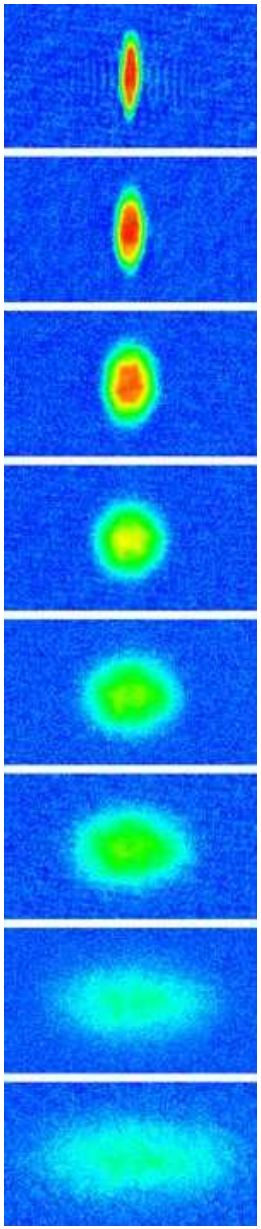
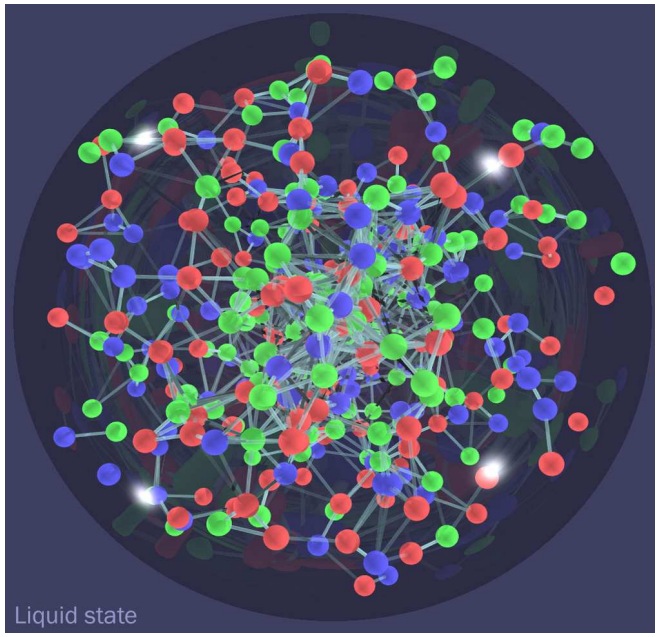
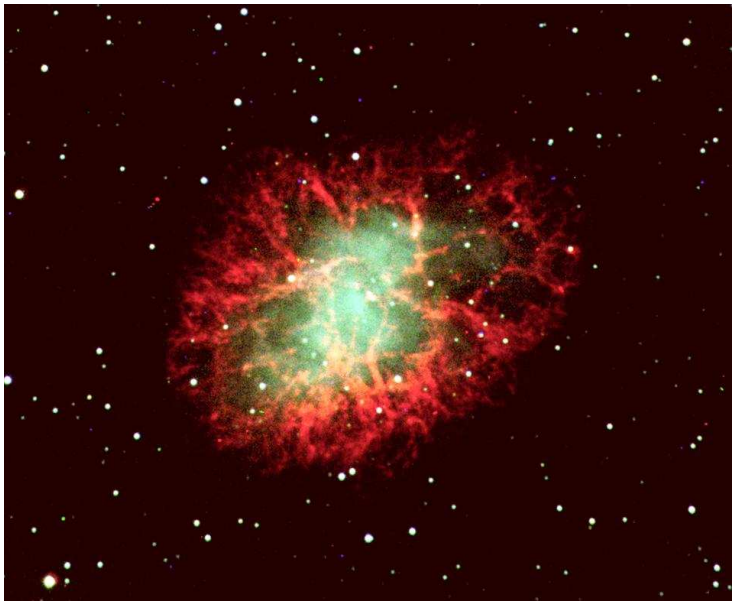
Neutron Matter

Perfect Liquids





sQGP



Neutron Star (Crab)

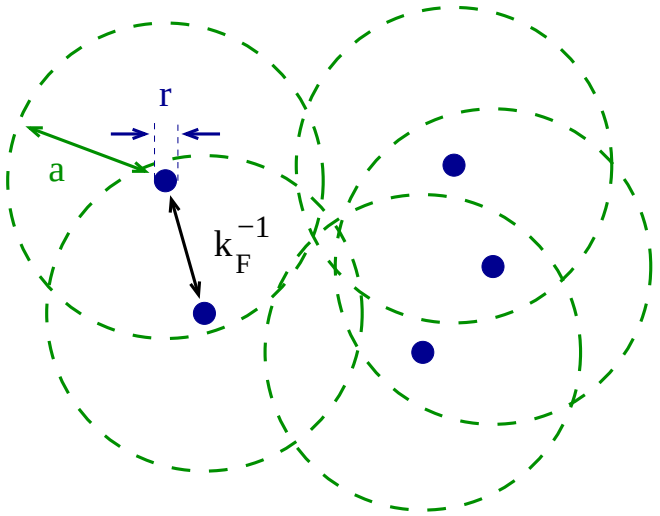
Trapped Fermi Gas

Universality

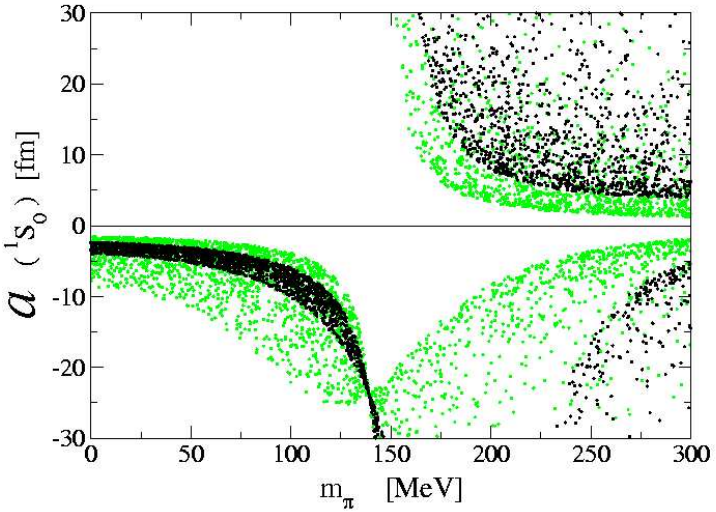
What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

