CHIBA-EP-188

A low-energy effective Yang-Mills theory for quark and gluon confinement

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We derive a gauge-invariant low-energy effective model of the Yang-Mills theory, exhibiting both quark and gluon confinement: the Wilson loop average has area law and the Schwinger function violates reflection positivity. The resulting gluon propagator has the Gribov-Stingl type, especially, the infrared finite case is reproduced when one includes a mass term breaking nilpotency of the BRST symmetry. However, quark and gluon confinement can be obtained even in the absence of such a mass term. This model achieves 100% magnetic monopole dominance in the string tension and is dual to the Ginzburg-Landau model, confirming the dual superconductor picture for confinement.

PACS numbers: 12.38.Aw, 21.65.Qr

It is well-known that the area law of the Wilson loop average is a gauge-invariant criterion for quark confinement. However, a gauge-invariant criterion for gluon confinement and color confinement is not yet achieved. In recent several years, nevertheless, great endeavors have been made to clarify the low-energy behavior of propagators for gluon and the Faddeev-Popov (FP) ghost in specific gauges, e.g., Landau, Coulomb and Maximally Abelian (MA) gauges. Especially, in the Landau gauge, it is still under debate to discriminate two different types of propagators, i.e., scaling and decoupling. See e.g. [1]. According to [2, 3], however, both solutions satisfy quark confinement criterion and positivity violation as a necessary condition for gluon confinement. The quark confinement [2] has been exhibited for *non-zero* temperature T below the deconfinement temperature T_c ($0 < T < T_c$), since vanishing Polyakov loop average was used as a gauge-invariant criterion for quark confinement [4, 5].

In this Letter we derive a confining low-energy effective model of the Yang-Mills theory at *zero temperature*. Then we discuss a relationship between quark confinement and gluon confinement via the infrared behavior of gluon propagator. We show that including a certain mass term violating the nilpotent BRST symmetry yields the gluon propagator of Gribov-Stingl type. However, such a mass term is not indispensable to obtain quark and gluon confinement, since the area law and positivity violation can be obtained even in the absence of such a mass term.

This Letter is organized as follows.

(Step 1) [Reformulating the Yang-Mills theory in terms of new variables] In a path-integral quantization for the Yang-Mills theory, we decompose the Yang-Mills field $\mathscr{A}_\mu(x)$ into two pieces $\mathscr{V}_\mu(x)$ and $\mathscr{X}_\mu(x)$, i.e., $\mathscr{A}_\mu(x) = \mathscr{V}_\mu(x) + \mathscr{X}_\mu(x)$, and rewrite the action $S_{\rm YM}[\mathscr{A}]$ and the integration measure $[d\mathscr{A}]$ in terms of new variables related to $\mathscr{V}_\mu(x)$ and $\mathscr{X}_\mu(x)$, according to [6–8] and [9–12].

(Step 2) [Deriving an effective model by eliminating high-energy modes] We integrate out $\mathscr{X}_\mu$ field as the high-energy mode ($p^2 \geq M^2$) with a certain mass scale M of the field $\mathscr{A}_\mu$. Therefore, the resulting model $S_{\rm YM}^{\rm eff}[\mathscr{V}]$ is written in terms of $\mathscr{V}_\mu(x)$, and is identified with a low-energy effective model for describing the low-energy regime $p^2 \leq M^2$. A physical reasoning behind this step is explained below. The full gauge invariance of the original Yang-Mills theory $S_{\rm YM}[\mathscr{A}]$ is retained also for $S_{\rm YM}^{\rm eff}[\mathscr{V}]$.

(Step 3) [Converting the Wilson loop to the surface-integral] In the new formulation using new variables, we can exactly rewrite the Wilson loop operator $W_C[\mathscr{A}]$ originally defined in terms of $\mathscr{A}_\mu(x)$ by using $\mathscr{V}_\mu(x)$ alone without any reference to $\mathscr{X}_\mu(x)$, according to [13–15]. This fact suggests that $S_{\rm YM}^{\rm eff}[\mathscr{V}]$ is suitable as a low-energy effective model for quark confinement.

