PHOTOPRODUCTION

H. Rollnik,
Physikalisches Institut der Universität, Bonn

I. INTRODUCTION

The aim of these lectures is to explain what the investigation of photoproduction processes can teach us about the structure of strongly interacting particles. In the beginning of meson physics, experiments on the photoproduction of pions have led to a series of important discoveries: the existence of neutral pions was finally established[1]; the ratio of π^+ and π^0 production led to the invention of the first pion-nucleon resonant state[2]; also the existence and properties of the second [N(1512)] and third [N(1688)] π-N resonance was for the first time discussed using results of Cal-Tech and Cornell Electron synchrotron[3].

Nowadays most of the structures in Elementary Particle Physics have been discovered by people working with hadrons - strongly interacting particles - only, but there are important properties of these resonances which can be studied only with photons.

Using photons for particle physics brings about experimental as well as theoretical complications. These lectures have to deal with the new theoretical problems which arise due to the peculiar properties of the photons. From a principal point of view there are only a few such peculiarities and these are of a more technical character:

the spin 1 of the photon makes formulae clumsy;
the vanishing mass of the photon introduces problems of gauge invariance;
the photon breaks isospin invariance.

The last point does not introduce such great changes as one might expect because of the fortunate fact that the photon is coupled to other particles only "electromagnetically" by the small fine structure constant $\alpha = 1/137$. Therefore it does not take part in the strong interconnection of the hadrons often referred to by the term "unitarity of the S matrix". More precisely, this fact allows the properties of the photons and the strongly interacting particles to be separated. In mathematical terms: any photoproduction amplitude T can be factorized according to

$$T = A^{\mu} < \ldots | j_{\mu} | \ldots > \qquad (1)$$

only if contributions of the order $\sqrt{\alpha}$ are taken into account. Here A^{μ} describes the photon wave function, and the Hilbert states in the matrix elements contain only hadrons. Equation (1) is valid for real as well as for virtual photons which are exchanged in electron processes. Table I gives examples for Eq. (1). All methods which have been invented for the hadron physics - analyticity methods, symmetries, etc. - can now be applied to the matrix elements of the current operator (with some minor change). Therefore a systematic theory exists for the matrix elements (a)*) and (b) of Table I, while for the multiple processes (c) only special

*) Cp. the preceding lectures by P. Beckmann.

models have been investigated. I shall not give a complete account of the systematic theory but I do intend to explain the general ideas and especially to point to the differences between photoproduction and hadronic two body-processes.

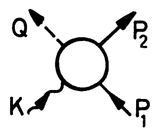
Table I

Examples for the matrix elements of the current operator

matrix elements	investigated by
(a) $\langle N' \| j_\mu \| N \rangle$	electron scattering
(b) $\langle \pi N' \| j_\mu \| N \rangle$ $\langle \eta N' \| j_\mu \| N \rangle$ $\langle KY \| j_\mu \| N \rangle$	photoproduction of single pions photoproduction of η particles photoproduction of strange particles
(c) $\langle 2\pi, N' \| j_\mu \| N \rangle$ especially $\langle \rho N' \| j_\mu \| N \rangle$ $\langle \omega N' \| j_\mu \| N \rangle$ . . .	multiple pion production ρ production ω production . . .

II. KINEMATICS AND INTERMEDIATE STATES FOR THE TWO-BODY PROCESSES

Let us start some brief remarks on the kinematics of the photoproduction of a meson (m) and a baryon (B):



$$\gamma + N \rightarrow m\ (\text{meson}) + B\ (\text{baryon}) \tag{2}$$

$$K + P_1 = Q + P_2 \ . \tag{2a}$$

The last equation contains four-momenta of the involved particles such that the different masses are given by

$$K^2 = 0,\ Q^2 = m^2;\ P_1^2 = M_1^2;\ P_2^2 = M_2^2 \ . \tag{2b}$$

As usual we introduce the Lorentz invariant quantities:

$$s = (K + P_1)^2 = W^2, \quad W = \text{total c.m.s. energy}$$

$$t = (Q - K)^2 = m^2 - 2KQ = m^2 - 2K(\omega - q\cos\Theta) \qquad (3)$$

$$u = (P_2 - K)^2 = M_2^2 - 2K(E_2 + q\cos\Theta),$$

$$s + t + u = M_1^2 + M_2^2 + m^2$$

and where $K^2 = \frac{(s - M_1^2)^2}{4s}$ = (c.m.s. momentum)^2 of the γ + N system (3a)

$$q^2 = \frac{|s - (M_2 + m)^2|\,|s - (M_2 - m)^2|}{4s} = \text{(c.m.s. momentum)}^2 \text{ of the m + B system} \qquad (3b)$$

$\omega = \sqrt{q^2 + m^2}$ = c.m.s. energy of the meson

$E_2 = \sqrt{q^2 + M_2^2}$ = c.m.s. energy of the final baryon (3c)

Θ = c.m.s. angle between meson and photon.

From these relations the "physical domain" in the s-t plane can easily be calculated; i.e. these s,t points which can be realized by the photoproduction process (2). A necessary condition is:

$$s \geq (M_2 + m)^2; \quad t < 0 .$$

Different from the elastic scattering case, the value t = 0 cannot be reached for finite s; compare Fig. 1 which illustrates the situation for pion photoproduction. This fact has brought about some difficulties for the application of dispersion relations[4] and the Regge pole hypothesis[5,6].

All attempts for a detailed theoretical description of the dynamics of photoproduction start from analyticity properties[7] of the photoproduction amplitude T which are assumed to be valid like in other two-body processes though they have never been proved. These assumptions maintain that T can be described by holomorphic functions in the physical s-t domain, the structure of which is determined by singularities with a simple physical meaning: their position and detailed properties can be traced back to the intermediate physical states which are allowed by the general conservation laws. To get a complete system of these singularities one must discuss the following three reactions at the same time[8]

$$\gamma + N \to m + B : \quad \text{s-channel} \qquad (4a)$$

$$\gamma + \bar{m} \to \bar{N} + B : \quad \text{t-channel} \qquad (4b)$$

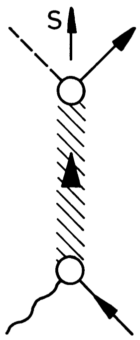
$$\gamma + \bar{B} \to m + \bar{N} : \quad \text{u-channel} . \qquad (4c)$$

They are connected with each other by the crossing relation. Each process is named according to the variable which describes its total c.m.s. energy.

At the present time it appears hopeless to take account of all intermediate states. Even a more modest approach has not been carried through completely which consists in retaining only the resonant intermediate states. Tables II through IV give the presently known resonances which in principle can play a role in photoproduction:

Table II

Intermediate states in the s-channel
(if the incoming photon is replaced by a pion the situation remains the same)

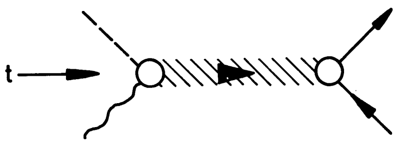


final state	intermediate (resonant) state	
π N	N(940)	Δ(1238)
	N(1480)?	
or KΣ	N(1512)	
	N(1688)	Δ(1920)
	N(2190)	Δ(2360)
	N(2700)	
η N	only N(I = ½) intermediate states	
KΛ		

s-channel (Table II) - All known isobars = non-strange baryon resonances can occur for the π N and KΣ final state. For the production of η N and KΛ conservation of isospin in the upper bubble allows only the I = ½ isobars.

Table III

Intermediate states in the u-channel



final state	intermediate (resonant) states
π N	all isobars
η N	only I = ½ - isobars
KΛ	Λ(1115) $Y_0^*(1405)$; $Y_0^*(1520)$; $Y_0^*(1815)$,
KΣ	Σ(1189) $Y_1^*(1385)$; $Y_1^*(1660)$; $Y_1^*(1765)$.

u-channel (Table III) - For π N production all isobars can occur; in the η N case only I = ½ isobars are allowed, while for strange particle production all hyperon states with strangeness -1 can occur.

In both cases the photon can be replaced by a pion without changing any result. The deeper reason for this lies in the fact that the photon can be regarded as a mixture of an isoscalar and an isovector particle[9]. This can be inferred directly from the Gell-Mann-Nishijima relation:

$$Q = I_3 + Y/2 \quad . \tag{5}$$

For a local field theory which is finally the basis of all our considerations we deduce from Eq. (5) because of $Q = \int j_0(x)\, d^3x$ a decomposition of the current operator into an isovector and an isoscalar component:

$$j_\mu = j_\mu^{V_3} + j_\mu^{s} \quad . \qquad (6)$$

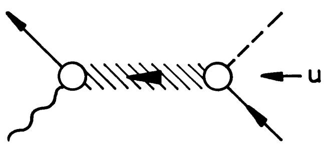
The first term behaves with respect to isospin transformations like a neutral pion, which proves our statement. Introducing Eq. (6) into the matrix element $< f | j_\mu | N >$ one finds by coupling the isospin of j_μ to the isospin of the initial nucleon, the <u>isospin decomposition for the photoproduction amplitude</u>

$$T = T^{(o)} + T^{(1/2)} + T^{(3/2)} \; , \qquad (7)$$

where $T^{(o)}$ originates in the isoscalar current and contains (in the s-channel) only the (total) isospin $I = 1/2$. The two other terms come from $j_\mu^{V_3}$ and belong to the isospin $I = 1/2$ and $I = 3/2$, respectively. For the production of the isoscalar η particles and for $K + \Lambda$ only $T^{(o)}$ and $T^{(1/2)}$ are different from zero while for pion and $K\Sigma$ production all three terms contribute.

