

# The Dichotomous Nucleon: Some Radical Conjectures for the Large $N_c$ Limit

Yoshimasa Hidaka<sup>a</sup>, Toru Kojo<sup>b</sup>, Larry McLerran<sup>b,c</sup>, and Robert D. Pisarski<sup>c</sup>

<sup>a</sup>*Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan*

<sup>b</sup>*RIKEN/BNL Research Center, Brookhaven National Laboratory, Upton, NY-11973, USA*

<sup>c</sup>*Department of Physics, Brookhaven National Laboratory, Upton, NY-11973, USA*

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## Abstract

We discuss some problems with the large  $N_c$  approximation for nucleons which arise if the axial coupling of the nucleon to pions is large,  $g_A \sim N_c$ . While  $g_A \sim N_c$  in non-relativistic quark and Skyrme models, it has been suggested that Skyrmions may collapse to a small size,  $r \sim 1/f_\pi \sim \Lambda_{\text{QCD}}^{-1}/\sqrt{N_c}$ . (This is also the typical scale over which the string vertex moves in a string vertex model of the baryon.) To construct a nucleon with a small axial coupling we suggest that most quarks are bound into colored scalar diquarks. For odd  $N_c$ , this leaves one unpaired quark, which carries the spin and isospin of the nucleon. If the unpaired quark is in a spatial wavefunction orthogonal to the wavefunctions of the scalar diquarks, then up to logarithms of  $N_c$ , the unpaired quark only costs an energy  $\sim \Lambda_{\text{QCD}}$ . This naturally gives  $g_A \sim 1$  and has other attractive features. In nature, the wavefunctions of the paired and unpaired quarks might only be approximately orthogonal; then  $g_A$  depends weakly upon  $N_c$ . This dichotomy in wave functions could arise if the unpaired quark orbits at a size which is parametrically large in comparison to that of the diquarks.

*Key words:* Axial charge, Coherent pions, Nuclear forces, Large  $N_c$  expansion

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## 1 Introduction

The large  $N_c$  limit of 't Hooft [1] for the description of baryons has been developed by Adkins, Nappi and Witten [2]. In this limit, the nucleon is a topological excitation of the pion field, where the pion field is described by a non-linear sigma model plus a Skyrme term [3]. This topological excitation is

described by a stable soliton solution of size  $r \sim 1/\Lambda_{\text{QCD}}$ , which is a Skyrmion;  $\Lambda_{\text{QCD}}$  is a mass scale typical of the strong interactions.

The action of the Skyrmion is  $\sim N_c$ , and so it contains of order  $N_c$  coherent pions. In the Skyrme model, the nucleon pion coupling constant is enhanced from its naive value,  $g_{\pi NN} \sim \sqrt{N_c}$ , which arises from counting the number of quarks inside a nucleon, to become  $g_{\pi NN} \sim N_c^{3/2}$ . This is a consequence of the coherent nature of the pions which compose the Skyrmion. By the Goldberger-Treiman relation [4], the axial coupling  $g_A$  is then of order  $N_c$ . Such a strong axial coupling generates strong spin-isospin dependent forces, of order  $N_c$ , out to distances which are large in comparison to the size of the nucleon,  $\sim 1/\Lambda_{\text{QCD}}$ . In the limit of massless pions, these interactions are of infinite range. In Monte-Carlo computations of the nucleon-nucleon force on the lattice, no strong long range tails are seen; indeed, even at intermediate ranges the forces do not appear to be large [5] (Some cautions on the interpretation of lattice results were raised in [6]). In addition, the magnetic moment of the proton would be of order  $N_c$ , which would also generate strong electromagnetic interactions [7].

Such a description of the nucleon at infinite  $N_c$  appears to be rather different from what we observe for  $N_c = 3$ . At finite  $N_c$ , these problems might be fixed by a fine tuning of parameters. For example, in the Skyrme model description of Ref. [2], the parameter  $1/e^2$  that controls the strength of the Skyrme term, and which stabilizes the Skyrmion at a non-zero radius, should be of order  $N_c$ . To provide a phenomenologically viable description of the nucleon for  $N_c = 3$ , though, it is taken to be  $3.3 \times 10^{-2}$ .

Another generic problem is the nature of nuclear matter. Some of the channels for the long distance spin-isospin dependent forces are attractive. This means that the ground state of nuclear matter is a crystal and the binding energy is of order  $N_c \Lambda_{\text{QCD}}$  [8]. On the other hand, ordinary nuclear matter is very weakly bound, with a binding energy  $\delta E \sim 16$  MeV [9]. This number seems to be closer to  $\Lambda_{\text{QCD}}/N_c$  than to  $N_c \Lambda_{\text{QCD}}$ , the value typical of a Skyrme crystal. Moreover, nuclear matter appears to be in a liquid state, and not a crystal.

An excellent discussion of the properties of the nucleon-nucleon force is found in Ref. [10–12]. Many of the relationships derived there are generic relationships between the magnitudes of various forces, and these seem to work quite well. Thus it is somewhat of a mystery why the large  $N_c$  limit for baryons can work well in some contexts, but provide qualitative disagreement in others.

Yet another problem is the mass splitting between the nucleon and  $\Delta$ . Consistency conditions at large  $N_c$  and standard large  $N_c$  counting indicate that this mass difference is  $\sim \Lambda_{\text{QCD}}/N_c$  [10,11]. In QCD, though, it is  $\sim 300$  MeV, which is  $\sim \Lambda_{\text{QCD}}$ .

A large value of  $g_A$  also generates problems in writing a chiral effective theory for the nucleon. In the linear sigma model, chiral symmetry implies that there is a large coupling to the sigma meson,  $g_{\sigma NN} = g_{\pi NN} \sim N_c^{3/2}$ . Such a large coupling generates self-energy corrections to the nucleon that would be larger than  $N_c$ . In addition, if the axial coupling is of order  $N_c$ , self-interactions associated with an axial-vector current should result in a significant contribution to the nucleon mass. If there is some way to lower the axial coupling, which does not greatly increase the mass of the nucleon, then it is plausible that nature would realize this possibility.

Ultimately, large self-energies for the nucleon might destabilize a nucleon of size  $\sim 1/\Lambda_{\text{QCD}}$ . One might be tempted to argue that this cannot happen in QCD, since the action in QCD is of order  $N_c$ , and a collapsed soliton, with a size other than  $\Lambda_{\text{QCD}}$ , should have a mass which is not linear in  $N_c$ . This would be a strong argument if the nucleon appeared as a purely classical solution of the QCD equations of motion, as a Skyrmionic soliton for example. Following others, however, we suggest that the Skyrmionic soliton may collapse [13–17]. If so, at short distances the nucleons are more naturally described by quarks rather than by coherent pions. The quarks cannot collapse to a small size without paying a price of order  $N_c/R$  in quark kinetic energy. The relevance of quark descriptions inside of the nucleon was also emphasized in [18].

A key observation in this paper is that such constituent quarks are the main origin of the axial charge  $g_A$ , which is the source of pion fields. If  $N_c$  constituent quarks have a small axial charge,  $g_A \sim 1$ , then the problems related to large coherent pions will be solved.

We suggest such a nucleon wavefunction. Most quarks are bound into colored diquarks [19]. For odd  $N_c$ , that leaves one unpaired quark. We then put that unpaired quark into a wavefunction which is approximately *orthogonal* to those of the paired quarks. This can be accomplished by making the spatial extent of the unpaired quark larger than that of the paired diquarks: it is “dichotomous”. Putting the additional quark into such a wavefunction costs an energy of order  $\Lambda_{\text{QCD}}$ , up to logarithms of  $N_c$  (as we show later). Such a construction results in small self-energies from the pion-nucleon self-interactions, as a result of  $g_A \sim 1$ . It is also clear that long-range nucleon-nucleon interactions are no longer strong.

This is a minimal modification of the naive non-relativistic quark model of the nucleon. There quarks are paired into diquarks, save for one quark that carries the quantum numbers of the nucleon. It is usually assumed, however, that *all* of the quarks, paired or not, have the same spatial wavefunction. This gives  $g_A = (N_c + 2)/3$ , and the problems discussed above [7,20].