(Step 4) [Choosing a gauge to simplify the calculation] The Wilson loop average $\langle W_C[\mathscr{A}]\rangle_{\rm YM}$, i.e., vacuum expectation value of the Wilson loop operator $W_C[\mathscr{A}]$ is evaluated by using the effective model $S_{\rm YM}^{\rm eff}[\mathscr{V}]$ as $\langle W_C[\mathscr{A}]\rangle_{\rm YM}^{\rm eff}$. We show that the Wilson loop average has the area law for sufficiently large loop C, leading to the non-vanishing string tension σ in the linear part for the static quark-antiquark potential $V(R)$.

However, from the physical viewpoint of clarifying what is the mechanism for confinement, we modify

(Step 1', 2') We introduce an antisymmetric tensor field $B_{\mu\nu}$ of rank 2 [16–18], which is interpreted as a composite field of the Yang-Mills field. Then we repeat the same procedures as before to obtain the effective model $S_{\rm YM}^{\rm eff}[\mathscr{V}, B]$ by integrating out $\mathscr{X}$ field. Although the area law of the Wilson loop average is obtained also in this model, this modification has the advantages:

(Step 5)[Gribov-Stingl form for the gluon propagator consistent with gluon confinement] The effective gluon propagator for $\mathscr{V}$ obtained from $S_{\rm YM}^{\rm eff}[\mathscr{V}, B]$ by eliminating B field has the Gribov-Stingl form, which exhibits positivity violation suggesting gluon confinement [21–23].

(bonus)[dual superconductivity as a mechanism of quark confinement] The low-energy effective action $S_{\rm YM}^{\rm eff}[\mathscr{V}, B]$ is dual to the Ginzburg-Landau (GL) model in the London limit. This confirms that the dual superconductivity due to magnetic monopole condensation is indeed the mechanism for quark confinement. This aspect cannot be shown without introducing $B_{\mu\nu}$.

Thus the derived effective model exhibits both quark confinement (area law) and gluon confinement (positivity violation).

In this Letter, we consider only the $SU(2)$ gauge group [9–11] and the extension to $SU(N)$ based on [12] will be given in a subsequent paper.

(Step 1) The new variable $\mathscr{V}_\mu(x)$ as a Lie-algebra $su(2)$-valued field $\mathscr{V}_\mu(x) = \mathscr{V}_\mu^A(x)T_A$ $(A = 1,2,3)$ is constructed so that (i) $\mathscr{V}_\mu$ has the same gauge transformation as the original field $\mathscr{A}_\mu$, i.e., $\mathscr{V}_\mu(x) \to \Omega(x)\mathscr{V}_\mu(x)\Omega(x)^\dagger + ig^{-1}\Omega(x)\partial_\mu\Omega(x)^\dagger$ and hence its field strength $\mathscr{F}_{\mu\nu}[\mathscr{V}] := \partial_\mu\mathscr{V}_\nu - \partial_\nu\mathscr{V}_\mu - ig[\mathscr{V}_\mu, \mathscr{V}_\nu]$ transforms as $\mathscr{F}_{\mu\nu}[\mathscr{V}](x) \to \Omega(x)\mathscr{F}_{\mu\nu}[\mathscr{V}]\Omega(x)^\dagger$, and (ii) $\mathscr{F}_{\mu\nu}[\mathscr{V}]$ is proportional to $\boldsymbol{n}$, i.e., $\mathscr{F}_{\mu\nu}[\mathscr{V}](x) := \boldsymbol{n}(x)G_{\mu\nu}(x)$. Consequently, $G_{\mu\nu} = \boldsymbol{n}\cdot\mathscr{F}_{\mu\nu}[\mathscr{V}]$ is gauge-invariant, since the field $\boldsymbol{n}$ is constructed so that it transforms as $\boldsymbol{n}(x) \to \Omega(x)\boldsymbol{n}(x)\Omega(x)^\dagger$. The explicit transformation rule from the original variables $\mathscr{A}_\mu$ to the new variables $\mathscr{V}_\mu$, $\mathscr{X}_\mu$ are given by

$$\begin{aligned}
\mathscr{V}_\mu(x) &= c_\mu(x)\boldsymbol{n}(x) + ig^{-1}[\boldsymbol{n}(x), \partial_\mu\boldsymbol{n}(x)], \\
c_\mu(x) &:= \mathscr{A}_\mu(x)\cdot\boldsymbol{n}(x), \\
\mathscr{X}_\mu(x) &= ig^{-1}[D_\mu[\mathscr{A}]\boldsymbol{n}(x), \boldsymbol{n}(x)],
\end{aligned} \tag{1}$$

