RG & EFT for nuclear forces

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- Low momentum interactions:
  Using the RG to simplify the nuclear force for many-body calculations.

- **Application of chiral perturbation theory to nuclear systems:**
  How to apply perturbation theory to a non-perturbative problem?

- Three-nucleon forces:
  importance of 3NF's for the quantitative description of (light) nuclei
  relation to low momentum interactions

Application of ChPT to Nuclei

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Nuclear forces are usually based on phenomenology.

1π exchange + short range ansatz + NN data ➡ NN force model

The outcome is not unique !

- the relation to QCD is lost!
  How to relate the NN system to other strong interaction processes ?

- 3NF's and MEC's are important for a quantitative description of many observables
  How to define **consistent** 3NF's and MEC's?

- Lattice QCD will be able to predict NN, 3N observables for high pion masses.
  How extrapolate to physical pion masses?
  How to make use of the results for more complex systems?

Find appropriate degrees of freedom and make an EFT for nuclear systems!

EFT of QCD

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What properties of QCD do we want use to build the EFT?

$$\mathcal{L}_{QCD} = \bar{q}_L\, i\not{D}\, q_L + \bar{q}_R\, i\not{D}\, q_R - \frac{1}{2}\,\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} - \bar{q}_R\, \mathcal{M}\, q_L - \bar{q}_L\, \mathcal{M}\, q_R$$

$$\not{D} = \not{\partial} - i g_s \not{G}^a\, T^a; \qquad T^a = \text{Gell-Mann matrices}$$

$$\mathcal{M} = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$ quark mass matrix; here only flavor SU(2) sector

$$q_{L,R} = \frac{1}{2}\,(1 \pm \gamma_5) q$$ projection of on left/right handed quarks

In this form chiral symmetry becomes apparent:

m_u and m_d (5 and 9 MeV for the usual renormalization scale of 1 GeV)
are very small compared to typical hadronic masses (approx. 1 GeV)

$\mathcal{M} \approx 0$ → QCD Lagragian becomes chirally symmetric

$$SU_L(2) \otimes SU_R(2)$$

EFT of QCD

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$SU_L(2) \otimes SU_R(2)$ is an approximate symmetry

m_u and m_d are finite.
But in the SU(2) sector chiral symmetry should be a good approximation.

Experimental observation is:

- There are isospin multiplets like p,n or $\Sigma^+, \Sigma^-, \Sigma^0$
  This means that isospin symmetry ("vectorial subgroup" with L=R is realized)

- There are no opposite parity partners for these states with at least approximately the same mass!

⇒ The "axial" part of chiral symmetry is spontaneously broken down!

EFT of QCD

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$SU_L(2) \otimes SU_R(2)$ is spontaneously broken down to $SU_V(2)$

(the Lagragian is invariant, but the vacuum is not)

→ Goldstone's theorem: there are massless bosons (Goldstone bosons)

Experimentally, we find π^+, π^-, π^0 !

Since chiral symmetry is also explicitly broken, the pions are not strictly massless, but at least approximately $m_\pi \approx 138\ \mathrm{MeV} \ll 1\ \mathrm{GeV}$

We again see that chiral symmetry is spontaneously broken.

EFT of QCD

Which degrees of freedom should we choose for an EFT for QCD?

- nucleons, if we want to use it for nuclear physics
- pions, since these are almost massless,
  mass of the pions is of the order of a typical momentum in a nucleus

Chiral symmetry constrains the possibly interactions of pions, and pions and nucleons.

→ A predictive effective theory for strongly interacting systems can be formulated: Chiral Perturbation Theory (ChPT)

The explicit breaking of chiral symmetry can be taken into account by pion mass dependent terms that break chiral symmetry the same way as it is broken in QCD!

Since the pion mass is small, these terms are suppressed.

The inclusion of nucleons requires special care, because of its large mass:
Heavy Baryon Chiral Perturbation Theory.

EFT of QCD

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Being an EFT, one can formulate an infinite number of terms in the Lagrangian.

The terms are ordered according to the number of derivatives and quark (pion) mass insertions (power counting).
This leads to an expansion in terms of a typical small momentum Q.

The leading order Lagrangians, e.g., read (the part of interest for the NN forces)

$$\mathcal{L}^{(0)} = \frac{1}{2}\partial_\mu\pi\cdot\partial^\mu\pi - \frac{1}{2}m_\pi{}^2\pi^2 + N^\dagger\left[i\partial_0 + \frac{g_A}{2f_\pi}\tau\vec{\sigma}\cdot\vec{\nabla}\pi - \frac{1}{4f_\pi{}^2}\tau\cdot(\pi\times\dot{\pi})\right]N - \frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger\vec{\sigma}N)(N^\dagger\vec{\sigma}N) + \ldots,$$

$$\mathcal{L}^{(1)} = N^\dagger\left[4c_1 m_\pi{}^2 - \frac{2c_1}{f_\pi{}^2}m_\pi{}^2\pi^2 + \frac{c_2}{f_\pi{}^2}\dot{\pi}^2 + \frac{c_3}{f_\pi{}^2}(\partial_\mu\pi\cdot\partial^\mu\pi) - \frac{c_4}{2f_\pi{}^2}\epsilon_{ijk}\,\epsilon_{abc}\,\sigma_i\tau_a(\nabla_j\,\pi_b)(\nabla_k\,\pi_c)\right]N - \frac{D}{4f_\pi}(N^\dagger N)(N^\dagger\vec{\sigma}\tau N)\cdot\vec{\nabla}\pi - \frac{1}{2}E\,(N^\dagger N)(N^\dagger\tau N)\cdot(N^\dagger\tau N) + \ldots$$

Based on the order of the Lagrangian, one can the estimate the order of a diagram

$$v = -4 + 2N + 2L + \sum_i\left(d_i + \frac{n_i}{2} - 2\right)$$

The infinite number of diagrams can be ordered, so that only a finite number contributes at each order.

Chiral potential

The infinite number of diagrams can be ordered, so that only a finite number contributes at each order.

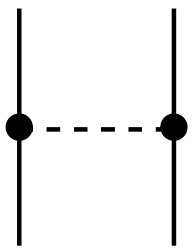
If this was strictly true, we would **not have bound states** within this framework.

Let's look at one example and do the naive counting explicitly:

$$v = -4 + 2N + 2L + \sum_i \left(d_i + \frac{n_i}{2} - 2 \right)$$

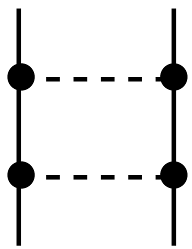
$$\Delta_i = d_i + \frac{n_i}{2} - 2 = 0$$

vertices



$N = 2, \quad L = 0$

$\nu = -4 + 2 \cdot 2 + 2 \cdot 0 + 2 \cdot 0 = 0$



$N = 2, \quad L = 1$

$\nu = -4 + 2 \cdot 2 + 2 \cdot 1 + 2 \cdot 0 = 2$

We naively find the one pion exchange in leading order, and the two pion exchange in subleading, etc.

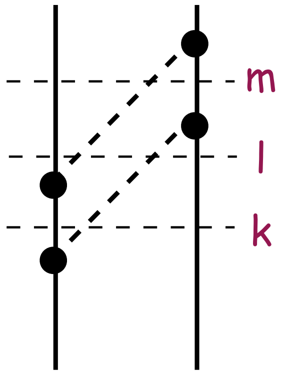
Chiral potential

Where is the pitfall?

