MAID
Analysis Techniques

- introduction
- a dynamical approach to meson electroproduction
- the unitary isobar model MAID
- comparison of multipoles between MAID and SAID
- resonance parameters and transition form factors
- summary and outlook

The MAID Project

http://www.kph.uni-mainz.de/MAID/

MAID $e + N \to e' + N + \pi$ unitary isobar model

S. Kamalov, D. Drechsel, L.T.

currently Maid03 on the web, Maid05 is ready

DMT $e + N \to e' + N + \pi$ dynamical model

S. Kamalov, S.N. Yang, D. Drechsel, L.T.

currently DMT 2001

KAON-MAID $e + N \to e' + \{\Lambda, \Sigma\} + K$ isobar model

T. Mart, C. Bennhold, H. Haberzettl, L.T.

KAONMAID 2000

ETA-MAID $e+N \rightarrow e'+N+\eta$ isobar model

W.-T. Chiang, C. Bennhold, D. Drechsel, L.T.

ETAMAID 2001

$\gamma+p \rightarrow p+\eta$ regge isobar model

W.T.-Chiang, M. Vanderhaeghen, L.T.

ETAMAID 2003

ETA'-MAID $\gamma+p \rightarrow p+\eta'$ regge isobar model

W.-T. Chiang, M. Vanderhaeghen, L.T.

ETAPRIMEMAID 2003

DR-MAID $e+N \rightarrow e'+N+\pi$ dispersion model

S. Kamalov, D. Drechsel, L.T.

in progress

why partial wave analysis

- learn about nucleon resonances
  in particular about their e.m. structure
  transition moments, form factors G_E, G_M, G_C
- study threshold amplitudes
  to compare with ChPT
- obtain cross sections and amplitudes
  over a wide range of energies for dispersive studies
  - sum rules: GDH, FFR, etc.
  - two-photon reactions: Compton, VCS, SSA, etc.
- find missing or exotic resonances

Isobar and Dynamical Models

dynamical approach to pion electroproduction

coupled channels Lippmann-Schwinger equation: $T = V + VG_0T$

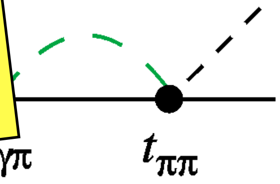
$$t_{\pi\pi} = v_{\pi\pi} + v_{\pi\pi}\, g_\pi\, t_{\pi\pi}$$

$$t_{\gamma\pi} = v_{\gamma\pi} + v_{\gamma\pi}\, g_\pi\, t_{\pi\pi}$$

for resonance analysis, we split the amplitude in background + resonance

the background is calculated dynamically with driving potential (Born+vector meson exchange)

$$t^\alpha_{\gamma\pi}(E) = \underbrace{v^\alpha_{\gamma\pi}(E)(1 + it^\alpha_{\pi\pi})}_{\text{K-matrix approx.}} + P\int \frac{v^\alpha_{\gamma\pi}(E')\, t^\alpha_{\pi\pi}(E,E')}{}\, dE'$$



ude) part

part of the resonance dressing
in K-matrix and isobar models
- as well as in PDG definition -

the resonance part is parametrized phenomenologically with Breit-Wigner functions

$$t^\alpha_{\gamma,\pi}(W,Q^2) = A_\lambda(Q^2)\, f_{\gamma N}(W)\, \frac{\Gamma_{tot}(W) M_R}{M_R^2 - W^2 - iM_R\Gamma_{tot}(W)}\, f_{\pi N}(W)\, e^{i\Psi(W)}$$

MAID

the Mainz-Dubna Unitary Isobar Model

$$t^{\alpha}_{\gamma,\pi} = v^{\alpha}_{\gamma,\pi}(Born+\omega,\rho)\,\left(1+i\,t^{\alpha}_{\pi,\pi}\right)$$

K-matrix unitarization

$$+\; t^{\alpha}_{\gamma,\pi}(Resonances)\; e^{i\Phi(W)}$$

unitarization phase
determined by the Watson theorem, below 2π threshold
relaxed above 2π threshold

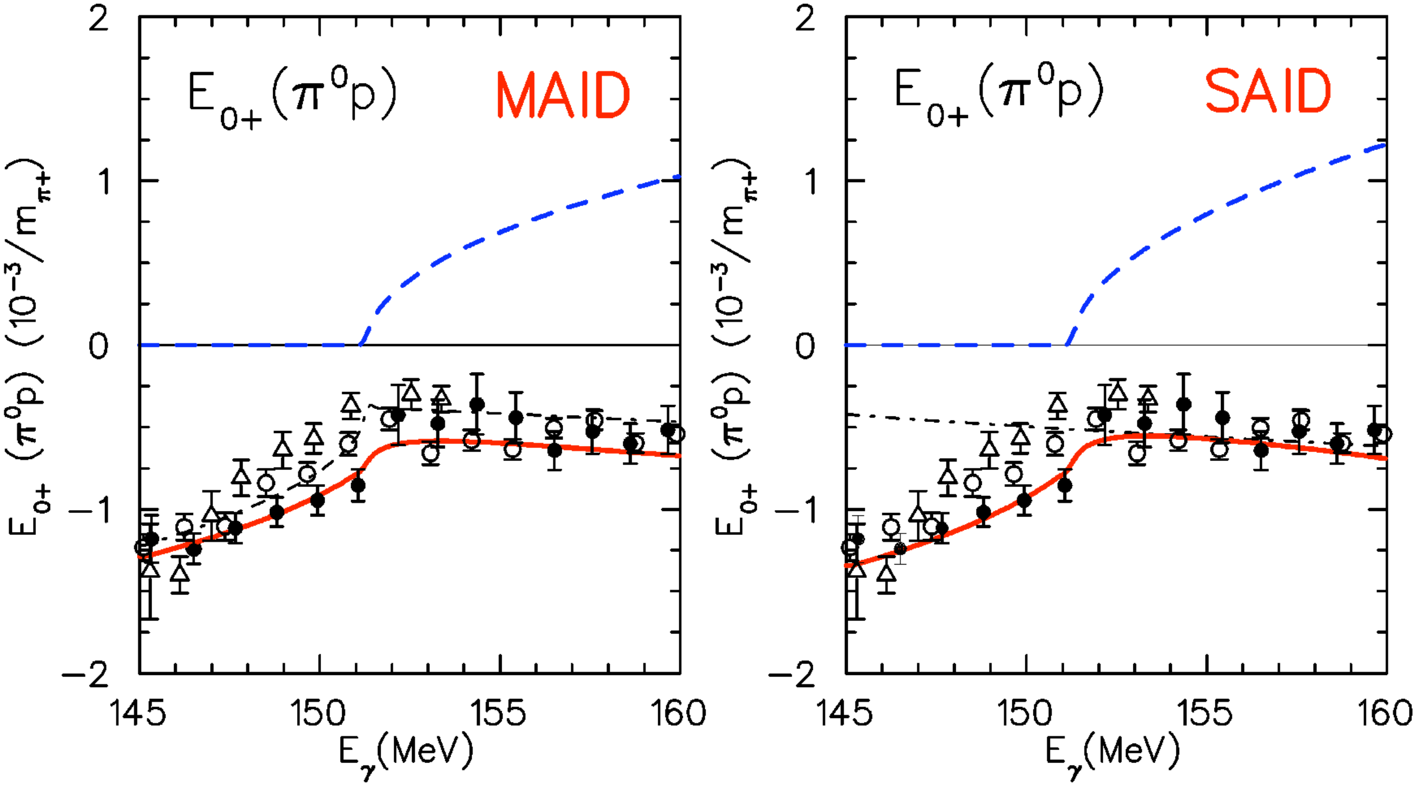
- the Resonances in MAID are dressed resonances

Unitarity at Pion Threshold

here demonstrated with dispersion relations: MAID and SAID groups, Phys. Rev. C66 (2002)

$$Re\, \widetilde{\mathcal{M}}_\alpha(W,Q^2) = \widetilde{\mathcal{M}}_\alpha^{Pole}(W,Q^2) + \frac{P}{\pi}\int_{W_{thr.}}^{\infty} dW' \frac{Im\, \widetilde{\mathcal{M}}_\alpha(W',Q^2)}{W'-W}$$

$$+ \frac{1}{\pi}\int_{W_{thr.}}^{\infty} dW' \sum_\beta \widetilde{\mathcal{K}}_{\alpha\beta}(W,W',Q^2)\, Im\, \widetilde{\mathcal{M}}_\beta(W',Q^2)$$