Here $\boldsymbol{n}(x)$ is the Lie-algebra $su(2)$-valued field $\boldsymbol{n}(x) = n^A(x)T_A$ $(A = 1,2,3)$ with a unit length, i.e., $n^A(x)n^A(x) = 1$. The so-called color field $\boldsymbol{n}$ must be obtained in advance as a functional of the original variable $\mathscr{A}_\mu$, e.g., by solving the reduction condition [9] $[\boldsymbol{n}(x), D_\mu[\mathscr{A}]D_\mu[\mathscr{A}]\boldsymbol{n}(x)] = 0$. $G_{\mu\nu}$ has the same form as the 't Hooft-Polyakov tensor for magnetic monopole:

$$G_{\mu\nu} = \partial_\mu c_\nu - \partial_\nu c_\mu + ig^{-1}\boldsymbol{n}\cdot[\partial_\mu\boldsymbol{n}, \partial_\nu\boldsymbol{n}]. \tag{2}$$

(Step 1') We can introduce a gauge-invariant antisymmetric tensor field $(^*B)_{\mu\nu}$ of rank 2 by inserting a unity into the path-integral [16–18]:

$$\begin{aligned}
1 = \int \mathcal{D}B \exp\Big[&- \int d^4x \frac{\gamma}{4}\{(^*B)_{\mu\nu} \\
&- (\alpha\boldsymbol{n}\cdot\mathscr{F}_{\mu\nu}[\mathscr{V}] - \beta\boldsymbol{n}\cdot ig[\mathscr{X}_\mu, \mathscr{X}_\nu])\}^2\Big],
\end{aligned} \tag{3}$$

where $*$ is the Hodge dual operation. Here (too many) parameters γ, α, β are introduced to see effects of each term. When $\beta = \gamma^{-1} = \tilde{G}$ and $\alpha = 0$, indeed, $(^*B)_{\mu\nu}$ is regarded as a collective field for the composite operator $\boldsymbol{n}\cdot ig[\mathscr{X}_\mu, \mathscr{X}_\nu]$ with the propagator $\tilde{G}$ obtainable in a self-consistent way [19] according to the Wilsonian renormalization group (RG) [20]. Then the Euclidean Yang-Mills Lagrangian is rewritten and modified into

$$\begin{aligned}
&\mathscr{L}_{\rm YM}[\mathscr{V}, \mathscr{X}, B] \\
&= \frac{1+\gamma\alpha^2}{4}G_{\mu\nu}^2 + \frac{\gamma}{4}(^*B)_{\mu\nu}^2 - \frac{\gamma\alpha}{2}(^*B)_{\mu\nu}G_{\mu\nu} \\
&+ \frac{1}{2}\mathscr{X}^{\mu A}Q_{\mu\nu}^{AB}\mathscr{X}^{\nu B} + \frac{1+\gamma\beta^2}{4}(ig[\mathscr{X}_\mu, \mathscr{X}_\nu])^2,
\end{aligned} \tag{4}$$

where we have defined

$$\begin{aligned}
Q_{\mu\nu}^{AB} :=& S^{AB}\delta_{\mu\nu} + (2+\gamma\alpha\beta)g\epsilon^{ABC}n^C G_{\mu\nu} \\
&- \gamma\beta g\epsilon^{ABC}n^C(*B)_{\mu\nu}, \\
S^{AB} :=& -(D_\rho[\mathscr{V}]D_\rho[\mathscr{V}])^{AB},
\end{aligned} \tag{5}$$

with the covariant derivative D_μ in the adjoint representation with $\mathscr{V}_\mu := \mathscr{V}_\mu^C T_C$, $(T_C)^{AB} = if^{ACB}$: $D_\mu^{AB} := \partial_\mu\delta^{AB} - gf^{ABC}\mathscr{V}_\mu^C = [\partial_\mu\mathbf{1} - ig\mathscr{V}_\mu]^{AB}$.

[On the effect and the role of the gluon mass term] The gluon "mass term" for the $\mathscr{X}$ field,

$$\frac{1}{2}M^2\mathscr{X}_\mu^2, \tag{6}$$

is gauge invariant in the new formulation [11]. Therefore, we can include this mass term in calculating the low-energy effective action. But we do not introduce this mass term explicitly. On the other hand, the inclusion of the gluon mass term for the $\mathscr{V}$ field,

$$\frac{1}{2}m^2\mathscr{V}_\mu^2 = \frac{1}{2}m^2c_\mu^2 + \frac{1}{2}m^2(\partial_\mu\boldsymbol{n})^2, \tag{7}$$

breaks gauge invariance and BRST invariance after taking specific gauges. However, we can modify the BRST such that the modified BRST is a symmetry of the Yang-Mills theory with the mass term at the cost of nilpotency.