Let's do time-ordered perturbation theory for the same two diagrams

$$T_{ij} = \langle i|H_I|j\rangle + \sum_k \frac{\langle i|H_I|k\rangle\langle k|H_I|j\rangle}{E_j + i\epsilon - E_k} + \sum_{kl} \frac{\langle i|H_I|k\rangle\langle k|H_I|l\rangle\langle l|H_I|j\rangle}{(E_j + i\epsilon - E_k)(E_j + i\epsilon - E_l)}$$
$$+ \sum_{klm} \frac{\langle i|H_I|k\rangle\langle k|H_I|l\rangle\langle l|H_I|m\rangle\langle m|H_I|j\rangle}{(E_j + i\epsilon - E_k)(E_j + i\epsilon - E_l)(E_j + i\epsilon - E_m)} + \ldots$$

For all particles having a typically small momentum Q, we can estimate the energy denominators



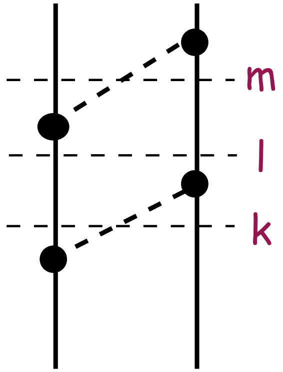
$$\frac{1}{E_j + i\epsilon - E_{k,l,m}} \propto \frac{1}{\frac{Q^2}{2m} + m_\pi} \propto \frac{1}{Q}$$

this diagram is irreducible in the sense that no two nucleon intermediate state appears

1/Q is in agreement with the power counting estimate

Chiral potential

The same estimate for a diagram with a two nucleon intermediate state is different!



$$\frac{1}{E_j + i\epsilon - E_{k,m}} \propto \frac{1}{\frac{Q^2}{2m} + m_\pi} \propto \frac{1}{Q}$$

$$\frac{1}{E_j + i\epsilon - E_l} \propto \frac{1}{\frac{Q^2}{2m}} \propto \frac{1}{Q}\,\frac{2m}{Q}$$

This diagram is reducible in the sense that purely nucleonic intermediate states appear

There is an enhancement of order m/Q !

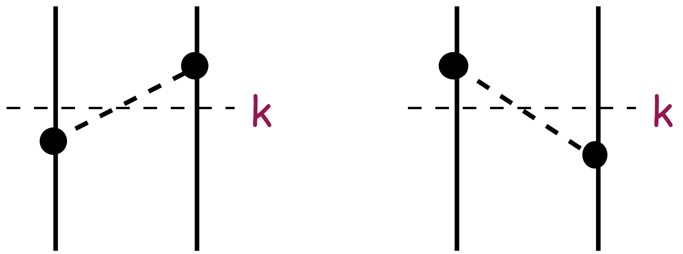
→ This enhancement is sufficient to make the theory **non-perturbative**.

Good news: the irreducible diagrams give a potential, which can be summed numerically using a LS equation

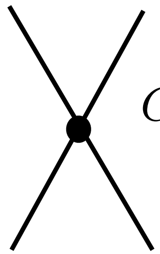
This defines a **chiral potential**.

LO interaction

Leading order (LO) interaction (in time-ordered perturbation theory):



$\frac{g_A}{2f_\pi}\tau\vec{\sigma}\cdot\vec{\nabla}\pi$ vertex



$C_T \;/\; C_S$ vertex

$$
\begin{aligned}
V_{1\pi}(\vec{q}) &= \left(\frac{g_a}{2f_\pi}\right)^2 \vec{\tau}_1\cdot\vec{\tau}_2 \,\frac{1}{2\omega_q}\,\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\frac{p_1^2}{2m}+\frac{p_2^2}{2m}-\frac{p_1'^{\,2}}{2m}-\frac{p_2^2}{2m}-\omega_q} \\
&\quad + \left(\frac{g_a}{2f_\pi}\right)^2 \vec{\tau}_1\cdot\vec{\tau}_2 \,\frac{1}{2\omega_q}\,\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\frac{p_1^2}{2m}+\frac{p_2^2}{2m}-\frac{p_1{}^2}{2m}-\frac{p_2'^{\,2}}{2m}-\omega_q} \\
&= -\left(\frac{g_a}{2f_\pi}\right)^2 \vec{\tau}_1\cdot\vec{\tau}_2 \,\frac{\vec{\sigma}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{\vec{q}^2+m_\pi^2}
\end{aligned}
$$

$$
V_C = C_S + C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2
$$

Other schemes (e.g. Okubo transformation) exist to obtain the potential!

LO interaction

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We need to solve the LS equation for

$$V(\vec{q}) = -\left(\frac{g_a}{2f_\pi}\right)^2 \vec{\tau}_1 \cdot \vec{\tau}_2 \, \frac{\vec{\sigma}_1 \cdot \vec{q} \, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^2 + m_\pi^2} + C_S + C_T \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Regularization is required !

$$V(\vec{p}\,', \vec{p}) \rightarrow e^{-\left(\frac{p'}{\Lambda}\right)^{2n}} \, V(\vec{p}\,', \vec{p}) \, e^{-\left(\frac{p}{\Lambda}\right)^{2n}}$$

This choices has the advantage that the counter terms only contribute in s-waves.

$$V_{ll'}(p', p) = \int d\hat{p} \, d\hat{p}\,' Y_l^*(\hat{p}\,') \, C \, Y_{l'}(\hat{p}) = \delta_{l\,0} \, \delta_{l'\,0} \, 4\pi \, C$$

Higher partial waves are a prediction,
if there are no counter terms contributing to them.

LO interaction

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A short note on the relation to yesterday's talk:

- The RG equation for the vlowk potential made the observables exactly cutoff independent.
  The numerical solution automatically put in an "infinite" number of counter terms.

- Here, I will follow Peter Lepage's approach:
  We only add a finite number of counter terms and will retain a residual cutoff dependence.
  These counter terms need to be fitted to data.

- The question, I want to address is:
  How many counter terms do I have do add additionally to the ones requires by naive power counting?
  How can I decide without knowing the experimental result?

I will show that studying the cutoff dependence for large cutoffs helps to decide on that.

LO interaction

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It is instructive to look at the potential in configuration space!

$$V_{1\pi}(\vec{r}) = \frac{m_\pi^3}{12\pi}\left(\frac{g_A}{2f_\pi}\right)^2 \vec{\tau}_1\cdot\vec{\tau}_2 \left[T(r)\; S_{12} + Y(r)\; \vec{\sigma}_1\cdot\vec{\sigma}_2\right]$$

$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$T(r) = \frac{e^{-m_\pi r}}{m_\pi r}\left[1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right] \propto \frac{1}{r^3}$$

$$Y(r) = \frac{e^{-m_\pi r}}{m_\pi r}$$

The potential is singular!

QM of singular potentials:

We might need a counter term in every partial wave, where the tensor force is attractive!

Singlets are not affected.

$\tau_1\cdot\tau_2\, S_{12}$

	$s=1$	$l=j-1$	$l=j$	$l=j+1$
$t=1$	$l'=j-1$	$-2\,\frac{j-1}{2j+1}$	0	$6\frac{\sqrt{j(j+1)}}{2j+1}$
	$l'=j$	0	2	0
	$l'=j+1$	$6\frac{\sqrt{j(j+1)}}{2j+1}$	0	$-2\,\frac{j+2}{2j+1}$
$t=0$	$l'=j-1$	$6\,\frac{j-1}{2j+1}$	0	$-18\frac{\sqrt{j(j+1)}}{2j+1}$
	$l'=j$	0	-6	0
	$l'=j+1$	$-18\frac{\sqrt{j(j+1)}}{2j+1}$	0	$6\,\frac{j+2}{2j+1}$

