# Centre-sector tunneling, confinement and the quark Fermi surface

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- Does centre-sector-tunneling take place in the hadronic phase?
  → `lattice gauge simulations`: YM + qHiggs
  → centre-sector-tunneling and the 't Hooft loop
  → tunneling coeffcient (*new!*)

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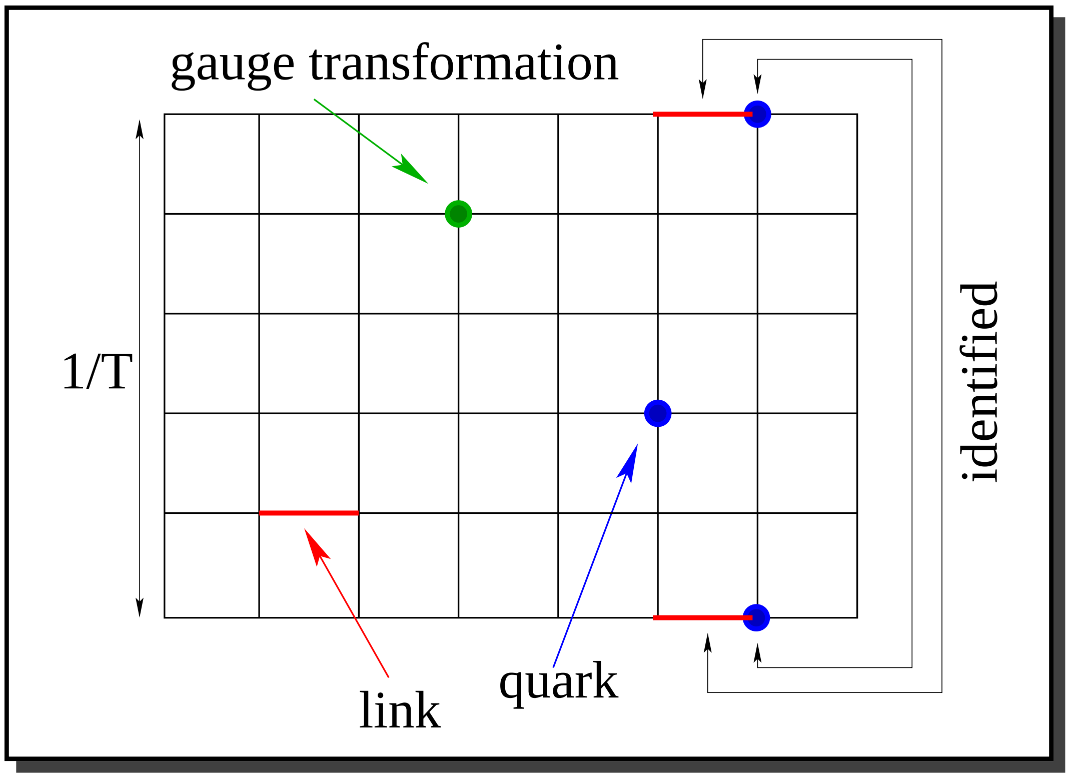
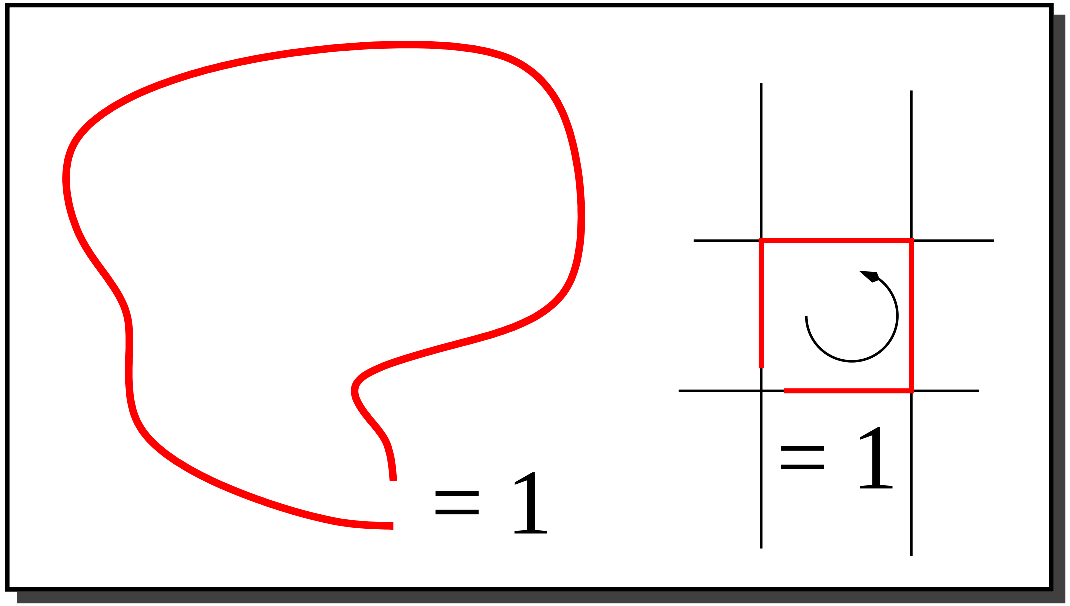



- gauge fields: links U
- matter fields: site q
- gauge transformations: site $\Omega$

# Yang-Mills moduli

- My name is vacuum - `the vacuum:`
  (pert.) vacuum $\leftrightarrow$ all contractable loops are $1$

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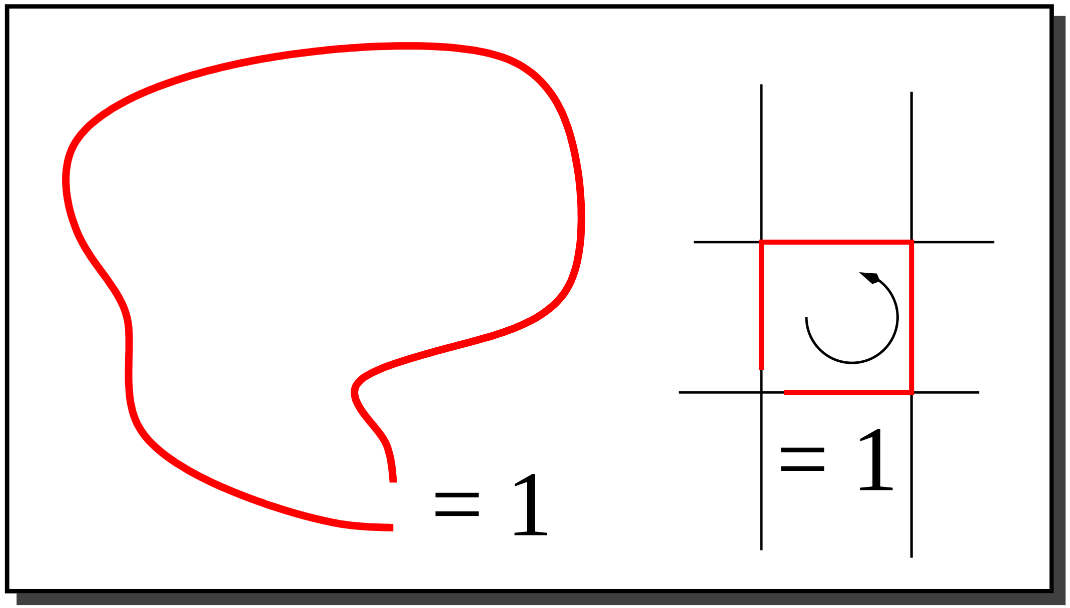


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more vacua?

- constructing the moduli space
  $\Rightarrow$ need to "devide out" the gauge transformations
  `[Keurentjes, Rosly, Smilga, PRD 58 (1998) 081701]`
  `[Schaden, PRD 71 (2005) 105012]`
  `[Langfeld, Lages, Reinhardt, PoS LAT2005:201,2006.]`

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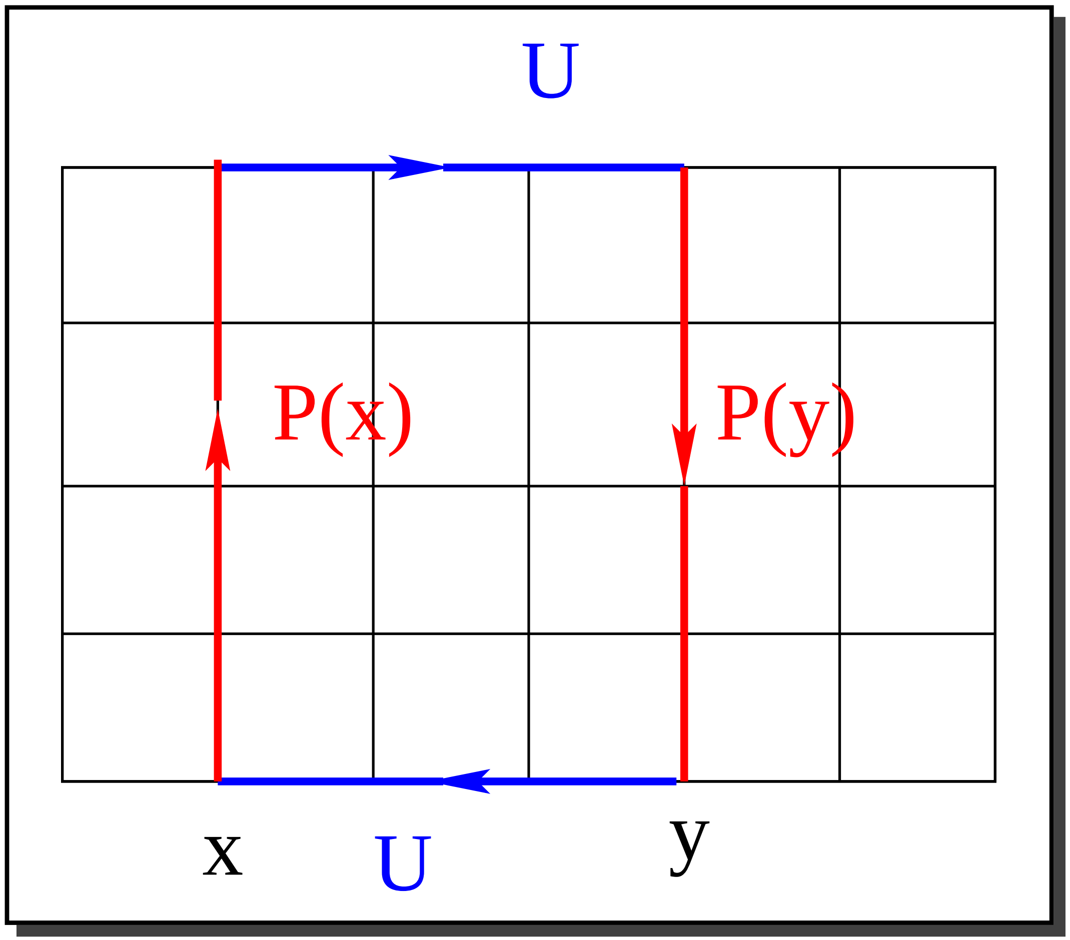
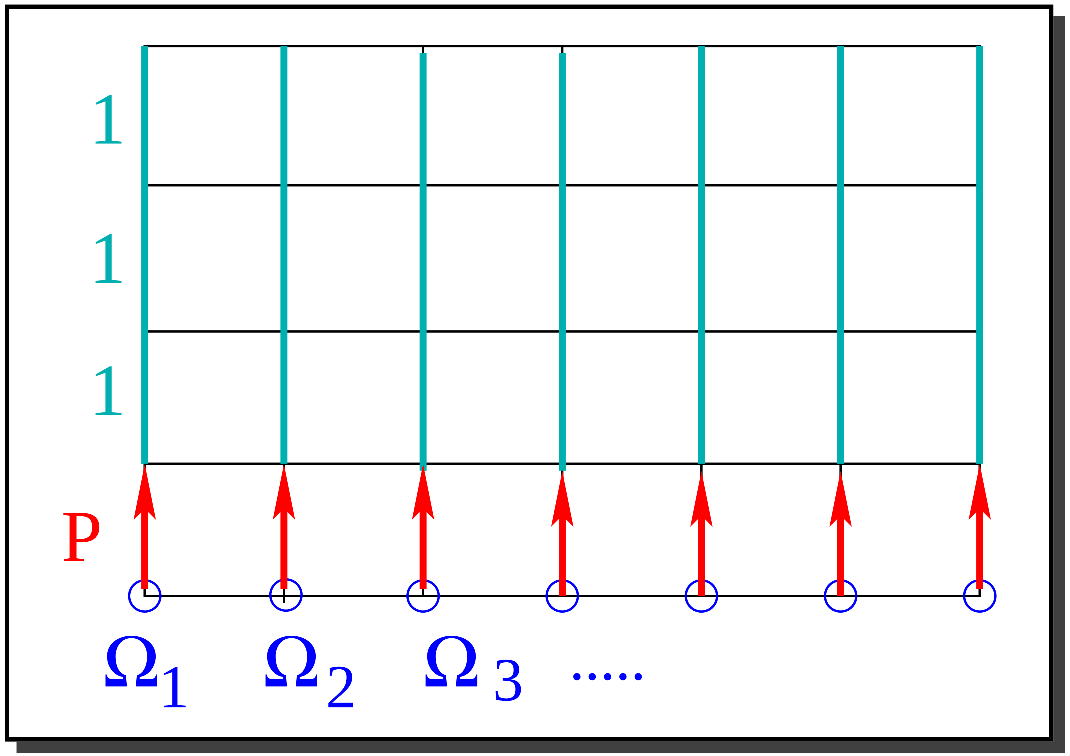
- $U^\dagger P(x) U P^\dagger(y) = 1 \quad \Rightarrow \quad P(y) = U^\dagger P(x) U$ `q.e.d.`



# Yang-Mills moduli

- complete gauge fixing $\longrightarrow$ moduli space
  step 1



# Yang-Mills moduli

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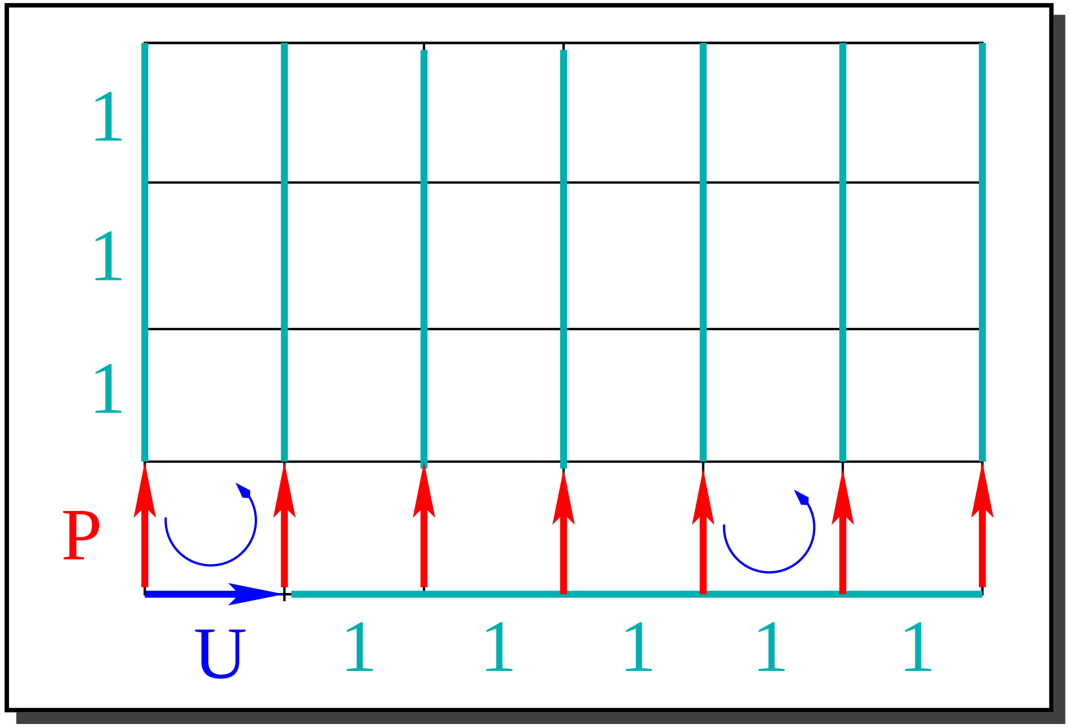


