Effective Field Theories of Dense (and Very Dense) Matter

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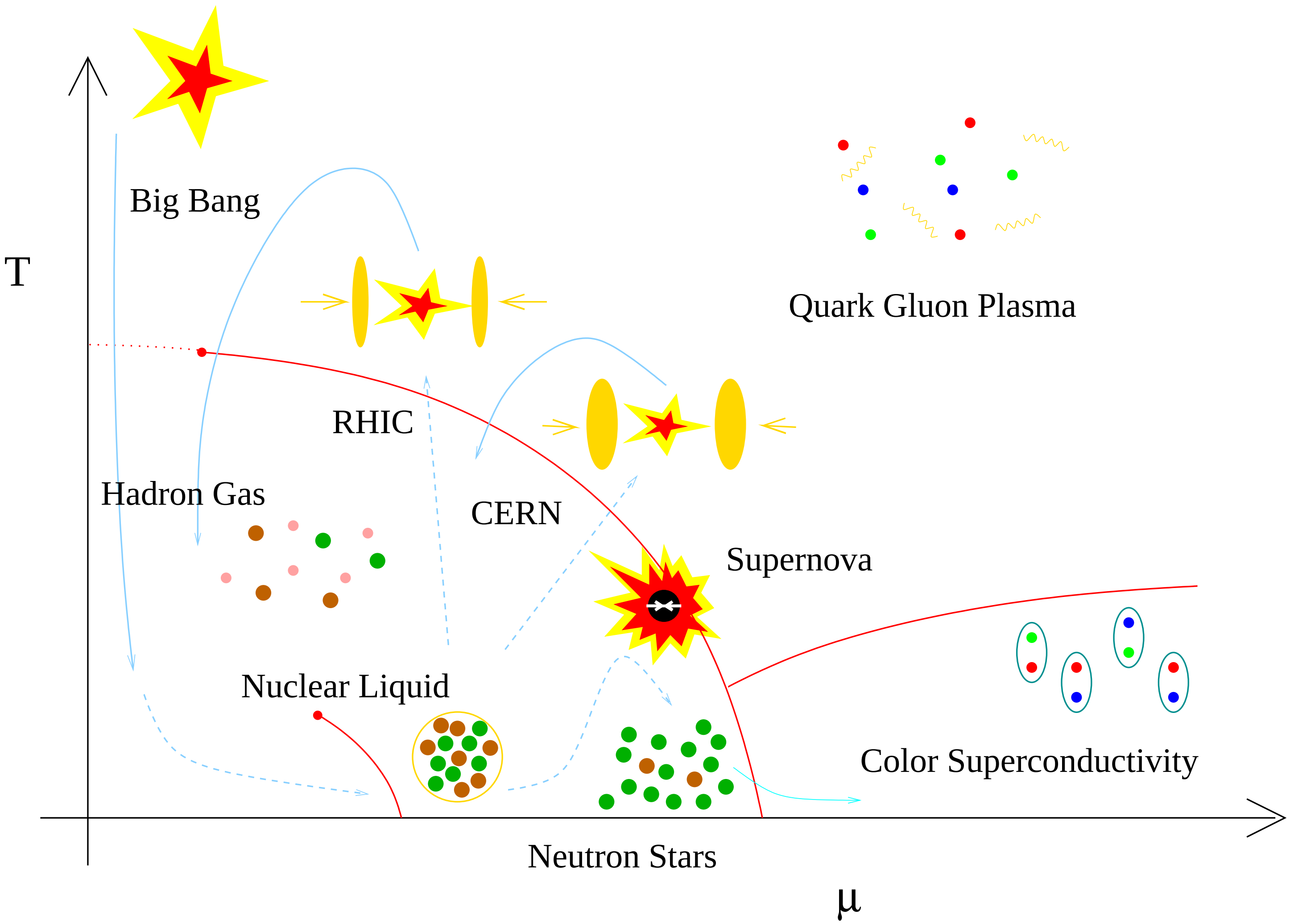
QCD Phase Diagram



Plan

1. Phases of QCD

2. Fermi/Bose liquids

3. Quark Matter

4. Neutron Matter

Quantumchromodynamics

Elementary fields: **Quarks** **Gluons**

$$(q_\alpha)^a_f \left\{ \begin{array}{ll} \text{color} & a = 1, \ldots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{array} \right.$$

