Physics of the Monopoles in QCD

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Abstract. We discuss implications of the recent measurements of the non-Abelian action density associated with the monopoles condensed in the confining phase of gluodynamics. The radius of the monopole determined in terms of the action was found to be small numerically. As far as the condensation of the monopoles is described in terms of a scalar field, a fine tuning is then implied. In other words, a hierarchy exists between the self energy of the monopole and the temperature of the confinement-deconfinement phase transition. The ratio of the two scales is no less than a factor of 10. Moreover, we argue that the hierarchy scale can well eventually extend to a few hundred GeV on the ultraviolet side. The corresponding phenomenology is discussed, mostly within the polymer picture of the monopole condensation.

1 Introduction

The monopole condensation is one of the most favored mechanisms [1] of the confinement, for review see, e.g., [2]. In the field theoretical language, one usually thinks in terms of a Higgs-type model:

$$S_{eff} \;=\; \int d^4x\big(|D_\mu\phi|^2 + \frac{1}{4}F^2_{\mu\nu} + \; V(|\phi|^2)\big) \qquad (1)$$

where ϕ is a scalar field with a non-zero magnetic charge, $F_{\mu\nu}$ is the field strength tensor constructed on the dual-gluon field B_μ, D_μ is the covariant derivative with respect to the dual gluon. Finally, $V(|\phi|^2)$ is the potential energy ensuring that $\langle\phi\rangle \neq 0$ in the vacuum. Relation of the "effective" fields ϕ, B_μ to the fundamental QCD fields is one of the basic problems of the approach considered but here we would simply refer the reader to [3] for further discussion of this problem. At this moment, it suffices to say that the "dual-superconductor" mechanism of confinement assumes formation of an Abrikosov-type tube between the heavy quarks introduced into the vacuum via the Wilson loop while the tube itself is a classical solution of the equations of motion corresponding to the effective Lagrangian (1).

By introducing scalar fields, one opens a door to the standard questions on the consistency, on the quantum level, of a $\lambda\phi^4$ theory. Here, we mean primarily the problem of the quadratic divergence in the scalar mass. At first sight, these problems are not serious in our case since (1) apparently represents an effective theory presumably valid for a limited range of mass scales.

However, if we ask ourselves, what are the actual limitations on the use of the effective theory (1) we should admit that there is no way at the moment to

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answer this question on pure theoretical grounds and we should turn instead to the experimental data, that is lattice measurements. This lack of understanding concerns first of all the nature of the non-perturbative field configurations that are defined as monopoles. First, it is not clear apriori which $U(1)$ subgroup of the $SU(2)$ [1] is to be picked up for the classification of the monopoles. Even if we make this choice on pure pragmatic basis and concentrate on the most successful scheme of the monopoles in the maximal Abelian projection [2] we still get very little understanding of the field configurations underlying the objects defined as monopoles in this projection, for discussion see, e.g., [4]. In particular, nothing can be said on the size of the monopole which presumably limits application of (1) on the ultraviolet side.

Direct measurements of the monopole size were reported recently [5] and brought an unexpectedly small value of the monopole radius:

$$R_{mon} \approx 0.06 \text{ fm}, \tag{2}$$

where the monopole radius is defined here in terms of the full non-Abelian action associated with the monopole and not in terms of the projected action. If we compare the radius (2) with the temperature of the confinement-deconfinement transition:

$$T_{deconf} \approx 300 \text{ MeV} \tag{3}$$

then we would come to the conclusion that there are different mass scales co-existing within the effective scalar-field theory (1). And the question, how this mass hierarchy is maintained is becoming legitimate.

Although comparison of (2) and (3) is instructive by itself, we will argue that the actual hierarchy mass scale can be much higher on the ultraviolet side. Namely, we will emphasize later that even at the size (2) the monopoles are very "hot", i.e. have action comparable to the action of the zero-point fluctuations. For physical interpretation, it is natural to understand by the radius such distances where the non-perturbative fields die away on the scale of pure perturbative fluctuations. And this radius is to be considerably smaller than (2).

