RG & EFT for nuclear forces

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- Low momentum interactions:
  Using the RG to simplify the nuclear force for many-body calculations.

- Application of chiral perturbation theory to nuclear systems:
  How to apply perturbation theory to a non-perturbative problem?

- **Three-nucleon forces:**
  importance of 3NF's for the quantitative description of (light) nuclei
  relation to low momentum interactions

Why are 3NF's important for nuclear physics?

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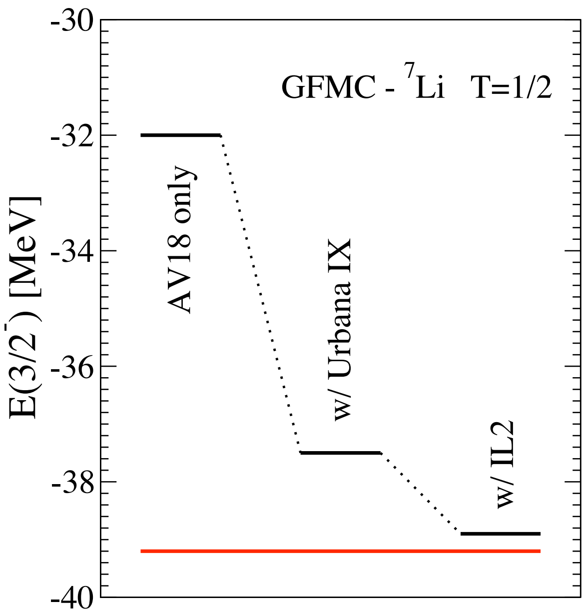
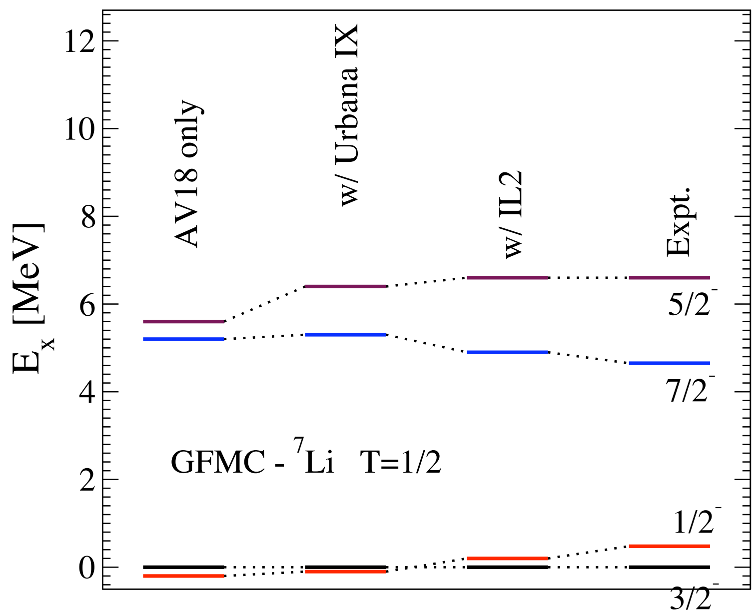
- e.g. the sensitivity of p-shell spectra & binding energies
  (here Green's Function Monte Carlo results from Pieper, Wiringa, Carlson et. al.)





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- p-shell spectra are sensitive to 3NF's and

➡ 3NF's are necessary to describe them

We want to understand the 3NF's based on chiral EFT!

Higher order interactions

- Chiral interactions have been worked out to higher orders

  → also three-nucleon force (3NF) terms are predicted

- EFT confirms the expectation that NN forces >> 3N forces > 4N forces > ...

- For NN, the terms up to $(Q/\Lambda)^4$ (=next-to-next-to-next-to-leading order=N3LO) have been worked out.

- Starting from this order, one is able to achieve a description of the data similar to the traditional interaction models (fitting 26 parameters).

- The fits of the counter terms have been performed for $\Lambda \approx 500$ MeV.

Higher order interactions

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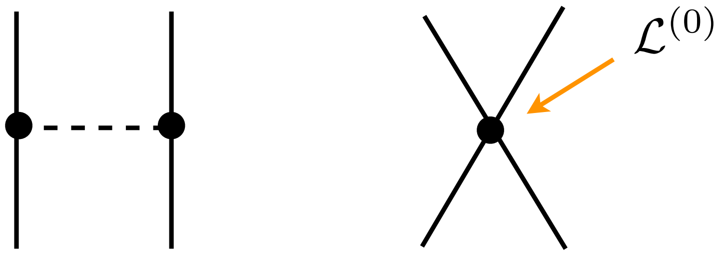
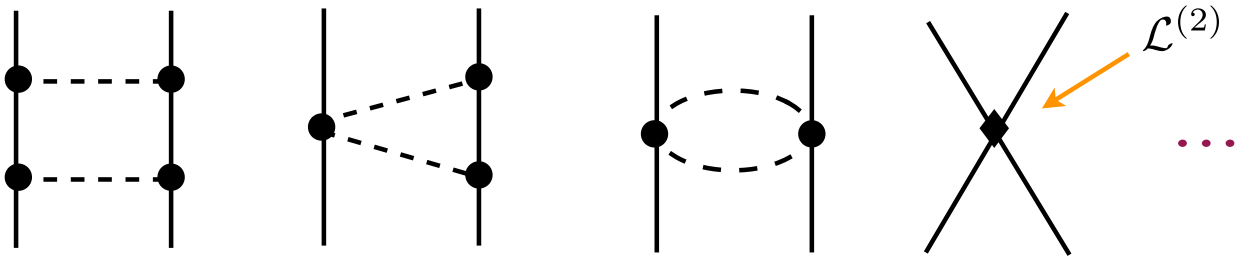
Contributions to the NN potential:

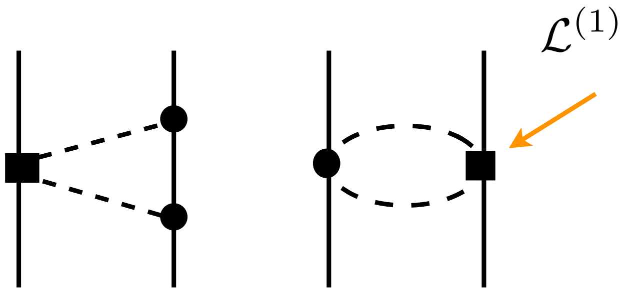
Q^0 (LO):



Q^2 (NLO):



Q^3 (NNLO):



...

$$v = -4 + 2N + 2L + \sum_i \left(d_i + \frac{n_i}{2} - 2 \right)$$

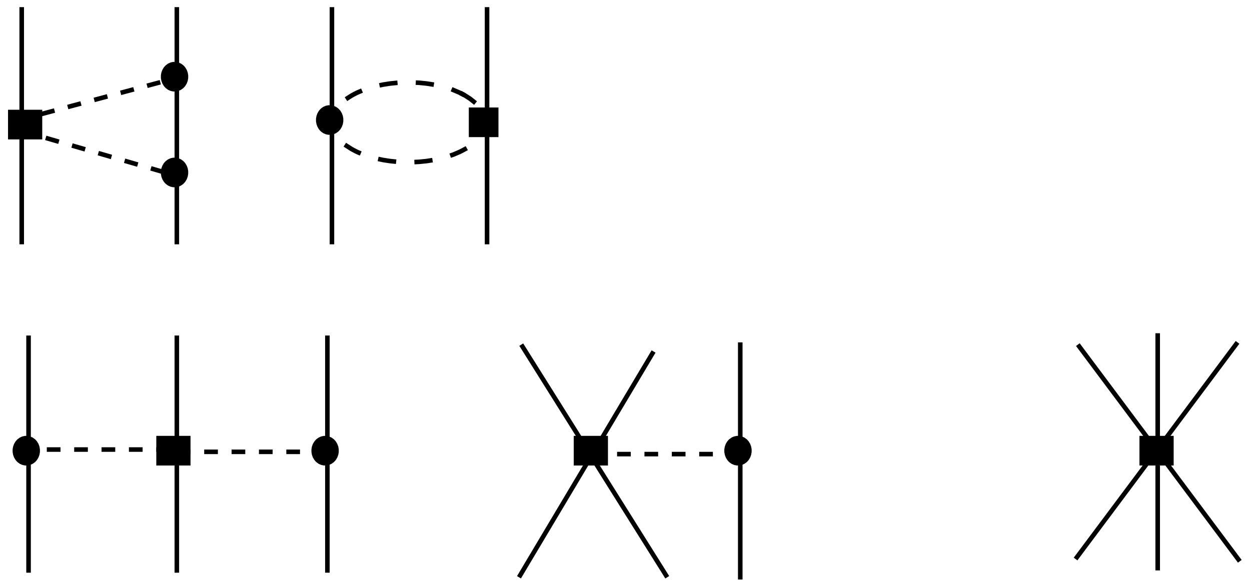
Note: No contribution at Q^1 and no counter terms at Q^3 !

In Q^4 : new pion exchanges & counter terms up to D-waves

3NF in EFT

At order Q^3 , we find the first contributions of 3NF's !

Q^3 (NNLO):



Most vertices of 3NF also appear in NN diagrams and are related to other processes !

3NF in EFT

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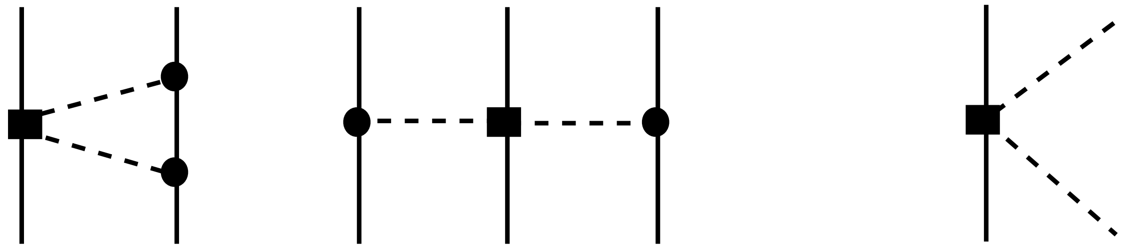
Example: "c_1, c_3 and c_4 terms" of the $\mathcal{L}^{(1)}$ Lagrangian

Q^3 (NNLO):



These are important diagrams for the 3NF, and they are subleading contributions to NN and πN scattering.

