where it is certainly four-dimensional) and known to a much lesser extent at short distances. In fact, below 0.1 mm or so the gravitational force has not been measured, and one cannot rule out that at such distances it is $(4 + n)$-dimensional.¶

Let us assume that the size of extra dimensions $R \sim 0.1$ mm and see to what consequences this assumption leads. If in the future gravity will be proved to be four-dimensional at such distances one can always downsize extra dimensions making $R \sim 0.01$ mm or less.

3.2. *Fundamental scale and the size of extra dimensions*

Somewhat symbolically, the action can be written as

$$S = \frac{M_{\mathrm{f}}^{2+n}}{2} \int d^4x \int_0^{2\pi R} d^n Z \sqrt{G}\, R_{4+n} + \int d^4x \sqrt{g}\,(T + \mathcal{L}_{\mathrm{SM}}(\Phi_{\mathrm{SM}})) \quad (7)$$

¶In 1998 measurements of the gravity extended down to 1 mm. Dedicated experiments[20] performed after the ADD suggestion gave rise to a considerable theoretical activity improved the above bound by an order of magnitude.

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where $M_{\rm f}$ is the *bona fide* fundamental constant of gravity, G and R_{4+n} are the $(4 + n)$-dimensional metric and scalar curvature, respectively, g is four-dimensional metric, and, finally, $\mathcal{L}_{\rm SM}$ is a Lagrangian describing all SM fields (which, remember, are trapped on the brane). A typical mass scale associated with $\mathcal{L}_{\rm SM}$ will be denoted as $M_{\rm SM}$,

$$M_{\rm SM} \sim 100 \text{ GeV}.$$

The constant T has to be adjusted in such a way that the overall cosmological term, which includes T plus all quantum loops, vanishes. This is the usual fine-tuning condition on the cosmological constant. The ADD scenario adds nothing new in this respect.

Now let us apply the Kaluza–Klein mode expansion to the graviton field and keep, for the time being, only the zero mode, neglecting all states in the KK tower with masses of the order of $1/R$. Since the zero mode is Z independent, we can perform the Z integration in the first term on the right-hand side thus obtaining

$$\frac{M_{\rm f}^{2+n}}{2} \int d^4x \int_0^{2\pi R} d^n Z \sqrt{G}\, R_{4+n} \longrightarrow \frac{1}{2}\, M_{\rm f}^{2+n}\, V_n \int d^4x\, \sqrt{g}\, R, \quad (8)$$

where g and R are four-dimensional metric and scalar curvature evaluated on the zero mode, and V_n is the volume of extra dimensions,

$$V_n = (2\pi R)^n . \quad (9)$$

The expression on the right-hand side of Eq. (8) is applicable at distances $r \gg R$.

At such distances the gravitational potential takes the standard Newton form

$$V(r) = -\frac{G_N m_1 m_2}{r}, \quad (10)$$

with the Newton constant G_N determined by Eq. (8).

$$G_N = (M_{\rm f}^{2+n}\, V_n)^{-1} . \quad (11)$$

At the same time, if $r \ll R$ we must return to Eq. (7) which implies the following static potential

$$V(r) = -\frac{m_1 m_2}{M_{\rm f}^{2+n}\, r^{1+n}} . \quad (12)$$

It is quite obvious that, upon inspecting Eq. (11), a four-dimensional observer will interpret $M_{\rm f}^{2+n}\, V_n$ as the Planck scale,

$$M_{\rm f}^{2+n}\, V_n = M_{\rm Pl}^2 \sim \left(10^{19} \text{ GeV}\right)^2 . \quad (13)$$

He or she will think that this is the fundamental scale at which gravity becomes of order unity and needs quantization. The "visible" fundamental scale, as established by such an observer, is separated from accessible energies $M_{\rm SM}$ by a huge interval, thus creating an enormous hierarchy of scales.

In fact, in the ADD scenario the genuine fundamental scale is $M_{\rm f}$; this is the energy at which $4 + n$ dimensional gravity becomes strong. $M_{\rm f}$ is related to the "visible" fundamental scale $M_{\rm Pl}$ as follows:

$$M_{\rm f} = \frac{1}{2\pi R} \left(2\pi R\, M_{\rm Pl}\right)^{\frac{2}{n+2}} \tag{14}$$

If $n = 1$ and $M_{\rm f} \sim 10$ TeV, then $R \lesssim 10^{12}$ cm which is definitely inconsistent with the Newton law well-established at such distances. Thus, a single extra dimension is ruled out in the ADD scenario. Then it is natural to assume that $n = 2$. If so, and $M_{\rm f} \sim 10$ TeV, we can use the formula

$$R = \frac{1}{2\pi\, M_{\rm f}} \left(\frac{M_{\rm Pl}}{M_{\rm f}}\right)^{2/n} \tag{15}$$

to deduce that $R \sim 0.1$ mm.