Also, estimate (2) means that the asymptotic freedom is not yet reached at quite small distances and the question arises as to how reconcile this observation with such phenomena as the precocious scaling.

We cannot claim at all understanding answers to these questions but feel that it is important to start discussing them. Our approach is mostly phenomenological and we are trying to formulate which measurements could help to find answers to the puzzles outlined above. The theoretical framework which we are using is mainly the polymer approach to the scalar field theory, see, e.g., [6,7,8].

[1] for simplicity we will confine ourselves to the case of $SU(2)$ as the color group.

2 Monopole Condensation: Overview of the Theory

2.1 Compact $U(1)$

The show case of the monopole condensation is the compact $U(1)$ [9]. The crucial role of the compactness is to ensure that the Dirac string does not cost energy (for a review see, e.g., [4]). The monopole self energy reduces then to the energy associated with the radial magnetic field $\mathbf{B}$. The self energy is readily seen to diverge linearly in the ultraviolet:

$$M_{mon}(a) \;=\; \frac{1}{8\pi}\int \mathbf{B}^2 d^3r \;\sim\; \frac{c}{8e^2}\frac{1}{a}\,, \tag{4}$$

where c is a constant, a is the lattice spacing, e is the electric charge and the magnetic charge is [2] $g_m = 1/2e$. Thus, the monopoles are infinitely heavy and, at first sight, this precludes any condensation since the probability to find a monopole trajectory of the length L is suppressed as

$$\exp(-S) \;=\; \exp\left(-\frac{c}{e^2}\cdot\frac{L}{a}\right)\,. \tag{5}$$

Note that the constant c depends on the details of the lattice regularization but can be found explicitly in any particular case.

However, there is an exponentially large enhancement factor due to the entropy. Namely, trajectory of the length L can be realized on a cubic lattice in $N_L = 7^{L/a}$ various ways. Indeed, the monopole occupies center of a cube and the trajectory consists of L/a steps. At each step the trajectory can be continued to an adjacent cube. In four dimensions there are 8 such cubes. However, one of them has to be excluded since the monopole trajectory is non-backtracking. Thus the entropy factor,

$$N_L \;=\; \exp\left(\ln 7\cdot\frac{\mathrm{L}}{\mathrm{a}}\right)\,, \tag{6}$$

cancels the suppression due to the action (5) if the coupling e^2 satisfies the condition

$$e^2_{crit} \;=\; c/\ln 7 \;\approx\; 1\,, \tag{7}$$

where we quote the numerical value of e^2_{crit} for the Wilson action and cubic lattice. At e^2_{crit} any monopole trajectory length L is allowed and the monopoles condense.

This simple theory works within about one percent accuracy in terms of e^2_{crit} [10]. Note that the energy-entropy balance above does not account for interaction with the neighboring monopoles.

[2] The notation g is reserved for the non-Abelian coupling, the magnetic coupling is denoted as g_m.

2.2 Monopole Cluster in the Field-Theoretical Language

The derivation of the previous subsection implies that the monopole condensation occurs when the monopole action is ultraviolet divergent. On the other hand, the onset of the condensation in the standard field theoretical language corresponds to the zero mass of the magnetically charged field ϕ. It is important to emphasize that this apparent mismatch between the two languages is not specific for the monopoles at all. Actually, there is a general kinematic relation between the physical mass of a scalar field m^2_{phys} and the mass M defined in terms of the (Euclidean) action, $M \equiv S/L$ where L is the length of the trajectory and S is the corresponding action [3]:

$$m^2_{phys} \cdot a \;\approx\; M - \frac{\ln 7}{\mathrm{a}}, \tag{8}$$

where terms of higher order in ma are omitted. Here by m^2_{phys} we understand the mass entering the propagator of a free particle,

$$D(p^2, m^2_{phys}) \;\sim\; (p^2 + m^2_{phys})^{-1} \;,$$

where p^2 is either Euclidean or Minkowskian momentum squared.