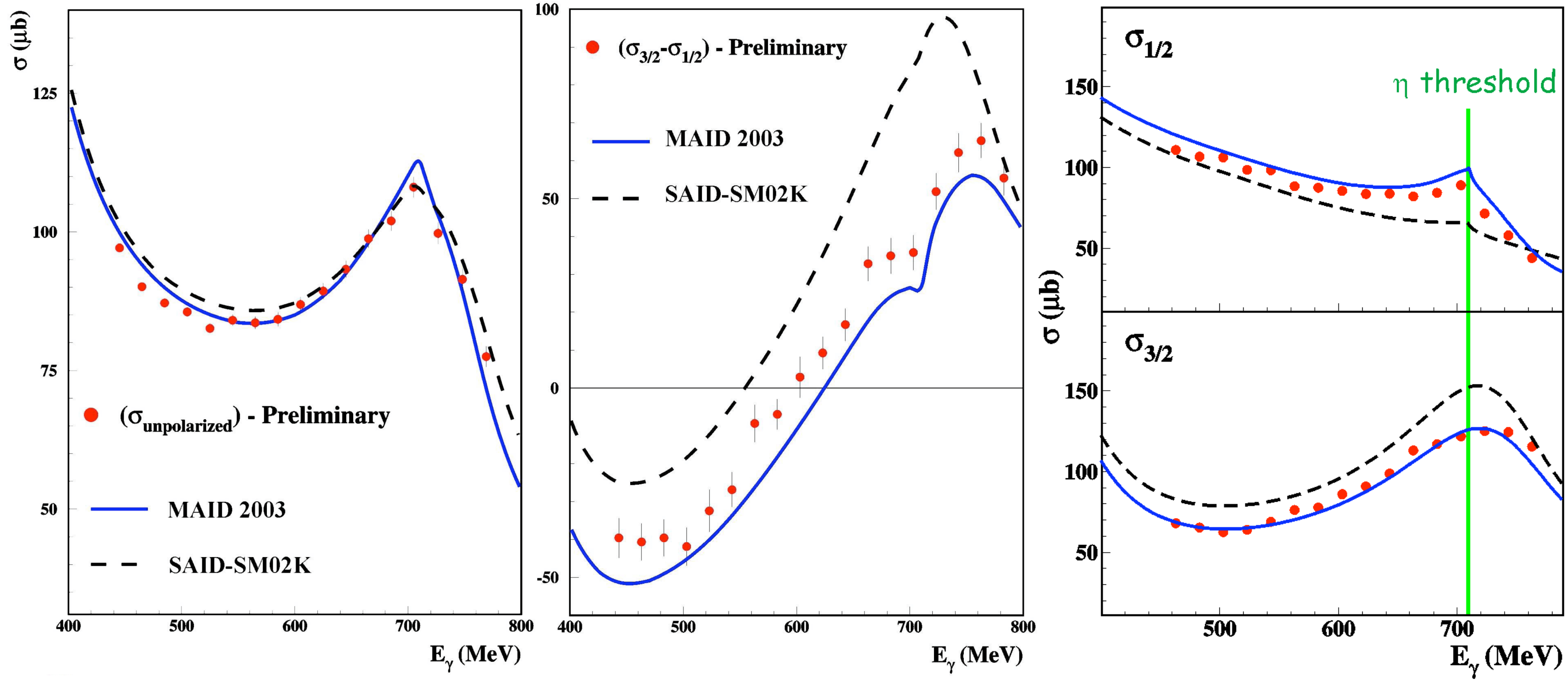


Unitarity at Eta Threshold

demonstrated with new exp. data of GDH collaboration, MAMI 2005

for further details see Allessandro Bragheri, Thursday morning

$$\vec{\gamma}+\vec{p}\rightarrow n+\pi^{+}$$



background potential $V^{bg}_{\pi\gamma}$

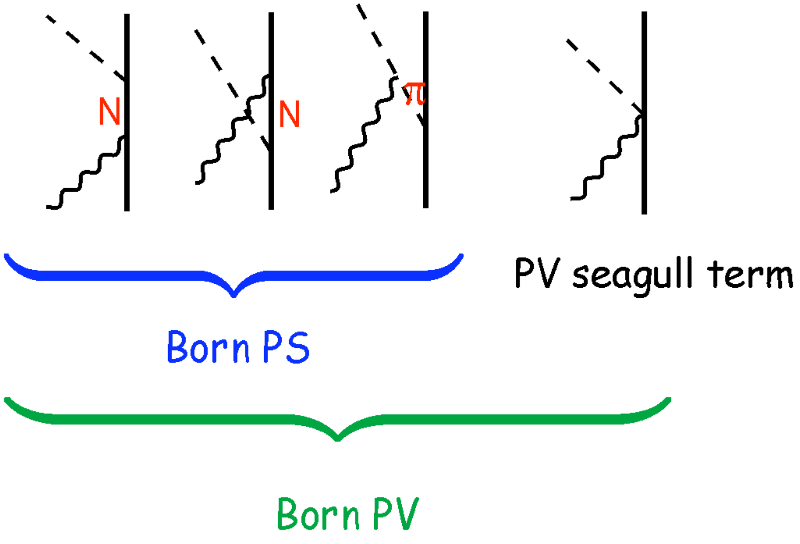
Born terms from field theoretical Lagrangians

e.m. current of nucleon: $\mathcal{L}_{\gamma NN} = -e\bar{\psi}\left[\gamma_\mu \mathcal{A}^\mu F_1^{p,n}(Q^2) + \frac{\sigma_{\mu\nu}}{2m_N}\partial^\mu \mathcal{A}^\nu F_2^{p,n}(Q^2)\right]\psi$

e.m. current of pion: $\mathcal{L}_{\gamma\pi\pi} = e\left[(\partial_\mu \pi)^\dagger \times \pi\right]_3 \mathcal{A}^\mu F_\pi(Q^2)$

pseudoscalar πN coupling: $\mathcal{L}^{\mathrm{PS}}_{\pi NN} = ig\bar{\psi}\gamma_5 \tau \cdot \psi\pi$

pseudovector πN coupling: $\mathcal{L}^{\mathrm{PV}}_{\pi NN} = -\frac{f}{m_\pi}\bar{\psi}\gamma_5\gamma_\mu \tau \cdot \partial^\mu \pi\psi$



PS-PV mixing:

$$\mathcal{L}^{HM}_{\pi NN} = \frac{\Lambda_m^2}{\Lambda_m^2 + q_0^2}\mathcal{L}^{\mathrm{PV}}_{\pi NN} + \frac{q_0^2}{\Lambda_m^2 + q_0^2}\mathcal{L}^{\mathrm{PS}}_{\pi NN}$$

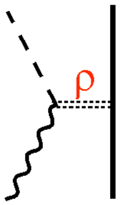
is equivalent to PV coupling plus a gauge invariant contact term:

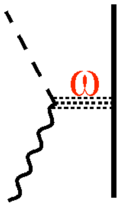
$$j^\mu_\kappa = \frac{ieg}{2m}\kappa_N F(q_0^2)\frac{\sigma^{\mu\nu}k_\nu}{2m}\gamma_5$$

similar approach for vector meson exchange in the t-channel

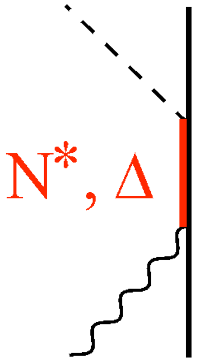
$$\mathcal{L}_{\gamma\pi V} = e\frac{\lambda_V}{m_\pi}\varepsilon_{\mu\nu\rho\sigma}(\partial^\mu A^\nu)\,\pi_i\,\partial^\rho(\delta_{i3}\omega^\sigma + \rho_i^\sigma)\,F_V(Q^2)$$

$$\mathcal{L}_{VNN} = \bar{\psi}\left(g_{V1}\gamma_\mu + \frac{g_{V2}}{2m_N}\sigma_{\mu\nu}\partial^\nu\right)(\omega^\mu + \tau\cdot\rho^\mu)\psi$$





s-channel resonance contributions with Breit-Wigner ansatz



advantage: no off-shell effects, no anti-resonances

each resonance with total spin $J = L \pm \frac{1}{2}$ and isospin I
contributes only to multipoles of the partial wave $L_{2I,2J}$

e.g. $\Delta(1232)P_{33}: \quad E_{1+}^{(3/2)},\ M_{1+}^{(3/2)},\ S_{1+}^{(3/2)}$

$$A_{l\pm}(W) = \bar{\mathcal{A}}_{l\pm}\, f_{\gamma N}(W)\, \frac{\Gamma_{tot}\, W_R\, e^{i\phi}}{W_R^2 - W^2 - iW_R\Gamma_{tot}}\, f_{\pi N}(W)\, C_{\pi N}$$

$$f_{\gamma N}(W) = \left(\frac{k_W}{k_R}\right)^n \left(\frac{X^2+k_R^2}{X^2+k_W^2}\right), \quad n \geqslant l_\gamma \qquad f_{\pi N}(W) = \left[\frac{1}{(2j+1)\pi}\,\frac{k_W}{|q|}\,\frac{m_N}{W}\,\frac{\Gamma_{\pi N}}{\Gamma_{tot}^2}\right]^{1/2}$$

energy dependent width:
(implicitly includes coupling
with 2π and η channels)

$$\Gamma_{tot} = \Gamma_{\pi N} + \Gamma_{in}$$

$$\Gamma_{\pi N} = \beta_\pi \Gamma_R \left(\frac{|q|}{q_R}\right)^{2l+1} \left(\frac{X^2+q_R^2}{X^2+q^2}\right)^{l} \frac{W_R}{W}$$

$$\Gamma_{in} = (1-\beta_\pi)\Gamma_R \left(\frac{q_{2\pi}}{q_0}\right)^{2l+4} \left(\frac{X^2+q_0^2}{X^2+q_{2\pi}^2}\right)^{l+2}$$

Nucleon Resonances Overview $(< 2\ \text{GeV},\ > **)$

resonance	partial wave $L_{2I,2J}$	overall status	πN	γN	new rating (BRAG)	in MAID since	multipoles
$\Delta(1232)\ \frac{3}{2}^+$	P_{33}	****	****	****	****	1998	E_{1+}, M_{1+}, L_{1+}
$N(1440)\ \frac{1}{2}^+$	P_{11}	****	****	***	***	1998	M_{1-}, L_{1-}
$N(1520)\ \frac{3}{2}^-$	D_{13}	****	****	****	****	1998	E_{2-}, M_{2-}, L_{2-}
$N(1535)\ \frac{1}{2}^-$	S_{11}	****	****	***	***	1998	E_{0+}, L_{0+}
$\Delta(1600)\ \frac{3}{2}^+$	P_{33}	***	***	**	**	–	E_{1+}, M_{1+}, L_{1+}
$\Delta(1620)\ \frac{1}{2}^-$	S_{31}	****	****	***	***	2000	E_{0+}, L_{0+}
$N(1650)\ \frac{1}{2}^-$	S_{11}	****	****	***	****	1998	E_{0+}, L_{0+}
$N(1675)\ \frac{5}{2}^-$	D_{15}	****	****	****	****	2003	E_{2+}, M_{2+}, L_{2+}
$N(1680)\ \frac{5}{2}^+$	F_{15}	****	****	****	****	1998	E_{3-}, M_{3-}, L_{3-}
$N(1700)\ \frac{3}{2}^-$	D_{13}	***	***	**	**	–	E_{2-}, M_{2-}, L_{2-}
$\Delta(1700)\ \frac{3}{2}^-$	D_{33}	****	****	***	**	1998	E_{2-}, M_{2-}, L_{2-}
$N(1710)\ \frac{1}{2}^+$	P_{11}	***	***	***	***	–	M_{1-}, L_{1-}
$N(1720)\ \frac{3}{2}^+$	P_{13}	****	****	****	**	2003	E_{1+}, M_{1+}, L_{1+}
$\Delta(1905)\ \frac{5}{2}^+$	F_{35}	****	****	***	**	2003	E_{3-}, M_{3-}, L_{3-}
$\Delta(1910)\ \frac{1}{2}^+$	P_{31}	****	****	*	**	2003	M_{1-}, L_{1-}
$\Delta(1950)\ \frac{7}{2}^+$	F_{37}	****	****	****	****	2003	E_{3+}, M_{3+}, L_{3+}