Thus, two or more extra dimensions in the ADD framework are not inconsistent with the existing data. What is important is that even at larger n the size of the extra dimensions R is large compared with δ^{-1} and the more so with $M_{\rm Pl}^{-1}$. There are no theoretical motives behind the choice of $M_{\rm f} \sim 10$ TeV. The only reason is the desire to have new physics in the accessible range of energies. If a typical scale of new physics is higher than 10 TeV, one can adjust $M_{\rm f}$ appropriately.

Several simplifying assumptions were made in the course of the above consideration, namely,

(i) The wall thickness δ (which is assumed to be $\delta \sim M_{\rm f}^{-1}$) is neglected;

(ii) The wall shape fluctuations are neglected (these are the Nambu–Goldstone bosons which couple to matter derivatively);

(iii) All extra dimensions are assumed to have equal size R;

(iv) It is postulated that only gravity propagates in the bulk.

One or more of the above assumptions can be easily lifted. For example, one can consider several extra dimensions with individual sizes, or let escape in the bulk other fields in addition to gravity (but only those which carry no charges with respect to SM). This will change technical details, leaving conceptual foundations of the approach intact.

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3.3. *Phenomenological implications*

Some basic phenomenological regularities that we observe in Nature are:

(i) Proton stability;
(ii) Mass hierarchies;
(iii) The existence of three and only three generations;
(iv) The lightness of neutrinos;
(v) The lightness of (yet to be discovered) Higgs particles;
(vi) A peculiar pattern of quark mixing angles (the CKM matrix elements); a peculiar pattern of the neutrino mixing angles.

In the years that elapsed after the creation of the standard model and before the advent of the LED paradigm these regularities received more or less satisfactory explanations. Some of them were understood at a conceptual level, while for others detailed technical explanations were worked out. At the very least, we believed that the above regularities presented no mysteries that could shake foundations of physics. For instance, the proton stability was explained by a very high mass scale of unification/strong gravity, of the order of $M_{\rm Pl}$. The lightness of the left-handed neutrino was thought to be due to a *seesaw* mechanism (see below), and so on.

With the advent of the LED paradigm a drastic rethinking of particle physics, and in particular, flavordynamics, was inevitable. Everything had to be questioned anew, and novel explanations had to be invented. Needless to say, they were suggested before long.

Below we will outline some mechanisms discussed in this context. The aspect which we will emphasize here is a natural conversion of dynamical regularities into geometric ones within the LED framework.

3.4. *Hierarchy of scales*

At terrestrial energies gravity is exceedingly weak. The gravitational interaction is characterized by the Newton constant $G_N \equiv M_{\rm Pl}^{-2}$. While a typical electroweak scale is $M_{\rm SM} \sim 100$ GeV, the Planck scale $M_{\rm Pl} \sim 10^{19}$ GeV, so that there is a huge hierarchy of scales, $M_{\rm Pl}/M_{\rm SM} \sim 10^{17}$.

In the standard model *per se*, the electroweak scale is not stable, since quantum loops drag the Higgs boson mass (which is supposed to be of the order of $M_{\rm SM}$) to the Planck scale. In the desert paradigm supersymmetry plays a protective stabilizing role — superpartners cancel quadratic divergences in the Higgs boson mass above the supersymmetry breaking scale $M_{\rm SUSY}$. Although supersymmetry is not yet discovered experimentally, the general belief is that $M_{\rm SUSY} \sim$ few hundred GeV, i.e. quite close to $M_{\rm SM}$.
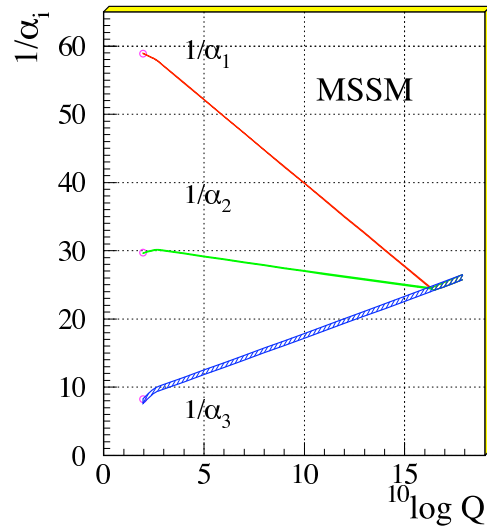
Fig. 9.   Evolution of the inverse of the three gauge coupling constants in MSSM.