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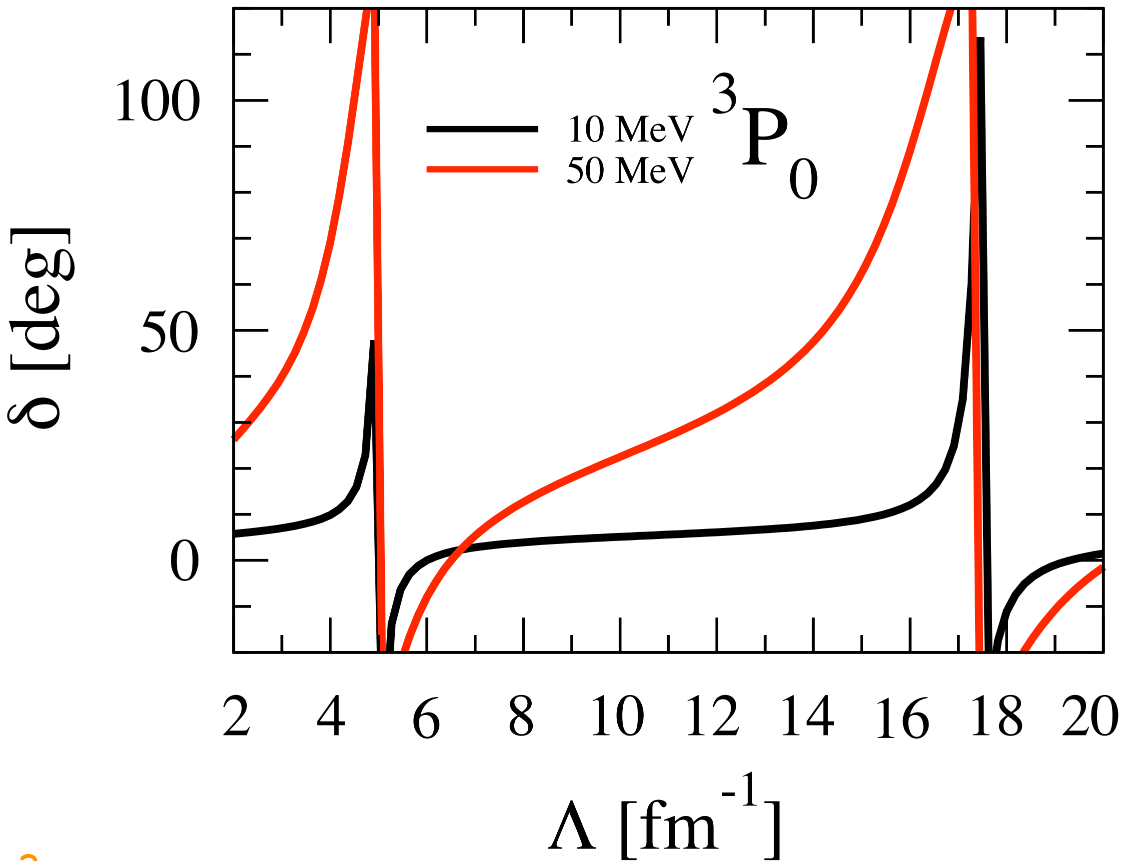
We find the expected cutoff dependence for attractive triplet channels



We confirm the limit cycle behavior for attractive singular interactions

Size and slope within the cycles depends on the partial wave

The slope of the "plateau region" also depends on the energy

$^3P_0$ is the worst case, because the variation is strongest for small cutoffs and the slope is the steepest one.

What is the problem here?

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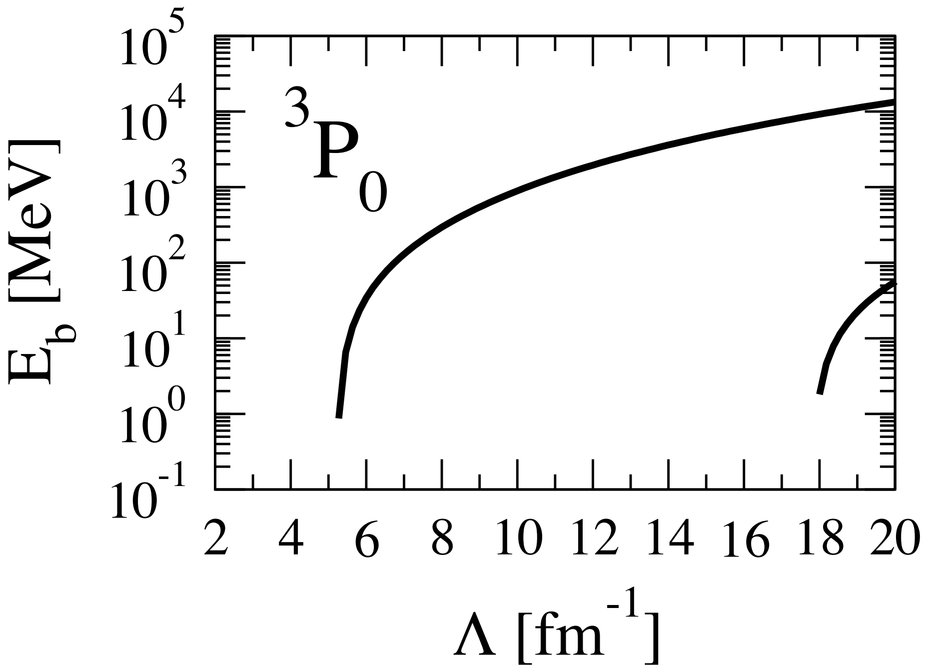
- This cutoff dependence is induced by spurious bound states coming in from threshold.
- For $\Lambda \leq 20\ \mathrm{fm}^{-1}$, we find bound states in $^3P_0$



-
- The binding energies increase very rapidly to several hundred MeV

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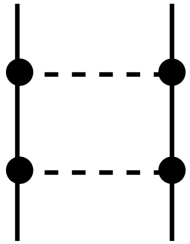
How can it happen that we apparently do not have cutoff independence of the results?

This can partly be understood looking at perturbation theory:



The 2π-exchange diagrams can not be renormalized without additional counter terms!
Doing perturbation theory, we find these counter terms at second order!

But using the LS equation for the potential, we do include them.

Then, based on perturbation theory, one finds that an **infinite number of counter terms** is necessary!

1) solution: KSW counting (treat pions perturbatively) → fails!

2) solution: let us look carefully what happens if we do not use perturbation theory

motivation: QM of singular potentials can be made well defined.
Iteration of the potential (Weinberg counting) works very well.

LO interaction

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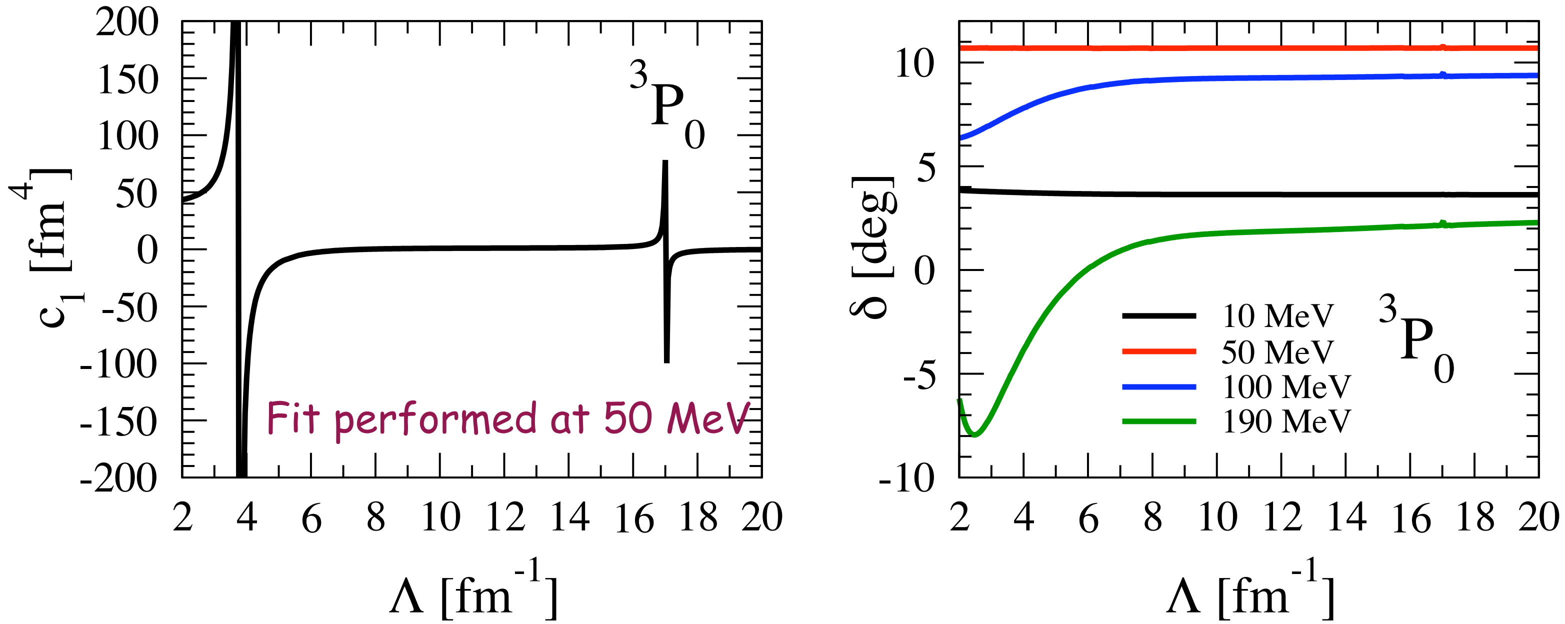
The renormalization of singular interactions is possible with
1 counter term (boundary condition) per partial wave (which is still an infinite number)

In LO, this requires the promotion of counter terms from naïvely higher orders.