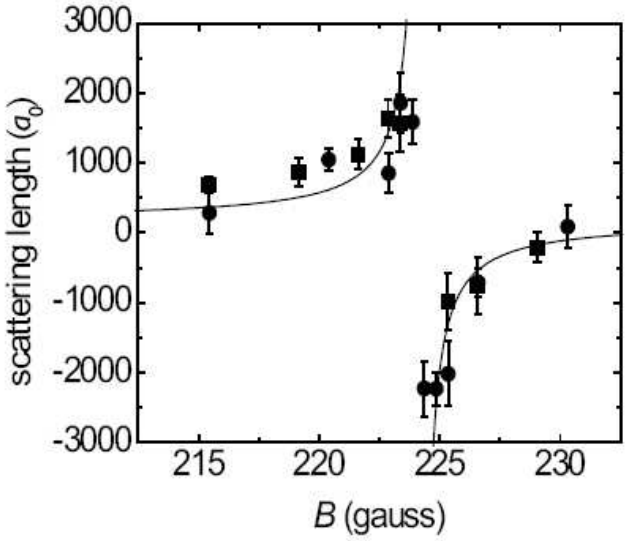
strongly correlated: $a\rho^{1/3} \gg 1$



Neutron Matter



Feshbach Resonance in ^{6}Li



Universality

Consider limiting case ("Bertsch" problem)

$$(k_F a) \to \infty \qquad\qquad (k_F r) \to 0$$

Universal equation of state

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0 = \xi \frac{3}{5}\left(\frac{k_F^2}{2M}\right)$$

How to find ξ?

Numerical Simulations

Experiments with trapped fermions

Analytic Approaches

Effective Field Theory

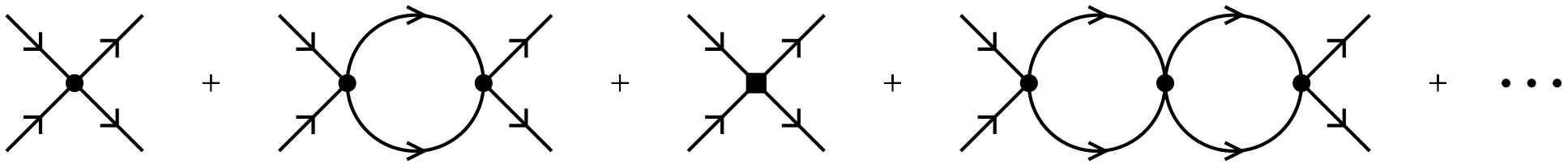
Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \ldots$$

Scattering amplitude

$$\mathcal{A} = \frac{1}{p \cot \delta - ip} \qquad p \cot \delta = -\frac{1}{a} + \frac{1}{2} \Lambda^2 \sum_n r_n \left(\frac{p^2}{\Lambda^2} \right)^{n+1}$$

Low energy expansion (natural case)



$$\mathcal{A} = -\frac{4\pi a}{M} \left[1 - iap + (ar_0/2 - a^2)p^2 + O(p^3/\Lambda^3) \right]$$

Modified Expansion

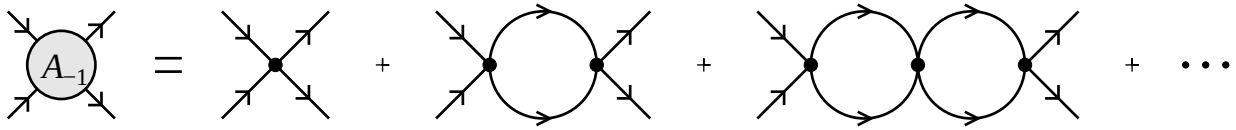
Coupling constants determined by nn interaction

$$C_0 = \frac{4\pi a}{M} \qquad a = -18 \text{ fm} \qquad\qquad C_2 = \frac{4\pi a^2}{M}\frac{r}{2} \qquad r = 2.8 \text{ fm}$$

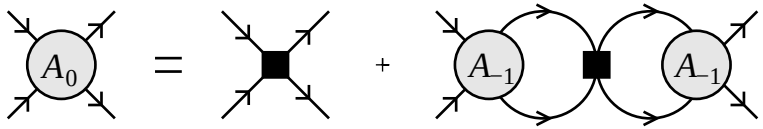
Problem: Large scattering length

$$(ap) \ll 1 \qquad\qquad p \ll 10 \text{ MeV}$$

Need to sum (ap) to all orders. Small parameter $Q \sim (a^{-1}, p, \ldots)$



$$\mathcal{A}_{-1} = -\frac{4\pi}{M}\frac{1}{a+ip}$$



$$\mathcal{A}_0 = -\frac{4\pi}{M}\frac{r_0 p^2/2}{(a+ip)^2}$$

Power Counting Made Manifest: PDS Scheme

Dimensional regularization in PDS scheme

$$\int \frac{d^3q}{(2\pi)^3} \frac{M}{k^2 - q^2 + i\epsilon} = -\frac{M}{4\pi}(\mu + ik)$$

Low energy constants

$$C_0 = -\frac{4\pi/M}{\mu - 1/a} \sim \frac{1}{Q} \qquad C_2\, k^2 = \frac{4\pi/M}{(\mu - 1/a)^2} \frac{r}{2}\, k^2 \sim Q^0.$$

