The Fall and Rise of Lattice QCD: Computational Effective Field Theory

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Effective Field Theory

Effective Field Theory

Quantum field theory with a finite UV cutoff Λ:

$$\mathcal{L}^{(\Lambda)} = \mathcal{L}_0^{(\Lambda)} + \sum_n \frac{1}{\Lambda^n} \mathcal{L}_n^{(\Lambda)}$$

Mimics $\mathcal{O}(p^n/\Lambda^n)$ effects of $p > \Lambda$ physics.

- $p \ll \Lambda \Rightarrow$ only finite number of terms needed.
- Local $\Rightarrow \mathcal{L}_n^{(\Lambda)}$ parameterized by finite number of Λ-dependent coupling constants.

Uses: Two Situations

1) Short-distance physics unknown.

- Systematically parameterize ignorance in terms of small number of coupling constants.
- Parameterization independent of underlying dynamics (symmetries critical).

$\Rightarrow$ Systematic calculations possible despite ignorance!

E.g., high-precision Standard Model calculations without understanding string theory.

2) Short-distance physics known.

- Separate long-distance from short-distance physics; analyze separately.

$\Rightarrow$ Best analysis tool for each.

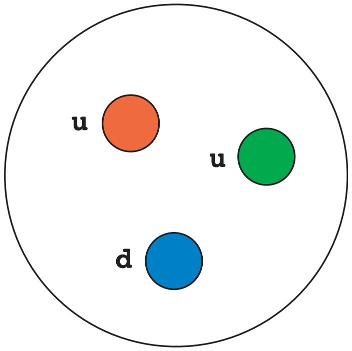
$\Rightarrow$ Highly efficient calculations.

E.g., NRQED for high-precision atomic physics, π-N theories for low-energy nuclear physics, lattice QCD.

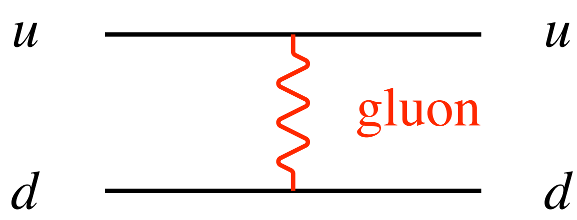
Strong Interactions — A History

Quark Model (1960s)

Proton =



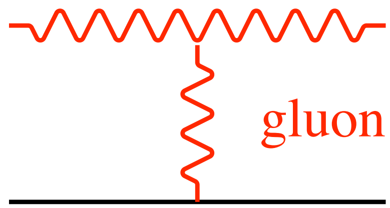
Interactions — QCD (1970s)



Gauge theory (like QED) $\Rightarrow$ Complete theory!

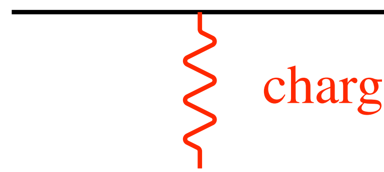
But. . .

Nonlinear:



gluon carries QCD charge

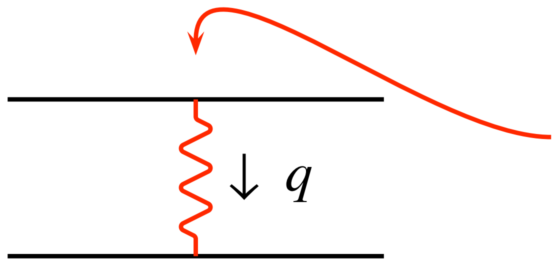
Strongly Interacting:



charge g big

- $\Rightarrow$ Couldn't solve QCD.
- $\Rightarrow$ QCD added nothing to understanding of proton structure.
- $\Rightarrow$ Theory useless?

Asymptotic Freedom (1973)



$g_{\text{eff}} = g(q) \to 0$ as $q \to \infty$.

$\Rightarrow$ Solved QCD for high-energy (short-distance) processes by expanding in powers of

$$\alpha_s(q) \equiv \frac{g^2(q)}{4\pi}.$$

$\Rightarrow$ Detailed experimental verification of QCD at high-energy accelerators (1980s–1990s).

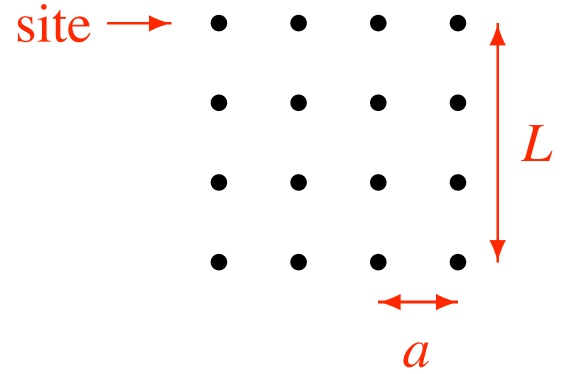
But still no insight into proton, neutron, pion. . . structure.

- Low-energy ($< 1\,\text{GeV}$) QCD is non-perturbative.

Lattice QCD

Lattice Approximation

Continuous
Space & Time $\longrightarrow$



$\Rightarrow$ Fields $\psi(x)$, $A_\mu(x)$ specified only at grid sites; interpolate for other points.

K. Wilson (1974)

$\Rightarrow$ QCD $\rightarrow$ multidimensional integration.

$$\int \mathcal{D}A_\mu \ \ldots \ \mathrm{e}^{-\int L \mathrm{d}t} \longrightarrow \int \prod_{x_j \epsilon \text{ grid}} \mathrm{d}A_\mu(x_j) \ \ldots \ \mathrm{e}^{-a \sum L_j}.$$

$\Rightarrow$ Millions of integration variables.

$\Rightarrow$ Numerical Monte Carlo integration.

$\Rightarrow$ Nonperturbative QCD.

Fall & Rise of LQCD

- Invented in 1974; "explains" confinement.
- Stalls for almost 20 years.
  - ◇ Ken Wilson declares it dead! (1986)
- Renaissance in 1990's.
  - ◇ Perturbation theory fixed.
  - ◇ Effective field theories for c, b quarks.
  - ◇ Improved discretizations $\Rightarrow$ larger a.
  - ◇ Unquenching! (2000)

$\Rightarrow$ High-prescision nonperturbative results.