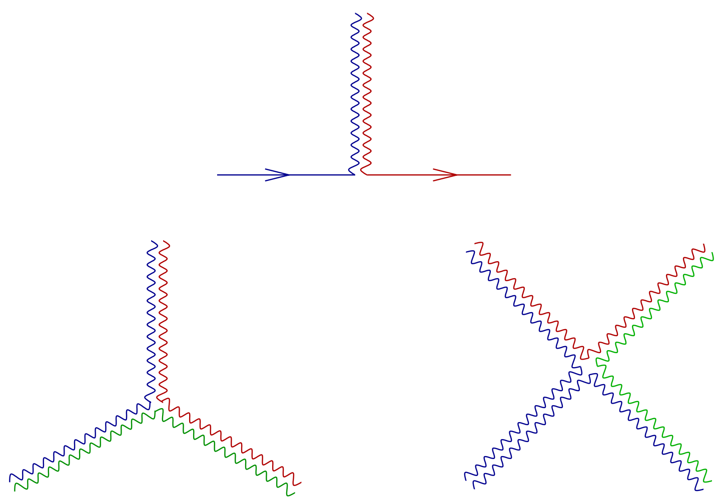
$$A^a_\mu \left\{ \begin{array}{ll} \text{color} & a = 1, \ldots, 8 \\ \text{spin} & \epsilon^\pm_\mu \end{array} \right.$$

Dynamics: non-abelian gauge theory

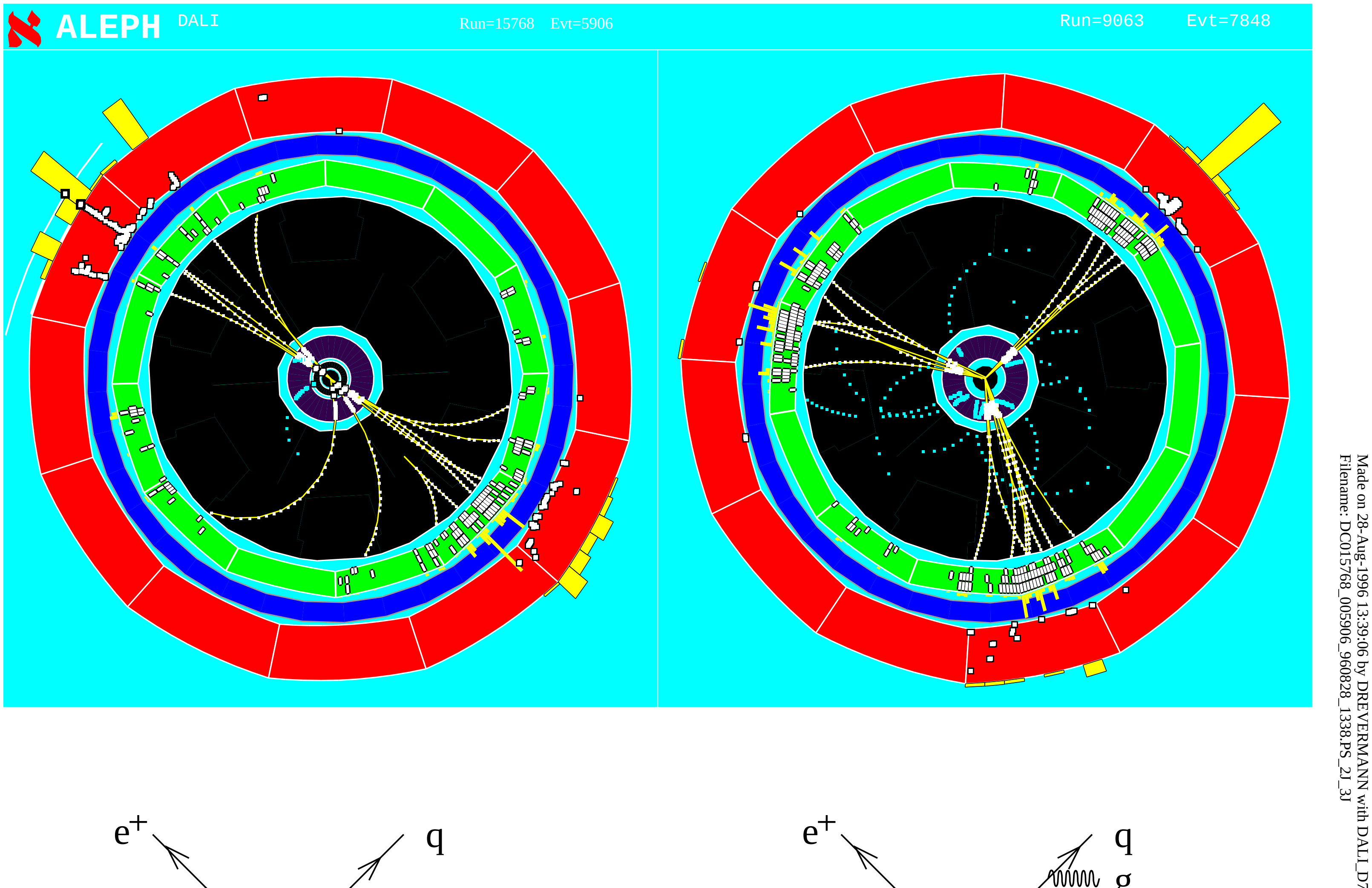
$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

