Feynman parametrization

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The **Feynman parametrization** is a nifty technology for evaluating loop integrals which crop up when evaluating Feynman diagrams with one or more loops.

Richard Feynman came up with the amazing observation that

$$\frac{1}{AB} = \int_0^1 \frac{du}{\left[uA + (1-u)B\right]^2}$$

and suddenly, evaluating integrals like

$$\int \frac{dp}{A(p)B(p)} = \int dp \int_0^1 \frac{du}{\left[uA(p) + (1-u)B(p)\right]^2} = \int_0^1 du \int \frac{dp}{\left[uA(p) + (1-u)B(p)\right]^2}$$

is a breeze!

More generally,

$$\frac{1}{A_1 \cdots A_n} = (n-1)! \int_0^1 du_1 \cdots \int_0^1 du_n \frac{\delta(u_1 + \ldots + u_n - 1)}{\left[u_1 A_1 + \ldots + u_n A_n\right]^n}$$

Even more generally,

$$\frac{1}{A_1^{\alpha_1} \cdots A_n^{\alpha_n}} = \frac{(\alpha_1 + \ldots + \alpha_n - 1)!}{(\alpha_1 - 1)! \cdots (\alpha_n - 1)!} \int_0^1 du_1 \cdots \int_0^1 du_n \frac{\delta(u_1 + \ldots + u_n - 1) u_1^{\alpha_1 - 1} \cdots u_n^{\alpha_n - 1}}{\left[u_1 A_1 + \ldots + u_n A_n\right]^{\alpha_1 + \ldots + \alpha_n}}$$

See also Schwinger parametrization.