We use $V_i = \frac{c_i}{(2\pi)^3} p'p$ in P-waves, which is supposedly suppressed by $(Q/\Lambda)^2$



As expected, we obtain Λ independence for all energies.
We, however, see that the “bare” counter terms get infinite!

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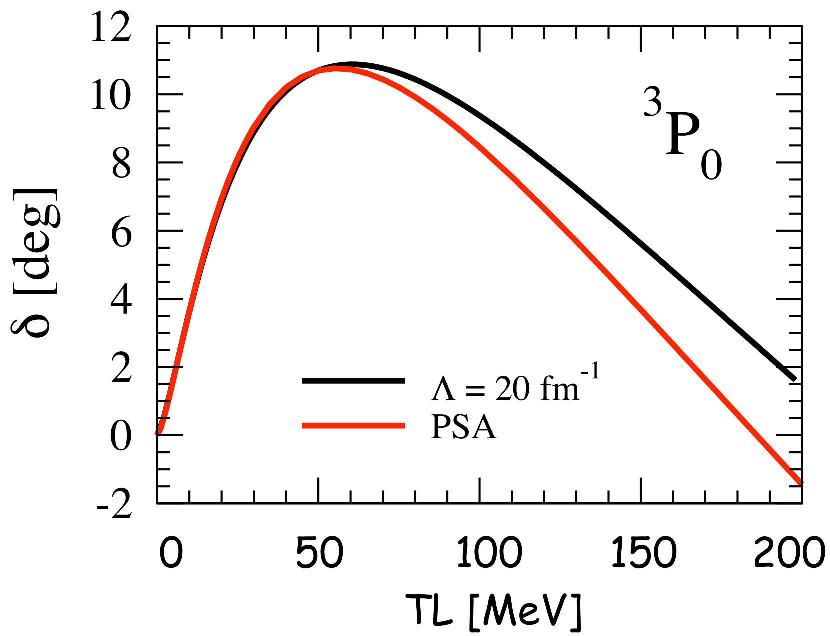
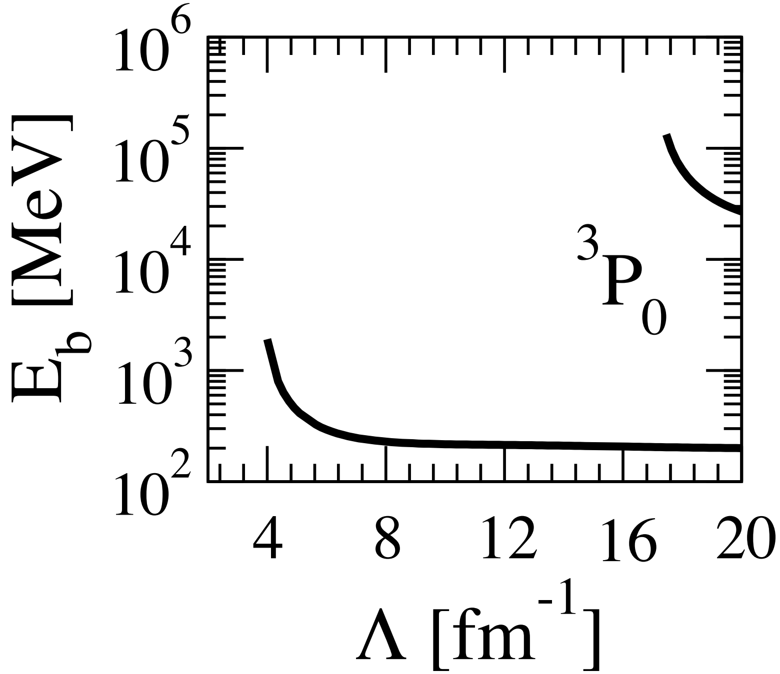
Adding this single counter term leads to an astonishingly good description of the data!
Bound states come in from infinity!

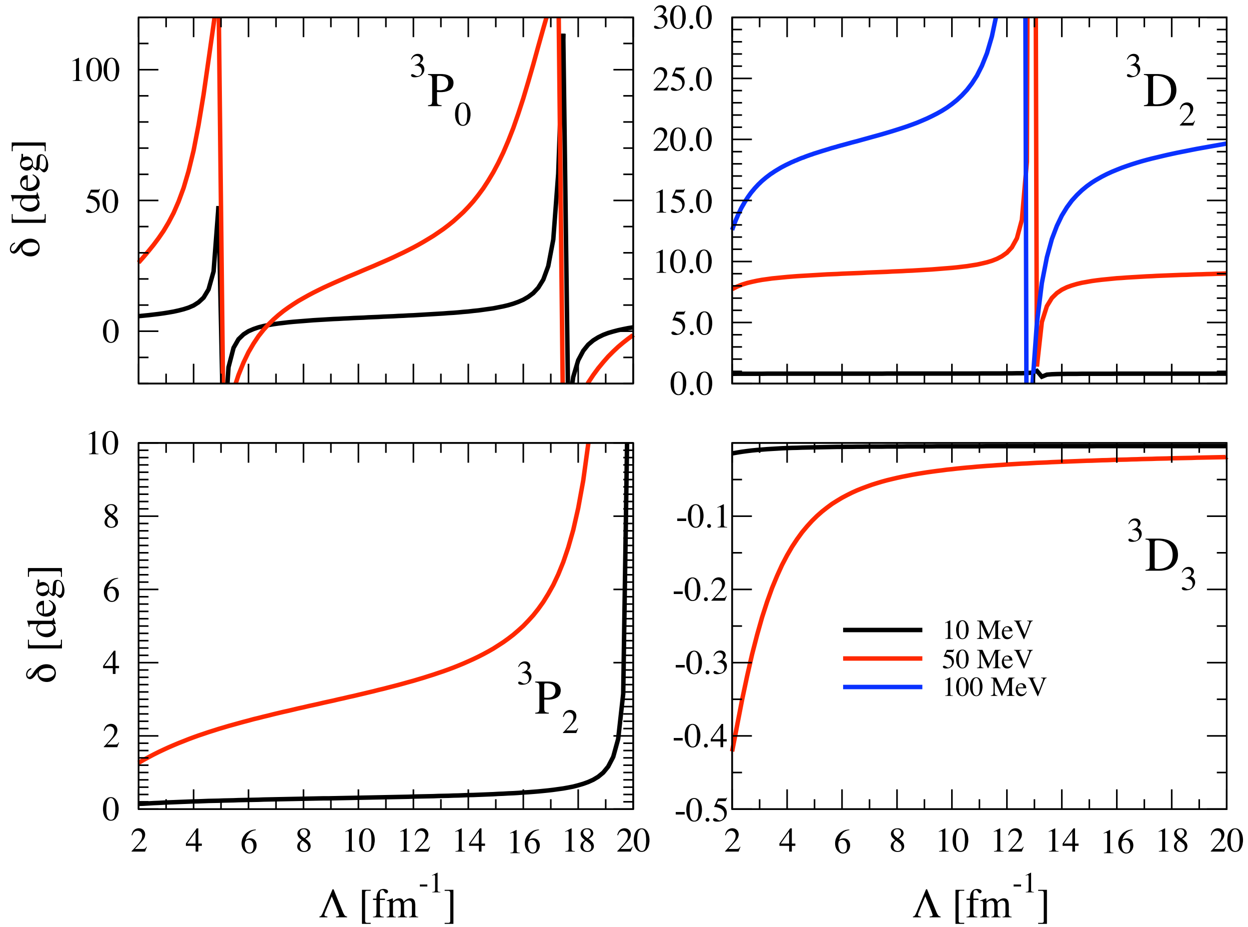




But this leaves us with the problem that we "require" a counter term in all attractive tensor force channels!