A trace of the collapsed Skyrmion might appear at a scale size of order  $1/f_\pi$ .

This size corresponds to the intrinsic scale of a quantum pion. Since  $f_\pi \sim \sqrt{N_c} \Lambda_{\text{QCD}}$  at large  $N_c$ , the size of the nucleon shrinks to zero as  $N_c \rightarrow \infty$ . We will also show that such a small size naturally arises in a string vertex model of the nucleon, as the root mean square fluctuations in the position of the string vertex. Of course, the contribution of the string vertex to the mass of the nucleon is of order  $f_\pi$ , as most of the mass of the nucleon is generated by a cloud of quarks and quark-antiquark pairs surrounding the collapsed Skyrmion, or string vertex. The picture we develop has some aspects in common with bag models [21], and particularly the hybrid descriptions of Brown and Rho [22,23].

The collapsed Skyrmion we conjecture has some features which are similar to the nucleon in the Sakai-Sugimoto model [24]. They suggest that the Skyrmion, computed in the action to leading order in strong coupling, is unstable with respect to collapse. It is stabilized by  $\omega$  vector meson interaction, which is of higher order correction in strong coupling. It is argued that the nucleon has a size of order  $1/(\sqrt{g^2 N_c} \Lambda_{\text{QCD}})$ . The methods used to derive this result are questionable at sizes  $\ll 1/\Lambda_{\text{QCD}}$ , but at least this shows that there is a small object in such theories. It is quite difficult for  $\omega$  exchange or other strong coupling effects to stabilize the nucleon once it acquires a size much less than  $1/\Lambda_{\text{QCD}}$ . The basic problem is that mesons will decouple from small objects due to form factor effects. Without form factors, the  $\omega$  interaction generates a term  $\sim 1/R$ , which resists collapse; form factors convert this into a factor of  $\sim R$ , which is harmless as  $R$  shrinks to zero.

The outline of this paper is as follows: In Sec. 2 we review the sigma model and its predictions for nucleon structure. We show that its predictions for the large  $N_c$  properties of the nucleon are at variance from the large  $N_c$  limit predicted for a Skyrmion of size  $\sim 1/\Lambda_{\text{QCD}}$ . In Sec. 3 we discuss the general form of nucleon-nucleon interactions in the sigma model and in the Skyrme model. In Sec. 4 we argue that the Skyrmion might collapse to a size scale of order  $1/f_\pi$  [13–17]. In Sec. 5 we discuss the string vertex model of Veneziano [25]. In particular, we argue that the spatial extent of the string vertex is typically of order  $1/f_\pi$ , which is the minimal size for the string vertex. Such a vertex might be thought of as the localization of baryon number. Quarks attached to the ends of strings will nevertheless have a spatial extent of order  $1/\Lambda_{\text{QCD}}$  to avoid paying a huge price in quark kinetic energies. In Sec. 6 we compute the contribution to  $g_A$  arising from quarks. Using the non-relativistic quark model, we find that if we make the wavefunction of those quarks paired as diquarks, and that of the unpaired quark have a small overlap, then  $g_A$  is parametrically smaller in  $N_c$  than the canonical value of  $g_A = (N_c + 2)/3$ . An explicit computation of  $g_A$  and the magnetic moments for such a variable overlap is carried out in Appendix A and B. In Sec. 7 we present arguments about how, dynamically, such a small overlap might be achieved. Sec. 8 is a summary.

## 2 The Sigma Model

Let us begin by reviewing how the long range nucleon-nucleon interaction depend on  $N_c$  in the sigma model. The linear sigma model is written in the form

$$S = \int d^4x \left\{ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi^a \partial^\mu \pi^a) - \frac{\mu^2}{2} (\sigma^2 + (\pi^a)^2) + \frac{\lambda}{4} (\sigma^2 + (\pi^a)^2)^2 + \bar{\psi} (-i \not{\partial} + g(\sigma + i\pi \cdot \tau \gamma^5)) \psi \right\}, \quad (1)$$

where  $\psi$  denotes the nucleon field. Our metric convention is  $g^{00} = -1$ . The naive arguments of large  $N_c$  QCD would have the mass term  $\mu$  of order  $\Lambda_{\text{QCD}}$ , the four meson coupling  $\lambda \sim 1/N_c$ , and the pion nucleon coupling  $g \sim \sqrt{N_c}$ .

Upon extremizing the action, we find that  $M_\sigma \sim \mu$ ,  $M_\pi = 0$ ,  $M_N \sim g\mu/\sqrt{\lambda} \sim N_c \mu$ . Therefore the typical large  $N_c$  assignments of couplings are consistent with the nucleon mass being of order  $N_c$ , the sigma mass of order one, and a weakly coupled pionic and sigma system. Note that the sigma is strongly coupled to the nucleon, consistent with large  $N_c$  phenomenology.

What about the pion coupling? It is naively of order  $\sqrt{N_c}$  but the  $\gamma^5$  matrix, because of the negative parity of the pion, suppresses pion emission when the momentum of the pion is much less than that of the nucleon. A non relativistic reduction of the pion nucleon interaction gives

$$g\pi_a \bar{\psi} \tau^a \gamma^5 \psi \sim \frac{g}{2M_N} (\partial_\mu \pi^a) \bar{\psi} \tau^a \gamma^\mu \psi. \quad (2)$$

This equation means that one pion emission is not of order  $\sqrt{N_c}$  at long distances, but of order  $1/\sqrt{N_c}$ . Thus the potential due to one pion exchange is of order  $1/N_c$ , and not of order  $N_c$ .

One might object that in higher orders this is not true, since one might expect the non-relativistic decoupling of the pions would disappear when one considers two pion exchange. If one considers the diagram in Fig. 1, this contribution is naively of order  $N_c$ . The sum of the two diagrams cancel to leading order when  $q, k \ll M$ , making it again naively of order one. However, when the diagram in Fig. 2 is included, which is also of order one in powers of  $N_c$ , there is a cancellation with the above two diagrams when the momentum of the pions is small compared to  $\mu$ . When all is said and done, we conclude that for momentum small compared to the QCD scale, the interaction is of order  $1/N_c$ . This corresponds to a suppression of  $1/\sqrt{N_c}$  for each pion emitted.

In fact, Weinberg proves by an operator transformation on the sigma model action, that this cancellation persists to all orders in perturbation in the theory,

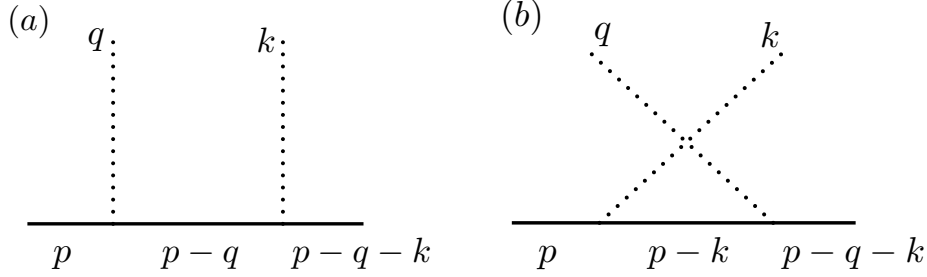


Fig. 1. a: One of the two pion exchanges. b: The crossed diagram.

and that pion emission when soft is always suppressed by  $1/\sqrt{N_c}$  for each emitted pion [26].

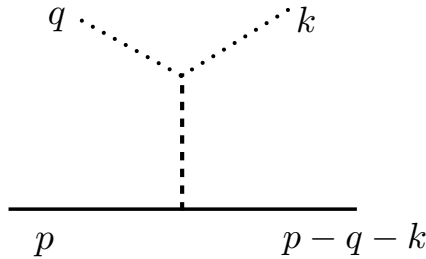


Fig. 2. Two pions produced by sigma exchange.