<u>Table IV</u>

Intermediate states in the t-channel for photoproduction and pion-nucleon scattering



final state	intermediate (resonant) states
$\pi^\pm N$	$\pi^\pm(140)$; $\rho^\pm(763)$; $A1^\pm(1090)$; $B^\pm(1215)$; $A2^\pm(1310)$
$\pi^0 N$ $\eta^0 N$	$\rho^0(763)$; $\omega^0(783)$; $\varphi^0(1020)$; $B^0(1215)$
$K^+ Y$ $K^0 Y$	$K^+(494)$; $K^+(725)$; $K^{*+}(891)$; $K_C^+(1215)$ K^0, K^{*0}, K_C^0
$\pi N \to \pi N$	ρ; $\eta 2\pi(960)$; B; $f^0(1250)$; E(1410)?

<u>t-channel</u> (Table IV) - Here conservation laws play an even more stringent role. Firstly we have to distinguish between neutral and charged mesons to take charge conservation into account. For the charged mode all charged meson resonances can enter as intermediate states. For the neutral mode, on the contrary, we have to pay attention to the <u>charge conjugation invariance</u>: because the C parities of the photon resp. of the neutral pion and of the η-particle are odd resp. even only meson resonances with odd C-parity are possible intermediate states. Thus the following particles are excluded: π^0; η^0; neutral components of A1 and A2, and finally the f^0 particle.

This last fact distinguishes the photoproduction of pions clearly from pion-nucleon scattering that is indicated in the last line of Table IV. The allowed states for π-N scattering are determined by their G parity; only even G parity states are allowed. The concept of G parity can also be applied to photoproduction if the decomposition (6) is used. The G operation

$$G = C\, e^{-i\pi I_2}$$

acts in the following way on the two components of j_μ:

$$G\, j_\mu^{V_3}\, G^{-1} = +\, j_\mu^{V_3} \qquad (8a)$$

$$G\, j_\mu^{S}\, G^{-1} = -\, j_\mu^{S} \;, \qquad (8b)$$

where the odd behaviour of the electromagnetic current under C has been taken into account. In the diagram of Table IV the current j_μ occurs only on the left hand bubble. Counting also the G parity of the pion resp. the η particle one finds the entries of Table V. Note, for example, that the exchange of a ρ particle contributes only to the isoscalar part in π production and only to the isovector part in η production.

Table V

Contribution of the intermediate particles in the t-channel to the different isospin components of the photoproduction amplitude for $\gamma + N \to \pi + N$. For $\gamma + N \to \eta + N$ the two lines have to be interchanged.

exchanged particle	G	contribution to
π, ω; φ, A1, A2	- 1	$T^{(1/2)}$ and $T^{(3/2)}$ resp. $T^{(\pm)}$ (isovector)
ρ; B	+ 1	$T^{(0)}$ (isoscalar)

For a theoretical description of the photoproduction process one must know the contribution of the different resonances. In the next section we start with qualitative discussion.

III. RELATIVE IMPORTANCE OF THE DIFFERENT RESONANCES: QUALITATIVE DISCUSSION

The first criteria to answer this question can be found by looking at the position of the resonances in the s-t plane: as a working principle we shall regard those resonances as most important which lie nearest to the discusse kinematical point[8]. Therefore each resonance in the s-channel will become important if the variable s passes through the mass value of the respective resonance. One encounters a usual resonance phenomenon. For the other channels the importance of the resonances can be easily discussed with the help of Fig. 2. Evidently the exchange of a pion should play an important role at least in the forward direction. Now pion exchange is by C invariance only possible for production of charged pions. We expect a marked difference between the cross-sections for charged and neutral pions which indeed shows up in the experimental data. For the backward direction the nearest singularity is due to the exchange of a nucleon in the u-channel. It must be taken into account at any rate together with the pion exchange and the nucleon pole in the s-channel to preserve gauge invariance (cp. section VIII). The contributions of these three poles can be calculated completely by the formulae of (renormalized) perturbation theory once the pion-nucleon coupling constant f^2 resp. the kaon-hyperon coupling constants and the magnetic moments of the baryons are known. Figure 3 gives the results for the best known case: photoproduction of pions[10]. Turning now to the role of the other mesonic resonances the situation is much more unclear. Especially the influence of the ρ meson is under vivid discussion in the present literature. In this section we only mention a reasoning using the Bronzan-Low quantum number A[11]. This quantum number should be conserved in strong and electromagnetic interactions as long as virtual baryon states can be neglected. Table VI shows the values of A for the mesons in question and the result of the application of A conservation to the electromagnetic vertices of mesons: only ω and the somewhat dubious A1 particle can be coupled to photon and pion (resp. η^0). If one could rely on this argument only the exchange of pions and very heavy subjects like A1 is allowed for charged pion production. On the other hand, in the π^0 and η production, ω exchange will presumably play a role. In connection with the questionable A parity of the A2 resonance doubts have been raised against its applicability[12].

Table VI

Electromagnetic couplings of mesons allowed by A parity conservation
(V means allowed, - means forbidden.
Empty places are forbidden by other reasons.)

	γ	π	η	K	ρ	ω	φ	A1	A2	B	κ	K^*	K_C
A parity	+	-	-	-	+	-	+	-	?	+	?	+	-

X	π	ρ	ω	φ	A1	A2	B
$\gamma\pi X$	V	-	V	-	V	?	-
$\gamma\eta X$		-	V	-	V	?	-
$\gamma K X$	V	?	-	V			
X	K	κ	K^*	K_C			

We conclude this section with some remarks for very high energies ($s \to \infty$). s-channel resonances should no longer be observed; they overlap completely if present at all. On the other hand, the exchange of particles should become increasingly more important especially for particles with higher spin. We recall the well-known fact that the differential cross-section due to the exchange of a spin j particle in the t-channel behaves like[13],

$$\frac{d\sigma}{dt} \propto s^{2j-2} \tag{9}$$

for small momentum transfers t.

Therefore vector particles like ρ, ω and φ are good candidates, even better, a spin 2 particle (perhaps A2 !). But it is generally believed that the s dependence of Eq. (9) will be "damped" in some way in order to avoid conflicts with analyticity and unitarity[14]. This is achieved in the simplest way by replacing j by a "Regge trajectory" $\alpha(t)$ thus introducing a "moving spin" of the exchanged particle. Making the not very convincing assumption that $\alpha(t)$ is a linear function of t with a slope given by proton-proton diffraction scattering one can draw some conclusions[15]: according to Fig. 4, ρ, ω and A2 have the largest exponents $\alpha(t)$ in the negative t region. The energy variation of the photopion cross-section in the forward direction is therefore approximatively given by [because of $\alpha(0) \approx 0.5$]

$$\frac{d\sigma}{dt} \propto s^{2[\alpha(0)-1]} \sim \frac{1}{s} \text{ for } \gamma + p \begin{array}{l} \nearrow N+\pi \\ \searrow N+\eta \end{array} . \tag{10a}$$

Strange particle production is determined by an analogous argument by K^* exchange leading to

$$\frac{d\sigma}{dt} \propto s^{2\alpha_{K^*}-2} \sim \frac{1}{s^{8/5}} . \tag{10b}$$

For the backward direction the exchange of excited baryons must be considered. The $\Delta(1235)$ trajectory seems to be the most important giving

$$\frac{d\sigma}{du} \propto s^{2\alpha_\Delta-2} \sim \frac{1}{s^2} \text{ for pion and } \eta\text{-production ,} \tag{10c}$$

while for strange particle production the $Y_1^*(1385)$ exchange suggests

$$\frac{d\sigma}{du} \propto \frac{1}{s^{2,6}} . \tag{10d}$$

Of course all these formulae are guesses and describe only how the situation could be. For the formal details of the application of Regge poles to photoproduction we refer to the literature[5,6,15-18].