Do we loose predictivity?

LO interaction



3P0 is the "worst" case. The centrifugal barrier screens the short range part in higher partial waves effectively.

LO potential

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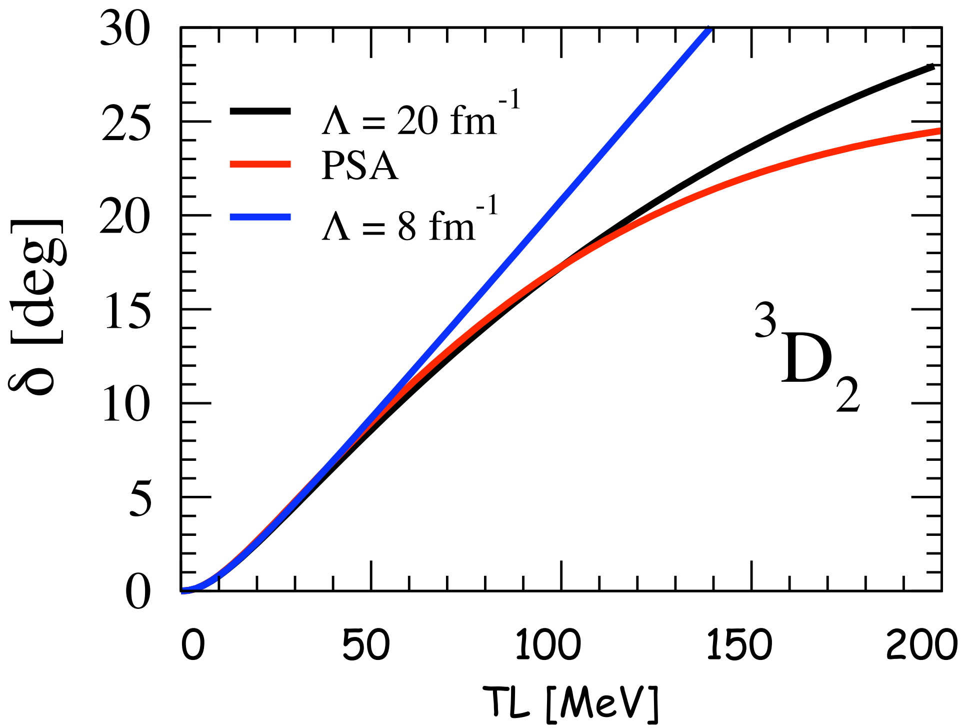
For low energy, the inclusion of a higher order counter term is not required!
But it does not hurt either!



We included counter terms in 3P0, 3P2-3F2 and 3D2,
but leave them out for all other channels.

This is sensible, since we will **not** put the cutoff to infinity and find a plateau!

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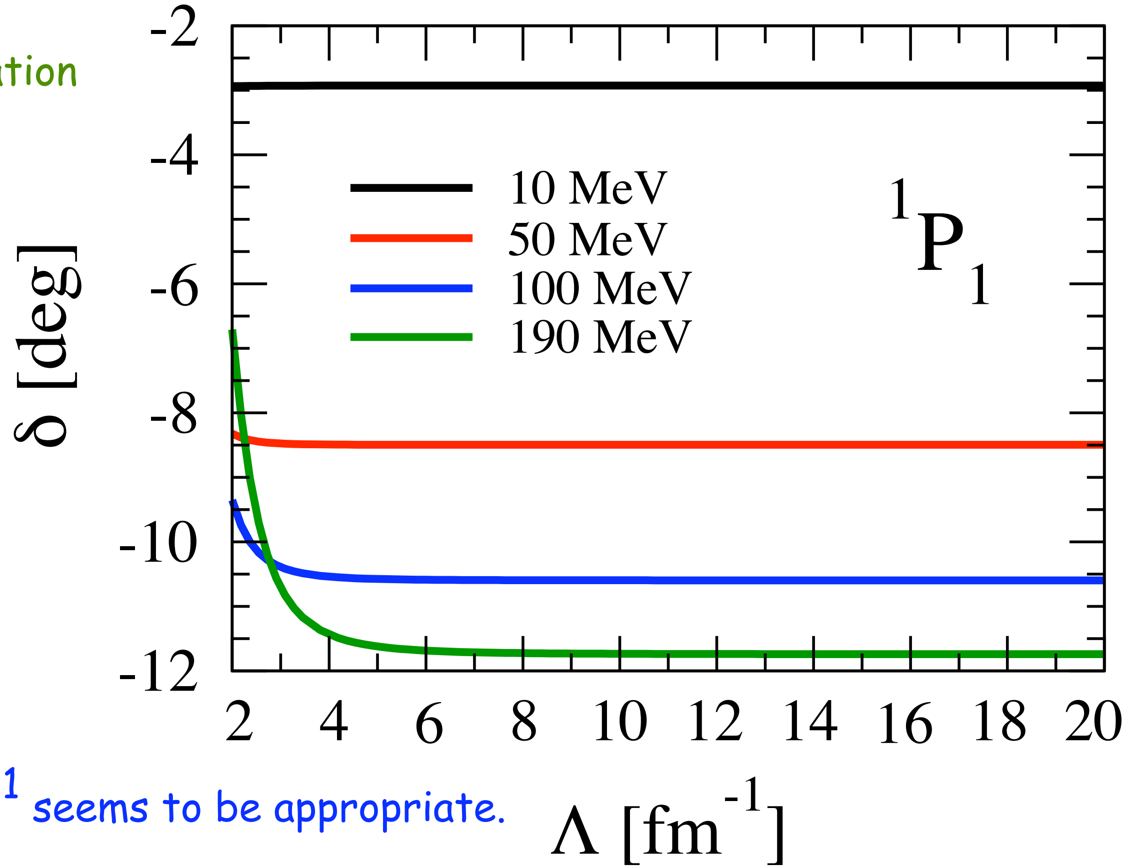
Numerically, one does not observe any cutoff dependence in singlets for large cutoffs.

This indicates that no renormalization is necessary in these channels.



Again, up to 100 MeV, Λ = 3-4 fm^{-1} seems to be appropriate.

Also Λ = 2.5 fm^{-1} leads to a reasonably independent result.

For 190 MeV, Λ = 5-6 fm^{-1} insures almost converged results.

Numerically, no inconsistency of the power counting is found in singlets.