resonance gap →

some different techniques in different channels

MAID	$t_{\gamma,\pi} = (Born + \underbrace{\omega,\rho}_{\text{poles}})\,(1 + i\,t_{\pi N}) + \sum_{k=1}^{13} Res_k\, e^{i\Phi_k}$
η − MAID	$t_{\gamma,\eta} = (Born + \underbrace{\omega,\rho}_{\text{poles/Regge}}) + \sum_{k=1}^{8} Res_k$ $S_{11}(1535)$, $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$ $D_{13}(1520)$, $D_{13}(1700)$, $D_{15}(1675)$, $F_{15}(1680)$
η' − MAID	$t_{\gamma,\eta'} = (Born + \underbrace{\omega,\rho}_{\text{Regge}}) + \sum_{k=1}^{3} Res_k$ $S_{11}(1930)$, $P_{11}(1950)$, $P_{13}(1950)$
K − MAID	$t_{\gamma,K} = (Born + \underbrace{K^*, K_1}_{\text{poles}}) + \sum_{k=1}^{5} Res_k$ for K, Λ $S_{11}(1650)$, $P_{11}(1710)$, $P_{13}(1720)$, $D_{13}(1900)$ for K, Σ $+\ S_{31}(1900)$, $P_{31}(1990)$

Results
on
photo- and electroproduction
of
nucleon resonances

Resonance Parameters

hadronic parameters: (MAID uses SAID pion nucleon analysis)

M_R : mass

Γ_R : total width

$\Gamma_\pi\,,\,\Gamma_\eta\,,\,\Gamma_{\pi\pi}\,,\,\cdots$: partial widths

electromagnetic parameters:

$A_{1/2},\,A_{3/2},\,S_{1/2}$: photon decay helicity couplings (amplitudes, form factors)

world database on pion photoproduction

GWU/SAID database from 1960 - 2001:

$d\sigma/d\Omega,\ \Sigma,\ T,\ P,\ \sigma_{tot}$

1500 pts of $p\pi^-$
6900 pts of $n\pi^+$
7700 pts of $p\pi^0$

more recent data:

Mainz/TAPS	(2001):	1500	pts of $p\pi^0$	$d\sigma_0, \Sigma$
Mainz/GDH	(2002):	300	pts of $p\pi^0$	$d\sigma_0, d\sigma_{31}$
Mainz/GDH	(2002):	300	pts of $n\pi^+$	$d\sigma_0, d\sigma_{31}$
GRAAL	(2001):	240	pts of $n\pi^+$	Σ
GRAAL	(2005):	1330	pts of $p\pi^0$	$d\sigma_0, \Sigma$
BONN	(2004):	670	pts of $p\pi^0$	$d\sigma_0$
BNL	(2004):	280	pts of $p\pi^-$	$d\sigma_0$

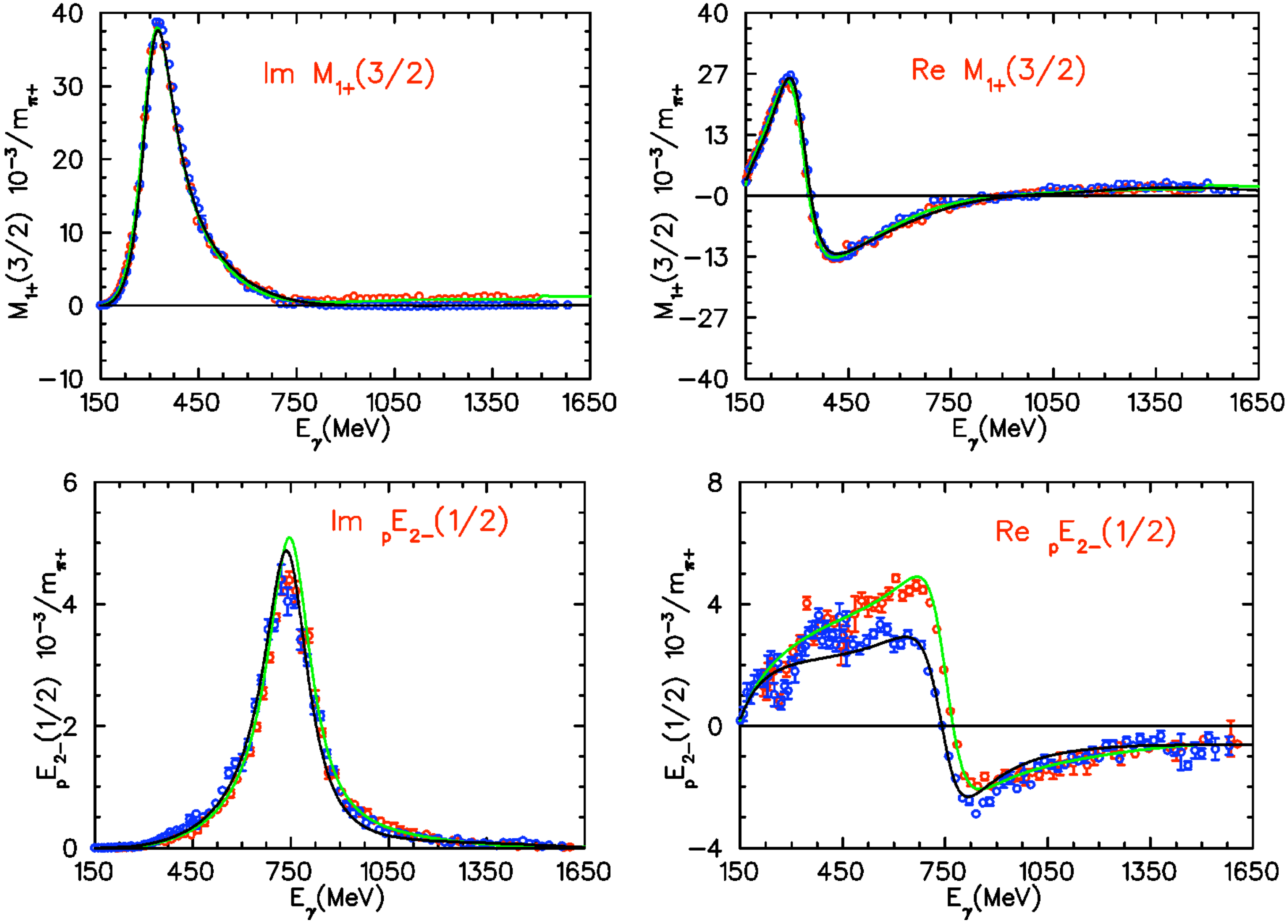
comparison between MAID and SAID

— MAID energy dep. sol.

○ MAID single-energy sol.

— SAID energy dep. sol.

○ SAID single-energy sol.



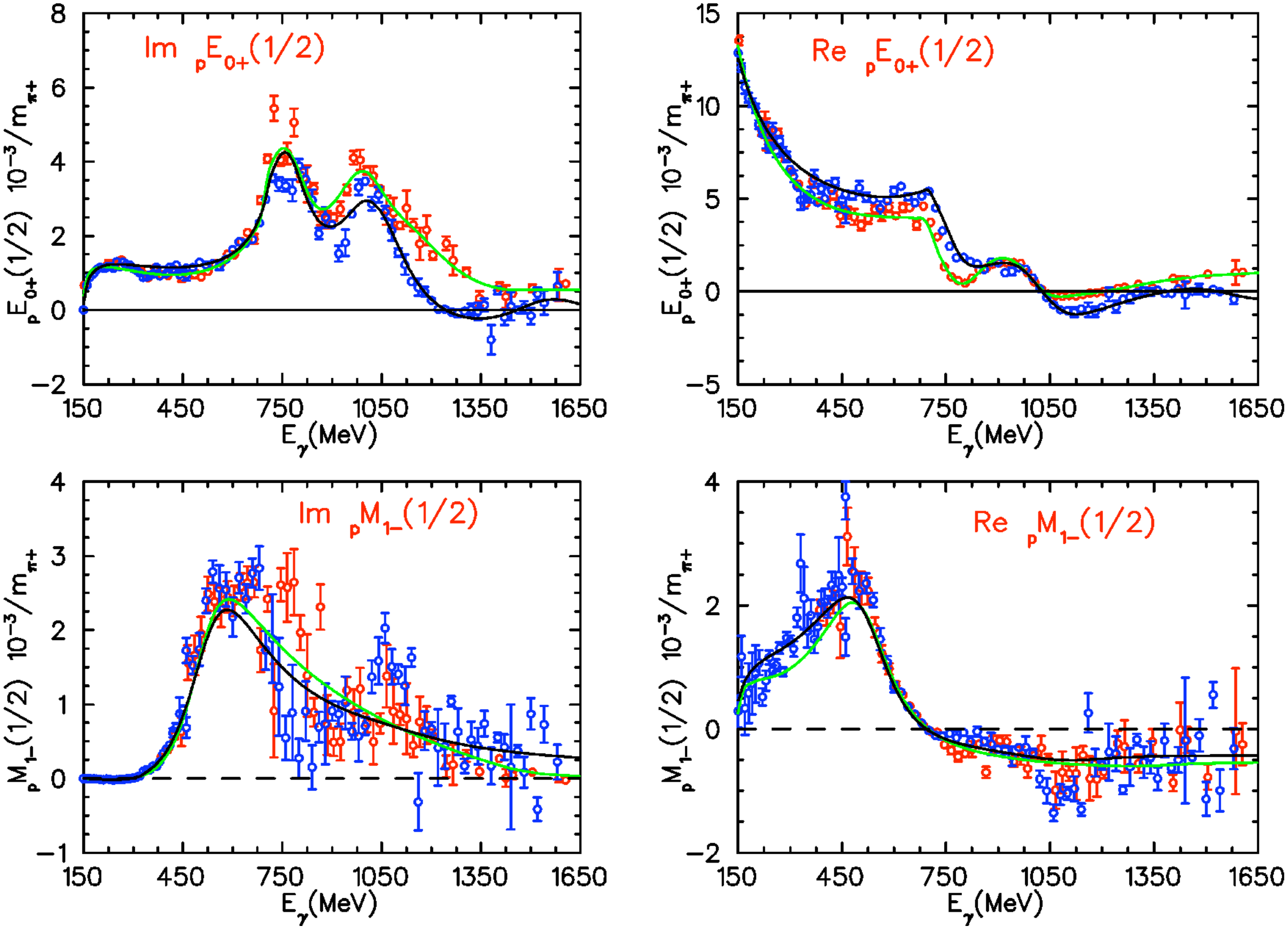
comparison between MAID and SAID

— MAID energy dep. sol.