How can we determine their strength ?

1) Fit to NN data: long range part of the chiral interactions
   + counter terms + NN data
   → phase shifts analysis + some constraints on c_i

2) Fit to phase shifts for high angular momenta (3F4)

3) Fit to πN data

3NF in EFT

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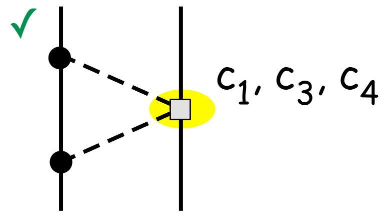
- Each approach has some disadvantages, which constrains the accuracy.
  But order of magnitude and signs are in agreement

	c_1	c_3	c_4
NN phase shift analysis	-0.76	-4.78	3.96
πN scattering (dispersion rel.)	-0.81	-4.70	3.40
πN scattering (directly)	-1.23	-5.94	3.47
NN pert. 3F4	-0.81	-3.40	3.40
NN potential fit to data	-0.81	-3.20	5.40



- Determination from πN scattering and fit to NN data agree approximately:
  connection of subleading 2π-exchange and πN supported!

- **but:** some determinations are highly controversial,
  sensitivity of the NN data is rather small,
  c_1 is not extracted from NN data, but input to the analysis

- A more accurate and independent determination is desirable!

➡ **Long term goal: Determination from 3N data!**

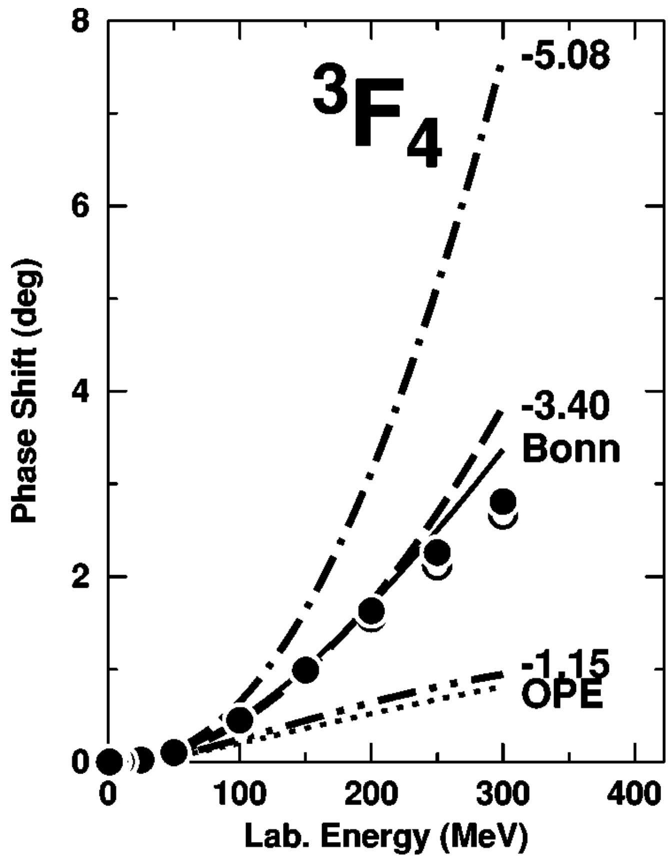
3NF of EFT

3F4 partial wave receives visible contributions from **2π-exchange**

and

is **perturbative!**

Leading counter terms are of order Q^6 !



from Entem & Machleidt (2002)

3NF in EFT

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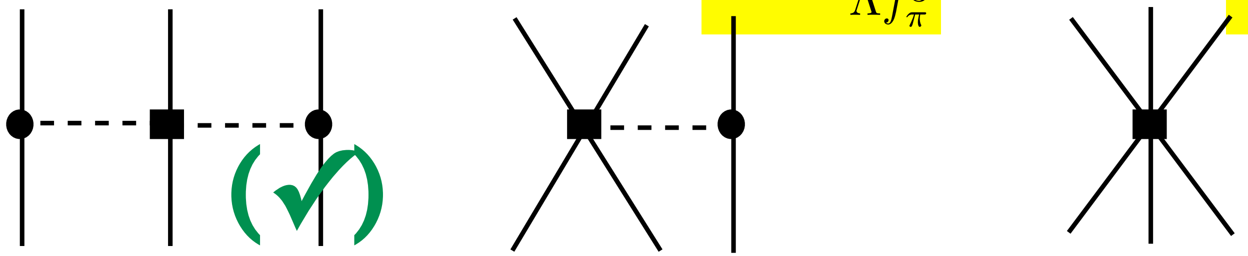
c_1, c_3 and c_4 terms are constrained



$$\propto c_D \cdot \frac{1}{\Lambda f_\pi^3}$$

$$\propto c_E \cdot \frac{1}{\Lambda f_\pi^4}$$

How can we determine the strength of the remaining two terms (D- and E-term)?

1) E-term: it is only related to nucleons, so we need to fit to e.g. 3N observable

2) D-term:

a) pion production in NN scattering
but: this has not been analyzed within this EFT yet.

b) Fit to another few-nucleon observable

3NF in EFT

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More specific: today's calculations were done with Idaho-N3LO

	c_1	c_3	c_4
NN phase shift analysis	-0.76	-4.78	3.96
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This insures the complete consistency of the NN force & 3NF.
Keep in mind that the determination is not perfect.