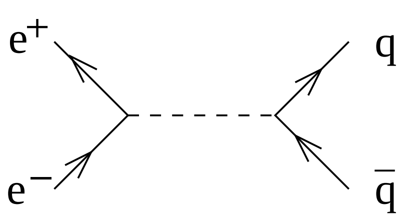
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

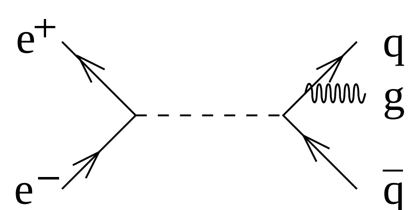
$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a \right) q$$



“Seeing” Quarks and Gluons



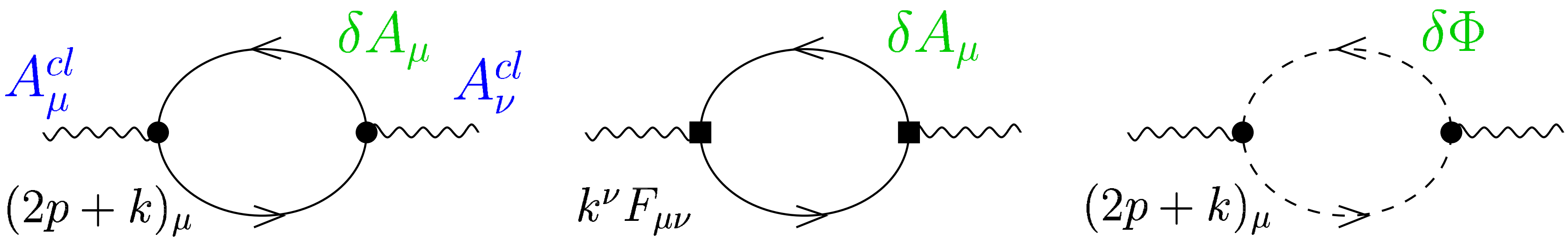




Asymptotic Freedom

Classical field $A_0^{cl} \sim g/r$. Modification due to quantum fluctuations:

$$A_\mu = A_\mu^{cl} + \delta A_\mu \qquad g \to g(\mu) \qquad \beta(g) = \frac{\partial g}{\partial \log(\mu)}$$