- ◇ Masses, decay rates, mixing amplitudes…
- ◇ Ken Wilson retracts. (1995)

QCD Revolution

Traditional wisdom	$\Rightarrow$	need $a < 0.05$ fm.
New simulation results	$\Rightarrow$	$a = 0.1$–0.4 fm works.

Simulation cost $\propto (1/a)^6$

$\Rightarrow$ New simulations cost 10^2–$10^6\times$ less!

$+$ New light-quark discretization 50–1000$\times$ faster!

Quantum Field Theory on a Lattice

Approximate Derivatives

Numerical Analysis $\Rightarrow$

$$\frac{\partial \psi(x_j)}{\partial x} = \Delta_x \psi(x_j) + \mathcal{O}(a^2)$$

$$\frac{\psi(x_j + a) - \psi(x_j - a)}{2a}$$

$\Rightarrow$ uses only ψ's at grid sites.

Large $a \Rightarrow$ need *improved discretizations*.

E.g.

$$\frac{\partial \psi}{\partial x} = \Delta_x \psi - \frac{a^2}{6} \Delta_x^3 \psi + \mathcal{O}(a^4)$$

10–15% for $a = 0.4\,\mathrm{fm}$

1–2% for $a = 0.4\,\mathrm{fm}$

$\Rightarrow a = 0.4\,\mathrm{fm}$ okay?

But quantum numerical analysis $\neq$ classical numerical analysis!

Ultraviolet Cutoff

$\lambda_{\text{min}} = 2a$ is smallest wavelength.

E.g.) $\psi = \ +1 \quad -1 \quad +1 \quad -1 \quad +1$

$\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$

$\Rightarrow$ all quark and gluon states with $p > \pi/a$ are excluded by the lattice since $p = 2\pi/\lambda$.

N.B. Lattice QCD $\equiv$ QCD + lattice UV regulator

$\equiv$ real QCD.

But $\forall\, p$s important in quantum field theory!
(Consider ultraviolet divergences.)

Renormalization Theory $\Rightarrow$ mimic effects of
$p > \pi/a$ excluded states by adding extra
a-dependent *local* terms to the field equations,
Lagrangian, currents, etc.

Lattice QCD $\equiv$ Effective Field Theory ($\Lambda = \pi/a$).

$$\Rightarrow \qquad \partial\psi \;\to\; \Delta\psi + c(a)\, a^2\, \Delta^3\psi + \cdots$$

where

$$c(a) \;=\; -\frac{1}{6} + \begin{array}{l}\text{Contribution for}\\ p > \pi/a \text{ physics}\end{array}$$

Numerical Analysis

Theory & context specific $\Rightarrow$ not universal!

Bad News: Need a^2 corrections when a large, but *Numerical Recipes* won't tell you values of $c(a)\ldots$

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Good News: $p \geq \pi/a$ QCD is perturbative if a *small* enough (asymptotic freedom).

$\Rightarrow$ compute $c(a)\ldots$ using perturbation theory.

Perturbation theory fills in gaps in lattice;
$\Rightarrow$ continuum results without $a \to 0$!

E.g.,

Renormalization constant.

$$\mathcal{L}^{(a)} = Z(a)\,\overline{\psi}(\Delta\cdot\gamma - m(a))\psi + c(a)\,a^2\,\overline{\psi}\,\Delta^3\cdot\gamma\,\psi + \cdots$$

Finite-a correction.

where

$$c(a) = -\frac{1}{6} + c_1\,\alpha_s(\pi/a) + \cdots$$

Numerical Analysis

Mimics effects of $p > \pi/a$ states excluded by grid.

Asymptotic freedom in QCD $\Rightarrow$

- short-distance physics simple (perturbative);
- long-distance physics difficult (nonperturbative).

Lattice separates “short” from “long”:

- $p > \pi/a$ QCD $\rightarrow$ corrections $\delta\mathcal{L}$ computed in perturbation theory (determines a);
- $p < \pi/a$ QCD $\rightarrow$ nonperturbative, numerical Monte Carlo integration.

Perturbation Theory

Improved discretizations and larger as — old ideas.

But perturbation theory is essential.

$\Rightarrow$ a small enough so that $p \approx \pi/a$ QCD is perturbative (determines a).

$\Rightarrow$ Before 1992: $a < 0.05$ fm.

$\Rightarrow$ After 1992: $a < 0.4$ fm works (smaller for high precision).

G.P. Lepage and P.B. Mackenzie (1992).

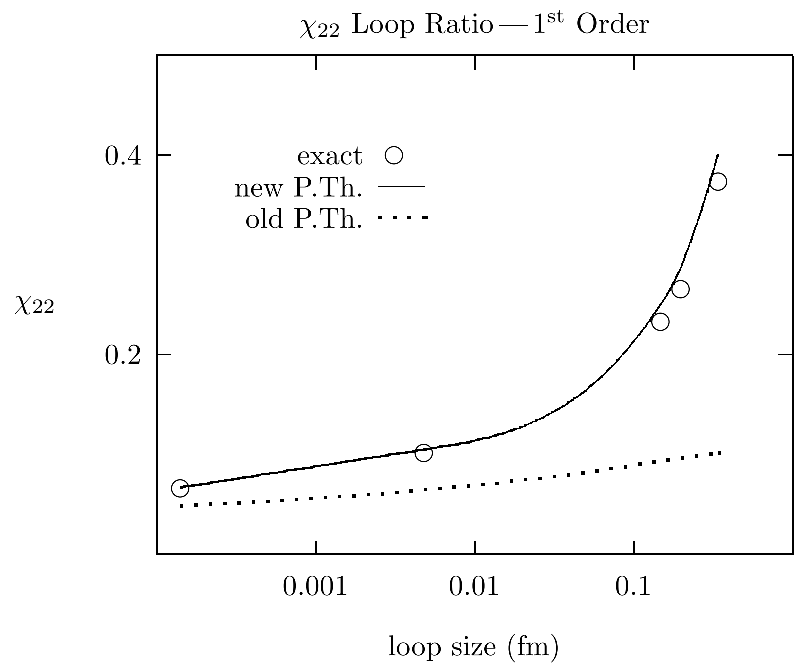
Test by comparing short-distance quantities from:

- perturbation theory;
- numerical Monte Carlo integration ($\Rightarrow$ exact result).