# Yang-Mills moduli

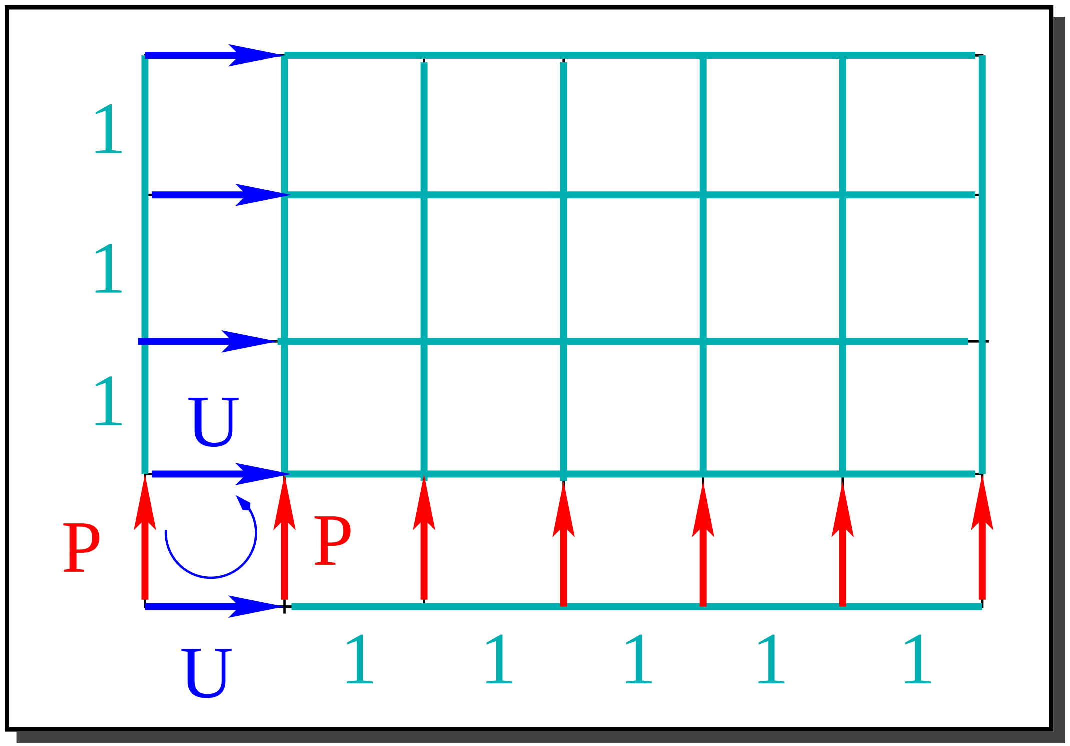
- complete gauge fixing $\longrightarrow$ moduli space

  step 3

$$U\,P\,U^\dagger\,P^\dagger = 1$$

$$[U, P] = 0$$

$U, P \in$ cartan



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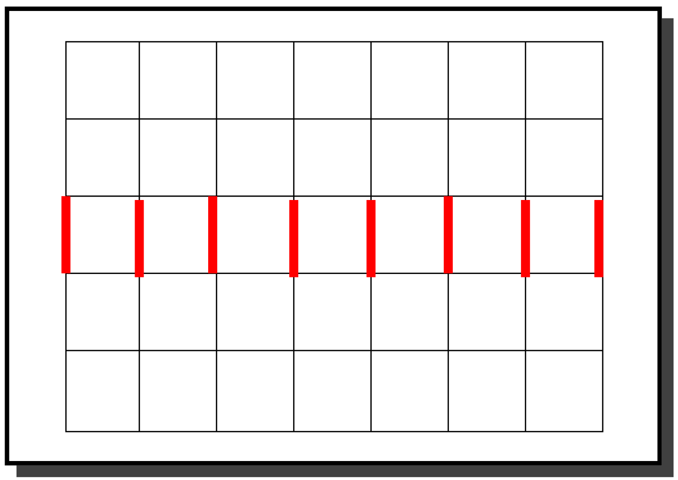

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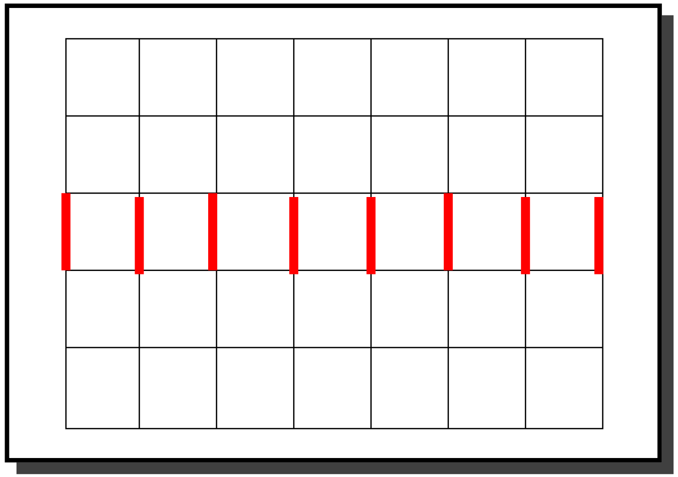

- symmetry of the action
- *mediates between vacua:*

  tr $P \rightarrow z$ tr $P$

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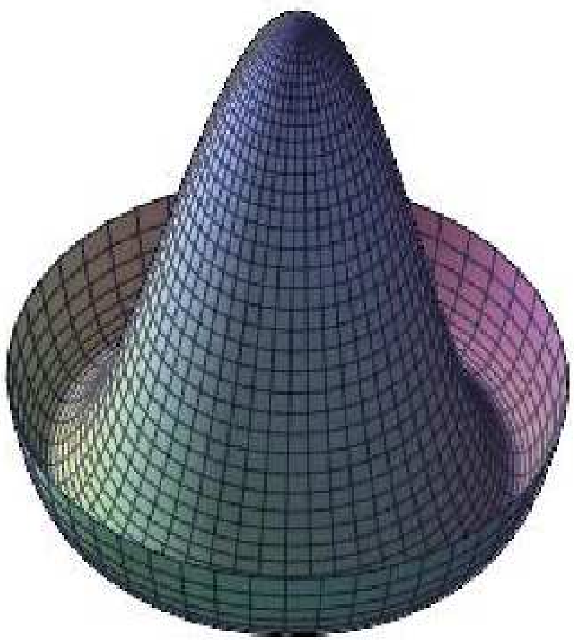
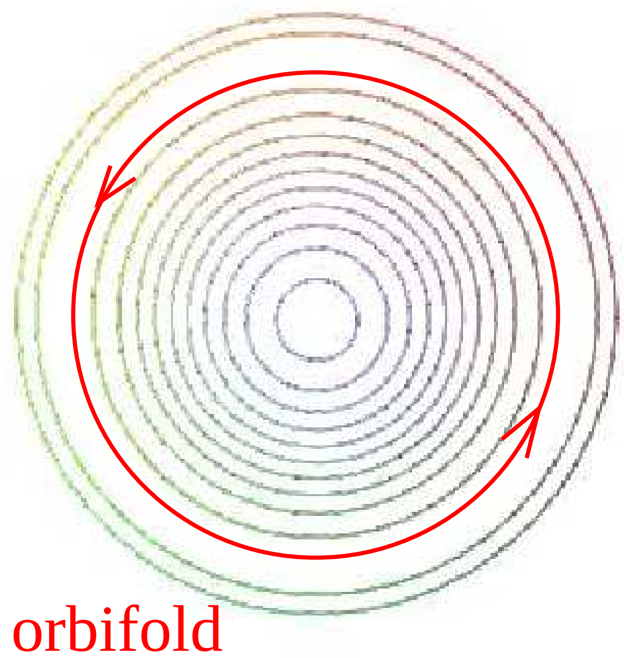
  [ $\langle \mathtt{tr}\ P \rangle = 0$ also for $T > T_{\rm deconf}$]
- dynamical matter (*QCD!*): flat directions of the vacuum are lifted, but centre sector tunneling still takes place in the hadronic phase
  extreme conditions: SSB of centre symmetry on top of explicit breaking

# Centre sector tunneling:

Illustration:

- classical action ⇒ moduli space

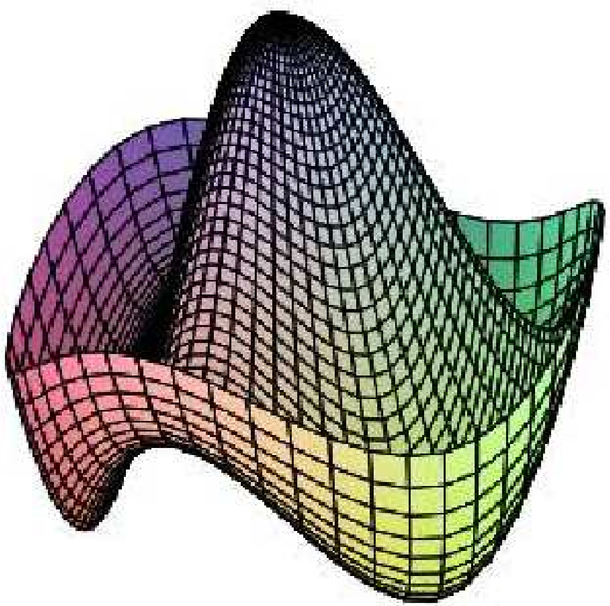


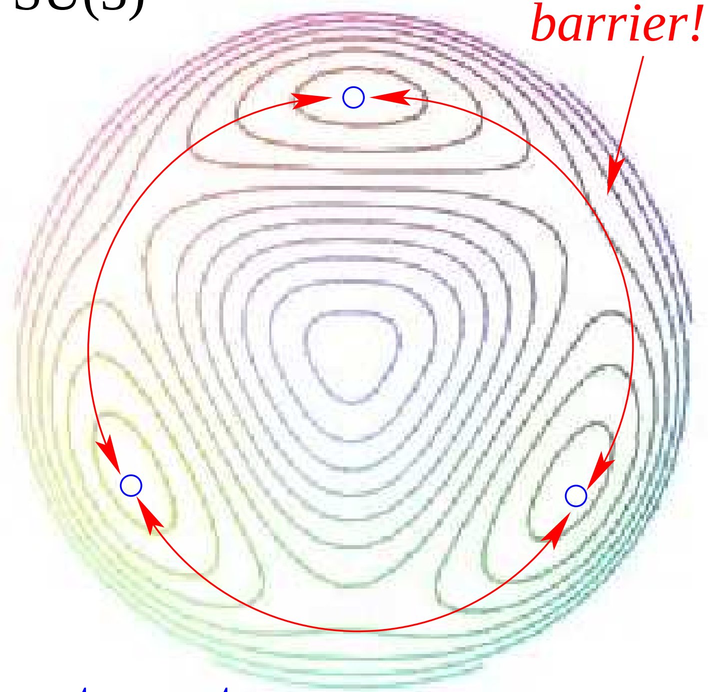


# Centre sector tunneling:

**Illustration:**

- quantum effective action, pure YM $\Rightarrow$ centre sectors





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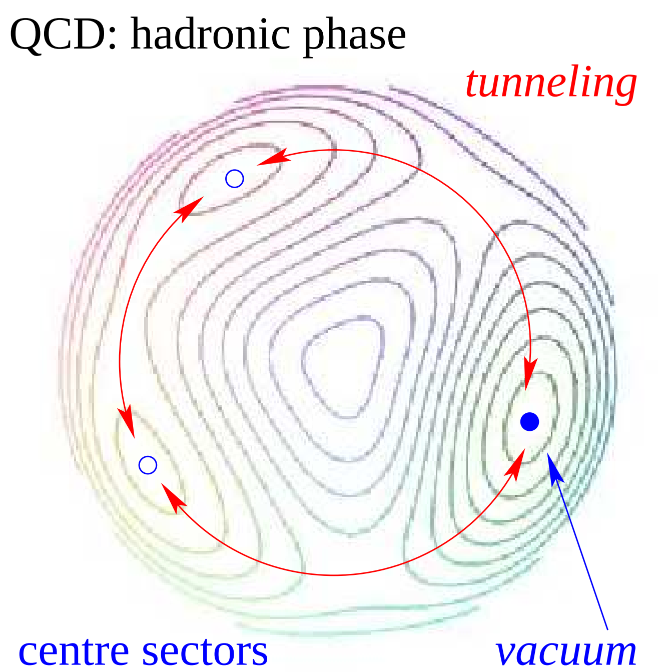

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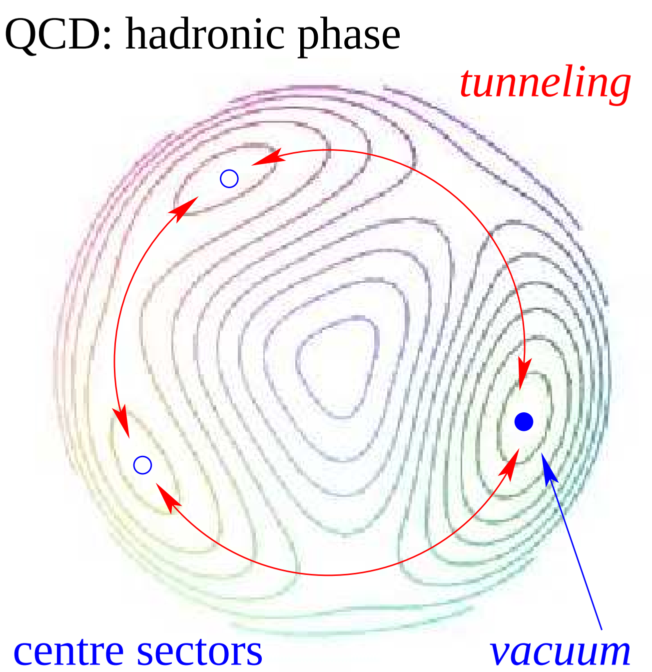


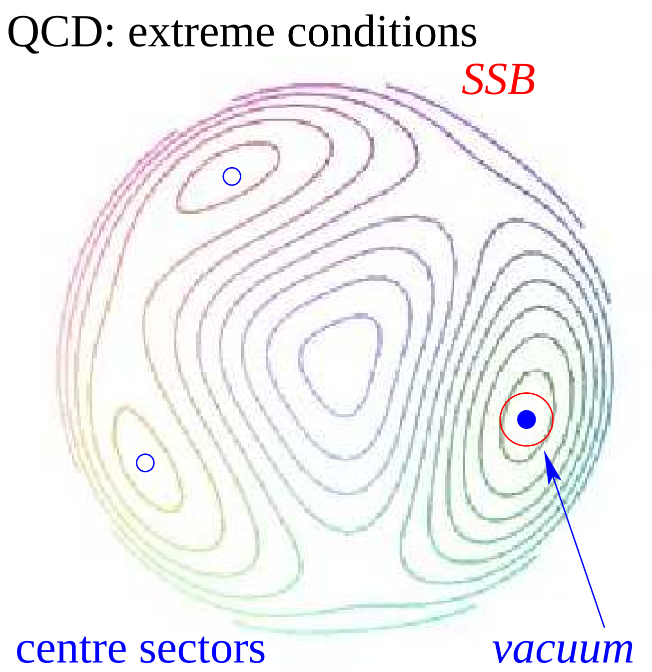

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- ...will talk about SU(3) + matter = QCD later

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## Model consideration:

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  $A_m$: moduli fields $\Rightarrow$ weighted sum over centre sectors

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- $q(x)$: quarks, $m$: mass, $\mu$: chemical potential
  $A_m$: moduli fields $\Rightarrow$ weighted sum over centre sectors
- partition function: $\exp\{iA_m\} = Z_m \in Z(N_C)$

$$Z = \sum_{m=1}^{N_c} p_m \int \mathcal{D}q \, \mathcal{D}\bar{q} \, \exp\{\bar{q}(i\partial\!\!\!/ + (A_m + i\mu)\gamma_0 + im)q\}$$