(Step2') We identify $\mathscr{X}_\mu$ with the "high-energy" mode in the range $p^2 \in [M^2, \Lambda^2]$, integrate out the "high-energy" modes $\mathscr{X}_\mu$ by taking into account the FP-like determinant [10] term associated with the reduction condition [9], but we neglect quartic self-interactions among $\mathscr{X}_\mu$, i.e., $(ig[\mathscr{X}_\mu, \mathscr{X}_\nu])^2$. Here Λ is the ultraviolet (UV) cutoff as the initial value for the Wilsonian RG and M is the infrared (IR) cutoff. The calculation is not exactly the one-loop level after introducing $B_{\mu\nu}$. In these approximations we obtain a *gauge-invariant* low-energy effective action $S_{\rm YM}^{\rm eff}[\mathscr{V}, B]$ *without mass terms* (6),(7):

$$\begin{aligned}
&S_{\rm YM}^{\rm eff}[\mathscr{V}, B] \\
&= \int \Big[\frac{1+\gamma\alpha^2}{4}G_{\rho\sigma}^2 + \frac{\gamma}{4}(^*B)_{\rho\sigma}^2 - \frac{\gamma\alpha}{2}(^*B)_{\rho\sigma}G_{\rho\sigma}\Big] \\
&\quad + \frac{1}{2}\ln\det Q_{\rho\sigma}^{AB} - \ln\det S^{AB},
\end{aligned} \tag{8}$$

with the functional logarithmic determinant

$$\begin{aligned}
&\frac{1}{2}\ln\det Q_{\rho\sigma}^{AB} - \ln\det S^{AB} \\
&= \int \frac{g^2\ln\frac{\mu^2}{M^2}}{(4\pi)^2}\Big[\frac{1}{6}G_{\rho\sigma}^2 - \frac{1}{2}\{(2+\gamma\alpha\beta)G_{\rho\sigma} - \gamma\beta(^*B)_{\rho\sigma}\}^2\Big] \\
&+ \int \frac{g^2}{(4\pi)^2}\frac{1}{M^2}\frac{1}{6}(D_\lambda^{AB}[\{(2+\gamma\alpha\beta)G_{\rho\sigma} - \gamma\beta(^*B)_{\rho\sigma}\}n^B])^2 \\
&+ O(\partial^4/M^4),
\end{aligned} \tag{9}$$

where $\int = \int d^4x$. This is one of main results. The gauge fixing is unnecessary in this calculation. Indeed, the resulting effective action (8) with (9) is manifestly gauge invariant. The correct RG β-function at the one-loop level $\beta(g) := \mu\frac{dg(\mu)}{d\mu} = -b_1g^3 + O(g^5)$, $b_1 = \frac{22}{3}/(4\pi)^2$ is reproduced in a gauge invariant way when $\gamma\alpha\beta = 0$ which follows from e.g. $\alpha = 0$ (mentioned above) or $\gamma = 0$ (in the case of no $B_{\mu\nu}$ field). To obtain (9), we used the heat kernel to calculate the *regularized* logarithmic determinant. Instead of using the standard regulator function R_M of the functional RG approach [20], we restrict the integration range of τ to $\tau \in [1/\Lambda^2, 1/M^2]$, which corresponds to the momentum-shell integration $p^2 \in [M^2, \Lambda^2]$

$$\ln\det\mathcal{O} = -\int d^D x \lim_{s\to 0}\frac{d}{ds}\Big[\frac{\mu^{2s}}{\Gamma(s)}\int_{1/\Lambda^2}^{1/M^2} d\tau\tau^{s-1} \times \mathrm{tr}\left\langle x|e^{-\tau\mathcal{O}}|x\right\rangle\Big], \tag{10}$$

where tr denotes the trace over Lorentz indices and group indices and μ is the renormalization scale. The limit $\Lambda \to \infty$ should be understood in what follows. These results extend previous works [16, 17, 24, 25].