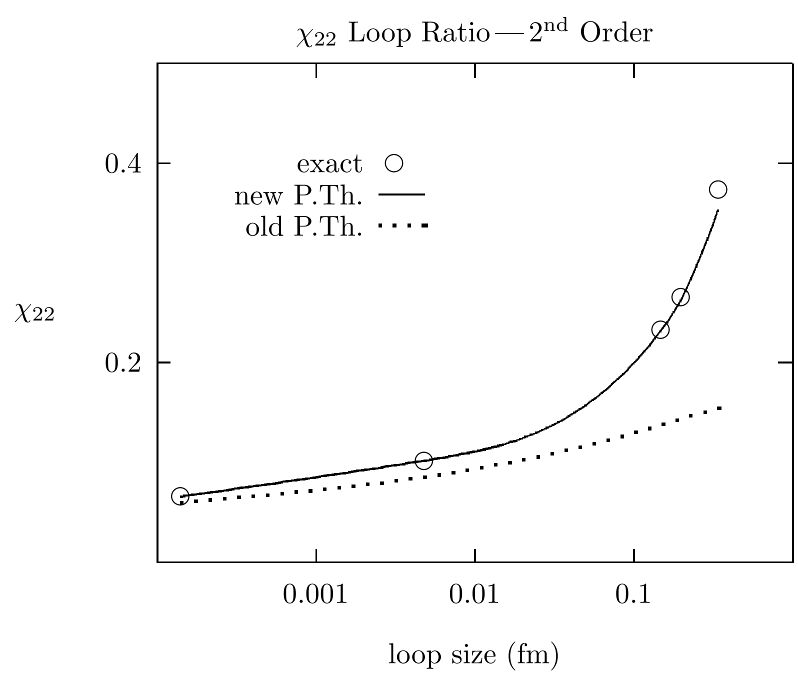
E.g., Wilson loops:

$$W(\mathcal{C}) \equiv \langle 0 | \tfrac{1}{3} \,\mathrm{Re}\ \mathrm{Tr}\ \mathrm{P}\, \mathrm{e}^{-ig\oint_{\mathcal{C}} A\cdot \mathrm{d}x} | 0 \rangle,$$

$\mathcal{C}$ = small, closed path.

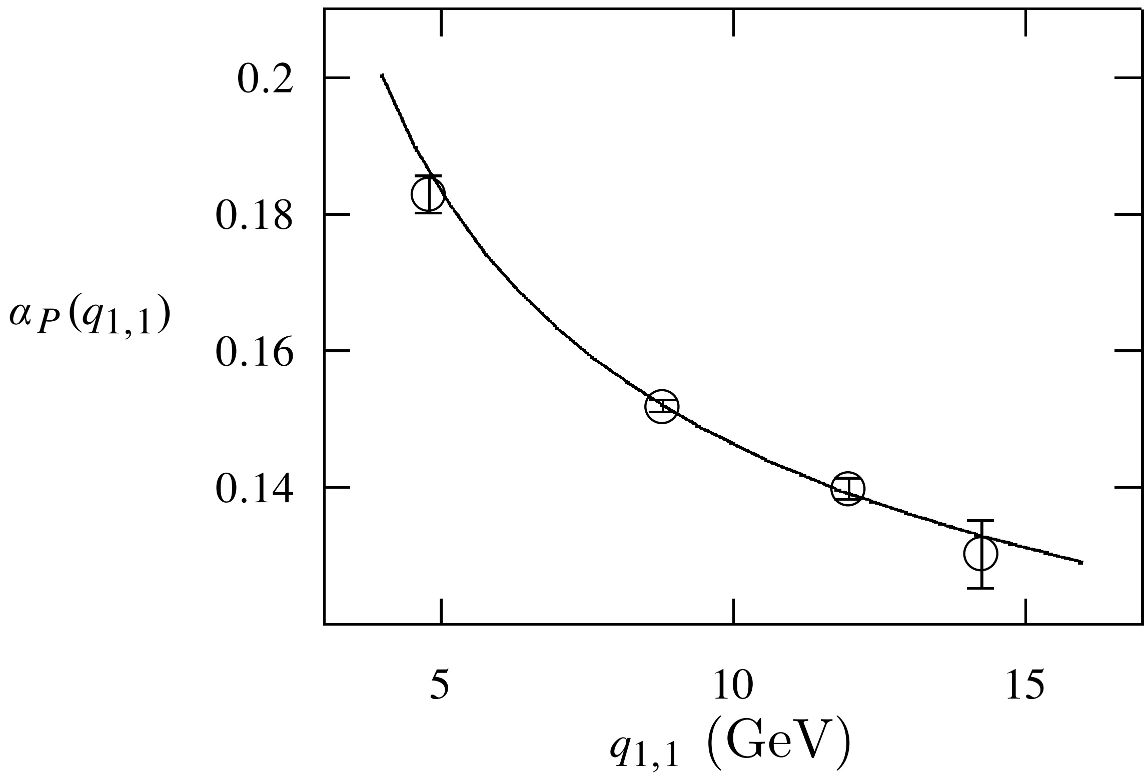


χ_{22} Loop Ratio — 1st Order
exact
new P.Th.
old P.Th.
0.4
0.2
χ_{22}
0.001
0.01
0.1
loop size (fm)



χ_{22} Loop Ratio — 2nd Order
exact
new P.Th.
old P.Th.
0.4
0.2
χ_{22}
0.001
0.01
0.1
loop size (fm)

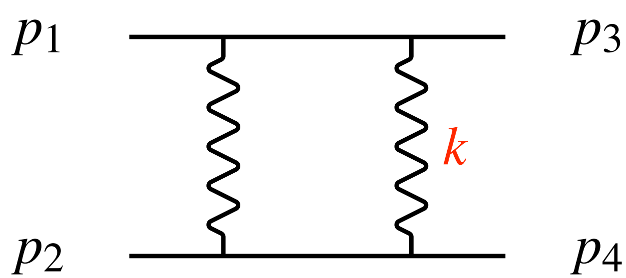
Running coupling constant:



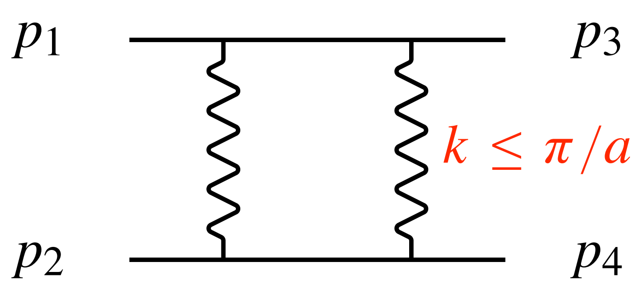
Computing Lattice Operators

Compute on-shell, low-momentum ($p_i \ll \pi/a$) scattering amplitudes for:

Continuum: T $=$



Lattice: $T^{(a)}$ $=$



$T - T^{(a)}$ = what is omitted by lattice QCD.

