On kinematical constraints in boson-boson systems

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(Dated: December 15, 2011)

We construct partial-wave amplitudes for boson-boson scattering that are free of kinematical constraints and frame independent. Explicit transformations from conventional helicity states to covariant states that eliminate all kinematical constraints are derived. The covariant partial-wave scattering amplitudes obtained are expected to satisfy dispersion relations and therefore provide a suitable basis for the construction of effective field theories and data analysis. Scattering amplitudes are decomposed into suitable sets of invariant functions. A novel algebra that permits the efficient computation of such functions in terms of computer algebra codes is presented.

PACS numbers: $11.55.-m$, $13.75.Cs$, $11.80.-m$

I. INTRODUCTION

In a strongly interacting quantum field theory like QCD an important challenge is the reliably and predictive treatment of final-state interactions. Given some effective degrees of freedom micro-causality and coupled-channel unitarity are crucial constraints that help to establish coupled-channel reaction amplitudes from effective Lagrangians (see e.g. [1–3]).

Although partial-wave scattering amplitudes can be straightforwardly introduced in the helicity formalism of Jacob and Wick [4], the derivation of transformations that lead to amplitudes free of kinematical constraints is, however, a nontrivial, but necessary, task. Partial-wave analysis of experimental data or effective field theory approaches that consider the consequences of micro-causality in terms of partial-wave dispersion-integral representations [1, 5–13] require such kinematically unconstraint amplitudes. Our aim is to derive such amplitudes for two-body systems with $J^P = 0^-$ or 1^- particles by means of suitable transformations of the helicity partial-wave amplitudes . In a previous work one of the authors studied the scattering of 0^- off 1^- particles [14] and fermion-antifermion annihilation processes with $\frac{1}{2}^+$ and $\frac{3}{2}^+$ particles [15]. So far reactions involving two-body states with two 1^- particles have not been dealt with. We note that previous studies of two-body scattering systems with photons, pions and nucleons [5, 16–24] have employed the technique applied in this work, and that a possibly related approach is that of Chung and Friedrich [25, 26]. The technique developed in this work will be relevant for data analysis of the PANDA experiment at FAIR, where protons and antiprotons may be annihilated into systems of spin 0 or 1 states [27].

In this work we consider partial-wave projections of the scattering amplitude of all two-body reactions possible with 0^- and 1^- bosons. Partial-wave amplitudes with proper analytic properties that justify the use of uncorrelated integral-dispersion relations are constructed. As in [15], we first decompose the scattering amplitude into a complete basis of invariant functions that are free of kinematical constraints, and are expected to satisfy a Mandelstam dispersion-integral representation [16, 28]. A given choice of basis is free of kinematical constraints if any additional structure can be decomposed into the basis with coefficients that are regular. The identification of such a basis is a nontrivial task as the spins of the involved particles increase. In a second step non-unitary transformations are identified, that map the initial, respectively final helicity states to new covariant states, such that the resulting partial-wave amplitudes are free of kinematical constraints.

The work is organized as follows. Section II introduces the conventions used for the kinematics and the helicity wave functions. The scattering amplitudes are decomposed into sets of invariant amplitudes free of kinematical constraints. In the following section the helicity partial-wave amplitudes are constructed within the given convention. The central results are presented in section IV, where the transformation to partial-wave amplitudes free of kinematical constraints are derived and discussed.

II. ON-SHELL SCATTERING AMPLITUDES

We consider two-body reactions involving pseudoscalar and vector particles. All derivations will be completely generic. We introduce the 4-momenta p_1 and $\bar{p}_1$ of the incoming and outgoing first particle and those of the second particle, p_2 and $\bar{p}_2$. In the center of mass

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frame we write

$$
\begin{aligned}
&p_1^\mu = (\omega_1, 0, 0, +p)\,, \quad &\bar{p}_1^\mu = (\bar{\omega}_1, +\bar{p}\sin\theta, 0, +\bar{p}\cos\theta)\,,\\
&\omega_1 = \sqrt{m_1^2 + p^2}\,, &\bar{\omega}_1 = \sqrt{\bar{m}_1^2 + \bar{p}^2}\,,\\
&p_2^\mu = (\omega_2, 0, 0, -p)\,, &\bar{p}_2^\mu = (\bar{\omega}_2, -\bar{p}\sin\theta, 0, -\bar{p}\cos\theta)\,,\\
&\omega_2 = \sqrt{m_2^2 + p^2}\,, &\bar{\omega}_2 = \sqrt{\bar{m}_2^2 + \bar{p}^2}\,,
\end{aligned}
\tag{1}
$$

where θ is the scattering angle, p and $\bar{p}$ are the magnitudes of the initial and final three-momenta. The relative momenta p and $\bar{p}$ can be expressed in terms of the total energy $\sqrt{s}$ of the system

$$
\begin{aligned}
&p^2 = \frac{1}{4\,s}\left(s - (m_1 + m_2)^2\right)\left(s - (m_1 - m_2)^2\right)\\
&w^\mu = p_1^\mu + p_2^\mu = \bar{p}_1^\mu + \bar{p}_2^\mu\,, \qquad s = w^2\,,\\
&\bar{p}^2 = \frac{1}{4\,s}\left(s - (\bar{m}_1 + \bar{m}_2)^2\right)\left(s - (\bar{m}_1 - \bar{m}_2)^2\right).
\end{aligned}
\tag{2}
$$

It is convenient to introduce some further notation

$$
\begin{aligned}
&k^\mu = \frac{1}{2}\,(p_1^\mu - p_2^\mu)\,, \qquad &r_\mu = k_\mu - \frac{1}{2}\,\frac{p_1^2 - p_2^2}{s}\,w_\mu\,,\\
&\bar{k}^\mu = \frac{1}{2}\,(\bar{p}_1^\mu - \bar{p}_2^\mu)\,, &\bar{r}_\mu = \bar{k}_\mu - \frac{1}{2}\,\frac{\bar{p}_1^2 - \bar{p}_2^2}{s}\,w_\mu\,,\\
&\hat{g}_{\mu\nu} = g_{\mu\nu} - \frac{1}{s}\,w_\mu\,w_\nu\,, &\bar{r}\cdot r = -\bar{p}\,p\,\cos\theta\,,
\end{aligned}
\tag{3}
$$

where the two 4-vectors r_μ and $\bar{r}_\mu$ have a transparent relation to the center-of-mass momenta p and $\bar{p}$.

We specify the spin-one wave functions

$$
\begin{aligned}
&\epsilon^\mu(\bar{p}_1, \pm 1) = \begin{pmatrix} 0 \\ \frac{\mp\cos\theta}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \\ \frac{\pm\sin\theta}{\sqrt{2}} \end{pmatrix}, \; \epsilon^\mu(\bar{p}_1, 0) = \begin{pmatrix} \frac{\bar{p}}{\bar{m}_1} \\ \frac{\bar{\omega}_1}{\bar{m}_1}\sin\theta \\ 0 \\ \frac{\bar{\omega}_1}{\bar{m}_1}\cos\theta \end{pmatrix},\\
&\epsilon^\mu(\bar{p}_2, \pm 1) = \begin{pmatrix} 0 \\ \frac{\pm\cos\theta}{\sqrt{2}} \\ \frac{-i}{\sqrt{2}} \\ \frac{\mp\sin\theta}{\sqrt{2}} \end{pmatrix}, \; \epsilon^\mu(\bar{p}_2, 0) = \begin{pmatrix} \frac{\bar{p}}{\bar{m}_2} \\ -\frac{\bar{\omega}_2}{\bar{m}_2}\sin\theta \\ 0 \\ -\frac{\bar{\omega}_2}{\bar{m}_2}\cos\theta \end{pmatrix},
\end{aligned}
$$

where the wave function of the corresponding initial states is recovered with $\theta = 0$.

The on-shell production and scattering amplitudes are defined in terms of plane-wave matrix elements of the scattering operator T. We represent the scattering amplitudes in terms of a complete set of invariant functions $F_n(s,t)$. The merit of the decomposition lies in the transparent analytic properties of such functions $F_n(s,t)$, which are expected to satisfy Mandelstam's dispersion integral representation [16, 28]. For reactions involving spin-one particles it is not straight forward to identify such amplitudes.

We begin with the elastic scattering of two pseudoscalar particles

$$
T_{0\,0\to 0\,0}(\bar{k},\,k,\,w) = F_1(s,t)\,,
\tag{4}
$$

which is characterized by one scalar function $F_1(s,t)$ depending on two Mandelstam variables, e.g. s and t with

$$
s + t + u = m_1^2 + m_2^2 + \bar{m}_1^2 + \bar{m}_2^2\,.
\tag{5}
$$

We suppress internal degrees of freedom like isospin or strangeness quantum numbers for simplicity.

A slightly more complicated process involves one vector particle in the final state

$$
\begin{aligned}
&T_{0\,0\to 0\,1}(\bar{k},\,k,\,w) = F_1(s,t)\,\langle T^{(1)}_{\bar{\nu}}\rangle^{\bar{\nu}}_{0\,0\to 0\,1}\,,\\
&\langle\, T^{(1)}_{\bar{\nu}}\,\rangle^{\bar{\nu}}_{0\,0\to 0\,1} = \epsilon^{\dagger,\bar{\nu}}(\bar{p}_2,\,\bar{\lambda}_2)\,T^{(1)}_{\bar{\nu}}\,,\\
&T^{(1)}_{\bar{\nu}} = i\,\epsilon_{\bar{\nu}\tau\alpha\beta}\,w^\tau\,\bar{p}_2^\alpha\,p_2^\beta\,,
\end{aligned}
\tag{6}
$$

where we use a notation analogous to the one introduced in [15]. For notational simplicity we do not introduce different notations for the invariant amplitudes $F_1(s,t)$ in the two reactions (4, 6). Further processes related to (6) are obtained by the exchange of the out or ingoing momenta.

The structure of the on-shell reaction amplitudes turns more complicated with increasing number of spin-1 particles involved. Consider the production of two vector particles

$$
\begin{aligned}
&T_{0\,0\to 1\,1}(\bar{k},\,k,\,w) = \sum_{n=1}^{5} F_n(s,t)\,\langle T^{(n)}_{\bar{\mu}\bar{\nu}}\rangle^{\bar{\mu}\bar{\nu}}_{0\,0\to 1\,1}\,,\\
&\langle\, T^{(n)}_{\bar{\mu}\bar{\nu}}\,\rangle^{\bar{\mu}\bar{\nu}}_{0\,0\to 1\,1} = \epsilon^{\dagger\,\bar{\mu}}(\bar{p}_1,\,\bar{\lambda}_1)\,\epsilon^{\dagger\,\bar{\nu}}(\bar{p}_2,\,\bar{\lambda}_2)\,T^{(n)}_{\bar{\mu}\bar{\nu}}\,,\\
&T^{(1)}_{\bar{\mu}\bar{\nu}} = \hat{g}_{\bar{\mu}\bar{\nu}}\,, \qquad T^{(2)}_{\bar{\mu}\bar{\nu}} = w_{\bar{\mu}}\,w_{\bar{\nu}}\,,\\
&T^{(3)}_{\bar{\mu}\bar{\nu}} = w_{\bar{\mu}}\,r_{\bar{\nu}}\,, \qquad T^{(4)}_{\bar{\mu}\bar{\nu}} = r_{\bar{\mu}}\,w_{\bar{\nu}}\,,\\
&T^{(5)}_{\bar{\mu}\bar{\nu}} = r_{\bar{\mu}}\,r_{\bar{\nu}}\,,
\end{aligned}
\tag{7}
$$

which is characterized by five invariant amplitudes, $F_n(s,t)$. The choice of Lorentz tensors in (7) is not unambiguous. Various linear combinations of the given tensors may be used. For instance we could have used the 5 tensors which follow from (7) by the replacements $r_\mu \to k_\mu$ and $\bar{r}_\mu \to \bar{k}_\mu$. The suggested form proves most convenient when calculating helicity matrix elements.

The number of invariant amplitudes is easily determined for the reaction $0\,0 \to 1\,1$. The task is to construct all rank two tensor in terms of the four vectors $\bar{k}, k, w$ and

$$
v_\mu = \epsilon_{\mu\alpha\tau\beta}\,\bar{k}^\alpha\,w^\tau\,k^\beta
\tag{8}
$$

At first there are $4 \times 4 + 1 = 17$ distinct tensors that one may construct. Parity conservation requires the pairwise occurrence of the vector v_μ introduced in (8). This eliminates 6 structures. The transversality of the spin-one wave functions with

$$
\begin{aligned}
&2\,\epsilon_\mu(\bar{p}_1)\,\bar{k}^\mu = +\epsilon_\mu(\bar{p}_1)\,w^\mu\,,\\
&2\,\epsilon_\mu(\bar{p}_2)\,\bar{k}^\mu = -\epsilon_\mu(\bar{p}_2)\,w^\mu\,,
\end{aligned}
\tag{9}
$$

eliminates additional 5 structures for on-shell conditions. Altogether there are 6 structures left. The five terms displayed in (7) and the Lorentz tensor $v_{\bar\mu}\, v_{\bar\nu}$. To show the on-shell redundance of the extra term requires an explicit computation of on-shell matrix elements.

In a practical application it is important to derive explicit expressions for the invariant amplitudes $F_n(s,t)$ (see e.g. [29]). In the general case this is may be a tedious exercise, which is considerably streamlined by the derivation and application of a set of projection tensors $P^{(n)}_{\mu\nu}$ with the following properties

$$\begin{aligned} &P^{(n)}_{\mu\nu}\, g^{\bar\mu\mu}\, g^{\bar\nu\nu}\, T^{(m)}_{\bar\mu\bar\nu} = \delta_{nm}\,, \\ &P^{(n)}_{\bar\mu\bar\nu}\, \bar p_1^{\bar\mu} = 0\,, \qquad\quad P^{(n)}_{\bar\mu\bar\nu}\, \bar p_2^{\bar\nu} = 0\,. \end{aligned} \tag{10}$$

Given any off-shell production amplitude the invariant function $F_n(s,t)$ is obtained by the contraction with the nth projection tensor. We decompose the projection tensors into a basis

$$\begin{aligned} &P^{(n)}_{\bar\mu\bar\nu} = \sum_{k=1}^{5} c^{(n)}_k\, Q^{(k)}_{\bar\mu\bar\nu}\,, \\ &Q_1^{\bar\mu\bar\nu} = v^{\bar\mu}\, v^{\bar\nu}/v^2\,, \\ &Q_2^{\bar\mu\bar\nu} = w_\rfloor{}^{\bar\mu}\, w_\lfloor{}^{\bar\nu}\,, \qquad Q_3^{\bar\mu\bar\nu} = w_\rfloor{}^{\bar\mu}\, r_\perp{}^{\bar\nu}\,, \\ &Q_4^{\bar\mu\bar\nu} = r_\perp{}^{\bar\mu}\, w_\lfloor{}^{\bar\nu}\,, \qquad Q_5^{\bar\mu\bar\nu} = r_\perp{}^{\bar\mu}\, r_\perp{}^{\bar\nu}\,, \end{aligned} \tag{11}$$

where the 4-vectors $r_\perp, w_\rfloor$ and $w_\lfloor$ are suitable linear combinations of $\bar r, r$ and w as to have the convenient properties

$$\begin{aligned} &r_\perp\cdot r = 1\,, \qquad r_\perp\cdot \bar r = 0 = r_\perp\cdot w\,, \\ &w_\rfloor\cdot w = 1\,, \qquad w_\rfloor\cdot r = 0 = w_\rfloor\cdot \bar p_1\,, \\ &w_\lfloor\cdot w = 1\,, \qquad w_\lfloor\cdot r = 0 = w_\lfloor\cdot \bar p_2\,. \end{aligned} \tag{12}$$

The index $\rfloor$ and $\lfloor$ of a vector indicates whether it is orthogonal to the 4 momentum of the first or second particle respectively. The patched symbol $\perp$ implies the orthogonality to both 4 momenta. Given such vectors the coefficients $c^{(n)}_k$ are readily determined. We find

$$\begin{aligned} &c^{(n)}_n = 1\,, \quad c^{(2)}_1 = -w_\rfloor\cdot w_\lfloor + 1/s\,, \qquad\qquad (13)\\ &c^{(3)}_1 = -w_\rfloor\cdot r_\perp\,, \quad c^{(4)}_1 = -r_\perp\cdot w_\lfloor\,, \quad c^{(5)}_1 = -r_\perp\cdot r_\perp\,, \end{aligned}$$

where we display non-vanishing elements only.