(Step 3) We use a non-Abelian Stokes theorem [13–15] to rewrite a non-Abelian Wilson loop operator

$$W_C[\mathscr{A}] := \mathrm{tr}\left[\mathscr{P}\exp\left\{ig\oint_C dx^\mu \mathscr{A}_\mu(x)\right\}\right], \tag{11}$$

into the area-integral over the surface Σ ($\partial\Sigma = C$):

$$W_C[\mathscr{A}] = \int d\mu_\Sigma(\xi)\exp\left[ig\int_{\Sigma:\partial\Sigma=C} G\right], \tag{12}$$

where the product measure $d\mu_\Sigma(\xi) := \prod_{x\in\Sigma} d\mu(\xi_x)$ is defined with an invariant measure $d\mu$ on $SU(2)$ normalized as $\int d\mu(\xi_x) = 1$, $\xi_x \in SU(2)$. In the two-form $G := \frac{1}{2}G_{\mu\nu}(x)dx^\mu \wedge dx^\nu$, $G_{\mu\nu}$ agrees with the field strength (2) under the identification of the color field $\boldsymbol{n}(x)$ with a normalized traceless field $\boldsymbol{n}(x) := \xi_x(\sigma_3/2)\xi_x^\dagger$.

(Step 4) We evaluate the Wilson loop average $W(C) = \langle W_C[\mathscr{A}]\rangle_{\mathrm{YM}}$ by using the effective action $S^{\mathrm{eff}}_{\mathrm{YM}}[\mathscr{V}, B]$, i.e., $\langle W_C[\mathscr{A}]\rangle_{\mathrm{YM}} \simeq \langle W_C[\mathscr{A}]\rangle^{\mathrm{eff}}_{\mathrm{YM}}$ with the aid of (12).

To obtain the propagator or correlation functions, we need to fix the gauge. In the Landau gauge, $\partial^\mu\mathscr{A}_\mu = 0$, correlation functions for new variables are studied in the numerical way in [28] based on [26, 27]. This justifies the identification of $\mathscr{X}_\mu$ as the high-energy mode negligible in the low-energy regime below $M \simeq 1.2$GeV.

In what follows, we take the unitary-like gauge

$$n^A(x) = \delta_{A3}, \tag{13}$$

which reproduces the same effect as taking the MA gauge [29] in the original theory. In this gauge, $\mathscr{X}_\mu^A(x)$ reduces to the off-diagonal component $A_\mu^a(x)$ ($a = 1, 2$), while $\mathscr{V}_\mu^A(x)$ reduces to the diagonal one $A_\mu^3(x) = a_\mu(x)$, i.e., $\mathscr{X}_\mu^A(x) = \mathscr{A}_\mu^a(x)\delta_{Aa}$, $\mathscr{V}_\mu^A(x) = \mathscr{A}_\mu^3(x)\delta_{A3} = c_\mu(x)\delta_{A3}$. The field strength reads

$$G_{\mu\nu}(x) \to F_{\mu\nu}(x) := \partial_\mu c_\nu(x) - \partial_\nu c_\mu(x), \quad F = dc. \tag{14}$$

The gauge (13) forces the color field to point to the same direction by gauge rotations. Hence the field c contains singularities (of hedge-hog type) similar to the Dirac magnetic monopole after taking the gauge (13). Therefore, $dF = ddc \neq 0$. If we do not take this gauge, such a contribution is contained also in the part $ig^{-1}\boldsymbol{n}\cdot[\partial_\mu\boldsymbol{n}, \partial_\nu\boldsymbol{n}]$ in a gauge-invariant combination $G_{\mu\nu}$, see [26, 27]. In this gauge, the effective action with an optional mass term (7) reads up to quadratic in the fields

$$\begin{aligned} &S^{\mathrm{eff}}_{\mathrm{YM}}[c, B] \\ &= \int \Big[\frac{1+\gamma\alpha^2}{4}F_{\rho\sigma}^2 + \frac{\gamma}{4}({}^*B)_{\rho\sigma}^2 - \frac{\gamma\alpha}{2}({}^*B)_{\rho\sigma}F_{\rho\sigma}\Big] \\ &\quad + \int \frac{g^2\ln\frac{\mu^2}{M^2}}{(4\pi)^2}\left[\frac{1}{6}F_{\rho\sigma}^2 - \frac{1}{2}[(2+\gamma\alpha\beta)F_{\rho\sigma} - \gamma\beta({}^*B)_{\rho\sigma}]^2\right] \\ &\quad + \int \frac{g^2}{(4\pi)^2}\frac{1}{M^2}\frac{1}{6}(\partial_\lambda[(2+\gamma\alpha\beta)F_{\rho\sigma} - \gamma\beta({}^*B)_{\rho\sigma}])^2 \\ &\quad + \int \frac{1}{2}m^2 c_\mu^2 + O(\partial^4/M^4). \end{aligned} \tag{15}$$

We can show that the mass term (6) plays the same role as the IR regulator mentioned above, see [18].