$T - T^{(a)}$ dominated by $k > \pi/a \gg p_i$

$\Rightarrow$ Taylor expand in powers of $p_i/k \approx p_i a \ll 1$:

$$\begin{aligned} T - T^{(a)} &= c(a)\, a^2\, \overline{u}(p_2)\gamma_\mu u(p_1)\, \overline{u}(p_4)\gamma^\mu u(p_3) \\ &+ c_A(a)\, a^2\, \overline{u}\gamma_\mu\gamma_5 u\, \overline{u}\gamma^\mu\gamma_5 u \\ &+ d(a)\; a^4\, (p_1 - p_2)^2\; \overline{u}\gamma_\mu u\, \overline{u}\gamma^\mu u + \cdots. \end{aligned}$$

Dimensionless Taylor coefficient; depends upon $\alpha_s(\pi/a)$, $m\,a$.

Expansion is in powers of $p_i a \ll 1$

Correct lattice $\mathcal{L}$: add new, local interactions (i.e., polynomial in aD) that generate the missing pieces from $T - T^{(a)}$:

$$\begin{aligned}
\delta\mathcal{L}^{(a)} &= \tfrac{1}{2}\,c(a)\,a^2\,\overline{\psi}\gamma_\mu\psi\,\overline{\psi}\gamma^\mu\psi \\
&+ \tfrac{1}{2}\,c_A(a)\,a^2\,\overline{\psi}\gamma_\mu\gamma_5\psi\,\overline{\psi}\gamma^\mu\gamma_5\psi \\
&+ d(a)\,a^4\,\overline{\psi}D^2\gamma_\mu\psi\,\overline{\psi}\gamma^\mu\psi \\
&+ \cdots .
\end{aligned}$$

Intermediate particles in $T - T^{(a)}$ have $k > \pi/a \gg p_i$

- $\Rightarrow$ highly virtual ($k^2 > 1/a^2$);
- $\Rightarrow$ propagate only short distances ($\approx a$);
- $\Rightarrow$ corrections must be local;
- $\Rightarrow$ only finite number to a given order in $p_i a$.

Correction terms needed for $\mathcal{L}$, currents, and all other operators. (C.f., effective field theory.)

N.B., page 2 of sample vertex rule in LQCD:


Printed by Quentin Mason

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2/435 Friday August 08, 2003

Does It Work?

Quarks

The standard discretization of the quark action has $\mathcal{O}(a^2)$ errors:

$$\mathcal{L}_{\mathrm{lat}} \approx \overline{\psi}(D\cdot\gamma + m)\psi + \frac{a^2}{6}\sum_\mu \overline{\psi} D_\mu^3 \gamma^\mu \psi + \cdots .$$

$\mathcal{O}(a^2)$ error violates rotation/Poincaré invariance; removed by adding correction term.