LO potential

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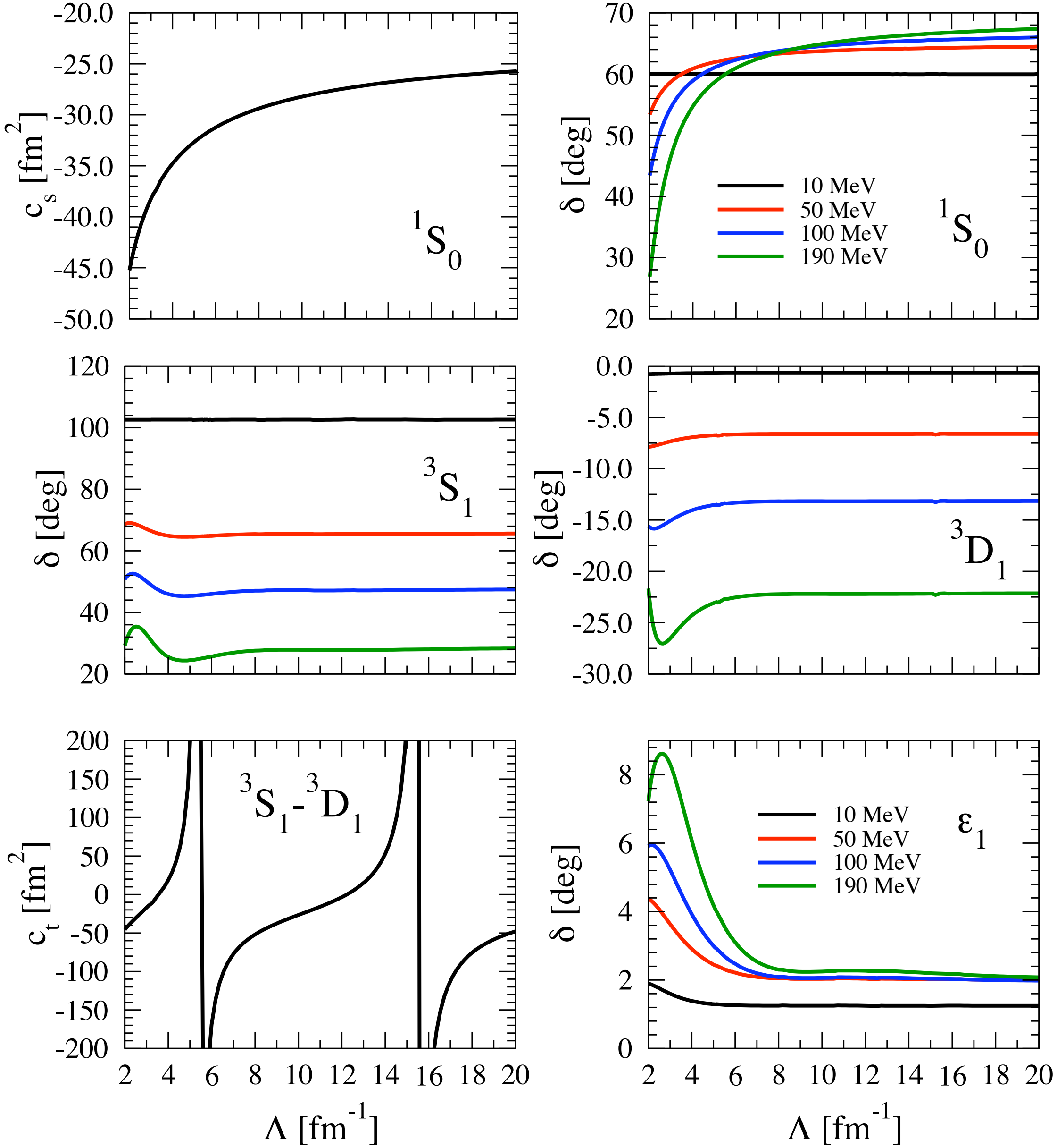


The s-wave channels do not need additional counter terms!

The "bare" counter term in 3S1 may be infinite, but the results stay cutoff independent.

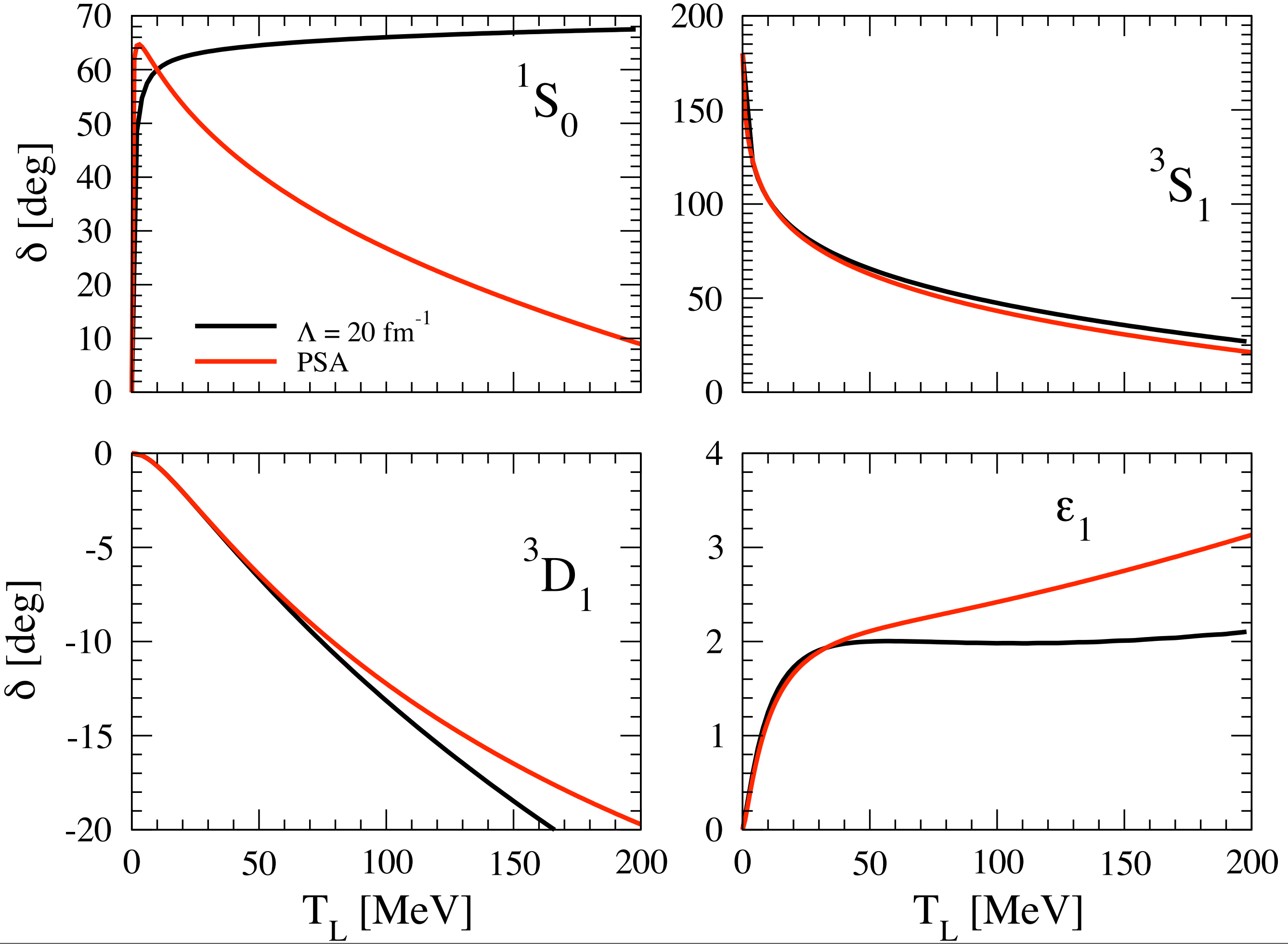
This is a "bare" coupling constant.

We observe that the mixing angle get cutoff independent for larger cutoffs as usual.

LO potential

The $^1S_0$ prediction is poor! ---- This will be resolved in higher orders!

Small cutoffs improve the predictions, but the result will strongly dependent on the cutoff ---- Higher order counter terms?