The simplest way to demonstrate the area law is to use the low-energy effective action $S^{\mathrm{eff}}_{\mathrm{YM}}[c, B]$ retained up to terms quadratic and bilinear in c and B and the Wilson loop operator reduced in the unitary-like gauge to

$$W_C[F] = \exp\left[ig\int_{\Sigma:\partial\Sigma=C} F\right] = \exp\left[ig(\Theta_\Sigma, F)\right], \tag{16}$$

where Θ_Σ is the vorticity tensor defined by $\Theta_\Sigma^{\mu\nu}(x) = \int_\Sigma d^2S^{\mu\nu}(x(\sigma))\delta^D(x - x(\sigma))$,which has the support on the surface Σ whose boundary is the loop C. Here $(\cdot,\cdot)$ is the L^2 inner product for two forms: $(\Theta_\Sigma, F) = \int d^D x \frac{1}{2}\Theta_\Sigma^{\mu\nu}(x)F_{\mu\nu}(x) = \int_{\Sigma:\partial\Sigma=C} F$. By integrating out B, we obtain the effective model $S^{\mathrm{eff}}_{\mathrm{YM}}[c] = \frac{1}{2}\left(c, \mathscr{D}_{\mathrm{cc}}^{-1}c\right) = \frac{1}{2}\left(F, \mathscr{D}_{\mathrm{FF}}^{-1}F\right)$ in terms of c or F. Then the Wilson loop average $W(C)$ is evaluated by integrating out $F = dc$:

$$W(C) = \exp\left[-\frac{1}{2}g^2(\Theta_\Sigma, \mathscr{D}_{\mathrm{FF}}\Theta_\Sigma)\right], \tag{17}$$

where $\mathscr{D}_{\mathrm{FF}} = \mathit{\Delta}\mathscr{D}_{\mathrm{cc}}$ and its Fourier transform $\tilde{\mathscr{D}}_{\mathrm{FF}}(p) = p^2\tilde{\mathscr{D}}_{\mathrm{cc}}(p)$. For concreteness, we choose Θ_Σ for a planar surface bounded by a rectangular loop C with side lengths T and R in the $x_3 - x_4$ plane. Then the Wilson loop average has the area law $W(C) \sim \exp[-\sigma RT]$ for large R with the string tension given by the formula:

$$\sigma = g^2\int_{p^2:=p_1^2+p_2^2\le M^2}\frac{dp_1dp_2}{(2\pi)^2}\tilde{\mathscr{D}}_{\mathrm{FF}}(p_1, p_2, 0, 0) > 0, \tag{18}$$

where the momentum integration is cutoff at the upper limit M. A positive and finite string tension $0 < \sigma < \infty$ follows from the condition of no real poles in the effective gluon propagator $\tilde{\mathscr{D}}_{\mathrm{cc}}(p)$ in the Euclidean region, $0 < \tilde{\mathscr{D}}_{\mathrm{FF}}(p) = p^2\tilde{\mathscr{D}}_{\mathrm{cc}}(p) < \infty$, which is connected to gluon confinement below. This is another of main results.

The effective propagator $\mathscr{D}_{\rm cc}$ has the Gribov-Stingl form (up to an overall normalization $C>0$):

$$\tilde{\mathscr{D}}_{\rm FF}(p)=p^2\tilde{\mathscr{D}}_{\rm cc}(p),\ \tilde{\mathscr{D}}_{\rm cc}(p)=C\frac{1+d_1p^2}{c_0+c_1p^2+c_2p^4}, \quad (19)$$