IV. MULTIPOLE ANALYSIS IN THE s-CHANNEL

In principle the s-channel resonances can be seen directly as maxima in the energy dependence of the production cross-sections. For the lower resonances this is indeed the case (see Fig. 5). The heavier isobars do not show up so clearly. Here the contributions of the different resonant states overlap each other and there are also non-resonant terms. To formulate a mathematical apparatus for such considerations we have to generalize the well-known partial wave expansion

$$\sum_{\ell} (2\ell + 1)\, e^{i\delta_\ell} \sin\delta_\ell\, P_\ell(\cos\Theta) \tag{11}$$

for the scattering of spinless particles for our case. Complications arise because of the spins of photons and baryons. In the older treatments[19] also the zero mass of the photon introduces troubles. But the brilliant "helicity-formalism" invented by Jacob and Wick[20] gives a simple solution for our problem. In this lecture we shall apply this formalism directly to the photoproduction, referring to the original papers for its deeper foundation.

We recall the basic definition: each spinning particle is described by its momentum $\vec{p}$ and its spin component in the direction of p = helicity λ:

$$|\vec{p}, \lambda> \quad . \tag{12}$$

In general λ can take the 2j + 1 values

$$\lambda = -j, \ldots + j \quad , \tag{12a}$$

but for massless particles only

$$\lambda = \pm j \tag{12b}$$

is allowed. The helicity does not change under rotations, but is reversed under space reflection. Moreover, it is not Lorentz invariant. Therefore we shall use throughout the c.m.s. system. The photoproduction process (2) for spin-zero mesons and spin ½ baryons can be described by the following states (see Fig. 6):

a) Initial state (in the c.m.s. frame !):

$|k, \Theta_i; \lambda, \nu_1>$

$k = |\vec{K}_{c.m.}|$ = c.m.s. momentum of the incoming particles [cp. Eq. (3a)]

Θ_i = initial angle (will be put equal to zero later on) (13)

λ = helicity of the photon; $\lambda = \pm 1$

ν_1 = helicity of the incoming baryon; $\nu_1 = \pm \frac{1}{2}$.

Note that the photon has no λ = 0 (longitudinal) component.

b) Final state:

$|q, \Theta_f; \nu_2>$

$q = |\vec{Q}_{c.m.}|$ = c.m.s. momentum of the final particles [cp. Eq. (3b)]

Θ_f = final angle

ν_2 = helicity of the final baryon; $\nu_2 = \pm \frac{1}{2}$.

The production amplitude

$$< q, \Theta; \nu_2|T| k, 0; \lambda, \nu_1 > \tag{15}$$

contains 8 functions of W and Θ resp. s and t (count all possibilities $\lambda = \pm 1$, $\nu_1 = \pm \frac{1}{2}$, $\nu_2 = \pm \frac{1}{2}$!) but only 4 are independent. Parity conservation ensures that the process with

$$\lambda, \nu_1; \nu_2 \quad \text{and} \quad -\lambda, -\nu_1; -\nu_2$$

are directly connected. One has only to apply a reflection on the production plane. Therefore we can restrict to

$$\nu_2 = + \tfrac{1}{2} \tag{16}$$

and must consider the following 4 amplitudes[21])

$$H^{\pm}(s,t) = \langle q, \Theta; \nu_2 = + \tfrac{1}{2} | T | k, 0; \lambda = \pm 1, \nu_1 = + \tfrac{1}{2} \rangle \tag{17a}$$

no helicity-flip amplitudes

$$\Phi^{\pm}(s,t) = \langle q, \Theta; \nu_2 = + \tfrac{1}{2} | T | k, 0; \lambda = \pm 1, \nu_1 = - \tfrac{1}{2} \rangle \tag{17b}$$

helicity-flip amplitudes.

By the way, these definitions can easily be generalized to virtual photons which occur in inelastic electron scattering. One has only the additional possibility of a longitudinal polarized photon: $\lambda = 0$ and therefore two longitudinal amplitudes

$$H^{0}(s,t) \quad \text{and} \quad \Phi^{0}(s,t) , \tag{17c}$$

which can be obtained from Eq. (15) by putting $\lambda = 0$.

In order to find the wanted partial wave decomposition one has to develop the states (13) and (15) in terms of eigenstates of the total angular momentum. This can be easily done because the helicity is a rotational invariant quantity. One gets

$$|q; \Theta_f; \nu_2 \rangle = \sum_{J,M} |W; JM, \nu_2 \rangle \sqrt{\frac{2J+1}{4\pi}}\, d^J_{M-\nu_2}(\Theta_f) , \tag{18a}$$

and

$$|k; \Theta_i; \lambda, \nu_1 \rangle = \sum_{J,M} |W; JM; \lambda\nu_1 \rangle \sqrt{\frac{2J+1}{4\pi}}\, d^J_{M\,\lambda-\nu_1}(\Theta_i) \tag{18b}$$

where the known functions[22])

$$\langle JM | e^{-i\Theta J_2} | J, M' \rangle = d^J_{MM'}(\Theta)$$

have been introduced and the energy dependence of the states has been indicated by the total c.m.s. energy W.

Contrary to the usual situation the angular momentum states occurring in equations (18a) and (18b) are not eigenstates of the parity operator. Instead one has the parity property:

$$P|W; JM; \nu_2 \rangle = -(-1)^{J-\frac{1}{2}} |W; JM; -\nu_2 \rangle \qquad \text{for the } \pi\text{ B state} \tag{19a}$$

$$P|W; JM; \lambda, \nu_1 \rangle = +(-1)^{J-\frac{1}{2}} |W; JM; -\lambda, \mp\nu_1 \rangle \quad \text{for the } \gamma\text{ B state ,} \tag{19b}$$

where we have used the proper phase normalizations of the state vectors and have assumed:

negative parity for the meson and positive parity for the baryons*). (19c)

Now parity eigenstates can be constructed in a simple way. Let us start with pion-baryon states:

$$|W;\ JM,\pi> = \frac{1}{\sqrt{2}}(|W;\ JM,\nu_2> \pm |W;\ JM,-\nu_2>) \equiv |W;\ JM,\ l> \tag{20}$$

where the parity π is given by

$$\pi = \mp (-1)^{J-\frac{1}{2}} \overset{\text{def}}{=} -(-1)^{l} \ . \tag{20a}$$

Here we have in addition introduced the orbital angular momentum 1 of the π N system which is formally defined by the second equation of (20a).

These state vectors (20) are the proper quantities to describe the isobars which by definition have a well-defined spin and parity.

Repeating this procedure for the photon-baryon state one encounters a somewhat more complicated situation. The appropriate eigenstates of $\vec{J}^2$ and P are

$$|W;\ JM;\ \pi;\lambda> = -\frac{1}{\sqrt{2}}(W;\ JM;\ \lambda,\nu_1 = +\tfrac{1}{2}> \pm |W;\ JM;\ -\lambda,\nu_1 = -\tfrac{1}{2}>) \tag{21}$$

with

$$\pi = \pm (-1)^{J-\frac{1}{2}} = +(-1)^{l} \ . \tag{21a}$$

The parameter λ in the l.h.s. of Eq. (21) no longer denotes the helicity of the states but distinguishes between two different eigenstates belonging to the same total angular momentum J and the same parity π. Therefore: for given spin J and parity there are two possible states of the γB system. Correspondingly we have two photoproduction amplitudes for each isobar:

$$A_{\lambda}^{J,\pi}(W) = <JM;\ \pi|T(W)|JM;\ \pi;\ \lambda> \quad \text{with } \lambda = \pm 1 \ . \tag{22}$$

The matrix elements on the r.h.s. are independent on M by rotational invariance**). These two amplitudes are usually characterized by the terms "electric and magnetic multipoles". But these quantities are not directly given by Eq. (22). They are therefore introduced by a consideration starting from the photon states only

$$|\vec{k},\lambda> \ , \tag{23}$$

which describe,a single photon with momentum $\vec{k}$ and helicity λ. Repeating the step leading to Eq. (18) we arrive at eigenstates of the total angular momentum of the photon which we characterize by the quantum numbers L, m†):

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*) The general rule is given by Eq. (21): $P|JM,\lambda_1\lambda_2> = \pi_1\pi_2(-1)^{J-s_1-s_2}|JM,-\lambda_1,-\lambda_2>$, where s_1 and s_2 are the spins of the two particles.