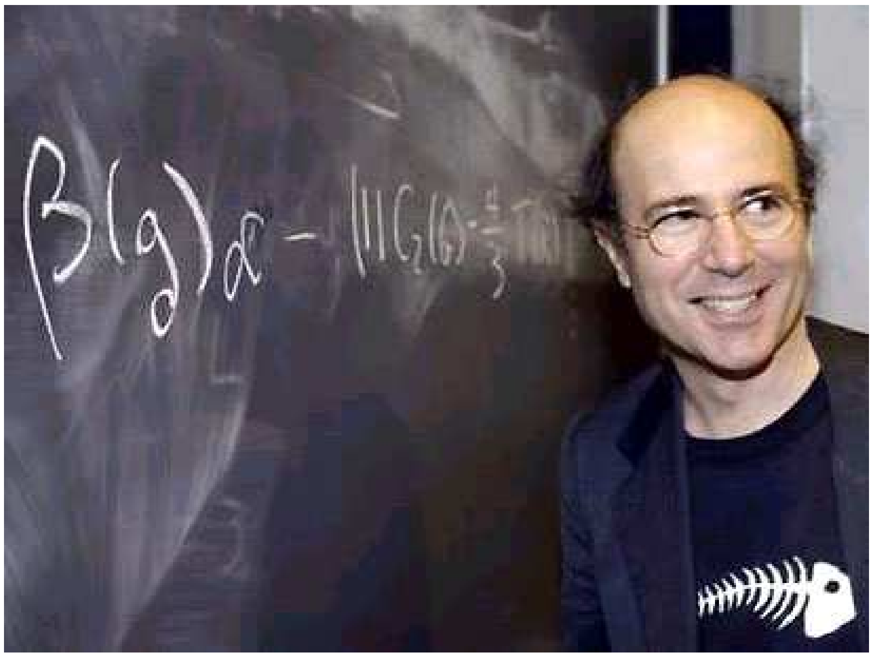


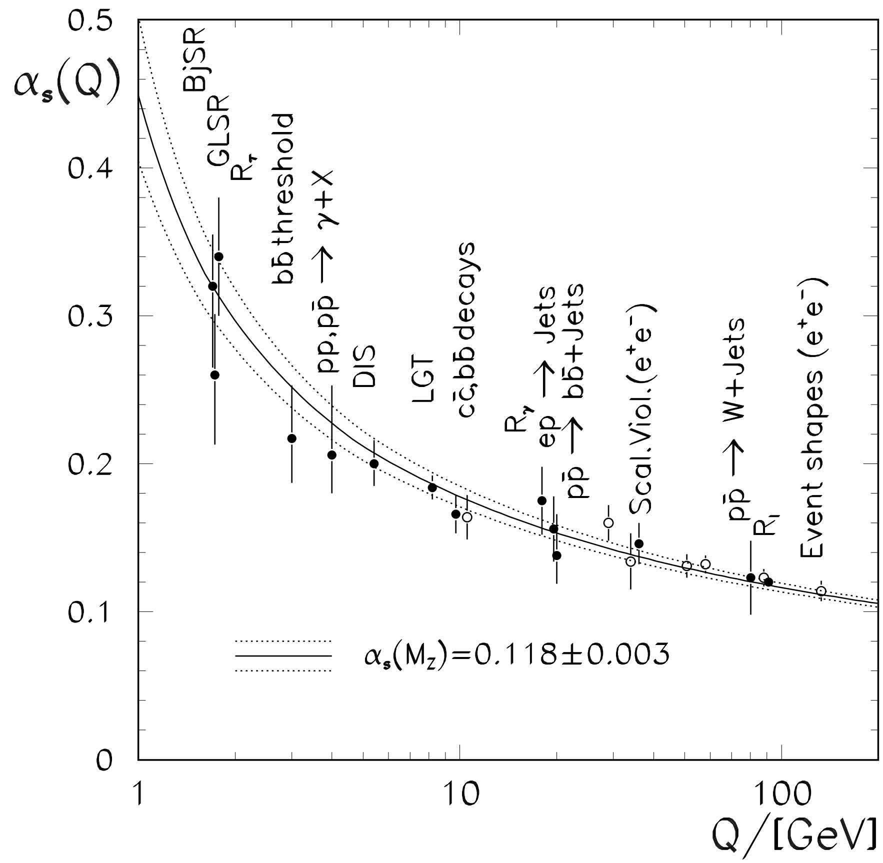
dielectric $\epsilon > 1$ paramagnetic $\mu > 1$ dielectric $\epsilon > 1$

$$\mu\epsilon = 1 \;\Rightarrow\; \epsilon < 1$$

$$\beta(g) = \frac{g^3}{(4\pi)^2}\left\{\left[\frac{1}{3} - 4\right]N_c + \frac{2}{3}N_f\right\}$$

Running Coupling Constant





About Units

QCD Lite* is a parameter free theory

The lagrangian has a coupling constant, g, but no scale.

After renormalization g becomes scale dependent

g is traded for a scale parameter Λ

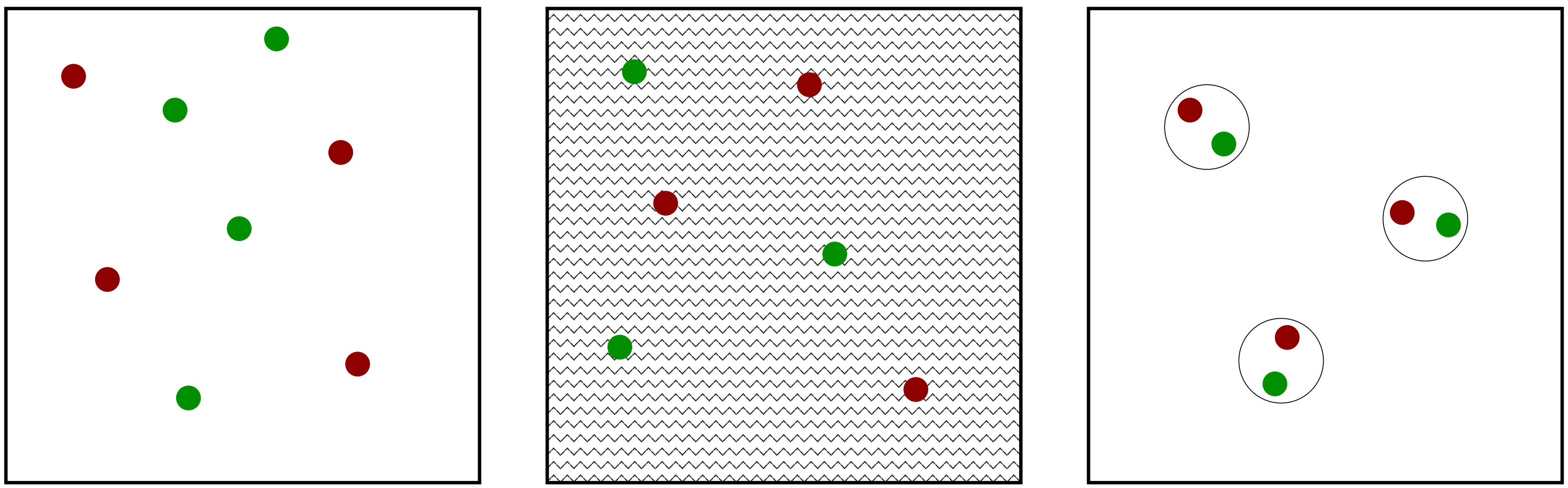
Λ is the only scale, the QCD "standard kilogram"