The cutoff will be 500 MeV ≈ 2.5 fm^{-1}.

Is this large enough for nuclei?

Indication: How strong is the dependence of the 3N binding energy on Λ?

E(^{3}H) changes by 40 keV out of 7.85 MeV,
when changes from Λ=500 MeV to Λ=600 MeV (without 3NF)

3NF in EFT

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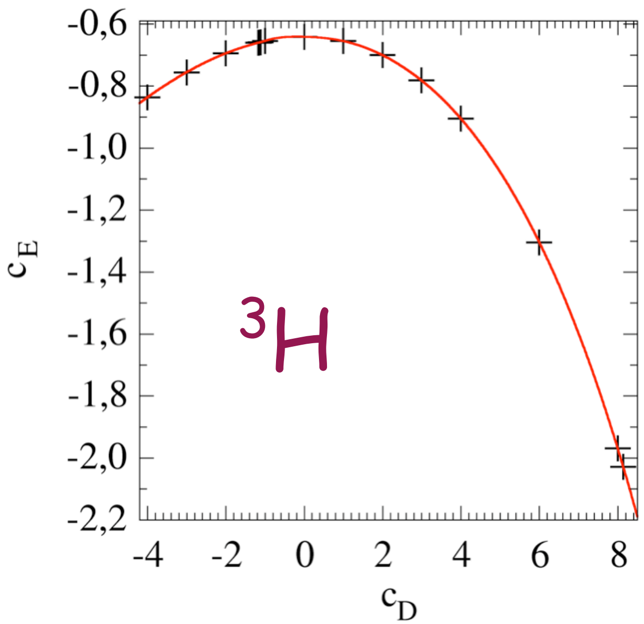
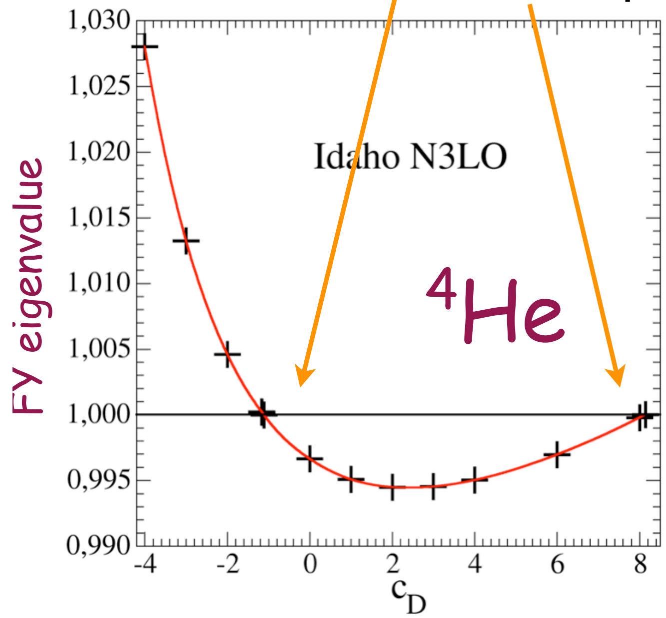
- Here, we use the ^{3}H and ^{4}He binding energies to fix the strength of the 3NF counter terms
- **we find two solutions that describe the 3N and 4N binding energies equally well**

	c_D	c_E
3NF-A	-1.11	-0.66
3NF-B	8.14	-2.02





3NF in EFT

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This completes the nuclear chiral EFT interaction.

Combination of N3LO NN interaction with N2LO 3NF !
More order by order calculations are necessary to confirm convergence.

Double check: Is the outcome for the 3NF contribution reasonable?

Again: check consistency with expectation values

$$\langle H \rangle = \langle T \rangle + \langle V_{NN} \rangle + \langle V_{3NF} \rangle$$

	VNN [MeV]	V3NF [MeV]	H [MeV]
^3H	-45	-0.83/-1.31	-8.48
^4He	105	-4.1/-7.1	-28.3

3NF is Q^3 contribution: we expect $V_{3NF} = V_{NN} \times \frac{m_\pi^3}{\Lambda^3}$.

One finds an estimate of 1 MeV for ^3H and 2 MeV for ^4He !

Predictions for p-shell nuclei

- 3NF can be investigated in
  - Nd scattering (only isospin T=1/2)
    Many experiments have been performed.

    Guided by calculations, one could identify observables that are sensitive to the 3NF structure.

  - 4N scattering (contributions of T=1/2 and T=3/2 channels)

    predictions based on NN forces indicate that 3NF effects could be significant

  - p-shell nuclei (contribution of T=1/2 and T=3/2 channels)

    binding energy generally require 3NF
    more important: at least spectra seem to be sensitive to the structure of 3NF's

Predictions for p-shell nuclei

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- The chiral interactions slightly underbind ^6Li and ^7Li, which might be more pronounced for 3NF-B
- There is no indication for an increasing overbinding in the p-shell
- 3NF-B increases radii, agreement with expt. is similar as the one for harder core interactions.