LO potential

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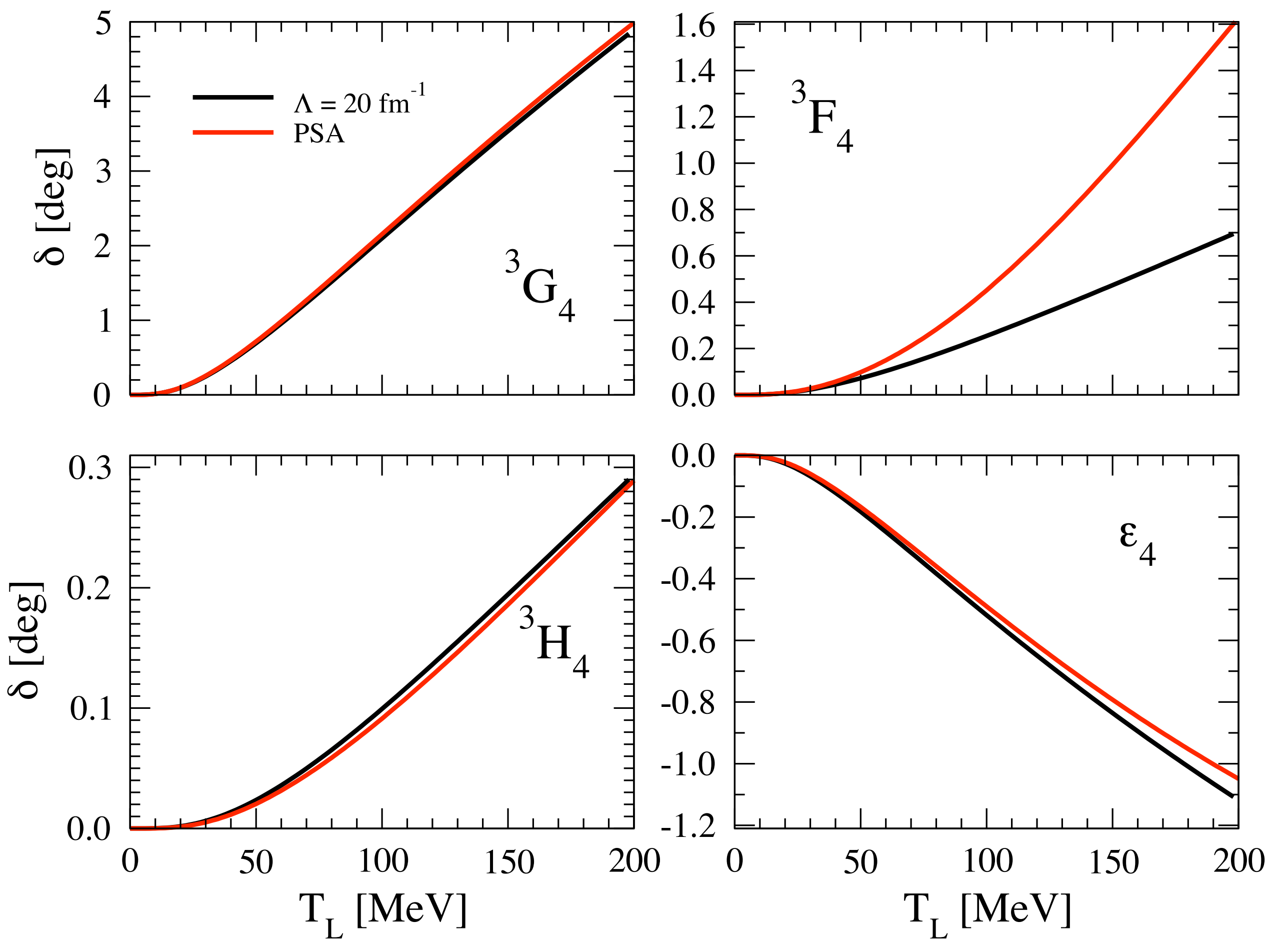
High partial waves remain predictions of the chiral potential.
E.g. some typical examples:



LO potential

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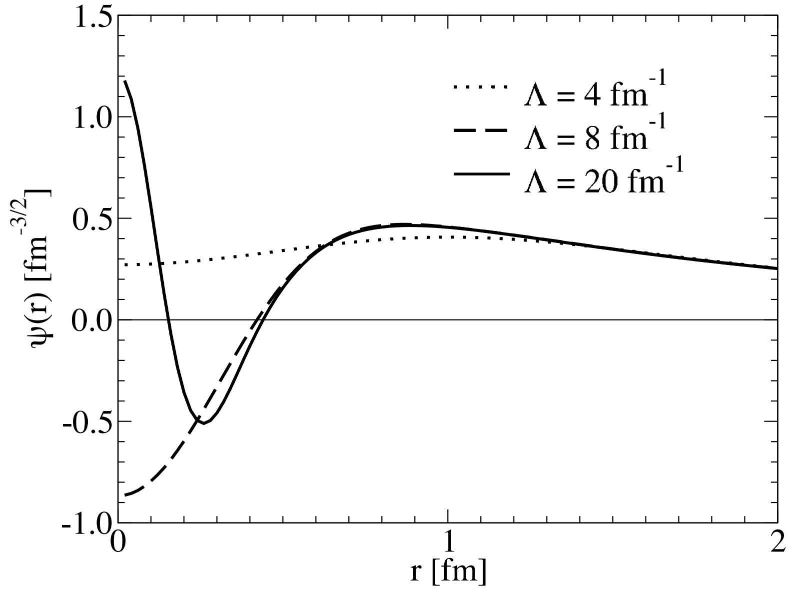
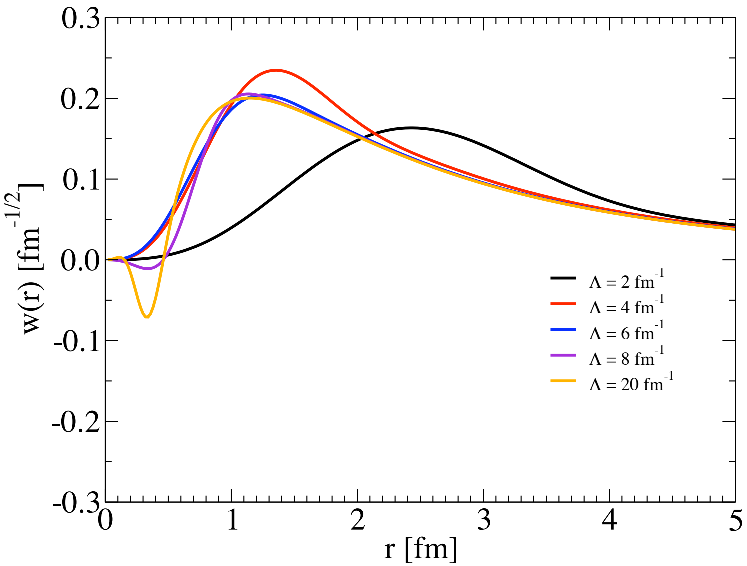
So far, we have looked at the scattering observables:

The deuteron is interesting, because it should be described well, since it is very loosely bound.

However, do the spurious bound states interfere somehow?





LO potential

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What are the predictions for the deuteron?

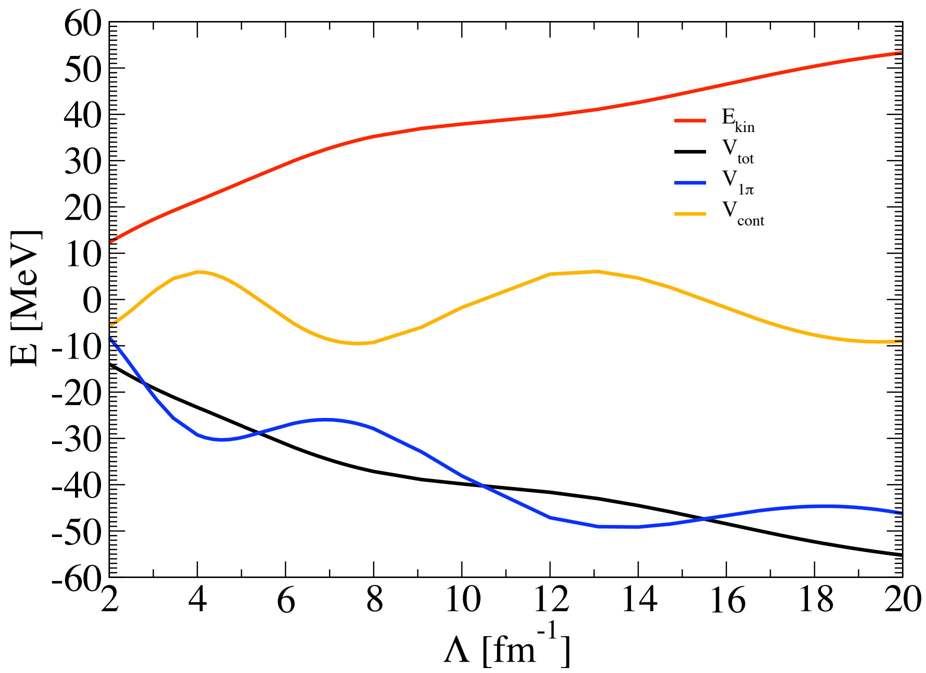
The deuteron is very loosely bound → high momentum components not important