$p_m$ : probability for centre sector $m$

pure YM-theory: $p_m = 1/N_c, \forall\, m$

high $T$ SSB phase: $p_{N_c} = 1$, $p_m = 0$ for $m = 1 \ldots N_c - 1$

hadronic phase: $p_{N_c} > p_m \neq 0$ for $m = 1 \ldots N_c - 1$

[Langfeld, Wellegehausen, Wipf, Phys. Rev. D81 (2010) 114502]

# *Fermi-Einstein-Condensation (FEC)*

## Results for baryon density:

- $$B == \frac{1}{\pi^2} \int_m^\infty dE \; E \, \sqrt{E^2 - m^2} \; \rho(E, T, \mu)$$

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- $$w_m = p_m \, \rho_m / \sum_i p_i \, \rho_i$$

$$\rho_i \; = \; \exp\Big\{ \frac{V}{\pi^2} \int_m^\infty dE \, E \, \sqrt{E^2 - m^2} \, \ln(1 + z_i \mathrm{e}^{-\frac{E-\mu}{T}}) \Big\}$$

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## (I) high temperature phase

- remember:
  sector probability: $p_{N_c} = 1$, $p_m = 0$ for $m = 1 \ldots N_c - 1$
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*free quarks with a Fermi surface !!*

# Fermi-Einstein-Condensation (FEC)

## (II) hadronic phase ($N_c$ even)

- sector probability: $p_{N_c} > p_m \neq 0$ for $m = 1 \ldots N_c - 1$

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  weights: $w_{N_c/2} \approx \frac{p_{N_c/2}\rho_{N_c/2}}{p_{N_c/2}\rho_{N_c/2}} = 1$, $w_m \approx 0$ else

$$\rho_{N_c/2} \;=\; \exp\left\{ \frac{V}{\pi^2} \int_m^\infty dE \, E \sqrt{E^2 - m^2} \, \ln(1 - \mathrm{e}^{-\frac{E-\mu}{T}}) \right\} \to \infty$$

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- $$\rho(E,T,\mu) \;=\; \sum_m \frac{z_m}{e^{[E-\mu]/T} + z_m} \; w_m \;=\; \frac{-1}{e^{[E-\mu]/T} - 1}$$

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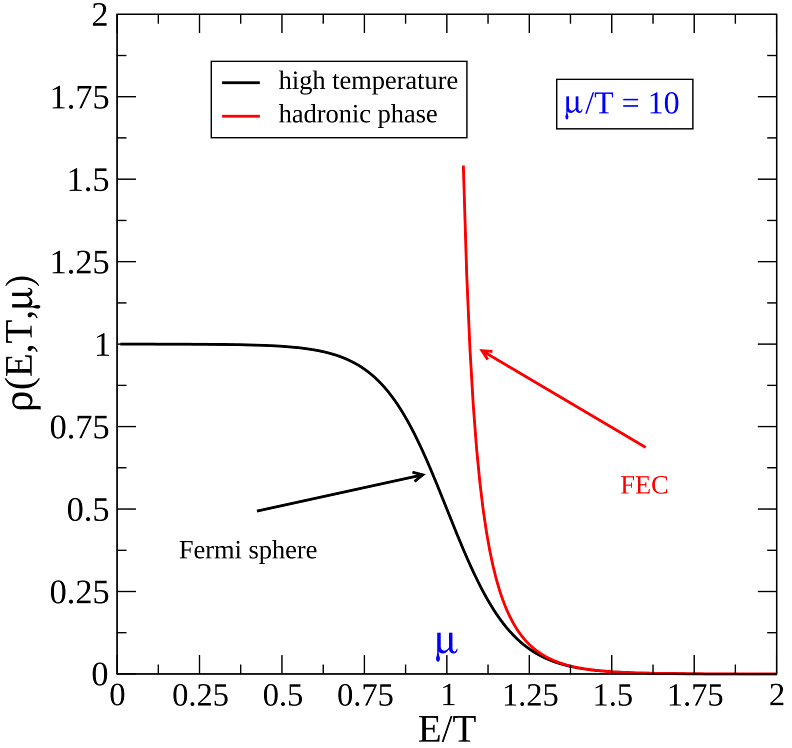
[Langfeld, Wellegehausen, Wipf, Phys. Rev. D81 (2010) 114502]

# Fermi-Einstein-Condensation (FEC)

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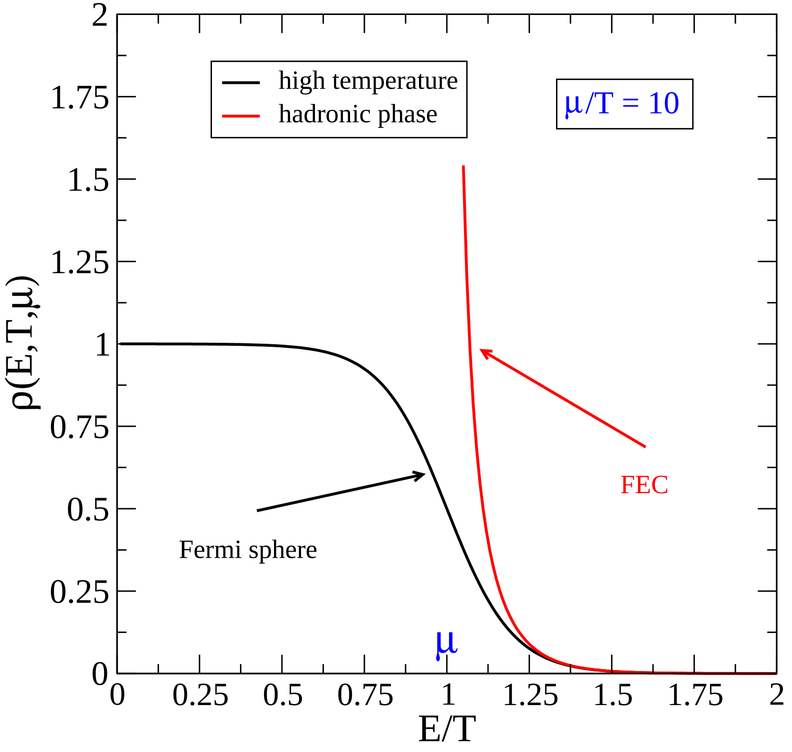


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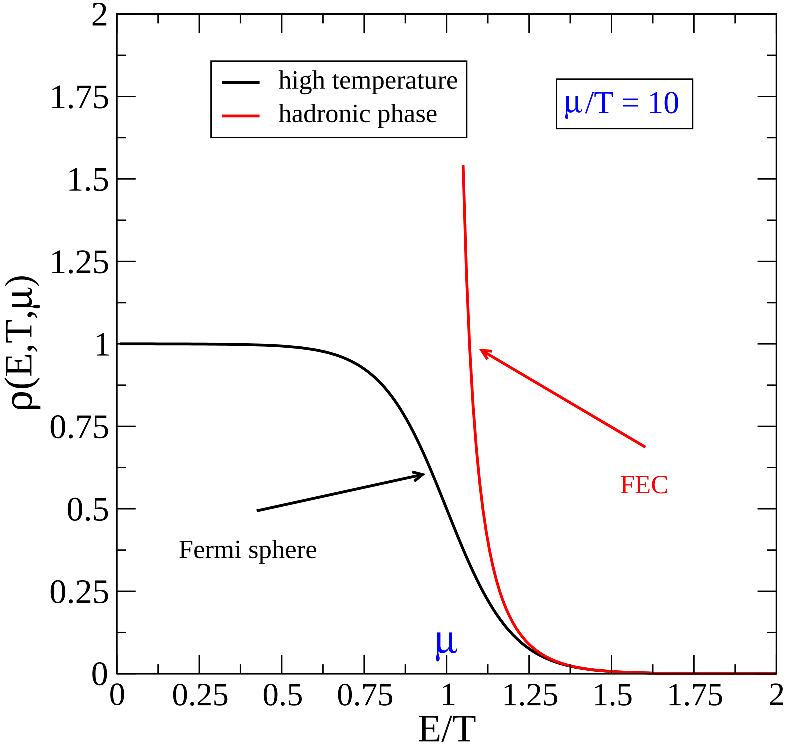


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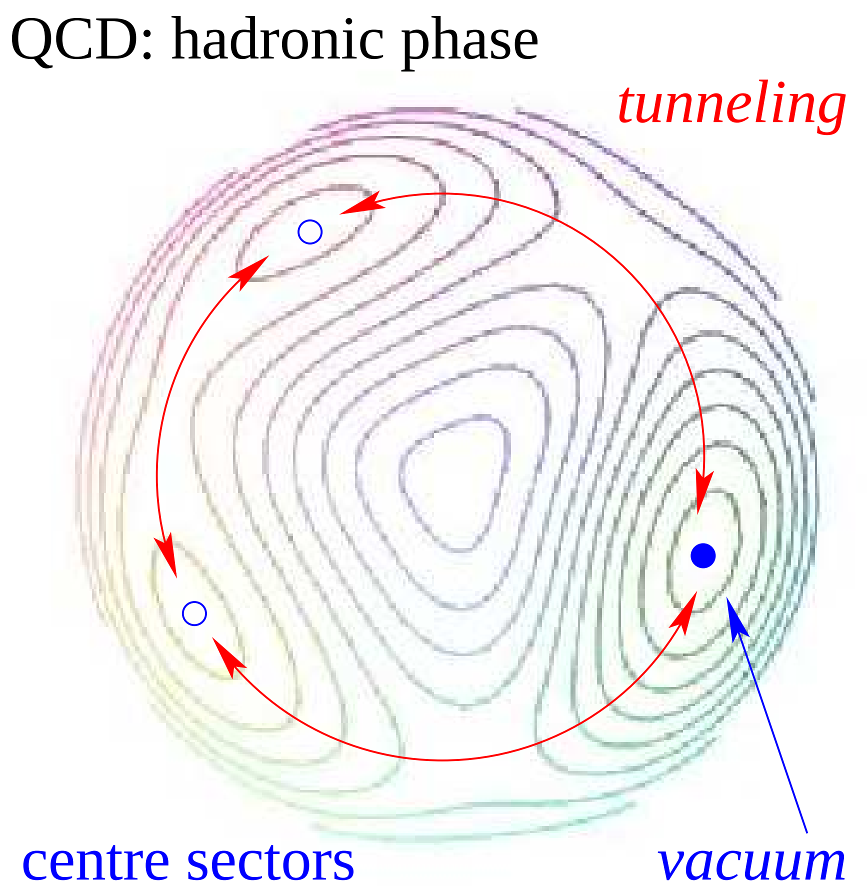
- quarks are still represented by Grassmann fields but the spin-statistic theorem does not apply as long as colour is confined

# Centre sector tunneling:

- Does centre sector tunneling take place in in the hadronic phase of QCD ?



# *Model considerations:*

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- need good ergodcity:

  `lHMC for the gluon sector`

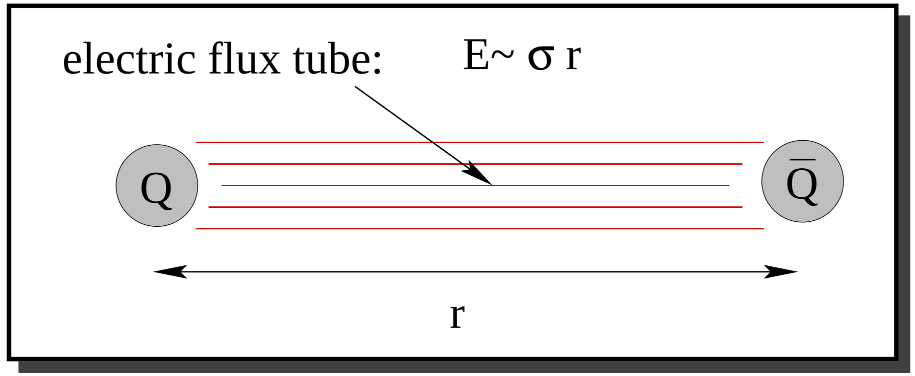
  `HMC for the Higgs sector`

  `simulations at the HPCC, Plymouth`

# Model consideration:

## String breaking

- Pure Yang-Mills theory (no dynamical quarks)



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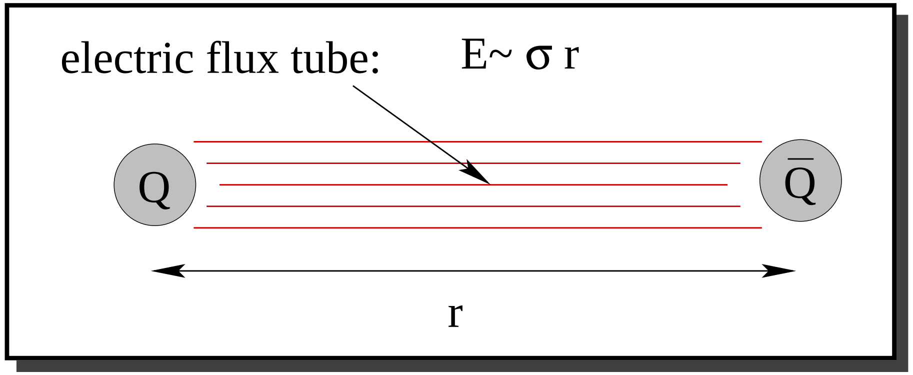
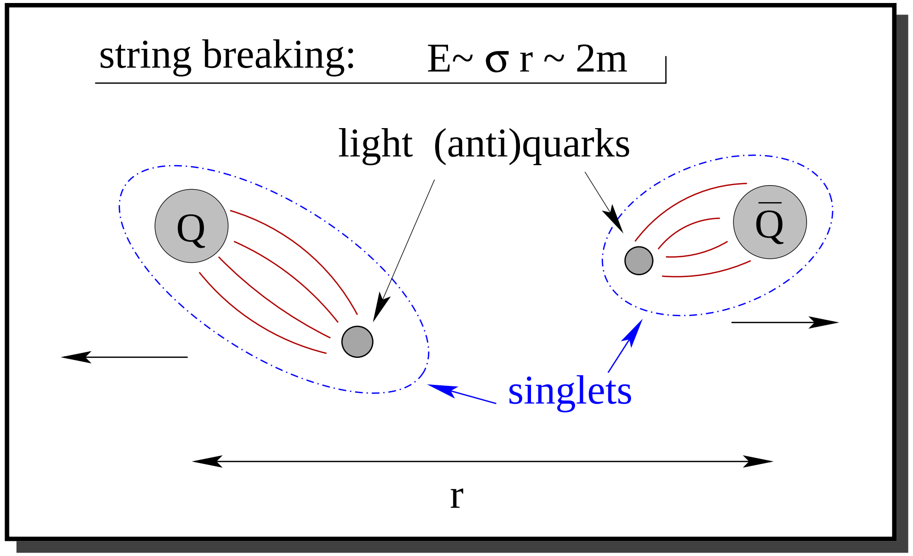



- QCD (with dynamical quarks)