This conclusion about the strength of the nucleon force is consistent with what we know about nuclear matter. Nuclear matter is weakly bound, and has a binding energy which is of order  $\Lambda^2/M_N \sim 1/N_c$ . Such a parametric dependence on  $N_c$  is seen in nuclear matter computations where pion exchange is augmented by a hard core interaction [27]. The hard core interaction presumably arises when momentum transferred is of order  $\Lambda_{\text{QCD}}$ , and interactions become of order one in powers of  $N_c$ . In nuclear matter computations, the hard core essentially tells the nucleons they cannot go there, and its precise form is not too important.

It is useful to consider the non-linear sigma model, as this is the basis of the Skyrme model treatment. The non-linear sigma model is essentially the infinite sigma particle mass limit of the linear sigma model. It should be valid at distance scales much larger than  $1/\Lambda_{\text{QCD}}$ , which is also the range of validity of the linear sigma model. The action for the non-linear sigma model is

$$S = \int d^4x \left\{ f_\pi^2 \text{tr} \partial_\mu U \partial^\mu U^\dagger + \bar{\Psi} (-i\not{\partial} + M\bar{U}) \Psi \right\}. \quad (3)$$

In this equation,

$$U = e^{i\tau \cdot \pi / f_\pi}, \quad (4)$$

and

$$\bar{U} = e^{i\tau \cdot \pi \gamma^5 / f_\pi}, \quad (5)$$

where  $f_\pi \sim \sqrt{N_c} \Lambda_{\text{QCD}}$ , and the nucleon mass  $M \sim N_c \Lambda_{\text{QCD}}$ .

Weinberg's trick is to rotate away the interactions in the mass term by a chiral rotation,

$$\bar{U} \rightarrow V^{-1/2} \bar{U} V^{-1/2} = 1. \quad (6)$$

After this rotation of the nucleon fields, the action becomes,

$$S = \int d^4x \left\{ f_\pi^2 \text{tr} \partial_\mu U \partial^\mu U^\dagger + \bar{\Psi} \left( \frac{1}{i} \gamma^\mu (\partial_\mu + \gamma^5 V^{1/2} \partial_\mu V^{-1/2}) + M \right) \Psi \right\}. \quad (7)$$

We do not need to know the explicit form of  $V$  to extract the essential physics. The point is that the expansion in powers of the pion-nucleon interaction involves a factor of  $1/\sqrt{N_c}$  for each power of the pion field. This arises because the coupling to the pions is a derivative coupling, and to get the dimensions right each power of the derivative times the pion fields must be accompanied by a factor of  $1/f_\pi$ . Notice also that the first term in the expansion of the pion field is  $1/f_\pi$ , and couples to the nucleonic axial-vector current. The nucleonic axial-vector current is one for free fermions, and the interactions in this theory, corresponding to decreasing powers of  $1/\sqrt{N_c}$ , do not change the parametric dependence upon  $N_c$ .

While in these models above  $g_A \sim 1$ , it is possible to obtain  $g_A \sim N_c$  by the addition of further terms to the effective Lagrangian. In the linear sigma model, consider adding a term [28,29]

$$\frac{\tilde{g}}{\Lambda_{\text{QCD}}^2} \left( \bar{\psi}_L (\Phi^\dagger \not{\partial} \Phi) \psi_L + \bar{\psi}_R (\Phi \not{\partial} \Phi^\dagger) \psi_R \right). \quad (8)$$

Here  $\psi_{L,R}$  are chiral projections of the nucleon field, and  $\Phi$  transforms under  $SU_L(2) \times SU_R(2)$ . This term is non-renormalizable, with the coupling having an overall dimension of inverse mass squared. We take this mass scale to be  $\Lambda_{\text{QCD}}$ , so the coupling  $\tilde{g}$  is dimensionless. Taking  $\Phi \sim f_\pi U$ , this term generates an axial vector coupling of the pion to the nucleon, and  $g_A \sim N_c \tilde{g}$ ; see, *e.g.*, Eqs. (19.5.48), (19.5.49), and (19.5.50) of Ref. [29].

Thus if we allow the addition of non-renormalizable terms to the linear sigma model,  $g_A$  can be treated as a free parameter. Our point, however, is that if one takes  $g_A \sim N_c$ , then at large distances, where the nucleon-nucleon interaction is determined by pion exchange, that the corresponding interactions are strong,  $\sim N_c$ . While certainly logically possible, this does not agree with the phenomenology of nucleon scattering, which sees no long range tails which are large in magnitude [5].

### 3 The General Structure of the Nucleon-Nucleon Force in the Sigma Model and the Skyrme Model

It is useful to compute the general form of the nucleon-nucleon force in both the Skyrme model and the sigma model. Let us first consider the Skyrme model,

$$S_{\text{skyrme}} = \int d^4x \left\{ \frac{f_\pi^2}{16} \text{tr} \partial_\mu U \partial^\mu U^\dagger + \frac{1}{32e^2} \text{tr} [\partial_\mu U, \partial_\nu U]^2 \right\}. \quad (9)$$

The last term in this equation is the Skyrme term. It has a coefficient  $1/e^2$  that is assumed to be of order  $N_c$ , and is positive. There is a topological winding number in the theory, and this winding number can be related to the total baryon number by an anomaly. The nucleon corresponds to the solution with winding number one. The size of the baryon is found to be  $R_{\text{baryon}} \sim 1/\sqrt{ef_\pi} \sim 1/\Lambda_{\text{QCD}}$ , and is independent of  $N_c$ . If the Skyrme term were zero or negative, the solution would collapse to zero size.

The two nuclear force is derived by considering a two Skyrmion solution and computing the energy of separation [30]. Since if we simply redefine scale sizes in the Skyrme action by defining a dimensionless pion field as  $\pi' = \pi/f_\pi$ , and rescaling coordinates by  $\Lambda_{\text{QCD}}$ , the Skyrme action becomes explicitly proportional to  $N_c$  when all dimensional quantities are so expressed. Therefore, the potential between two nucleons is of the form

$$V_{\text{skyrme}}(r) \sim \frac{N_c}{r} F_{\text{skyrme}}(\Lambda_{\text{QCD}}r). \quad (10)$$

This is clearly inconsistent with the result one gets from the Weinberg action. Here the lowest order diagram which contributes at distances much larger than  $1/\Lambda_{\text{QCD}}$  is due to one pion emission. Its strength is of order  $1/r(rf_\pi)^2 \sim 1/N_c$ . In the Skyrme model this difficulty is evaded by arguing that the strength of the axial coupling is of order  $N_c$  rather than of order one. Since the derivative of the pion field couples to the axial-vector current, and in the potential, there are two such vertices, one can get a long distance force of order  $N_c$ . Therefore the strong force due to pion exchange at long distances and the large value of the axial coupling in the Skyrme model are related.

It is useful to understand the nature of the potential in the sigma model, due to higher order pion exchanges. First, let us look at the contributions to the potential. Note that if the vertices were not derivatively coupled, each pion exchange would bring in a factor of  $1/r$ . Due to the derivative coupling at the vertices, there are two derivatives for each exchange. There is also a factor of  $1/f_\pi^2$ . This means the potential predicted by the non-linear sigma model is of



the form

$$V_\sigma = \frac{1}{r} V_\sigma(f_\pi r). \quad (11)$$

Note that this potential has the scale  $R \sim 1/f_\pi \sim 1/(\sqrt{N_c}\Lambda_{\text{QCD}})$ , so that it is much smaller than that of the standard nucleonic Skyrmion.

Perhaps it is easier to think about the pion field. In the linear limit when we treat the nucleon field as a point source, the pion field satisfies the equation

$$-\nabla^2 \pi^a = \frac{1}{f_\pi} \nabla^i \delta^{(3)}(\vec{r}) \langle \sigma^i \tau^a \rangle, \quad (12)$$

where  $\sigma^i$  is a Pauli matrix. This means that in lowest order, the pion field is of the order of  $\pi \sim f_\pi (1/r f_\pi)^2$ . Higher corrections give

$$\pi_\sigma = f_\pi G_\sigma(f_\pi r). \quad (13)$$

This is to be compared to that for the pion field of the Skyrme model

$$\pi_{\text{Skyrme}} = f_\pi G_{\text{Skyrme}}(\Lambda_{\text{QCD}} r). \quad (14)$$

We see that in the Skyrme case, that  $\pi/f_\pi \sim 1$  for  $r \sim 1/\Lambda_{\text{QCD}}$  while for the sigma model this occurs at the much smaller distance scale  $r \sim 1/f_\pi$ .