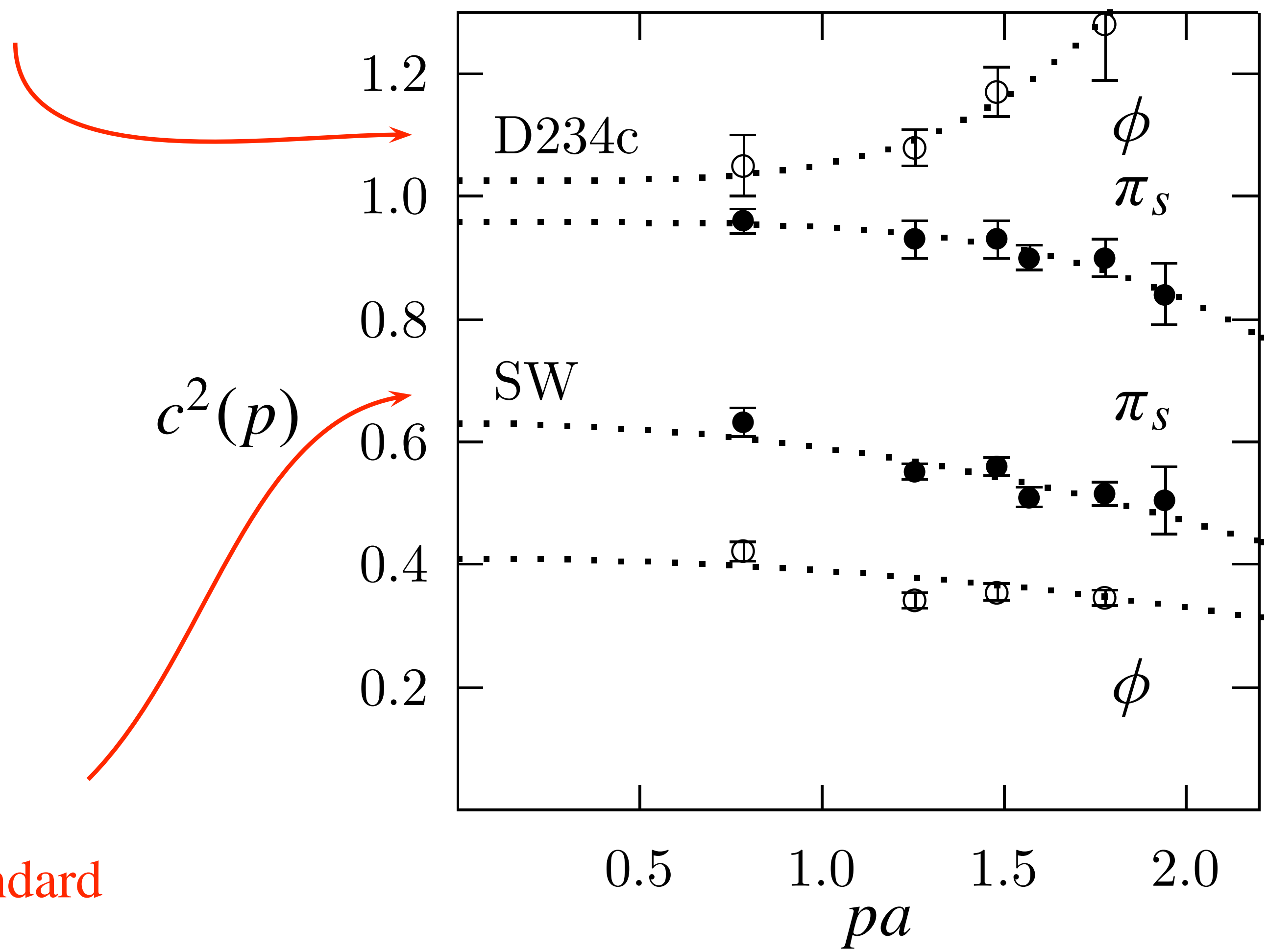
Test by computing

$$c^2(\mathbf{p}) \equiv \frac{E^2(\mathbf{p}) - m^2}{\mathbf{p}^2};$$

Lorentz invariance implies:

$$c^2(\mathbf{p}) = 1 \qquad \forall\, \mathbf{p}.$$

Improved



Standard

Alford et al (1997).

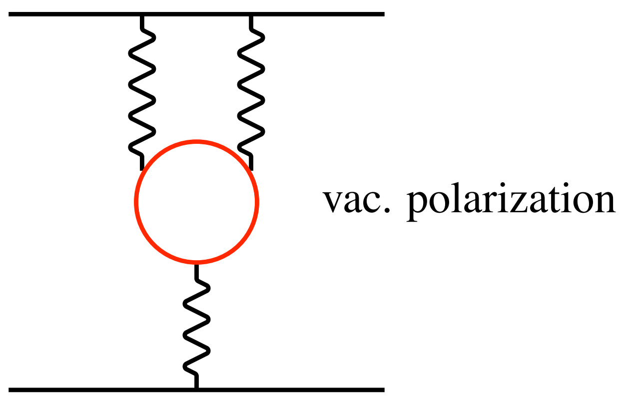
“Unquenching”

Unquenched LQCD

"Quenched" QCD $\equiv$ QCD without quark vacuum polarization.

$\Rightarrow$ 15–30% errors in most calculations;

$\Rightarrow$ *the* major limitation of LQCD *until 2000*.



Naive/staggered quarks + improved discretization

$\Rightarrow$ 50–1000 times faster
& smallest finite-a errors
& best behavior in chiral limit!

$\Rightarrow$ High-precision (few %) LQCD possible *now!*

$\Rightarrow$ Already have thousands of configurations (MILC):

- $n_f = 3$;
- smallest ($m_u = m_d$) ever: $m_s \ldots m_s/5,\ m_s/7,\ m_s/10$;
- small as: $1/8$ fm, $1/11$ fm;
- large Ls: 2.5 fm, 3.0 fm.

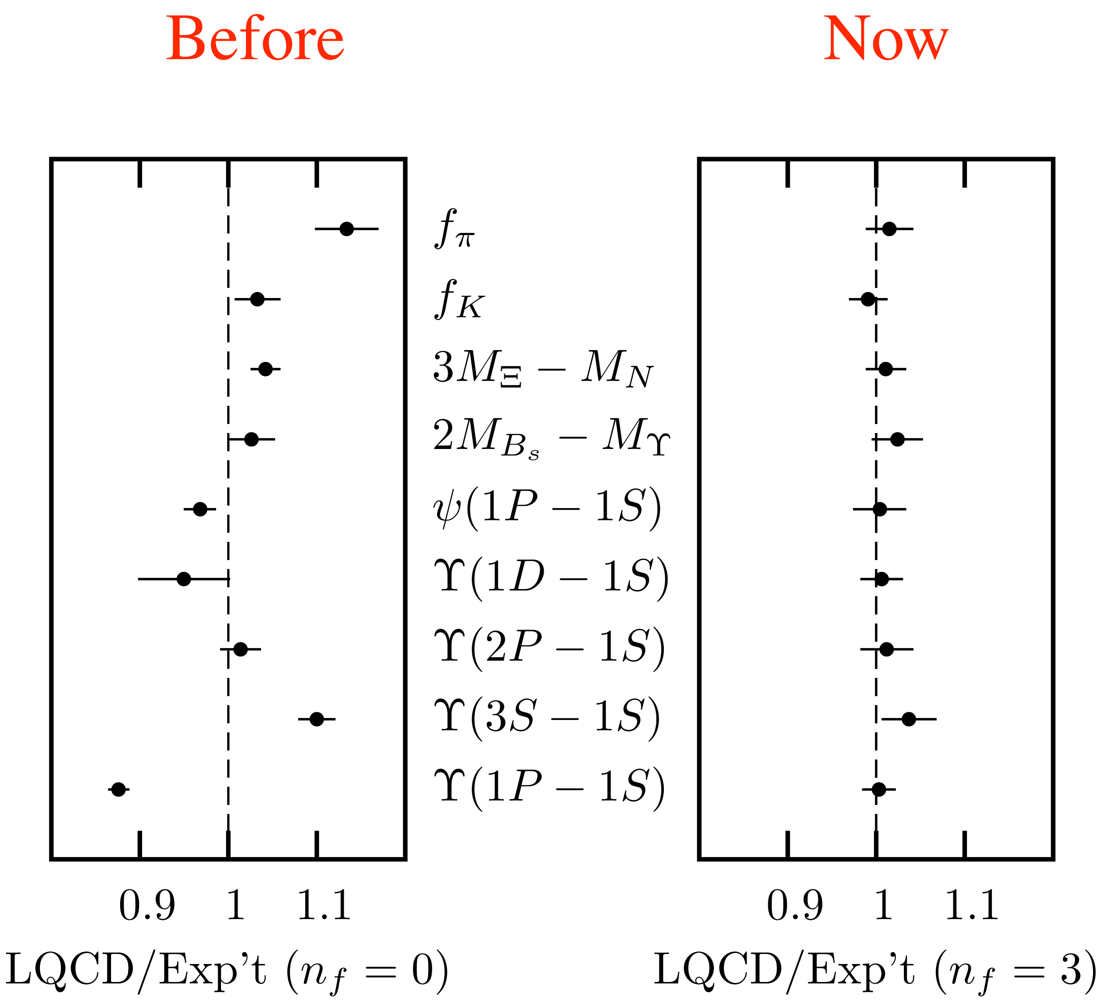
High-Precision Test

1) Tune 5 free parameters (bare $m_u = m_d$, m_s, m_c, m_b and α_s) using m_π, m_K, m_ψ, m_Υ, and $\Delta E_\Upsilon(1P - 1S)$.

