Three pion Deck amplitude in photoproduction

straight clone of the Ascoli study

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The basic change required is that the 'upper vertex' scattering process is now $\gamma\pi_t \to \pi_2\pi_3$, which will now have the complication of the photon helicity index. I'll consider two ways to set this up

1 General $2 \to 2$ helicity amplitude

Perl equation (10-9a,b) tells us that in the CM frame of the $2 \to 2$ scattering process we can write the helicity amplitude

$$T_{\lambda_\gamma}(s,\theta,\phi) = \frac{8\pi\sqrt{s}}{\sqrt{p_i^* p_f^*}} e^{i\lambda_\gamma\phi} \sum_J (2J+1) d^J_{\lambda_\gamma,0}(\theta) \langle 00|T^J(s)|\lambda 0\rangle \tag{1}$$

where the angles θ, ϕ are those of one of the pions relative to the z-axis defined by the incoming photon. In the case that the photon is actually a pion and has no helicity index this reduces to the first line of Ascoli equation (2.7). Parity invariance is manifested in Perl equation (10-19a) and gives the constraint $\langle 00|T^J(s)|-\lambda 0\rangle = -\langle 00|T^J(s)|\lambda 0\rangle$

If one performs an Euler rotation, $(\alpha_1,\beta_1,\gamma_1)$, followed by another, $(\alpha_2,\beta_2,\gamma_2)$, the resulting net rotation is (α,β,γ). In the basis of angular momentum eigenstates we have the representation

$$D^J_{m',m}(\alpha,\beta,\gamma) = \sum_{m''} D^J_{m',m''}(\alpha_2,\beta_2,\gamma_2) D^J_{m'',m}(\alpha_1,\beta_1,\gamma_1). \tag{2}$$

The $m = m' = 0$ case of this expression is used in Ascoli equation (2.8) to relate the scattering amplitude in the $[23]_{RF}$ to some standard angle set. I think it works something like this:

- π_3 is at angle (χ_1,γ) to the 23 direction (*not sure this is how γ enters?*)
- hence in the $[23]_{RF}$ π_2 is at angle $(\pi-\chi_1, \gamma+\pi)$
- from Ascoli figure 5 there is an angle ψ between A and 23

So then we can compose the angles (not sure about the ordering though) as

$$P_S(\theta_{\pi\pi}) = D^{S*}_{0,0}(0,\theta_{\pi\pi},0) = \sum_\lambda D^{S*}_{0,\lambda}(0,\psi,0) D^{S*}_{\lambda,0}(\gamma+\pi,\pi-\chi_1,0) \tag{3}$$

$$= \sum_\lambda d^S_{0\lambda}(\psi)(-1)^\lambda e^{i\lambda\gamma}(-1)^{S-\lambda} d^S_{\lambda 0}(\chi_1) \tag{4}$$

$$= (-1)^S \sum_\lambda d^S_{0\lambda}(\psi) D^{S*}_{\lambda,0}(\gamma,\chi_1,0). \tag{5}$$

In the photon case the only difference appears to be the extra helicity index

$$d_{\lambda_\gamma 0}(\theta_{\gamma\pi}) = (-1)^S \sum_\lambda d^S_{\lambda_\gamma \lambda}(\psi) D^{S*}_{\lambda,0}(\gamma,\chi_1,0). \tag{6}$$

I'll assume we can put the $2 \to 2$ reaction in the zx plane ($\phi = 0$) and deal with the photon polarisation direction (which we could define using the angle ϕ in the first equation) later (via the density matrix). Then we have

$$T_{\lambda_\gamma}(s,\theta,\phi) = \frac{8\pi\sqrt{s}}{\sqrt{p_i^* p_f^*}} \sum_J (2J+1)\langle 00|T^J(s)|\lambda 0\rangle (-1)^S \sum_\lambda d^S_{\lambda_\gamma \lambda}(\psi) D^{S*}_{\lambda,0}(\gamma,\chi_1,0), \tag{7}$$

which is in a form that allows us to follow through the rest of the Ascoli manipulations easily.