The axial vector coupling  $g_A$  is estimated from the pion behavior at long distance,  $g_A \sim f_\pi^2 R^2$ , where  $R$  is the size of the pion cloud [2]. In the Skyrme case,  $g_A \sim f_\pi^2/\Lambda_{\text{QCD}}^2 \sim N_c$ , while in the sigma model  $g_A \sim f_\pi^2/f_\pi^2 \sim 1$ .

There are several subtleties in extracting the result for the Skyrmion case. Note that for any solution with a size scale  $R \ll 1/\Lambda_{\text{QCD}}$ , the argument which led to the Skyrme term has broken down. For such solutions, the Skyrme term itself is very small compared to the zeroth order non-linear sigma model contribution at the size scale  $1/\Lambda_{\text{QCD}}$ . This is because in addition to the derivatives, there are four powers of the field, which are very small. Nevertheless, there should be a breakdown of the Skyrmion model at such a scale, arising from QCD corrections of the underlying theory. The sigma model solution sits at a distance scale small compared to where the Skyrmion action is applicable, and one should ask what is the nature of the corrections to the Skyrme model action at such distance scales. In addition, there is good room for skepticism about the Skyrme model treatment of the nucleon. In the Skyrme model,  $1/e^2 \sim N_c$ , but phenomenologically is of the order  $3 \times 10^{-2}$ . If we were to naively take parametrically  $1/e^2 \sim 1$ , the nucleon-nucleon force of the Skyrme model would be parametrically the same as that in the sigma model. The mass would not be correct however as it would be of order  $f_\pi$ . In later sections we will see that this picture has features of what we have in mind: a string vertex whose size is  $1/f_\pi$ , surrounded by a cloud of quarks.

## 4 What Might Be Wrong with the Skyrme Term?

What might be wrong with the Skyrme model solution other than it is not consistent with the sigma model? Is there any problem with internal inconsistency? When attempts were made to derive the Skyrme term from QCD, one found a Skyrme term which was generated, but also other terms [13–16]. When these terms were added together and only terms of fourth order in derivatives were retained, the Skyrmion was found to be unstable to collapse. If this tendency to collapse were maintained to all orders, then the Skyrmion might collapse to sizes much less than the QCD distance scale. One could not describe the nucleon within the conventional assumptions of the Skyrme model. (Strictly speaking, the Skyrme model comes from derivative expansion and keeping only lowest order terms is justified only when size scales  $R \gg 1/\Lambda_{\text{QCD}}$  are considered.)

One can ask whether or not higher order terms might stabilize the Skyrmion. Following Refs. [13–16], we postulate that such corrections are generated by a quark determinant in the presence of a background pion field. We might hope that such a description would be valid down to a scale of Skyrmion size of order  $1/f_\pi$ . It is at this scale that high order terms in the pion nucleon sigma model generate quantum corrections which are large. This is also the natural size scale for a pion since pion-pion interactions are of order  $1/N_c$ , and even deep inelastic scattering off of a pion is suppressed by  $1/N_c$ . Interactions with other mesons are suppressed by  $1/N_c$ . If we assume that the quark-pion interaction is parameterized by a vertex that is pointlike to a distance scale of order  $1/f_\pi$ , then this interaction strength is of order  $g_{\pi QQ} \sim 1/\sqrt{N_c}$ , then one finds a contribution to the Skyrme term that is leading order in  $N_c$ . This is because there are  $N_c$  quark loops. Evaluating the leading term in the derivative expansion of the pion field, there is the Skyrme term plus two others, that have signs that cause the Skyrmion to collapse. (It should be noted that the intrinsic size scale over which quarks are distributed inside the meson is more likely  $1/\Lambda_{\text{QCD}}$ , and the small apparent size of the pion arises from the nature of interactions of these quarks in the large  $N_c$  limit, rather than their intrinsic scale of spatial distribution.)

Subsequent to this [17], it was argued that in chiral soliton models, that the chiral soliton is stable against collapse when the full quark determinant is computed. This happens when there are bound fermions in the presence of a non-trivial background field, and the energy of the bound quarks is included. This suggests that the Skyrmion could be metastable if all orders in the fermion determinant are included.

We shall argue below that the Skyrmion at a size scale  $R \sim 1/\Lambda_{\text{QCD}}$  is absolutely unstable. Sufficiently small Skyrmions,  $R \ll 1/\Lambda_{\text{QCD}}$  always collapse

and they have an energy parametrically small compared to that of the nucleon.

The quark contribution to the non-linear sigma model action modifies the non-linear sigma model by

$$\delta S = N_c \ln\{\det(-i\cancel{\partial} + M\bar{U})\}. \quad (15)$$

Here the quark mass  $M$  is the constituent quark mass. We now use Weinberg's trick to rewrite this as a coupling  $a$  to an axial-vector background field that is a pure gauge transform of vacuum:

$$A_5^\mu = \frac{1}{i} V^{1/2} \partial^\mu V^{-1/2}. \quad (16)$$

Here  $V^{1/2}$  is a function of the pion field and is a unitary matrix. The determinant becomes

$$\delta S = N_c \ln(\det(\frac{1}{i} \gamma^\mu (\partial_\mu - \gamma_5 A_{\mu 5}) + M)). \quad (17)$$

In the limit there  $M = 0$ , the fermion determinant is gauge invariant. This means that all functions of  $A$  generated by the determinant are gauge invariant and they vanish when evaluated on  $A_5^\mu$  which is a gauge transformation of vacuum field.

Now for fields that are slowly varying, this determinant may be computed by the method of Refs. [13–16]. This yields the result that in leading order the Skyrmion collapses. We also see that if the Skyrmion is parametrically small compared to the scale size of  $\Lambda_{\text{QCD}}$ , we can ignore the mass term in the fermion propagator, and no potential is generated to resist the collapse of the Skyrmion. Since a Skyrmion with size much less than  $1/\Lambda_{\text{QCD}}$  has an energy arising from the non-linear sigma model contribution to the action that is parametrically small compared to  $N_c \Lambda_{\text{QCD}}$ , the Skyrmion is absolutely unstable.

It should be noted that it would be very difficult to resist the collapse on very general grounds. The collapse is prevented by fields that are singular at short distances. It is very difficult to generate such singular terms on scale sizes much less than  $1/\Lambda_{\text{QCD}}$  since QCD interactions are typically spread out on a distance scale of order  $1/\Lambda_{\text{QCD}}$ . The exception to this is pion self-interactions, which are presumably special because the pion is a Goldstone boson. The reason for a lack of a short distance singularity on scale sizes much less than  $1/\Lambda_{\text{QCD}}$  is that the nucleonic core is color singlet and interactions that would produce a  $1/r$  singularity would need to couple to a non-zero color charge. The evasion to this conclusion arises from quark kinetic energies. If the quarks were confined to a size scale which is very small would generate a  $1/R$  term.

## 5 The String Vertex Model and a Collapsed Skyrmion

If the Skyrmion is unstable against collapse, a reasonable conjecture for its minimum size is when quantum corrections to the non-linear sigma model for the pions is large, when  $R \sim 1/f_\pi$ . This limiting size can be understood from the Skyrme model itself. Recall that the energy of a Skyrmion of size  $R$  is

$$E = \int d^3x \left[ \frac{f_\pi^2}{16} \text{tr} \nabla U \nabla U^\dagger \right] \sim f_\pi^2 R. \quad (18)$$

Here we have ignored the possibility of a Skyrme term, since for small Skyrmions, we have argued there is no such term. The energy of each constituent of the Skyrmion is  $1/R$ , so that the number of quanta in the Skyrmion is

$$N = f_\pi^2 R^2. \quad (19)$$

For a Skyrmion of size  $R \sim 1/\Lambda_{\text{QCD}}$  this is  $N \sim N_c$ . For the collapsed Skyrmion, where  $R \sim 1/f_\pi$ ,  $N \sim 1$ . This is the limit where the quantum nature of the Skyrmion cannot be ignored.