2) Compute other quantities and compare with experiment.

Davies et al, Phys. Rev. Lett. 92:022001, 2004. (HPQCD, MILC, Fermilab, UKQCD)

Lattice QCD/Experiment (no free parameters!):



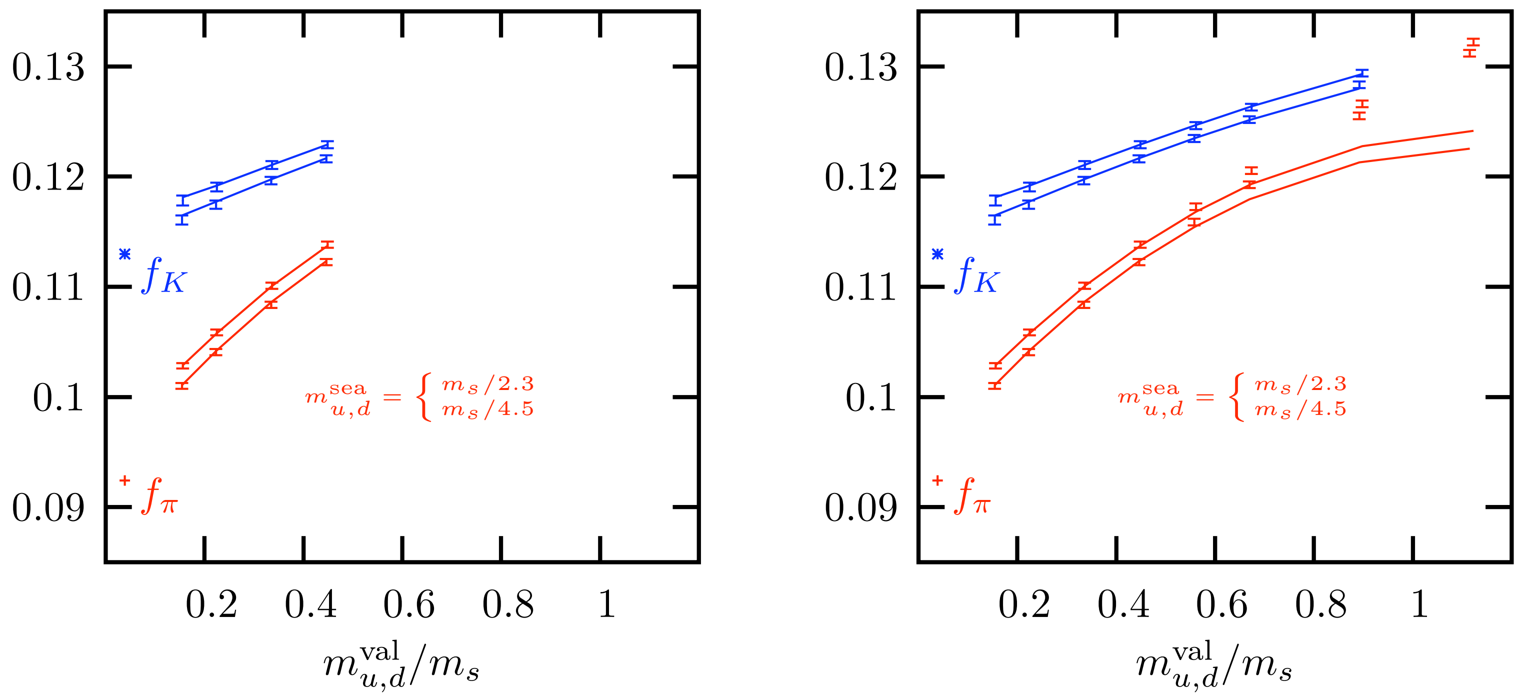
Tests:

- $m_{u,d}$ extrapolation;
- masses and wavefunctions;
- s quark;
- light-quark baryons;
- light-heavy mesons;
- heavy quarks (no potential model...);
- improved staggered quark vacuum polarization.

$\Rightarrow$ Most accurate strong interaction calculation in history!

Quark Mass Problem (Solved)

f_π and f_K fits versus valence u, d mass:



⇒ New values for quark masses

$$
\begin{aligned}
m_s^{\overline{\rm MS}} &= 76\,(0)(3)(7)(0)\ \text{MeV} \\
m_{u,d}^{\overline{\rm MS}} &= 2.8\,(0)(1)(3)(0)\ \text{MeV} \\
m_s^{\overline{\rm MS}}/m_{u,d}^{\overline{\rm MS}} &= 27.4\,(1)(4)(0)(1)
\end{aligned}
$$

errors = (stat.) (extrap.) (pert. th.) (elect-mag.)

Soon: m_b and m_c, 2-loop pert'n theory (2–3× smaller error)

Aubin et al (HPQCD, MILC, UKQCD) (2004)

QCD Coupling

Tuned LQCD simulation $\equiv$ real QCD.

$\Rightarrow$ Measure short-distance quantity Y in simulation and compare with perturbation theory,

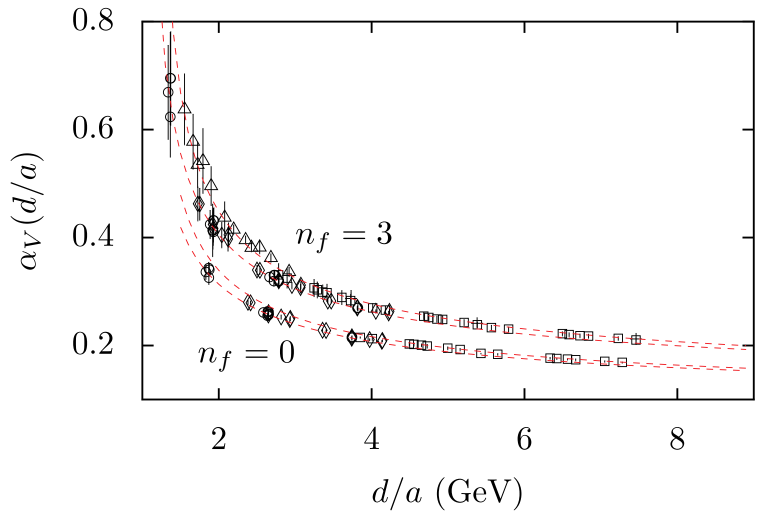
$$Y = \sum_{n=1}^{\infty} c_n \, \alpha_V^n(d/a),$$