In view of the crucial role of the (8) for our discussion, let us reiterate the statement. We consider propagator of a free scalar particle in terms of the path integral:

$$D(x_i, x_f) \;\sim\; \Sigma_{paths} exp(\;-S_{cl}(path)), \tag{9}$$

where for the classical action associated with the path we would like to substitute simply the action of a point-like classical particle, $S_{cl} = M \cdot L$ where M is the mass of the particle and L is the length of the path. Then we learn that there is no such representation (with replacement of S_{cl} by iS_{cl})) for the propagator of a relativistic particle in the Minkowski space because of the backward-in-time motions [4]. However, in the Euclidean space the representation (9) works. The physical mass is, however, gets renormalized compared to M according to (8).

Derivation of the Eq (8) is in textbooks [5] , see, e.g., [12]. The central point is that the action for a point-like particle in the Euclidean space looks exactly the same as that of a non-interacting polymer with a non-vanishing chemical potential for the constituent atoms. The transition from the polymer to the field theoretical language is common in the statistical physics (see, e.g., [13]). The first applications to the monopole physics are due to the authors in [7]. For the

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[3] It is worth emphasizing that the results of the lattice measurements are commonly expressed in terms of Higgs masses and interaction constants, see [11]. However, these masses are obtained without subtracting the ln7 term (compare Eq (8)) and, to our belief, are not the physical mass for this reason. Where by the physical masses we understand the masses in the continuum limit. In particular, the physical masses determine the shape of the Abrikosov-like string confining the heavy quarks.

[4] I am indebted to L. Stodolsky for an illuminating discussions on this topic.

[5] Actually, one finds mostly ln2D ≡ ln8 instead of ln7. We do think that ln7 is the correct number but in fact this difference is not important for further discussion.

sake of completeness we reproduce here the main points crucial for our discussion later. Mostly, we follow the second paper in [7].

The scalar particle trajectory represented as a random walk and the corresponding partition function is:

$$Z = \int \mathrm{d}^4 x \sum_{N=1}^{\infty} \frac{1}{N} e^{-\mu N} Z_N(x,x)\,, \tag{10}$$

where μ is the chemical potential and $Z_N(x_0, x_f)$ is the partition function of a polymer broken into N segments:

$$Z_N(x_0,x_f) = \Big[\prod_{i=1}^{N-1} \int \mathrm{d}^4 x_i\Big] \prod_{i=1}^{N} \left[\frac{\delta(|x_i - x_{i-1}| - a)}{2\pi^2 a^3}\right] \exp\Big\{-\sum_{i=1}^{N} gV(x_i)\Big\}\,. \tag{11}$$

This partition function represents a summation over all atoms of the polymer weighted by the Boltzmann factors. The δ–functions in (11) ensure that each bond in the polymer has length a. The starting point of the polymer (11) is x_0 and the ending point is $x_f \equiv x_N$.

In the limit $a \to 0$ the partition function (11) can be treated analogously to a Feynman integral. The crucial step is the coarse–graining: the N–sized polymer is divided into m units by n atoms ($N = mn$), and the limit is considered when both m and n are large while a and $\sqrt{n}a$ are small. We get,

$$\prod_{i=\nu n}^{(\nu+1)n-1} \frac{1}{2\pi^2 a^3} \delta(|x_i - x_{i+1}| - a) \to \Big(\frac{2}{\pi n a^2}\Big)^2 \exp\Big\{-\frac{2}{n\,a^2}(x_{(\nu+1)n} - x_{\nu n})^2\Big\}\,, \tag{12}$$

where the index i, $i = \nu n \cdots (\nu+1)n - 1$, labels the atoms in ν^{th} unit. The polymer partition function becomes [7]:

$$Z_N(x_0,x_f) = \mathrm{const}\cdot \Big[\prod_{\nu=1}^{m-1} \mathrm{d}^4 x\Big] \left[\Big(\frac{2}{\pi n a^2}\Big)^{2m} \exp\Big\{\sum_{\nu=1}^{m} \frac{(x_\nu - x_{\nu-1})^2}{na^2}\Big\}\right] \cdot \exp\Big\{-\sum_{\nu=1}^{m} n(\mu + V(x_\nu))\Big\}\,. \tag{13}$$