Scattering matrix

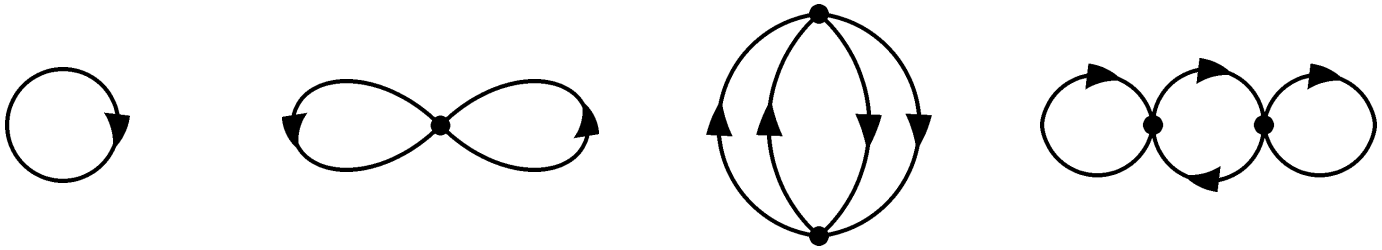
$$T(k) = \frac{C_0 + C_2 k^2}{1 - \frac{M}{4\pi}(\mu + ik)(C_0 + C_2 k^2)}.$$

Low Density Expansion

Finite density: $\mathcal{L} \to \mathcal{L} - \mu\psi^\dagger\psi \Rightarrow$ Modified propagator

$$G_0(k)_{\alpha\beta} = \delta_{\alpha\beta}\left(\frac{\theta(k-k_F)}{k_0 - k^2/2M + i\epsilon} + \frac{\theta(k_F-k)}{k_0 - k^2/2M - i\epsilon}\right) \qquad \frac{k_F^2}{2M} = \mu$$

Perturbative expansion

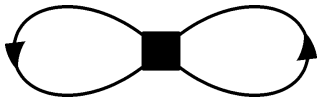


$\epsilon_F\rho$ $\epsilon_F\rho\,(k_Fa)$ $\epsilon_F\rho\,(k_Fa)^2$

$$\frac{E}{A} = \frac{k_F^2}{2M}\left[\frac{3}{5} + \left(\frac{2}{3\pi}(k_Fa) + \frac{4}{35\pi^2}(11-2\log(2))(k_Fa)^2\right) + \ldots\right]$$

Low Density Expansion: Higher orders

Effective range corrections

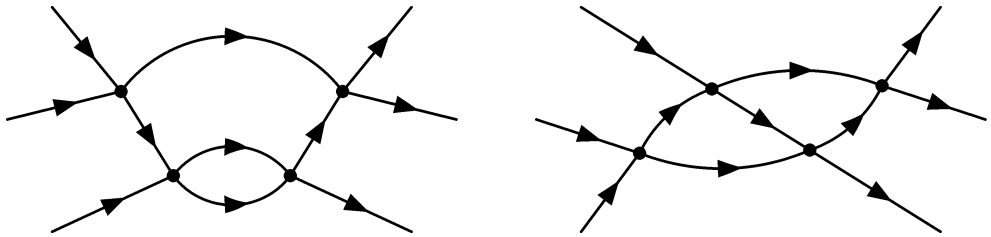


$$\frac{E}{A} = \frac{k_F^2}{2M}\frac{1}{10\pi}(k_F a)^2(k_F r)$$

Logarithmic terms

$$\frac{E}{A} = \frac{k_F^2}{2M}(g-1)(g-2)\frac{16}{27\pi^3}(4\pi - 3\sqrt{3})(k_F a)^4 \log(k_F a)$$

related to log divergence in $3 \to 3$ scattering amplitude



local counterterm $D(\psi^\dagger\psi)^3$ exists if $g \geq 3$

Lattice Calculation

Free fermion action

$$S^{free} = \sum_{\vec{n},i} \left[e^{(m_N-\mu)\alpha_t} c_i^*(\vec{n}) c_i(\vec{n}+\hat{0}) - (1-6h) c_i^*(\vec{n}) c_i(\vec{n}) \right]$$

$$- h \sum_{\vec{n},l_s,i} \left[c_i^*(\vec{n}) c_i(\vec{n}+\hat{l}_s) + c_i^*(\vec{n}) c_i(\vec{n}-\hat{l}_s) \right]$$

Contact interaction: Hubbard-Stratonovich

$$\exp\left[-C\alpha_t a_\uparrow^\dagger a_\uparrow a_\downarrow^\dagger a_\downarrow \right] = \int \frac{ds}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} s^2 \right)$$

$$\exp\left[\left(s\sqrt{-C\alpha} + \frac{C\alpha_t}{2} \right) (a_\uparrow^\dagger a_\uparrow + a_\downarrow^\dagger a_\downarrow) \right]$$

Path Integral

$$\mathrm{Tr} \exp\left[-\beta(H-\mu N) \right] = \int DsDcDc^* \exp\left[-S \right]$$

Lattice Fermions

Introduce pseudo fermions: $S = \psi_i^* Q_{ij} \psi_j + V(s)$

$$Z = \int DsD\phi D\phi^* \exp\left[-S'\right], \qquad S' = \phi_i^* Q_{ij}^{-1} \phi_j + V(s)$$

$C < 0$ (attractive): $\det(Q) \geq 0$

Hybrid Monte Carlo method

(4+1)-d Hamiltonian	$H(\phi, s, p) = \frac{1}{2} p_\alpha^2 + S'(\phi, s)$
Molecular Dynamics	$\dot{s}_\alpha = p_\alpha \qquad \dot{p}_\alpha = -\frac{\partial H}{\partial s_\alpha}$
Metropolis acc/rej	$P([s_\alpha, p_\alpha] \to [s'_\alpha, p'_\alpha]) = \exp(-\Delta H)$