○ MAID single-energy sol.

— SAID energy dep. sol.

○ SAID single-energy sol.



helicity amplitudes $A_{1/2}$ and $A_{3/2}$ at $Q^2=0$

		proton				neutron			
		PDG	GW 02	Maid 03	Maid 05	PDG	GW 02	Maid 03	Maid 05
$P_{33}(1232)$	$A_{1/2}$	-135 ± 6	-129 ± 1	-140	-137				
	$A_{3/2}$	-255 ± 8	-243 ± 1	-265	-260				
$P_{11}(1440)$	$A_{1/2}$	-65 ± 4	-67 ± 4	-77	-61	40 ± 10	47 ± 5	52	54
$S_{11}(1535)$	$A_{1/2}$	90 ± 30	30 ± 4	73	66	-46 ± 27	-16 ± 5	-42	-51
$D_{13}(1520)$	$A_{1/2}$	-24 ± 9	-24 ± 4	-30	-27	-59 ± 9	-67 ± 4	-85	-77
	$A_{3/2}$	166 ± 5	135 ± 4	166	161	-139 ± 11	-112 ± 3	-148	-154
$S_{11}(1650)$	$A_{1/2}$	53 ± 16	74 ± 4	32	33	-15 ± 21	-28 ± 4	27	9
$D_{15}(1675)$	$A_{1/2}$	19 ± 8	33 ± 4	23	15	-43 ± 12	-50 ± 4	-61	-62
	$A_{3/2}$	15 ± 9	9 ± 4	24	22	-58 ± 13	-71 ± 5	-74	-84
$F_{15}(1680)$	$A_{1/2}$	-15 ± 6	-13 ± 4	-25	-25	29 ± 10	29 ± 6	25	28
	$A_{3/2}$	133 ± 12	129 ± 4	134	134	-33 ± 9	-58 ± 9	-35	-38
$P_{13}(1720)$	$A_{1/2}$	18 ± 30		55	73	1 ± 15		17	-3
	$A_{3/2}$	-19 ± 20		-32	-11	-29 ± 61		-75	-31
$S_{31}(1620)$	$A_{1/2}$	27 ± 11	-13 ± 4	71	66				

electroproduction analysis of nucleon resonances with MAID

$$A^{res}(W,Q^2) = \overline{\mathrm{A}}(Q^2)\, f_{\gamma N}(W) \frac{\Gamma_{tot}(W)\, M_R}{M_R^2 - W^2 - iM_R\Gamma_{tot}(W)} f_{\pi N}(W) e^{i\Psi(W)}$$

a) single-Q^2 analysis : (Q^2=0.12, 0.4, 0.52, 0.65, 0.7, 0.9, 1.0, 1.15, 1.45, 2.8, 4.0 GeV^2)

uses only data at fixed Q^2 and gives helicity amplitudes

as "data points" $A_{1/2}$, $A_{3/2}$, $S_{1/2}$

b) Q^2-dependent analysis : (superglobal fit)

uses all data at any Q^2 and gives Q^2-dependent helicity amplitudes

as smooth functions $A_{1/2}(Q^2)$, $A_{3/2}(Q^2)$, $S_{1/2}(Q^2)$

(transition form factors)

typical parametrization for Q^2 region of $0 \le Q^2 < 3\ (GeV/c)^2$

$$A_{1/2}(Q^2) = A_{1/2}(0)\ (1-\alpha Q^2)\ e^{-\beta Q^2}$$

etc.

with 1 or 2 parameters α, β for each form factor

experimental data on pion electroproduction

mostly $d\sigma_V / d\Omega$ and $d\sigma_T + \varepsilon\, d\sigma_L$, $d\sigma_{LT}$, σ_{TT}

GWU/SAID database from 1971 - 1999:

$d\sigma_V / d\Omega$ and $d\sigma_T + \varepsilon\, d\sigma_L$, $d\sigma_{LT}$, σ_{TT} for $0 < Q^2 < 4$ GeV^2

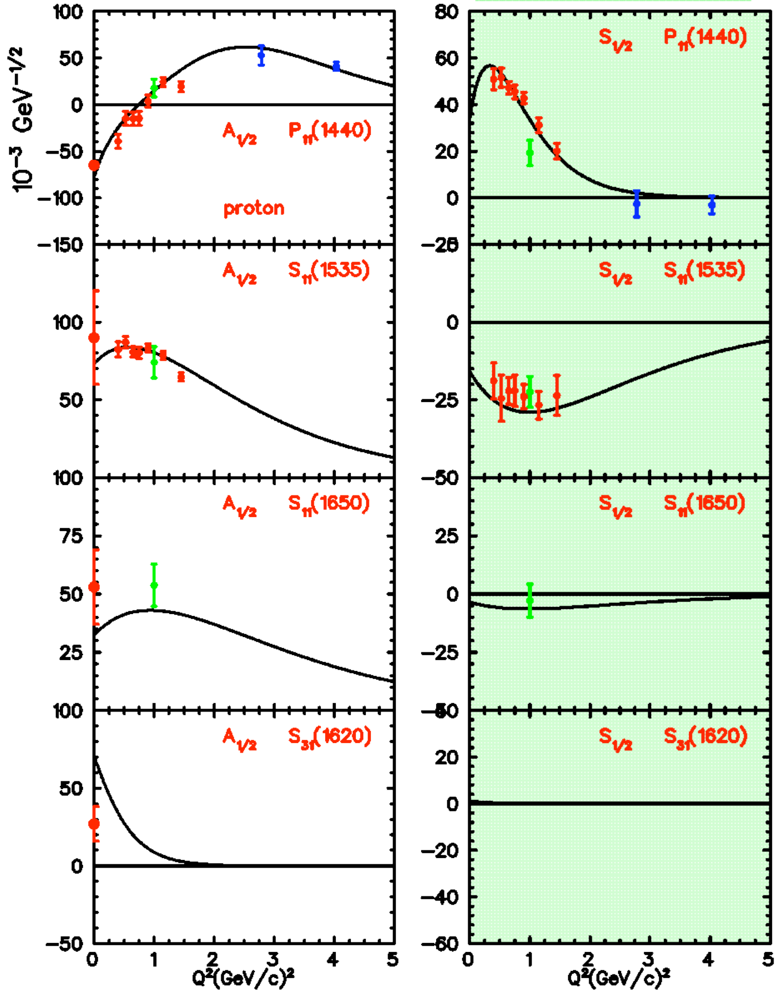
800	pts of $p\pi^-$
6500	pts of $n\pi^+$
13000	pts of $p\pi^0$

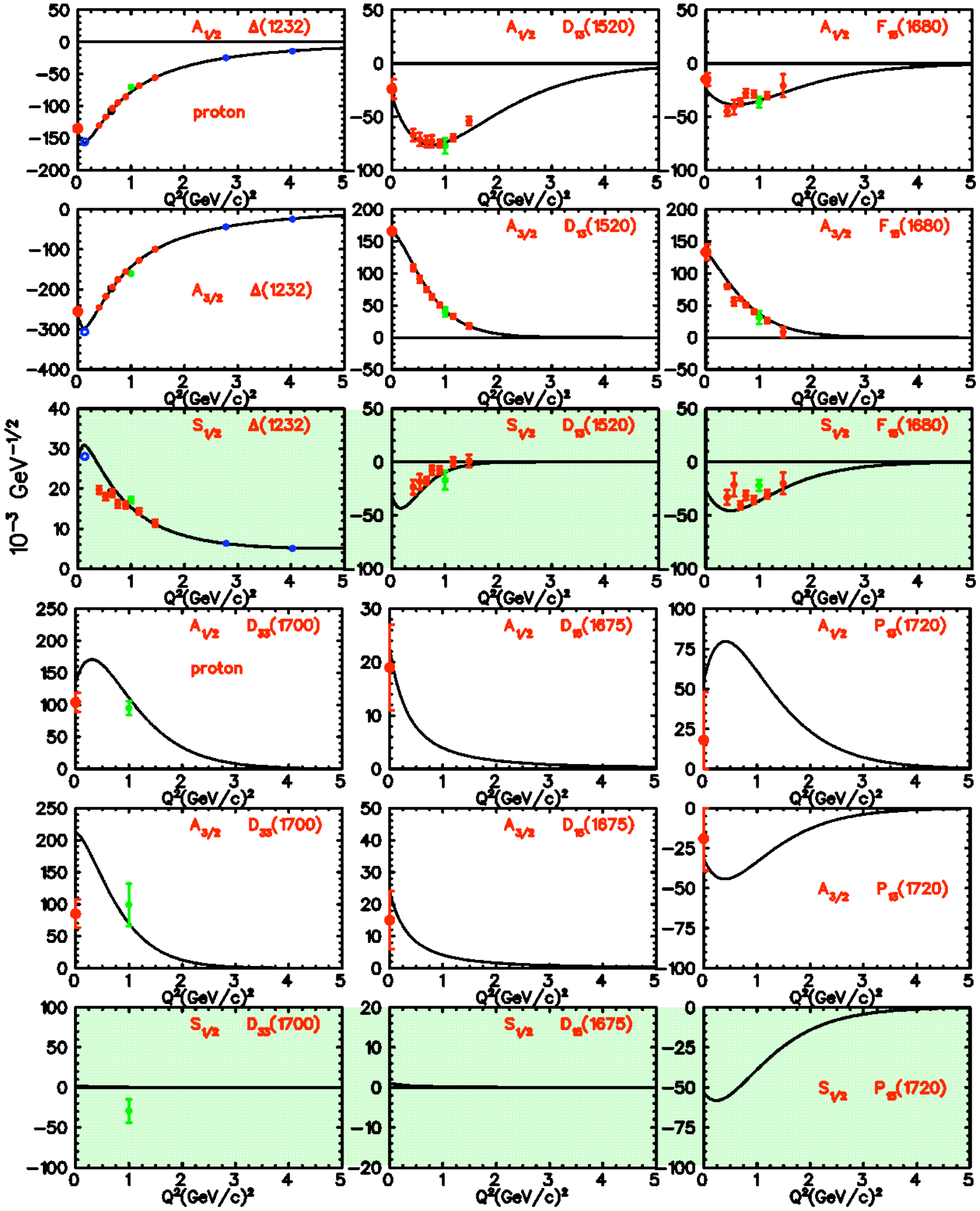
Bates (2000):	80	pts of $p\pi^0$
Bonn (2002):	4900	pts of $p\pi^0$
JLab C (1999):	1600	pts of $p\pi^0$
JLab B (2002):	22000	pts of $p\pi^0$
JLab A (2003):	360	pts of $p\pi^0$