A minimally supersymmetrized version of SM (minimal supersymmetric standard model, or MSSM for short) has three gauge groups, U(1), SU(2), and SU(3), and, correspondingly, three gauge coupling constants, α_1, α_2, and α_3. These coupling constants run; the running formula being logarithmic in energy. This slow logarithmic running introduces another huge scale — the scale of grand unification. In MSSM the scale of grand unification $M_{\text{GUT}} \sim 10^{16}$ GeV. It turns out that all three gauge coupling constants unify to within a few percent at M_{GUT}, see Fig. 9. The grand unification scale is, in turn, quite close to the Planck scale M_{Pl}, where gravity becomes strong,

$$M_{\text{GUT}} \sim 10^{-3} M_{\text{Pl}}. \tag{16}$$

There is a vast desert between M_{SM} and M_{SUSY} on the one hand, and M_{GUT} and M_{Pl} on the other hand, stretching in energy over 16 to 17 orders of magnitudes.

In the LED scenario there is only one fundamental scale, $M_{\text{f}} \sim 10$ TeV. That's the scale where gravity must become strong, and all interactions unify.$^{\parallel}$ Where has the hierarchy gone?

$^{\parallel}$Beyond M_{f} physics is $4 + n$ dimensional, and the law of running of the gauge couplings is non-logarithmic.[21]

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The enormous hierarchy did not disappear. The energy hierarchy is converted into a geometric hierarchy of the "transverse" sizes: the radius of the extra dimension versus the brane thickness. Why the compact dimensions are so large in the LED paradigm? It is impossible to answer this question without complete understanding of the compactification dynamics, which is well beyond the scope of this lecture.

3.5. *Proton stability*

Why this issue is potentially dangerous for the very existence of the ADD scenario?

To properly set a reference point, let us start from the desert paradigm, where two mechanisms were considered in connection with the problem of the proton decay. First, in the grand unification theories (GUT's), there exist gauge bosons with mass $\sim M_{\text{GUT}}$ and leptoquark quantum numbers (in the Russian literature they are sometimes called "elephants"). These elephant bosons mediate quark annihilation into leptons, see Fig. 10, leading to the proton decay ($B - L$ is still conserved). Experimentally the proton lifetime is known to exceed 10^{32} years. The appropriate suppression of the "elephant" contribution is due to the fact that their masses are very large, $\gtrsim M_{\text{GUT}}$.

The proton decay rate associated with this mechanism (for a review see e.g. [22,23].) is easy to estimate, [**]

$$\Gamma_{\text{proton}} \sim \alpha^2 \, m_{\text{proton}} \left(\frac{m_{\text{proton}}}{M_{\text{GUT}}} \right)^4 , \qquad (17)$$

where α is the common value of the three gauge couplings at the unification scale. Given Eq. (16), the proton lifetime is predicted to be longer than 10^{33} years, which is compatible with experiment.

The second mechanism discussed in connection with the proton decay is possible non-conservation of global quantum numbers, such as the baryon number B or lepton number L, in the presence of quantum gravity. The argumentation goes as follows:[24] global charges can be swallowed by black holes — say, virtual black holes which are formed non-perturbatively at the distance scales where gravity becomes strong — which then eventually evaporate (Fig. 11). Or a wormhole may suck in a global charge from our

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[**]The estimate given below refers to the so-called dimension-six operators, see Fig. 10; dimension-five operators result in a different (more contrived) expression, which is close numerically, however.
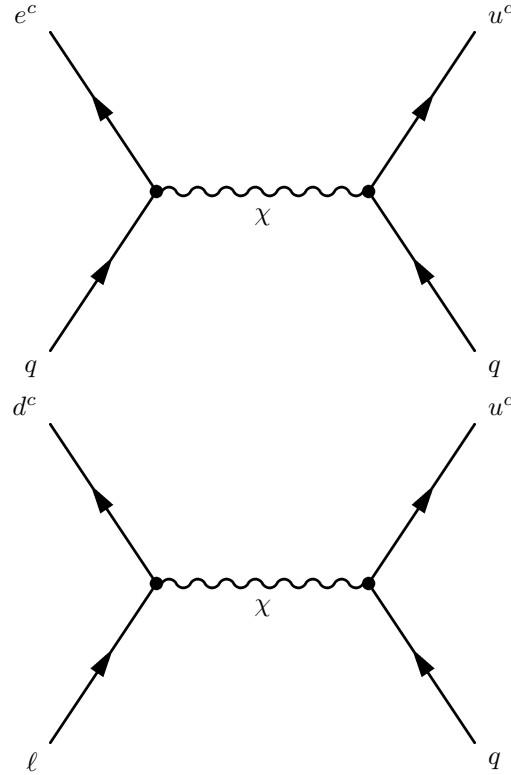
Fig. 10.   Diagrams responsible for the proton decay of the type $p \to \pi^0 e^+$, or $p \to K^0 e^+$, or $p \to K^+ \nu$ in supersymmetric GUT's. "Elephants" are denoted by χ.

universe (Fig. 12) and spit it out into another one.[25] In both cases such event will be interpreted by "our" observer as a proton decay. In the "old" four-dimensional picture the contribution of this mechanism is of the order of

$$\Gamma_{\text{proton}} \sim m_{\text{proton}} \left(\frac{m_{\text{proton}}}{M_{\text{Pl}}} \right)^4 , \tag{18}$$

a rate which is even smaller than that in Eq. (17). What makes the proton instability due to gravity effects phenomenologically acceptable is the huge value of the mass scale where gravity becomes strong.