We construct the auxiliary vectors $r_\perp, w_\rfloor$ and $w_\lfloor$. For this purpose we introduce an intermediate notation. Given three 4-vectors a_μ, b_μ and c_μ we introduce a vector, $a^\mu_{b\,c} = a^\mu_{c\,b}$, as follows

$$\begin{aligned} \frac{a^\mu_{b\,c}}{a_{b\,c}\cdot a_{b\,c}} &= a^\mu - \frac{a\cdot c}{c\cdot c}\, c^\mu \\ &\quad - \frac{a\cdot (b - \frac{c\cdot b}{c\cdot c}\, c)}{(b-\frac{c\cdot b}{c\cdot c}\, c)^2}\left(b^\mu - \frac{c\cdot b}{c\cdot c}\, c^\mu\right)\,, \\ a^\mu_{b\,c}\, a_\mu = 1\,, &\qquad a^\mu_{b\,c}\, b_\mu = 0\,, \qquad a^\mu_{b\,c}\, c_\mu = 0\,. \end{aligned} \tag{14}$$

In the notation of (14) the desired vectors are identified with

$$\begin{aligned} &r_\perp{}^\mu = r^\mu_{\bar r\, w}\,, \qquad w_\rfloor{}^\mu = w^\mu_{r\,\bar p_1}\,, \qquad w_\lfloor{}^\mu = w^\mu_{r\,\bar p_2}\,, \\ &\bar r_\perp{}^\mu = \bar r^\mu_{r\, w}\,, \qquad \bar w_\rfloor{}^\mu = w^\mu_{\bar r\, p_1}\,, \qquad \bar w_\lfloor{}^\mu = w^\mu_{\bar r\, p_2}\,, \end{aligned} \tag{15}$$

where we introduced the additional vectors $\bar r_\perp, \bar w_\rfloor$ and $\bar w_\lfloor$ that will turn useful below.

It remains the question why did we select the five tensors in (7) and did not include the extra structure $v_{\bar\mu}\, v_{\bar\nu}$ into our basis? The reason is our request that the invariant amplitudes should be free of kinematical constraints. The issue is nicely illustrated at hand of the on-shell identity

$$\begin{aligned} v_{\bar\mu}\, v_{\bar\nu} &= v^2\Big[g_{\bar\mu\bar\nu} - (w_\rfloor\cdot w_\lfloor)\, w_{\bar\mu}\, w_{\bar\nu} - (w_\rfloor\cdot r_\perp)\, w_{\bar\mu}\, r_{\bar\nu} \\ &\qquad - (w_\rfloor\cdot r_\perp)\, r_{\bar\mu}\, w_{\bar\nu} - (r_\perp\cdot r_\perp)\, r_{\bar\mu}\, r_{\bar\nu}\Big]\,, \\ v^2 &= s\left[(\bar r\cdot r)^2 - \bar r^2\, r^2\right]\,, \end{aligned} \tag{16}$$

with the tensor basis introduced in (7). Eliminating any of the five tensors in favor of the structure $v_{\bar\mu}\, v_{\bar\nu}$ leads to invariant functions singular at various kinematical conditions. This is evident from the regularity of the expressions

$$\begin{aligned} &v^2\,(\,r_\perp\cdot r_\perp) = -s\,\bar r^2\,, \\ &v^2\,(w_\rfloor\cdot w_\lfloor) = (\bar r\cdot r)^2 + r^2\,\bar p_1\cdot \bar p_2\,, \\ &v^2\,(\,r_\perp\cdot w_\rfloor) = \tfrac{1}{2}\,(\bar m_2^2 - \bar m_1^2 - s)\,(\bar r\cdot r)\,, \\ &v^2\,(\,r_\perp\cdot w_\lfloor) = \tfrac{1}{2}\,(\bar m_2^2 - \bar m_1^2 + s)\,(\bar r\cdot r)\,. \end{aligned} \tag{17}$$

We continue with the scattering of pseudo-scalar off vector particles. There are again five invariant amplitudes needed to characterize the scattering amplitude

$$\begin{aligned} &T_{0\,1\to 0\,1}(\bar k,\, k,\, w) = \sum_{n=1}^{5} F_n(s,t)\, \langle T^{(n)}_{\bar\nu\nu}\rangle^{\bar\nu\nu}_{0\,1\to 0\,1}\,, \\ &\langle\, T^{(n)}_{\bar\nu\nu}\,\rangle^{\bar\nu\nu}_{0\,1\to 0\,1} = \epsilon^{\dagger\,\bar\nu}(\bar p_2,\, \bar\lambda_2)\, T^{(n)}_{\bar\nu\nu}\, \epsilon^\nu(p_2,\, \lambda_2)\,, \\ &T^{(1)}_{\bar\nu\nu} = \hat g_{\bar\nu\nu}\,, \qquad T^{(2)}_{\bar\nu\nu} = w_{\bar\nu}\, w_\nu\,, \\ &T^{(3)}_{\bar\nu\nu} = w_{\bar\nu}\, \bar r_\nu\,, \qquad T^{(4)}_{\bar\nu\nu} = r_{\bar\nu}\, w_\nu\,, \\ &T^{(5)}_{\bar\nu\nu} = r_{\bar\nu}\, \bar r_\nu\,, \end{aligned} \tag{18}$$

where it suffices to assume the second particles with momenta p_2 and $\bar p_2$ to carry the spin. The type arguments that lead to the given choice of tensors in (18) are identical to those given for the two vector production process (7). The construction of the associated projection tensors

$$\begin{aligned} &P^{(n)}_{\bar\mu\mu}\, g^{\mu\nu}\, g^{\bar\mu\bar\nu}\, T^{(m)}_{\bar\nu\nu} = \delta_{nm}\,, \\ &P^{(m)}_{\bar\nu\nu}\, \bar p_2^{\bar\nu} = 0\,, \qquad\quad P^{(m)}_{\bar\nu\nu}\, p_2^{\nu} = 0\,, \end{aligned} \tag{19}$$

is analogous to (11). We find

$$P^{(n)}_{\bar\nu\nu} = \sum_{k=1}^{5} c^{(n)}_k\, Q^{(k)}_{\bar\nu\nu}\,, \tag{20}$$

$$\begin{aligned} &Q_1^{\bar\nu\nu} = v^{\bar\nu}\, v^\nu/v^2\,, \\ &Q_2^{\bar\nu\nu} = w_\lfloor{}^{\bar\nu}\, \bar w_\lfloor{}^\nu\,, \qquad Q_3^{\bar\nu\nu} = w_\lfloor{}^{\bar\nu}\, \bar r_\perp{}^\nu\,, \\ &Q_4^{\bar\nu\nu} = r_\perp{}^{\bar\nu}\, \bar w_\lfloor{}^\nu\,, \qquad Q_5^{\bar\nu\nu} = r_\perp{}^{\bar\nu}\, \bar r_\perp{}^\nu\,, \end{aligned}$$

k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$
1	11	1	2	2	$\bar r^2\, s$	2	3	$-(\bar r\cdot r)\, s$
2	4	$(\bar r\cdot r)$	2	5	$-(\bar r\cdot r)$	2	7	$\bar r^2$
2	8	$-\bar r^2$	2	13	$\bar\alpha_+$	3	2	$-\frac{1}{2}\,\bar\alpha_+\,\bar r^2\, s$
3	3	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\, s$	3	4	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)$	3	5	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)$
3	7	$\frac{1}{2}\,\bar\alpha_-\,\bar r^2$	3	8	$\frac{1}{2}\,\bar\alpha_+\,\bar r^2$	3	10	$-\frac{1}{2}\,\bar\alpha_+\, s$
3	13	$-\frac{1}{2}\,\bar\alpha_+^2$	4	12	1	5	2	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\, s$
5	3	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\, s$	5	4	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)$	5	5	$\frac{1}{4}\,\bar\alpha_+\,(\alpha_-\,(\bar r\cdot r)+2\,r^2)$
5	6	$(\bar r\cdot r)$	5	7	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2$	5	8	$\frac{1}{4}\,\bar\alpha_+\,(\alpha_-\,\bar r^2+2\,(\bar r\cdot r))$
5	9	$\bar r^2$	5	13	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+$	6	2	$\bar r^2\, s$
6	3	$-(\bar r\cdot r)\, s$	6	4	$(\bar r\cdot r)$	6	5	$-(\bar r\cdot r)$
6	7	$\bar r^2$	6	8	$-\bar r^2$	6	13	$-\bar\alpha_-$
7	2	$\frac{1}{2}\,\bar\alpha_-\,\bar r^2\, s$	7	3	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\, s$	7	4	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)$
7	5	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)$	7	7	$\frac{1}{2}\,\bar\alpha_-\,\bar r^2$	7	8	$\frac{1}{2}\,\bar\alpha_+\,\bar r^2$
7	10	$\frac{1}{2}\,\bar\alpha_-\, s$	7	13	$\frac{1}{2}\,\bar\alpha_-\,\bar\alpha_+$	8	2	$-(\bar r\cdot r)\, s$
8	3	$r^2\, s$	8	4	$-r^2$	8	5	r^2
8	7	$-(\bar r\cdot r)$	8	8	$(\bar r\cdot r)$	8	12	1
9	1	$(\bar r\cdot r)^2-\bar r^2\, r^2$	9	2	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\, s$	9	3	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\, s$
9	4	$\frac{1}{4}\,(-\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)-2\,\bar\alpha_-\, r^2)$	9	5	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)$	9	6	$(\bar r\cdot r)$
9	7	$\frac{1}{4}\,(-\alpha_-\,\bar\alpha_+\,\bar r^2-2\,\bar\alpha_-\,(\bar r\cdot r))$	9	8	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2$	9	9	$\bar r^2$
9	13	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+$	10	2	$\bar r^2\, s$	10	3	$-(\bar r\cdot r)\, s$
10	4	$(\bar r\cdot r)$	10	5	$-(\bar r\cdot r)$	10	7	$\bar r^2$
10	8	$-\bar r^2$	10	10	s	10	13	$\bar\alpha_+$
11	2	$\frac{1}{2}\,\alpha_-\,\bar r^2\, s$	11	3	$-\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)\, s$	11	4	$\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)$
11	5	$-\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)-r^2$	11	7	$\frac{1}{2}\,\alpha_-\,\bar r^2$	11	8	$-\frac{1}{2}\,\alpha_-\,\bar r^2-(\bar r\cdot r)$
11	13	$-\frac{1}{2}\,\bar\alpha_-\,\alpha_-$	12	2	$\frac{1}{2}\,\alpha_-\,\bar r^2\, s$	12	3	$-\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)\, s$
12	4	$\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)-r^2$	12	5	$-\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)$	12	7	$\frac{1}{2}\,\alpha_-\,\bar r^2-(\bar r\cdot r)$
12	8	$-\frac{1}{2}\,\alpha_-\,\bar r^2$	12	13	$\frac{1}{2}\,\alpha_-\,\bar\alpha_+$	13	6	$-r^2$
13	9	$-(\bar r\cdot r)$	13	12	$\frac{1}{2}\,\alpha_-$			

TABLE I: The non-vanishing coefficients $c_k^{(n)}$ in the expansion (24). Here is $\alpha_\pm = 1\pm\frac{m_1^2-m_2^2}{s}$ and $\bar\alpha_\pm = 1\pm\frac{\bar m_1^2-\bar m_2^2}{s}$.

$$
\begin{aligned}
&c_n^{(n)} = 1\,, \quad c_1^{(2)} = -w_\llcorner\cdot\bar w_\llcorner + 1/s\,,\\
&c_1^{(3)} = -w_\llcorner\cdot\bar r_\perp, \quad c_1^{(4)} = -r_\perp\cdot\bar w_\llcorner, \quad c_1^{(5)} = -r_\perp\cdot\bar r_\perp,
\end{aligned}
$$

where we display non-vanishing elements only and use the 4-vectors introduced in (15).

While the construction of a suitable basis was almost trivially implied for the reactions $0\,0\to 1\,1$ and $0\,1\to 0\,1$ by choosing the tensors that involve the minimal number of momenta, the task is considerably more complicated once there are three vector particles involved. It suffices to construct tensors composed out of the metric tensor $g_{\mu\nu}$ and the three 4 momenta $\bar r_\mu, r_\mu$ and w_μ. Owing to the relation (16) any structure involving an even number of v_μ vectors is redundant. Altogether there are 61 Lorentz structures with the proper parity transformation. Due to the Schouten identity [30, 31]

$$
\begin{aligned}
g_{\sigma\tau}\epsilon_{\alpha\beta\gamma\delta} &= g_{\alpha\tau}\epsilon_{\sigma\beta\gamma\delta} + g_{\beta\tau}\epsilon_{\alpha\sigma\gamma\delta} + g_{\gamma\tau}\epsilon_{\alpha\beta\sigma\delta}\\
&\quad + g_{\delta\tau}\epsilon_{\alpha\beta\gamma\sigma}\,,
\end{aligned}
\tag{21}
$$

only a subset of 28 structures are off-shell independent. This is in contrast to the number of independent helicity amplitudes, which there are 13. Thus using on-shell conditions out of the 28 tensors only 13 are linear independent. The task is to find a subset which is free of kinematical constraints. The construction of such a set is quite tedious and to the best knowledge of the authors such amplitudes did not exist for the considered reaction. The on-shell scattering amplitude may be parameterized in terms of the following 13 scalar amplitudes $F_{1,\dots,13}$ with

$$
\begin{aligned}
&T_{0\,1\to 1\,1}(\bar k,\,k,\,w) = \sum_{n=1}^{13} F_n(s,t)\,\langle\, i\,T^{(n)}_{\bar\mu\bar\nu,\nu}\rangle^{\bar\mu\bar\nu,\nu}_{0\,1\to 1\,1}\,, \qquad (22)\\
&\langle\, T^{(n)}_{\bar\mu\bar\nu,\nu}\,\rangle^{\bar\mu\bar\nu,\nu}_{0\,1\to 1\,1} = \epsilon^{\dagger\,\bar\mu}(\bar p_1,\,\bar\lambda_1)\,\epsilon^{\dagger\,\bar\nu}(\bar p_2,\,\bar\lambda_2)\\
&\qquad\qquad \times T^{(n)}_{\bar\mu\bar\nu,\nu}\,\epsilon^{\nu}(p_2,\,\lambda_2)\,,
\end{aligned}
$$

$$
\begin{aligned}
T^{(1)}_{\bar\mu\bar\nu,\nu} &= \epsilon_{\bar\mu\bar\nu\nu\alpha}\, w^\alpha\,, & T^{(2)}_{\bar\mu\bar\nu,\nu} &= \epsilon_{\bar\mu\bar\nu\nu\alpha}\, r^\alpha\,,\\
T^{(3)}_{\bar\mu\bar\nu,\nu} &= \epsilon_{\bar\mu\bar\nu\nu\alpha}\, \bar r^\alpha\,, & T^{(4)}_{\bar\mu\bar\nu,\nu} &= w_{\bar\nu}\, \epsilon_{\bar\mu\nu\alpha\beta}\, \bar r^\alpha\, w^\beta\,,\\
T^{(5)}_{\bar\mu\bar\nu,\nu} &= w_{\bar\mu}\, \epsilon_{\bar\nu\nu\alpha\beta}\, \bar r^\alpha\, w^\beta\,, & T^{(6)}_{\bar\mu\bar\nu,\nu} &= r_{\bar\nu}\, \epsilon_{\bar\mu\nu\alpha\beta}\, \bar r^\alpha\, w^\beta\,,\\
T^{(7)}_{\bar\mu\bar\nu,\nu} &= w_{\bar\nu}\, \epsilon_{\bar\mu\nu\alpha\beta}\, w^\alpha\, r^\beta\,, & T^{(8)}_{\bar\mu\bar\nu,\nu} &= w_{\bar\mu}\, \epsilon_{\bar\nu\nu\alpha\beta}\, w^\alpha\, r^\beta\,,\\
T^{(9)}_{\bar\mu\bar\nu,\nu} &= r_{\bar\nu}\, \epsilon^{\bar\mu\nu\alpha\beta}\, w^\alpha\, r^\beta\,, & T^{(10)}_{\bar\mu\bar\nu,\nu} &= w_{\bar\nu}\, \epsilon_{\bar\mu\nu\alpha\beta}\, \bar r^\alpha\, r^\beta\,,\\
T^{(11)}_{\bar\mu\bar\nu,\nu} &= \hat g_{\bar\nu\nu}\, \epsilon_{\bar\mu\alpha\tau\beta}\, \bar r^\alpha\, w^\tau\, r^\beta\,,\\
T^{(12)}_{\bar\mu\bar\nu,\nu} &= r_{\bar\nu}\, w_\nu\, \epsilon_{\bar\mu\alpha\tau\beta}\, \bar r^\alpha\, w^\tau\, r^\beta\,,\\
T^{(13)}_{\bar\mu\bar\nu,\nu} &= \tfrac{1}{2}\left(w_{\bar\nu}\, w_\nu\, \epsilon_{\bar\mu\alpha\tau\beta} - w_{\bar\mu}\, w_\nu\, \epsilon_{\bar\nu\alpha\tau\beta}\right) \bar r^\alpha\, w^\tau\, r^\beta\,.
\end{aligned}
$$

We assure that our choice of amplitudes in (22) excludes the occurrence of kinematical constraints with the possible exception at $s = 0$. Using slightly modified amplitudes as implied by the replacement $r_\mu \to k_\mu$ and $\bar r_\mu \to \bar k_\mu$ removes the constraints at $s = 0$.