The obvious problem with the collapsed Skyrmion is that it has a size parametrically small compared to  $1/\Lambda_{\text{QCD}}$ . On such size scales, surely quark degrees of freedom are important. Since quarks carry a conserved charge they cannot be collapsed to small sizes without paying a price in kinetic energy  $E \sim 1/R$ , and so to keep the baryon mass from growing larger than  $N_c \Lambda_{\text{QCD}}$ , the quarks cannot be compressed to smaller than the QCD scale. Therefore if there is some remnant of the collapsed Skyrmion it must include quarks and quark-antiquark pairs at the QCD size scale. It is in these degrees of freedom that the energy of the nucleon must reside. The collapsed Skyrmion can only have an energy of order  $f_\pi$  and so does not contribute much to the energy.

How can this picture of the nucleon be consistent with that of the quark model? Imagine that the nucleon is produced by the operator

$$O_B(x) = \int d^3x_1 \cdots d^3x_N q^{a_1}(x_1) U_{a_1, b_1}(x_1, x) \cdots q^{a_N}(x_N) U_{a_N, b_N}(x_N, x) \epsilon_{b_1, \dots, b_N}. \quad (20)$$

Here, a path ordered phase along some path that connects the quark operator and the position of the baryon is denoted by  $U(x, y)$ . This operator is the topological baryon number operator of Veneziano [25]. It is shown pictorially in Fig. 3.

In this picture, quarks are joined together by lines of colored flux tubes at a central point. The quark operators are at a distance of order  $1/\Lambda_{\text{QCD}}$  away from the central point. We can identify the central point as the place where the baryon number sits. This is natural if we think about hadronizing mesons

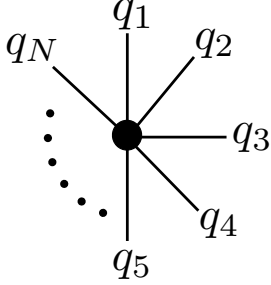


Fig. 3. The topological baryon number operator of Veneziano.

along the lines of color flux. This happens by quark-antiquark pairs and so it is ambiguous to think about the baryon number as either centered at the multiple string junction or at the ends of the strings. As far as baryon number is concerned, there is a symmetry between thinking about the baryon as made of quarks or as a topological object is a fundamental dualism of the theory: We can either think about the baryon number as being delocalized on quark degrees of freedom. This is reflected in Cheshire Cat models of the baryon [31,32].

In fact, it is easy to see that the degree of localization of the strong vertex is the same as that of the collapsed Skyrmion. Let us identify the string vertex position with the average center of mass coordinate of the quarks,

$$\vec{R} = \frac{1}{N_c}(\vec{r}_1 + \dots + \vec{r}_{N_c}). \quad (21)$$

We work in a frame where  $\langle \vec{r}_i \rangle = 0$ . The typical dispersion in the position of the center of the string is therefore  $\langle \vec{R}^2 \rangle \sim \langle \vec{r}_1^2 \rangle / N_c = 1/f_\pi^2$ . In the Skyrmion picture, one imagines the collapsed Skyrmion as corresponding to the string junction and having a high average density of baryon number in a localized region, a picture that is dual to the quark model description.

The topological string model generates lines of colored electric flux from the position of the string vertex. This presumably results in linear confinement of the quarks at distances far from the vertex. Close to the vertex, each quark feels a strong color Coulombic interactions that can be computed as the mean field of the color Coulombic fields of the other quarks. The Coulombic energy of all the quarks would be of order  $N_c/R$  times the 't Hooft coupling, and the kinetic energy for relativistic quarks would be of order  $N_c/R$ . The quarks sit at  $R \sim 1/\Lambda_{\text{QCD}}$  in order not to make the nucleon energy larger than  $N_c\Lambda_{\text{QCD}}$ .

## 6 The Quark Distributions

The picture above does not directly resolve the problems associated with a large axial-vector coupling or large matrix elements of the vector isospin currents. To understand what happens, we will take the non-relativistic quark model as a starting point. We will consider a matrix element of the non-relativistic expression of axial-vector current,  $\bar{q}\gamma_5\gamma_3\tau_3q$ , which takes the form

$$\langle N|R_3|N\rangle \equiv \langle N|\sum_{q=1}^{N_c} I_3^{(q)} S_3^{(q)}|N\rangle, \quad (22)$$

where the operator  $O^{(q)}$  acts on  $q$ -th quark wavefunction contained in nucleon wavefunctions.

In nonrelativistic limit, spins can characterize the irreducible representation of Hamiltonian, so wavefunctions can be characterized as  $|\text{color}\rangle \otimes |\text{flavor}\rangle \otimes |\text{spin}\rangle \otimes |\text{space}\rangle$ . Since color is totally antisymmetric, we should totally symmetrize spin-flavor-space wavefunction.

The frequently used construction of baryon wavefunctions is to use spin and isospin singlet diquark wavefunctions [33]. We will denote a number of diquark pair as  $n_d$ . We take a direct product of such a diquark state (and an extra quark if  $N_c$  odd), then totally symmetrize spin-flavor-space wavefunctions. In this construction,  $N_c = 2n_d$  baryon is a spin-isospin singlet, while spin-isospin quanta of  $N_c = 2n_d + 1$  baryon is solely determined by an extra quark. There is nothing nontrivial when we compute matrix elements related to spin and isospin operators.

The situation differs for the computation of  $\langle R_3\rangle$ . The reason is that both of diquark and nucleon states are not eigenstates of  $R_3$ , in contrast to spin and isospin. Below we will see this explicitly in terms of  $SU(2N_f)$  representation of states.

To compute  $\langle R_3\rangle$ , it is useful to use representations of the nonrelativistic  $SU(4)$  symmetry [34]. The  $SU(4)$  algebra is formed by the following fifteen generators

$$T_a = \sum_q I_3^{(q)}, \quad S_j = \sum_q S_j^{(q)}, \quad R_{aj} = \sum_q T_a^{(q)} S_j^{(q)}, \quad (23)$$

where  $j = 1, 2, 3$  and  $a = 1, 2, 3$ . The Cartan subalgebra of  $SU(4)$  is formed by three generators  $I_3, S_3$  and  $R_3 \equiv R_{33}$ , and states are characterized by eigenvalues of these generators and the dimension  $D$  of the irreducible representations.

We will denote such a state as  $|I_3, S_3, R_3; D\rangle$ . In the following we often omit  $D$  as far as it brings no confusions.

As a preparation, let us express our spin-isospin singlet diquark state in terms of  $SU(4)$  representations. It can be expressed as

$$\begin{aligned} |D\rangle &= \frac{1}{2} \{ |u\uparrow d\downarrow\rangle + |d\downarrow u\uparrow\rangle - |u\uparrow d\downarrow\rangle - |d\downarrow u\uparrow\rangle \} \\ &= |0, 0, 1/2\rangle_{\text{TS}} - |0, 0, -1/2\rangle_{\text{TS}}, \end{aligned} \quad (24)$$

where the subscript TS means that wavefunctions are totally symmetrized. Note that the diquark wavefunction has an eigenvalue of definite 3-component of isospin and spin, while it is a mixture of different  $R_3$  eigenstates. This is the origin to make computations of  $\langle N|R_3|N\rangle$  nontrivial.