With the advent of the LED paradigm this situation dramatically changes. First, the extended desert (14 orders of magnitude in energy!) preceding the unification point of the gauge couplings disappears. Indeed, in $4 + n$ dimensions, at energies $E \gtrsim 1/\delta$, the logarithmic law of running gives place to a power law. (Here δ is the wall thickness which is usually
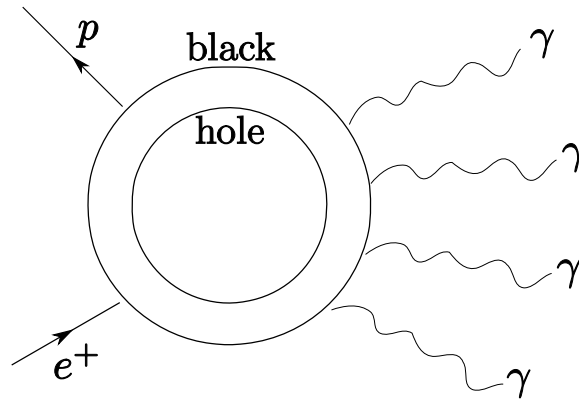
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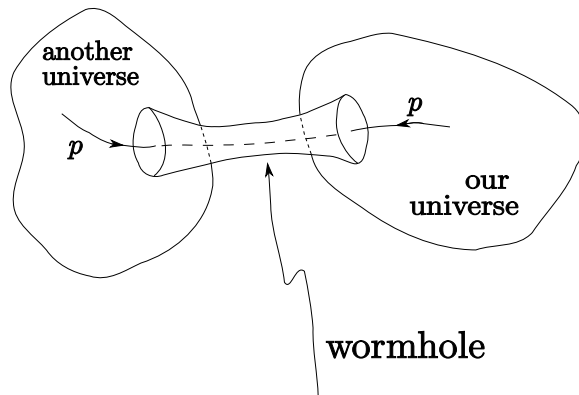Fig. 11.   Proton decay through a virtual black hole.

Fig. 12.   A proton escaping from our universe via a wormhole.

assumed to be related to $M_{\rm f}$.) If unification of the gauge couplings takes place,[21] it occurs at energies close to $M_{\rm f}$. With the fundamental scale $M_{\rm f}$ as low as 10 TeV in the ADD scenario, a disaster seems inevitable. With new interactions and particles (mediating the proton decay) as light as 10 TeV protons will decay immediately unless special protective mechanisms are found.

The proton instability due to quantum gravity is, at the very least, as severe a problem as the one discussed above. If strong gravity occurs at the scale $M_{\rm f} \sim 10$ TeV, virtual black holes will be abundant and they

will destroy all globally conserved quantum numbers (such as the baryon number) much faster than in 10^{32} years. This is seen from Eq. (18) where M_{Pl} is to be replaced by M_{f}. Needless to say, this would be a mortal blow to the ADD scenario.

One of possibilities to suppress the proton decay is to associate a gauge symmetry with the baryon number, more exactly, a discrete gauge symmetry.[††] This mechanism of protection was invented long ago.[26] As was noted by Krauss and Wilczek,[26] "neither black-hole evaporation, wormholes, nor anything else can violate discrete gauge symmetries." With the advent of the LED paradigm people used it first to guarantee the proton stability. This method was shown to be viable, i.e. its phenomenological implications are compatible with experimental data.[23]

Here we will focus on an alternative, geometric protection, one of many manifestations of a universal geometrical idea which, being combined with the ADD idea, brings lavish fruits. It goes under the name *fat branes*, or branes with a substructure, and was put forward by Arkani-Hamed and Schmaltz[27] in the context of the problem of proton stability.[‡‡]

Let us examine the transverse structure of the domain wall. A hypothetical slice through the wall is schematically shown in Fig. 13. The overall wall thickness is δ. A crucial observation is that quarks and leptons need not be localized at one and the same point on the Z axis, and, moreover, the localization width need not coincide with δ. Assume that quarks are localized on the left edge of the wall, with the localization width $\ell \ll \delta$, while leptons are localized on the right edge. If W and other gauge bosons are smeared everywhere inside the wall — i.e. their degree of localization is δ — this will ensure that "normal" SM quark/lepton decays proceed as they should. At the same time, the baryon number changing transition of quarks into an appropriate lepton will be suppressed by an exponentially small overlap of the wave functions in the Z direction. The suppression factor in the amplitude is proportional to

$$\int d^n Z \, \Psi_q^3(Z) \, \Psi_l^*(Z) \, \sim \, e^{-\delta/\ell} \,, \tag{19}$$

where the subscripts q and l stand for quarks and leptons, respectively. A relatively modest geometric hierarchy of $\delta/\ell \sim 30$, after the exponentiation,

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[††]What is a discrete gauge symmetry? One first gauges the baryon charge in a regular way coupling it, say, to a U(1) gauge boson. Then this continuous gauge symmetry must be spontaneously broken by a Higgs mechanism down to a discrete subgroup.