Again we provide the convenient projection tensors that streamline the computation of the invariant amplitudes $F_n(s,t)$ by means of algebraic computer codes. We find

$$
\begin{aligned}
& P^{(n)}_{\alpha\beta,\tau}\, g^{\alpha\bar\mu}\, g^{\beta\bar\nu}\, g^{\tau\nu}\, T^{(m)}_{\bar\mu\bar\nu,\nu} = \delta_{nm}\,, \qquad (23)\\
& P^{(n)}_{\bar\mu\bar\nu,\nu}\, \bar p_1^{\bar\mu} = 0\,, \qquad P^{(n)}_{\bar\mu\bar\nu,\nu}\, \bar p_2^{\bar\nu} = 0\,, \qquad P^{(n)}_{\bar\mu\bar\nu,\nu}\, p_2^{\nu} = 0\,.
\end{aligned}
$$

$$
\begin{aligned}
& P^{(n)}_{\bar\mu\bar\nu,\nu} = \sum_{k=1}^{13} c^{(n)}_k\, Q^{(k)}_{\bar\mu\bar\nu,\nu}\,, \qquad (24)\\
& Q_1^{\bar\mu\bar\nu,\nu} = v^{\bar\mu}\, v^{\bar\nu}\, v^\nu/v^2/v^2\,,\\
& Q_2^{\bar\mu\bar\nu,\nu} = v^{\bar\mu}\, w_{\lfloor}{}^{\bar\nu}\, \bar w_{\lfloor}{}^{\nu}/v^2 - (w_\lfloor \cdot \bar w_\lfloor)\, Q_1^{\bar\mu\bar\nu,\nu}\,,\\
& Q_3^{\bar\mu\bar\nu,\nu} = v^{\bar\mu}\, w_{\lfloor}{}^{\bar\nu}\, \bar r_{\perp}{}^{\nu}/v^2 - (w_\lfloor \cdot \bar r_\perp)\, Q_1^{\bar\mu\bar\nu,\nu}\,,\\
& Q_4^{\bar\mu\bar\nu,\nu} = v^{\bar\mu}\, r_{\perp}{}^{\bar\nu}\, \bar w_{\lfloor}{}^{\nu}/v^2 - (r_\perp \cdot \bar w_\lfloor)\, Q_1^{\bar\mu\bar\nu,\nu}\,,\\
& Q_5^{\bar\mu\bar\nu,\nu} = v^{\bar\mu}\, r_{\perp}{}^{\bar\nu}\, \bar r_{\perp}{}^{\nu}/v^2 - (r_\perp \cdot \bar r_\perp)\, Q_1^{\bar\mu\bar\nu,\nu}\,,\\
& Q_6^{\bar\mu\bar\nu,\nu} = w_{\rfloor}{}^{\bar\mu}\, v^{\bar\nu}\, \bar w_{\lfloor}{}^{\nu}/v^2\,, \qquad Q_7^{\bar\mu\bar\nu,\nu} = w_{\rfloor}{}^{\bar\mu}\, v^{\bar\mu}\, \bar r_{\perp}{}^{\nu}/v^2\,,\\
& Q_8^{\bar\mu\bar\nu,\nu} = r_{\perp}{}^{\bar\mu}\, v^{\bar\nu}\, \bar w_{\lfloor}{}^{\nu}/v^2\,, \qquad Q_9^{\bar\mu\bar\nu,\nu} = r_{\perp}{}^{\bar\mu}\, v^{\bar\nu}\, \bar r_{\perp}{}^{\nu}/v^2\,,\\
& Q_{10}^{\bar\mu\bar\nu,\nu} = w_{\rfloor}{}^{\bar\mu}\, w_{\lfloor}{}^{\bar\nu}\, v^\nu/v^2\,, \qquad Q_{11}^{\bar\mu\bar\nu,\nu} = w_{\rfloor}{}^{\bar\mu}\, r_{\perp}{}^{\bar\nu}\, v^\nu/v^2\,,\\
& Q_{12}^{\bar\mu\bar\nu,\nu} = r_{\perp}{}^{\bar\mu}\, w_{\lfloor}{}^{\bar\nu}\, v^\nu/v^2\,, \qquad Q_{13}^{\bar\mu\bar\nu,\nu} = r_{\perp}{}^{\bar\mu}\, r_{\perp}{}^{\bar\nu}\, v^\nu/v^2\,,
\end{aligned}
$$

where the explicit form of the coefficients $c^{(n)}_k$ can be found in Tab. I.

We turn to the most complicated reaction $1\,1 \to 1\,1$. Excluding Lorentz structures involving an even number of v_μ vectors there are 138 structures with the proper parity transformation that one may write down. A subset of 136 structures are off-shell independent. Using on-shell conditions out of the 136 tensors only 41 are linear independent. We constructed a subset which is free of kinematical constraints with the possible exception of $s = 0$. To the best knowledge of the authors such amplitudes did not exist for the considered scattering process. The on-shell scattering amplitude may be parameterized in terms of the following 41 scalar amplitudes $F_{1,\ldots,41}$ with

$$
\begin{aligned}
& T_{1\,1\to 1\,1}(\bar k,\, k,\, w) = \sum_{n=1}^{41} F_n(s,t)\, \langle T^{(n)}_{\bar\mu\bar\nu,\mu\nu}\rangle^{\bar\mu\bar\nu,\mu\nu}_{1\,1\to 1\,1}\,, \qquad (25)\\
& \langle T^{(n)}_{\bar\mu\bar\nu,\mu\nu}\rangle^{\bar\mu\bar\nu,\mu\nu}_{1\,1\to 1\,1} = \epsilon^{\dagger\,\bar\mu}(\bar p_1,\, \bar\lambda_1)\, \epsilon^{\dagger\,\bar\nu}(\bar p_2,\, \bar\lambda_2)\\
& \qquad\qquad \times T^{(n)}_{\bar\mu\bar\nu,\mu\nu}\, \epsilon^\nu(p_1,\, \lambda_1)\, \epsilon^\nu(p_2,\, \lambda_2)\,,
\end{aligned}
$$

$$
\begin{aligned}
T^{(1)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\mu}\, \hat g_{\bar\nu\nu}\,, & T^{(2)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\nu}\, \hat g_{\bar\nu\mu}\,,\\
T^{(3)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\bar\nu}\, \hat g_{\mu\nu}\,, & T^{(4)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\nu}\, w_{\bar\mu}\, w_\mu\,,\\
T^{(5)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\nu}\, r_{\bar\mu}\, w_\mu\,, & T^{(6)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\nu}\, w_{\bar\mu}\, \bar r_\mu\,,\\
T^{(7)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\nu}\, r_{\bar\mu}\, \bar r_\mu\,, & T^{(8)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\mu}\, w_{\bar\nu}\, w_\nu\,,\\
T^{(9)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\mu}\, r_{\bar\nu}\, w_\nu\,, & T^{(10)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\mu}\, w_{\bar\nu}\, \bar r_\nu\,,\\
T^{(11)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\mu}\, r_{\bar\nu}\, \bar r_\nu\,, & T^{(12)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\mu}\, w_{\bar\mu}\, w_\nu\,,\\
T^{(13)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\mu}\, r_{\bar\mu}\, w_\nu\,, & T^{(14)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\mu}\, w_{\bar\mu}\, \bar r_\nu\,,\\
T^{(15)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\nu\mu}\, r_{\bar\mu}\, \bar r_\nu\,, & T^{(16)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\nu}\, w_{\bar\nu}\, w_\mu\,,\\
T^{(17)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\nu}\, r_{\bar\nu}\, w_\mu\,, & T^{(18)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\nu}\, w_{\bar\nu}\, \bar r_\mu\,,\\
T^{(19)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\nu}\, r_{\bar\nu}\, \bar r_\mu\,, & T^{(20)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\bar\nu}\, w_\mu\, w_\nu\,,\\
T^{(21)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\bar\nu}\, \bar r_\mu\, w_\nu\,, & T^{(22)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\bar\nu}\, w_\mu\, \bar r_\nu\,,\\
T^{(23)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\bar\mu\bar\nu}\, \bar r_\mu\, \bar r_\nu\,, & T^{(24)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\mu\nu}\, w_{\bar\mu}\, w_{\bar\nu}\,,\\
T^{(25)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\mu\nu}\, r_{\bar\mu}\, w_{\bar\nu}\,, & T^{(26)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\mu\nu}\, w_{\bar\mu}\, r_{\bar\nu}\,,\\
T^{(27)}_{\bar\mu\bar\nu,\mu\nu} &= \hat g_{\mu\nu}\, r_{\bar\mu}\, r_{\bar\nu}\,, & T^{(28)}_{\bar\mu\bar\nu,\mu\nu} &= w_{\bar\mu}\, w_{\bar\nu}\, w_\mu\, w_\nu\,,\\
T^{(29)}_{\bar\mu\bar\nu,\mu\nu} &= r_{\bar\mu}\, r_{\bar\nu}\, w_\mu\, w_\nu\,, & T^{(30)}_{\bar\mu\bar\nu,\mu\nu} &= w_{\bar\mu}\, w_{\bar\nu}\, \bar r_\mu\, \bar r_\nu\,,\\
T^{(31)}_{\bar\mu\bar\nu,\mu\nu} &= r_{\bar\mu}\, w_{\bar\nu}\, w_\mu\, w_\nu\,, & T^{(32)}_{\bar\mu\bar\nu,\mu\nu} &= w_{\bar\mu}\, r_{\bar\nu}\, w_\mu\, w_\nu\,,\\
T^{(33)}_{\bar\mu\bar\nu,\mu\nu} &= r_{\bar\mu}\, w_{\bar\nu}\, \bar r_\mu\, \bar r_\nu\,, & T^{(34)}_{\bar\mu\bar\nu,\mu\nu} &= w_{\bar\mu}\, r_{\bar\nu}\, \bar r_\mu\, \bar r_\nu\,,\\
T^{(35)}_{\bar\mu\bar\nu,\mu\nu} &= w_{\bar\mu}\, w_{\bar\nu}\, \bar r_\mu\, w_\nu\,, & T^{(36)}_{\bar\mu\bar\nu,\mu\nu} &= r_{\bar\mu}\, r_{\bar\nu}\, \bar r_\mu\, w_\nu\,,\\
T^{(37)}_{\bar\mu\bar\nu,\mu\nu} &= w_{\bar\mu}\, w_{\bar\nu}\, w_\mu\, \bar r_\nu\,, & T^{(38)}_{\bar\mu\bar\nu,\mu\nu} &= r_{\bar\mu}\, r_{\bar\nu}\, w_\mu\, \bar r_\nu\,,\\
T^{(39)}_{\bar\mu\bar\nu,\mu\nu} &= \tfrac14 \left(r_{\bar\mu}\, w_{\bar\nu} + w_{\bar\mu}\, r_{\bar\nu}\right) \left(\bar r_\mu\, w_\nu - w_\mu\, \bar r_\nu\right),\\
T^{(40)}_{\bar\mu\bar\nu,\mu\nu} &= \tfrac14 \left(r_{\bar\mu}\, w_{\bar\nu} - w_{\bar\mu}\, r_{\bar\nu}\right) \left(\bar r_\mu\, w_\nu + w_\mu\, \bar r_\nu\right),\\
T^{(41)}_{\bar\mu\bar\nu,\mu\nu} &= \tfrac14 \left(r_{\bar\mu}\, w_{\bar\nu} - w_{\bar\mu}\, r_{\bar\nu}\right) \left(\bar r_\mu\, w_\nu - w_\mu\, \bar r_\nu\right),
\end{aligned}
$$

where our invariant functions are kinematically correlated at $s = 0$ only. The latter constraint can be eliminated by the use of the modified vectors $r_\mu \to k_\mu$ and $\bar r_\mu \to \bar k_\mu$ in (25). An algebra to project onto the invariant amplitudes $F_n(s,t)$ is developed in Appendix A.

We emphasize that all amplitudes $F_n(s,t)$ introduced in this section are truly uncorrelated and satisfy Mandelstam's dispersion integral representation [16, 28].

III. PARTIAL-WAVE DECOMPOSITION

The helicity matrix elements of the scattering operator, T, are decomposed into partial-wave amplitudes characterized by the total angular momentum J. Given a specific process together with our convention of the helicity wave functions it suffices to specify the helicity projection λ_1, λ_2 and $\bar\lambda_1, \bar\lambda_2$ as introduced in the previous section. We write

$$
\begin{aligned}
& \langle \bar\lambda_1 \bar\lambda_2 |\, T\, | \lambda_1 \lambda_2 \rangle = \sum_J (2\,J+1)\, \langle \bar\lambda_1 \bar\lambda_2 |\, T_J | \lambda_1 \lambda_2 \rangle\, d^{(J)}_{\lambda,\bar\lambda}(\theta)\,,\\
& d^{(J)}_{\lambda,\bar\lambda}(\theta) = (-)^{\lambda-\bar\lambda}\, d^{(J)}_{-\lambda,-\bar\lambda}(\theta)\\
& \qquad = (-)^{\lambda-\bar\lambda}\, d^{(J)}_{\bar\lambda,\lambda}(\theta) = d^{(J)}_{-\bar\lambda,-\lambda}(\theta)\,, \qquad (26)\\
& \langle \bar\lambda_1 \bar\lambda_2 |\, T_J | \lambda_1 \lambda_2 \rangle = \int_{-1}^{-1} \frac{\mathrm{d}\cos\theta}{2}\, \langle \bar\lambda_1 \bar\lambda_2 |\, T\, | \lambda_1 \lambda_2 \rangle\, d^{(J)}_{\lambda,\bar\lambda}(\theta)\,,
\end{aligned}
$$

with $\lambda = \lambda_1 - \lambda_2$ and $\bar\lambda = \bar\lambda_1 - \bar\lambda_2$. Wigner's rotation functions, $d^{(J)}_{\lambda,\bar\lambda}(\theta)$, are used in a convention as characterized by (26). The phase conventions assumed in this

a	b	$[U^J_{+,0\,1}]_{ab}$	a	b	$[U^J_{+,0\,1}]_{ab}$	a	b	$[U^J_{+,0\,1}]_{ab}$
1	1	$(M_+ - M_-)\,\frac{\sqrt{s}}{2\,p}$	2	1	$\alpha_-\,\frac{s}{2\,p}\,\sqrt{\frac{J}{J+1}}$	2	2	$-p$

a	b	$[U^J_{+,1\,1}]_{ab}$	a	b	$[U^J_{+,1\,1}]_{ab}$	a	b	$[U^J_{+,1\,1}]_{ab}$
1	1	$-\sqrt{2}\,\frac{s}{p}$	2	1	$-(M_+ + M_-)\,\frac{\sqrt{s}}{p}\,\sqrt{\frac{J}{J+1}}$	2	2	$\alpha_-\,(M_+ + M_-)\,\frac{\sqrt{s}}{4\,p}$
2	3	$(M_+ + M_-)\,\frac{p}{2\,\sqrt{s}}$	3	1	$-(M_+ - M_-)\,\frac{\sqrt{s}}{p}\,\sqrt{\frac{J}{J+1}}$	3	2	$-\alpha_+\,(M_+ - M_-)\,\frac{\sqrt{s}}{4\,p}$
3	3	$(M_+ - M_-)\,\frac{p}{2\,\sqrt{s}}$	4	1	$M_+\,M_-\,\frac{1}{p}\,\sqrt{\frac{2\,(J-1)\,J}{(J+1)\,(J+2)}}$	4	2	$-\alpha_-\,\alpha_+\,\frac{s}{4\,p}\,\sqrt{\frac{2\,(J-1)}{J+2}}$
4	3	$-M_+\,M_-\,\frac{p}{2\,s}\,\sqrt{\frac{2\,(J-1)}{J+2}}$	4	4	$\frac{p}{\sqrt{2}}$			

a	b	$[U^J_{-,1\,1}]_{ab}$	a	b	$[U^J_{-,1\,1}]_{ab}$	a	b	$[U^J_{-,1\,1}]_{ab}$
1	1	$(M_-^2 - M_+^2)\,\frac{s}{2\,p^2}\,\sqrt{\frac{J+1}{J}}$	2	1	$\alpha_-\,\alpha_+\,\frac{s^2}{4\,p^2}\,\sqrt{\frac{2\,(J+1)}{J}}$	2	2	$-s\,\sqrt{\frac{2\,(J+1)}{J}}$
3	1	$-\alpha_-\,(M_- + M_+)\,\frac{\sqrt{s}\,s}{2\,p^2}$	3	3	$(M_- + M_+)\,\frac{\sqrt{s}}{2}$	4	1	$\alpha_+\,(M_+ - M_-)\,\frac{\sqrt{s}\,s}{2\,p^2}$
4	3	$(M_+ - M_-)\,\frac{\sqrt{s}}{2}$	4	4	$\sqrt{s}\,(M_- - M_+)$	5	1	$\alpha_-\,\alpha_+\,\frac{s^2}{4\,p^2}\,\sqrt{\frac{2\,(J-1)}{J+2}}$
5	2	$s\,\sqrt{\frac{2\,(J-1)}{J+2}}$	5	3	$-M_+\,M_-\,\sqrt{\frac{2\,(J-1)}{J+2}}$	5	4	$-\alpha_-\,s\,\sqrt{\frac{2\,(J-1)}{J+2}}$
5	5	$-p^2\,\frac{\sqrt{2}\,(2\,J+1)}{\sqrt{J-1}\,\sqrt{J+2}}$						