The x_i's have been re-labeled so that x_ν is the average value of x in at the coarser cell. Using the variables:

$$s \;=\; \frac{1}{8} n a^2 \nu, \quad \tau \;=\; \frac{1}{8} a^2\, N\,, \quad m_0^2 \;=\; \frac{8\mu}{a^2}\,, \tag{14}$$

one can rewrite the partition function (10) as

$$Z = \mathrm{const}\cdot \int_0^\infty \frac{\mathrm{d}\tau}{\tau} \int\limits_{x(0)=x(\tau)=x} Dx\, \exp\left\{-\int_0^\tau \Big[\frac{1}{4}\dot{x}_\mu^2(s) + m_0^2 + g_0 V(x(s))\Big]\,\mathrm{d}s\right\}. \tag{15}$$

The next step is to rewrite the integral over trajectories $x(\tau)$ as the standard path integral representation for a free scalar field. For us it is important only that the m_0^2 term in (15) is becoming the standard mass term in the field theoretical language:

$$\mathcal{Z} = \sum_{M=0}^{\infty} \frac{1}{M!} Z^M$$
$$= \text{const} \cdot \int D\phi \, \exp\Big\{ - \int \mathrm{d}^4x \, \Big[(\partial_\mu \phi)^2 + m_0^2 \, \phi^2 + g_0 V(x) \phi^2 \Big] \Big\} \, . \qquad (16)$$

The whole machinery can be easily generalized to the case of charged particles (monopoles) with Coulomb-like interactions.

2.3 Monopole Condensation in Non-Abelian Case: Expectations

If we try to adjust the lessons from the compact $U(1)$ to the non-Abelian case then the good news is that, indeed, all the $U(1)$ subgroups of the color $SU(2)$ are compact. Moreover, dynamics of any subgroup of the $SU(2)$ is governed by the same running coupling $g^2(r)$. Thus, we could hope that the following simple picture might work: if the lattice spacing a is small we would not see monopoles because $g^2(a)$ falls below e^2_{crit}. However, going to a coarser lattice a la Wilson we come to the point where $g^2(a^2) \approx e^2_{crit}$. Then we apply the entropy-energy balance which works so well in case of the compact $U(1)$ and conclude that the monopoles of a critical size a_{crit} such that $g^2(a_{crit}) \sim 1$ condense in the QCD vacuum.

This simple picture is open, however, to painful questions. First, monopoles are defined topologically within a $U(1)$ subgroup [6]. However, it is only the $U(1)$ invariant action which has a non-vanishing minimum for a $U(1)$ topologically non-trivial object. There is no relation, generally speaking, between the full non-Abelian action and a $U(1)$-subgroup topology. For illustrations of this general rule see [3].

Therefore, there is no reason, at least at first sight, for the saturation of the functional integral at the classical solution with infinite action, see (4). This observation brings serious doubts on the validity of our simple dynamical picture.

3 Monopoles, as They Are Seen

3.1 Monopole Dominance

On the background of the theoretical turmoil, the data on the monopoles indicate a very simple and solid picture. We will constrain ourselves to the monopoles in the so called Maximal Abelian gauge and the related projection (MAP). We

[6] Note that a $SU(2)$-invariant definition of the monopoles is also possible [14]. However, their dynamical characteristics have not been measured yet and such monopoles are not considered here.

just mention some facts, a review and further references can be found, e.g., in [2].

Since the monopoles of the non-Abelian theory are expected to actually be $U(1)$ objects one first uses the gauge freedom to bring the non-Abelian fields as close to the Abelian ones as possible. The gauge is defined by maximization of a functional which in the continuum limit corresponds to $R(\hat{A})$ where

$$R(\hat{A}) \;=\; -\int d^4x\big[(A^1_\mu)^2+(A^2_\mu)^2\big] \tag{17}$$

where $1,2$ are color indices.

As the next step, one projects the non-Abelian fields generated on the lattice into their Abelian part, essentially, by putting $A^{1,2}\equiv 0$. In this Abelian projection one defines the monopole currents k_μ for each field configuration. Note that the original configurations which are used for a search of the monopoles are generated within the full non-Abelian theory. Upon performing the projection one can introduce also the corresponding Abelian, or projected action.