**) For electroproduction of mesons there exists for each J,π value a third longitudinal amplitude with $\lambda = 0$.

†) We use a small "m" to avoid confusion with M occurring in Eq. (18) ff.

$$|L,m;\lambda> \quad \text{with}^{*)} \quad \begin{matrix} L = 1,2,\dots \quad m = -L,\dots,+L \\ \lambda = \pm 1 \ . \end{matrix} \tag{24}$$

Now the electric resp. magnetic multipole states of order L are defined by (20)

$$|EL,m> = \frac{1}{\sqrt{2}}\left(|L,m;\ \lambda = +1> + |L,m;\ -1>\right) \tag{25a}$$

$$|ML,m> = \frac{1}{\sqrt{2}}\left(|L,m;\ +1> - |L,m;\ -1>\right)\ . \tag{25b}$$

They belong to the parities (cp. footnote of p. 27)

$$(-1)^L \quad \text{for the electric multipoles} \tag{26a}$$

$$-(-1)^L \quad \text{for the magnetic multipoles}\ . \tag{26b}$$

To get angular momentum states for the photon-baryon system we couple to Eqs. (25a) and (25b) spin ½ states u_s according to the well-known recipe

$$|JM;\ EL> = \sum_{m,s} (L,m;\ \tfrac{1}{2},s|JM)|EL,m>\ u_s \tag{27a}$$

$$\text{with } J = L \pm \tfrac{1}{2}\ .$$

$$|JM;\ ML> = \sum_{m,s} (L,m;\ \tfrac{1}{2},\ s|JM)|EL,m>\ u_s \tag{27b}$$

These states are the basis for the usual definition of the electric and magnetic multipole amplitudes. Using the π N states (20) the <u>electric multipole amplitudes</u> are given by

$$\begin{matrix} \text{meson-baryon} & \gamma\text{-baryon} \\ <JM;\ 1|T(W)| & JM;\ EL> \end{matrix}\ .$$

Because of parity conservation we find, using Eq. (20a) and Eq. (26a),

$$L = \ell \pm 1\ ,$$

therefore by $J = L \pm \frac{1}{2}$ only the matrix elements

$$<J = \ell + \tfrac{1}{2}, M;\ \ell|T(W)|\ell + \tfrac{1}{2},\ M;\ E(\ell+1)> \equiv \sqrt{(\ell+1)(\ell+2)}\,E_{\ell+}(W) \tag{28a}$$

(electric multipole of order $L = \ell + 1$)

and

$$<J = \ell - \tfrac{1}{2}, M;\ \ell|T(W)|\ \ell - \tfrac{1}{2}, M;\ E(\ell-1)> \equiv -\sqrt{\ell(\ell-1)}\ E_{\ell-}(W)$$

(electric multipole of order $L = \ell - 1$)

are different from zero.

---

*) Because of $\lambda > 0$ the value $L = 0$ is excluded by the analogue of Eq. (18).

On the r.h.s. we have introduced conventional factors and a notation due to Feld[23]. Observe that the index in $E_{l\pm}$ does not coincide with the multipole order L ! Analogously we have for the magnetic multipole amplitudes by parity conservation

$$L = l$$

and the non-vanishing matrix elements are given by

$$< J = l \pm \tfrac{1}{2}, M;\ l|T(W)|l \pm \tfrac{1}{2}, M_l > \equiv \sqrt{l(l+1)}\, M_{l\pm}(W) \qquad (29)$$

(magnetic multipole of order $L = l$) .

In this case the index l coincides with the multipole order.

It remains to establish the connection with earlier introduced amplitudes $A_\lambda^{J,\pi}$ [Eq. (22)]. We shall not go into the details of the calculation but merely quote the results[24]:

$$A_\lambda^{J=l+\frac{1}{2},\pi} = \frac{(l+2)(2l+3)}{2}\,(l+1,\lambda;\ \tfrac{1}{2},-\tfrac{1}{2}|l+\tfrac{1}{2},\lambda-\tfrac{1}{2})\,E_{l+}$$
$$+\ \frac{l(2l+1)}{2}\,(l,\lambda;\ \tfrac{1}{2}-\tfrac{1}{2}|l+\tfrac{1}{2},\lambda-\tfrac{1}{2})\,M_{l+} \qquad (30a)$$

and

$$A_\lambda^{J=l-\frac{1}{2},\pi} = -\frac{(l-1)(2l-1)}{2}\,(l-1,\lambda;\ \tfrac{1}{2}-\tfrac{1}{2}|l-\tfrac{1}{2},\lambda-\tfrac{1}{2})\,E_{l-}$$
$$+\ \frac{(l+1)(2l+1)}{2}\,(l,\lambda;\ \tfrac{1}{2}-\tfrac{1}{2}|l-\tfrac{1}{2},\lambda-\tfrac{1}{2})\,M_{l-}\ . \qquad (30b)$$

[In both equations the parity π is given by Eq. (20a).]

Now we have written down all necessary definitions and formulae to find the wanted expansion of the helicity amplitudes H^{+}, $\Phi^{\pm}$ by a straightforward calculation. We give the result in Table VII in an explicit form where all Clebsch-Gordan coefficients have already been worked out and the $d^J_{MM'}$ functions expressed in terms of Legendre polynomials $P_l(\cos\Theta)$[25].

A great advantage of the amplitudes $H^{\pm}$ and $\Phi^{\pm}$ lies in the fact that differential cross-sections and polarizations can be expressed in a simple way. We give three examples:

a) The differential cross section for photons with circular polarization but unpolarized baryons is given by

$$\frac{d\sigma^{\pm}}{d\Omega} = \frac{q}{k}\,(|H^{\pm}|^2 + |\Phi^{\pm}|^2)\ , \qquad (31a)$$

while the cross-section for unpolarized γ rays follows from

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{d\sigma^{+}}{d\Omega} + \frac{d\sigma^{-}}{d\Omega}\right)\ . \qquad (31b)$$

Table VII

Multipole expansion of the helicity amplitude

$$\frac{d\sigma^{\pm}}{d\Omega} = \frac{q}{k}\left(|H^{\pm}|^2 + |\Phi^{\pm}|^2\right); \quad P(\Theta)\frac{d\sigma}{d\Omega} = \frac{q}{k}\,\mathrm{Im}(\Phi^{-*}H^{+} + \Phi^{+*}H^{-})$$

$$H^{+} = -\frac{1}{\sqrt{2}}\sin\frac{\Theta}{2}\sum_{\ell}\left(P'_{\ell+1} + P'_{\ell}\right)\left\{(\ell+2)[E_{\ell+} + M_{(\ell+1)-}] + \ell\,[M_{\ell+} - E_{(\ell+1)-}]\right\}$$

$$H^{-} = -\frac{1}{\sqrt{2}}\sin\frac{\Theta}{2}\sum_{\ell}\left(\ell\,P'_{\ell+1} + (\ell+2)\,P'_{\ell}\right)\left[E_{\ell+} - M_{(\ell+1)-} - M_{\ell+} - E_{(\ell+1)-}\right]$$

$$\Phi^{+} = -\frac{1}{\sqrt{2}}\cos\frac{\Theta}{2}\sum_{\ell}\left(\ell\,P'_{\ell+1} - (\ell+2)\,P'_{\ell}\right)\left[E_{\ell+} + M_{(\ell+1)-} - M_{\ell+} + E_{(\ell+1)-}\right]$$

$$\Phi^{-} = -\frac{1}{\sqrt{2}}\cos\frac{\Theta}{2}\sum_{\ell}\left(P'_{\ell+1} - P'_{\ell}\right)\left[(\ell+2)\left(E_{\ell+} - M_{(\ell+1)-}\right) + \ell\left(M_{\ell+} + E_{(\ell+1)-}\right)\right]$$

$$H^{0} = -\frac{1}{\sqrt{2}}\cos\frac{\Theta}{2}\sum_{\ell}\left(\ell+1\right)\left(P'_{\ell+1} - P'_{\ell}\right)\left[L_{\ell+} - L_{(\ell+1)-}\right]\frac{\sqrt{k^2}}{k_0}$$

$$\Phi^{0} = -\sin\frac{\Theta}{2}\sum_{\ell}\left(\ell+1\right)\left(P'_{\ell+1} + P'_{\ell}\right)\left[L_{\ell+} + L_{(\ell+1)-}\right]\sqrt{\frac{k^2}{k_0}}$$

where $k^2 = k_0^2 - \vec{k}^2$ is the mass of the virtual photon

[Note: a factor ½ in Eq. (31a) has been cancelled because we use only four amplitudes instead of eight !]

b) The polarization $P(\Theta)$ of the final baryon which is perpendicular to the production plane can be calculated from

$$\frac{d\sigma}{d\Omega}P(\Theta) = \frac{q}{k}\,\mathrm{Im}(\Phi^{-*}H^{+} + \Phi^{+*}H^{-}) \; ; \qquad (32)$$

c) For linearly polarized photons linear combinations of $H^{\pm}$ resp. $\Phi^{\pm}$ occur. We describe these photons by a polarization vector $\vec{\epsilon}$ which is perpendicular to the photon momentum $\vec{k}$ and has an angle Φ with respect to $-(\vec{k}\times\vec{q})$

$$\cos\varphi = -\frac{\vec{\epsilon}(\vec{k}\times\vec{q})}{|\vec{k}\times\vec{q}|} \; .$$

Expressing $\vec{\epsilon}$ in terms of $\vec{\epsilon}\pm$ which corresponds to the helicity states, one arrives by simple algebra at the following amplitudes

$$H(\vec{\epsilon}) = \frac{1}{\sqrt{2}}\left(H^{+}e^{+i\varphi} - H^{-}e^{-i\varphi}\right);$$

$$\Phi(\vec{\epsilon}) = \frac{1}{\sqrt{2}}\left(\Phi^{+}e^{+i\varphi} - \Phi^{-}e^{-i\varphi}\right) . \qquad (33a)$$

The differential cross-section for linearly polarized photons is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{\vec{\epsilon}} = \frac{q}{k}\left(|H(\vec{\epsilon})|^2 + |\Phi(\vec{\epsilon})|^2\right) . \tag{33b}$$

For further use we collect in Table VIII the differential cross-sections for pure multipoles, and in Table IX the cross section and polarization if only the values $\ell = 0$ and 1 of the angular momentum of the pion are important.