TABLE II: Non-zero elements of the transformation matrices $U^J_{+,0\,1}$ and $U^J_{\pm,1\,1}$. We use the notation of (35).

work imply the parity relations

$$\langle -\bar\lambda_1, -\bar\lambda_2|\,T\,|-\lambda_1, -\lambda_2\rangle = (-)^{\Delta}\,\langle \bar\lambda_1, \bar\lambda_2|\,T\,|\lambda_1, \lambda_2\rangle\,,$$
$$\Delta = S_1 - S_2 + \bar S_1 - \bar S_2 + \lambda_1 - \lambda_2 - \bar\lambda_1 + \bar\lambda_2\,. \tag{27}$$

It is useful to decouple the two parity sectors by introducing parity eigenstates of good total angular momentum J, formed in terms of the helicity states [4]. Following (26) we introduce the angular momentum projection, $|\lambda_1, \lambda_2\,\rangle_J$, of the helicity state $|\lambda_1, \lambda_2\,\rangle$. We write

$$|\lambda_1, \lambda_2\,\rangle_J\,, \quad \text{with} \quad T\,|\lambda_1, \lambda_2\,\rangle_J = T_J|\lambda_1, \lambda_2\,\rangle\,. \tag{28}$$

We introduce parity eigenstates states, $|n_\pm, J\,\rangle$, that are eigenstates of the total angular momentum. We will be applying the following state convention

$$|1_-, J\,\rangle_{0\,0} = |\,0\,,\,0\rangle_J\,, \tag{29}$$

$$|1_-, J\,\rangle_{0\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|\,0\,,-\rangle_J - |\,0\,,+\rangle_J\Big)\,,$$

$$|1_-, J\,\rangle_{1\,1} = |\,0\,,\,0\rangle_J\,,$$
$$|2_-, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|+,+\rangle_J + |-,-\rangle_J\Big)\,,$$
$$|3_-, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|\,0\,,-\rangle_J + |\,0\,,+\rangle_J\Big)\,,$$
$$|4_-, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|+,\,0\rangle_J + |-,\,0\rangle_J\Big)\,,$$
$$|5_-, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|+,-\rangle_J + |-,+\rangle_J\Big)\,,$$

and

$$|1_+, J\,\rangle_{0\,1} = |\,0\,,\,0\rangle_J\,, \tag{30}$$
$$|2_+, J\,\rangle_{0\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|\,0\,,-\rangle_J + |\,0\,,+\rangle_J\Big)\,,$$

$$|1_+, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|+,+\rangle_J - |-,-\rangle_J\Big)\,,$$
$$|2_+, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|\,0\,,-\rangle_J - |\,0\,,+\rangle_J\Big)\,,$$
$$|3_+, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|+,\,0\rangle_J - |-,\,0\rangle_J\Big)\,,$$
$$|4_+, J\,\rangle_{1\,1} = \tfrac{1}{\sqrt{2}}\,\Big(|+,-\rangle_J - |-,+\rangle_J\Big)\,,$$

where we suppress the sector index 0 0, 0 1 or 1 1 on the right hand sides and use the short-hand notation $\pm \equiv \pm 1$. The states have the following property

$$P\,|n_\pm, J\,\rangle = \pm(-1)^{J+1}\,|n_\pm, J\,\rangle\,. \tag{31}$$

The partial-wave helicity amplitudes $t^J_{\pm,ab}$ that carry good angular momentum J and good parity are defined with

$$t^J_{\pm,ab} = \langle a_\pm, J\,|\,T\,|b_\pm, J\rangle\;, \tag{32}$$

where a and b label the states. For sufficiently large s the unitarity condition takes the simple form

$$\Im\big[t^{(J)}\big]^{-1}_{ab} = -\frac{p_a}{8\pi\,\sqrt{s}}\,\delta_{ab}\,, \tag{33}$$

where the index a and b spans the basis of two-particle helicity states in the $(0,0), (0,1)$ and $(1,1)$ sectors.

Helicity-partial-wave amplitudes are correlated at specific kinematical conditions. This is seen once the amplitudes $t^J_{\pm,ab}(s)$ are expressed in terms of the invariant functions $F_n(s,t)$. This is a well know problem related to the use of the helicity basis in covariant models, see for example the review [23]. In contrast covariant-partial wave amplitudes $T^J_\pm(s)$ are free of kinematical constraints and can therefore be used efficiently in partial-wave dispersion relation. They are associated to covariant states and a covariant projector algebra which diagonalizes the Bethe-Salpeter two-body scattering equation for local interactions [9, 10, 14, 15]. We introduce

$$T^J_\pm(s) = \left(\frac{s}{\bar{p}\,p}\right)^J \left[\bar{U}^J_\pm(s)\right]^T t^J_\pm(s)\, U^J_\pm(s)\,, \tag{34}$$

with nontrivial matrices $U^J_\pm(s)$ and $\bar{U}^J_\pm(s)$ characterizing the transformation for the initial and final states from the helicity basis to the new kinematic-free basis. The matrices $U^J_\pm(s)$ are given in Tab. III, where we use the conventions

$$\begin{aligned} M_\pm &= m_1 \pm m_2\,, & \bar{M}_\pm &= \bar{m}_1 \pm \bar{m}_2\,, \\ \delta &= \frac{M_+\,M_-}{s}\,, & \bar{\delta} &= \frac{\bar{M}_+\,\bar{M}_-}{s}\,, \\ \alpha_\pm &= 1 \pm \delta\,, & \bar{\alpha}_\pm &= 1 \pm \bar{\delta}\,. \end{aligned} \tag{35}$$

The transformation (34) implies a change in the phase-space distribution:

$$\begin{aligned} \rho^J_\pm(s) &= -\Im\Big[T^J_\pm(s)\Big]^{-1} \\ &= \frac{1}{8\,\pi}\left(\frac{p}{\sqrt{s}}\right)^{2\,J+1} \Big[U^J_\pm(s)\Big]^{-1}\Big[U^J_\pm(s)\Big]^{T,-1}\,. \end{aligned} \tag{36}$$

We adapt a convention for the transformation matrices that lead to an asymptotically bounded phase-space matrix, i.e. we require

$$\lim_{s\to\infty} \det \rho^J_\pm(s) = \text{const} \neq 0\,. \tag{37}$$

In contrast to the helicity states the phase-space matrix in the covariant states does have off-diagonal elements. The quest for the elimination of kinematical constraints leads necessarily to off-diagonal elements. To the best knowledge of the authors transformation matrices for the '1 1' case in Tab. III are novel and not presented in the literature before.

We provide particularly detailed results for the $0\,1 \to 0\,1$, $0\,1 \to 1\,1$ and $0\,0 \to 1\,1$ reactions, but refrain from giving the tedious details for the remaining cases. There are two one-dimensional cases with

$$U^J_{-,0\,0} = 1\,, \qquad U^J_{-,0\,1} = 1\,. \tag{38}$$

It follows

$$\begin{aligned} T^J_{-,0\,0\to 0\,0}(s) &= A^J_1(s)\,, \\ T^J_{-,0\,0\to 0\,1}(s) &= \frac{\sqrt{J\,(J+1)}}{(2\,J+1)\,\sqrt{s}}\Big[s^2\,A^{J-1}_1(s) \\ &\qquad - \bar{p}^2\,p^2 A^{J+1}_1(s)\Big]\,, \\ T^J_{-,0\,1\to 0\,1}(s) &= -A^J_1(s) + \frac{\bar{p}^2\,p^2}{(2\,J+1)\,s}\,A^{J+1}_5(s) \\ &\qquad - \frac{s}{2\,J+1}\,A^{J-1}_5(s)\,, \end{aligned} \tag{39}$$

with

$$A^J_n(s) = \left(\frac{s}{\bar{p}\,p}\right)^J \int_{-1}^{1} \frac{\mathrm{d}\cos\theta}{2}\, F_n(s,t)\, P_J(\cos\theta)\,. \tag{40}$$

The important merit of (39) is the absence of kinematical constraints, with the repeatedly discussed exception at $s=0$. A potential singularity at $\bar{p}\,p = 0$ in (40) is not realized due to the properties of the Legendre polynomials $P_J(\cos\theta)$.

The corresponding decomposition of the partial-wave scattering amplitudes $T^J(s)$ into integrals over the invariant amplitude $F_n(s,t)$ for the $0\,1 \to 0\,1$, $0\,1 \to 1\,1$, $0\,0 \to 1\,1$ and $1\,1 \to 1\,1$ reactions is considerably more involved. We write

$$T^J_\pm(s) = \sum_{k,n} a^{J+k}_{\pm n}(s)\, A^{J+k}_n(s)\,, \tag{41}$$

with coefficients $a^{J+k}_{\pm n}(s)$.

In Tab. III we detail those results for the coefficients $a^{J+k}_{\pm n}(s)$ which demonstrate the absence of kinematical singularities in all partial-wave amplitudes considered in this work. We note that the transformation matrices in Tab. III are derived from a study of the three reactions $0\,1 \to 0\,1$, $0\,1 \to 1\,1$ and $0\,0 \to 1\,1$ with appropriate parity selection only. From the regularity of the $a^{J+k}_{\pm n}(s)$ in Tab. III it follows the absence of kinematical singularities. Additional and tedious computations reveal that there are also no correlations of the various partial-wave amplitudes $T^J_\pm(s)$ at any kinematical point but at $s=0$. The covariant amplitudes are associated to a projector algebra for the Bethe-Salpeter scattering equation [9, 10, 14, 15]. A consistency check was performed that confirm the claimed properties for the remaining cases in particular the most tedious $1\,1 \to 1\,1$ reaction.

k	n	$[a^{J+k}_{+n}]_{11}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{11}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{11}$ $0\,1 \to 0\,1$
0	2	s^2	0	5	$\bar\alpha_-\,\alpha_-\,s^2\,\frac{1}{4}\,\frac{1}{(J+1)(2\,J+3)}$	1	1	$-\bar\alpha_-\,\alpha_-\,s\,\frac{1}{4}\,\frac{2\,J+1}{J+1}$
1	3	$\alpha_-\,\bar p^2\,s\,\frac{1}{2}$	1	4	$\bar\alpha_-\,p^2\,s\,\frac{1}{2}$	2	5	$\bar\alpha_-\,\alpha_-\,\bar p^2\,p^2\,\frac{1}{4}\,\frac{(J+2)(2\,J+1)}{(J+1)(2\,J+3)}$

k	n	$[a^{J+k}_{+n}]_{12}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{12}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{12}$ $0\,1 \to 0\,1$
−1	3	$s^2\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	0	5	$\bar\alpha_-\,p^2\,s\,\frac{1}{2}\,\frac{\sqrt{J}\,(J+2)}{\sqrt{J+1}\,(2\,J+3)}$	1	1	$\bar\alpha_-\,p^2\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
1	3	$-\bar p^2\,p^2\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	2	5	$-\bar\alpha_-\,\bar p^2\,p^4\,\frac{1}{2\,s}\,\frac{\sqrt{J}\,(J+2)}{\sqrt{J+1}\,(2\,J+3)}$			

k	n	$[a^{J+k}_{+n}]_{21}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{21}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{21}$ $0\,1 \to 0\,1$
−1	4	$s^2\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	0	5	$\alpha_-\,\bar p^2\,s\,\frac{1}{2}\,\frac{\sqrt{J}\,(J+2)}{\sqrt{J+1}\,(2\,J+3)}$	1	1	$\alpha_-\,\bar p^2\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
1	4	$-\bar p^2\,p^2\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	2	5	$-\alpha_-\,\bar p^4\,p^2\,\frac{1}{2\,s}\,\frac{\sqrt{J}\,(J+2)}{\sqrt{J+1}\,(2\,J+3)}$			

k	n	$[a^{J+k}_{+n}]_{22}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{22}$ $0\,1 \to 0\,1$	k	n	$[a^{J+k}_{+n}]_{22}$ $0\,1 \to 0\,1$
−2	5	$s^2\,\frac{J^2-1}{4J^2-1}$	−1	1	$-s\,\frac{J+1}{2\,J+1}$	0	5	$-\bar p^2\,p^2\,\frac{2\,J\,(J+1)-3}{4J\,(J+1)-3}$
1	1	$-\bar p^2\,p^2\,\frac{1}{s}\,\frac{J}{2\,J+1}$	2	5	$\bar p^4\,p^4\,\frac{1}{s^2}\,\frac{J\,(J+2)}{4J\,(J+2)+3}$			

k	n	$[a^{J+k}_{+n}]_{11}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{11}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{11}$ $0\,1 \to 1\,1$
−1	10	$2\sqrt{s}\,s^2\,\frac{J}{2\,J+1}$	−1	11	$-2\sqrt{s}\,s^2\,\frac{J}{2\,J+1}$	0	3	$-2\sqrt{s}\,s$
0	6	$\alpha_-\,\sqrt{s}\,s^2\,\frac{\bar\alpha_+\,J+1}{(J+1)(2\,J+3)}$	0	7	$\alpha_-\,\sqrt{s}\,s^2\,\frac{J}{J+1}$	0	8	$\alpha_-\,\sqrt{s}\,s^2\,\frac{J}{J+1}$
0	11	$\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{(2\,J+1)(\bar\alpha_-\,J+2)}{(J+1)(2\,J+3)}$	1	1	$-\alpha_-\,\sqrt{s}\,s\,\frac{2\,J+1}{J+1}$	1	2	$-2\,p^2\,\sqrt{s}$
1	4	$-\alpha_-\,\bar p^2\,\sqrt{s}\,s\,\frac{J}{J+1}$	1	5	$-\alpha_-\,\bar p^2\,\sqrt{s}\,s\,\frac{J}{J+1}$	1	9	$\bar\alpha_-\,\alpha_-\,p^2\,\sqrt{s}\,s\,\frac{1}{2}\,\frac{J}{J+1}$
1	10	$-2\,\bar p^2\,p^2\,\sqrt{s}\,\frac{J}{2\,J+1}$	1	11	$2\,\bar p^2\,p^2\,\sqrt{s}\,\frac{J}{2\,J+1}$	2	6	$-\alpha_-\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{(2\,J+1)(\bar\alpha_-\,J+2)}{(J+1)(2\,J+3)}$
2	11	$-\alpha_-\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{(2\,J+1)(\bar\alpha_-\,J+2)}{(J+1)(2\,J+3)}$						