[‡‡]These authors also considered the hierarchy problem following the same line of reasoning.

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will suppress the proton decay to the acceptable level, with no protection in the form of additional (gauged) conservation laws.
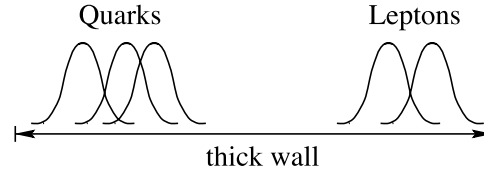

Fig. 13.   A layered domain wall with a substructure.

A possible substructure of the domain wall defining our brane world may provide geometric solutions to other problems from flavordynamics, in particular those listed in Sect. 3.3. For instance, the four-layer structure shown in Fig. 14 may explain[28] not only the fact of three distinct generations, but also the pattern of quark masses and mixing angles.[28,29] Four domains correspond to localizations of the Higgs field and quark-lepton fields belonging to generations three, two, and one. It is clear that the suppression of mass terms due to overlap of the wave functions similar to Eq. (19) will depend on the number of the generation at hand — the closer it is to the Higgs brane, the larger is the overlap with the Higgs field Z-profile, implying a larger mass mass term. In this way a typical pattern of the quark masses in three generations naturally emerges.

3.6. *The lightness of the left-handed neutrino*

The masses of "our" left-handed neutrinos are believed to lie in the ballpark of 10^{-2} or 10^{-3} eV. How does the good old paradigm cope with such small masses?

The standard explanation relies on the seesaw mechanism invented in the late 1970's.[30] The essence of the seesaw mechanism is as follows. Let us assume that the Dirac neutrino mass is described by the term $\mu\,\bar{\nu}_R\,\nu_L$ where the mass term μ is of the order of M_{SM}, its natural order of magnitude. The right-handed neutrino which is (nearly) sterile has a Majorana mass term $M\,\bar{\nu}_R\,\nu_R^c$ where the superscript c stands for the charge conjugation. It is natural to assume that $M \sim M_{\text{GUT}}$. Then, upon diagonalization of the mass matrix, one finds that the true left-handed neutrino is a mixture of ν_L and ν_R^c (the admixture of ν_R^c is very small, $\sim \mu/M$) and its mass is

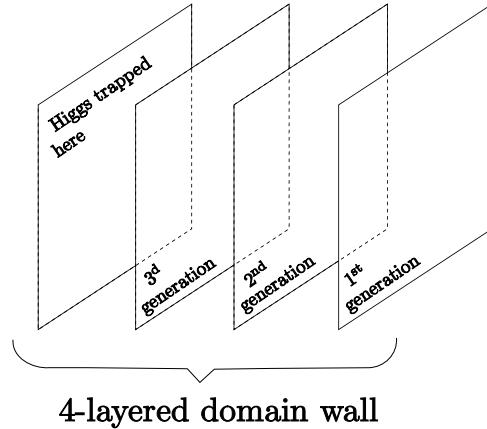$$m_{\nu_L} \sim \frac{\mu^2}{M}\,. \tag{20}$$

Fig. 14.   Four-layer structure of the domain wall generates a reasonable pattern of the quark masses and mixing angles.

With the above assumptions regarding μ and M, we get "our" neutrino mass in the right ballpark. The lightness of "our" neutrino is due to the enormity of M_{GUT}.

As we remember, in the ADD scenario the *bona fide* fundamental scale M_{f} is much lower. Gone with M_{GUT} is the seesaw mechanism. A question that immediately comes to one's mind is: "Can one engineer a LED-based mechanism (preferably, geometrical) that would naturally explain the lightness of the left-handed neutrino?"

The answer to this question is positive, and, in fact, there exists more than one solution. One of the nicest ideas belongs to the authors of the ADD scenario themselves.[31] The right-handed neutrino carries no charges with respect to the SM gauge bosons. Therefore, nothing precludes us from letting the right-handed neutrino escape to the bulk, unlike the left-handed neutrino, which has to be localized on the wall. The very existence of the wall may be responsible for the disparity between ν_R and ν_L. Indeed, the topology of the wall solution (i.e. wall vs. anti-wall) is typically correlated with the chirality of the fermion zero modes. Thus, it is quite natural to expect that a wall traps ν_L, while ν_R is free to go in the bulk. An anti-wall would trap ν_R.