The relation of the monopoles to the confinement is revealed through evaluation of the Wilson loop for the quarks in the fundamental representation. Namely it turns out, first, that the string tension in the Abelian projection is close to the string tension in the original $SU(2)$ theory [15]:

$$\sigma_{U(1)} \;\approx\; \sigma_{SU(2)}\,. \tag{18}$$

Moreover, one can define also the string tension which arises due to the monopoles alone. To this end, one calculates the field created by a monopole current:

$$A^{mon}_\mu(x)=\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\sum_y \Delta^{-1}(x-y)\,\partial_\nu m_{\alpha\beta}[y;k]\,, \tag{19}$$

where Δ^{-1} is the inverse Laplacian, and sums up (numerically) over the Dirac surface, $m[k]$, spanned on the monopole currents k. The resulting string tension is again close to that of the un-projected theory:

$$\sigma_{mon} \;\approx\; \sigma_{SU(2)}\,. \tag{20}$$

It might worth mentioning that these basic features remain also true upon inclusion of the dynamical fermions in $SU(3)$ case (full lattice QCD) [16].

3.2 Gauge-Invariant Properties of the Monopoles

Despite of the apparent gauge-dependence of the monopoles introduced within the MAP, they encode gauge-invariant information. In particular, we would mention two points: scaling of the monopole density and full non-Abelian action associated with the monopoles.

According to the measurements (see [17] and references therein) the monopole density ρ_{mon} in three-dimensional volume (that is, at any given time) is given in the physical units. In other words, the density scales according to the renormgroup as a quantity of dimension 3. Numerically:

$$\rho_{mon} \; = \; 0.65(2) \; (\sigma_{SU(2)})^{3/2} \, . \tag{21}$$

One important remark is in order here. While discussing the monopole density one should distinguish between what is sometimes called ultraviolet (UV) and infrared (IR) clusters [18]. The infrared, or percolating cluster fills in the whole lattice while the UV clusters are short. There is a spectrum of the UV clusters, as a function of their length, while the percolating cluster is in a single copy. The statement on the scaling (21) applies only to the IR cluster. We do not consider the UV clusters in this note.

Also, upon identification of the monopoles in the Abelian projection, one can measure the non-Abelian action associated with these monopoles. For practical reasons, the measurements refer to the plaquettes closest to the center of the cube containing the monopole. Since the self energy is UV divergent, it might be a reasonable approximation. The importance of such measurements is that we expect that it is the non-Abelian action which enters the energy-entropy balance for the monopoles.

The results of one of the latest measurements of this type are reproduced in Fig. 1 (see [5]).

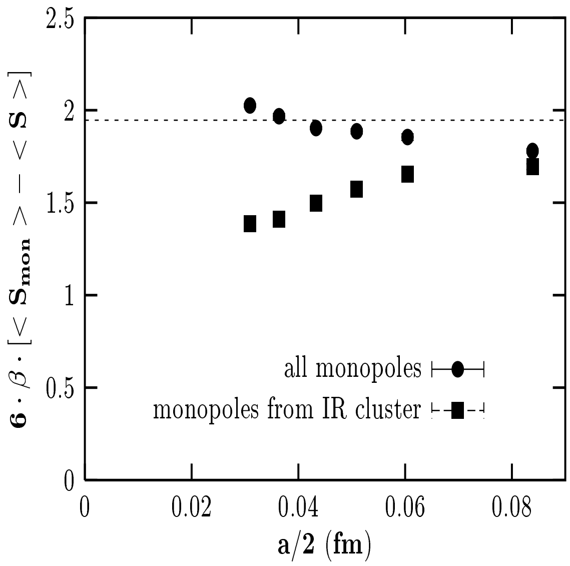

Fig. 1. The average excess of the full non-Abelian action on the plaquettes closest to the monopole, as a function of a half of the lattice spacing $a/2$. The data are reproduced from the first paper in [5]

What is plotted here is the average excess of the action on the plaquettes closest to the monopole (monopoles are positioned at centers of cubes). The action is the lattice units. In other words, the corresponding mass of the monopole $M_{mon}(a)$ of order $1/a$ if the action of order unit.