$$\Lambda_{QCD} \simeq 200\,\mathrm{MeV} \simeq 1\,\mathrm{fm}^{-1}$$

*QCD Lite is QCD in the limit $m_q \to 0$, $m_Q \to \infty$

Phases of Gauge Theories

Coulomb | Higgs | Confinement



$$V(r) \sim \frac{e^2}{r} \qquad V(r) \sim \frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

Standard Model: $U(1) \times SU(2) \times SU(3)$

Phases of Matter

phase	order param	broken symmetry	rigidity phenomenon	Goldstone boson
crystal	ρ_k	translations	rigid	phonon
magnet	$\vec{M}$	rotations	magnetization	magnon
superfluid	$\langle \Phi \rangle$	particle number	supercurrent	phonon
supercond.	$\langle \psi \psi \rangle$	gauge symmetry	supercurrent	none (Higgs)

Gauge Symmetry

Local gauge symmetry $U(x) \in SU(3)_c$

$$\psi \rightarrow U\psi \qquad D_\mu\psi \rightarrow UD_\mu\psi$$

$$A_\mu \rightarrow UA_\mu U^\dagger + iU\partial_\mu U^\dagger \qquad F_{\mu\nu} \rightarrow UF_{\mu\nu}U^\dagger$$

Local gauge "symmetries" cannot be broken (Elitzur's theorem)

Gauge "symmetries" can be realized in different modes

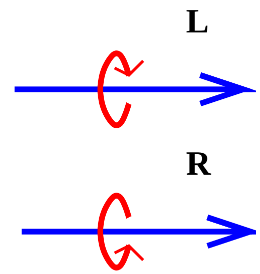
	Coulomb	Higgs	confined
d.o.f:	2 (massless)	3 (massive)	3 (massive)

Distinction between Higgs and confinement phase not always sharp

Chiral Symmetry

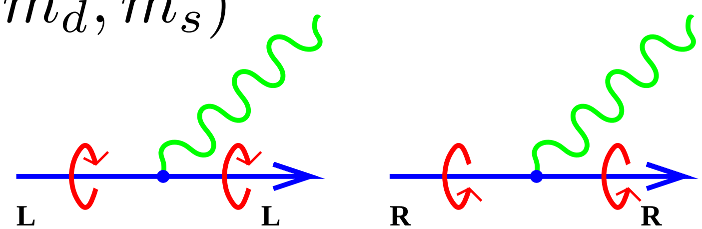
Define left and right handed fields

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

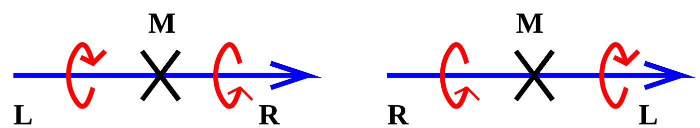


Fermionic lagrangian, $M = \mathrm{diag}(m_u, m_d, m_s)$

$$\mathcal{L} \; = \; \bar{\psi}_L (i\not{D})\psi_L + \bar{\psi}_R (i\not{D})\psi_R$$



$$+ \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$M = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

$$\psi_L \to L\psi_L, \qquad \psi_R \to R\psi_R$$

Chiral Symmetry Breaking

Chiral symmetry implies massless, degenerate fermions

$$m_N^{(1/2)^+} = 935\,\mathrm{MeV} \qquad m_{N^*}^{(1/2)^-} = 1535\,\mathrm{MeV}$$

Chiral symmetry is spontaneously broken

$$\langle \bar\psi_L^f \psi_R^g + \bar\psi_L^f \psi_R^g \rangle \simeq -(230\,\mathrm{MeV})^3\, \delta^{fg}$$

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad (G \to H)$$

Consequences: dynamical mass generation $m_Q = 300\,\mathrm{MeV} \gg m_q$

$$m_N = 890\,\mathrm{MeV} + 45\,\mathrm{MeV} \qquad (\mathrm{QCD},\ 95\%) + (\mathrm{Higgs},\ 5\%)$$