2 Covariant method for $S = 1$

This provides a cross check to the above, but note that it is still reliant on me having correctly understood all the angles. Discrete and Lorentz symmetries limit the structures of $\pi\pi$ and $\gamma\pi$ couplings to the ρ,

$$\langle \pi_2(p_2)\pi_3(p_3)|\rho(\lambda, p_2+p_3)\rangle = g\epsilon_\mu (p_2-p_3)^\mu \tag{8}$$

$$\langle \rho(\lambda, p_A+p_t)|\gamma(\lambda_\gamma, p_A)\pi_t(p_t)\rangle = f\epsilon^{\alpha\beta\gamma\delta}\epsilon_\alpha(\lambda_\gamma,p_A)(p_A+p_t)_\beta(p_A-p_t)_\gamma \epsilon^*_\delta(\lambda, p_A+p_t). \tag{9}$$

Hence the helicity amplitude

$$\begin{aligned} T_{\lambda_\gamma} &= \langle \pi_2(p_2)\pi_3(p_3)|\gamma(\lambda_\gamma,p_A)\pi_t(p_t)\rangle && (10)\\ &\propto \sum_\lambda \langle \pi_2(p_2)\pi_3(p_3)|\rho(\lambda,p_2+p_3)\rangle\langle\rho(\lambda,p_A+p_t)|\gamma(\lambda_\gamma,p_A)\pi_t(p_t)\rangle + \dots && (11)\\ &= fg(p_2-p_3)^\mu\left(-g_{\mu\alpha}+\frac{P_\mu P_\alpha}{P^2}\right)\epsilon^{\alpha\beta\gamma\delta}P_\beta(p_A-p_t)_\gamma\epsilon^*_\delta(\lambda,p_A+p_t), && (12)\end{aligned}$$

where $P = p_2 + p_3 = p_A + p_t$. In the CM frame $P^\mu = (\sqrt{s}, \vec{0})$, $\vec{p}_2 = -\vec{p}_3$ ... so that

$$T_{\lambda_\gamma} \propto -fg\sqrt{s}(-4)(-1)\vec{p}_3 \cdot \vec{p}_\gamma \times \vec{\epsilon}(\lambda_\gamma). \tag{13}$$

Let's evaluate this in a couple of simple frames. Firstly the frame in which the photon is along the z axis and p_2 is at some angle θ, ϕ. Then the basis of photon polarisation vectors is $\vec{\epsilon}(\lambda_\gamma = \pm) = \mp\frac{1}{\sqrt{2}}(1, \pm i, 0)$. Hence

$$T_\pm(s,\theta) \propto -4fg\sqrt{s}\, i\frac{\sin\theta}{\sqrt{2}}e^{\pm i\phi}. \tag{14}$$

Since $d^1_{\pm 0}(\theta) = \mp\frac{\sin\theta}{\sqrt{2}}$ we see that this agrees with our general form above including satifying the parity constraint.

The other obvious frame is the one used by Ascoli where I think the appropriate angles are

$$\hat{p}_3 = (\sin\chi_1\cos\gamma, \sin\chi_1\sin\gamma, \cos\chi_1) \tag{15}$$

$$\hat{p}_A = (-\sin\psi, 0, \cos\psi). \tag{16}$$

(I'm assuming Ascoli figure 5 is telling me the x-compt of $\hat{p}_A$ is negative).

We can rotate the photon polarisation basis for z-directed photons into the direction $\hat{p}_A$:

$$\vec{\epsilon}(\hat{p}_A, \pm) = \mp\tfrac{1}{\sqrt{2}}(\cos\psi, \pm i, \sin\psi). \tag{17}$$

Hence we can evaluate

$$T_\pm(s,\ldots) \propto -4fg\sqrt{s}\,\frac{i}{\sqrt{2}}(\cos\psi\sin\chi_1\cos\gamma \pm i\sin\chi_1\sin\gamma + \sin\psi\cos\chi_1), \tag{18}$$

which can be compared to the case $S = 1$ in

$$T_{\lambda_\gamma} = \frac{8\pi\sqrt{s}}{\sqrt{p_i^* p_f^*}}\sum_J (2J+1)\langle 00|T^J(s)|\lambda 0\rangle(-1)^S\sum_\lambda d^S_{\lambda_\gamma\lambda}(\psi)D^{S*}_{\lambda,0}(\gamma,\chi_1,0), \tag{19}$$

with the result that they agree.