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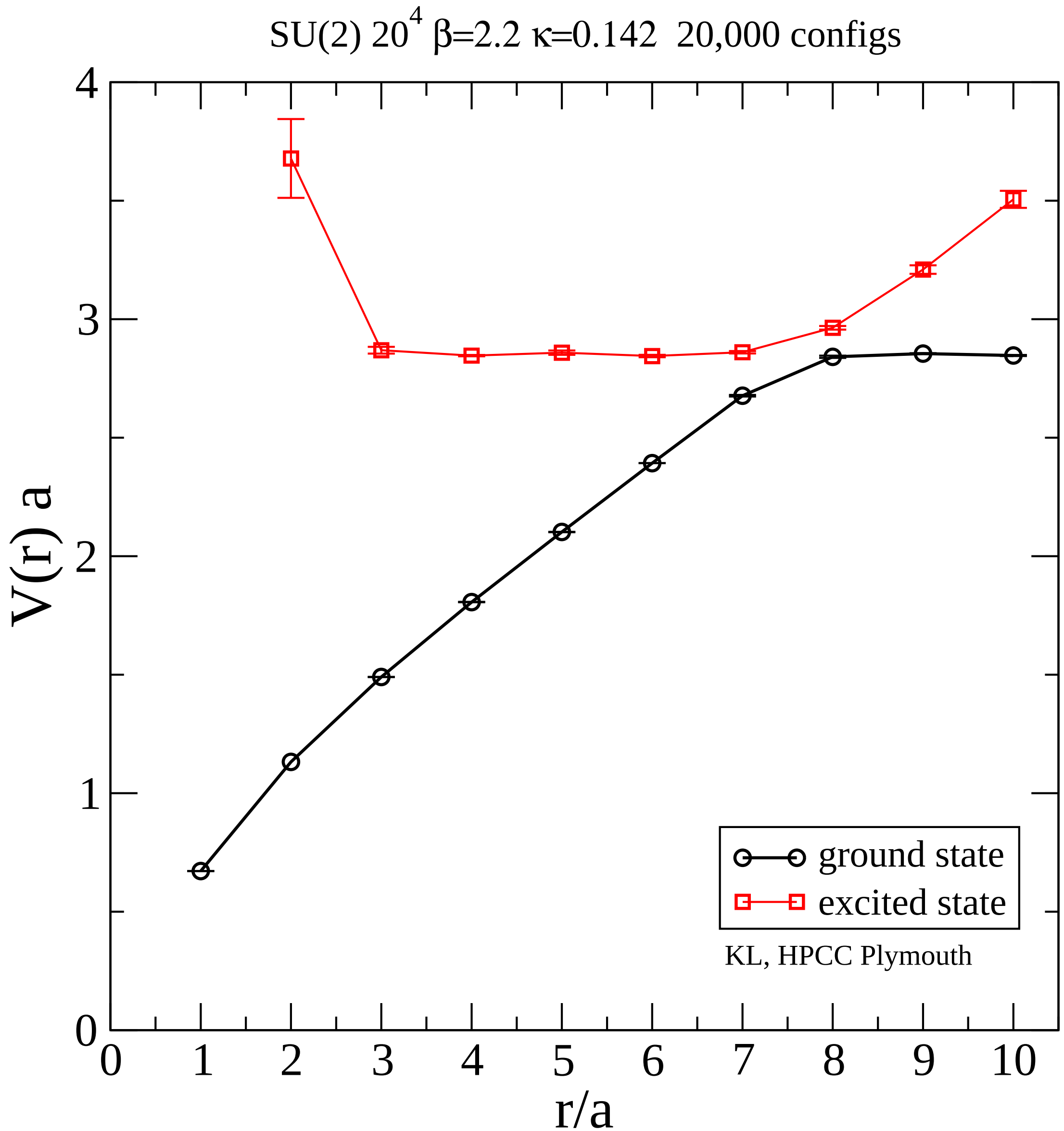


## SU(2) + qHiggs:

carefully tune the bare mass ($\kappa$)

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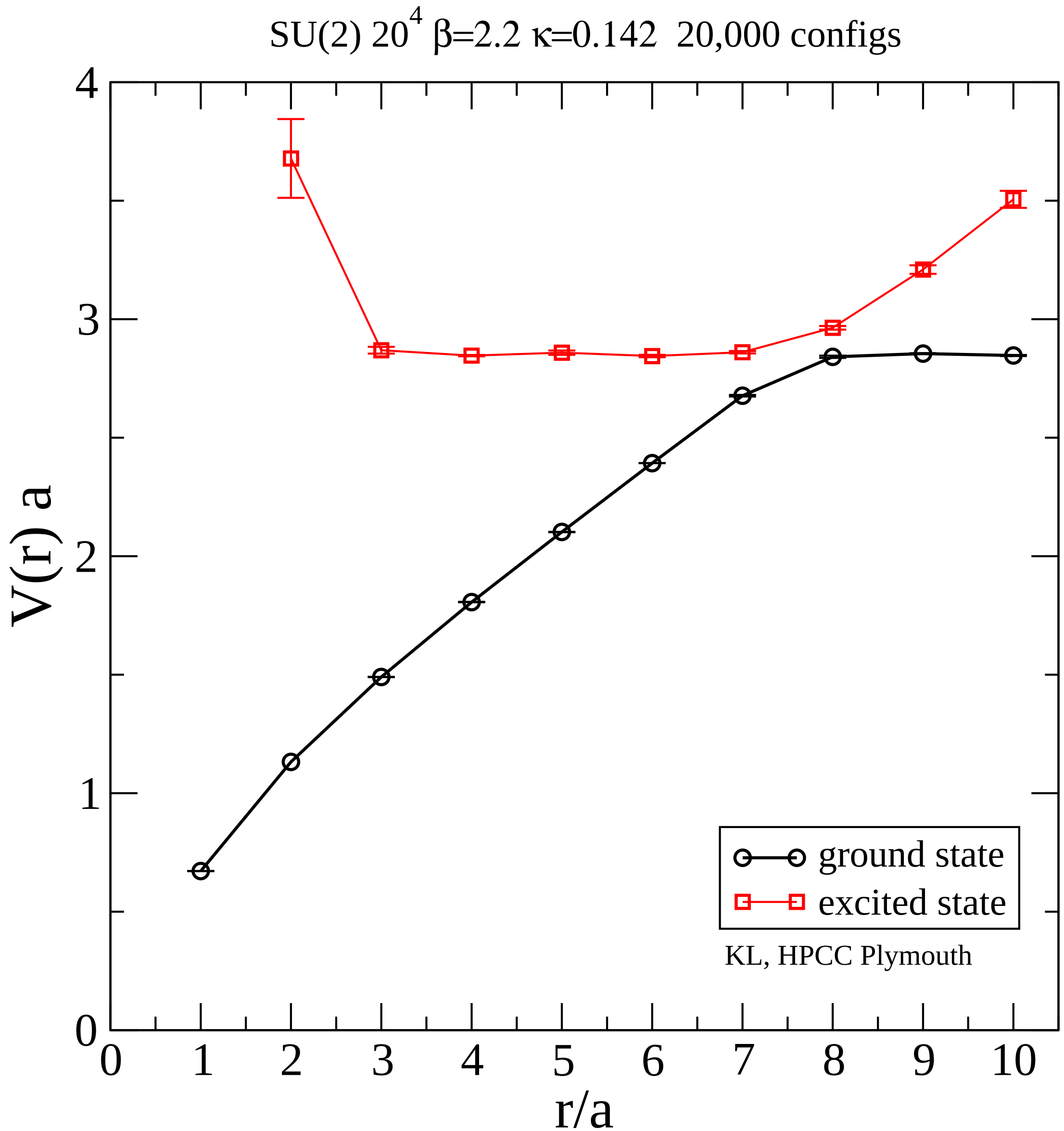



SU(2) + qHiggs:

carefully tune the bare mass ($\kappa$)

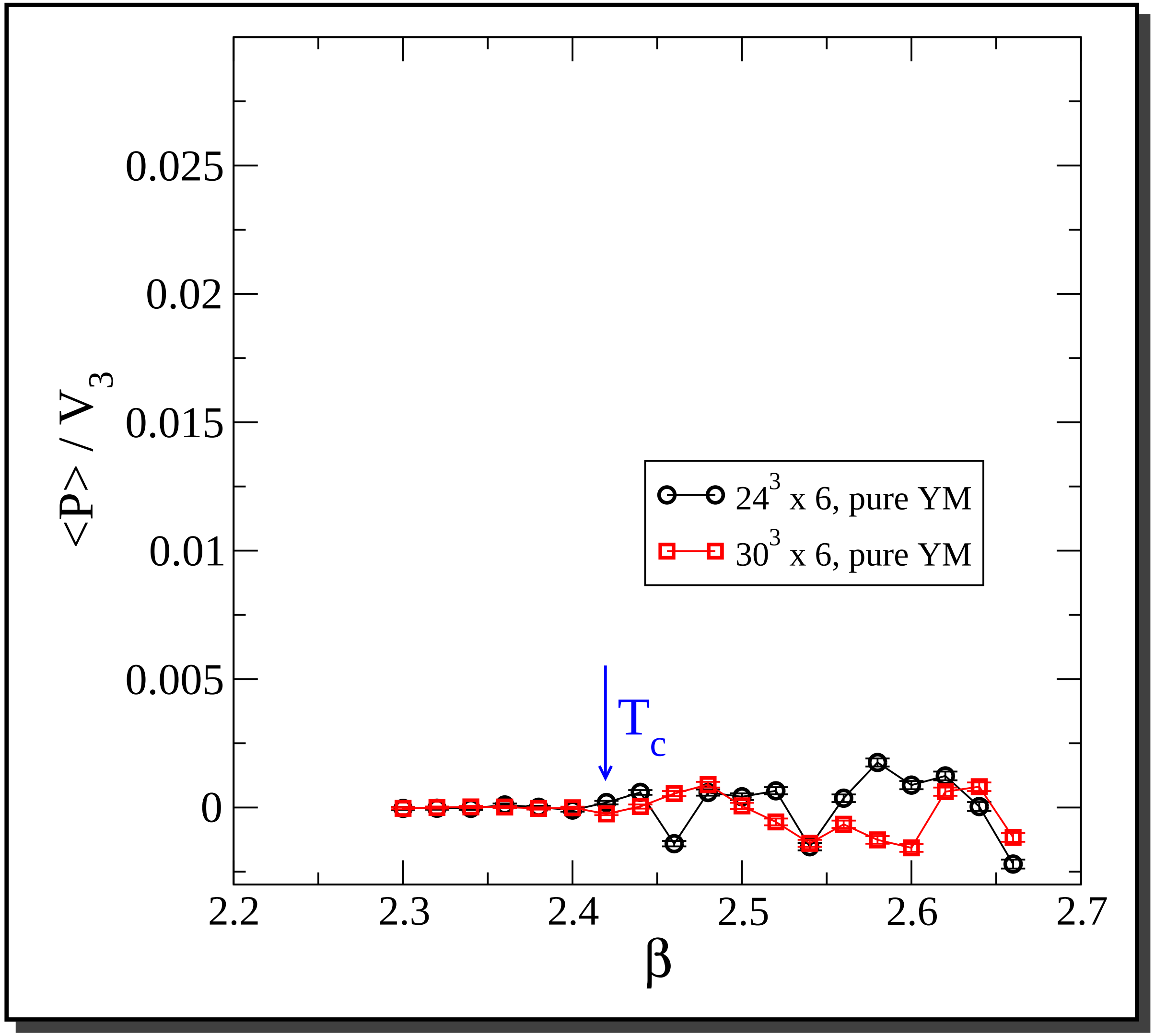
shows string breaking

breaks center symmetry

abundance of configs

# Centre sector tunneling: YM + qHiggs

- Polyakov line: $P = \left\langle \sum_{\vec{x}} \prod_t U_0(\vec{x}, t) \right\rangle$



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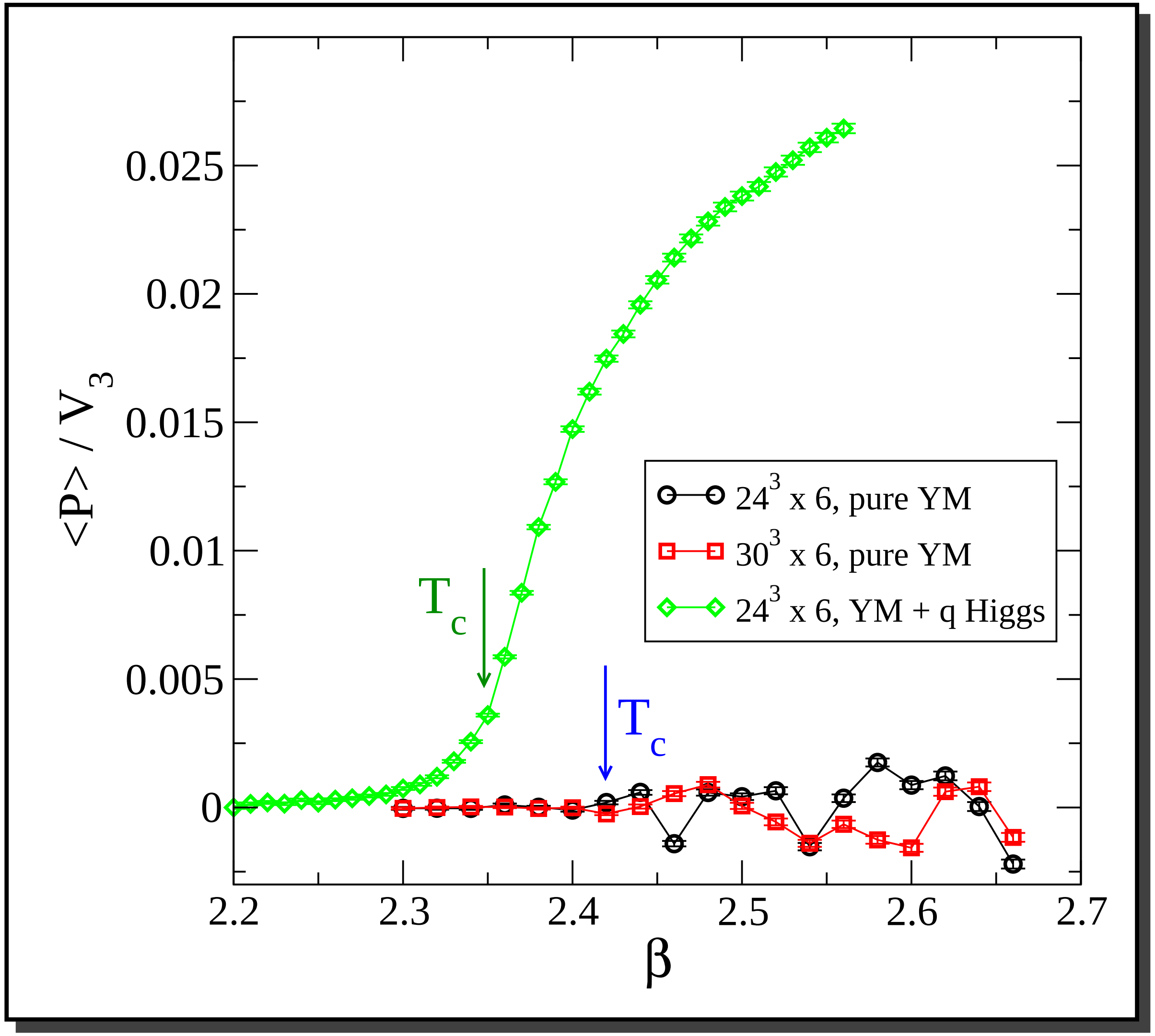
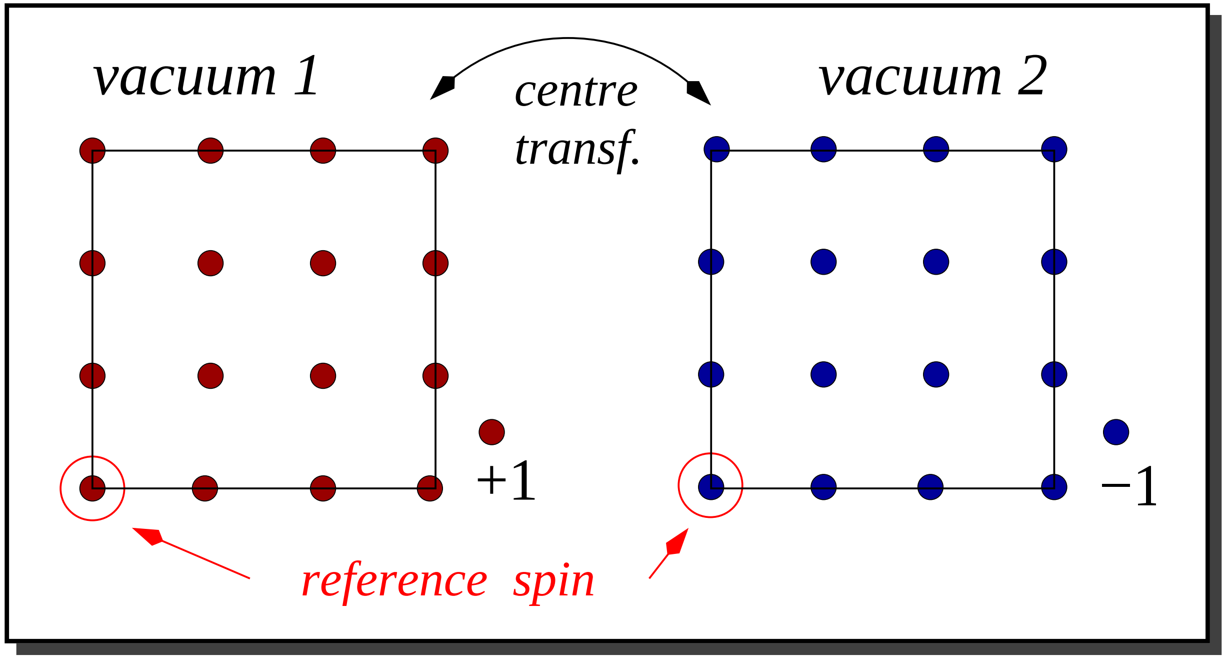



shift of $T_c$:

$300 MeV \longrightarrow 170 MeV$

# Centre sector tunneling: YM + qHiggs

- A lesson from the Ising model:



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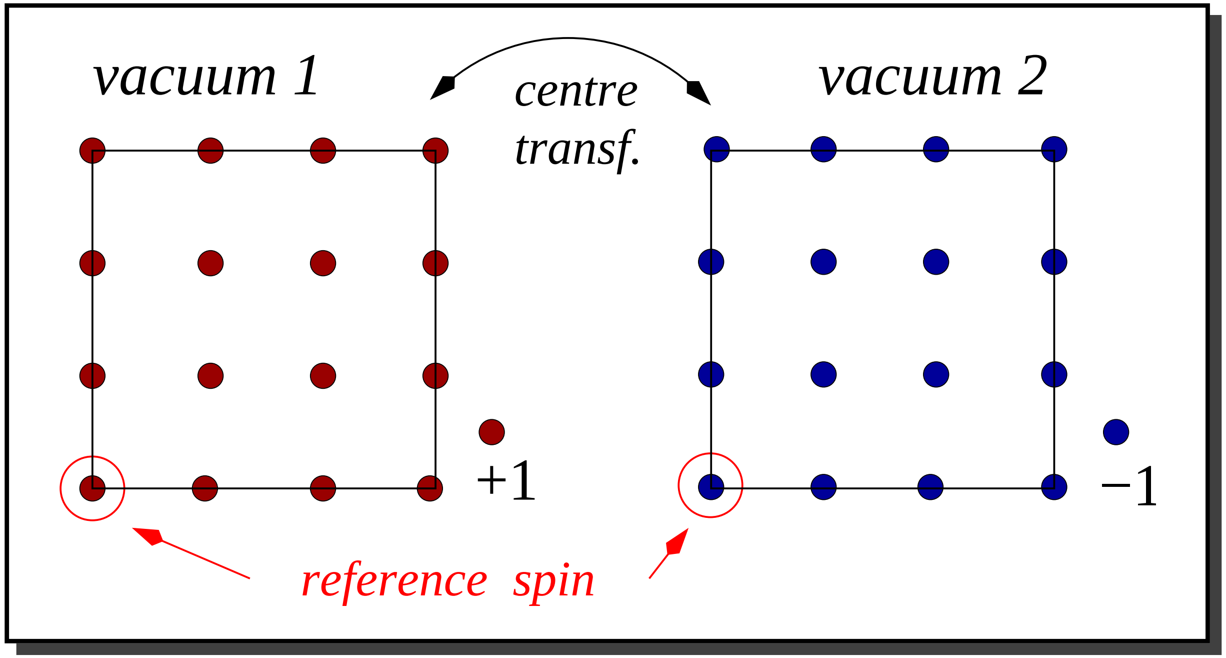


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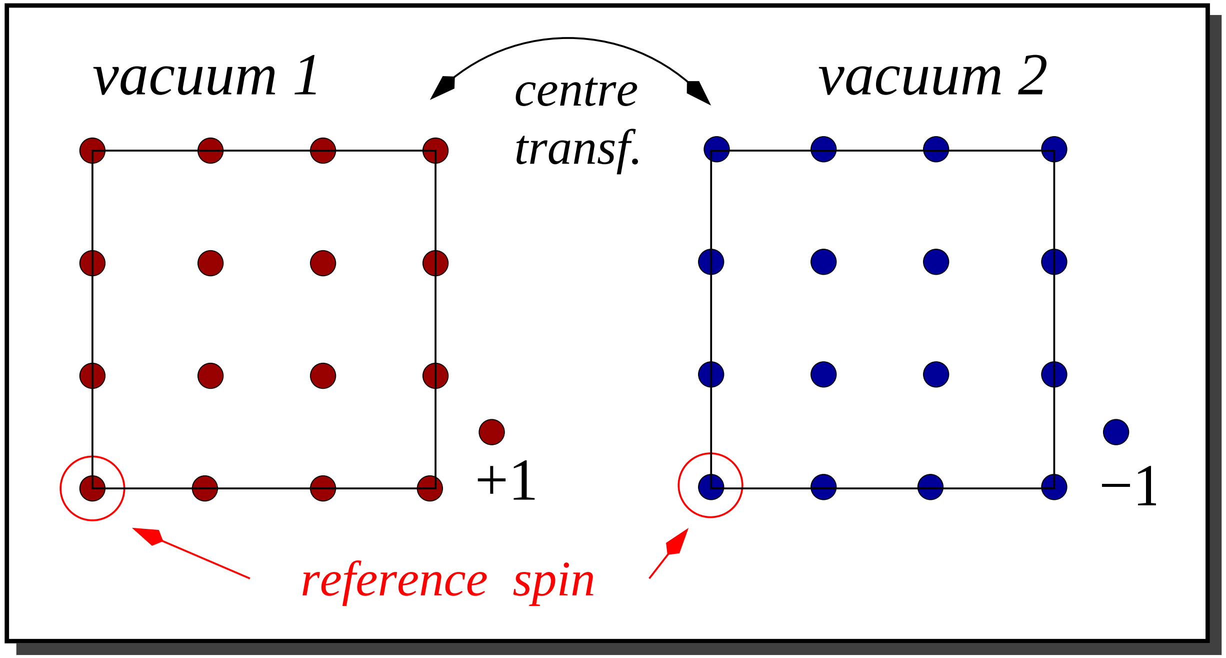


- magnetisation: $\langle M \rangle = 0$ (Swendsen-Wang cluster alg)

- use: $\left\langle M(0) \sum_x M(x) \right\rangle \propto \begin{cases} \xi^3 = \text{finite, symm. phase} \\ V_3 \text{ SSB phase} \end{cases}$

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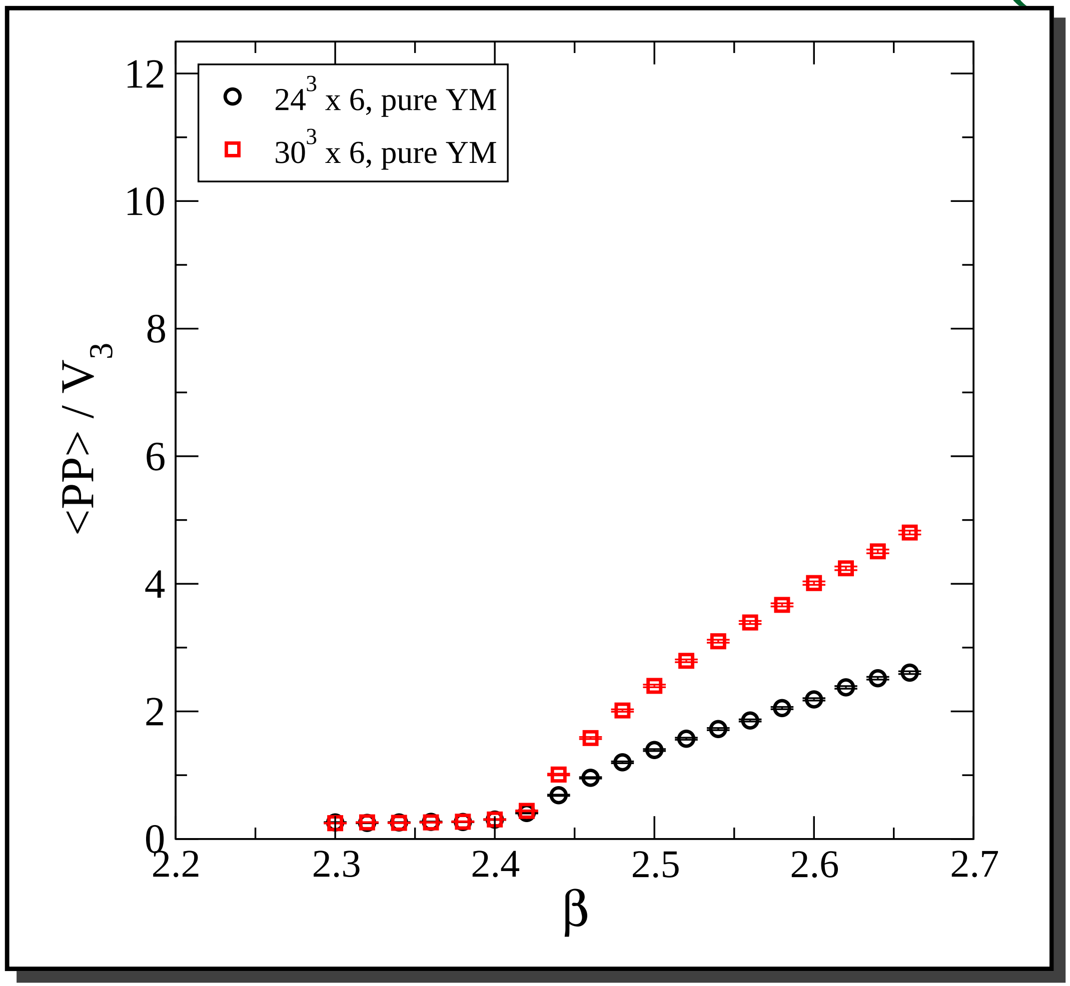

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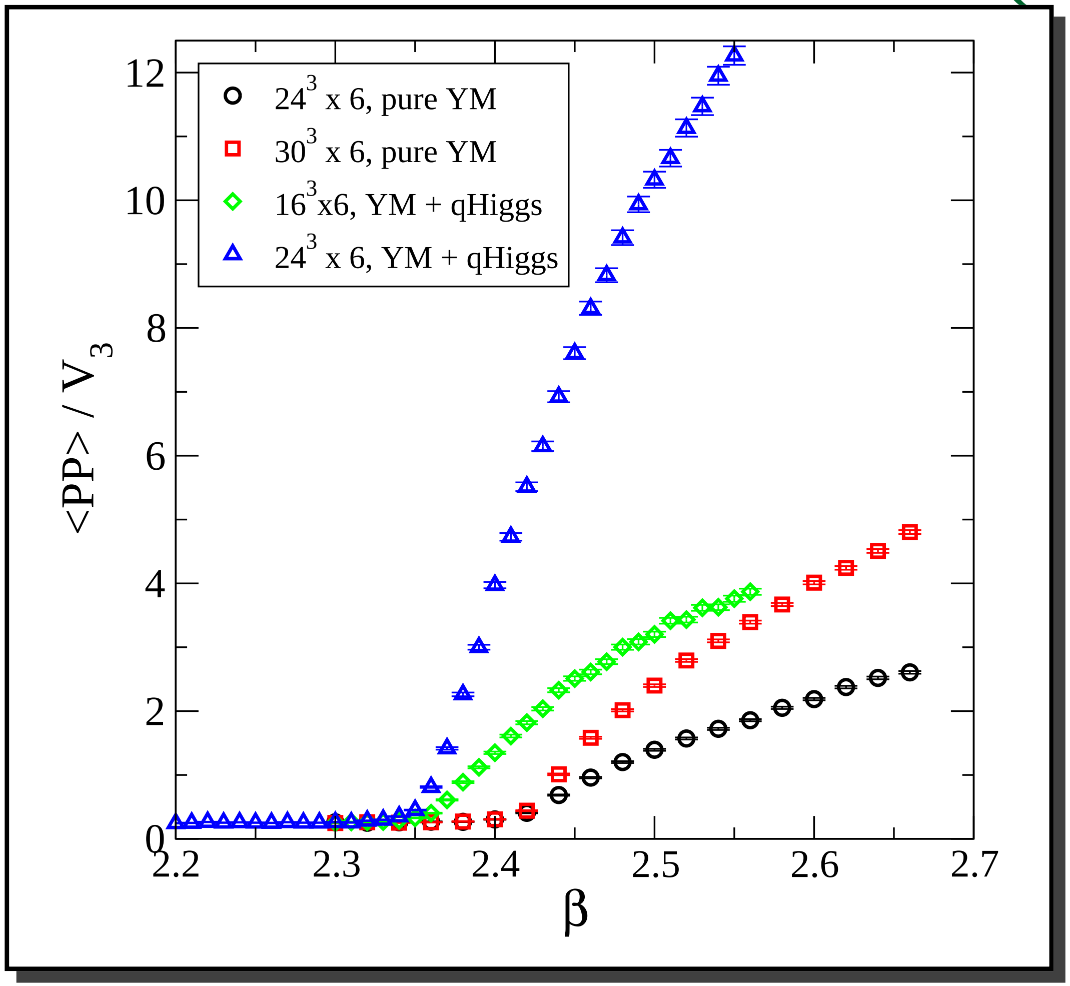


data suggest tunneling for $\beta < \beta_c$ depsite of explicit centre symm. breaking

# *Centre sector tunneling: 't Hooft loop*

- centre sector transformation: $|\psi\rangle \to |^Z\psi\rangle$
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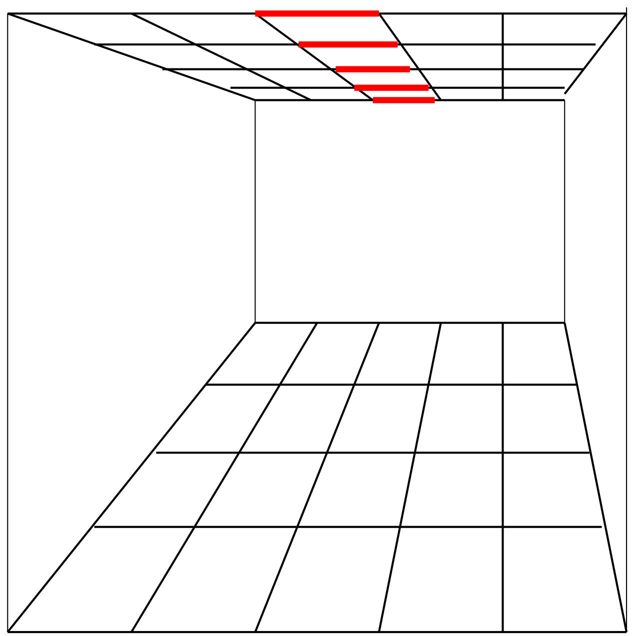

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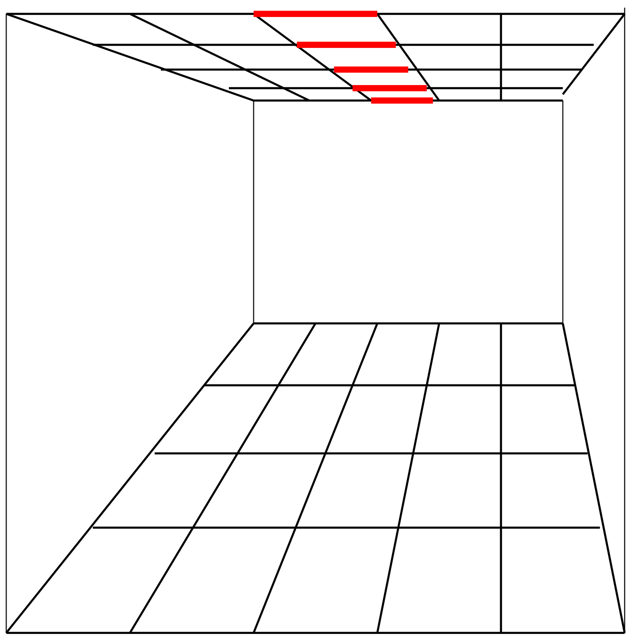
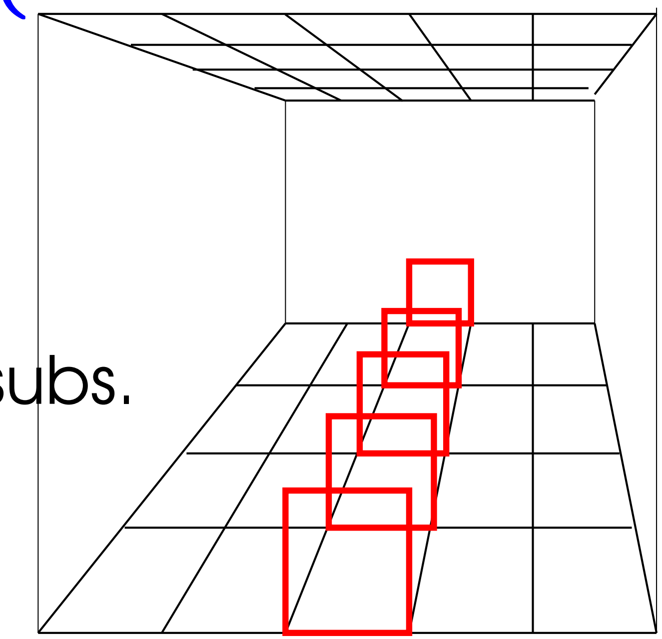


variable subs.