Goldstone Bosons: Consider broken generator Q_5^a

$$[H, Q_5^a] = 0 \qquad Q_5^a|0\rangle = |\pi^a\rangle \qquad\qquad H|\pi^a\rangle = HQ_5^a|0\rangle = Q_5^a H|0\rangle = 0$$

Low energy effective theory for the Goldstone modes

Step 1: Parameterize G/H =pseudoscalar GB's

$$U(x): \qquad U \to LUR^\dagger \qquad\qquad (L, R) \in SU(3)_L \times SU(3)_R$$

Vacuum $U^{fg} = \delta^{fg}$. Massless fluctuations (G/H)

$$U(x) = \exp(i\phi^a\lambda^a/f_\pi) \qquad\qquad \phi^a = (\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta)$$

Step 2: Write most general G invariant effective lagrangian

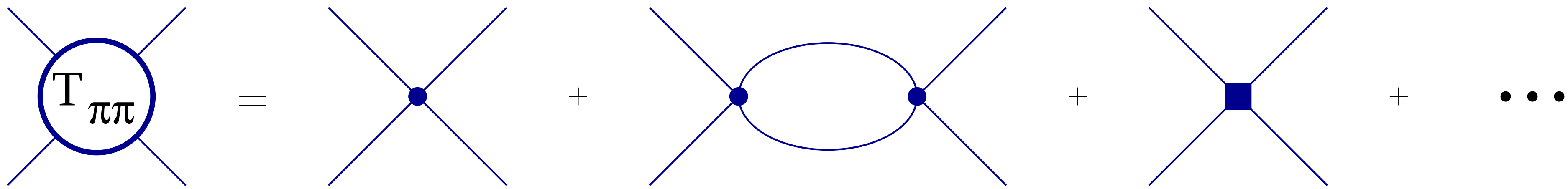
$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \ldots$$

Non-linear sigma model

Expand lagrangian ($SU(2)$ sector)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 + \frac{1}{6f_\pi^2}\left[(\phi^a \partial_\mu \phi^a)^2 - (\phi^a)^2(\partial_\mu \phi^b)^2\right] + O\left(\frac{\partial^4}{f_\pi^4}\right)$$

Step 3: Low energy expansion

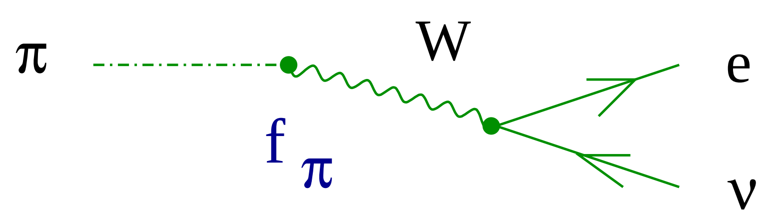


$$T_{\pi\pi} \quad \sim \quad O(k^2/f_\pi^2) \;+ \qquad\qquad O(k^4/f_\pi^4) \qquad\qquad + \quad \ldots$$

Relation to f_π: Couple weak gauge fields

$$\partial_\mu U \rightarrow (\partial_\mu + igW_\mu^\pm \tau^\mp)U$$

$$\mathcal{L} = g f_\pi W_\mu^\pm \partial^\mu \pi^\mp$$



Quark Masses

Non-zero quark masses: $\mathcal{L} = \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_R$

$$M \rightarrow L M R^\dagger \qquad\qquad \text{spurion field } M$$

Chiral lagrangian at leading order in M

$$\mathcal{L} = B\mathrm{Tr}[MU] + h.c.$$

Mass matrix $M = \mathrm{diag}(m_u, m_d m_s)$. Minimize effective potential

$$U_{vac} = 1, \qquad E_{vac} = -B\mathrm{Tr}[M] \qquad \langle\bar{\psi}\psi\rangle = -B$$

Expand around U_{vac}: pion mass

$$m_\pi^2 f_\pi^2 = (m_u + m_d)\langle\bar{\psi}\psi\rangle$$

Chiral expansion

$$\mathcal{L} = f_\pi^4 \left(\frac{\partial U}{\Lambda_\chi}\right)^m \left(\frac{m_\pi}{\Lambda_\chi}\right)^n \qquad\qquad \Lambda_\chi = 4\pi f_\pi$$

Symmetries of the QCD Vacuum: Summary

Local $SU(3)$ gauge symmetry

confined: $V(r) \sim kr$

Chiral $SU(3)_L \times SU(3)_R$ symmetry

spontaneously broken to $SU(3)_V$

Axial $U(1)_A$ symmetry

anomalous : $\partial_\mu A^0_\mu = \frac{N_f}{16\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$