k	n	$[a^{J+k}_{+n}]_{12}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{12}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{12}$ $0\,1 \to 1\,1$
−1	4	$2\sqrt{s}\,s^2\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	−1	5	$2\sqrt{s}\,s^2\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	0	6	$p^2\,\sqrt{s}\,s\,\frac{\sqrt{J}\,(\bar\alpha_-\,J-3\,\bar\delta+1)}{\sqrt{J+1}\,(2\,J+3)}$
0	7	$-2\,p^2\,\sqrt{s}\,s\,\frac{\sqrt{J}}{\sqrt{J+1}}$	0	8	$-2\,p^2\,\sqrt{s}\,s\,\frac{\sqrt{J}}{\sqrt{J+1}}$	0	11	$-p^2\,\sqrt{s}\,s\,\frac{\sqrt{J}\,(\bar\alpha_-\,J+2)}{\sqrt{J+1}\,(2\,J+3)}$
1	1	$2\,p^2\,\sqrt{s}\,\frac{\sqrt{J}}{\sqrt{J+1}}$	1	4	$2\,\bar p^2\,p^2\,\sqrt{s}\,\frac{\sqrt{J}\,J}{\sqrt{J+1}\,(2\,J+1)}$	1	5	$2\,\bar p^2\,p^2\,\sqrt{s}\,\frac{\sqrt{J}\,J}{\sqrt{J+1}\,(2\,J+1)}$
1	9	$-\bar\alpha_-\,p^4\,\sqrt{s}\,\frac{\sqrt{J}}{\sqrt{J+1}}$	2	6	$\bar p^2\,p^4\,\frac{1}{\sqrt{s}}\,\frac{\sqrt{J}\,(\bar\alpha_-\,J+2)}{\sqrt{J+1}\,(2\,J+3)}$	2	11	$\bar p^2\,p^4\,\frac{1}{\sqrt{s}}\,\frac{\sqrt{J}\,(\bar\alpha_-\,J+2)}{\sqrt{J+1}\,(2\,J+3)}$

k	n	$[a^{J+k}_{+n}]_{21}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{21}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{21}$ $0\,1 \to 1\,1$
−1	10	$\bar\alpha_+\,\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	−1	11	$-\bar\alpha_+\,\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	0	2	$-\alpha_-\,\sqrt{s}\,s\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
0	6	$-\bar\alpha_-\,\bar\alpha_+\,\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{4}\,\frac{\sqrt{J}}{\sqrt{J+1}\,(2\,J+3)}$	0	7	$\bar\alpha_+\,\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{4}\,\frac{\sqrt{J}}{\sqrt{J+1}}$	0	8	$-\bar\alpha_-\,\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{4}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
0	11	$\bar\alpha_-\,\bar\alpha_+\,\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{8}\,\frac{\sqrt{J}\,J\,(2\,J+1)}{\sqrt{J+1}\,(2\,J+3)}$	1	3	$-\alpha_-\,\bar p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$	1	4	$-\bar\alpha_+\,\alpha_-\,\bar p^2\,\sqrt{s}\,s\,\frac{1}{4}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
1	5	$\bar\alpha_-\,\alpha_-\,\bar p^2\,\sqrt{s}\,s\,\frac{1}{4}\,\frac{\sqrt{J}}{\sqrt{J+1}}$	1	9	$\bar\alpha_-\,\bar\alpha_+\,\alpha_-\,p^2\,\sqrt{s}\,s\,\frac{1}{8}\,\frac{\sqrt{J}\,J}{\sqrt{J+1}}$	1	10	$-\bar\alpha_+\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$
1	11	$\bar\alpha_+\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	2	6	$-\bar\alpha_-\,\bar\alpha_+\,\alpha_-\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{8}\,\frac{\sqrt{J}\,(2\,J+1)}{\sqrt{J+1}\,(2\,J+3)}$	2	11	$-\bar\alpha_-\,\bar\alpha_+\,\alpha_-\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{8}\,\frac{\sqrt{J}\,(2\,J+1)}{\sqrt{J+1}\,(2\,J+3)}$

k	n	$[a^{J+k}_{+n}]_{22}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{22}$ $0\,1 \to 1\,1$	k	n	$[a^{J+k}_{+n}]_{22}$ $0\,1 \to 1\,1$
−1	3	$\sqrt{s}\,s\,\frac{J+1}{2\,J+1}$	−1	4	$\bar\alpha_+\,\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{J+1}{2\,J+1}$	−1	5	$-\bar\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{J+1}{2\,J+1}$
0	2	$p^2\,\sqrt{s}$	0	6	$\bar\alpha_-\,\bar\alpha_+\,p^2\,\sqrt{s}\,s\,\frac{1}{4}\,\frac{J+3}{2\,J+3}$	0	7	$-\bar\alpha_+\,p^2\,\sqrt{s}\,s\,\frac{1}{2}$
0	8	$\bar\alpha_-\,p^2\,\sqrt{s}\,s\,\frac{1}{2}$	0	11	$-\bar\alpha_-\,\bar\alpha_+\,p^2\,\sqrt{s}\,s\,\frac{1}{4}\,\frac{J}{2\,J+3}$	1	3	$\bar p^2\,p^2\,\frac{1}{\sqrt{s}}\,\frac{J}{2\,J+1}$
1	4	$\bar\alpha_+\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{J}{2\,J+1}$	1	5	$-\bar\alpha_-\,\bar p^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{J}{2\,J+1}$	1	9	$-\bar\alpha_-\,\bar\alpha_+\,p^4\,\sqrt{s}\,\frac{1}{4}$
2	6	$\bar\alpha_-\,\bar\alpha_+\,\bar p^2\,p^4\,\frac{1}{4\sqrt{s}}\,\frac{J^2}{2\,J+3}$	2	11	$\bar\alpha_-\,\bar\alpha_+\,\bar p^2\,p^4\,\frac{1}{4\sqrt{s}}\,\frac{J}{2\,J+3}$			

TABLE III: Non-vanishing coefficients $a^{J+k}_{\pm n}$ in the expansion (41) for the reactions $0\,1 \to 0\,1$, $0\,1 \to 1\,1$ and $0\,0 \to 1\,1$.

$k\ n$	$\left[a^{J+k}_{+n}\right]_{31}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{31}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{31}$ $0\,1 \to 1\,1$
−1 2	$-\sqrt{s}\,s\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	−1 9	$-\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{4}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	−1 10	$-\bar{p}^2\,\sqrt{s}\,s\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$
−1 11	$\bar{p}^2\,\sqrt{s}\,s\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	0 6	$-\alpha_-\,\bar{p}^2\,\sqrt{s}\,s\,\frac{1}{4}\,-\,\frac{\sqrt{J}\,(-2\,\bar{\delta}+J+1)}{\sqrt{J+1}\,(2\,J+3)}$	0 7	$-\alpha_-\,\bar{p}^2\,\sqrt{s}\,s\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
0 8	$-\alpha_-\,\bar{p}^2\,\sqrt{s}\,s\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$	0 11	$\alpha_-\,\bar{p}^2\,\sqrt{s}\,s\,\frac{1}{4}\,\frac{\sqrt{J}\,(2\,\bar{\delta}\,J+\bar{\delta}-J-2)}{\sqrt{J+1}\,(2\,J+3)}$	1 1	$\alpha_-\,\bar{p}^2\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
1 2	$\bar{p}^2\,p^2\,\frac{1}{\sqrt{s}}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	1 4	$\alpha_-\,\bar{p}^4\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$	1 5	$\alpha_-\,\bar{p}^4\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J}}{\sqrt{J+1}}$
1 9	$\alpha_-\,\bar{p}^2\,p^2\,\sqrt{s}\,\frac{1}{4}\,\frac{\sqrt{J}\,(2\,\bar{\delta}\,J+\bar{\delta}-J)}{\sqrt{J+1}\,(2\,J+1)}$	1 10	$\bar{p}^4\,p^2\,\frac{1}{\sqrt{s}}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$	1 11	$-\bar{p}^4\,p^2\,\frac{1}{\sqrt{s}}\,\frac{\sqrt{J}\,\sqrt{J+1}}{2\,J+1}$
2 6	$-\alpha_-\,\bar{p}^4\,p^2\,\frac{1}{4\,\sqrt{s}}\,\frac{\sqrt{J}\,(2\,\bar{\delta}\,J+\bar{\delta}-J-2)}{\sqrt{J+1}\,(2\,J+3)}$	2 11	$-\alpha_-\,\bar{p}^4\,p^2\,\frac{1}{4\,\sqrt{s}}\,\frac{\sqrt{J}\,(2\,\bar{\delta}\,J+\bar{\delta}-J-2)}{\sqrt{J+1}\,(2\,J+3)}$		

$k\ n$	$\left[a^{J+k}_{+n}\right]_{32}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{32}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{32}$ $0\,1 \to 1\,1$
−2 6	$-\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{J^2-1}{4\,J^2-1}$	−2 11	$-\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{J^2-1}{4\,J^2-1}$	−1 1	$-\sqrt{s}\,s\,\frac{J+1}{2\,J+1}$
−1 4	$-\bar{p}^2\,\sqrt{s}\,s\,\frac{J+1}{2\,J+1}$	−1 5	$-\bar{p}^2\,\sqrt{s}\,s\,\frac{J+1}{2\,J+1}$	−1 9	$p^2\,\sqrt{s}\,s\,\frac{1}{2}\,\frac{J+1}{2\,J+1}$
0 6	$\bar{p}^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{\bar{\delta}(J+3)(2\,J-1)-2\,J\,(J+1)}{4\,J\,(J+1)-3}$	0 7	$\bar{p}^2\,p^2\,\sqrt{s}$	0 8	$\bar{p}^2\,p^2\,\sqrt{s}$
0 11	$-\bar{p}^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{2\,(\bar{\delta}-1)J^2-(\bar{\delta}+2)J+3}{4\,J\,(J+1)-3}$	1 1	$-\bar{p}^2\,p^2\,\frac{1}{\sqrt{s}}\,\frac{J}{2\,J+1}$	1 4	$-\bar{p}^4\,p^2\,\frac{1}{\sqrt{s}}\,\frac{J}{2\,J+1}$
1 5	$-\bar{p}^4\,p^2\,\frac{1}{\sqrt{s}}\,\frac{J}{2\,J+1}$	1 9	$-\bar{p}^2\,p^4\,\frac{1}{2\,\sqrt{s}}\,\frac{2\,\bar{\delta}\,J+\bar{\delta}-J}{2\,J+1}$	2 6	$\bar{p}^4\,p^4\,\frac{1}{2\,\sqrt{s}\,s}\,\frac{J\,(2\,\bar{\delta}\,J+\bar{\delta}-J-2)}{4\,J\,(J+2)+3}$
2 11	$\bar{p}^4\,p^4\,\frac{1}{2\,\sqrt{s}\,s}\,\frac{J\,(2\,\bar{\delta}\,J+\bar{\delta}-J-2)}{4\,J\,(J+2)+3}$				

$k\ n$	$\left[a^{J+k}_{+n}\right]_{41}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{41}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{41}$ $0\,1 \to 1\,1$
−1 9	$\alpha_-\,\sqrt{s}\,s^2\,\frac{1}{4}\,\frac{\sqrt{J}\,\sqrt{J\,(J+1)-2}}{\sqrt{J+1}\,(2\,J+1)}$	0 6	$-\alpha_-\,\bar{p}^2\,\sqrt{s}\,s\,\frac{1}{4}\,\frac{\sqrt{J}\,\sqrt{J\,(J+1)-2}}{\sqrt{J+1}\,(2\,J+3)}$	0 11	$-\alpha_-\,\bar{p}^2\,\sqrt{s}\,s\,\frac{1}{4}\,\frac{\sqrt{J}\,\sqrt{J\,(J+1)-2}}{\sqrt{J+1}\,(2\,J+3)}$
1 9	$-\alpha_-\,\bar{p}^2\,p^2\,\sqrt{s}\,\frac{1}{4}\,\frac{\sqrt{J}\,\sqrt{J\,(J+1)-2}}{\sqrt{J+1}\,(2\,J+1)}$	2 6	$\alpha_-\,\bar{p}^4\,p^2\,\frac{1}{4\,\sqrt{s}}\,\frac{\sqrt{J}\,\sqrt{J\,(J+1)-2}}{\sqrt{J+1}\,(2\,J+3)}$	2 11	$\alpha_-\,\bar{p}^4\,p^2\,\frac{1}{4\,\sqrt{s}}\,\frac{\sqrt{J}\,\sqrt{J\,(J+1)-2}}{\sqrt{J+1}\,(2\,J+3)}$

$k\ n$	$\left[a^{J+k}_{+n}\right]_{42}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{42}$ $0\,1 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{+n}\right]_{42}$ $0\,1 \to 1\,1$
−2 6	$\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{(J+1)\sqrt{J\,(J+1)-2}}{(2\,J-1)(2\,J+1)}$	−2 11	$\sqrt{s}\,s^2\,\frac{1}{2}\,\frac{(J+1)\sqrt{J\,(J+1)-2}}{(2\,J-1)(2\,J+1)}$	−1 9	$-p^2\,\sqrt{s}\,s\,\frac{1}{2}\,\frac{\sqrt{J\,(J+1)-2}}{2\,J+1}$
0 6	$-3\,\bar{p}^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J\,(J+1)-2}}{(2\,J-1)(2\,J+3)}$	0 11	$-3\,\bar{p}^2\,p^2\,\sqrt{s}\,\frac{1}{2}\,\frac{\sqrt{J\,(J+1)-2}}{(2\,J-1)(2\,J+3)}$	1 9	$\bar{p}^2\,p^4\,\frac{1}{2\sqrt{s}}\,\frac{\sqrt{J\,(J+1)-2}}{2\,J+1}$
2 6	$-\bar{p}^4\,p^4\,\frac{1}{2\,\sqrt{s}\,s}\,\frac{J\sqrt{J\,(J+1)-2}}{(2\,J+1)(2\,J+3)}$	2 11	$-\bar{p}^4\,p^4\,\frac{1}{2\,\sqrt{s}\,s}\,\frac{J\sqrt{J\,(J+1)-2}}{(2\,J+1)(2\,J+3)}$		

$k\ n$	$\left[a^{J+k}_{-n}\right]_{11}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{11}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{11}$ $0\,0 \to 1\,1$
0 2	$-2\,s^2\,\frac{\sqrt{J+1}}{\sqrt{J}}$	1 3	$-\bar{\alpha}_-\,p^2\,s\,\frac{\sqrt{J+1}}{\sqrt{J}}$	1 4	$\bar{\alpha}_+\,p^2\,s\,\frac{\sqrt{J+1}}{\sqrt{J}}$
2 5	$\bar{\alpha}_-\,\bar{\alpha}_+\,p^4\,\frac{1}{2}\,\frac{\sqrt{J+1}}{\sqrt{J}}$				

$k\ n$	$\left[a^{J+k}_{-n}\right]_{21}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{21}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{21}$ $0\,0 \to 1\,1$
0 1	$-2\,s\,\frac{\sqrt{J+1}}{\sqrt{J}}$	0 5	$2\,p^2\,s\,\frac{\sqrt{J+1}}{\sqrt{J}\,(2\,J+3)}$	2 5	$-2\,\bar{p}^2\,p^4\,\frac{1}{s}\,\frac{\sqrt{J+1}}{\sqrt{J}\,(2\,J+3)}$

$k\ n$	$\left[a^{J+k}_{-n}\right]_{31}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{31}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{31}$ $0\,0 \to 1\,1$
−1 3	$-s^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+3)}{4\,J\,(J+2)+3}$	−1 4	$-s^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+3)}{4\,J\,(J+2)+3}$	0 5	$\bar{M}_-\,\bar{M}_+\,p^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+1)}{4\,J\,(J+2)+3}$
1 3	$\bar{p}^2\,p^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+3)}{4\,J\,(J+2)+3}$	1 4	$\bar{p}^2\,p^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+3)}{4\,J\,(J+2)+3}$	2 5	$-\bar{M}_-\,\bar{M}_+\,\bar{p}^2\,p^4\,\frac{1}{s^2}\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+1)}{4\,J\,(J+2)+3}$

$k\ n$	$\left[a^{J+k}_{-n}\right]_{41}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{41}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{41}$ $0\,0 \to 1\,1$
−1 4	$2\,s^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+3)}{4\,J\,(J+2)+3}$	0 5	$\bar{\alpha}_-\,p^2\,s\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+1)}{4\,J\,(J+2)+3}$	1 4	$-2\,\bar{p}^2\,p^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+3)}{4\,J\,(J+2)+3}$
2 5	$-\bar{\alpha}_-\,\bar{p}^2\,p^4\,\frac{1}{s}\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+1)}{4\,J\,(J+2)+3}$				

$k\ n$	$\left[a^{J+k}_{-n}\right]_{51}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{51}$ $0\,0 \to 1\,1$	$k\ n$	$\left[a^{J+k}_{-n}\right]_{51}$ $0\,0 \to 1\,1$
−2 5	$-s^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+3)}{4\,J\,(J+1)-3}$	0 5	$2\,\bar{p}^2\,p^2\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J+1)}{4\,J\,(J+1)-3}$	2 5	$-\bar{p}^4\,p^4\,\frac{1}{s^2}\,\frac{\sqrt{J}\,\sqrt{J+1}\,(2\,J-1)}{4\,J\,(J+1)-3}$

TABLE IV: Continuation of Tab. III. We use the notation of (35).