Continuum Limit

Fix coupling constant at finite lattice spacing

$$\frac{M}{4\pi a} = \frac{1}{C_0} + \frac{1}{2}\sum_{\vec{p}} \frac{1}{E_{\vec{p}}}$$

Take lattice spacing b, b_τ to zero

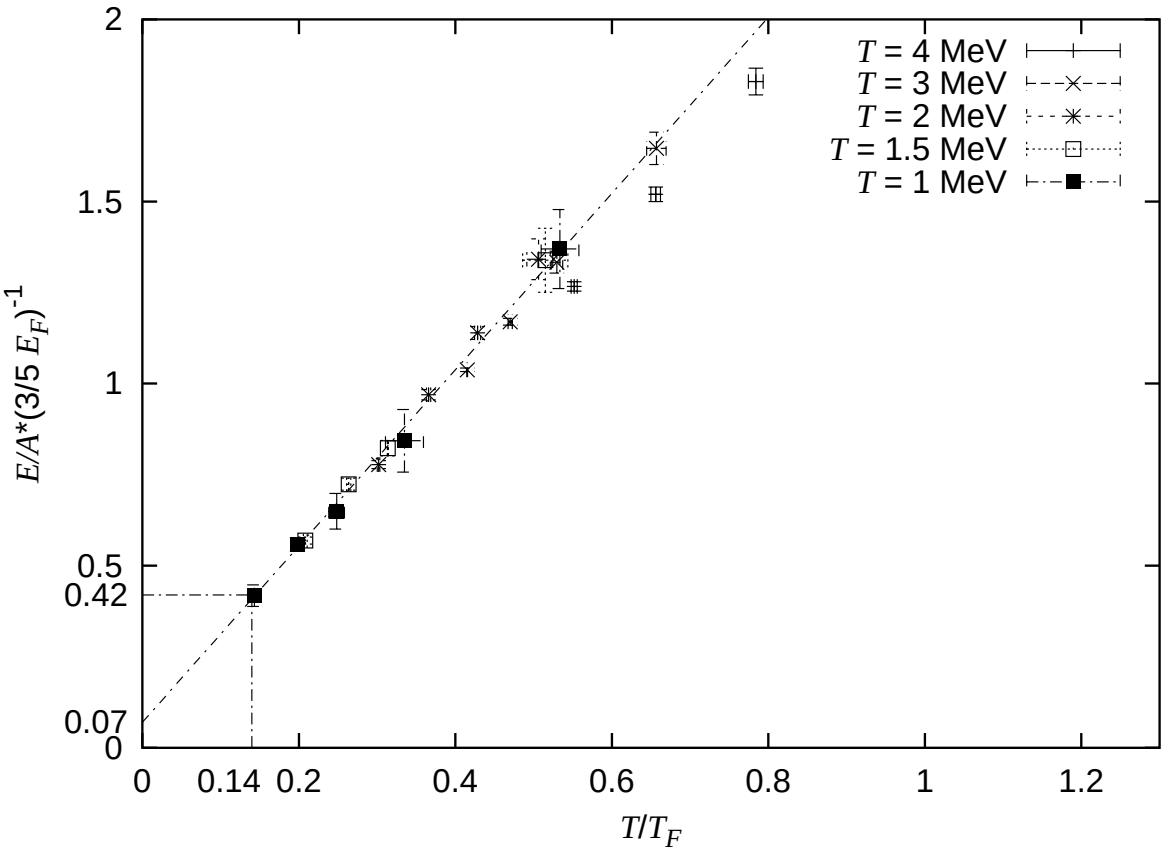
$$\mu b_\tau \to 0 \qquad n^{1/3} b \to 0 \qquad n^{1/3} a = const$$