If the right-handed neutrino wave function is smeared all over the bulk then the neutrino mass term, which is proportional to the overlap of Ψ_{ν_L}

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and $\Psi_{\nu_R}^*$, takes the form

$$\int d^4x\, H(x)\, \Psi_{\nu_L}(x)\, \Psi_{\nu_R}^*(x, Z = 0)\,, \tag{21}$$

where $H(x)$ is the Higgs field wave function. Due to the fact that Ψ_{ν_R} is totally delocalized in the extra dimensions,

$$\Psi_{\nu_R}(x, Z) \sim \frac{1}{\sqrt{V}_n}\,, \tag{22}$$

so that our neutrino mass gets a natural suppression factor $V_n^{-1/2}$,

$$m_\nu \sim \frac{v}{\sqrt{V_n M_{\mathrm{f}}^n}} \sim \frac{v\, M_{\mathrm{f}}}{M_{\mathrm{Pl}}}\,, \tag{23}$$

where v is the expectation value of the Higgs field and we used Eq. (13). Given the uncertainty in numerical factors, this estimate places the neutrino mass in the right ballpark.

Thus, the lightness of the left-handed neutrino is due to the same reason why gravity is weak — large volume of the extra dimensions.

3.7. *Downside of the ADD scenario*

Precision electroweak measurements firmly established the validity of the standard model. What will come beyond the standard model? Although theoretical speculations are abundant, experimental support is scarce. In fact, the only semiquantitative achievement in this direction is the success of the gauge coupling unification.

The gauge interactions unified by the standard model are SU(3)×SU(2)× U(1). The first is the color group, the last two represent electroweak interactions. At low energies the corresponding three gauge couplings are very different in their values, see Fig. 9. The SU(2) and U(1) couplings are measured with high precision; the accuracy of the SU(3) coupling is not that high, mainly due to theoretical uncertainties in its determination. The logarithmic running brings them closer, and eventually all three intersect — nearly exactly at one point — M_{GUT}, which, in turn, turns out to be rather close to M_{Pl}. Figure 9 illustrates this statement. It is important to note that the intersection occurs only provided that SM is supersymmetrized. Thus, the above success may be viewed, simultaneously, as a semidirect indication that supersymmetry is relevant to nature. It would be a pity to loose this encouraging indication.

The unification and the desert paradigm are closely connected. Being honest, we should admit that the ADD scenario erases the above success.

The logarithmic running stops at M_f, where the three gauge couplings are still very far from each other. Will a power-like running, which replaces the logarithmic one above M_f, unify the couplings, and at what scale? With the loss of the great desert the answer to this question becomes model-dependent and almost completely devalued.[21]

4. A few words on other scenarios

Next, we will outline, very briefly other scenarios, in which extra dimensions are instrumental in understanding the world we observe around us. Chronologically they appear later than ADD and are "ideologically" related.

4.1. *Warped Extra Dimensions*

In the ADD scenario gravity of the domain wall as such plays little role. Given that the brane tensions are small, this is a good approximation. However, this need not be the case. In the scenario suggested by Randal and Sundrum (RS)[32,33] the brane-induced gravity strongly warps extra dimensions which is instrumental in ensuring an appropriate localization.
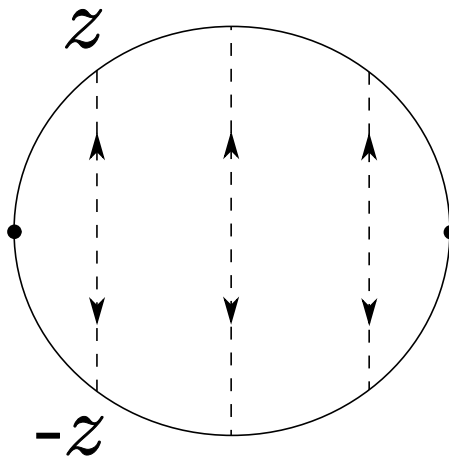

Fig. 15.   Graphical representation of S_1/Z_2. The dashed lines indicate the points of the circle which are to be identified.

A general observation on which the RS construction is based is as follows: if one has 3-branes in five dimensions, one can, in principle, balance

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the gravitational effects of the above branes by a five-dimensional bulk cosmological constant to get a theory in which an effective cosmological constant of our four-dimensional world will vanish. Our universe will seem static and flat for an observer on our brane.[14] The price to pay for this fine-tuning is a highly curved five-dimensional background. This phenomenon is called "off-loading." We off-load curvature from the brane on which we live onto the bulk. Needless to say, there is no theoretical rationale for the above fine-tuning. We just take the fact that the world we observe is (nearly) flat as given.