	^6Li		^7Li	
	E	r_p	E	r_p
no 3NF	30.0	2.20	34.6	2.15
3NF-A	32.3	2.16	38.0	2.11
3NF-B	31.1	2.25	36.7	2.23
AV18+Illinois 2	32.3	2.39	38.9	2.25
AV18+Urbana	31.1	2.57	37.5	2.33
Expt.	32.0	2.43	39.2	2.27

(AV18 results from Pieper et al. PRC 64,014001)

Predictions for p-shell nuclei

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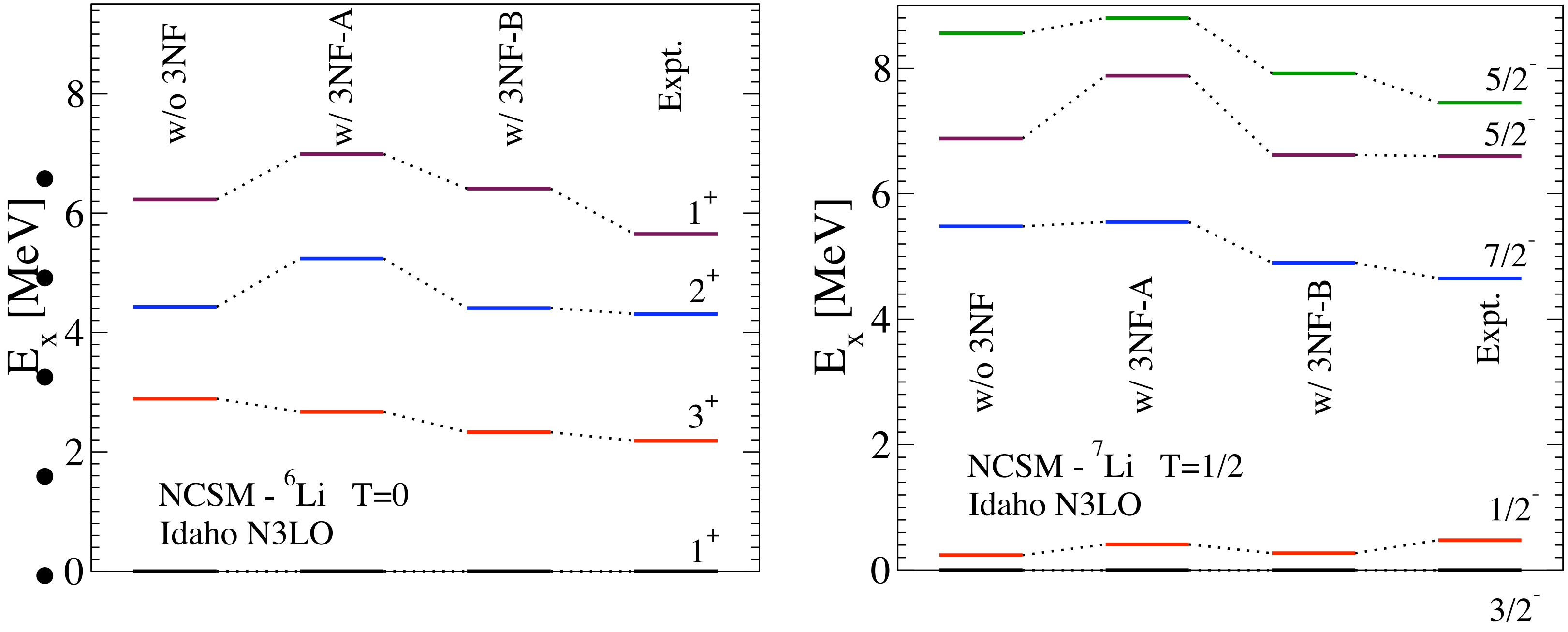
- Convergence for the spectra is better than for the binding energies
- The results show clearly deviating predictions for the two 3NF parameter sets
- Results prefer set 3NF-B for both nuclei



- Fine tuning of c_i's should be performed to find better binding energies.

3NF from EFT summary

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- High order chiral interactions have been developed, which describe the NN data perfectly again.

- Consistent 3NF's are formulated and can be fitted to few-body data.

- This chiral nuclear interaction model can be used to predict, e.g. p-shell nuclei.

- The approach shows explicitly, how very different strongly interacting systems are related to each other

- State of the art is the combination of N3LO NN interaction & N2LO 3NF: an order by order calculation is necessary to show rate of convergence.

- Strength of the subleading two-pion exchange? Uncertainty should be reduced.

- Soft interactions do describe p-shell nuclei quantitatively!

RG for nuclear potentials

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Idea for the combination of 3NF and vlowk's:

We are able to evolve down the NN interaction to arbitrary small cutoffs in the NN system!

→ Few-body observables become mildly cutoff dependent.

We are **not** able to evolve down interactions for the 3N system:

- It is hard, because we would require many 3N scattering solutions.
- At least starting from the phenomenological NN forces, we do not have appropriate 3NF's to start with.

What can we do?

Note: vlowk for small cutoffs is model independent!

→ add 3NF based on chiral perturbation theory and use the leading counter term to approximate RG (exactly in the spirit of Peter Lepage's lectures).