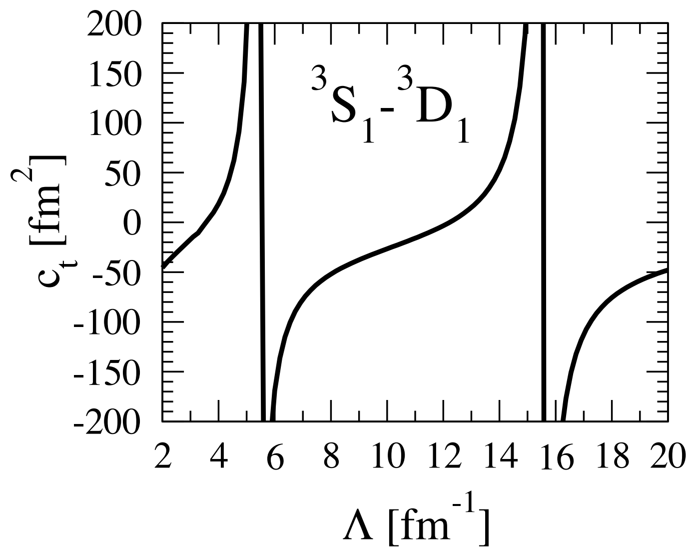
Λ[fm^{-1}]	E [MeV]	T [MeV]	P_D[%]	A_S [fm$^{-1/2}$]	η	r [fm]	Q_d [fm^2]	n
2	2.225	28.91	5.24	0.839	0.030	1.889	0.3005	1
3	2.225	38.45	8.09	0.855	0.028	1.913	0.2942	1
4	2.225	45.48	8.23	0.866	0.027	1.933	0.2827	1
5	2.225	53.53	7.49	0.867	0.025	1.935	0.2747	1
6	2.224	62.33	6.94	0.866	0.025	1.932	0.2704	2
7	2.225	70.16	6.73	0.865	0.025	1.928	0.2683	2
8	2.225	75.95	6.76	0.864	0.026	1.926	0.2676	2
10	2.227	81.99	7.00	0.864	0.026	1.925	0.2674	2
12	2.227	85.80	7.14	0.864	0.026	1.925	0.2675	2
14	2.224	91.94	7.14	0.863	0.026	1.926	0.2675	2
Expt.	2.225	—	—	0.8846	0.026	1.9671	0.2859	1

LO potential

We find that the "bare" coupling constants maybe be infinite.
However, the expectation value of the contact terms remain finite.

$$\langle H \rangle = \langle T \rangle + \langle V_{1\pi} \rangle + \langle V_c \rangle$$





3N system

For purely short range potentials, one finds that short 3NF's are needed to get well defined results.

Chiral Perturbation Theory does not predict a 3NF's in LO!
and the LO chiral potential has got a finite range.

Does this really imply that no 3NF's are necessary at LO?

We need to studying the cutoff dependence for the 3H bound state to find out, whether we are missing some 3N counter terms (namely 3NF's) ...

3N system

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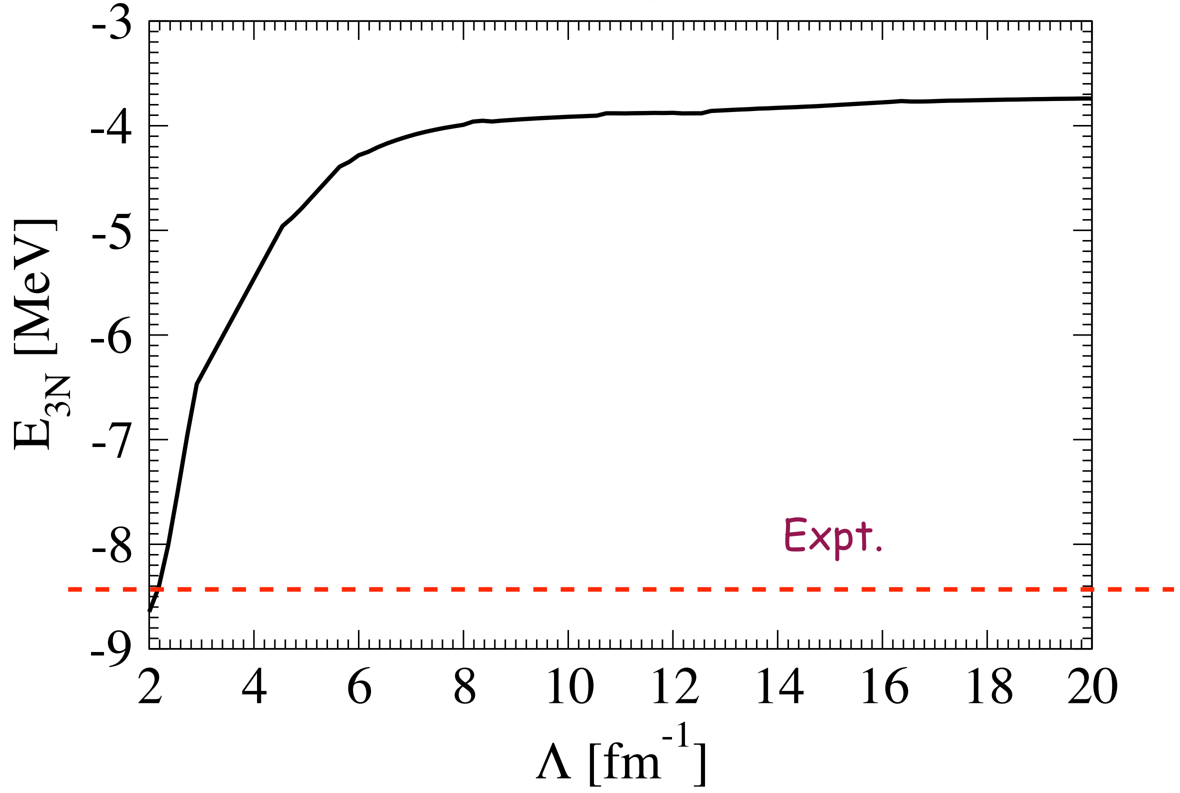
Indeed, the binding energy becomes cutoff independent!
We find approx. 4 MeV. The discrepancy to the experimental value is 4 MeV!
To we observe a breakdown of the theory?



Estimate the higher order effect should be based on potential energy

$$\Delta E \propto \left(\frac{Q}{700\ \mathrm{MeV}}\right)^2 \langle V \rangle \approx \left(\frac{1}{5}\right)^2 \ 50\ \mathrm{MeV} \approx 2\mathrm{MeV}$$

Summary

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- The appropriate effective field theory of QCD at low energies is
  Chiral Perturbation Theory.

- In NN systems, some kind of non-perturbativity is obvious,
  because we find a bound deuteron!

  The EFT expressions show that iterated, irreducible diagrams are enhanced

  → We define a chiral potential.

- Naive counting cannot absorb all cutoff dependence, because
  of the unphysical behavior of the LO potential.
  To define a reasonable cutoff is difficult, because no plateau regions are seen.
  Additional counter terms resolve the problem.

- Then spurious bound states do not interfere with low energy physics!

- The extension to the 3N system is predictive!