Let us discuss some basic elements of the RS construction. One starts from a finite length extra dimension[32] assumed to be an S_1/Z_2 orbifold. What is this? Take a sphere S_1 and identify opposite points, as shown in Fig. 15. The points 0 and π are called fixed points since they are identified with themselves. Two branes are placed at these points in the extra dimension. The solution of five-dimensional gravity – I will not derive it here – is self-consistent (and consistent with the fine tuning discussed above) provided that one of the branes has a positive tension while the tension of the other brane is negative (and the same in the absolute value).

Since we want the Lorentz invariance to be preserved in our four-dimensional world we have to assume that the induced metric (its $\mu\nu$ part) at every point along the extra dimension is proportional to the four-dimensional Minkowski metric $\eta_{\mu\nu}$, while the components of the five-dimensional metric depend only on the fifth coordinate Z in the following way:

$$ds^2 = e^{-A(Z)}dx^\mu dx^\nu \eta_{\mu\nu} - dZ^2 \,. \tag{24}$$

The degree of warping along the extra dimension depends on the factor $e^{-A(Z)}$, which is therefore called the warp factor. In the RS solution the warp factor depends on Z exponentially,

$$A(Z) = 2\,k\,|Z| \tag{25}$$

where k is a constant of dimension of mass which sets the scale of warping. It turns out that it is related to the five-dimensional cosmological constant Λ_{5D} and the five-dimensional Planck constant $M_{5D}^{\rm Pl}$ by the formula[32]

$$k^2 = -\frac{\Lambda_{5D}}{\left(M_{5D}^{\rm Pl}\right)^3} \,. \tag{26}$$

This relation implies, in particular, that Λ_{5D} is negative. Thus, the five-dimensional space-time one deals with in the RS scenario is anti-de Sitter.

Now finally we are in a position to understand the impact of two branes – a positive tension brane at $Z = 0$ and a negative tension brane at $Z = b$ – as well as the impact of warping. Equations (24) and (25) show that the induced metric on the negative tension brane is exponentially smaller than that on the positive tension brane provided that $kb \gg 1$,

$$g_{\mu\nu}^{\text{ind}}\Big|_{Z=b} = e^{-2kb}\,\eta_{\mu\nu}\,. \tag{27}$$

This suppression sets the scale for all other mass parameters. Indeed, consider the Higgs field part of the action. Proceeding to the canonically normalized kinetic term of the Higgs field we see that the physical value of its vacuum condensate is "warped down" to

$$v_{\text{phys.Higgs}} = e^{-kb}v_0\,, \tag{28}$$

which shows, in turn, that all masses following from (28) are exponentially suppressed on the negative tension brane (but not on the positive tension brane). Then it is natural to refer to the positive tension brane at $Z = 0$ as the *Planck brane*, since the fundamental mass scale there is of the order of the Planck scale. The negative tension brane is said to be the *TeV brane* which follows from Eq. (28) where we use $kb \sim 17$ and $M_{5D}^{\text{Pl}} \sim M_{4D}^{\text{Pl}}$.

In fact, there are two popular versions of the RS scenario. The one discussed above (RS1), has a finite size extra dimension with two branes, one at each end. The second version (RS2) is similar to the first, but one brane has been placed infinitely far away, so that there is only one brane left in the model. The particles of the standard model are placed on the Planck brane. This model was originally of interest because it represented an infinite five-dimensional model resembling, in many respects, a four-dimensional model.

For a more detailed consideration the reader is referred to numerous reviews, for instance [34].

4.2. *Braneworlds with Infinite Volume Extra Dimensions*

Here we will briefly outline a model proposed by Dvali, Gabadadze, and Porrati (DGP)[35] in 2000 in which, strictly speaking, gravity never becomes four-dimensional, no matter how far in the infrared we go. Therefore, this is not a genuine compactification. Unlike the ADD scenario in this construction gravity only approximately imitates four-dimensional behavior at large distances. A residual "tail" associated with extra dimensions produces an effect which could, in principle, reproduce the cosmic acceleration of dark

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energy.[36] Then the four-dimensional cosmological constant need not be non-vanishing. The DGP mechanism allows the volume of the extra space to be infinite,

$$V_N \equiv \int d^N Z \sqrt{G} \to \infty \,, \tag{29}$$

where N is the number of extra dimensions and G is the metric tensor in the $4 + N$ dimensional space. For orientation below we will assume $N = 1$, although other versions of the DGP scenario, with $N > 1$, were also considered.