*'t Hooft loop!*

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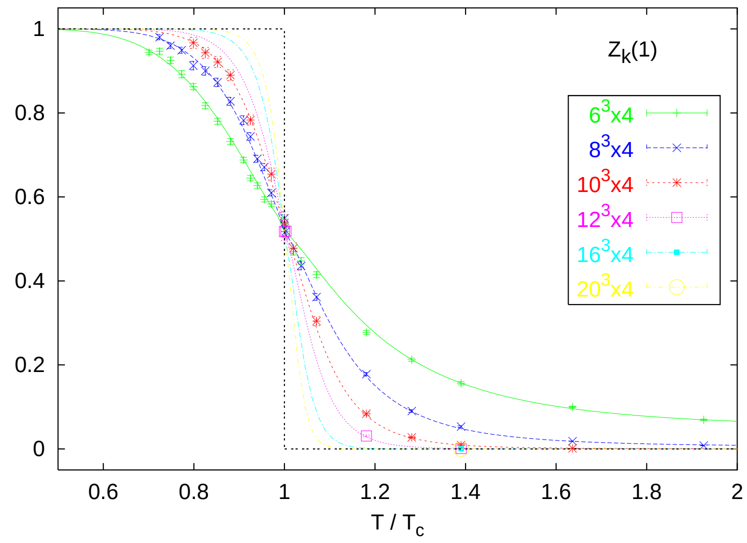

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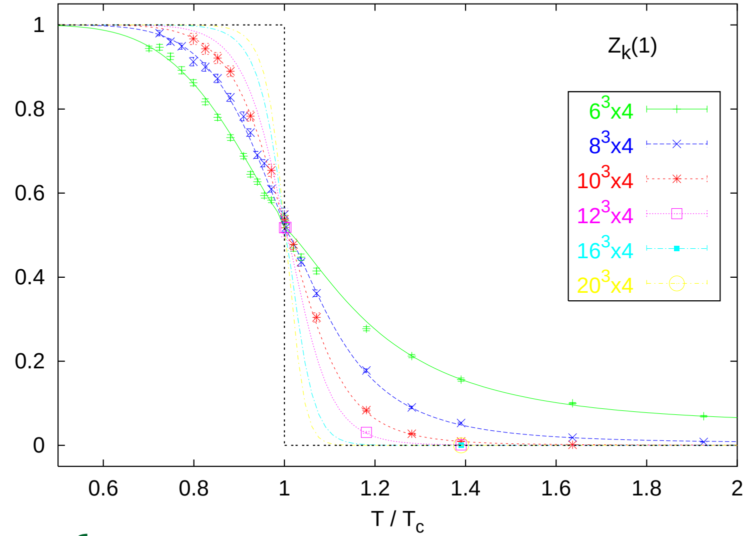


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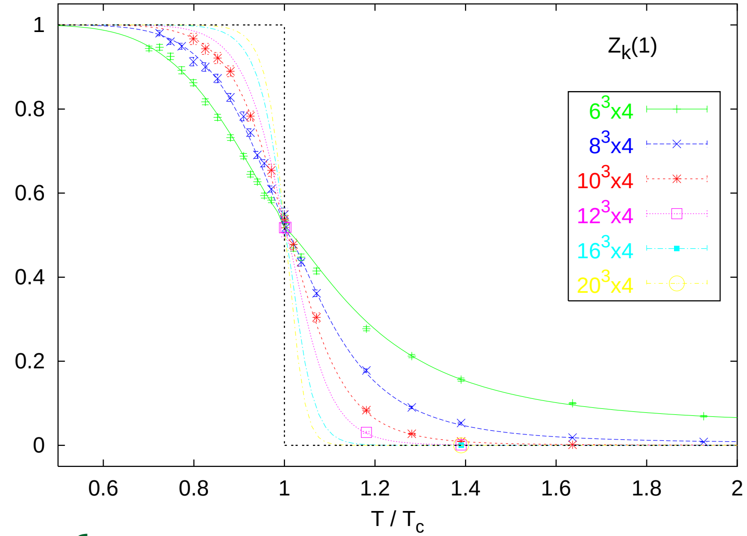


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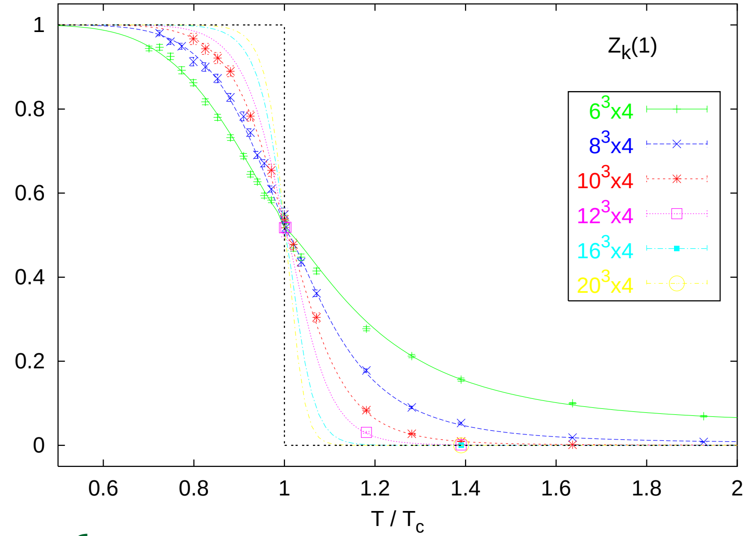


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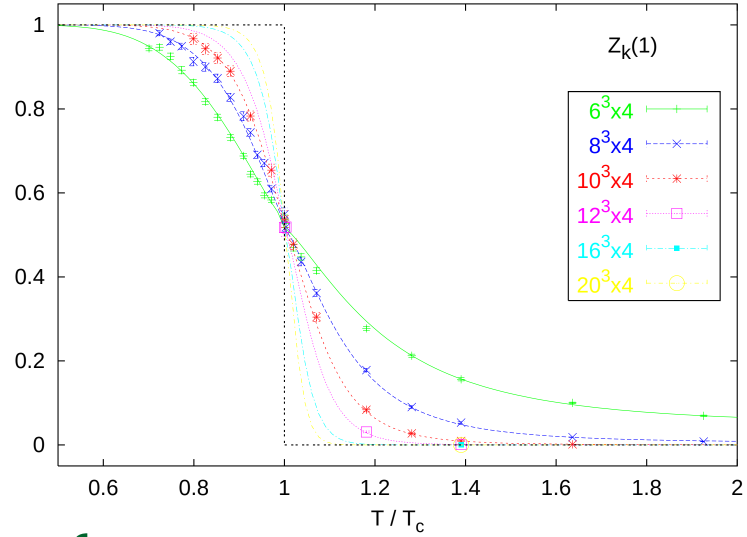


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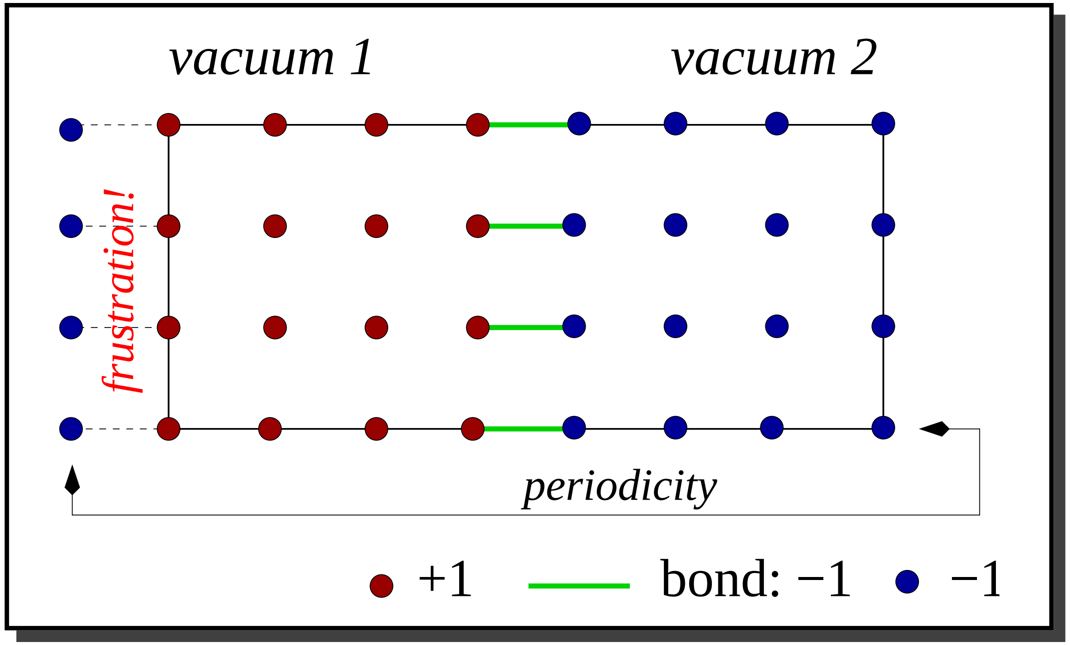
- need something else....

# Interface tension: YM + qHiggs

- A lesson from the Ising model:



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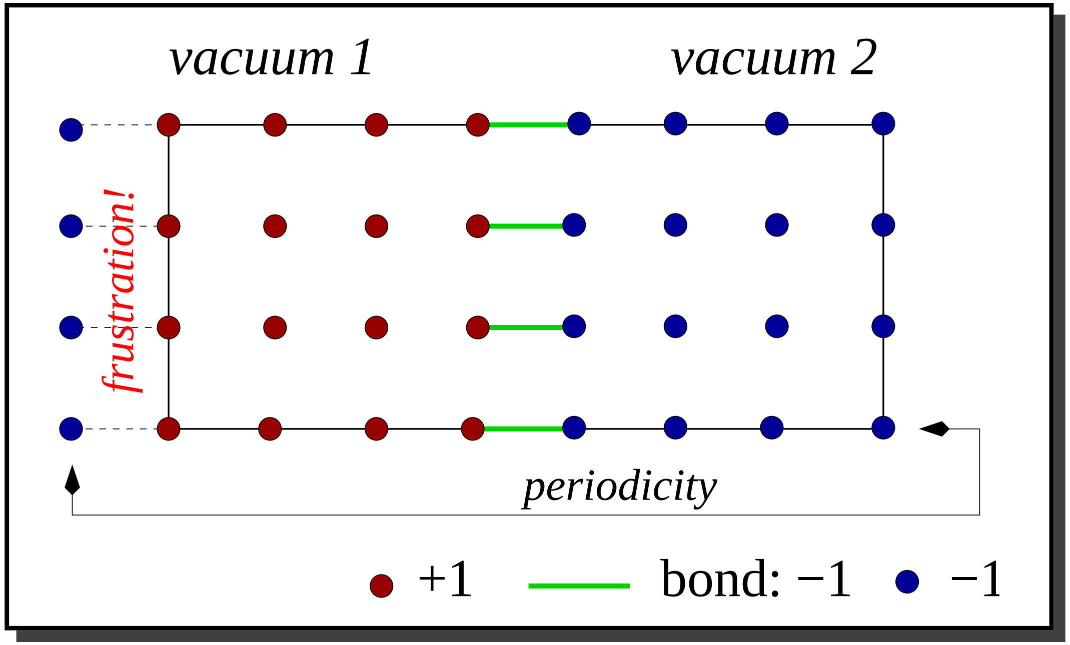


- Interface energy: $F$

$$\exp\{-F/T\} = \frac{\text{partion function}}{\text{partion function}}$$

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- Define:

  $P_< = \sum_{V_<} \text{tr} \prod_t U_0(x)$

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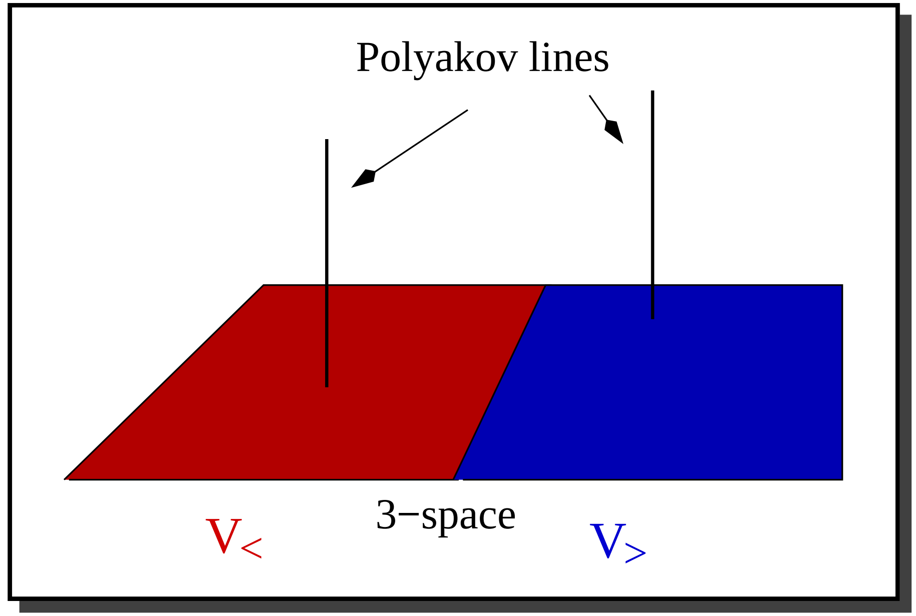