Physical density fixed, lattice filling $\to 0$

Consider universal (unitary) limit

$$n^{1/3} a \to \infty$$

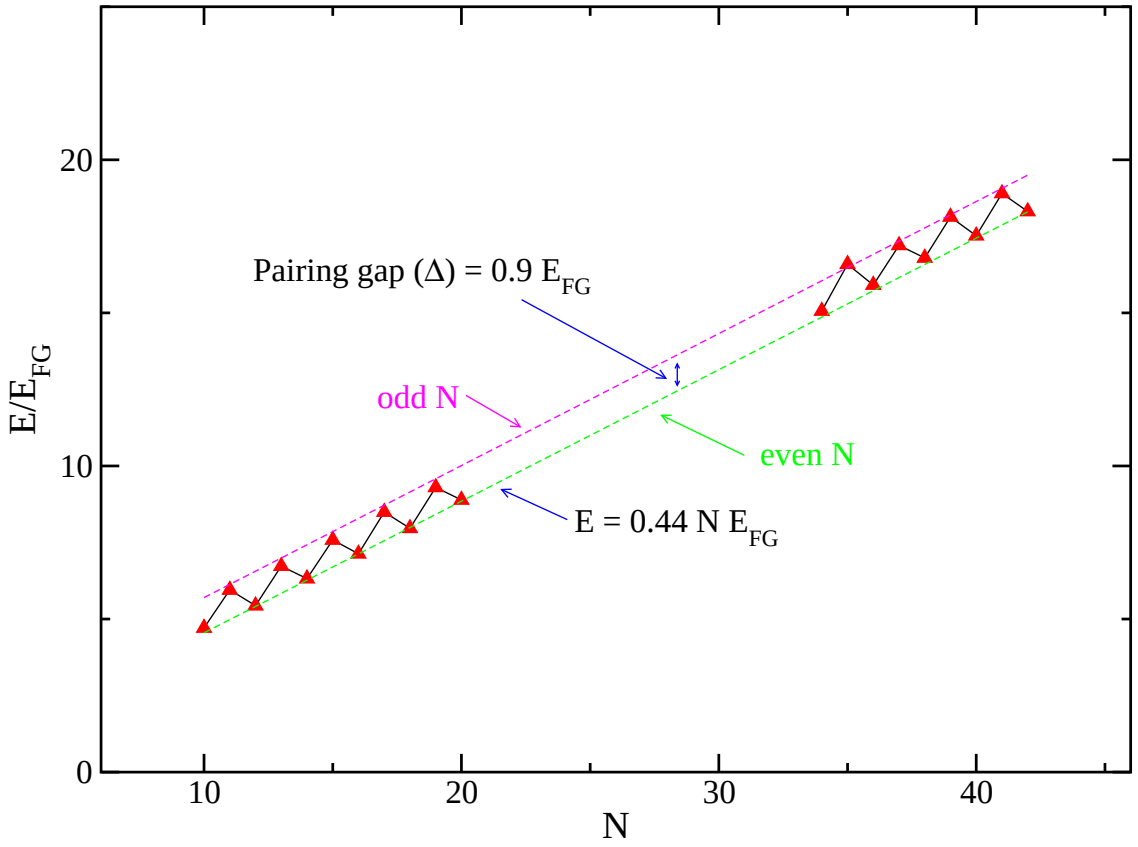
Lattice Results



Canonical $T = 0$ calculation: $\xi = 0.25(3)$ (D. Lee)

Not extrapolated to zero lattice spacing

Green Function Monte Carlo



Other lattice results: $\xi = 0.42$ (Bulgac et al. ,UMass)

Experiment: $\xi = 0.27^{+0.12}_{-0.09}$ [1], 0.51 ± 0.04 [2], 0.74 ± 0.07 [3]

[1] Bartenstein et al., [2] Kinast et al., [3] Gehm et al.

Other Lattice Calculations

Neutron matter with realistic interactions (pions)

Sign problem returns; can be handled at $T \neq 0$

Neutron matter with finite polarization

Sign problem returns

Nuclear Matter (neutrons and protons)

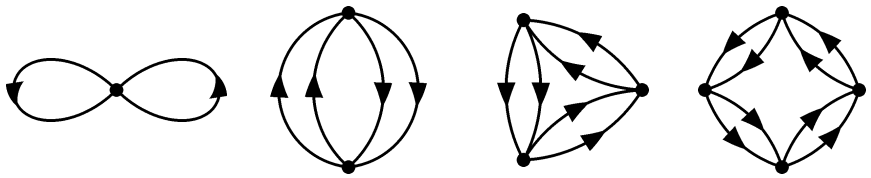
No sign problem in $SU(4)$ limit (Wigner symmetry)

Need a three body force (can be handled with HS)

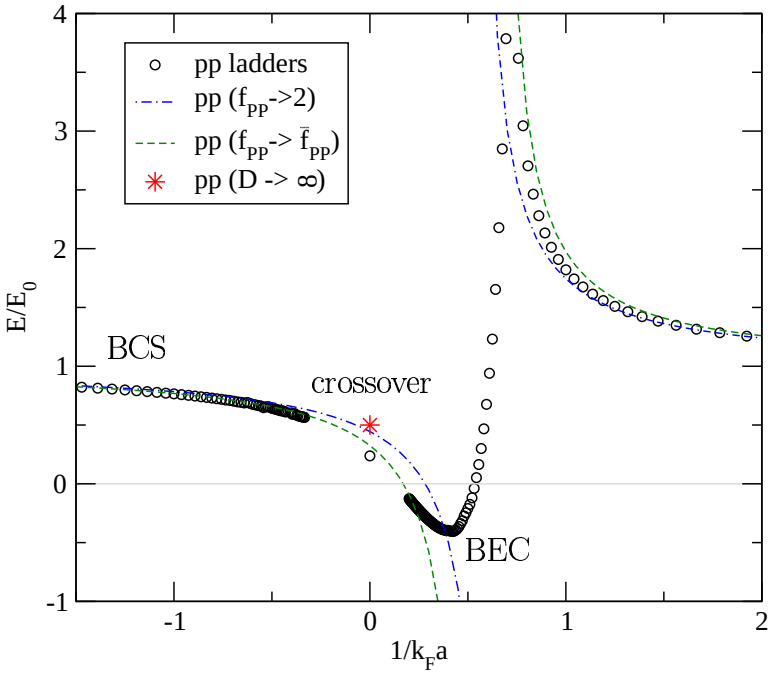
Isospin asymmetry possible

Analytic Approaches: Ladder Diagrams

Sum ladder diagrams at finite density (Brueckner theory)



$$\frac{E}{A} = \frac{k_F^2}{2M} \times \left[\frac{3}{5} + \frac{2(k_F a)/(3\pi)}{1 - \frac{6}{35\pi}(11 - 2\log(2))(k_F a)} \right]$$



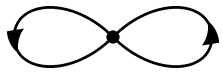
Independent of renormalization scale μ_{PDS}