MAID analysis
of proton
helicity amplitudes and
transition form factors

transverse

longitudinal

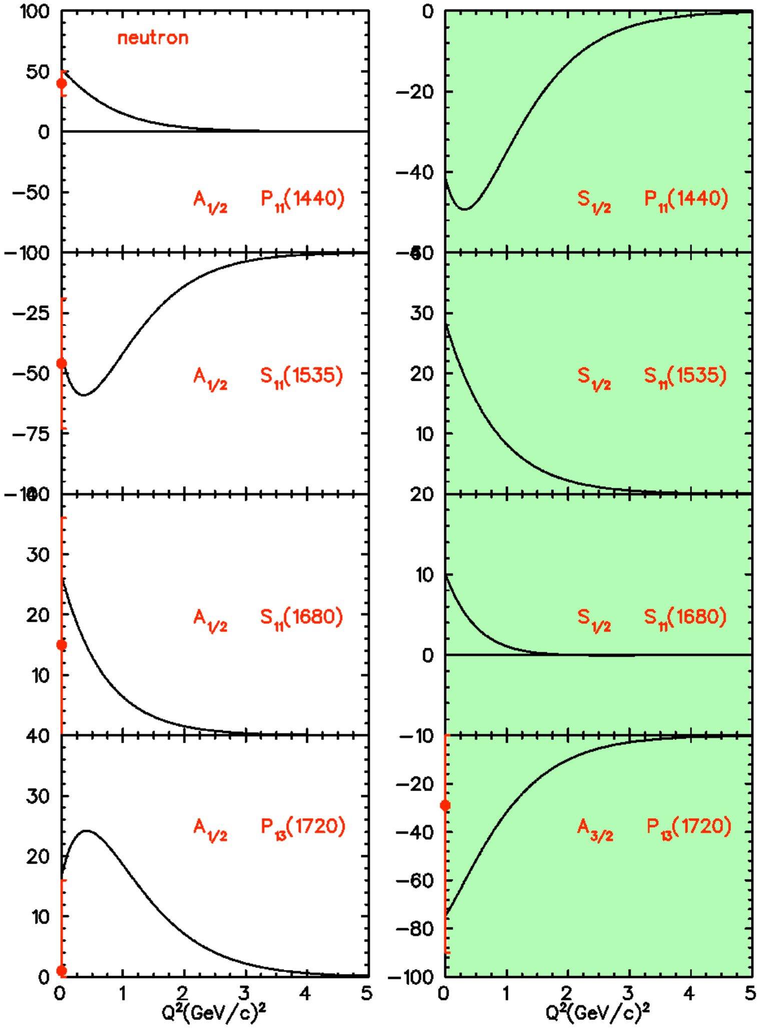


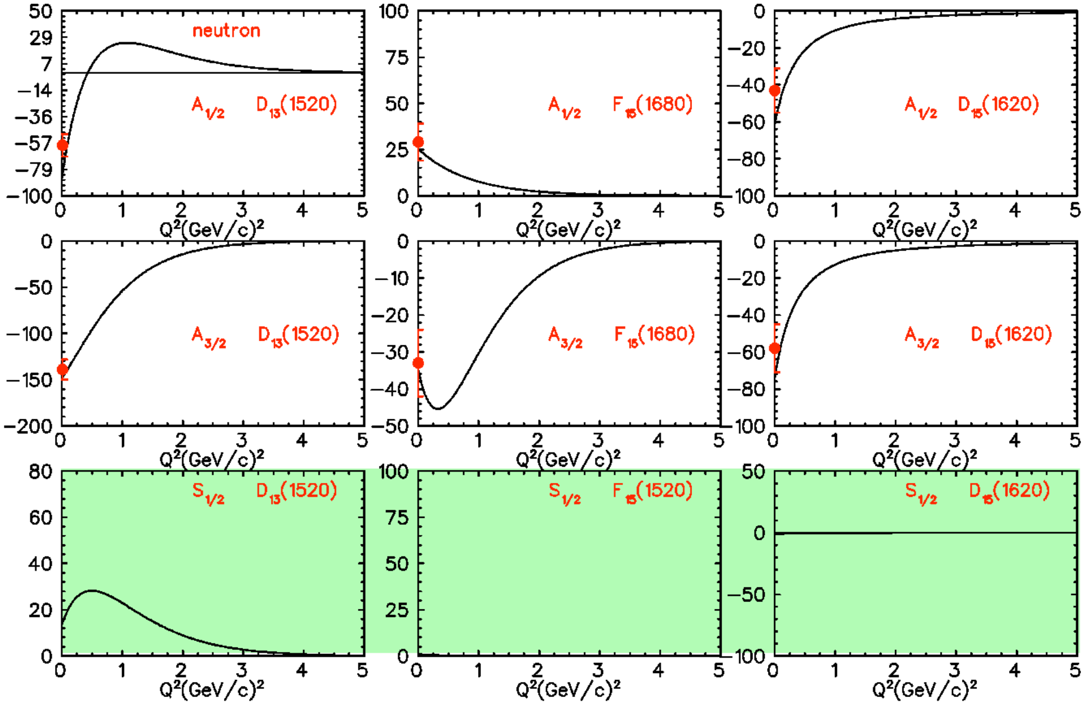


MAID analysis of neutron helicity amplitudes and transition form factors

transverse

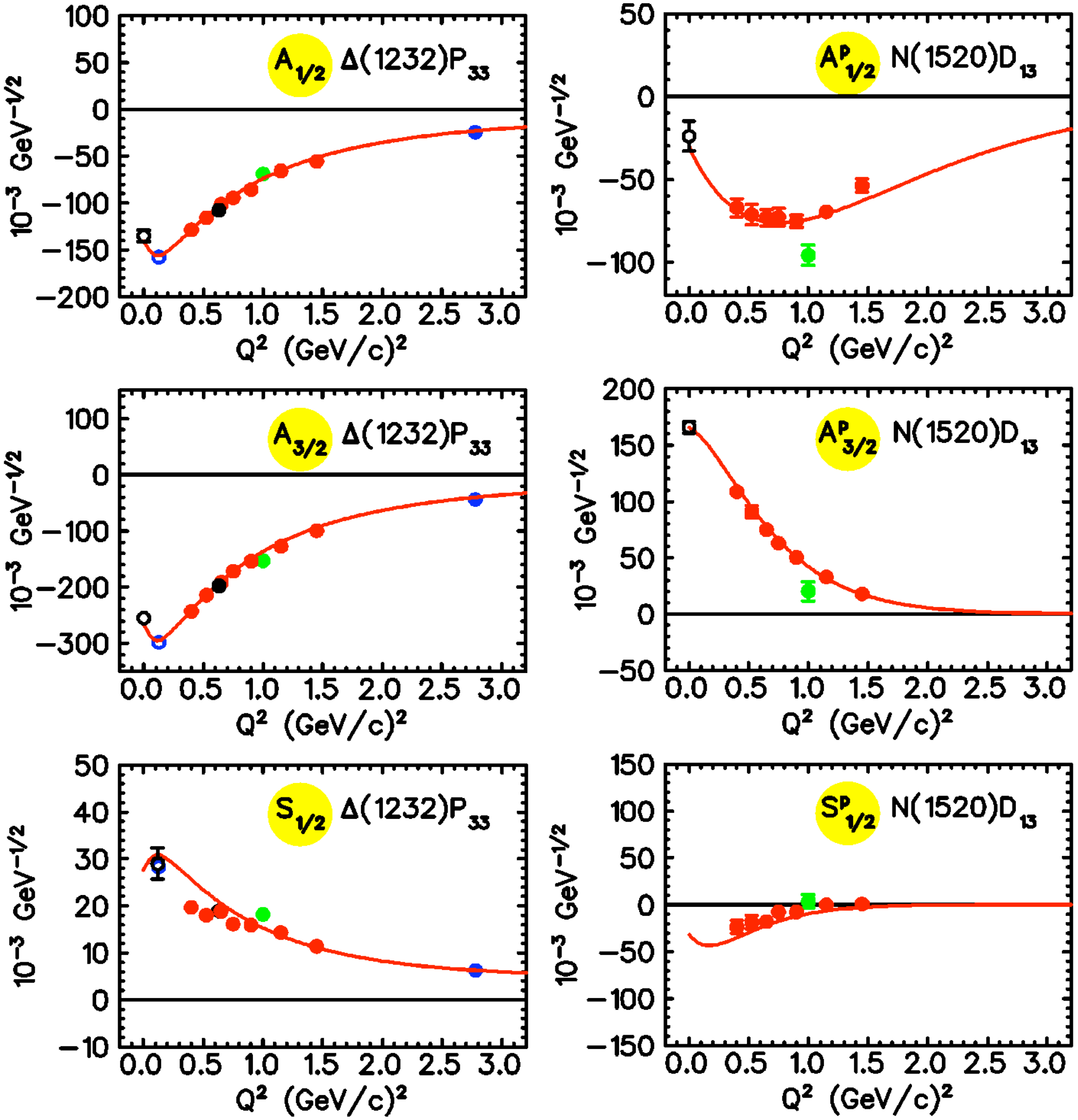
longitudinal





Delta P_{33}(1232) and D_{13}(1520) resonances

$\Delta(1232)$ P_{33} N(1520) D_{13}



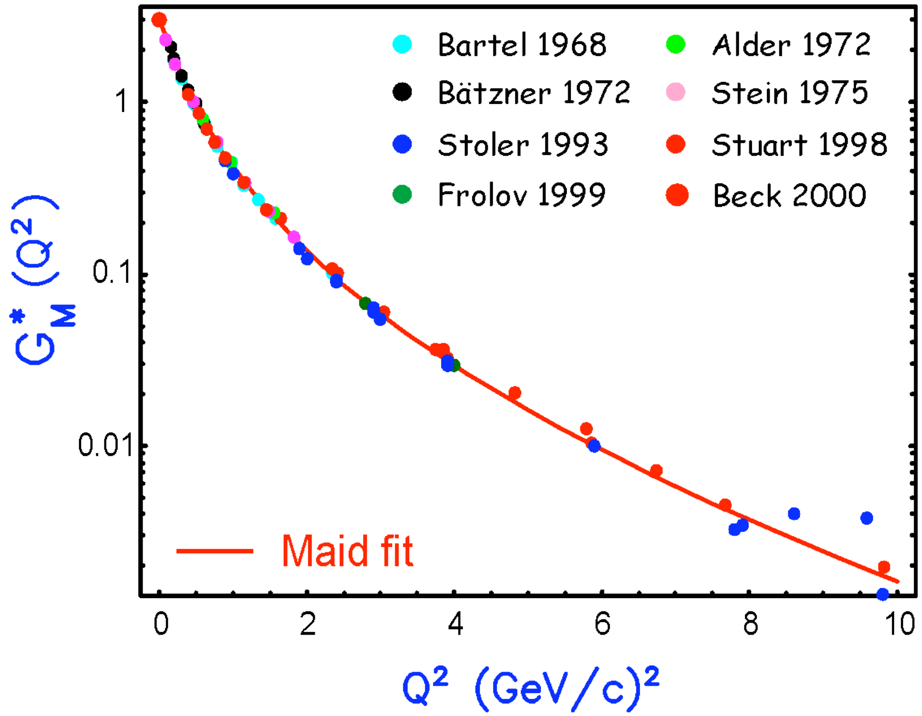
— Q^2 dependent fit
(superglobal fit)

single-Q^2 fits with:

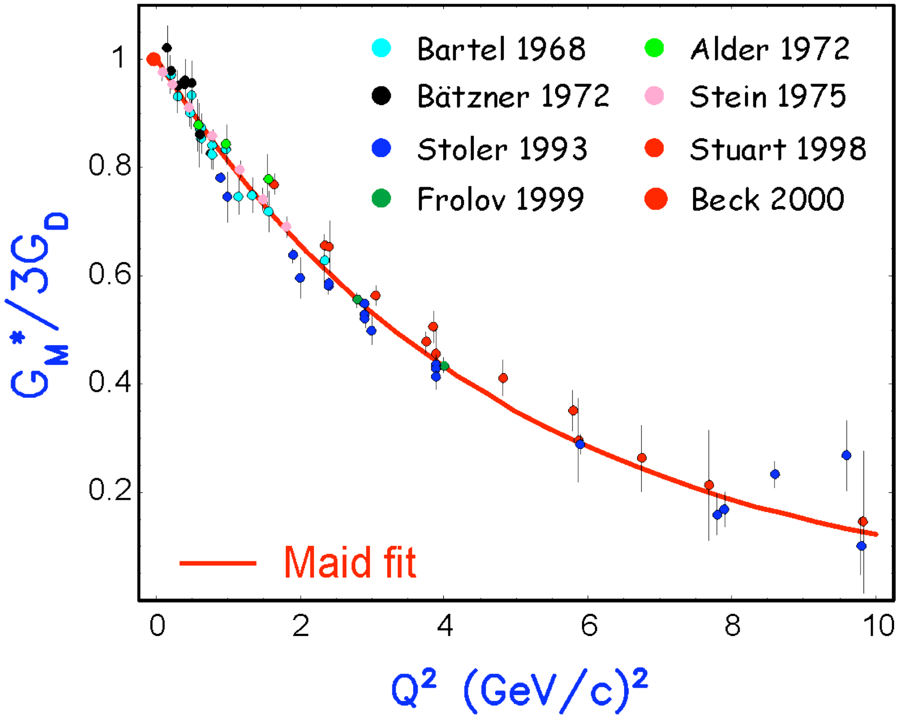
● JLab/Hall C data (Frolov 99)
● JLab/Hall A data (Laveissiere 01/03)
● JLab/Hall B data (Joo 02)
● Bonn data (Gothe 02)
○ Bates data (Mertz 00)