where $c_0=m^2$, $c_1=1+\frac{\gamma\beta^2}{3}\frac{g^2}{(4\pi)^2}\frac{m^2}{M^2}$, $c_2=\frac{g^2}{(4\pi)^2}\frac{1}{M^2}[(2+\gamma\alpha\beta)^2+(1+\gamma\alpha^2)\gamma\beta^2+2(2+\gamma\alpha\beta)\gamma\alpha\beta]/3$, and $d_1=\frac{\gamma\beta^2}{3}\frac{g^2}{(4\pi)^2}\frac{1}{M^2}$. The precise values of the parameters m,γ,α,β and M are to be determined by the functional RG [20] following [5], which is a subject of a subsequent paper. According to numerical simulations in MA gauge [31–33], the diagonal gluon propagator is well fitted to the form (19): e.g. [33] give $c_0=0.064(2)\text{GeV}^2$, $c_1=0.125(9)$, $c_2=0.197(9)\text{GeV}^{-2}$, and $d_1=0.13(1)\text{GeV}^{-2}$. This indeed leads to a good estimate for the string tension $\sigma\simeq(0.4\text{GeV})^2$ according to (18) for $M\simeq1.2\text{GeV}$ ($C\simeq1$) and $\alpha(\mu=M)=g^2(\mu=M)/(4\pi)\simeq1.0$.

The Gribov-Stingl form is obtained only when $c_0\neq0$ (i.e., $m\neq0$) and $d_1\neq0$ ($B_{\mu\nu}$ is included). Even in the limit $m^2\to0$ ($c_0\to0$), the area law survives according to (18), provided that $\tilde{\mathscr{D}}_{\rm FF}(p)$ remains positive and finite: $\tilde{\mathscr{D}}_{\rm FF}(p)\to C\frac{1+d_1p^2}{c_1+c_2p^2}$, while $\tilde{\mathscr{D}}_{\rm cc}(p)$ behaves as $\tilde{\mathscr{D}}_{\rm cc}(p)\to C\frac{1+d_1p^2}{p^2(c_1+c_2p^2)}$. Hence, it does not matter to quark confinement whether $m=0$ or $m\neq0$.

(Step5) The positivity violation is examined. In the case of $c_2\neq0$, $\tilde{\mathscr{D}}_{\rm cc}(p)$ has a pair of complex conjugate poles at $p^2=z$ and $p^2=z^*$, $z:=x+iy$, $x:=-c_1/(2c_2)$, $y:=\sqrt{c_0/c_2-(c_1/(2c_2))^2}$ We find that the Schwinger function $\Delta(t):=\int_{-\infty}^{+\infty}\frac{dp_4}{2\pi}e^{ip_4t}\tilde{\mathscr{D}}_{\rm cc}(\boldsymbol{p}=0,p_4)$ is negative over finite intervals in the Euclidean time $t>0$:

$$\Delta(t)=\frac{1}{2c_2|z|^{3/2}\sin(2\varphi)}e^{-t|z|^{1/2}\sin\varphi}[\cos(t|z|^{1/2}\cos\varphi-\varphi)$$
$$+d_1|z|\cos(t|z|^{1/2}\cos\varphi+\varphi)], \quad (20)$$

where $z=|z|e^{2i\varphi}$ with $|z|=(c_0/c_2)^{1/2}$, $\cos(2\varphi)=-\sqrt{c_1^2/(4c_0c_2)}$, and $\sin(2\varphi)=\sqrt{1-c_1^2/(4c_0c_2)}$. Therefore, the reflection positivity is violated for the gluon propagator (19), as long as $0<\frac{c_1^2}{4c_0c_2}<1$, irrespective of d_1. The special case $c_0=0$ also violates the positivity:

$$\Delta(t)=-\frac{t}{2\sqrt{2\pi}c_1}+\frac{c_2^{1/2}}{2c_1^{3/2}}\left(\frac{c_1d_1}{c_2}-1\right)e^{-t\sqrt{\frac{c_1}{c_2}}}. \quad (21)$$

Thus the diagonal gluon can be confined. In the case of $c_2=0$, there is no positivity violation, as far as $c_0/c_1>0$.

(bonus) The area law originates from magnetic monopoles. In our effective model, 100% monopole dominance in the string tension is achieved. For $m=0$, the path-integral duality transformation of our model agrees with the GL model in the London limit, and quark confinement is caused by the dual Meissner effect induced by spontaneous breaking of the dual U(1) symmetry, as demonstrated in [17, 18]. For $m\neq0$, there appears a deviation from the dual GL model. More details will be given in a subsequent paper.

Acknowledgements — This work is supported by Grant-in-Aid for Scientific Research (C) 21540256 from Japan Society for the Promotion of Science (JSPS).

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