Unitary Limit $(k_F a) \to \infty$: $\xi = 0.32$

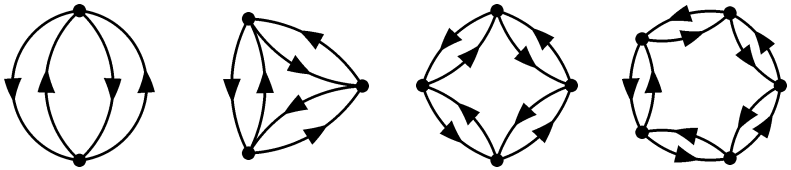
Large N approximation: Ring Diagrams

Consider N fermion species. Define $x \equiv N k_F a/\pi$

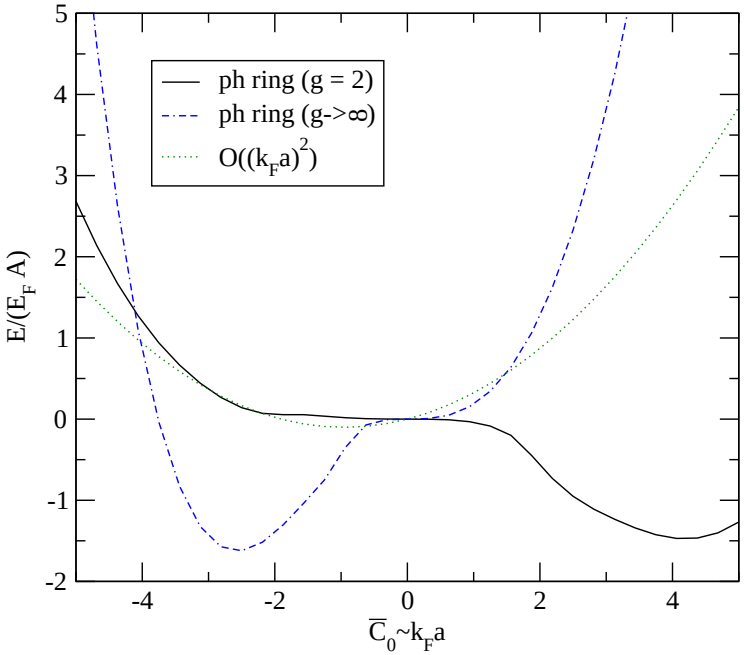
$$\frac{E}{A} = \frac{k_F^2}{2M} \times \left[\left(\frac{3}{5} + \frac{2x}{3} \right) + \frac{1}{N} \left(\frac{3}{\pi} H(x) - \frac{2x}{3} + \frac{4}{35}(22 - 2\log(2))x^2 \right) \right]$$