Table VIII

Angular distribution of photoproduction for different pure multipoles
(This distribution depends only on J and L but not
on the parity of the π N state.)

πN system	multipole	matrix element	$\frac{d\sigma}{d\Omega}$
$\frac{1}{2}^*$, $P_{1/2}$	M1	M_{1-}	$\|M_{1-}\|^2$
$\frac{1}{2}^-$, $S_{1/2}$	E1	E_{0+}	$\|E_{0+}\|^2$
$\frac{3}{2}^+$, $P_{3/2}$	M1	M_{1+}	$\|M_{1+}\|^2\ (5-3\cos^2\Theta)$
	E2	E_{1+}	$\|E_{1+}\|^2\ (1+\cos^2\Theta)$
$\frac{3}{2}^-$, $D_{3/2}$	E1	E_{2-}	$\|E_{2-}\|^2\ (5-3\cos^2\Theta)$
	M2	M_{2-}	$\|M_{2-}\|^2\ (1+\cos^2\Theta)$
$\frac{5}{2}^-$, $D_{5/2}$	M2	M_{2+}	$\|M_{2+}\|^2\ (1+6\cos^2\Theta-5\cos^4\Theta)$
	E3	E_{2+}	$\|E_{2+}\|^2\ (5+6\cos^2\Theta+5\cos^4\Theta)$
$\frac{5}{2}^+$, $F_{5/2}$	E2	E_{3-}	$\|E_{3-}\|^2\ (1+6\cos^2\Theta-5\cos^4\Theta)$
	M3	M_{3-}	$\|M_{3-}\|^2\ (5+6\cos^2\Theta+5\cos^4\Theta)$

Table IX

Differential cross-section and the polarization of the second baryon including all multipoles with $\ell = 0,1$

$\frac{d\sigma}{d\Omega} = A + B\cos\Theta + C\cos^2\Theta$
$A = \frac{q}{k}\left[\frac{5}{2}\lvert M_{1+}\rvert^2 + \lvert E_{0+}\rvert^2 + \lvert M_{1-}\rvert^2 + \frac{3}{2}\lvert E_{1+}\rvert^2 + \mathrm{Re}\left\{M^*_{1+}(M_{1-} - 3E_{1+}) + 3M^*_{1-}E_{1+}\right\}\right]$ $B = \frac{q}{k}\, 2\,\mathrm{Re}\left\{E^*_{0+}(M_{1+} - M_{1-} + 3E_{1+})\right\}$ $C = \frac{q}{k}\left[-\frac{3}{2}\lvert M_{1+}\rvert^2 + \frac{9}{2}\lvert E_{1+}\rvert^2 - 3\,\mathrm{Re}\left\{M^*_{1+}(M_{1-} - 3E_{1+}) - 3M^*_{1-}E_{1+}\right\}\right]$
$a = -\frac{q}{k}\,\mathrm{Im}\left[E^*_{0+}(M_{1+} + 2M_{1-} + 3E_{1+})\right]$ $\frac{d\sigma}{d\Omega}P(\Theta) = \sin\Theta[a + b\cos\Theta];$ $b = 3\frac{q}{k}\,\mathrm{Im}\left[M^*_{1-}(M_{1+} + 3E_{1+})\right]$

For linearly polarized photons one finds from Eqs. (33a) and (33b) for this case

$$\frac{d\sigma}{d\Omega}\bigg|_{\vec{\epsilon}} = \frac{d\sigma}{d\Omega}\bigg|_{\mathrm{unpol.}} + \alpha\sin^2\Theta\cos^2\varphi$$

with

$$\alpha = -\frac{3}{2}\lvert M_{1+}\rvert^2 + \frac{9}{2}\lvert E_{1+}\rvert^2 - 3\,\mathrm{Re}\left[M^*_{1-}(M_{1+} - E_{1+}) + M^*_{1+}E_{1+}\right].$$

We stress the important rules which are contained in these results:

1) the angular distribution depends only on the spin J and the multipole order L but not on the parity;

2) polarizations occur only if different multipoles interfere which leads to an over-all factor $\sin\Theta$;

3) the formulae are invariant under the simultaneous replacement of

$$\Theta \to \pi - \Theta,\ \text{i.e. } \cos\Theta \to -\cos\Theta;\ \sin\Theta \to +\sin\Theta$$

and of

$$E_{1\pm} \text{ by } -(-1)^\ell E_{1\pm}$$

resp.

$$M_{1\pm} \text{ by } -(-1)^\ell M_{1\pm}\ .$$

(By this rule the asymmetry coefficient B is due to an interference of different "parity" multipoles.) This invariance based on parity conservation helps very much when discussing the multipole expansion qualitatively.

V. PHENOMENOLOGICAL DISCUSSION OF THE s-CHANNEL RESONANCES

According to the results of the last section each s-channel resonance with a given spin and parity can lead to two different multipole amplitudes and we have to ask for each isobar: which multipole resonates ? In this section we discuss the empirical evidence for an answer to this question. In order to avoid the complications due to the near singularity in the t-channel introduced by one-pion exchange we restrict ourselves to the production of neutral pions. Figure 7 shows the coefficients of the angular distribution

$$\frac{d\sigma}{d\Omega} = A + B\cos\Theta + C\cos^2\Theta + D\cos^3\Theta + E\cos^4\Theta \qquad (34a)$$

for energies below 1 GeV[26]. One clearly recognizes the maxima corresponding to the first and second isobar, while the third πN resonance (with E_γ = 1050 MeV) lies just outside the region.

1. $\Delta(1235) \equiv N^*_{3/2\,3/2}$ with $3/2^+$ (E_γ = 350 MeV)

Here our question has been answered uniquely. The angular distribution can be well represented by

$$5 - 3\cos^2\Theta \qquad (34b)$$

with a small asymmetry coefficient B which goes through zero at resonance. Assuming the spin-parity assignment $3/2^+$ we immediately deduce with the help of Table VIII:

$$\text{the magnetic dipole amplitude } M_{1+} \text{ resonates at the } \Delta(1235) \text{ resonance .} \qquad (35)$$

Moreover one finds from experiment the behaviour

$$|M_{1+}|^2 \propto q^3$$

for small energies. This agrees with the expected threshold law[27])

$$M_{1\pm} \propto q^{2l+1}$$

for the expected value l = 1.

2. $N(1512) = N^*_{1/2\,3/2}$ with $3/2^-$ (E_γ = 750 MeV)

In this case the situation is not quite as clear. From the angular distribution which is given again by Eq. (34) we already conclude that the spin must be $3/2$. If, in addition, we again rely on the result of the detailed analysis of π N scattering[28]) and accept the negative parity we are led to an

$$\text{electric dipole } E_{2-} \text{ resonant amplitude.} \qquad (36)$$

This assignment fits very well with the polarization measurements of the recoiling proton. These have been done for several energies below 900 MeV for the c.m.s. angle Θ = 90°. In the region of the first resonance the polarization is quite small[29]),

$$\text{e.g.} \quad P\left(\frac{\pi}{2}\right) = (14 \pm 6)\% \quad \text{for } E_\gamma = 320 \text{ MeV ,} \qquad (37)$$

but approaching the second resonance it increases up to 60-80% and stays so in the measured energy range (≤ 850 MeV) [cp. Fig. 8[30]]. The small polarization can be understood from the formula of Table IX: it is due to the interference of the small non-resonant amplitude E_{0+} with M_{1+}. The large polarization between the first and the second resonance will be just expected by the assignment (36); it is due to an interference between two resonant amplitudes E_{2-} and M_{1+}

$$P\left(\frac{\pi}{2}\right) \frac{d\sigma}{d\Omega} = - 4 \frac{q}{k} \, \mathrm{Im}(E^*_{2-} \, M_{1+}) \quad . \qquad (38)$$

On the other hand, it has been argued that Eq. (36) is in contradiction with polarization measurements in the process

$$\gamma + n \rightarrow p + \pi^- \quad , \qquad (39)$$

where one observes again a negative polarization[31] *) ;

$$P\left(\frac{\pi}{2}\right) = - 0.26 \pm 0.06 \quad \text{for } E_\gamma = 715 \text{ MeV.} \qquad (40)$$

These authors[31] expect a change of sign in the isospin I = ½ amplitude E_{2-} if one goes over from $\gamma + p \rightarrow \pi^0 + p$ to Eq. (39) and arrivesat a contradiction to the experimental result. This is indeed the case if the second resonance has an isovector character (cp. Table X). In addition even in π N scattering the situation around the second resonance is controversal[33,34].