As is emphasized in [5], the IR and UV monopoles are distinguishable through their non-Abelian actions. For the UV monopoles the action is larger, in accordance with the fact that they do not percolate (condense). This is quite a dramatic confirmation that the condensation of the monopoles in the Maximal Abelian projection is driven by the full non-Abelian action, not by its projected counterpart.

4 Fine Tuning

Let us pause here to reiterate our strategy. We are assuming that the monopole condensation can be described within an effective Higgs-type theory like (1). In fact, even this broad assumption can be wrong but at this time it is difficult to suggest a framework alternative to the field theory. Next, we would like to fix the effective theory using results of the lattice measurements. Moreover we are interested first of all in interpreting data which can be expressed in gauge independent way. As the first step, we will argue in this section that the data on the monopole action [5] imply a fine tuning. By which we understand that

$$|M_{mon}(a) - \frac{ln7}{a}| \ \ll \ M_{mon}(a) \tag{22}$$

where $M_{mon}(a)$ is the monopole self energy [7] and ln7 is of pure geometrical origin (see (6)). Note that (22) looks similar to the fine tuning condition in the Standard Model.

4.1 Evidence

There are a few pieces of evidence in favor of the fine tuning (22):

a) Direct measurements indicate that the excess of the action is indeed related to the ln7, as is obvious from Fig. 1. Let us also emphasize that it is only the full non-Abelian action which “knows” about the ln7. The Abelian projected action is not related at all to the ln7 [5]. This illustrates once again that the dynamics of the monopoles in MAP is driven by the total $SU(2)$ action.

b) It is difficult to be more quantitative about the excess of the action basing on the direct data quoted above. In particular, we should have in mind that for finite a there are geometrical corrections to the equation (6). Indirect evidence could be more precise. In particular, it is rather obvious that the scaling of the monopole density (see (21)) implies:

$$|M_{mon}(a) - \frac{\ln 7}{\mathrm{a}}| \ \sim \ \Lambda_{QCD} \tag{23}$$

[7] We hope that the notations are not confusing: there are two monopole masses being discussed. One is the standard magnetic field energy (see (4)) and the other is what we call physical mass, m^2_{phys} and this mass determines propagation of a free monopole.

so that the action per unit length of the monopole trajectory does not depend on the lattice spacing a.

c) Also, independence on the lattice spacing of the temperature (3) of the phase transition suggests strongly validity of (23). Indeed, the measurements at the smallest a available, $a \sim 0.06 fm$, see Fig. 1, suggest

$$M_{mon} \ > \ 4\, GeV, \quad M_{mon} \ \gg \ T_{deconf}, \tag{24}$$

Moreover, it is well known that at the point of the phase transition the monopole trajectories change drastically. Such a sensitivity of the monopoles to the temperature is possible only if the effect of the self energy of the monopole is mainly canceled by the entropy factor, see (23).

Also, an analysis of the data in [19] suggests that

$$T_{deconf} \ \sim \ d_{mon}^{-1}, \tag{25}$$

where d_{mon} is the distance between the monopoles in the infrared cluster, $d_{mon} \sim 0.5 fm$ [5]. Thus the temperature is not sensitive to our ultraviolet parameter which is the size of the monopole.

d) Phenomenological fits suggest [11]:

$$M_{mon} \ \approx \ M^{Coul}_{mon}(a) + const, \quad const > 0\,, \tag{26}$$

where by M_{mon} we understand the action associated with the monopole. Note also that the Coulombic part of the mass, $M^{Coul}_{mon}(a)$ is of order $1/g^2 a$.

Let us recall the reader that on the theoretical side our main concern was that there is no reason why $M_{mon}(a)$ cannot drop to zero. Now we see that our fears are not justified: the monopole self energy is even higher than it would be in the pure Coulomb-like case! As far as we concentrate on a single monopole there is no way to understand (26). But this is indeed numerically necessary for the fine tuning.