$N(C_0 N)$



$(C_0 N)^k$

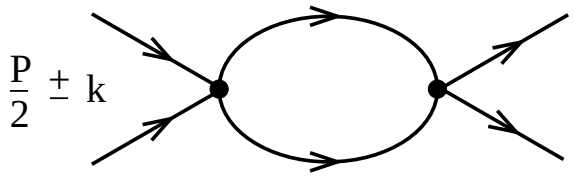


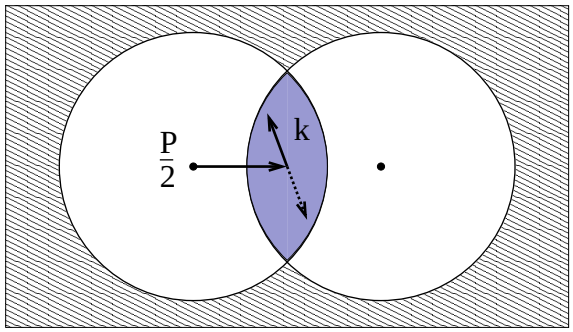
depends on PDS scale parameter μ_{PDS}

not suitable for $(k_F a) \to \infty$

Large d Limit

In medium scattering strongly restricted by phase space

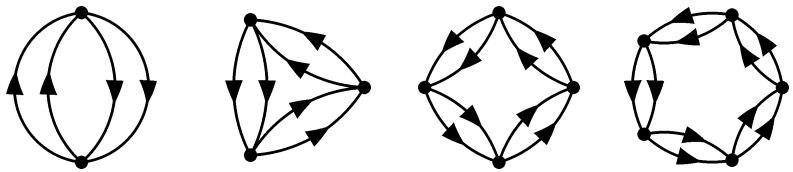




Find limit in which ladders are leading order

$(C_0/d) \cdot 1/d$

$$\lambda \equiv \left[\frac{\Omega_d C_0 k_F^{d-2} M}{d(2\pi)^d} \right]$$

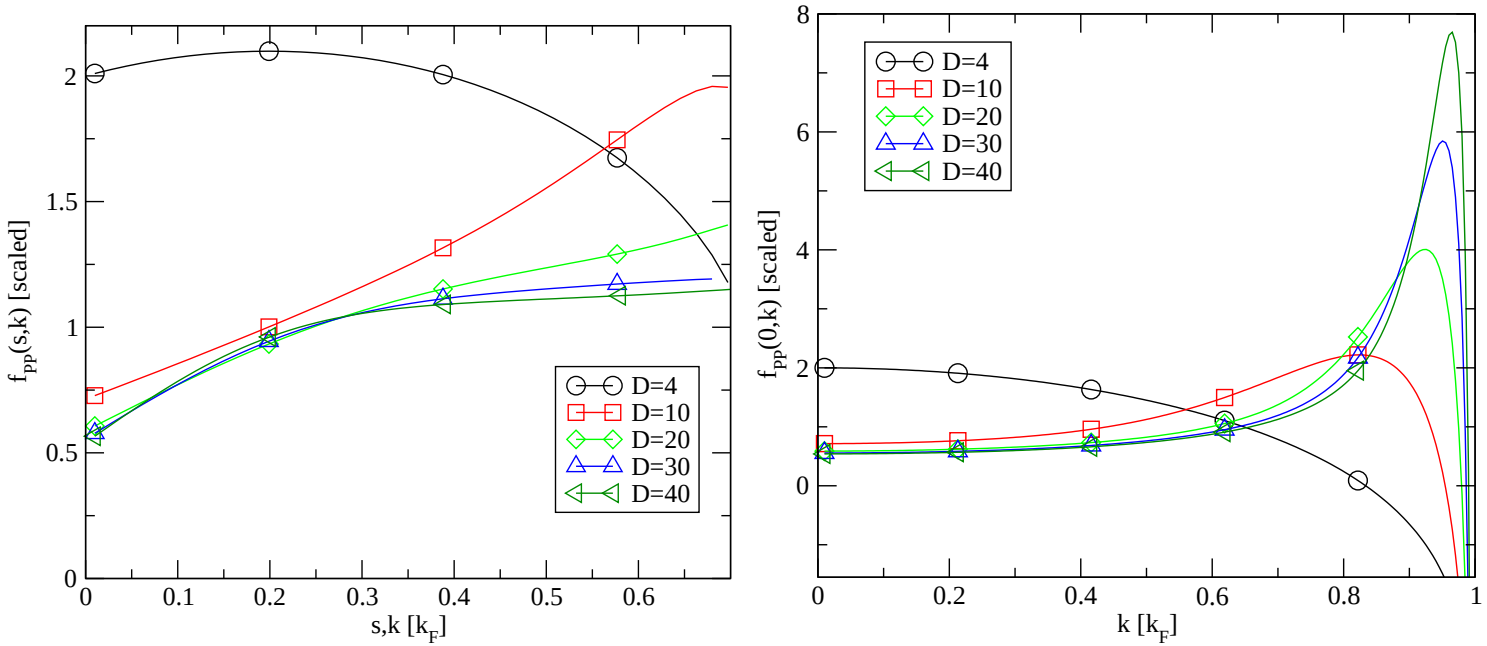


$(C_0/d)^k \cdot 1/d$

$$\lambda = const \quad (d \to \infty)$$

Particle-Particle Scattering Amplitude

$$\int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{\theta_q^+}{k^2 - q^2 + i\epsilon} = f_{vac}(k) + \frac{k_F^{d-2}\Omega_d}{2(2\pi)^d} f_{PP}^d(\kappa, s),$$



$$f_{PP}^{(d)}(s,\kappa) = \frac{1}{d} f_{PP}^{(0)}(s,\kappa)\left(1 + O\left(\frac{1}{d}\right)\right).$$

Example: 2nd order diagram

$$\int \frac{d^d P}{(2\pi)^d} \int \frac{d^d k}{(2\pi)^d} \theta_k^- f_{PP}^{(d)}(\kappa, s) = \frac{k_F^{2d}}{(d+1)^2} \left[\frac{\Omega_d}{(2\pi)^d}\right]^2 \frac{4}{d+1} + \cdots$$

Energy per particle is given by

$$\frac{E_2}{A} = 2 \left[\frac{\Omega_d C_0 k_F^{d-2} M}{(d+1)(2\pi)^d}\right]^2 \left(\frac{k_F^2}{2M}\right).$$

Ladder diagrams form geometric series

$$\frac{E}{A} = \left\{1 + \frac{\lambda}{1-2\lambda} + O\left(\frac{1}{d}\right)\right\} \left(\frac{k_F^2}{2M}\right)$$

$$\lambda \to \infty\text{: } \xi = 1/2 + O(1/d)$$

Pairing in the Large d Limit

BCS gap equation

$$\Delta = \frac{|C_0|}{2} \int \frac{d^d p}{(2\pi)^d} \frac{\Delta}{\sqrt{\epsilon_p^2 + \Delta^2}}$$

Solution

$$\Delta = \frac{2e^{-\gamma} E_F}{d} \exp\left(-\frac{1}{d\lambda}\right)\left(1 + O\left(\frac{1}{d}\right)\right),$$

Pairing Energy

$$\frac{E}{A} = -\frac{d}{4} E_F \left(\frac{\Delta}{E_F}\right)^2 \sim \frac{1}{d}.$$

Polarized Fermions: From BEC to BCS

BEC limit: Tightly bound bosons, no polarization for $\delta\mu < \Delta$

$\delta\mu > \Delta$: Mixture of Fermi and Bose liquid, no phase separation

Stable against current formation? Consider EFT for gapless fermions

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 - \epsilon(-i\vec{\partial}) - (\vec{\partial}\varphi) \cdot \frac{\overleftrightarrow{\partial}}{2m} \right) \psi + \frac{f_t^2}{2}\dot{\varphi}^2 - \frac{f^2}{2}(\vec{\partial}\varphi)^2$$

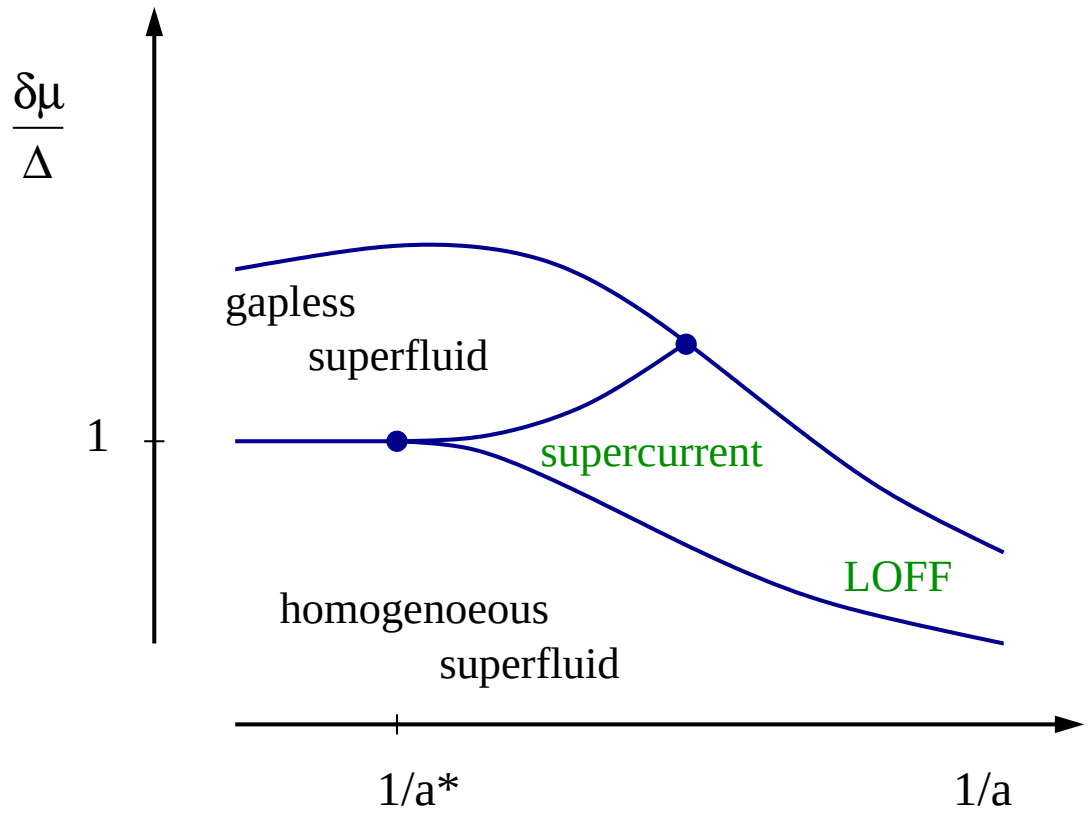
$$\epsilon_v(\vec{p}) = \epsilon(\vec{p}) + \vec{v}_s \cdot \vec{p} - \delta\mu$$