<u>Table X</u>

Isospin decomposition for $\pi^{+,0}$ and η production on nucleons

$\gamma + p \rightarrow \pi^0 + p$	$T^{(0)} + \frac{1}{3} T^{(1/2)} + \frac{2}{3} T^{(3/2)}$	$T^{(0)} + T^{(+)}$
$\gamma + p \rightarrow \pi^+ + n$	$\sqrt{2}\,(T^{(0)} + \frac{1}{3} T^{(1/2)} - \frac{1}{3} T^{(3/2)})$	$\sqrt{2}\,(T^{(0)} + T^{(-)})$
$\gamma + n \rightarrow \pi^0 + n$	$- T^{(0)} + \frac{1}{3} T^{(1/2)} + \frac{2}{3} T^{(3/2)}$	$- T^{(0)} + T^{(+)}$
$\gamma + n \rightarrow \pi^- + p$	$\sqrt{2}\,(T^{(0)} - \frac{1}{3} T^{(1/2)} + \frac{1}{3} T^{(3/2)})$	$\sqrt{2}\,(T^{(0)} - T^{(-)})$
$\gamma + p \rightarrow \eta + p$	$T^{(0)} + T^{(1/2)}$	
$\gamma + n \rightarrow \eta + n$	$T^{(0)} - T^{(1/2)}$	

*) cp. J.J. Sakurai[32].

3. $N(1688) = N^*_{\frac{1}{2}\frac{5}{2}}$ with $\frac{5}{2}^+$ (E_γ = 1050 MeV)

This is the first isobar where presumably both possible multipoles, the electric quadrupole E_{3-} and the magnetic octupole M_{3-}, contribute appreciably. From the angular distribution (Fig. 7) one finds E< 0 which, according to Table VIII, indicates J = $\frac{5}{2}$. On the other hand, the cross-section in forward direction (see Fig. 9) has been found to be quite small[35]. This can be understood[36,37] if the ratio of E_{3-} and M_{3-} is:

$$R = \frac{E_{3-}}{M_{3-}} = 2 \; . \tag{41}$$

In this case the differential cross-section contains a factor $\sin^2 \Theta$.

The argument for the positive parity is rather weak[38]. Again the large observed polarization is in favour of this assignment: it makes possible a large contribution to $P(\pi/2)$ because of an interference between the third and second resonance.

4. Higher isobars

The CEA results[38] on the pion photoproduction between 1 and 4 GeV show some structures (Fig. 10) but it seems premature to conclude anything about the existence and properties of isobars.

Concluding this section we remark that in the photoproduction of strange particles $\gamma + p \to K^+ + \Lambda$, $K^0 + \Sigma^+$ only the third and higher isobars can be seen directly. The experimental results[40] are too meagre to identify any resonance though the N(1688) lies in the accessible region. But theoretical studies indicate that s-channel resonances below the strange particle threshold and exchanges of resonances in the t- and u-channel play an important role[41].

VI. DETAILED THEORETICAL DISCUSSION OF THE s-CHANNEL RESONANCES

The empirical evidence summerized in the last section shows maxima in the multipole amplitudes of energies where also the π N scattering has a resonance. This is true at least for the first three isobars. Qualitatively this can be understood with the "compound nucleus" picture taken over from nuclear physics[41a]. If the isobar can be understood as such a compound system according to Bohr's independence assumption[42] its properties should be the same whether it is produced in π N or in γ N collisions.

For a more detailed theoretical development of these ideas the Watson Theorem has played an important role[43]. For energies below the two-meson threshold it gives an exact relation between the multipole amplitudes of a given order $E_{l\pm}$, $M_{l\pm}$ and the corresponding meson-nucleon scattering amplitudes:

$$f^I_{\ell\pm}(s) = \frac{1}{q(s)}\, e^{i\delta^I_{\ell\pm}} \sin \delta_{\ell\pm} \text{ refers to } J = \ell \pm \tfrac{1}{2} \; , \tag{42}$$

(I denotes the isospin).

This connection follows from time reversal invariance and the unitarity of the S matrix under the assumption that the only energetically possible states are the π N and γ N systems and if higher orders in the electric charge e are neglected.

For a given value of spin J, parity π resp. ℓ and isospin I, the S matrix can be written in the form:

$$\begin{array}{c|ccc|} & \pi N & \gamma N & \\ \hline \pi N & S^I_{\ell\pm}, & i\,E^I_{\ell\pm}, & i\,M^I_{\ell\pm} \\ \gamma N \left\{ \begin{array}{c} \\ \\ \end{array} \right. & i\,E^I_{\ell\pm}, & 1+\ldots & .. \\ & i\,M^I_{\ell\pm}, & .. & 1+.. \end{array} \quad \text{for } s \leq (M_2 + 2m)^2 . \qquad (43)$$

Use has already been made of time reversal invariance from which the symmetry of the S matrix follows[44]. $S_{\ell\pm}$ denotes the S-matrix element for π N scattering with given ℓ and $J = \ell \pm 1/2$. (By the helicity consideration of Section V one can prove that only one independent scattering amplitude exists for each J, ℓ !) The dots indicate Compton scattering amplitudes which are of second order in e. The factors "i" have been introduced in accordance with the definition S = 1 + iT.

The unitarity condition gives for the first line of Eq. (43):

$$|S^I_{\ell\pm}|^2 = 1 \rightarrow S_{\ell\pm} = e^{2\,i\delta^I_{\ell\pm}} \quad \text{with real } \delta^I_{\ell\pm}$$

neglecting second order terms in e. The orthogonality between the first and the second resp. the first and the third line gives (retaining only the linear terms in e):

$$S^{I*}_{\ell\pm}\,E^I_{\ell\pm} = E^{I*}_{\ell\pm}\,, \quad S^{I*}_{\ell\pm}\,M^I_{\ell\pm} = M^{I*}_{\ell\pm} \quad .$$

Introducing Eq. (42) one gets:

$$\text{Im}\,E^I_{\ell\pm} = q \cdot f^{I*}_{\ell\pm}\,E^I_{\ell\pm}\ ; \quad \text{Im}\,M^I_{\ell\pm} = q \cdot f^{I*}_{\ell\pm}\,M^I_{\ell\pm}\ . \qquad (45)$$

By observing that the l.h.s. of these relations must be real one arrives at the equivalent result:

$$E^I_{\ell\pm} = \pm\,|E_{\ell\pm}|\,e^{i\delta^I_{\ell\pm}}\ ; \quad M^I_{\ell\pm} = \pm\,|M_{\ell\pm}|\,e^{i\delta^I_{\ell\pm}}\ . \qquad (46)$$

The phases of the photoproduction amplitudes are equal to the corresponding scattering phases up to a multiple of π.

Returning now to the <u>description of resonances</u> we find that all dynamical theories maintain: for resonances, not only the phase of the multipole amplitude is given by the scattering amplitude but the multipole is proportional to $f_{\ell\pm}$:

$$E^I_{\ell\pm}(s) \ \text{or}\ M^I_{\ell\pm}(s) = C\,f^I_{\ell\pm}(s) \quad \text{for resonances}\ , \qquad (47)$$

where the factor C depends only weakly on the energy.

The most elaborate theory exists, of course, for the $\frac{3}{2}^+$ isobar. Here the proportionality

$$M^{3/2}_{1+}(s) = \frac{k}{q} \frac{\mu_v}{f/m\pi} f^{3/2}_{1+}(s) \tag{48}$$

has been first proven in the static (Chew-Low-theory[45]).

Here we have introduced the isovector magnetic moment of the nucleons

$$\mu_v = \tfrac{1}{2}(\mu_p - \mu_n) = 2{,}35 \frac{e}{2M} \tag{49a}$$

and the (renormalized) πN coupling constant

$$\frac{f^2}{4\pi} = 0.08 \quad . \tag{49b}$$

All other (1+) multipoles vanish, especially

$$E^{3/2}_{1+}(s) \approx 0 \quad . \tag{50}$$

The physical reason for these results (46) and (47) can be found in large magnetic coupling to the isovector magnetic moment which already gives the largest contribution to π^0 production in the Born approximation (cp. Fig. 3). After the advent of relativistic dispersion theory[46] it was found that Eq. (46) is also consistent with the relevant dispersion relations if all terms of order ω/M are neglected.