Now we first argue simpler case,  $N_c = 2n_d$  nucleons. Such nucleons are characterized by states with isospin and spin zero, while they are mixture of states with different  $R_3$  eigenvalues. We assume that spatial wavefunctions are common for all quarks, so that spin-flavor (SF) wavefunctions of our nucleons are obtained by totally symmetrizing a direct product of diquark's spin-flavor wavefunctions:

$$|N\rangle_{2n_d}^{\text{SF}} = \left\{ \left( |0, 0, 1/2\rangle - |0, 0, -1/2\rangle \right)^{n_d} \right\}_{\text{TS}}. \quad (25)$$

If we use  $SU(4)$  expressions and omit subscripts on isospin and spin components (since they are zero),

$$\begin{aligned} |N\rangle_{2n_d}^{\text{SF}} &= \left| \frac{n_d}{2} \right\rangle_{\text{TS}} - {}_{n_d}C_1 \left| \frac{n_d}{2} - 1 \right\rangle_{\text{TS}} + \cdots + (-1)^{n_d} \left| \frac{-n_d}{2} \right\rangle_{\text{TS}} \\ &= \sum_{m=0}^{[n_d/2]} (-1)^m {}_{n_d}C_m \left\{ \left| \frac{n_d}{2} - m \right\rangle_{\text{TS}} + (-1)^{n_d} \left| - \left( \frac{n_d}{2} - m \right) \right\rangle_{\text{TS}} \right\}, \end{aligned} \quad (26)$$

where  $[n_d/2]$  equals to  $n_d/2$  ( $n_d/2 - 1$ ) for  $n_d$  even (odd) case. Now it is easy to see

$$\begin{aligned} R_3 |N\rangle_{2n_d}^{\text{SF}} &= \sum_{m=0}^{[n_d/2]} (-1)^m {}_{n_d}C_m \times \left( \frac{n_d}{2} - m \right) \\ &\quad \times \left\{ \left| \frac{n_d}{2} - m \right\rangle_{\text{TS}} + (-1)^{n_d+1} \left| - \left( \frac{n_d}{2} - m \right) \right\rangle_{\text{TS}} \right\}. \end{aligned} \quad (27)$$

Note that relative sign in the second term is flipped after  $R_3$  operation. This gives  $\langle N|R_3|N\rangle_{2n_d}^{\text{SF}} = 0$  due to cancellations for each indices  $m$ .

From the above discussion, we saw that the matrix element of  $R_3$  vanishes not because nucleon states have small  $R_3$ , but because cancellations occur among contributions from different eigenstates. Such cancellations are subtle, and strongly depend on the fact that  $N_c$  is even. Once we consider  $N_c$  odd baryons by adding one extra quark with the same spatial wavefunction as others, this situation completely changes: terms which avoid cancellations lead to a huge value,  $\langle N | R_3 | N \rangle_{2n_d+1}^{\text{SF}} = (N_c + 2)/12$ . So we now come back to the original problem, a large value of  $g_A$ .

However, a comparison of  $N_c$  odd and even baryons suggests us the following way to avoid a large  $g_A$ . In the above, we always assume all quarks occupy the same spatial wavefunction. Here let us see what happens when we adopt a spatial wavefunction for the unpaired quark which is different from that of quarks paired into diquarks.

We call spatial wavefunction of quarks inside of diquark as  $A(\vec{r})$ , and that of an extra quark as  $B(\vec{r})$ . If we introduce a quantity  $x$  which characterizes the overlap between wavefunction  $A$  and  $B$ ,

$$x \equiv |\langle A | B \rangle| = \left| \int d\vec{r} A^*(\vec{r}) B(\vec{r}) \right|, \quad (28)$$

then the expectation value of  $R_3$  in the  $|p \uparrow\rangle$  state is

$$\langle p \uparrow | R_3 | p \uparrow \rangle^{\text{SFS}} = \frac{1}{12} \frac{(N_c - 1)(N_c + 6)x^2 + 12}{(N_c - 1)x^2 + 4}. \quad (29)$$

The derivation is a little bit cumbersome, so we give it in Appendix A.

Let us see the physical implications of this result. First the reason why  $x^2$ , not  $x$ , appears is that a permutation of  $A$  and  $B$  always makes two  $\langle A | B \rangle$ . For instance,  $\langle AAB | ABA \rangle = x^2$ .

For  $x = 1$ , the matrix element reproduces conventional result,  $(N_c + 2)/12$ , as it should.

On the other hand, for  $x = 0$ , or when  $A$  and  $B$  are completely orthogonal each other, the cancellations analogous to  $N_c = 2n_d$  baryon take place among the diquark part, so that  $\langle R_3 \rangle$  is merely characterized by  $R_3$  for the leftover quark,  $1/4$ .

To get  $g_A$  of  $\sim N_c^0$ ,  $x$  must be of order of  $1/N_c$ . Note also that  $g_A$  is of order  $N_c$  until the overlap  $x \sim 1/\sqrt{N_c}$ . This means that in order to reduce  $g_A$  from  $\sim N_c$ , the overlap must be very small. This disparity in wavefunctions suggests the term ‘‘dichotomous’’ baryon.



## 7 Why Small Overlap?

Apparently the above small  $g_A$  baryon are not energetically favored in the shell model picture of quarks. But so far we did not take into account the contributions from fields surrounding valence quarks. They affects the nucleon self-energy via virtual mesonic loops (polarization effects of the media). If the axial charge of the valence quark is  $\sim N_c$ , such a large charge induces a big change in the effective mass.

As argued above, we expect that the mass of the baryon is affected by a large value of  $N_c$ . A leading contribution to the  $g_A$  dependent self-energy comes from the vertex  $\partial^\mu \pi_a / f_\pi \times g_A \bar{N} \gamma_\mu \gamma_5 \tau_a N$ , and it generates  $g_A^2 / f_\pi^2 \sim N_c$  for  $g_A \sim N_c$  [11] (no additional  $N_c$  dependence arises from nucleon propagator.) We would expect other vector mesons might generate similar self-energies through coupling to the axial-vector current. From this self-energy dependence and the  $x^2$  dependence of  $g_A$ , we suggests that self-energy effects generate a term like

$$H_{g_A} \sim N_c \left( |\psi_{\text{paired}}|^2 + |\psi_{\text{unpaired}}|^2 \right), \quad (30)$$

in the effective Hamiltonian for the valence quarks. Let us minimize this effective term. Since there are of order  $N_c$  paired quarks, deforming their wavefunctions costs a lot of energy. However, deforming the wavefunction of the unpaired quark only costs an energy of order  $\Lambda_{\text{QCD}}$ , while the gain in  $H_{g_A}$  is  $\sim N_c$ .

This deformation is most easily accomplished by having the unpaired quarks exist in the region outside of the paired quarks. The paired quark wavefunction in a string model falls as  $\exp(-\kappa(r\Lambda_{\text{QCD}})^{3/2})$  at large distances. If the unpaired quark is excluded, due to its hard core interaction with the paired quarks, from a size scale  $r \leq \ln^{2/3}(N_c)/\Lambda_{\text{QCD}}$ , then  $g_A$  can be reduced.

How large is the reduction in  $g_A$ ? If the self-energy is of order  $g_A^2/N_c$ , then we would expect that when  $g_A \sim \sqrt{N_c}$ , the trade off in energy associated with deforming the unpaired quark wavefunction is balanced by self-energy effects. Such a reduction most likely allows for a phenomenologically acceptable large  $N_c$  limit.

Magnetic moments have been computed in Appendix B, and are proportional to  $g_A$ . Magnetic interactions will be of order  $\alpha_{\text{em}} g_A^2$ , so that for sufficiently large  $N_c \sim 1/\alpha_{\text{em}}$ , these effects would also work toward reducing  $g_A$  to a value of order 1.

It is also possible that a large value of  $g_A$  might mean even more singular self-energy terms for large  $N_c$  resulting is a greater reduction of  $g_A$ . For example, there might in principle be effects that contribute to the energy that

correspond to higher powers than linear in  $N_c$  when  $g_A \sim N_c$ . These terms will tend to further reduce the parametric dependence of  $g_A$  upon  $N_c$ . We have not been able to find a compelling argument for such effects from strong interactions, though.