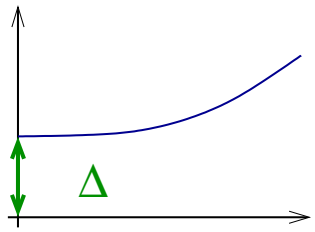
Free energy of state with non-zero current

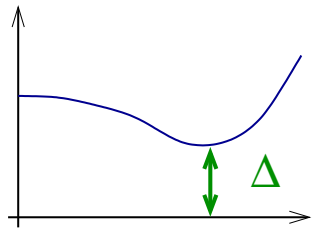
$$F(v_s) = \frac{1}{2} n m v_s^2 + \int \frac{d^3p}{(2\pi)^3}\, \epsilon_v(\vec{p}) \Theta\left(-\epsilon_v(\vec{p})\right)$$

Unstable for BCS-type dispersion relation

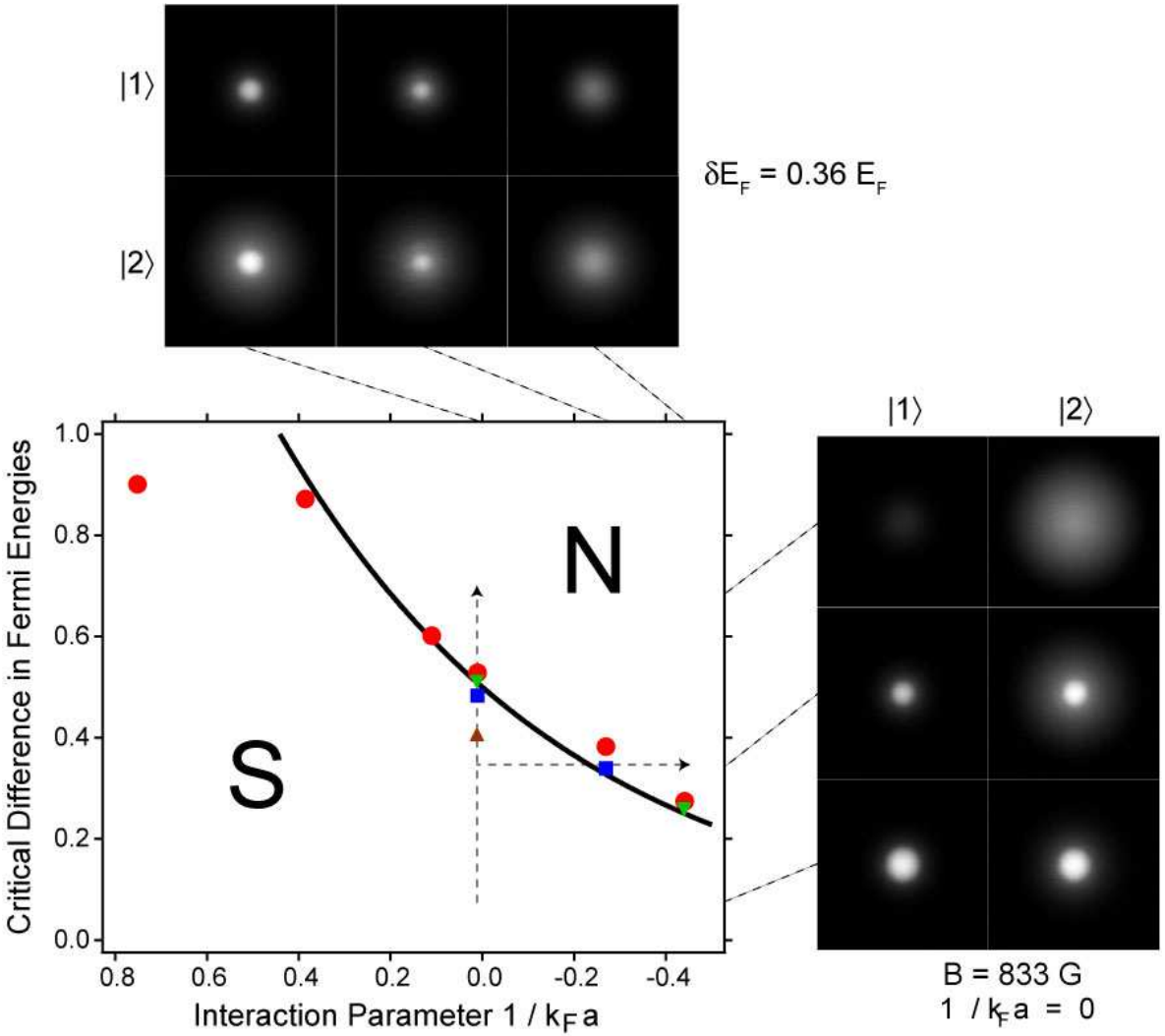
Schematic Phase Diagram







Experimental Situation (MIT group)



Summary

Effective Field Theory (EFT) methods provide

systematic and efficient calculational tools

unified approach to very different physical systems (cold atoms, neutron matter, quark matter)

EFT in many body systems

Low energy expansion: FL, NonFL, Goldstone bosons, ...

EFT for free space interaction. Need extra tools: lattice, large N, large d, exact RG