○ PDG 2002 @ Q^2=0

N -> Δ transition form factor M1



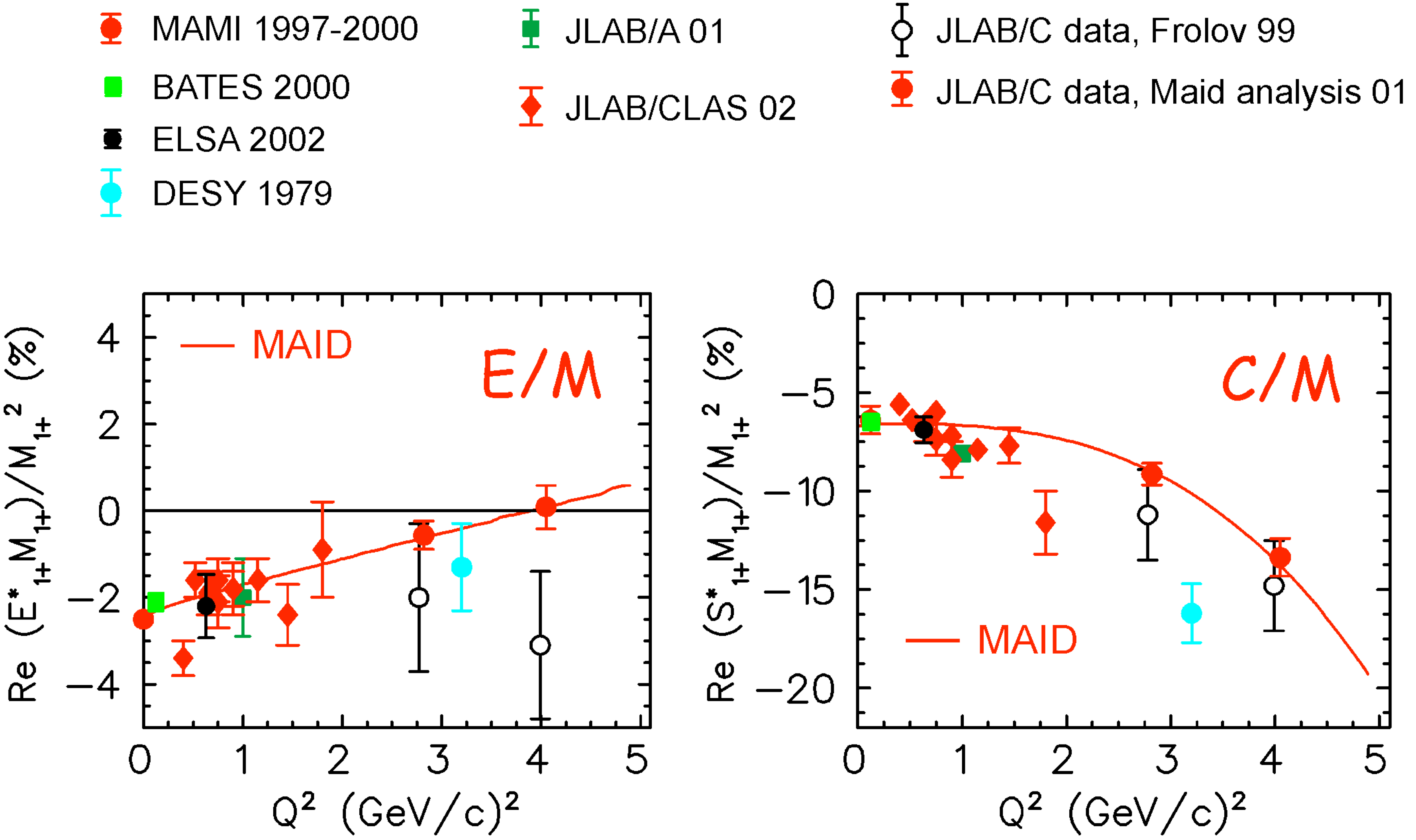
G_M^* form factor



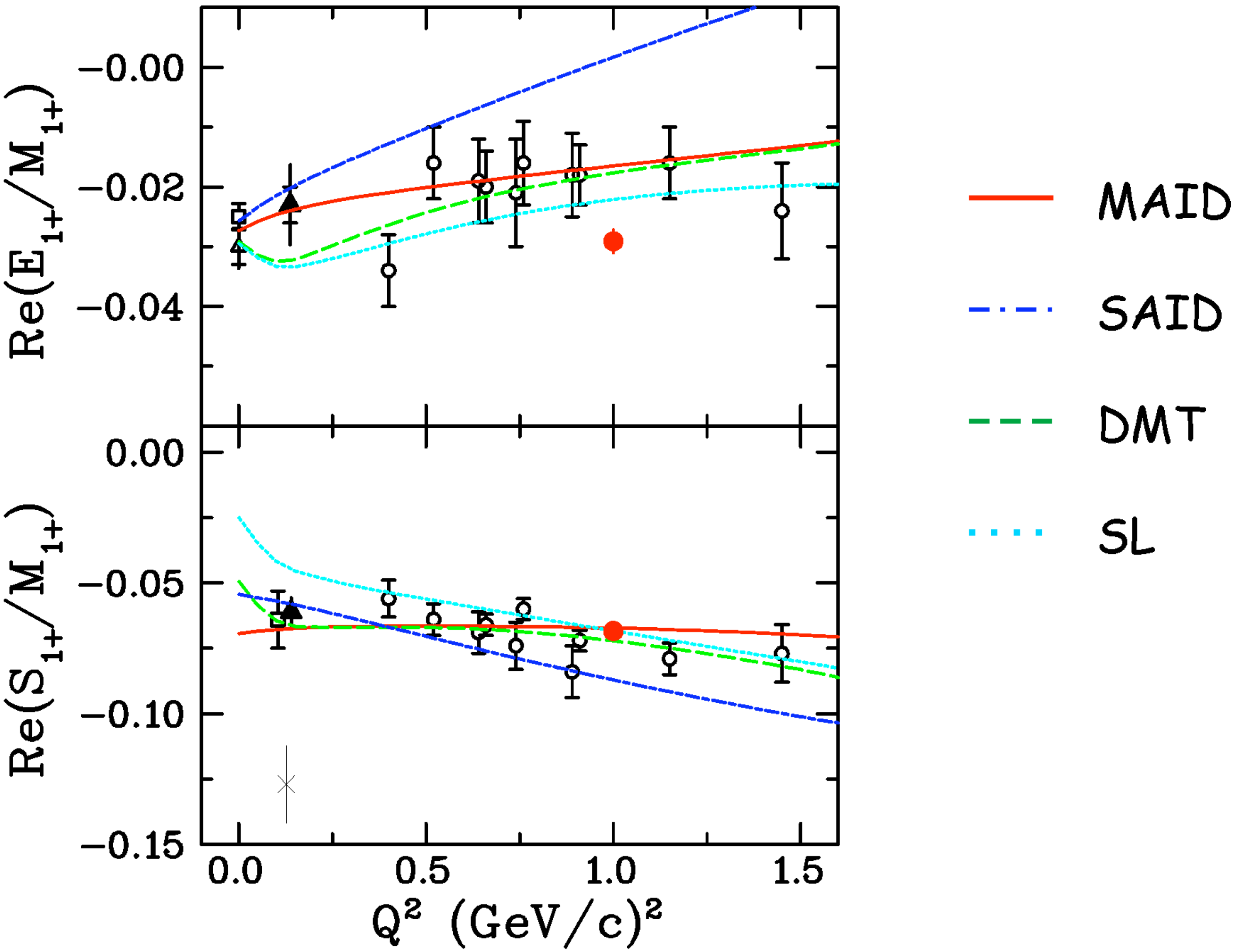
G_M^* / $3G_{dipole}$

Maid fit: $G_M^*(Q^2) = 3\,G_{dip}(Q^2)\,e^{-0.21\,Q^2}$

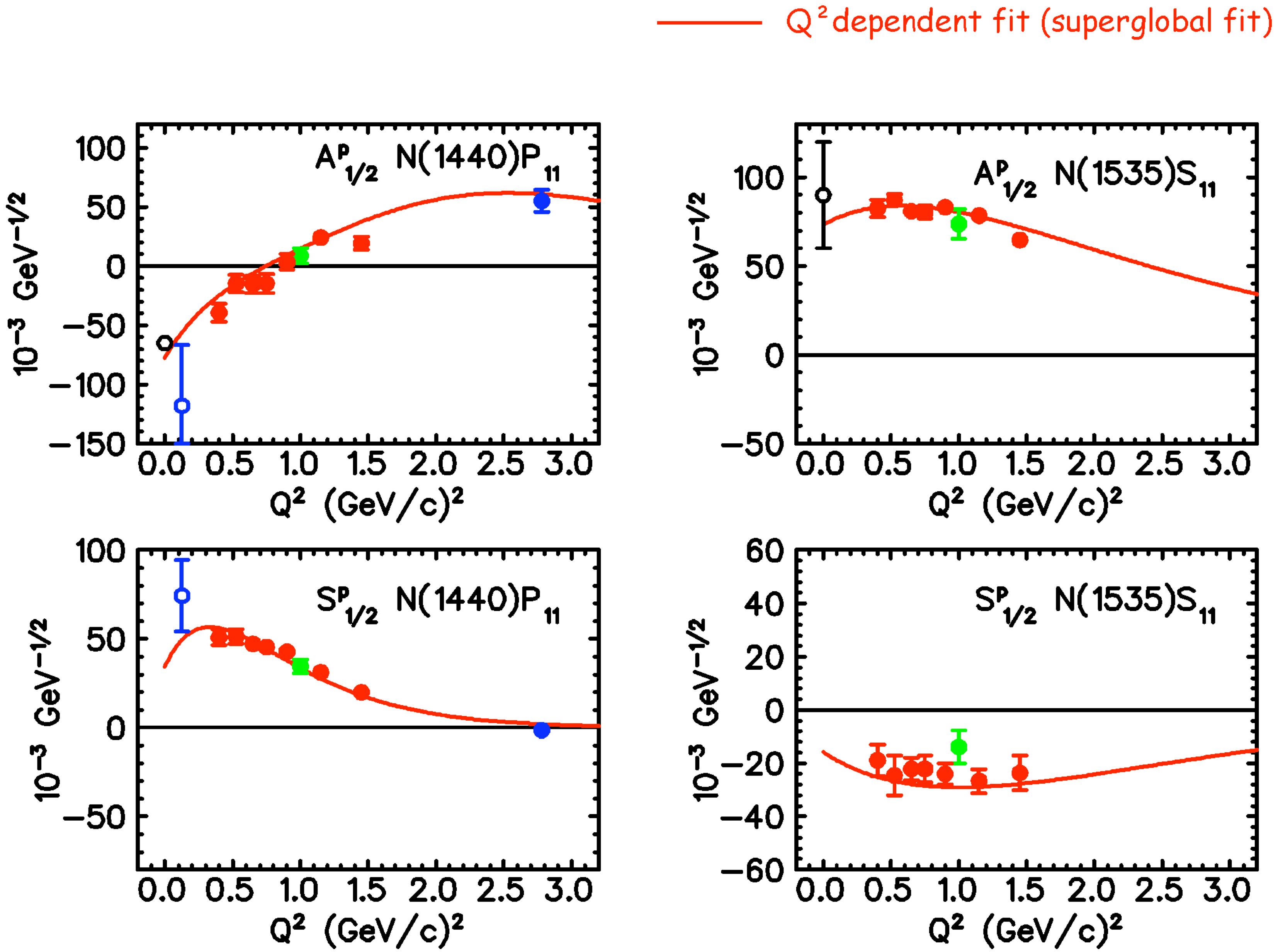
N -> Δ transition form factor ratios



J. Kelly et al. (Hall A), Phys. Rev. Lett. 95 (2005)