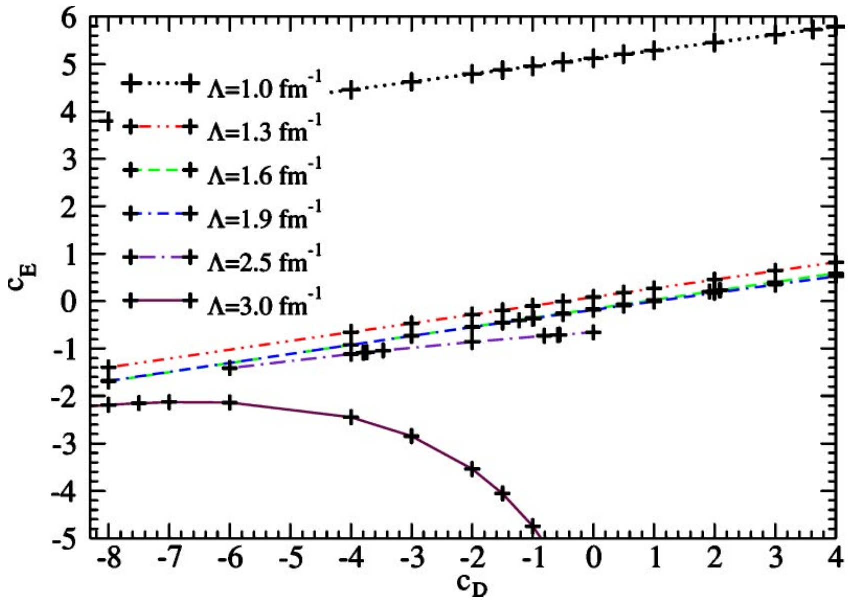
RG for nuclear potentials

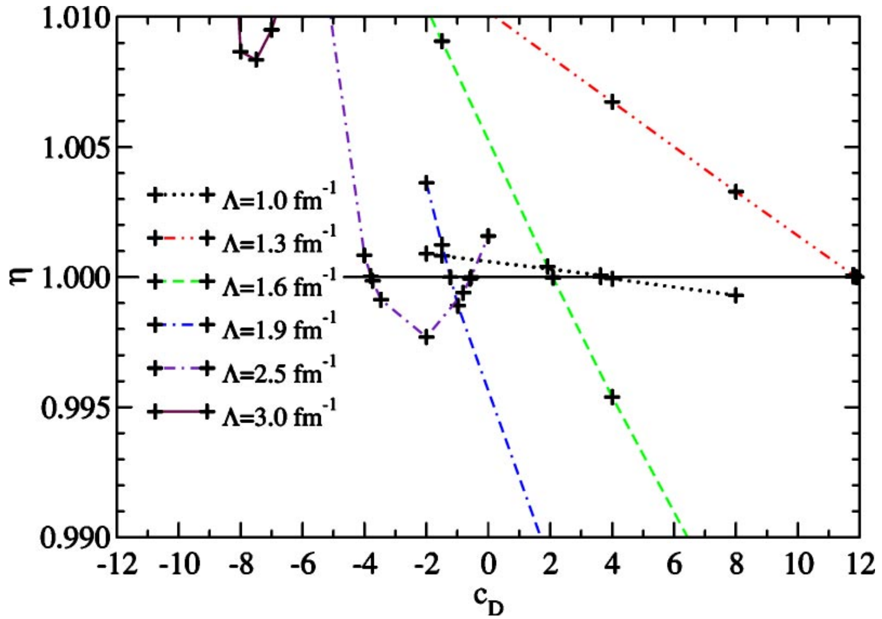
The procedure for the determination of c_D and c_E is the same as for chiral interactions.

Of course, this has to be done for several cutoffs.

We chose the range 1.0 to 3.0 fm^{-1}.

Note the perturbativity of the 3NF calculation for $\Lambda < 2.0\ fm^{-1}$.





RG for nuclear potentials

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	^{3}H					^{4}He				
Λ (fm^{-1})	T	$V_{\mathrm{low}\ k}$	c terms	D term	E term	T	$V_{\mathrm{low}\ k}$	c terms	D term	E term
1.0	21.06	−28.62	0.02	0.11	−1.06	38.11	−62.18	0.10	0.54	−4.87
1.3	25.71	−34.14	0.01	1.39	−1.46	50.14	−78.86	0.19	8.08	−7.83
1.6	28.45	−37.04	−0.11	0.55	−0.32	57.01	−86.82	−0.14	3.61	−1.94
1.9	30.25	−38.66	−0.48	−0.50	0.90	60.84	−89.50	−1.83	−3.48	5.68
2.5(*a*)	33.30	−40.94	−2.22	−0.11	1.49	67.56	−90.97	−11.06	−0.41	6.62
2.5(*b*)	33.51	−41.29	−2.26	−1.42	2.97	68.03	−92.86	−11.22	−8.67	16.45
3.0(*)	36.98	−43.91	−4.49	−0.73	3.67	78.77	−99.03	−22.82	−2.63	16.95

The first consistency check: Is the order of magnitude of the individual 3NF terms in agreement with the EFT estimate $V_{3NF} = V_{NN} \times \frac{m_\pi^3}{\Lambda^3}$?

The contribution seems to be unnaturally large in the regime, where the 3NF becomes non-perturbative and vlowk does not agree with the EFT interaction!