Thus, the fine tuning (22) seems to be granted by the data.

4.2 The Origin of the Huge Mass Scale

We are talking actually about small distances, by all the standards of QCD. The numerical value [5] of the size of the monopole (2) is much smaller than the inverse temperature of the phase transition.

The radius (2) is defined in terms of the derivative from the monopole action with respect to a, see [5]. What we would like to emphasize here is that the actual "physical size" of the monopole can be much smaller than (2). By the physical size R_{phys} we understand now the distances where the excess of the monopole action is parametrically smaller than the action associated with the zero-point fluctuations. It is the R_{phys} where the asymptotic freedom actually reigns, not R_{mon} quoted in (2).

No evidence exists at the moment that reaching R_{phys} is in sight, see Fig. 1. Indeed, in the lattice units used in Fig. 1 the excess of the action density of order Λ^4_{QCD} would look like having zero at $a = 0$ and approaching this zero as a^4. Having in mind the data showed in Fig 1 it is tempting to speculate that the onset of such a behavior is still far off from the presently available lattice spacings.

Moreover, as we will argue now it looks plausible that the R_{phys} is shifted to the scale

$$R_{phys} \sim (100\ GeV)^{-1} . \tag{27}$$

Before giving arguments in favor of (27) let us ask ourselves, why the estimate (27) is difficult to accept, at least at first sight so. The reason is obvious: one thinks usually about non-perturbative effects in quasi-classical terms, which work in the instanton case. Thus, one assumes that the probability to find non-perturbative effects is exponentially small at small $g^2(a)$, $exp(\ -c/g^2(a))$.

But the failure of such a logic in the monopole case is evident from the case of the compact $U(1)$, see above. Even the monopoles with infinite (Euclidean) action condense. Moreover, R_{phys} is naturally determined by the running of the coupling which is logarithmic and can result in huge factors in the linear scale.

Let us make simple estimates. Namely, the $U(1)$ critical coupling is well known, $e^2_{crit} \sim 1$. In the QCD case we can rewrite the condition (7) as a condition on the R_{phys}. In the realistic case we have at the LEP energies $E^2 \sim (100\ \text{GeV})^2$, $\alpha \approx 0.1$. Then

$$M_{phys} \sim \text{TeV} \tag{28}$$

and, remarkably enough, we are getting rather the weak interactions scale than $\sim \Lambda_{QCD}$.

Also, the $SU(2)$ lattice measurements typically refer to $\beta \sim 2.6$ while our guess about R_{phys} asks for measurements at $\beta \sim 4$ which are absolutely unrealistic at the moment.

Thus, we come to a paradoxical conclusion that the presently available β are too low to see dissolution of the monopoles at small distances. Moreover, because the running of the coupling is only logarithmic the scale of of the onset asymptotic freedom – which is defined now as the vanishing of the excess of the monopole action compared to the zero-point-fluctuations action– can be very far off.

It is amusing to notice [8] that in case of the $SU(3)$ gluodynamics on the lattice $g^2 = 1$, or $\beta = 6$ corresponds to the lattice spacing $a \approx 0.1\ \ fm$ and the scale is:

$$R^{SU(3)}_{phys} \sim (2\text{GeV})^{-1} .$$

Thus, through dedicated studies of the monopoles in the $SU(3)$ case it is possible to clarify whether there is a crucial change in the monopole structure at the point $g^2(a) \approx 1$.

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[8] The observation is due to M.I. Polikarpov.

5 Conclusions

We have argued that data are emerging which indicate that QCD, when projected onto the scalar-field theory via monopoles corresponds to a fine tuned theory. Which is if course extremely interesting, if true, in view of the mystery of the fine tuning in the Standard Model. The monopoles which we considered are defined ("detected") through the Maximal Abelian projection. However, the mass scales which exhibit mass hierarchy are gauge independent. The scales are provided by the $SU(2)$ invariant action per unit length of the monopole trajectory, on one hand, and by the temperature of the phase transition, on the other. More generally, we have found that the polymer approach allows to get a new insight into the mechanism of the monopole condensation.

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