Roper P_{11}(1440) and S_{11}(1535) resonances


Q²dependent fit (superglobal fit)
A^p_1/2 N(1440)P11
A^p_1/2 N(1535)S11
S^p_1/2 N(1440)P11
S^p_1/2 N(1535)S11
10⁻³ GeV^−1/2
Q² (GeV/c)²
100
50
0
−50
−100
−150
60
40
20
−20
−40
−60
0.0
0.5
1.0
1.5
2.0
2.5
3.0

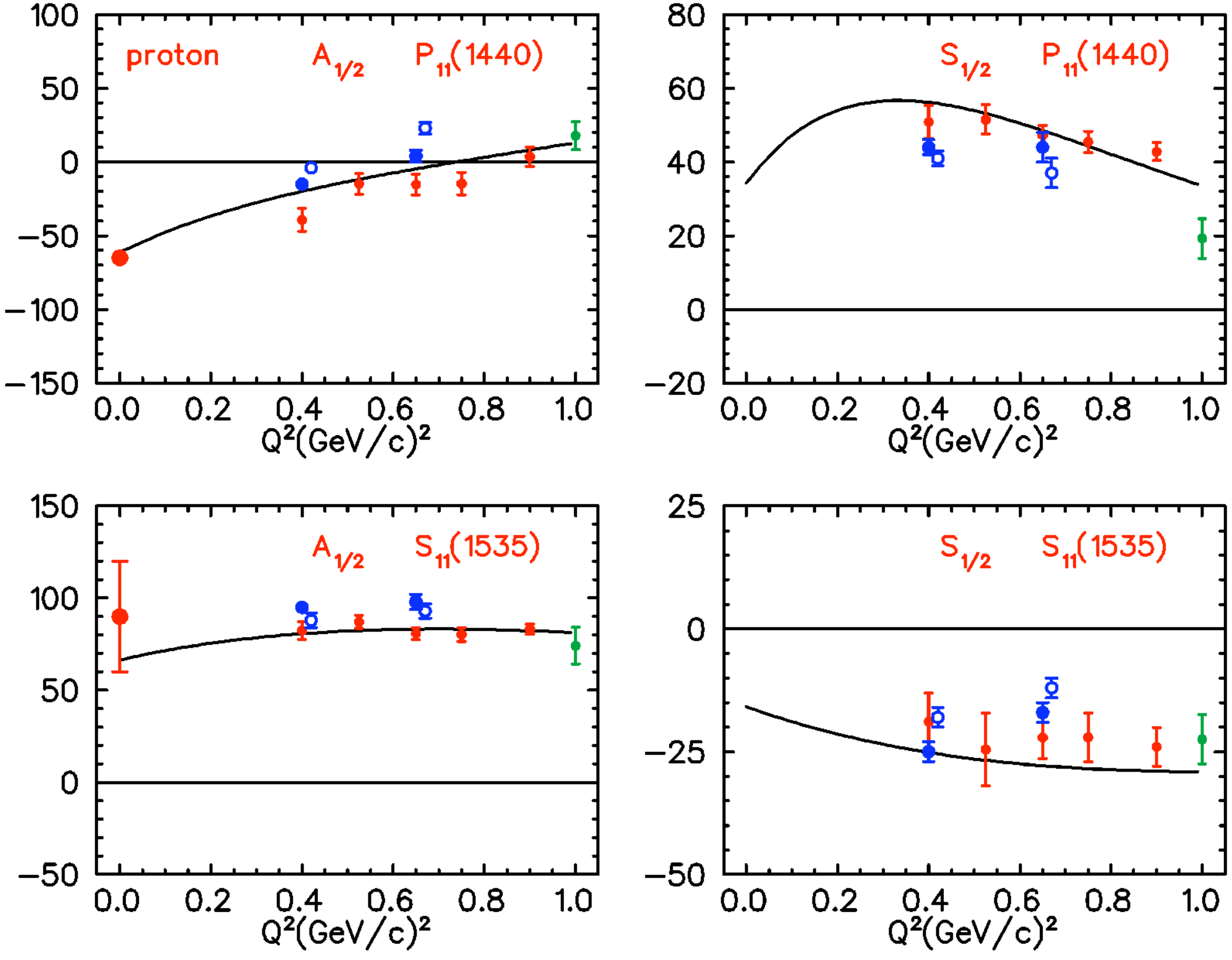
transition form factors for Roper and S_{11} at low Q^2