(be careful interpreting results at these cutoffs)

But in the region, where the 3NF is perturbative, it works out fine,

e.g. for D-term in ^{4}He at 1.3 fm^{-1}:

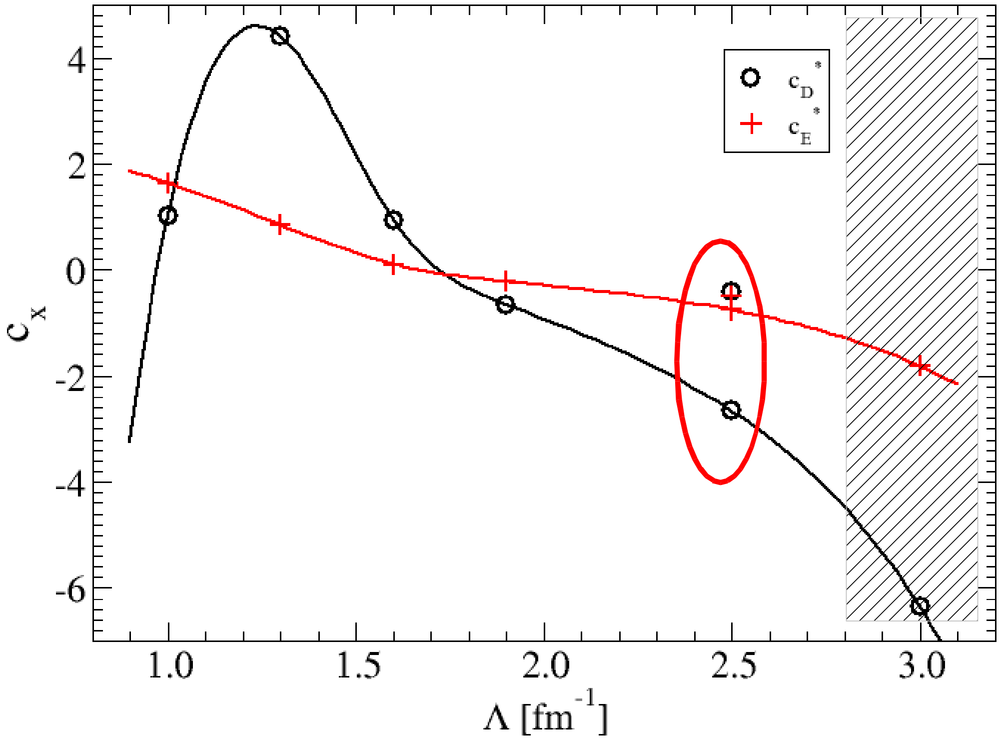
$$V_{3NF} = V_{NN} \times \frac{m_\pi^3}{\Lambda^3} \approx 12\ \mathrm{MeV}.$$

RG for nuclear potentials

This natural size also shows up in the natural coupling constants.

For this figure, we rescale c_D and c_E properly.

It is remarkable that c_D and c_E are natural even for for large cutoffs.



RG for nuclear potentials

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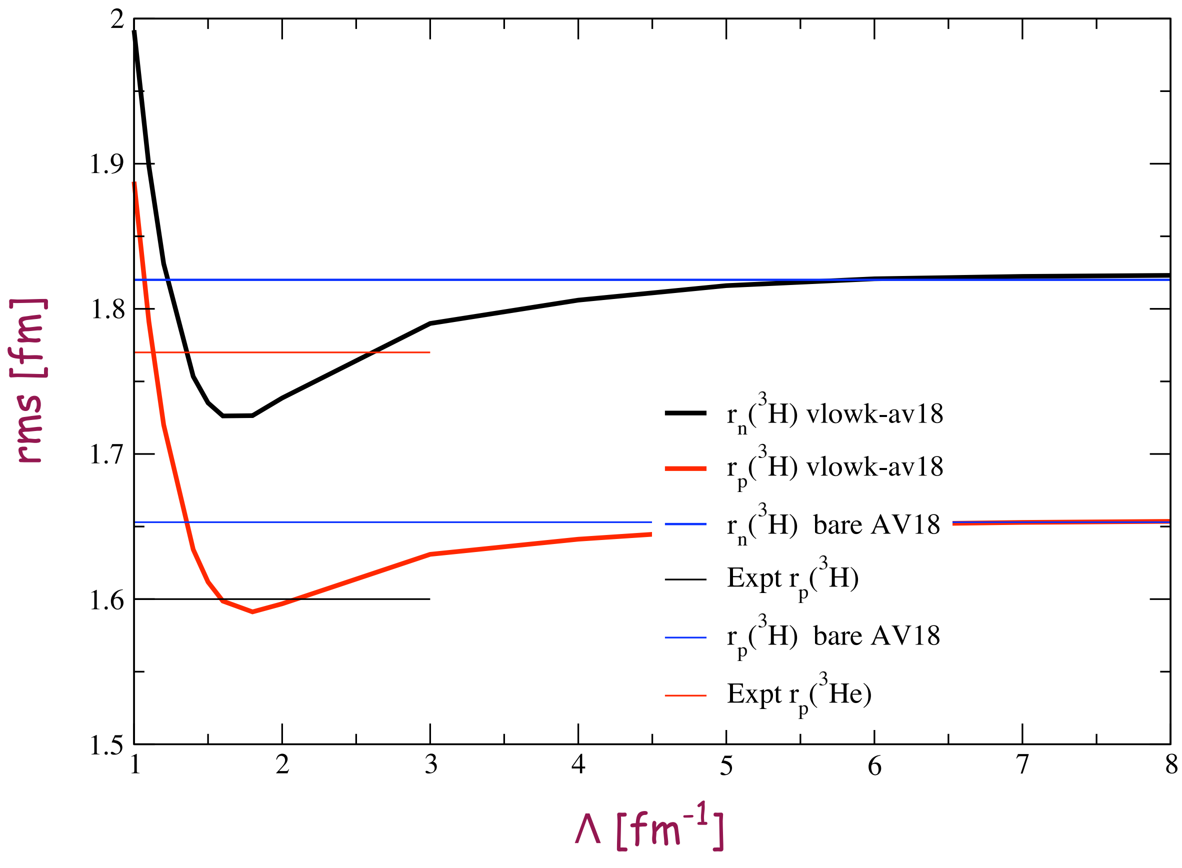
The binding energies are described by construction.