We turn next to a discussion of the splitting between the nucleon and the  $\Delta$ . In a conventional non-relativistic quark model, there is a  $SU(2N_f)$  symmetry which requires the  $N - \Delta$  masses to be equal to  $\sim \Lambda_{\text{QCD}}$ . This degeneracy is split by color hyperfine interaction,

$$\sum_{i \neq j} V_{\text{ss}}(\vec{r}_{ij}) \sim \frac{\lambda}{N_c} \sum_{i \neq j} \frac{\vec{S}_i \cdot \vec{S}_j}{M_i M_j} \delta(r_{ij}); \quad (31)$$

$\lambda = g^2 N_c$ , and the  $M_i \sim 1$  are constituent quark masses. Assuming all quark masses are the same, the expectation value for a state with spin  $S$  is

$$\langle \sum_{i \neq j} V_{\text{ss}}(\vec{r}_{ij}) \rangle \sim \frac{\lambda}{N_c} \left[ S(S+1) - \frac{3}{4} N_c \right] \times \frac{|\phi_{\text{relative}}(\vec{0})|^2}{M^2}. \quad (32)$$

Masses are split by the first term. If the difference in spins is  $\sim 1$ , as for the nucleon and the  $\Delta$ , the mass splitting is  $\sim 1/N_c$ . This agrees with the Skyrme model, identifying the  $\Delta$  as the first spin excitation of the nucleon. More general arguments can be found in [11,35].

In contrast, there is no  $SU(2N_f)$  symmetry in the model of a Dichotomous Baryon. The masses of the nucleon and the  $\Delta$  are not nearly degenerate, but split  $\sim \Lambda_{\text{QCD}}$ . This arises from polarization effects via the axial coupling of the  $\Delta$ ,  $g_{\Delta A}$ .

Consider what a dichotomous  $\Delta$ , with  $I_3 = S_3 = 3/2$ , is like. This can be obtained by breaking apart one diquark pair:

$$|D\rangle \longrightarrow |D'\rangle = \frac{1}{\sqrt{2}} \{ |u \uparrow u \uparrow\rangle + |u \uparrow u \uparrow\rangle \} = |1, 1, 1/2\rangle_{\text{TS}}. \quad (33)$$

Suppose that these  $|u \uparrow\rangle$  occupy the same spatial wavefunctions as those in the diquark pairs. Then  $g_{\Delta A}$  is  $\sim N_c$ , and  $\Delta$  has a large vacuum polarization of  $\sim N_c$ . As with the unpaired quark in the nucleon, this can be avoided by putting both  $u$  quarks into a spatial wavefunction which is orthogonal to that of the paired quarks. This costs an excitation energy  $\sim \Lambda_{\text{QCD}}$ , not  $\sim \Lambda_{\text{QCD}}/N_c$ .

If  $M_\Delta - M_N \sim \Lambda_{\text{QCD}}$  and  $g_{\Delta NA} \sim 1$ , the width of the  $\Delta$  is

$$\Gamma_\Delta \sim \frac{g_{\Delta NA}^2}{f_\pi^2} \left( \frac{M_\Delta^2 - M_N^2}{M_\Delta} \right)^3 \sim \frac{g_{\Delta NA}^2}{N_c} \Lambda_{\text{QCD}}. \quad (34)$$

Thus whether  $g_{\Delta NA}$  is  $\sim 1$  or  $\sim \sqrt{N_c}$ , the  $\Delta$  remains a narrow resonance at large  $N_c$ . We note that in QCD, the  $\Delta$  is not broad,  $\Gamma_\Delta \sim 118\text{MeV}$  [36]. By the Adler-Weisberger relation [37],  $g_A$  is of the same order as  $g_{\Delta NA}$  [38].

## 8 Summary and Conclusions

In order to temper the growth of  $g_A$  for large  $N_c$ , we have argued that the unpaired and paired quarks in a nucleon wavefunction exist in approximately non-overlapping regions. Hence the name Dichotomous Baryon. While this proposal appears radical, we note that we are only changing the wave function of a *one* quark out of  $N_c$ . This proposal also solves related problems in the phenomenology of baryons at large  $N_c$ . While our proposal might well generate other problems which we have not anticipated, perhaps it is worth further consideration. Our conjecture of a Dichotomous Baryon can be tested through lattice simulations of baryons at large  $N_c$  [39]; for example, simulations in the quenched approximation with five colors might be adequate to address the change of  $g_A$  with  $N_c$ .

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## A Computation of $g_A$

The purpose of this appendix is to reproduce a well-known results of  $\langle R_3 \rangle = 1/4 \times (N_c + 2)/3$  for  $N_c$  odd nucleons which is composed of nonrelativistic quarks occupying the same space wavefunction. We will also extend this result to the case with different space wavefunctions.

If we assume that all quarks have the same space wavefunction, we have only to completely symmetrize spin-flavor wavefunctions to satisfy the Pauli's principle. Here we will consider  $|p \uparrow\rangle$  which, in our construction, takes the form:

$$|p \uparrow\rangle^{\text{SF}} = \left\{ \left( |0, 0, 1/2\rangle - |0, 0, -1/2\rangle \right)^{n_d} \otimes |u \uparrow\rangle \right\}_{\text{TS}}. \quad (\text{A.1})$$

As in the text, we will omit third component of spin and isospin of the diquark wavefunction for notational simplicity. The expression is

$$|p \uparrow\rangle^{\text{SF}} = \sum_{m=0}^{n_d} (-1)^m {}_{n_d} C_m \left\{ \left| \frac{n_d}{2} - m \right\rangle \otimes |u \uparrow\rangle \right\}_{\text{TS}}. \quad (\text{A.2})$$

First we have to give a correct normalization. It is crucial to count a number of independent states which are contained in maximally symmetrized  $R_3$  eigenstates.

For instance,  $|u \uparrow u \uparrow u \uparrow\rangle_{\text{TS}}$  has only one independent state, and degeneracy factor  $3!$  for symmetrization, so  $\langle u \uparrow u \uparrow u \uparrow | u \uparrow u \uparrow u \uparrow \rangle_{\text{TS}} = (3!)^2 = 36$ . On the other hand,  $|u \uparrow u \uparrow d \uparrow\rangle_{\text{TS}}$  has three independent states and degeneracy factor  $3!/3 = 2!$ , so  $\langle u \uparrow u \uparrow d \uparrow | u \uparrow u \uparrow d \uparrow \rangle_{\text{TS}} = 3 \times (2!)^2 = 12$ .

Since our diquark state  $|0, 0, 1/2\rangle$  contains  $(u \uparrow, d \downarrow)$  and  $|0, 0, -1/2\rangle$  contains  $(u \downarrow, d \uparrow)$ , a state  $\left\{ \left| \frac{n_d}{2} - m \right\rangle \otimes |u \uparrow\rangle \right\}_{\text{TS}}$  includes  $(n_d - m + 1)$  number of  $u \uparrow$ ,  $(n_d - m)$  number of  $d \downarrow$ , and  $m$  number of  $u \downarrow$  and  $d \uparrow$ . This state can be written in the following way:

$$\left\{ \left| \frac{n_d}{2} - m \right\rangle \otimes |u \uparrow\rangle \right\}_{\text{TS}} = D(n_d - m + 1; n_d - m; m; m) \{ |u \uparrow, \dots\rangle + \dots \}, \quad (\text{A.3})$$

where  $D(n_d - m + 1; n_d - m; m; m) = (n_d - m + 1)!(n_d - m)!(m!)^2$  is a degeneracy factor coming from multiple counting of the same quarks. A number of independent states in the bracket,  $C(n_d - m + 1; n_d - m; m; m)$ , is given by a number of permutation  $(2n_d + 1)!$  divided by degeneracy factor  $D$ . From these observations, we get

$$\left\langle \left( \frac{n_d}{2} - m \right) \otimes u \uparrow \left| \left( \frac{n_d}{2} - m \right) \otimes u \uparrow \right\rangle_{\text{TS}} = D^2 \times C = (2n_d + 1)! \times D. \quad (\text{A.4})$$

