2.6 Correspondence Naturality ↔ Reflectivity)

The paper quoted to say

> states of $\eta = +1(-1)$ can be shown to correspond to natural (unnatural) parity exchange in a t-channel model in the limit of small $1/s$.

(quote from [28] (p. 289)) is [29]. So far I have failed to find the statement in there. The necessary requisites for the proof are contained in the predecessor paper [30]. My (trivial) proof based on it goes as follows: one has for t-channel helicity amplitudes in a notation similar to [31] (phase factors differ, also compare [32]):

$$\mathscr{M}^J_{\lambda_3\lambda_1\lambda_4\lambda_2} = \eta\eta_4\eta_2(-1)^{J+s_4-s_2}\mathscr{M}^J_{\lambda_3\lambda_1-\lambda_4-\lambda_2}, \tag{7}$$

for the Jth partial wave in the scattering process $1+2 \to 3+4$ where a particle with intrinsic parity η is exchanged. λ_i, s_i and η_i refer to the helicity, spin and intrinsic parity of particle i. The t-channel helicity amplitude (leaving aside other partial waves to avoid clutter) is then

$$\mathscr{M}_{\lambda_3\lambda_1\lambda_4\lambda_2} = (2J+1)d^J_{\lambda\mu}(\cos\theta_t)\mathscr{M}^J_{\lambda_3\lambda_1\lambda_4\lambda_2} \tag{8}$$

with $\lambda = \lambda_1 - \lambda_3$, $\mu = \lambda_2 - \lambda_4$. Now in the forward-region of the s-channel process, $s \gg t$ and thus $\cos\theta_t$ large. The asymptotic behavior of the Wigner functions is

$$d^J_{-\lambda\mu}(\cos\theta_t) \xrightarrow[\cos\theta_t\to\infty]{} (-1)^\lambda d^J_{\lambda\mu}(\cos\theta_t). \tag{9}$$

In this limit one then has

$$\mathscr{M}_{\lambda_3\lambda_1\lambda_4\lambda_2} \to \eta\eta_4\eta_2(-1)^J(-1)^{\lambda_4-\lambda_2}(-1)^{s_4-s_2}\mathscr{M}_{\lambda_3\lambda_1-\lambda_4-\lambda_2}. \tag{10}$$

With this at hand, we can prove the statement that we're after. Taking the reaction $p(\frac{1}{2}^+) + \pi(0^-) \to p(\frac{1}{2}^+) + X((s_4)^{\eta_4})$, where X is produced in a state of defined reflectivity ϵ (where the sign convention is as above i.e. Suh-Urk's reflectivity = naturality convention, make sure this is the case or flip signs!), and assuming that X decays into two pseudoscalars, which implies $\eta_4 = (-1)^{s_4}$, one then writes the scattering amplitude with defined reflectivity of X (and hence λ_4 non-negative),

$$\mathscr{M}^\epsilon_{\lambda_3\lambda_1\lambda_4 0} \equiv \mathscr{M}_{\lambda_3\lambda_1\lambda_4 0} - \epsilon(-1)^{\lambda_4}\mathscr{M}_{\lambda_3\lambda_1-\lambda_4 0}. \tag{11}$$

Inserting (10), this reduces to

$$\mathscr{M}^\epsilon_{\lambda_3\lambda_1\lambda_4 0} \to (1\pm\epsilon)\mathscr{M}_{\lambda_3\lambda_1\lambda_4 0} \tag{12}$$

where $+$ $(-)$ corresponds to natural (unnatural) exchange. This proves the statement. For $\lambda_4 = 0$, the statement holds exactly because eq. (9) trivially holds for all values of $\cos\theta_t$.

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