Let's look at the charge radius as an example, e.g. for the 3N system.



RG for nuclear potentials

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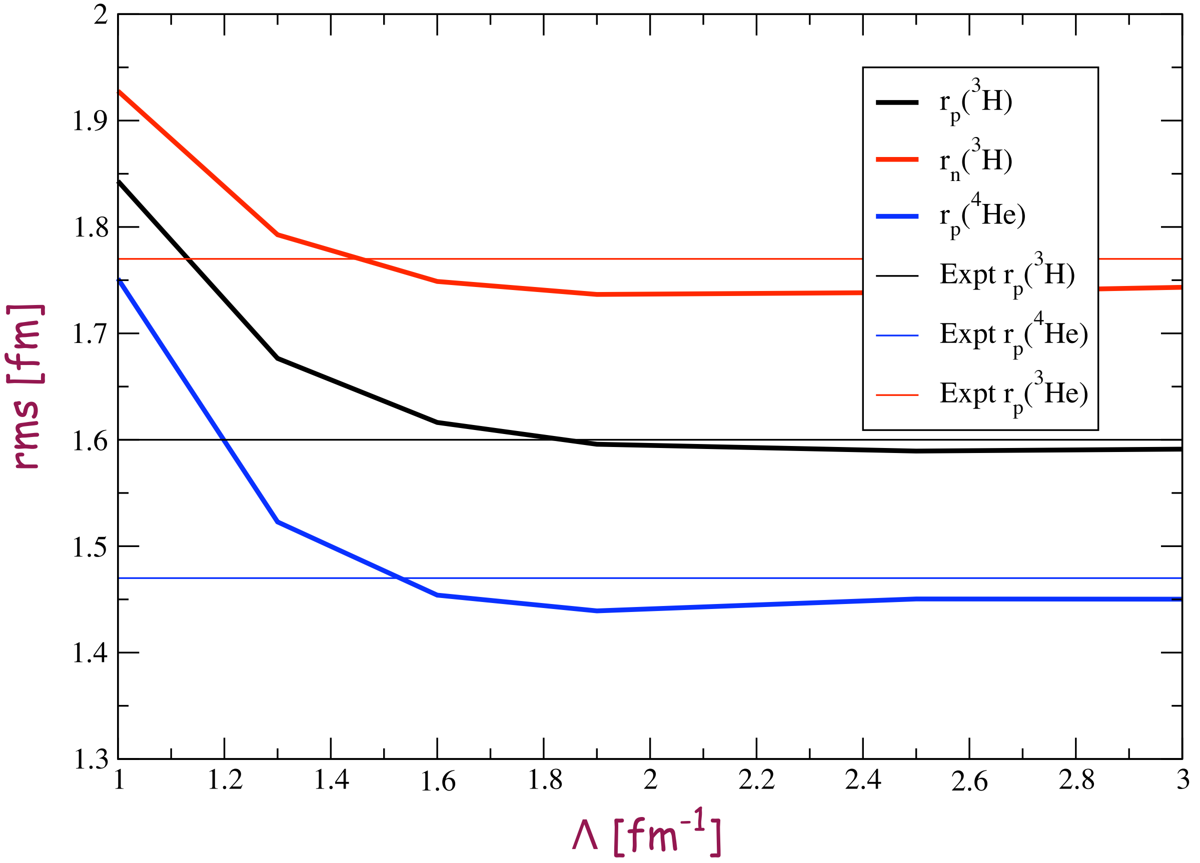
- Cutoff independence is reached for $\Lambda > 1.6\ \mathrm{fm}^{-1}$.
- The results are in good agreement with the experimental values!



Summary RG for nuclear potentials

- The model independence of "vlowk" motivates a strong connection between chiral EFT interactions and "vlowk" at small cutoffs.

- There is a practical way to augment "vlowk" interactions by 3NF's !

  The contribution of 3NF's is reasonable for small cutoffs and the similarity of EFT and "vlowk" suggests that the combination is consistent.

- The interaction can be applied to many-body systems, e.g. to nuclear matter