IV. CONCLUSIONS

We have constructed kinematically unconstrained and frame independent partial-wave amplitudes for two-body reactions involving $J^P = 0^-$ and $J^P = 1^-$ particles. Such amplitudes satisfy partial-wave dispersion relations and are well suited for data analysis.

To eliminate kinematical constraints in the helicity-partial wave amplitudes, we have derived non-unitary transformations from the conventional helicity states to covariant states. As an intermediate result we have identified complete bases of invariant functions, which are expected to satisfy a Mandelstam's dispersion integral representation and characterize the scattering and reaction amplitudes unambiguously. We have also derived explicit expressions for the covariant partial-wave scattering amplitudes in terms of integrals over the invariant functions. In addition we have constructed a convenient projection algebra that streamlines the derivation of the invariant functions by means of computer algebra codes significantly.

In conclusion, the present work offers a convenient framework for analyzing boson-boson scattering in a covariant coupled-channel approach that considers the constraints set by micro-causality and coupled-channel unitarity.

Acknowledgments

This work has been partially supported by FEDER and FCT (Portugal) under projects PTDC/FIS/113292/2009, CERN/FP/109316/2009, and CERN/FP/116366/2010. M.F.M.L. thanks the Centro de Física Computacional at the University of Coimbra for its kind hospitalilty.

APPENDIX A

We provide projection tensors for the $1\,1 \to 1\,1$ reaction. They streamline the computation of the 41 invariant amplitudes $F_n(s,t)$ introduced in (25) by means of algebraic computer codes. We find

$$
\begin{aligned}
& P^{(n)}_{\alpha\beta,\tau\sigma}\, g^{\alpha\bar\mu}\, g^{\beta\bar\nu}\, g^{\tau\mu}\, g^{\sigma\nu}\, T^{(m)}_{\bar\mu\bar\nu,\mu\nu} = \delta_{nm}\,,\\
& P^{(n)}_{\bar\mu\bar\nu,\mu\nu}\, \bar p_1^{\bar\mu} = 0\,, \qquad P^{(n)}_{\bar\mu\bar\nu,\mu\nu}\, \bar p_2^{\bar\nu} = 0\,,\\
& P^{(n)}_{\bar\mu\bar\nu,\mu\nu}\, p_1^{\mu} = 0\,, \qquad P^{(n)}_{\bar\mu\bar\nu,\mu\nu}\, p_2^{\nu} = 0\,.
\end{aligned}
$$

$$
\begin{aligned}
& P^{(n)}_{\bar\mu\bar\nu,\mu\nu} = \sum_{k=1}^{41} c_k^{(n)}\, Q^{(k)}_{\bar\mu\bar\nu,\nu\mu}\,, \qquad\qquad (A1)\\
& Q_n^{\bar\mu\bar\nu,\mu\nu} = \bar L^{\bar\mu\bar\nu}_{\lceil n/5\rceil}\, L^{\mu\nu}_{n+5-5\,\lceil n/5\rceil} \quad \text{for} \quad n \le 25\,,\\
& Q_{26}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\mu}\, w_{\lfloor}{}^{\bar\nu}\, \bar w_{\lfloor}{}^{\nu}\,, \qquad Q_{27}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\mu}\, w_{\lfloor}{}^{\bar\nu}\, \bar r_{\perp\!\!\!\perp}{}^{\nu}\,,\\
& Q_{28}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\mu}\, r_{\perp\!\!\!\perp}{}^{\bar\nu}\, \bar w_{\lfloor}{}^{\nu}\,, \qquad Q_{29}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\mu}\, r_{\perp\!\!\!\perp}{}^{\bar\nu}\, \bar r_{\perp\!\!\!\perp}{}^{\nu}\,,\\
& Q_{30}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\nu}\, w_{\rfloor}{}^{\bar\mu}\, \bar w_{\rfloor}{}^{\mu}\,, \qquad Q_{31}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\nu}\, w_{\rfloor}{}^{\bar\mu}\, \bar r_{\perp\!\!\!\perp}{}^{\mu}\,,\\
& Q_{32}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\nu}\, r_{\perp\!\!\!\perp}{}^{\bar\mu}\, \bar w_{\rfloor}{}^{\mu}\,, \qquad Q_{33}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\nu}\, r_{\perp\!\!\!\perp}{}^{\bar\mu}\, \bar r_{\perp\!\!\!\perp}{}^{\mu}\,,\\
& Q_{34}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\mu}\, w_{\rfloor}{}^{\bar\mu}\, \bar w_{\lfloor}{}^{\nu}\,, \qquad Q_{35}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\mu}\, w_{\rfloor}{}^{\bar\mu}\, \bar r_{\perp\!\!\!\perp}{}^{\nu}\,,\\
& Q_{36}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\mu}\, r_{\perp\!\!\!\perp}{}^{\bar\mu}\, \bar w_{\lfloor}{}^{\nu}\,, \qquad Q_{37}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\nu}\, \hat v^{\mu}\, r_{\perp\!\!\!\perp}{}^{\bar\mu}\, \bar r_{\perp\!\!\!\perp}{}^{\nu}\,,\\
& Q_{38}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\nu}\, w_{\lfloor}{}^{\bar\nu}\, \bar w_{\rfloor}{}^{\mu}\,, \qquad Q_{39}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\nu}\, w_{\lfloor}{}^{\bar\nu}\, \bar r_{\perp\!\!\!\perp}{}^{\mu}\,,\\
& Q_{40}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\nu}\, r_{\perp\!\!\!\perp}{}^{\bar\nu}\, \bar w_{\rfloor}{}^{\mu}\,, \qquad Q_{41}^{\bar\mu\bar\nu,\mu\nu} = \hat v^{\bar\mu}\, \hat v^{\nu}\, r_{\perp\!\!\!\perp}{}^{\bar\nu}\, \bar r_{\perp\!\!\!\perp}{}^{\mu}\,,
\end{aligned}
$$

with $\hat v_\mu = v_\mu / v^2$. The ceiling function,

$$
\lceil x \rceil - 1 \le x \le \lceil x \rceil\,, \qquad (A2)
$$

maps a real number x onto an integer number $\lceil x \rceil$ as defined by (A2). The tensors $L_n^{\mu\nu}$ and $\bar L_n^{\bar\mu\bar\nu}$ used in (A1) are

$$
\begin{aligned}
L_1^{\mu\nu} &= v^\mu\, v^\nu / v^2\,,\\
L_2^{\mu\nu} &= \bar w_{\rfloor}{}^{\mu}\, \bar w_{\lfloor}{}^{\nu}\\
&\quad - \big[(\bar w_{\rfloor} \cdot \bar w_{\lfloor}) - (\bar w_{\rfloor} \cdot w)\,(w \cdot \bar w_{\lfloor})/w^2 \big]\, L_1^{\mu\nu}\,,\\
L_3^{\mu\nu} &= \bar w_{\rfloor}{}^{\mu}\, \bar r_{\perp\!\!\!\perp}{}^{\nu} - (\bar w_{\rfloor} \cdot \bar r_{\perp\!\!\!\perp})\, L_1^{\mu\nu}\,,\\
L_4^{\mu\nu} &= \bar r_{\perp\!\!\!\perp}{}^{\mu}\, \bar w_{\lfloor}{}^{\nu} - (\bar r_{\perp\!\!\!\perp} \cdot \bar w_{\lfloor})\, L_1^{\mu\nu}\,,\\
L_5^{\mu\nu} &= \bar r_{\perp\!\!\!\perp}{}^{\mu}\, \bar r_{\perp\!\!\!\perp}{}^{\nu} - (\bar r_{\perp\!\!\!\perp} \cdot \bar r_{\perp\!\!\!\perp})\, L_1^{\mu\nu}\,,\\
\bar L_1^{\bar\mu\bar\nu} &= v^{\bar\mu}\, v^{\bar\nu} / v^2\,,\\
\bar L_2^{\bar\mu\bar\nu} &= w_{\rfloor}{}^{\bar\mu}\, w_{\lfloor}{}^{\bar\nu}\\
&\quad - \big[(w_{\rfloor} \cdot w_{\lfloor}) - (w_{\rfloor} \cdot w)\,(w \cdot w_{\lfloor})/w^2 \big]\, \bar L_1^{\bar\mu\bar\nu}\,,\\
\bar L_3^{\bar\mu\bar\nu} &= w_{\rfloor}{}^{\bar\mu}\, r_{\perp\!\!\!\perp}{}^{\bar\nu} - (w_{\rfloor} \cdot r_{\perp\!\!\!\perp})\, \bar L_1^{\bar\mu\bar\nu}\,,\\
\bar L_4^{\bar\mu\bar\nu} &= r_{\perp\!\!\!\perp}{}^{\bar\mu}\, w_{\lfloor}{}^{\bar\nu} - (r_{\perp\!\!\!\perp} \cdot w_{\lfloor})\, \bar L_1^{\bar\mu\bar\nu}\,,\\
\bar L_5^{\bar\mu\bar\nu} &= r_{\perp\!\!\!\perp}{}^{\bar\mu}\, r_{\perp\!\!\!\perp}{}^{\bar\nu} - (r_{\perp\!\!\!\perp} \cdot r_{\perp\!\!\!\perp})\, \bar L_1^{\bar\mu\bar\nu}\,. \qquad (A3)
\end{aligned}
$$

The derivation of the coefficient matrix $c_k^{(n)}$ appears prohibitively cumbersome at first. A 41×41 matrix needs to be inverted. The merit of the algebra developed in this work are concise and manageable expressions for the coefficients $c_k^{(n)}$. They are presented in terms of the building blocks

$$
\begin{aligned}
& \delta = \frac{m_1^2 - m_2^2}{s}\,, \qquad \alpha_\pm = 1 \pm \delta\,, \qquad (A4)\\
& \bar\delta = \frac{\bar m_1^2 - \bar m_2^2}{s}\,, \qquad \bar\alpha_\pm = 1 \pm \bar\delta\,,\\
& \beta_\pm = \pm 3\,\bar\delta\,\delta + \bar\delta - \delta\,, \qquad \gamma_\pm = \pm 3\,\bar\delta\,\delta + \bar\delta + \delta\,.
\end{aligned}
$$

In Tab. V we specify k and n for all coefficients with $c_k^{(n)} = 1$. In Tab. VI we detail all remaining non-vanishing elements in the expansion (A1).

k	1	2	3	4	5	6	7	8	9	10
n	3	20	22	21	23	24	28	37	35	30
k	11	12	15	16	17	20	21	22	23	24
n	26	32	34	25	31	33	27	29	38	36

TABLE V: The non-vanishing coefficients $c_k^{(n)} = 1$.