Now normalization factor of  $|p \uparrow\rangle^{\text{SF}}$  can be computed as

$$\begin{aligned} \langle p \uparrow | p \uparrow \rangle^{\text{SF}} &= (2n_d + 1)! \sum_{m=0}^{n_d} ({}_{n_d}C_m)^2 \times D(n_d - m + 1; n_d - m; m; m) \\ &= (2n_d + 1)! (n_d!)^2 \times \frac{(n_d + 1)(n_d + 2)}{2}. \end{aligned} \quad (\text{A.5})$$

Now computation of  $\langle p \uparrow | R_3 | p \uparrow \rangle^{\text{SF}}$  is straightforward. We have only to multiply an eigenvalue  $(n_d/2 - m + 1/4)$  when we take the sum of  $m$ ,

$$\begin{aligned} \langle p \uparrow | R_3 | p \uparrow \rangle^{\text{SF}} &= (2n_d + 1)! \sum_{m=0}^{n_d} \left( \frac{n_d}{2} - m + \frac{1}{4} \right) \times ({}_{n_d}C_m)^2 \times D \\ &= (2n_d + 1)! (n_d!)^2 \times \frac{(n_d + 1)(n_d + 2)(2n_d + 3)}{24}. \end{aligned} \quad (\text{A.6})$$

Using  $N_c = 2n_d + 1$ , we reproduce the well-known result,  $\langle p \uparrow | R_3 | p \uparrow \rangle^{\text{SF}} / \langle p \uparrow | p \uparrow \rangle^{\text{SF}} = (N_c + 2)/12$ .

Next we little bit extend the results to the case where all diquark wavefunctions occupy the same space wavefunction,  $A(\vec{r})$ , while extra quark occupies a different space wavefunction,  $B(\vec{r})$ . In such a case, it is no longer useful to separate treatments of spin-flavor and space. Rather we will totally symmetrize spin-flavor-space (SFS) wavefunctions with explicitly expressing space dependence in such a way that  $|u \uparrow, A\rangle$ ,  $|u \uparrow, B\rangle$ , ..., and so on.

Here we introduce a quantity  $x$  which characterizes the overlap between wavefunction  $A$  and  $B$ ,

$$x \equiv |\langle u \uparrow, A | u \uparrow, B \rangle|, \quad (\text{A.7})$$

We will consider the following nucleon states

$$|p \uparrow\rangle^{\text{SFS}} = \sum_{m=0}^{n_d} (-1)^m {}_{n_d}C_m \left\{ \left| \frac{n_d}{2} - m, A \right\rangle \otimes |u \uparrow, B\rangle \right\}_{\text{TS}}. \quad (\text{A.8})$$

where

$$\begin{aligned}
& \left\{ \left| \frac{n_d}{2} - m, A \right\rangle \otimes |u \uparrow, B \rangle \right\}_{\text{TS}} \\
& = D(n_d - m; n_d - m; m; m) \times \left\{ |u \uparrow, A; \dots; u \uparrow, B \rangle + \dots \right\}. \tag{A.9}
\end{aligned}$$

We distinguish  $|u \uparrow, A \rangle$  and  $|u \uparrow, B \rangle$ , so that, compared to previous case, the degeneracy factor  $D$  decreases by a factor  $1/(n_d - m + 1)$  while a number of independent states  $C$  increases by a factor  $(n_d - m + 1)$ .

Now to see how nonzero overlap of  $A$  and  $B$  arises, let us take the inner-product of bracket in (A.9):

$$\begin{aligned}
& \left\{ \langle u \uparrow, A; \dots; u \uparrow, B | + \dots \right\} \left\{ |u \uparrow, A; \dots; u \uparrow, B \rangle + \dots \right\} \\
& = C(n_d - m; n_d - m; m; m) \times \left\{ 1 + x^2 \times (n_d - m) \right\}. \tag{A.10}
\end{aligned}$$

The first term comes from diagonal matrix elements, while second term comes from offdiagonal terms. (Perhaps the simplest way to determine a coefficient of  $x^2$  is to see that  $x = 1$  reproduce the previous results (A.4) ).

Remaining calculations are just a repetition of the previous calculations. A normalization factor is

$$\begin{aligned}
\langle p \uparrow | p \uparrow \rangle^{\text{SFS}} & = (2n_d + 1)! \sum_{m=0}^{n_d} (n_d C_m)^2 \times D(n_d - m; n_d - m; m; m) \\
& \quad \times \left\{ 1 + x^2(n_d - m) \right\} \\
& = (2n_d + 1)!(n_d!)^2 \times \frac{(n_d + 1)(x^2 n_d + 2)}{2}, \tag{A.11}
\end{aligned}$$

which of course reproduces previous results for  $x = 1$ . And also the expectation value of  $R_3$  is

$$\begin{aligned}
\langle p \uparrow | R_3 | p \uparrow \rangle^{\text{SFS}} & = (2n_d + 1)! \sum_{m=0}^{n_d} (n_d C_m)^2 \times D(n_d - m; n_d - m; m; m) \\
& \quad \times \left\{ 1 + x^2(n_d - m) \right\} \times (n_d/2 - m + 1/4) \\
& = (2n_d + 1)!(n_d!)^2 \times \frac{(n_d + 1)(2n_d^2 x^2 + 7n_d x^2 + 6)}{24}, \tag{A.12}
\end{aligned}$$

Finally, taking into account the normalization factor, we get

$$\frac{\langle p \uparrow | R_3 | p \uparrow \rangle^{\text{SFS}}}{\langle p \uparrow | p \uparrow \rangle^{\text{SFS}}} = \frac{2n_d^2 x^2 + 7n_d x^2 + 6}{12(x^2 n_d + 2)} = \frac{1}{12} \frac{(N_c - 1)(N_c + 6)x^2 + 12}{(N_c - 1)x^2 + 4}. \tag{A.13}$$

## B Magnetic Moments

The purpose of this section is to give a relationship between  $\langle R_3 \rangle$  and magnetic moments:

$$\mu_N = \langle N | \left[ \sum_u \mu_u S_3^{(u)} + \sum_d \mu_d S_3^{(d)} \right] | N \rangle. \quad (\text{B.1})$$

We assume  $|N\rangle$  for spin  $\uparrow$  case in the following. By introducing isospin projector, the sum of  $(u, d)$  indices can be extended, so we can rewrite the sum in terms of total spin and  $R_3$  operators,

$$\begin{aligned} \mu_N &= \langle N | \left[ \sum_q \mu_q S_3^{(q)} \left( \frac{1}{2} + I_3^{(q)} \right) + \sum_q \mu_d S_3^{(q)} \left( \frac{1}{2} - I_3^{(q)} \right) \right] | N \rangle \\ &= \frac{\mu_u + \mu_d}{2} \langle N | \sum_q S_3^{(q)} | N \rangle + (\mu_u - \mu_d) \langle N | \sum_q R_3^{(q)} | N \rangle \\ &= \frac{(\mu_u + \mu_d) \pm g_A (\mu_u - \mu_d)}{4}, \end{aligned} \quad (\text{B.2})$$

where  $+$  ( $-$ ) signs for protons (neutrons).

Assuming constituent masses of  $(u, d)$  quarks are almost same, and using  $Q = I_3 + B/2 = I_3 + 1/2N_c$ , we denote quark magnetic moments  $Q\bar{\mu} \equiv Qe/2M_q$ :

$$\mu_u = \frac{N_c + 1}{2N_c} \bar{\mu}, \quad \mu_d = -\frac{N_c - 1}{2N_c} \bar{\mu}. \quad (\text{B.3})$$

Therefore proton and neutron magnetic moments are

$$\mu_{p,n} = \frac{\bar{\mu}}{4} \left( \frac{1}{N_c} \pm g_A \right). \quad (\text{B.4})$$

For conventional baryon wavefunctions,  $N_c = 3$  and  $g_A = 5/3$ , which gives  $\mu_n/\mu_p = -2/3$  (exp:  $-0.685$ ). For our wavefunction with  $x^2 = 0$ ,  $g_A = 1$  and  $\mu_n/\mu_p = -1/2$  ( $-1$ ) for  $N_c = 3$  ( $\infty$ ).

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