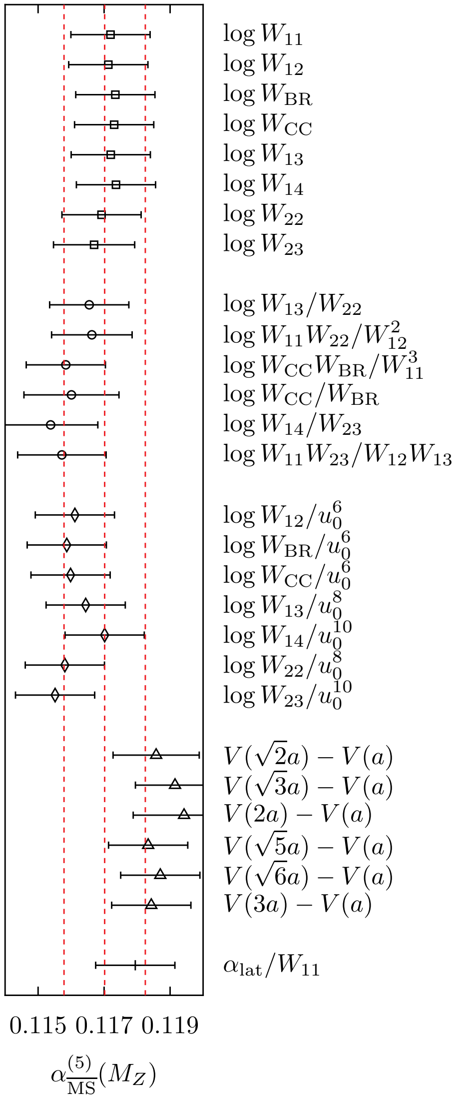
to extract QCD coupling constant α_V.

- Compute c_ns using Feynman diagrams; fit multiple as to extract estimates for uncalculated c_ns (4$^{\text{th}}$ order possible).

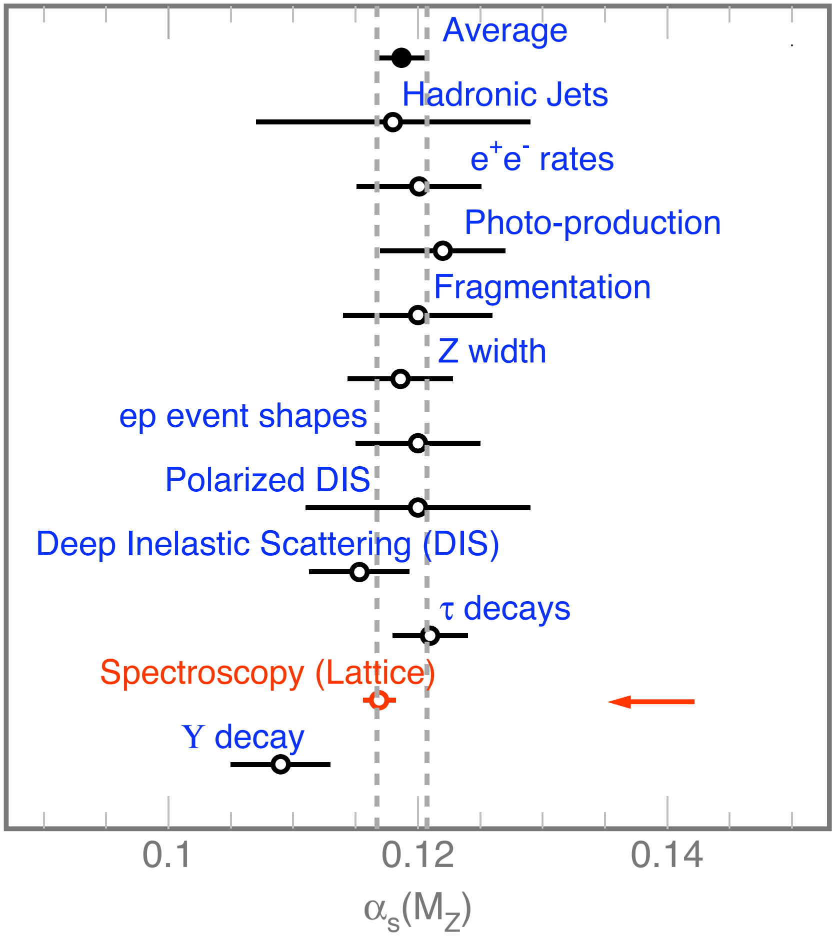
Final results:

- 28 short-distance quantities.
- $\alpha_{\overline{\mathrm{MS}}}^{(5)}(M_Z) = 0.1170(12)$.
- PDG world average is 0.1187(20).
- No light-quark vacuum polarization $\Rightarrow$ 0.0900(4).