General mathematical conditions which can lead to Eq. (45) have been discussed by P. Stichel[47]. He introduces the frequently used "irreducible" amplitudes[48]

$$f_{irr}(s) = \frac{f(s)}{1 + i\,q\,f(s)} \quad \text{and} \quad M_{irr}(s) = \frac{M(s)}{1 + i\,q\,f(s)} \tag{51}$$

where we have dropped for a moment all indices. As long as Eq. (44) and Eq. (46) are valid one has*)

$$\mathrm{Im}\, f_{irr}(s) = \mathrm{Im}\, M_{irr}(s) = 0 \quad ,$$

i.e. these functions are real up to threshold for two-particle production. But they can have pole singularities if the denominator in Eq. (51) vanishes. Indeed, if we assume

$$f_{irr}(s) = \frac{\gamma(s)}{s - s_0} \quad [\gamma(s): \text{ slowly varying near } s_0] \ , \tag{52a}$$

we find a Breit-Wigner behaviour for the scattering amplitude:

$$f(s) = \frac{\gamma(s)}{s - s_0 - i\,q\,\gamma(s)} \quad . \tag{52b}$$

*) To prove the first result one conveniently uses $\mathrm{Im}\, f(s) = q|f(s)|^2$ which follows from the reality of $\delta(s)$ in Eq. (42).

Now the trivial relation

$$\frac{M(s)}{f(s)} = \frac{M_{irr}(s)}{f_{irr}(s)} \tag{53}$$

leads to

$$M(s) = \frac{(s-s_0)\, M_{irr}(s)}{\gamma(s)}\, f(s)\ . \tag{52c}$$

This formula provides us with a necessary and sufficient condition for the validity of Eq. (47): $M_{irr}(s)$ must have a pole at the same position s_0:

$$M_{irr}(s) = \frac{\Gamma(s)}{s-s_0}\ ,\quad \Gamma(s):\ \text{slowly varying around } s_0\ . \tag{54}$$

$M^{3/2}_{1+}$ evidently exhibits such a behaviour while $E^{3/2}_{1+}$ does not.

This last statement has to be modified somewhat for the semi-phenomenological "isobar model" proposed by Gourdin and Salin[49]. The authors treat the unstable resonant states N with the same Feynman rules as stable particles only replacing in the propagators

$$\frac{1}{M^2 - s} \quad \text{by} \quad \frac{1}{M^{*2} - s - i\Gamma M} \quad \text{with} \quad \begin{matrix} M^* = \text{Mass} \\ \\ \Gamma = \text{width} \end{matrix} \quad \text{of the resonance.}$$

In addition, one has to introduce a variety of coupling constants: for the γNN^* and $NN^*\pi$ vertices. Counting helicity states*) one finds for the γ-coupling two parameters and for the pion-coupling one parameter independent of the spin of N^* (assuming, of course, spin ½ for N[50]). For the spin 3/2 resonances the Rarita-Schwinger formalism[51] has been used. Thereby introducing the following interaction Hamiltonians

$$H_{\pi NN^*} = \frac{\lambda_1}{m_\pi}\, \bar\Psi \Psi_\mu\, \partial^\mu \Phi + \text{h.c.} \tag{55}$$

$$H_{\gamma NN^*} = e\, C_1\, \bar\Psi \gamma_5 \Psi_\mu\, A^\mu + e\, \frac{C_2}{m_\pi}\, \bar\Psi \gamma_\mu \gamma_5 \Psi_\nu\, \partial^\nu A^\mu + \text{h.c.}\ , \tag{56}$$

where

Ψ = nucleon spin or operator,

Φ = π-meson operator,

A^μ = electromagnetic potential.

The $3/2^-$ particle has been described by four Dirac spinors Ψ_μ ($\mu = 0, \ldots, 3$) obeying the auxiliary conditions

$$\gamma^\mu \Psi_\mu = \partial^\mu \Psi_\mu = 0\ , \tag{56a}$$

thus reducing the arbitrary components to four. The authors[51] obtain:

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*) This can be done as in Section V using the Breit system.

$$\frac{f^{3/2}_{1+}}{q^2} = \frac{1}{12\pi}\left(\frac{\lambda_1}{m_\pi}\right)^2 (E_1 + M_N)\,\frac{1}{M^{*2} - s - i\Gamma_\pi M} \tag{57}$$

$$\frac{M^{3/2}_{1+}}{qk} = \frac{\mu_{1+}}{M^{*2} - s - i\Gamma_\gamma M}\ ; \quad \frac{E^{3/2}_{1+}}{qk} = \frac{\epsilon^*_{1+}}{M^{*2} - s - i\Gamma_\gamma M} \tag{58}$$

with

$$\mu_1 = \frac{-1}{24\pi}\frac{e\lambda_1}{m_\pi}\sqrt{\frac{E_2 + M}{E_1 + M}}\left\{C_1 + \frac{E_1 + M}{m_\pi} C_2\right\} \tag{58a}$$

$$\epsilon^*_{1+} = \frac{-1}{24\pi}\frac{e\lambda_1}{m_\pi}\sqrt{\frac{E_2 + M}{E_1 + M}}\left\{-C_1 + \frac{E_1 + M}{m_\pi} C_2\right\}. \tag{58b}$$

From a theoretical point of view the great number of parameters is a bad feature of the model, especially if one varies also the width Γ_γ as the authors do. On the other hand, the simple formulae of the model are convenient for practical calculation and can be easily generalized for several isobars.

Taking the first three isobars into account, in addition to the Born approximation, Salin found an over-all fit to the existing data (Fig. 11 a and b). The best fit parameters lead to

$$\frac{E^{3/2}_{1+}}{M^{3/2}_{1+}} = -\,0.045$$

in a certain contradiction to Eq. (50).

As has been stressed by Höhler[52] the model contains an over-simplication which we like to mention because of its general significance. In the formulae used, the isobar makes contributions only to the resonant amplitudes M_{1+} resp. E_{1+}. But we must expect an influence on non-resonant multipole amplitudes for two reasons:

a) The isobar in the s-channel is in general a virtual off-shell particle and such particles with spin $J > 1/2$ are known to contain also lower spin values. This is due to the failure of the subsidiary condition (56a) for virtual particles[53]. Relativity also allows changes in parity which is brought about by the small components of the Dirac spinors;

b) The isobars can also occur in the u-channel.

Both effects are not small as shown by Höhler and co-workers[52] using the dispersion relation approach. For an illustration we show in Fig. 12 the influence of the first isobar on the E_{0+} electric dipole matrix element: $(E_{0+})_{33}$. In fact, also Gourdin and Salin need such an E_{0+} matrix element which they introduce by a so-called subtraction term whereby another parameter enters*). In their discussion of the π^+ photoproduction these authors have of course also taken account of the influence of one-pion exchange. This process will be discussed in the next section.

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*) For a more complete discussion, cp. the seminar given by Professor Höhler.

VII. THE ONE-PION EXCHANGE, GAUGE INVARIANCE AND THE COMPLETE BORN APPROXIMATION

Turning now to the t-channel we have at first to deal with the one-pion intermediate state. The connected one-pion exchange processes have been studied extensively for inelastic hadronic processes[54)]. For photoreactions a new problem arises in this connection which is due to the gauge invariance of the photoproduction amplitude.

Because of the continuity equation for the electric current $\partial^\mu j_\mu = 0$ the matrix element of Eq. (1) must obey the condition

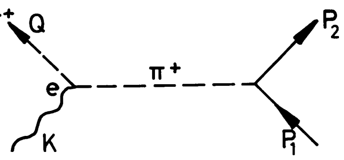
$$K^\mu < \ldots | j_\mu | \ldots > = 0 \quad . \tag{59}$$

For a real photon we can replace A_μ in Eq. (1) by the four-vector ϵ_μ of the photon polarization so that each photoproduction amplitude can be written in the form

$$T = \epsilon^\mu < \ldots | j_\mu | \ldots > \quad . \tag{60}$$

Incidentally this is true exactly to each order in α. Because of Eq. (59) we have the rule: by replacing ϵ^μ in the photoproduction amplitude by the photon four-momentum K^μ one must get zero.