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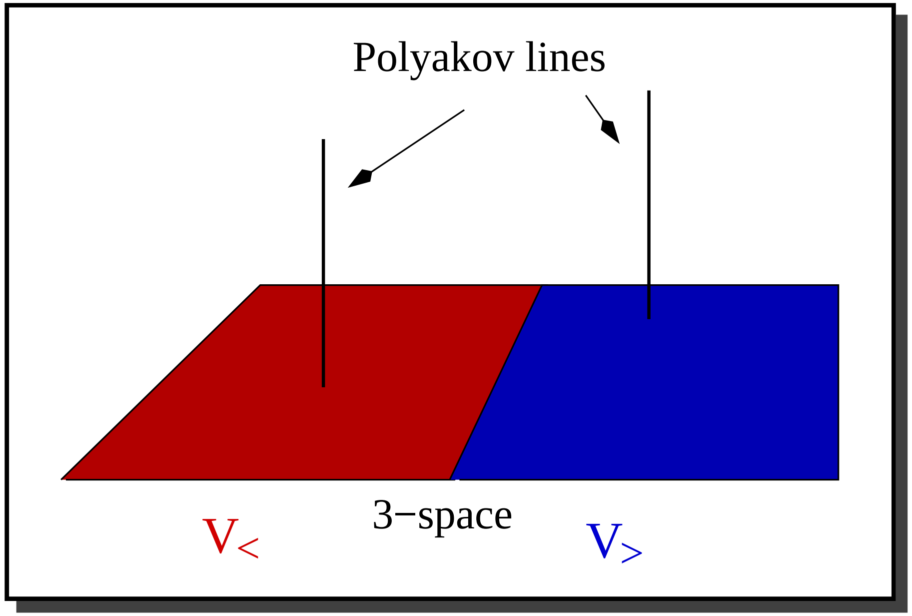

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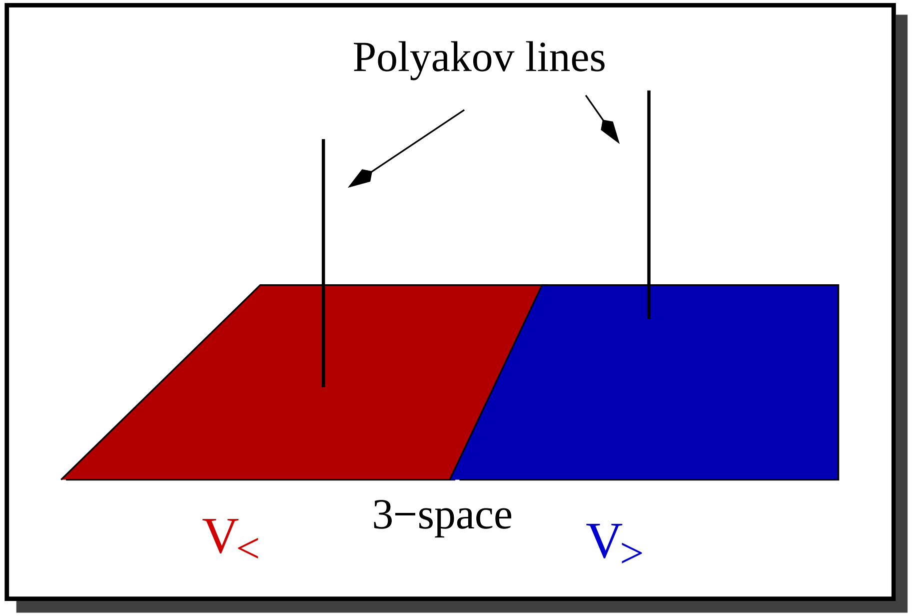


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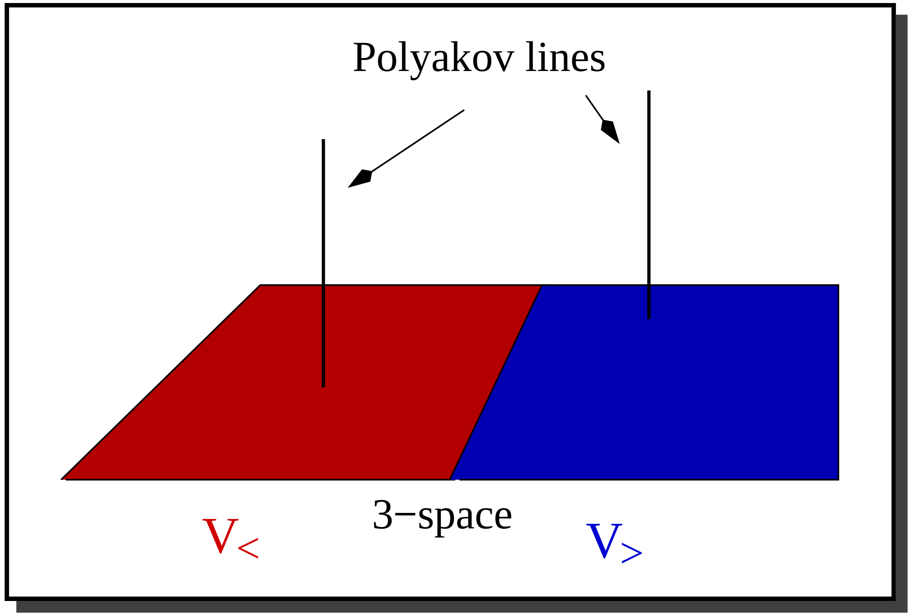


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- **Tunneling coefficient:**

  probability $\Big(C(P_<)$ and $C(P_>)\Big)$ are different

# *Interface tension: YM + qHiggs*

- tunneling coefficient for SU(2):

| | | | | |
|---|---|---|---|---|
| $C(P_<)$ | $-1$ | $-1$ | $+1$ | $+1$ |
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tunneling $\rightarrow 1/2$

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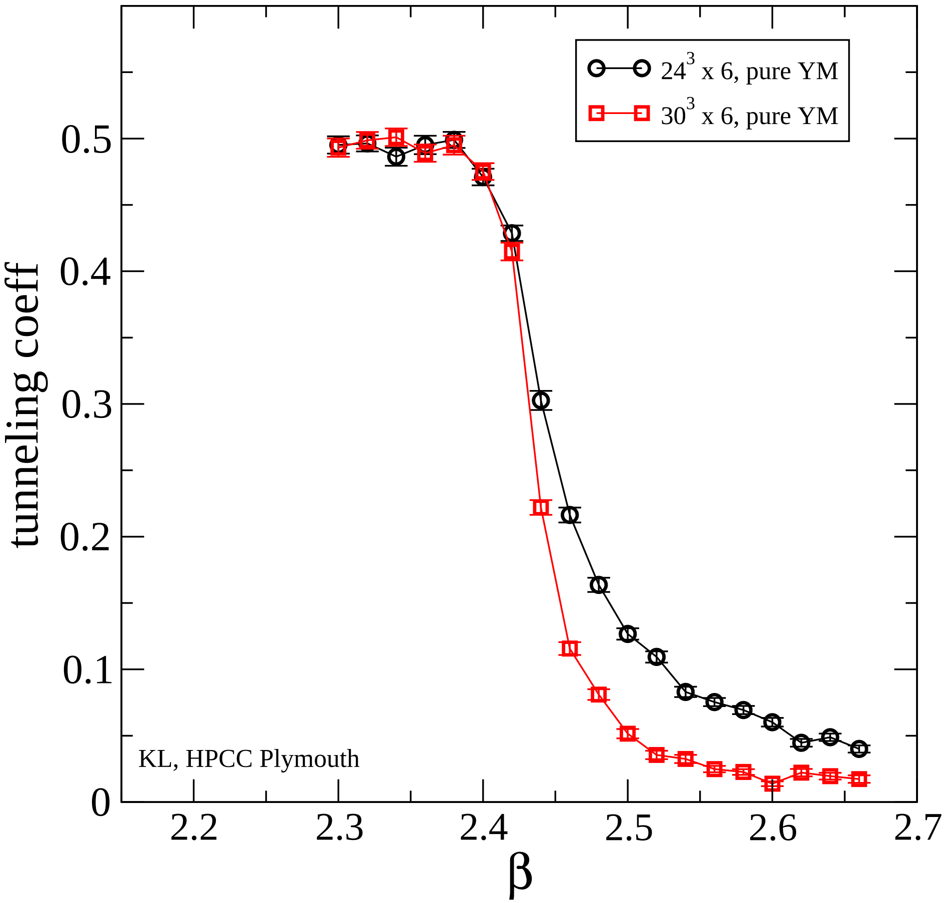

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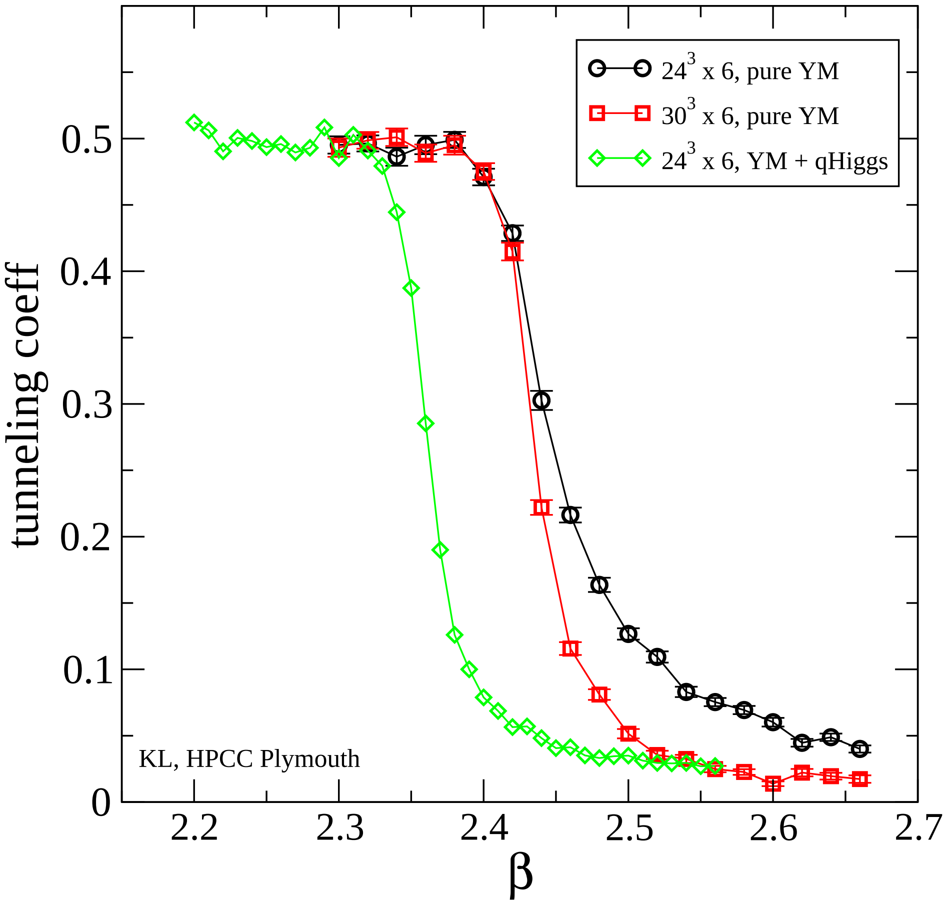

# *Interface tension: SU(3) + quarks*

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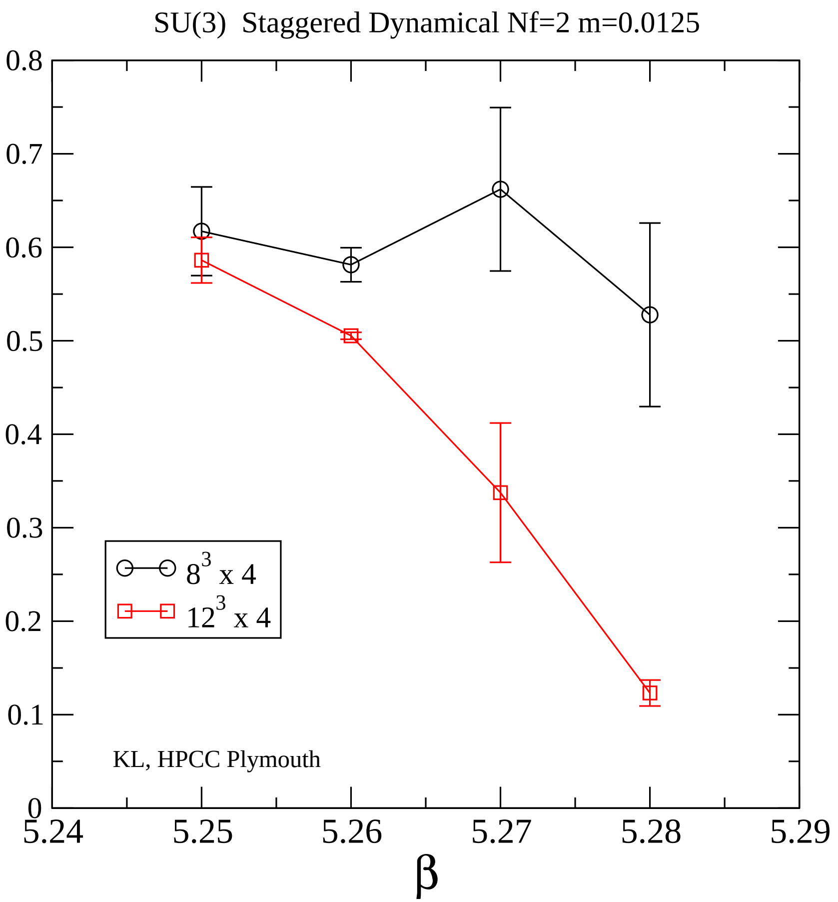


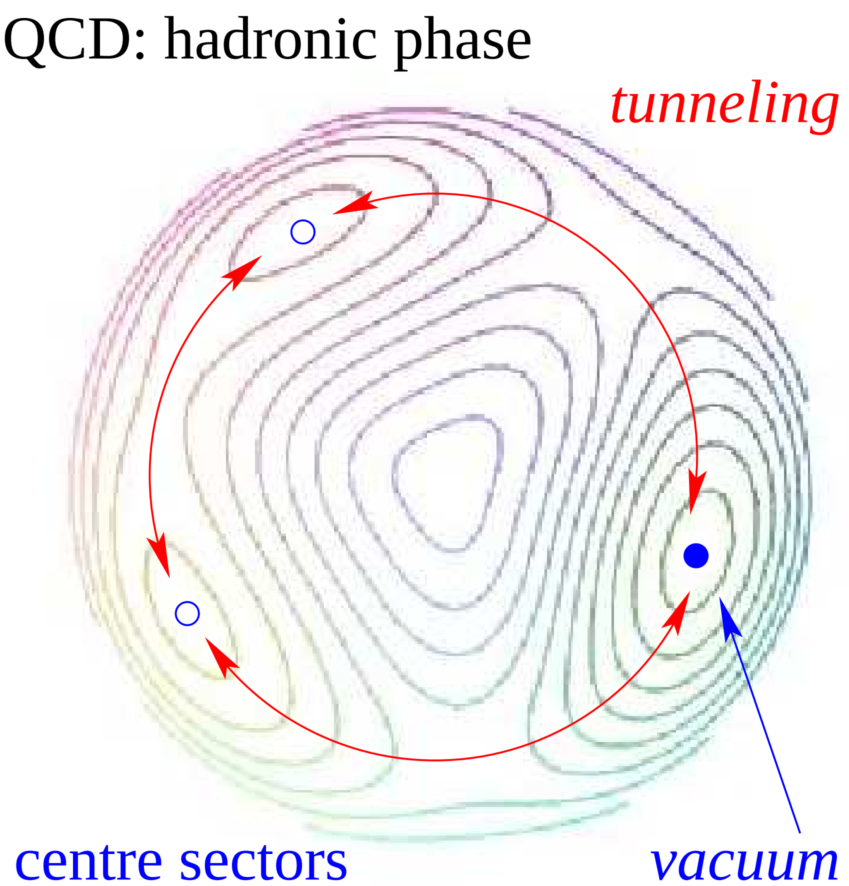

using MILC configs

$N_f = 2, m\,a = 0.0125$

$\beta_c \approx 5.26$

# Centre sector tunneling:

- evidence that centre sector tunneling take places in in the hadronic phase of QCD !