Maid2005 superglobal fit

Maid2005 single-Q^2 analysis

comparison with isobar model ● and dispersion analysis ○

of Aznauryan and Burkert, Phys. Rev. C 71 (2005)



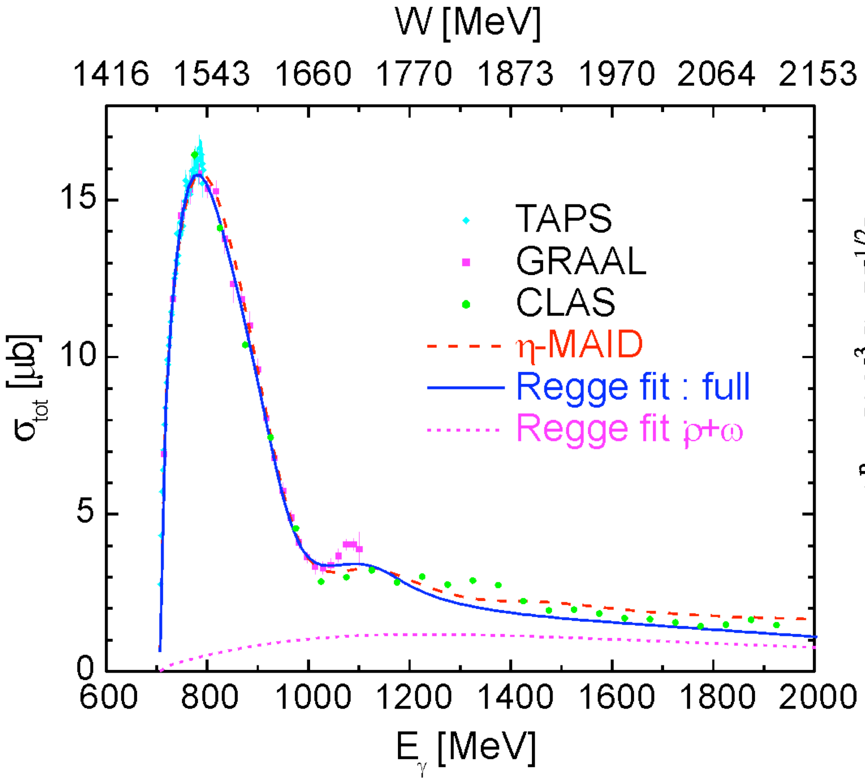
some results from Eta-MAID

S_{11}(1535) analysis from η electroproduction

W.-T. Chiang, S.N. Yang, L. Tiator and D. Drechsel, Nucl. Phys. A 700 (2002) 429

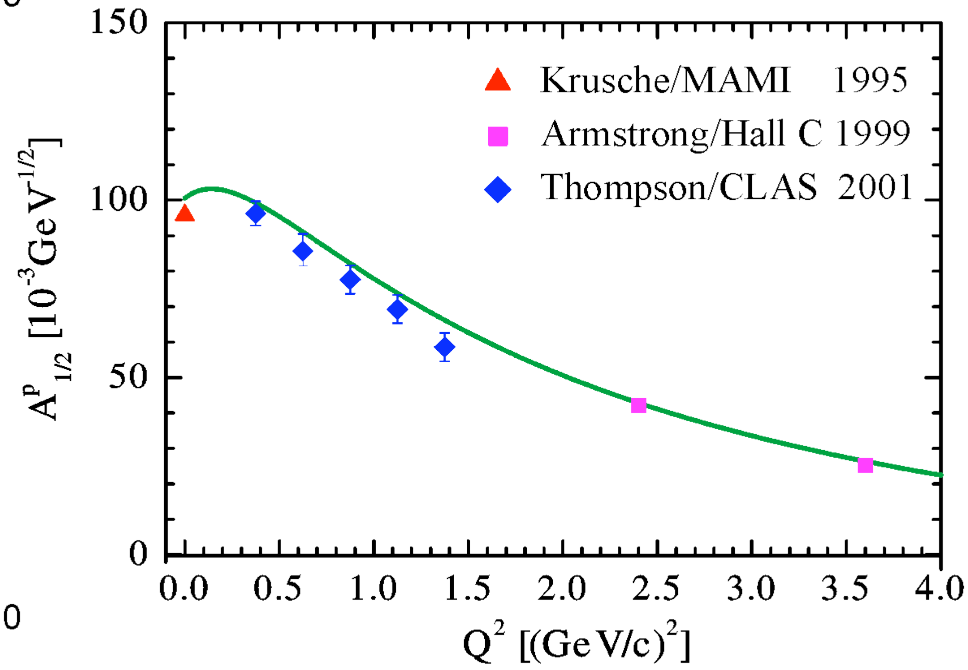
$\gamma p \rightarrow \eta p$

total cross sections



transition form factor

(normalized $A_{1/2}$ amplitude)



Resonance Parameters from η-MAID

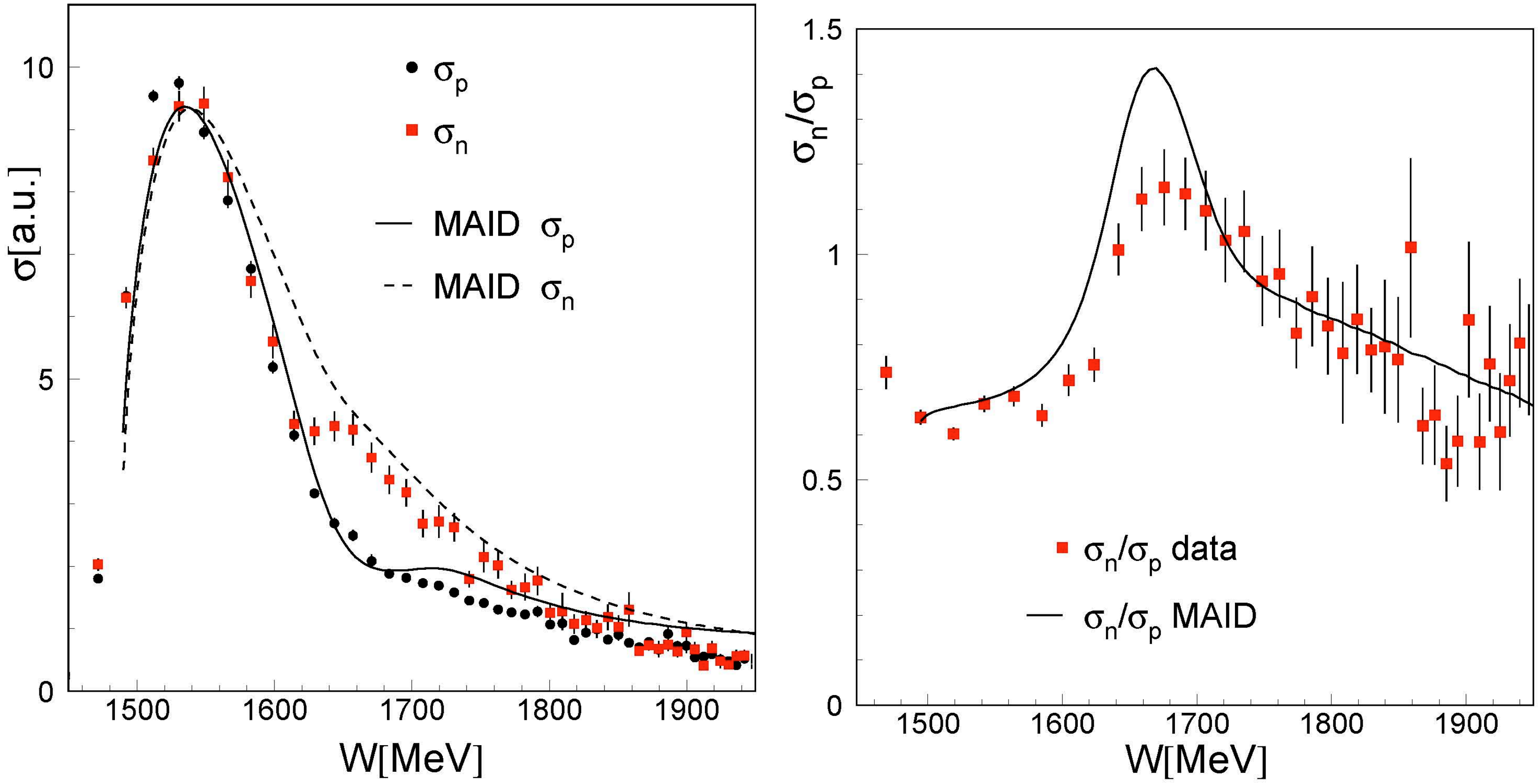
N^*	Mass	Width	$\beta_{\eta N}$	$A^p_{1/2}$	$A^p_{3/2}$
$D_{13}(1520)$	1520	120	0.05%	-39	166
$S_{11}(1535)$	1545	203	50%	125	
$S_{11}(1650)$	1640	130	10%	73	
$D_{15}(1675)$	1682	150	17%	17	24
$F_{15}(1680)$	1670	130	0.04%	-9	145
$D_{13}(1700)$	1700	100	0.7%	-18	-2
$P_{11}(1710)$	1725	100	26%	22	
$P_{13}(1720)$	1720	150	6.6%	18	-19

[MeV]　　　　[10^{-3} GeV$^{-1/2}$]

Numbers in green are fitting parameters

$D_{15}(1675)$ mainly a neutron resonance with large branching into ηN

quasifree $\gamma N \rightarrow N\eta$ total cross sections



summary on MAID techniques

- field theoretical background in tree approximation
- pion loop contributions in K-matrix approximation guarantees a unitarized background
- non-Born bg terms account for missing loop contributions
  a) empirical terms near threshold for E_{0+}, S_{0+}
  b) PS-PV mixing at higher energies in E_{0+}, S_{0+}, M_{1-}, S_{1-}
- nucleon resonance excitation in s-channel Breit-Wigner parametrization
  (MAID strategy: as few as necessary, **in Maid05: 13 res.**)
- Q^2 dependent transition form factors lead to superglobal energy- and Q^2 dependent solutions
- **Outlook:** PWA groups should work together in order to determine and eventually to reduce model dependence