The basic idea behind this scenario is as follows. We start from $4 + N$-dimensional space, with $4+N$-dimensional gravity governed by the standard Einstein–Hilbert action. A four-dimensional domain wall is embedded in N dimensions. As in the ADD scenario, all matter fields are assumed to be localized on the wall. The central point of the DGP construction is the emergence of the induced four-dimensional Einstein–Hilbert term on the wall due to a loop of virtual matter localized on the wall, see Fig. 16. The bare action has no such term. However, as soon as the localized matter fields are coupled to $4 + N$ dimensional gravity, we obtain 4-dimensional gravity action on the wall from loops. This mechanism described by Sakharov[37] long ago is usually referred to as Sakharov's induced gravity.
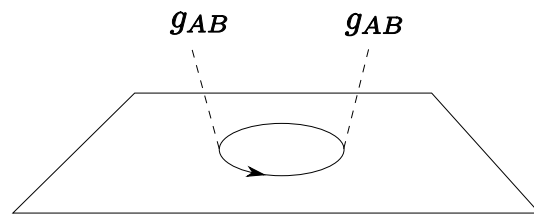
$$-\frac{1}{16\pi} M_{\mathrm{P}}^2 \int d^4 x \sqrt{-g^{(4)}} R^{(4)}$$

Fig. 16.   One-loop contribution of the matter fields to the effective action of gravitons. The matter fields are localized on the brane yielding the four-dimensional Einstein–Hilbert term which is also localized on the brane.

Then the DGP action takes the form

$$S = \frac{\left(M_{5D}^{\mathrm{Pl}}\right)^3}{2} \int d^4x \int_{-\infty}^{+\infty} dy \sqrt{G}\, R_5$$

$$+ \int d^4x \sqrt{g} \left[\frac{\left(M_{4D}^{\mathrm{Pl}}\right)^2}{2}\, R_4 + \Lambda + \mathcal{L}_{\mathrm{matter}}(\Phi_{\mathrm{SM}}, \Psi_{\mathrm{SM}}) \right], \qquad (30)$$

where G and g stand for five- and four-dimensional metrics, respectively,

$$g = G(Z = 0),$$

R_5 and R_4 are the corresponding Ricci tensors, while Φ_{SM}, Ψ_{SM} is a generic notation for all boson and fermion matter fields. The parameter $\left(M_{4D}^{\mathrm{Pl}}\right)^2$ is obtained in Eq. (30) as an ultraviolet cut-off in loops of the brane-confined matter, representing the scale of energies at which the standard-model physics drastically changes. In the DGP scenario it is assumed that

$$M_{5D}^{\mathrm{Pl}}/M_{4D}^{\mathrm{Pl}} \gg 1.$$

Let us discuss the value of the domain wall tension. One assumes that the $4 + N$-dimensional theory is supersymmetric, and that supersymmetry is spontaneously broken only on the brane. Non-breaking of supersymmetry in the bulk is only possible due to the infinite volume of the extra space; supersymmetry breaking is not transmitted from the brane into the bulk since the breaking effects are suppressed by an infinite volume factor. Then, the bulk cosmological term can be set to zero, without any fine-tuning. As for the four-dimensional cosmological constant Λ in Eq. (30), the natural value of Λ can be as low as TeV^4, since the brane tension can be protected above this value by $\mathcal{N} = 1$ supersymmetry. Please, note that Λ must be fine-tuned in such a way that $\Lambda + \langle \mathcal{L}_{\mathrm{matter}} \rangle = 0$. This is a usual fine-tuning of the cosmological constant. The DGP construction adds nothing new in this respect.

With the action (30) gravity from the sources localized on the domain wall will propagate both, in the bulk and in the brane. Interplay of these two modes of propagation leads to quite a peculiar gravitational dynamics on the brane. Unlike the ADD scenario, in which gravity deviates from its four-dimensional form only at short distances, in the DGP case deviations from the four-dimensional Newton law occur both at short and large distances. Despite the fact that the volume of extra space is infinite, an observer on the

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brane measures four-dimensional gravitational interaction up to distances

$$r_c \sim \frac{\left(M_{4D}^{\mathrm{Pl}}\right)^2}{\left(M_{5D}^{\mathrm{Pl}}\right)^3} \,. \tag{31}$$

At distances larger than r_c the Newton potential losses its four-dimensional form. In order for the late-time cosmology to be standard we must require r_c to be of the order of the universe size, i.e. $r_c \sim H_0^{-1} \sim 10^{28}$ cm. This, in turn, requires M_{5D}^{Pl} to be very small. Small M_{5D}^{Pl} means that gravity in the bulk is strong. However, the large Einstein–Hilbert term on the brane "shields" matter localized on the brane from strong bulk gravity.

4.3. *Universal extra dimensions*

For completeness I mention the universal extra dimension (UED) scenario.[38] It assumes compactification of the ADD type. However, all standard matter fields are free to propagate through all of the extra-dimensional space (which is essentially flat), rather than being confined to a brane. The size of the extra dimensions is assumed to be in the ballpark of TeV^{-1}. To my mind this scenario lacks elegant theoretical features that can be found in ADD, RS and DGP. Phenomenology that ensues was discussed in the literature, see e.g. the review papers [39].

Acknowledgments

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