Vectorial $U(1)_B$ symmetry

unbroken: $B = \int d^3x \, \psi^\dagger \psi$ conserved

QCD at non-zero temperature

Basic object: Partition function

$$Z = \mathrm{Tr}[e^{-\beta H}], \quad \beta = 1/T \qquad\qquad F = T\log(Z)$$

Basic trick

$$Z = \mathrm{Tr}[e^{-i(-i\beta)H}] \qquad\qquad \text{imaginary time evolution}$$

Path integral representation

$$Z = \int DA_\mu D\psi \exp\left(-\int_0^\beta d\tau \int d^3x\ \mathcal{L}_E\right)$$

$$A_\mu(\vec{x},\beta) = A_\mu(\vec{x},0);\ \psi(\vec{x},\beta) = -\psi(\vec{x},0)$$

Starting point of perturbative and lattice approaches

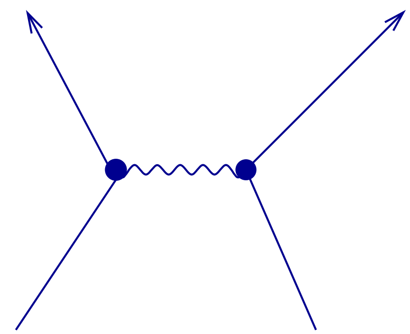
The High T Phase: Qualitative Argument

High T phase: Weakly interacting gas of quarks and gluons?

typical momenta $p \sim 3T$

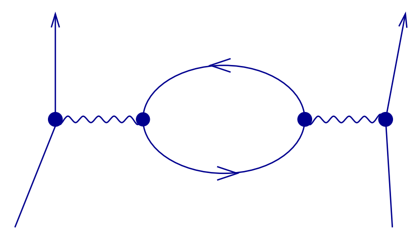
Large angle scattering involves large momentum transfer

effective coupling is small



Small angle scattering is screened (not anti-screened!)