k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$
13	1	$-v^2/s$	13	2	v^2/s	13	4	$\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)$
13	5	$\frac{1}{2}\,\alpha_+\,\bar r^2$	13	6	$\frac{1}{2}\,\bar\alpha_+\,r^2$	13	7	$(\bar r\cdot r)$
13	8	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)$	13	9	$-\frac{1}{2}\,\alpha_-\,\bar r^2$	13	10	$-\frac{1}{2}\,\bar\alpha_-\,r^2$
13	11	$(\bar r\cdot r)$	13	12	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)$	13	13	$\frac{1}{2}\,\alpha_-\,\bar r^2$
13	14	$-\frac{1}{2}\,\bar\alpha_+\,r^2$	13	15	$-(\bar r\cdot r)$	13	16	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)$
13	17	$-\frac{1}{2}\,\alpha_+\,\bar r^2$	13	18	$\frac{1}{2}\,\bar\alpha_-\,r^2$	13	19	$-(\bar r\cdot r)$
13	39	$-\alpha_-$	13	40	$-\bar\alpha_-$	13	41	$1-\bar\delta\,\delta$
14	1	$-v^2/s$	14	2	v^2/s	14	4	$\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)$
14	5	$\frac{1}{2}\,\alpha_+\,\bar r^2$	14	6	$\frac{1}{2}\,\bar\alpha_+\,r^2$	14	7	$(\bar r\cdot r)$
14	8	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)$	14	9	$-\frac{1}{2}\,\alpha_-\,\bar r^2$	14	10	$-\frac{1}{2}\,\bar\alpha_-\,r^2$
14	11	$(\bar r\cdot r)$	14	12	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)$	14	13	$\frac{1}{2}\,\alpha_-\,\bar r^2$
14	14	$\frac{1}{2}\,-\,\bar\alpha_+\,r^2$	14	15	$-(\bar r\cdot r)$	14	16	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)$
14	17	$\frac{1}{2}\,-\,\alpha_+\,\bar r^2$	14	18	$\frac{1}{2}\,\bar\alpha_-\,r^2$	14	19	$-(\bar r\cdot r)$
14	39	α_+	14	40	$-\bar\alpha_-$	14	41	$-\bar\delta\,\delta-1$
18	1	$-v^2/s$	18	2	v^2/s	18	4	$\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)$
18	5	$\frac{1}{2}\,\alpha_+\,\bar r^2$	18	6	$\frac{1}{2}\,\bar\alpha_+\,r^2$	18	7	$(\bar r\cdot r)$
18	8	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)$	18	9	$-\frac{1}{2}\,\alpha_-\,\bar r^2$	18	10	$-\frac{1}{2}\,\bar\alpha_-\,r^2$
18	11	$(\bar r\cdot r)$	18	12	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)$	18	13	$\frac{1}{2}\,\alpha_-\,\bar r^2$
18	14	$-\frac{1}{2}\,\bar\alpha_+\,r^2$	18	15	$-(\bar r\cdot r)$	18	16	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)$
18	17	$-\frac{1}{2}\,\alpha_+\,\bar r^2$	18	18	$\frac{1}{2}\,\bar\alpha_-\,r^2$	18	19	$-(\bar r\cdot r)$
18	39	$-\alpha_-$	18	40	$\bar\alpha_+$	18	41	$-\bar\delta\,\delta-1$
19	1	$-v^2/s$	19	2	v^2/s	19	4	$\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)$
19	5	$\frac{1}{2}\,\alpha_+\,\bar r^2$	19	6	$\frac{1}{2}\,\bar\alpha_+\,r^2$	19	7	$(\bar r\cdot r)$
19	8	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)$	19	9	$-\frac{1}{2}\,\alpha_-\,\bar r^2$	19	10	$-\frac{1}{2}\,\bar\alpha_-\,r^2$
19	11	$(\bar r\cdot r)$	19	12	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)$	19	13	$\frac{1}{2}\,\alpha_-\,\bar r^2$
19	14	$-\frac{1}{2}\,\bar\alpha_+\,r^2$	19	15	$-(\bar r\cdot r)$	19	16	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)$
19	17	$-\frac{1}{2}\,\alpha_+\,\bar r^2$	19	18	$\frac{1}{2}\,\bar\alpha_-\,r^2$	19	19	$-(\bar r\cdot r)$
19	39	α_+	19	40	$\bar\alpha_+$	19	41	$1-\bar\delta\,\delta$
25	1	$-\frac{1}{2}\,(\bar\delta\,\delta+1)\,v^2/s$	25	2	$\frac{1}{2}\,(\bar\delta\,\delta-1)\,v^2/s$	25	3	v^2/s
25	4	$\frac{1}{8}\,\bar\alpha_+\,\alpha_+\,(\bar\delta\,\delta+1)\,(\bar r\cdot r)$	25	5	$\frac{1}{4}\,\alpha_+\,(\bar\delta\,\delta+1)\,\bar r^2$	25	6	$\frac{1}{4}\,\bar\alpha_+\,(\bar\delta\,\delta+1)\,r^2$
25	7	$\frac{1}{2}\,(\bar\delta\,\delta+1)\,(\bar r\cdot r)$	25	8	$\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,(\bar\delta\,\delta+1)\,(\bar r\cdot r)$	25	9	$-\frac{1}{4}\,\alpha_-\,(\bar\delta\,\delta+1)\,\bar r^2$
25	10	$-\frac{1}{4}\,\bar\alpha_-\,(\bar\delta\,\delta+1)\,r^2$	25	11	$\frac{1}{2}\,(\bar\delta\,\delta+1)\,(\bar r\cdot r)$	25	12	$\frac{1}{8}\,\alpha_-\,\bar\alpha_+\,(\bar\delta\,\delta-1)\,(\bar r\cdot r)$
25	13	$\frac{1}{4}\,\alpha_-\,(\bar\delta\,\delta-1)\,\bar r^2$	25	14	$-\frac{1}{4}\,\bar\alpha_+\,(\bar\delta\,\delta-1)\,r^2$	25	15	$-\frac{1}{2}\,(\bar\delta\,\delta-1)\,(\bar r\cdot r)$
25	16	$\frac{1}{8}\,\bar\alpha_-\,\alpha_+\,(\bar\delta\,\delta-1)\,(\bar r\cdot r)$	25	17	$-\frac{1}{4}\,\alpha_+\,(\bar\delta\,\delta-1)\,\bar r^2$	25	18	$\frac{1}{4}\,\bar\alpha_-\,(\bar\delta\,\delta-1)\,r^2$
25	19	$-\frac{1}{2}\,(\bar\delta\,\delta-1)\,(\bar r\cdot r)$	25	20	$-\frac{1}{4}\,\alpha_-\,\alpha_+\,\bar r^2$	25	21	$-\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)$
25	22	$\frac{1}{2}\,\alpha_+\,(\bar r\cdot r)$	25	23	r^2	25	24	$-\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,r^2$
25	25	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)$	25	26	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)$	25	27	$\bar r^2$
25	28	$-\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,\alpha_+$	25	39	$-\frac{1}{2}\,\alpha_-\,\alpha_+\,\bar\delta$	25	40	$-\frac{1}{2}\,\bar\alpha_-\,\bar\alpha_+\,\delta$
25	41	$-\frac{1}{2}\,(\bar\delta^2\,\delta^2-1)$	26	1	$(\bar r\cdot r)\,v^2$	26	2	$-(\bar r\cdot r)\,v^2$
26	4	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)^2\,s$	26	5	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	26	6	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$
26	7	$-(\bar r\cdot r)^2\,s$	26	8	$\frac{1}{4}\,(4\,v^2-\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)^2\,s)$	26	9	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$
26	10	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	26	11	$(-(\bar r\cdot r)^2)\,s$	26	12	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)^2\,s$
26	13	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	26	14	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	26	15	$(\bar r\cdot r)^2\,s$
26	16	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)^2\,s$	26	17	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	26	18	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$
26	19	$(\bar r\cdot r)^2\,s$	26	28	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,s$	26	31	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,s$
26	35	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,s$	26	39	$-\alpha_+\,(\bar r\cdot r)\,s$	26	40	$-\bar\alpha_+\,(\bar r\cdot r)\,s$
26	41	$(\bar\delta\,\delta-1)\,(\bar r\cdot r)\,s$	27	1	$\frac{1}{2}\,\alpha_+\,\bar r^2\,v^2)$	27	2	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,v^2$
27	4	$-\frac{1}{8}\,\bar\alpha_+\,\alpha_+^2\,\bar r^2\,(\bar r\cdot r)\,s$	27	5	$-\frac{1}{4}\,\alpha_+^2\,(\bar r^2)^2\,s$	27	6	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s$
27	7	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	27	8	$-\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	27	9	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$
27	10	$\frac{1}{4}\,(\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s+4\,v^2)$	27	11	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	27	12	$-\frac{1}{8}\,\alpha_-\,\bar\alpha_+\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
27	13	$-\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$	27	14	$\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s$	27	15	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
27	16	$-\frac{1}{8}\,\bar\alpha_-\,\alpha_+^2\,\bar r^2\,(\bar r\cdot r)\,s$	27	17	$\frac{1}{4}\,\alpha_+^2\,(\bar r^2)^2\,s$	27	18	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s$
27	19	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	27	30	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,s$	27	33	$-(\bar r\cdot r)\,s$
27	37	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,s$	27	39	$\frac{1}{2}\,\alpha_-\,\alpha_+\,\bar r^2\,s$	27	40	$-\frac{1}{2}\,\bar\alpha_+\,\alpha_+\,\bar r^2\,s$
27	41	$\frac{1}{2}\,\alpha_+\,(\bar\delta\,\delta+1)\,\bar r^2\,s$	28	1	$\frac{1}{2}\,\bar\alpha_+\,r^2\,v^2$	28	2	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,v^2$
28	4	$-\frac{1}{8}\,\bar\alpha_+^2\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	28	5	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s$	28	6	$-\frac{1}{4}\,\bar\alpha_+^2\,(r^2)^2\,s$
28	7	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	28	8	$-\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	28	9	$\frac{1}{4}\,(\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s+4\,v^2)$
28	10	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$	28	11	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	28	12	$-\frac{1}{8}\,\alpha_-\,\bar\alpha_+^2\,(\bar r\cdot r)\,r^2\,s$
28	13	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s$	28	14	$\frac{1}{4}\,\bar\alpha_+^2\,(r^2)^2\,s$	28	15	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$

TABLE VI: The non-vanishing coefficients $c_k^{(n)}$ in the expansion (A1).

k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$
28	16	$-\frac{1}{8}\,\bar\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	28	17	$\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s$	28	18	$-\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$
28	19	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	28	29	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,s$	28	32	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,s$
28	36	$-(\bar r\cdot r)\,s$	28	39	$-\frac{1}{2}\,\bar\alpha_+\,\alpha_+\,r^2\,s$	28	40	$\frac{1}{2}\,\bar\alpha_-\,\bar\alpha_+\,r^2\,s$
28	41	$\frac{1}{2}\,\bar\alpha_+\,(\bar\delta\,\delta+1)\,r^2\,s$	29	1	$\frac{1}{4}\,(\gamma_++3)\,(\bar r\cdot r)\,v^2$	29	2	$-\frac{1}{4}\,(\gamma_+-1)\,(\bar r\cdot r)\,v^2$
29	3	$-(\bar r\cdot r)\,v^2$	29	4	$-\frac{1}{16}\,\bar\alpha_+\,\alpha_+\,(\gamma_++3)\,(\bar r\cdot r)^2\,s$	29	5	$-\frac{1}{8}\,\alpha_+\,(\gamma_++3)\,\bar r^2\,(\bar r\cdot r)\,s$
29	6	$-\frac{1}{8}\,\bar\alpha_+\,(\gamma_++3)\,(\bar r\cdot r)\,r^2\,s$	29	7	$-\frac{1}{4}\,(\gamma_++3)\,(\bar r\cdot r)^2\,s$	29	8	$-\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,(\gamma_++3)\,(\bar r\cdot r)^2\,s$
29	9	$\frac{1}{8}\,\alpha_-\,(\gamma_++3)\,\bar r^2\,(\bar r\cdot r)\,s$	29	10	$\frac{1}{8}\,\bar\alpha_-\,(\gamma_++3)\,(\bar r\cdot r)\,r^2\,s$	29	11	$-\frac{1}{4}\,((\gamma_+-1)\,(\bar r\cdot r)^2+4\,\bar r^2\,r^2)\,s$
29	12	$-\frac{1}{16}\,\alpha_-\,\bar\alpha_+\,(\gamma_+-1)\,(\bar r\cdot r)^2\,s$	29	13	$-\frac{1}{8}\,\alpha_-\,(\gamma_+-1)\,\bar r^2\,(\bar r\cdot r)\,s$	29	14	$\frac{1}{8}\,\bar\alpha_+\,(\gamma_+-1)\,(\bar r\cdot r)\,r^2\,s$
29	15	$\frac{1}{4}\,(\gamma_+-1)\,(\bar r\cdot r)^2\,s$	29	16	$-\frac{1}{16}\,\bar\alpha_-\,\alpha_+\,(\gamma_+-1)\,(\bar r\cdot r)^2\,s$	29	17	$\frac{1}{8}\,\alpha_+\,(\gamma_+-1)\,\bar r^2\,(\bar r\cdot r)\,s$
29	18	$-\frac{1}{8}\,\bar\alpha_-\,(\gamma_+-1)\,(\bar r\cdot r)\,r^2\,s$	29	19	$\frac{1}{4}\,(\gamma_+-1)\,(\bar r\cdot r)^2\,s$	29	20	$\frac{1}{4}\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
29	21	$\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)^2\,s$	29	22	$-\frac{1}{2}\,\alpha_+\,(\bar r\cdot r)^2\,s$	29	23	$-(\bar r\cdot r)\,r^2\,s$
29	24	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	29	25	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)^2\,s$	29	26	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)^2\,s$
29	27	$-\bar r^2\,(\bar r\cdot r)\,s$	29	28	$\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,s$	29	34	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,s$
29	38	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,s$	29	39	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(3\,\bar\delta+1)\,(\bar r\cdot r)\,s$	29	40	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(3\,\delta+1)\,(\bar r\cdot r)\,s$
29	41	$\frac{1}{4}\,(\bar\delta\,\delta-1)\,(\gamma_++3)\,(\bar r\cdot r)\,s$	30	1	$(\bar r\cdot r)\,v^2$	30	2	$-(\bar r\cdot r)\,v^2$
30	4	$\frac{1}{4}\,(4\,v^2-\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)^2\,s)$	30	5	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	30	6	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$
30	7	$-(\bar r\cdot r)^2\,s$	30	8	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)^2\,s$	30	9	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$
30	10	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	30	11	$-(\bar r\cdot r)^2\,s$	30	12	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)^2\,s$
30	13	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	30	14	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	30	15	$(\bar r\cdot r)^2\,s$
30	16	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)^2\,s$	30	17	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	30	18	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$
30	19	$(\bar r\cdot r)^2\,s$	30	28	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)\,s$	30	32	$\frac{1}{2}\,\alpha_-\,\bar r^2\,s$
30	37	$\frac{1}{2}\,\bar\alpha_-\,r^2\,s$	30	39	$\alpha_-\,(\bar r\cdot r)\,s$	30	40	$\bar\alpha_-\,(\bar r\cdot r)\,s$
30	41	$(\bar\delta\,\delta-1)\,(\bar r\cdot r)\,s$	31	1	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,v^2$	31	2	$\frac{1}{2}\,\alpha_-\,\bar r^2\,v^2$
31	4	$\frac{1}{8}\,\alpha_-\,\bar\alpha_+\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	31	5	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$	31	6	$\frac{1}{4}\,(\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s+4\,v^2)$
31	7	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	31	8	$\frac{1}{8}\,\bar\alpha_-\,\alpha_-^2\,\bar r^2\,(\bar r\cdot r)\,s$	31	9	$-\frac{1}{4}\,\alpha_-^2\,(\bar r^2)^2\,s$
31	10	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s$	31	11	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	31	12	$\frac{1}{8}\,\alpha_-^2\,\bar\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
31	13	$\frac{1}{4}\,\alpha_-^2\,(\bar r^2)^2\,s$	31	14	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s$	31	15	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$
31	16	$\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	31	17	$-\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$	31	18	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s$
31	19	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	31	30	$(\bar\alpha_-\,r^2\,s/2$	31	34	$-(\bar r\cdot r)\,s$
31	35	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)\,s$	31	39	$\frac{1}{2}\,\alpha_-\,\alpha_+\,\bar r^2\,s$	31	40	$-\frac{1}{2}\,\bar\alpha_-\,\alpha_-\,\bar r^2\,s$
31	41	$-\frac{1}{2}\,\alpha_-\,(\bar\delta\,\delta+1)\,\bar r^2\,s$	32	1	$-\frac{1}{2}\,\bar\alpha_-\,r^2\,v^2$	32	2	$(\bar\alpha_-\,r^2\,v^2/2$
32	4	$\frac{1}{8}\,\bar\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	32	5	$\frac{1}{4}\,(\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s+4\,v^2)$	32	6	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$
32	7	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	32	8	$\frac{1}{8}\,\bar\alpha_-^2\,\alpha_-\,(\bar r\cdot r)\,r^2\,s$	32	9	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s$
32	10	$-\frac{1}{4}\,\bar\alpha_-^2\,(r^2)^2\,s$	32	11	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	32	12	$\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$
32	13	$\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s$	32	14	$-\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$	32	15	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$
32	16	$\frac{1}{8}\,\bar\alpha_-^2\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	32	17	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s$	32	18	$\frac{1}{4}\,\bar\alpha_-^2\,(r^2)^2\,s$
32	19	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	32	29	$(\alpha_-\,\bar r^2\,s/2$	32	31	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)\,s$
32	38	$-(\bar r\cdot r)\,s$	32	39	$-\frac{1}{2}\,\bar\alpha_-\,\alpha_-\,r^2\,s$	32	40	$\frac{1}{2}\,\bar\alpha_-\,\bar\alpha_+\,r^2\,s$
32	41	$-\frac{1}{2}\,\bar\alpha_-\,(\bar\delta\,\delta+1)\,r^2\,s$	33	1	$-\frac{1}{4}\,(\gamma_--3)\,(\bar r\cdot r)\,v^2$	33	2	$\frac{1}{4}\,(\gamma_-+1)\,(\bar r\cdot r)\,v^2$
33	3	$-(\bar r\cdot r)\,v^2$	33	4	$\frac{1}{16}\,\bar\alpha_+\,\alpha_+\,(\gamma_--3)\,(\bar r\cdot r)^2\,s$	33	5	$\frac{1}{8}\,\alpha_+\,(\gamma_--3)\,\bar r^2\,(\bar r\cdot r)\,s$
33	6	$\frac{1}{8}\,\bar\alpha_+\,(\gamma_--3)\,(\bar r\cdot r)\,r^2\,s$	33	7	$\frac{1}{4}\,((\gamma_-+1)\,(\bar r\cdot r)^2-4\,\bar r^2\,r^2)\,s$	33	8	$\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,(\gamma_--3)\,(\bar r\cdot r)^2\,s$
33	9	$-\frac{1}{8}\,\alpha_-\,(\gamma_--3)\,\bar r^2\,(\bar r\cdot r)\,s$	33	10	$-\frac{1}{8}\,\bar\alpha_-\,(\gamma_--3)\,(\bar r\cdot r)\,r^2\,s$	33	11	$\frac{1}{4}\,(\gamma_--3)\,(\bar r\cdot r)^2\,s$
33	12	$\frac{1}{16}\,\alpha_-\,\bar\alpha_+\,(\gamma_-+1)\,(\bar r\cdot r)^2\,s$	33	13	$\frac{1}{8}\,\alpha_-\,(\gamma_-+1)\,\bar r^2\,(\bar r\cdot r)\,s$	33	14	$-\frac{1}{8}\,\bar\alpha_+\,(\gamma_-+1)\,(\bar r\cdot r)\,r^2\,s$
33	15	$-\frac{1}{4}\,(\gamma_-+1)\,(\bar r\cdot r)^2\,s$	33	16	$\frac{1}{16}\,\bar\alpha_-\,\alpha_+\,(\gamma_-+1)\,(\bar r\cdot r)^2\,s$	33	17	$-\frac{1}{8}\,\alpha_+\,(\gamma_-+1)\,\bar r^2\,(\bar r\cdot r)\,s$
33	18	$\frac{1}{8}\,\bar\alpha_-\,(\gamma_-+1)\,(\bar r\cdot r)\,r^2\,s$	33	19	$-\frac{1}{4}\,(\gamma_-+1)\,(\bar r\cdot r)^2\,s$	33	20	$\frac{1}{4}\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
33	21	$\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)^2\,s$	33	22	$-\frac{1}{2}\,\alpha_+\,(\bar r\cdot r)^2\,s$	33	23	$-(\bar r\cdot r)\,r^2\,s$
33	24	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	33	25	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)^2\,s$	33	26	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)^2\,s$
33	27	$-\bar r^2\,(\bar r\cdot r)\,s$	33	28	$\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,s$	33	33	$(\bar\alpha_-\,r^2\,s/2$
33	36	$\frac{1}{2}\,\alpha_-\,\bar r^2\,s$	33	39	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(3\,\bar\delta-1)\,(\bar r\cdot r)\,s$	33	40	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(3\,\delta-1)\,(\bar r\cdot r)\,s$
33	41	$-\frac{1}{4}\,(\bar\delta\,\delta-1)\,(\gamma_--3)\,(\bar r\cdot r)\,s$	34	1	$(\bar r\cdot r)\,v^2$	34	2	$-(\bar r\cdot r)\,v^2$
34	4	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)^2\,s$	34	5	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	34	6	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$
34	7	$-(\bar r\cdot r)^2\,s$	34	8	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)^2\,s$	34	9	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$
34	10	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	34	11	$-(\bar r\cdot r)^2\,s$	34	12	$\frac{1}{4}\,(4\,v^2-\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)^2\,s)$
34	13	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	34	14	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	34	15	$(\bar r\cdot r)^2\,s$
34	16	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)^2\,s$	34	17	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	34	18	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$
34	19	$(\bar r\cdot r)^2\,s$	34	28	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)\,s$	34	32	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,s$
34	35	$\frac{1}{2}\,\bar\alpha_-\,r^2\,s$	34	39	$-\alpha_+\,(\bar r\cdot r)\,s$	34	40	$\bar\alpha_-\,(\bar r\cdot r)\,s$
34	41	$(\bar\delta\,\delta+1)\,(\bar r\cdot r)\,s$	35	1	$\alpha_+\,\bar r^2\,v^2/2$	35	2	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,v^2$
35	4	$-\frac{1}{8}\,\bar\alpha_+\,\alpha_+^2\,\bar r^2\,(\bar r\cdot r)\,s$	35	5	$-\frac{1}{4}\,\alpha_+^2\,(\bar r^2)^2\,s$	35	6	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s$
35	7	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	35	8	$-\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	35	9	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$