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- is there FEC for $SU(N)$, $N$ odd, such as $SU(3)$?

$$Z = -\frac{1}{2} + i\,\frac{\sqrt{3}}{2}$$
$$Z = 1$$
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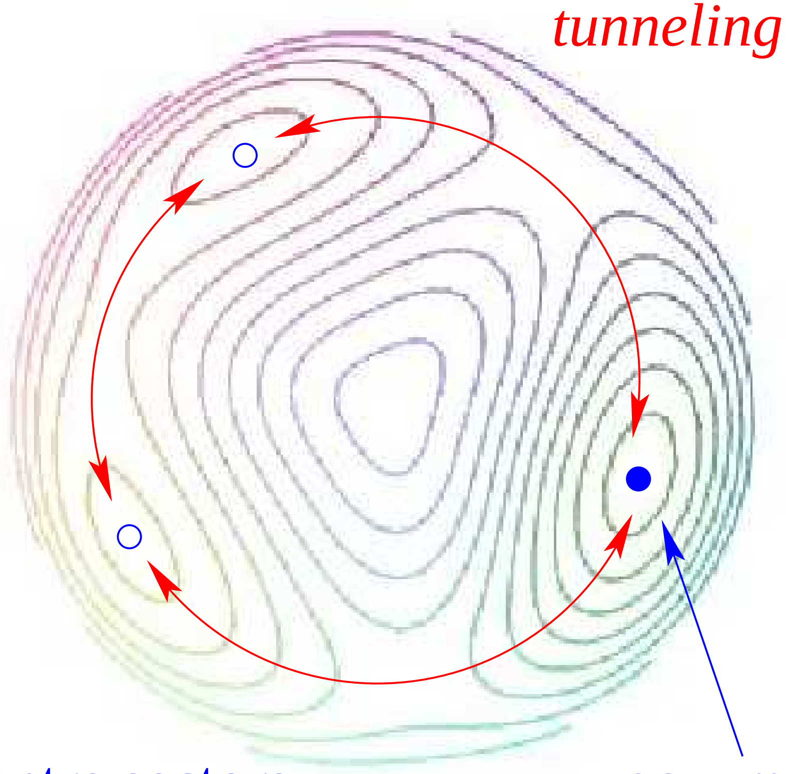

# Fermi-Einstein-Condensation (FEC)

## Model consideration: $SU(3)$

- $q(x)$: quarks, $m$: mass, $\mu$: chemical potential
  $A_m$: moduli fields $\Rightarrow$ weighted sum over centre sectors

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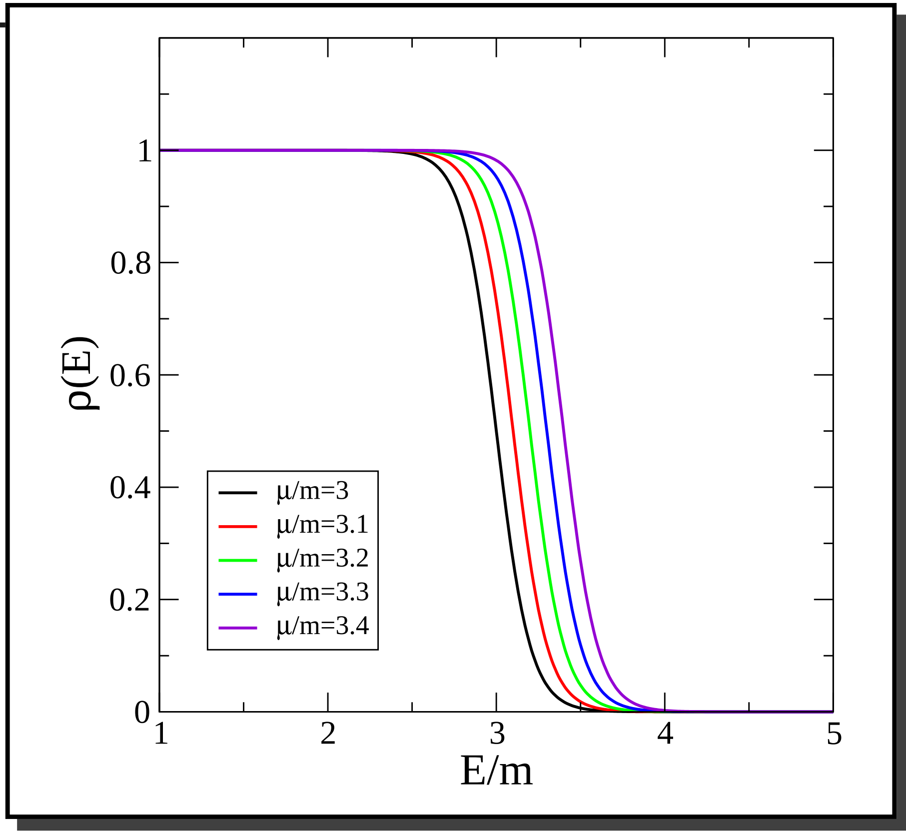
- partition function: $\exp\{iA_m\} = Z_m \in Z(N_C)$

$$Z \;=\; \sum_{m=1}^{N_c} p_m \int \mathcal{D}q\,\mathcal{D}\bar{q}\,\exp\{\bar{q}(i\partial\!\!\!/ + (A_m + i\mu)\gamma_0 + im)q\}$$

$p_m$ : probability for centre sector $m$
hadronic phase: $p_m \approx 1/N_c, \forall\, m$
high $T$ SSB phase: $p_{N_c} = 1$, $p_m = 0$ for $m = 1 \ldots N_c - 1$

# *Fermi-Einstein-Condensation (FEC)*

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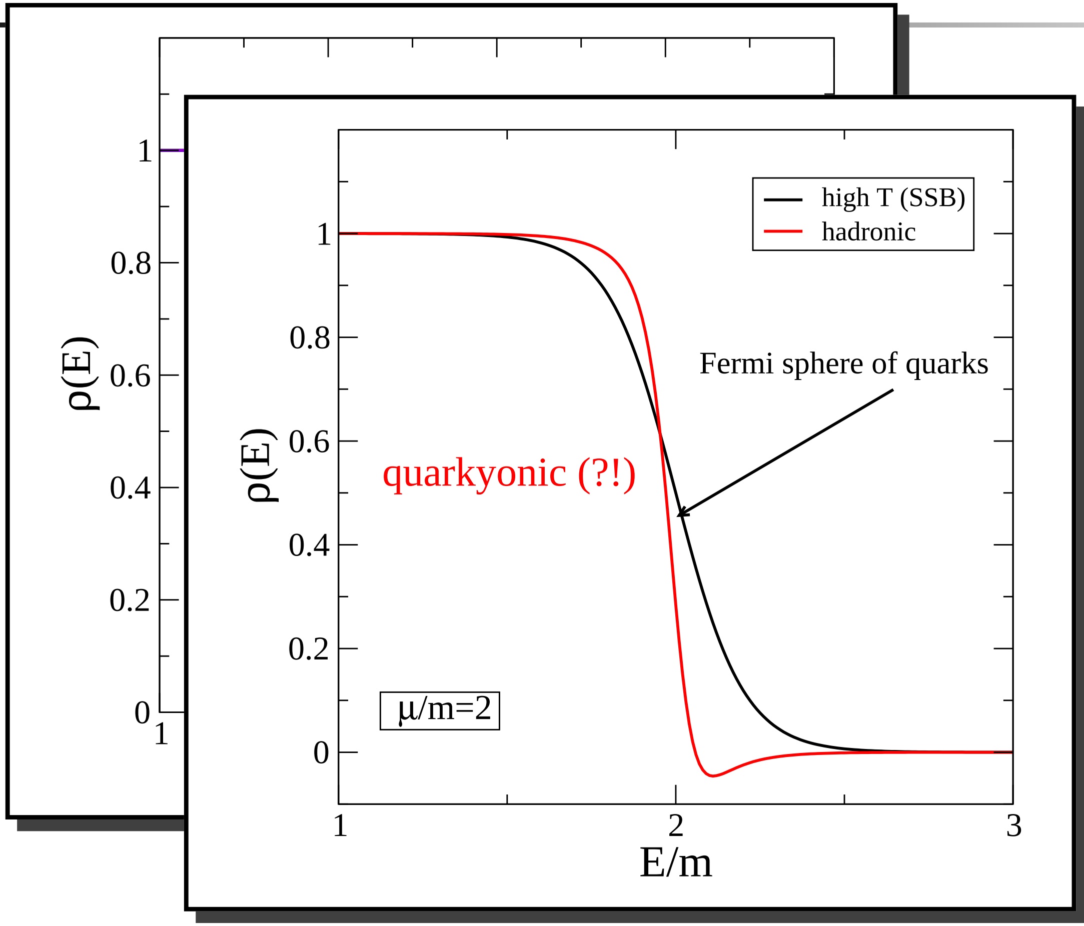


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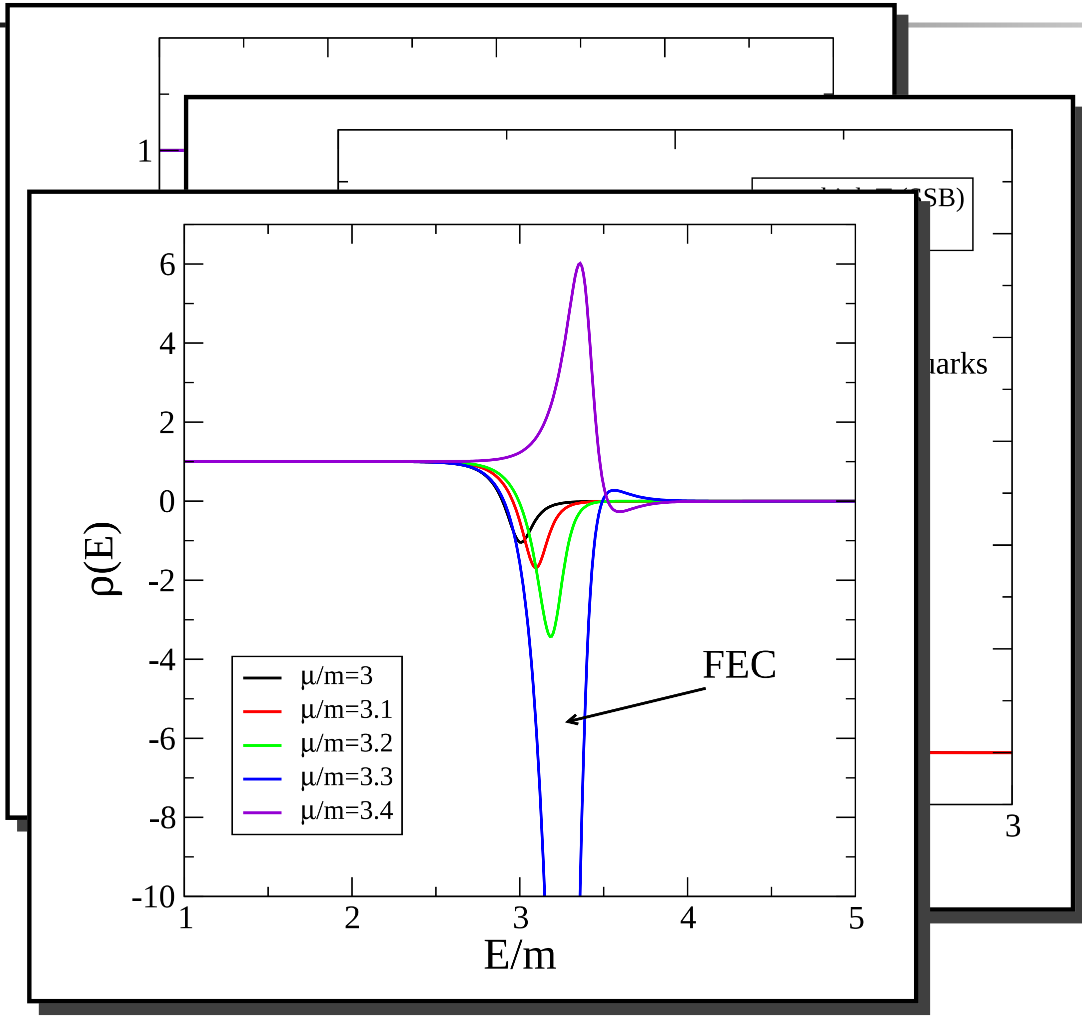


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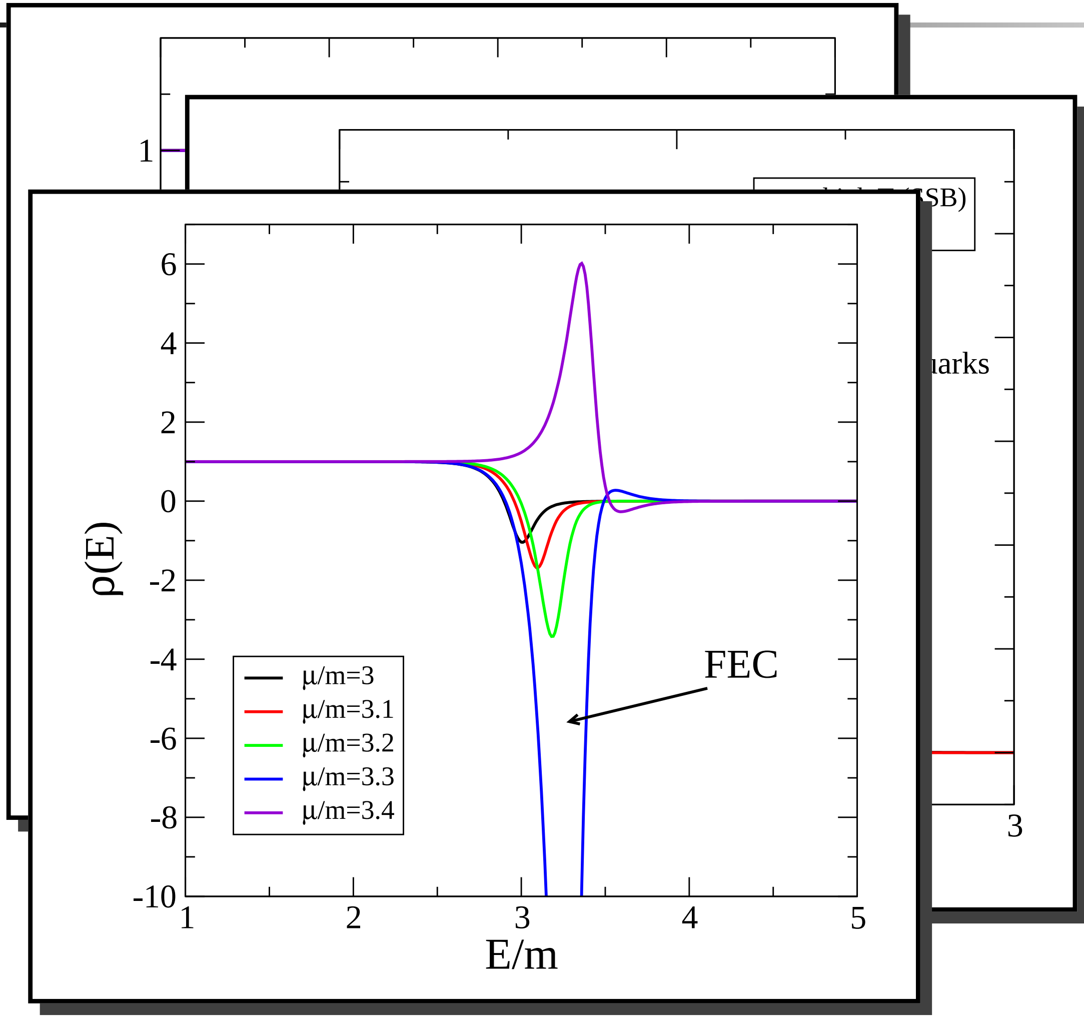


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- orgin of the Cooper instability:

$$\exp\{-F/T\} = Z = \sum_{m=1}^{N_c} p_m \int \mathcal{D}q\,\mathcal{D}\bar{q}\,\ldots \to 0$$

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- Evidence that centre-sector-tunneling takes place

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  `lattice gauge simulations`: $SU(2)$ + qHiggs, SU(3) + $N_f = 2$
  $\rightarrow$ tunneling coeffcient
  *generalised 't Hooft loop* $(\langle\psi|^Z\psi\rangle)$ with dynamical matter...