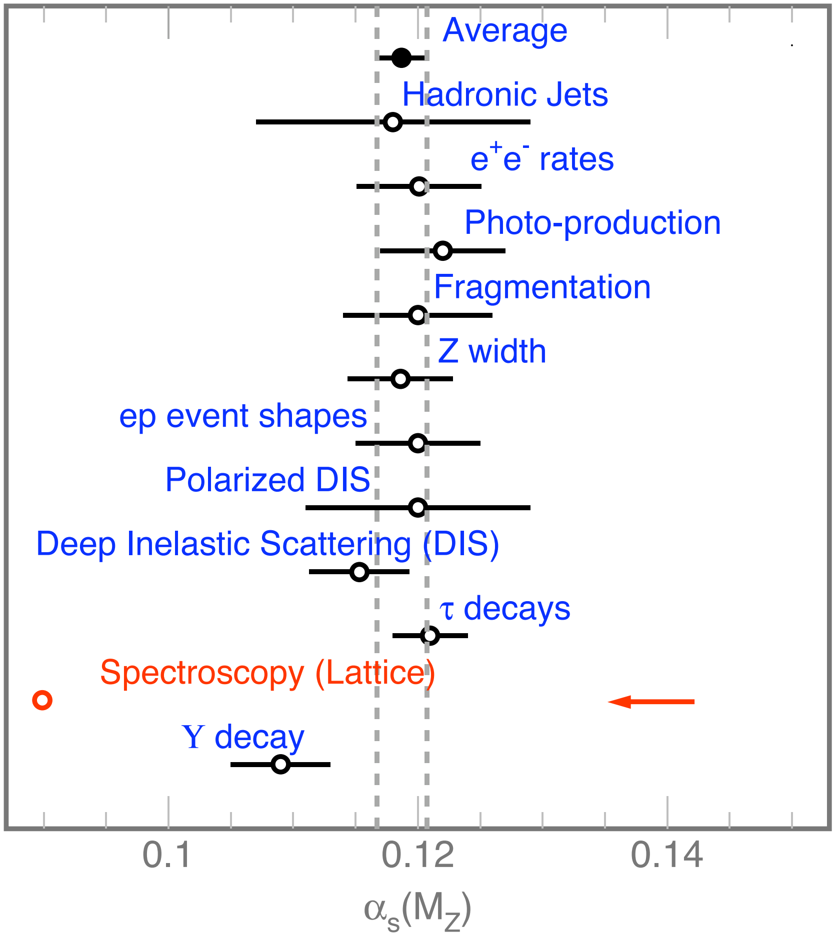


Context:



Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004)

And without light-quark vacuum polarization:



Mason et al, hep-lat/0503005v1 (2005); Particle Data Group (2004)

The Future

High-Precision Now

Few % accuracy for dozens of "gold-plated" calculations:

- Masses, form factors, decay constants, mixing amplitudes for π, K, p, n (but not ρ, ϕ, $\Delta\ldots$).
- Masses, decay constants, semileptonic form factors, and mixing for D, D_s, B, B_s (but not $D^*\ldots$).
- Masses, leptonic widths, electromagnetic form factors, and mixing for any meson in ψ and Υ families well below D/B threshold.
- High-precision $\Rightarrow$ masses and amplitudes with at most one hadron in the initial and/or final state, for stable or nearly stable hadrons.

HPQCD Plan

Focus on physics of heavy quarks:

- Major experimental program to measure weak-interaction decays of c and b quarks to few % (BaBar, Belle, CLEO-c).

$\Rightarrow$ Standard Model pushed to point of failure (supersymmetry, extra dimensions. . . ?).

$\Rightarrow$ Lattice QCD essential (for high-precision):

$$\text{quark decay} = \text{weak-interaction} \times \text{QCD}.$$

- Gold-plated quantities for almost every CKM matrix elements (and K-$\overline{K}$ mixing):

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to l\nu & B \to l\nu \\ & K \to \pi l\nu & B \to \pi l\nu \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to Dl\nu \\ D \to \pi l\nu & D \to Kl\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

- Extensive cross-checks for error calibration: Υ, B, ψ, $D\ldots$.

HPQCD: $B/D/K$ Physics Status

- Decay constants:

$$\begin{aligned} f_{B_s} &= 260\,(7)(26)(8)(5)\ \mathrm{MeV} \\ f_{D_s} &= 290\,(20)(29)(29)(6)\ \mathrm{MeV} \end{aligned}$$

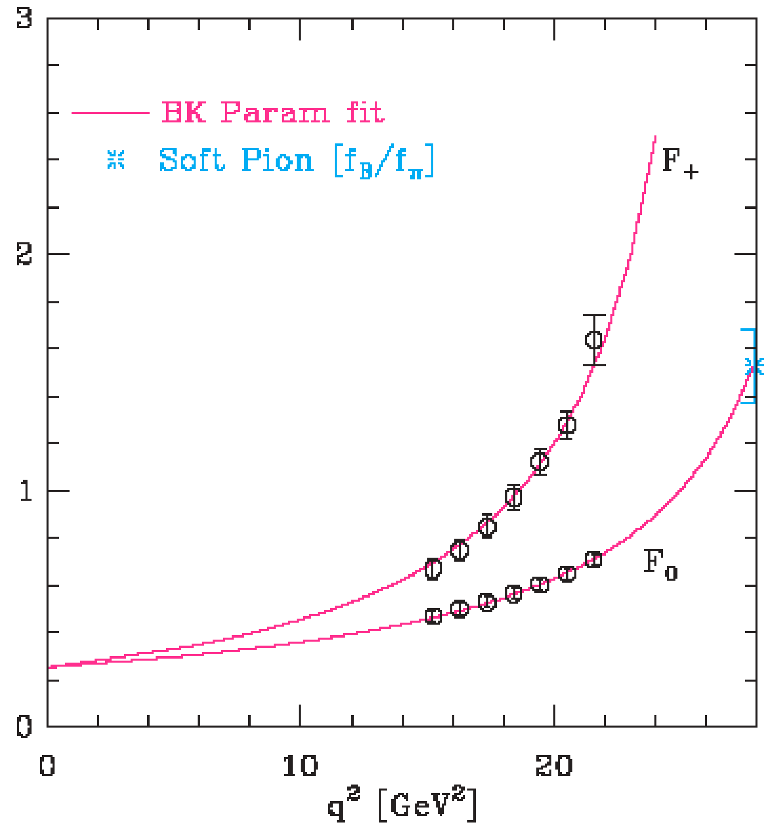
  where errors are due to: statistics/fitting, perturbation theory, relativistic corrections, and finite-a. Two-loop perturbation theory $\Rightarrow$ 2–3$\times$ reduction in error.

- $B-\overline{B}$ Mixing: Preliminary results completed; needs perturbation theory.

- B_K: Scaling violations reduced by 3–4$\times$ using improved staggered quarks; unquenching effects seem small. Needs perturbation theory.

Wingate et al (2004); Gámez et al (2004); Gray et al (2004).

- $B \to \pi \ell \nu$:



Needs perturbation theory.

Shigemitsu et al (2004).

Conclusion

Few percent precision ⇒ superb opportunity for lattice QCD to have an impact on particle physics.

- LQCD essential to high-precision B/D physics at BaBar, Belle, CLEO-c, Fermilab. . .
- *Predicting* CLEO-c, BaBar/Belle results ⇒ much needed credibility for LQCD.
- Critical to focus on gold-plated quantities.
- Landmark in history quantum field theory: quantitative verification of nonperturbative technology (c.f., 1950s).
- Ready for beyond the Standard Model, strong coupling beyond QCD?