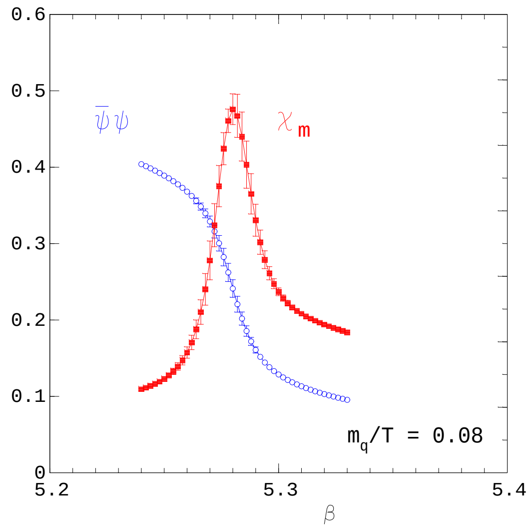
coupling does not become large

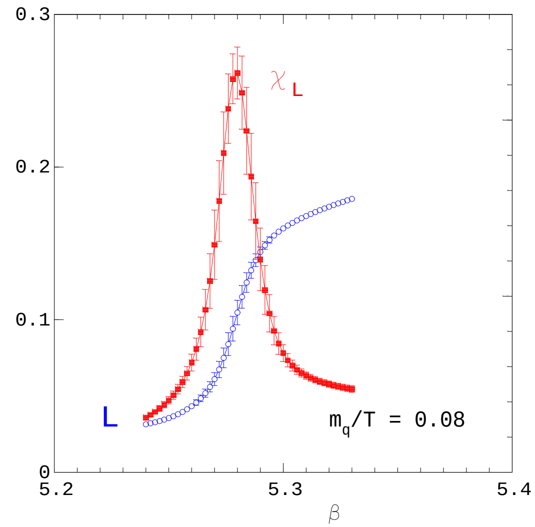


Quark Gluon Plasma

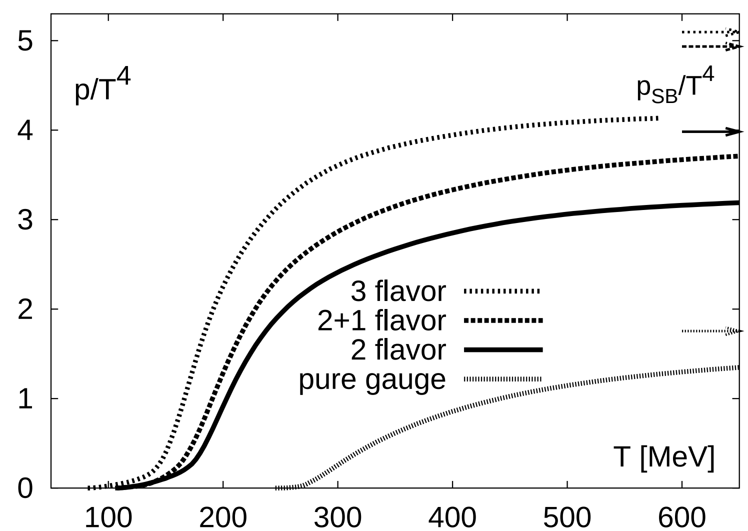
Lattice Results

order parameters





equation of state



QCD at Finite Density

Partition function

$$Z = \mathrm{Tr}\left[e^{-\beta(H-\mu N)}\right] \qquad \beta = 1/T \qquad N = \int d^3x\, \psi^\dagger \psi$$

Path integral representation (euclidean)

$$Z = \int DA_\mu \det(i\not{D} + i\mu\gamma_4)e^{-S} = \int DA_\mu e^{i\phi}|\det(i\not{D} + i\mu\gamma_4)|e^{-S}$$

Sign problem: importance sampling does not work

Also: No general theorems (a la Vafa-Witten)

Phase structure much richer

(breaking of translational, rotational, parity, isospin, ... symmetry)

Evading the Sign Problem I

QCD like theories with extra “C” symmetry

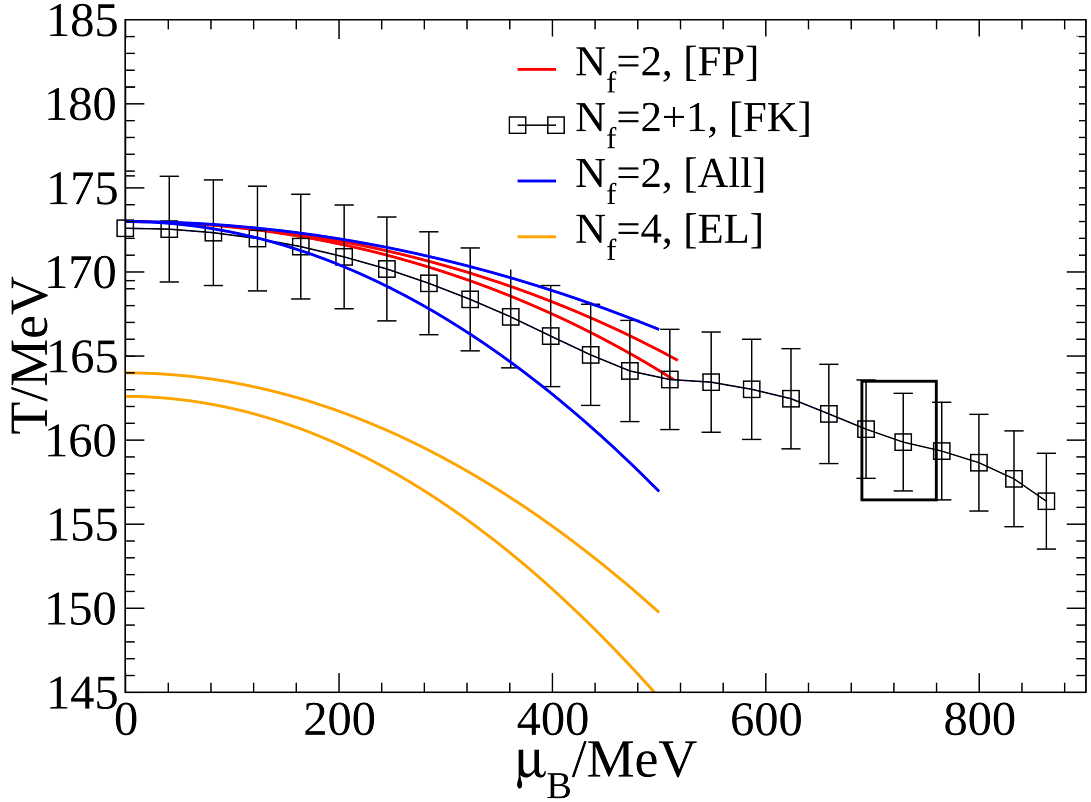
$$\det(D)\,\det(CDC^{-1}) = \det(D)\,\det(D)^* = |\det(D)|^2$$

- QCD with $SU(2)_F$ symmetry at non-zero μ_{I_3}
- QCD with $N_c = 2$ colors
- QCD with fermions in the adjoint representation

These theories have some common features

- Charged Goldstone Bosons, Bose condensation
- No Higgs phase (color superconductivity)

Evading the Sign Problem II



[FK] Improved re-weighting, [FP] imaginary chemical potential

[All] Taylor expansions