TABLE VII: Continuation of Tab. V.

k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$	k	n	$c_k^{(n)}$
35	10	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s$	35	11	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	35	12	$-\frac{1}{8}\,\alpha_-\,\bar\alpha_+\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
35	13	$-\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$	35	14	$\frac{1}{4}\,(\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s+4\,v^2)$	35	15	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
35	16	$-\frac{1}{8}\,\bar\alpha_-\,\alpha_+^2\,\bar r^2\,(\bar r\cdot r)\,s$	35	17	$\frac{1}{4}\,\alpha_+^2\,(\bar r^2)^2\,s$	35	18	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s$
35	19	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	35	30	$\frac{1}{2}\,\bar\alpha_-\,r^2\,s)$	35	34	$-(\bar r\cdot r)\,s$
35	37	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)\,s$	35	39	$\frac{1}{2}\,\alpha_-\,\alpha_+\,\bar r^2\,s$	35	40	$\frac{1}{2}\,\bar\alpha_-\,\alpha_+\,\bar r^2\,s$
35	41	$\frac{1}{2}\,\alpha_+\,(\bar\delta\,\delta-1)\,\bar r^2\,s$	36	1	$-\frac{1}{2}\,\bar\alpha_-\,r^2\,v^2$	36	2	$\frac{1}{2}\,\bar\alpha_-\,r^2\,v^2$
36	4	$\frac{1}{8}\,\bar\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	36	5	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s$	36	6	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$
36	7	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	36	8	$\frac{1}{8}\,\bar\alpha_-^2\,\alpha_-\,(\bar r\cdot r)\,r^2\,s$	36	9	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s$
36	10	$-\frac{1}{4}\,\bar\alpha_-^2\,(r^2)^2\,s$	36	11	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	36	12	$\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$
36	13	$\frac{1}{4}\,(\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s+4\,v^2)$	36	14	$-\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$	36	15	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$
36	16	$\frac{1}{8}\,\bar\alpha_-^2\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	36	17	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,\bar r^2\,r^2\,s$	36	18	$\frac{1}{4}\,\bar\alpha_-^2\,(r^2)^2\,s$
36	19	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	36	29	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,s$	36	31	$\frac{1}{4}\,\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)\,s$
36	36	$-(\bar r\cdot r)\,s$	36	39	$\frac{1}{2}\,\bar\alpha_-\,\alpha_+\,r^2\,s$	36	40	$\frac{1}{2}\,\bar\alpha_-\,\bar\alpha_+\,r^2\,s$
36	41	$-\frac{1}{2}\,\bar\alpha_-\,(\bar\delta\,\delta-1)\,r^2\,s$	37	1	$\frac{1}{4}\,(\beta_++1)\,(\bar r\cdot r)\,v^2$	37	2	$-\frac{1}{4}\,(\beta_+-3)\,(\bar r\cdot r)\,v^2$
37	3	$-(\bar r\cdot r)\,v^2$	37	4	$-\frac{1}{16}\,\bar\alpha_+\,\alpha_+\,(\beta_++1)\,(\bar r\cdot r)^2\,s$	37	5	$-\frac{1}{8}\,\alpha_+\,(\beta_++1)\,\bar r^2\,(\bar r\cdot r)\,s$
37	6	$-\frac{1}{8}\,\bar\alpha_+\,(\beta_++1)\,(\bar r\cdot r)\,r^2\,s$	37	7	$-\frac{1}{4}\,(\beta_++1)\,(\bar r\cdot r)^2\,s$	37	8	$-\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,(\beta_++1)\,(\bar r\cdot r)^2\,s$
37	9	$\frac{1}{8}\,\alpha_-\,(\beta_++1)\,\bar r^2\,(\bar r\cdot r)\,s$	37	10	$\frac{1}{8}\,\bar\alpha_-\,(\beta_++1)\,(\bar r\cdot r)\,r^2\,s$	37	11	$-\frac{1}{4}\,(\beta_++1)\,(\bar r\cdot r)^2\,s$
37	12	$-\frac{1}{16}\,\alpha_-\,\bar\alpha_+\,(\beta_+-3)\,(\bar r\cdot r)^2\,s$	37	13	$-\frac{1}{8}\,\alpha_-\,(\beta_+-3)\,\bar r^2\,(\bar r\cdot r)\,s$	37	14	$\frac{1}{8}\,\bar\alpha_+\,(\beta_+-3)\,(\bar r\cdot r)\,r^2\,s$
37	15	$\frac{1}{4}\,((\beta_++1)\,(\bar r\cdot r)^2-4\,\bar r^2\,r^2)\,s$	37	16	$-\frac{1}{16}\,\bar\alpha_-\,\alpha_+\,(\beta_+-3)\,(\bar r\cdot r)^2\,s$	37	17	$\frac{1}{8}\,\alpha_+\,(\beta_+-3)\,\bar r^2\,(\bar r\cdot r)\,s$
37	18	$-\frac{1}{8}\,\bar\alpha_-\,(\beta_+-3)\,(\bar r\cdot r)\,r^2\,s$	37	19	$\frac{1}{4}\,(\beta_+-3)\,(\bar r\cdot r)^2\,s$	37	20	$\frac{1}{4}\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
37	21	$\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)^2\,s$	37	22	$-\frac{1}{2}\,\alpha_+\,(\bar r\cdot r)^2\,s$	37	23	$-(\bar r\cdot r)\,r^2\,s$
37	24	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	37	25	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)^2\,s$	37	26	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)^2\,s$
37	27	$-\bar r^2\,(\bar r\cdot r)\,s$	37	28	$\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,s$	37	33	$\frac{1}{2}\,\bar\alpha_-\,r^2\,s$
37	38	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,s$	37	39	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(3\,\bar\delta-1)\,(\bar r\cdot r)\,s$	37	40	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(3\,\delta+1)\,(\bar r\cdot r)\,s$
37	41	$\frac{1}{4}\,(\beta_+-3)\,(\bar\delta\,\delta+1)\,(\bar r\cdot r)\,s$	38	1	$(\bar r\cdot r)\,v^2$	38	2	$-(\bar r\cdot r)\,v^2$
38	4	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)^2\,s$	38	5	$-\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	38	6	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$
38	7	$-(\bar r\cdot r)^2\,s$	38	8	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,(\bar r\cdot r)^2\,s$	38	9	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$
38	10	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$	38	11	$(-(\bar r\cdot r)^2)\,s$	38	12	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)^2\,s$
38	13	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	38	14	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	38	15	$(\bar r\cdot r)^2\,s$
38	16	$\frac{1}{4}\,(4\,v^2-\bar\alpha_-\,\alpha_+\,(\bar r\cdot r)^2\,s)$	38	17	$\frac{1}{2}\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	38	18	$-\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)\,r^2\,s$
38	19	$(\bar r\cdot r)^2\,s$	38	28	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,s$	38	31	$\frac{1}{2}\,\alpha_-\,\bar r^2\,s)$
38	37	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,s$	38	39	$\alpha_-\,(\bar r\cdot r)\,s$	38	40	$-\bar\alpha_+\,(\bar r\cdot r)\,s$
38	41	$(\bar\delta\,\delta+1)\,(\bar r\cdot r)\,s$	39	1	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,v^2$	39	2	$\frac{1}{2}\,\alpha_-\,\bar r^2\,v^2)$
39	4	$\frac{1}{8}\,\alpha_-\,\bar\alpha_+\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	39	5	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$	39	6	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s$
39	7	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	39	8	$\frac{1}{8}\,\bar\alpha_-\,\alpha_-^2\,\bar r^2\,(\bar r\cdot r)\,s$	39	9	$-\frac{1}{4}\,\alpha_-^2\,(\bar r^2)^2\,s$
39	10	$-\frac{1}{4}\,\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s$	39	11	$\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	39	12	$\frac{1}{8}\,\alpha_-^2\,\bar\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
39	13	$\frac{1}{4}\,\alpha_-^2\,(\bar r^2)^2\,s$	39	14	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s$	39	15	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$
39	16	$\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$	39	17	$-\frac{1}{4}\,\alpha_-\,\alpha_+\,(\bar r^2)^2\,s$	39	18	$\frac{1}{4}\,(\bar\alpha_-\,\alpha_-\,\bar r^2\,r^2\,s+4\,v^2)$
39	19	$-\frac{1}{2}\,\alpha_-\,\bar r^2\,(\bar r\cdot r)\,s$	39	30	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,s$	39	33	$-(\bar r\cdot r)\,s$
39	35	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,s$	39	39	$\frac{1}{2}\,\alpha_-\,\alpha_+\,\bar r^2\,s$	39	40	$\frac{1}{2}\,\alpha_-\,\bar\alpha_+\,\bar r^2\,s$
39	41	$-\frac{1}{2}\,\alpha_-\,(\bar\delta\,\delta-1)\,\bar r^2\,s$	40	1	$\frac{1}{2}\,\bar\alpha_+\,r^2\,v^2$	40	2	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,v^2$
40	4	$-\frac{1}{8}\,\bar\alpha_+^2\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	40	5	$-\frac{1}{4}\,\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s$	40	6	$-\frac{1}{4}\,\bar\alpha_+^2\,(r^2)^2\,s$
40	7	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	40	8	$-\frac{1}{8}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	40	9	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s$
40	10	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$	40	11	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	40	12	$-\frac{1}{8}\,\alpha_-\,\bar\alpha_+^2\,(\bar r\cdot r)\,r^2\,s$
40	13	$-\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,\bar r^2\,r^2\,s$	40	14	$\frac{1}{4}\,\bar\alpha_+^2\,(r^2)^2\,s$	40	15	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$
40	16	$-\frac{1}{8}\,\bar\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,r^2\,s$	40	17	$\frac{1}{4}\,(\bar\alpha_+\,\alpha_+\,\bar r^2\,r^2\,s+4\,v^2)$	40	18	$-\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(r^2)^2\,s$
40	19	$\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	40	29	$\frac{1}{2}\,\alpha_-\,\bar r^2\,s$	40	32	$\frac{1}{4}\,\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,s$
40	38	$-(\bar r\cdot r)\,s$	40	39	$\frac{1}{2}\,\alpha_-\,\bar\alpha_+\,r^2\,s$	40	40	$\frac{1}{2}\,\bar\alpha_-\,\bar\alpha_+\,r^2\,s$
40	41	$\frac{1}{2}\,\bar\alpha_+\,(\bar\delta\,\delta-1)\,r^2\,s$	41	1	$-\frac{1}{4}\,(\beta_--1)\,(\bar r\cdot r)\,v^2$	41	2	$\frac{1}{4}\,(\beta_-+3)\,(\bar r\cdot r)\,v^2$
41	3	$-(\bar r\cdot r)\,v^2$	41	4	$\frac{1}{16}\,\bar\alpha_+\,\alpha_+\,(\beta_--1)\,(\bar r\cdot r)^2\,s$	41	5	$\frac{1}{8}\,\alpha_+\,(\beta_--1)\,\bar r^2\,(\bar r\cdot r)\,s$
41	6	$\frac{1}{8}\,\bar\alpha_+\,(\beta_--1)\,(\bar r\cdot r)\,r^2\,s$	41	7	$\frac{1}{4}\,(\beta_--1)\,(\bar r\cdot r)^2\,s$	41	8	$\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,(\beta_--1)\,(\bar r\cdot r)^2\,s$
41	9	$-\frac{1}{8}\,\alpha_-\,(\beta_--1)\,\bar r^2\,(\bar r\cdot r)\,s$	41	10	$-\frac{1}{8}\,\bar\alpha_-\,(\beta_--1)\,(\bar r\cdot r)\,r^2\,s$	41	11	$\frac{1}{4}\,(\beta_--1)\,(\bar r\cdot r)^2\,s$
41	12	$\frac{1}{16}\,\alpha_-\,\bar\alpha_+\,(\beta_-+3)\,(\bar r\cdot r)^2\,s$	41	13	$\frac{1}{8}\,\alpha_-\,(\beta_-+3)\,\bar r^2\,(\bar r\cdot r)\,s$	41	14	$-\frac{1}{8}\,\bar\alpha_+\,(\beta_-+3)\,(\bar r\cdot r)\,r^2\,s$
41	15	$-\frac{1}{4}\,(\beta_-+3)\,(\bar r\cdot r)^2\,s$	41	16	$\frac{1}{16}\,\bar\alpha_-\,\alpha_+\,(\beta_-+3)\,(\bar r\cdot r)^2\,s$	41	17	$-\frac{1}{8}\,\alpha_+\,(\beta_-+3)\,\bar r^2\,(\bar r\cdot r)\,s$
41	18	$\frac{1}{8}\,\bar\alpha_-\,(\beta_-+3)\,(\bar r\cdot r)\,r^2\,s$	41	19	$-\frac{1}{4}\,((\beta_--1)\,(\bar r\cdot r)^2+4\,\bar r^2\,r^2)\,s$	41	20	$\frac{1}{4}\,\alpha_-\,\alpha_+\,\bar r^2\,(\bar r\cdot r)\,s$
41	21	$\frac{1}{2}\,\alpha_-\,(\bar r\cdot r)^2\,s$	41	22	$-\frac{1}{2}\,\alpha_+\,(\bar r\cdot r)^2\,s$	41	23	$-(\bar r\cdot r)\,r^2\,s$
41	24	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(\bar r\cdot r)\,r^2\,s$	41	25	$\frac{1}{2}\,\bar\alpha_-\,(\bar r\cdot r)^2\,s$	41	26	$-\frac{1}{2}\,\bar\alpha_+\,(\bar r\cdot r)^2\,s$
41	27	$-\bar r^2\,(\bar r\cdot r)\,s$	41	28	$\frac{1}{16}\,\bar\alpha_-\,\alpha_-\,\bar\alpha_+\,\alpha_+\,(\bar r\cdot r)\,s$	41	34	$-\frac{1}{2}\,\bar\alpha_+\,r^2\,s$
41	36	$\frac{1}{2}\,\alpha_-\,\bar r^2\,s)$	41	39	$\frac{1}{4}\,\alpha_-\,\alpha_+\,(3\,\bar\delta+1)\,(\bar r\cdot r)\,s$	41	40	$\frac{1}{4}\,\bar\alpha_-\,\bar\alpha_+\,(3\,\delta-1)\,(\bar r\cdot r)\,s$
41	41	$-\frac{1}{4}\,(\beta_-+3)\,(\bar\delta\,\delta+1)\,(\bar r\cdot r)\,s$						

TABLE VIII: Continuation of Tab. V.

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