Let us now go back to the one-pion exchange diagram. The application of Feynman rules gives the following result (we have omitted the nucleon spinors for simplicity)



$$J_\alpha^{(-)} \; e \; 2\epsilon^\mu Q_\mu \frac{1}{m_\pi^2 - t} \, g \cdot \gamma_5 \quad , \tag{61}$$

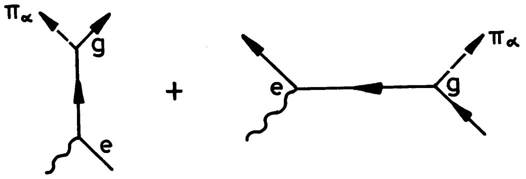
where $g^2/4\pi = 15$ is the usual pseudoscalar coupling constant and the isospin factor

$$J_\alpha^{(-)} = (-i) \, \epsilon_{\alpha\beta 3} \, \tau_\beta = \tfrac{1}{2}[\tau_\alpha \tau_3] \tag{62a}$$

describes the fact that only charged pions can be exchanged. Replacing ϵ^μ in Eq. (61) by K^μ one gets

$$J_\alpha^{(-)} \; e \; g\gamma_5 \quad . \tag{61'}$$

Thus gauge invariance is violated in the one-pion exchange approximation. This result seems plausible if one recalls that in this process a proton emits a π^+ changing into a neutron. We have to take account of the current of the proton also to get a result which obeys charge conservation. Therefore we write down the relevant contributions of the nucleon pole diagram:



The electric γ N coupling is usually described by $e\gamma_\mu$ which contains an orbital current contribution $e\,P_\mu/M$ and a current due to the normal magnetic moment[55]. For our purposes we need only the first so that we are led to

$$g\tau_\alpha\gamma_5\frac{1}{M^2-s}\,e\,\frac{1+\tau_3}{2}\,2\epsilon^\mu P_{1\mu}+e\,\frac{1+\tau_3}{2}\,2\epsilon^\mu P_{2\mu}\frac{1}{M^2-u}\,g\tau_\alpha\gamma_5 =$$

$$= e\,g\gamma_5\left\{\left(J_\alpha^{(o)}+J_\alpha^{(+)}\right)\left[\frac{-\epsilon^\mu P_{1\mu}}{M^2-s}+\frac{\epsilon^\mu P_{2\mu}}{M^2-u}\right]+J_\alpha^{(-)}\left[\frac{-\epsilon^\mu P_{1\mu}}{M^2-s}-\frac{\epsilon^\mu P_{2\mu}}{M^2-u}\right]\right\}\ , \tag{63}$$

where in addition to Eq. (62a) the isospin quantities

$$J_\alpha^{(o)}\equiv\tau_\alpha,\ J_\alpha^{(+)}=\tfrac{1}{2}(\tau_\alpha\tau_3+\tau_3\tau_\alpha)=\delta_{\alpha 3} \tag{62b}$$

have been introduced which are different from zero also for the neutral pions. In accordance with this fact the factor of $J_\alpha^{(o,+)}$ in Eq. (63) vanishes if ϵ^μ is replaced by K^μ, while the factor of $J_\alpha^{(-)}$ just leads to a result which compensates Eq. (61′). [Looking for the origin of the isospin factors in Eq. (63) we observe that $J_\alpha^{(o)}$ is due to first term in the electric charge e $1+\tau_3/2$ of the proton and thus describes the isoscalar part while $J_\alpha^{(\pm)}$ due to τ_3 has an isovector character.] These gauge properties can be expressed most conveniently with the help of the "invariant"

$$M_2\equiv 2\,i\,\gamma_5\,F^{\mu\nu}P_\mu Q_\nu = 2\,i\,\gamma_5(P\cdot\epsilon\,Q\cdot K\cdot P\cdot K\,Q\cdot\epsilon)\ , \tag{64}$$

where

$$P=\tfrac{1}{2}(P_1+P_2)\quad\text{and}\quad F^{\mu\nu}=\epsilon^\mu K^\nu-\epsilon^\nu K^\mu$$

denote the antisymmetric tensor of the electric and magnetic field strength for a plane wave photon. Therefore M_2 is an evident gauge invariant quantity. By an elementary calculation (61) and (63) can be found to be proportional to M_2:

$$eg\left\{\left(J_\alpha^{(o)}+J_\alpha^{(+)}\right)\frac{1}{(M^2-s)(M^2-u)}+J_\alpha^{(-)}\frac{u-s}{(m_\pi^2-t)(M^2-s)(M^2-u)}\right\}M_2\ . \tag{63′}$$

This result is contained in Table XII where the complete result of an evaluation of the pion and nucleon pole diagrams can be found. In Fig. 3 we already have given the cross-sections following from this result. I would like now to interrupt the formal theory and discuss a very practical application of these considerations. Several years ago S. Drell proposed the diagram of Fig. 13 as a source of a high intense pion beam produced parallel to the incoming photon[56]. Evidently this one-pion exchange diagram again violates gauge invariance.

Table XII

Pole-terms = Born approximation for pion photoproduction

$\left(F^{\mu\nu} = \epsilon^{\mu} k^{\nu} - \epsilon^{\nu} k^{\mu} ;\quad P = \tfrac{1}{2}(P_1 + P_2)\right);\quad \kappa_s = -0.065;\quad \kappa_v = 1.845$

	normal magnetic moment contribution	electric orbital current	anomalous magnetic moment contributions	
	$M_1 = i\gamma_5 \not{\epsilon}\not{k} = \tfrac{1}{2}\gamma_5 F^{\mu\nu}\gamma_\mu\gamma_\nu$	$M_2 = 2i\gamma_5(P\cdot\epsilon\ q\cdot k - P\cdot k\ q\cdot\epsilon) = 2i\gamma_5 F^{\mu\nu} P_\mu q_\nu$	$M_3 = \gamma_5(\not{\epsilon} P\cdot k - \not{k} P\cdot\epsilon) = \gamma_5 F^{\mu\nu}\gamma_\mu q_\nu$	$M_4 = 2\gamma_5(\not{\epsilon} P\cdot k - \not{k} P\cdot\epsilon - iM\not{\epsilon}\not{k}) = 2\gamma_5 F^{\mu\nu}\gamma_\mu P_\nu - 2MM_1$
$J_\alpha^{(0)} = \tau_\alpha$	$\frac{-eg}{2}\left[\frac{1}{M^2-s} + \frac{1}{M^2-u}\right]$	$eg\frac{1}{(M^2-s)(M^2-u)} = eg\frac{1}{t-m_\pi^2}\left[\frac{1}{M^2-s} + \frac{1}{M^2-u}\right]$	$\frac{\kappa_s eg}{2M}\left[\frac{1}{M^2-s} - \frac{1}{M^2-u}\right]$	$\frac{\kappa_s eg}{2M}\left[\frac{1}{M^2-s} + \frac{1}{M^2-u}\right]$
$J_\alpha^{(+)} = \delta_{\alpha 3}$	$\frac{-eg}{2}\left[\frac{1}{M^2-s} - \frac{1}{M^2-u}\right]$	$eg\frac{1}{t-m_\pi^2}\left[\frac{1}{M^2-s} + \frac{1}{M^2-u}\right]$	$\frac{\kappa_v eg}{2M}\left[\frac{1}{M^2-s} - \frac{1}{M^2-u}\right]$	$\frac{\kappa_v eg}{2M}\left[\frac{1}{M^2-s} + \frac{1}{M^2-u}\right]$
$J_\alpha^{(-)} = \tfrac{1}{2}[\tau_\alpha \tau_3]$	$\frac{-eg}{2}\left[\frac{1}{M^2-s} - \frac{1}{M^2-u}\right]$	$eg\frac{u-s}{(m_\pi^2-t)(M^2-s)(M^2-u)} = eg\frac{1}{t-m_\pi^2}\left[\frac{1}{M^2-s} - \frac{1}{M^2-u}\right]$	$\frac{\kappa_v eg}{2M}\left[\frac{1}{M^2-s} + \frac{1}{M^2-u}\right]$	$\frac{\kappa_v eg}{2M}\left[\frac{1}{M^2-s} - \frac{1}{M^2-u}\right]$

Stichel and Scholz[57)] have investigated a model where this lack can be remedied - quite analogously to the way described above. They restricted themselves to a production of a $\Delta(1235)$ isobar on the nucleon bubble (see Fig. 14) which they again describe by the Rarita-Schwinger formalism [cp. Eqs. (55) and (56)]. Due to the derivative coupling in Eq. (55) they have to calculate four diagrams (see Fig. 15) in order to arrive at gauge invariance. The coupling of the new "catastrophic" diagram (III) follows from Eq. (55) by replacing ∂^μ by ieA^μ. The evaluation for high energies ($s \to \infty$) gives a remarkably large change relative to the simple Drell diagram (Fig. 16).

VIII. DETAILED FORMULATION OF THE ANALYTICITY PROPERTIES

The decomposition for the nucleon and pion pole terms of Table XII which just give the renormalized Born approximation is very convenient also in the general case. It can therefore be proven that a photoproduction amplitude can in general be written as

$$T = \sum_{i=1}^{4} \left[A_i^{(o)}(s,t)\, J_\alpha^{(o)} + A_i^{(+)}(s,t)\, J_\alpha^{(+)} + A_i^{(-)}\, J_\alpha^{(-)} \right] M_i \, . \tag{65}$$

Accordingly we have for a specified charge mode four independent amplitudes in agreement with the existence of four helicity amplitudes. The connection of $H^\pm$, $\Phi^\pm$ with the A_i - s is somewhat involved. One normally uses in an intermediate step the functions F_i and $\mathcal{F}_i$ defined in Table XIII. With their help the helicity amplitudes can be calculated according to Table XIV.

People trained in analyticity properties will suspect that the functions $A_i(s,t)$ obey a Mandelstam representation in the variables s, t and u. Unfortunately, gauge invariance again lead to complications. Of course, we can only guess the validity of this representation. But perturbation theory gives a good tool to guess the correct answer. Looking at Table XII one observes that $A_1^{(\pm,o)}$, $A_3^{(\pm,o)}$, $A_4^{(\pm,o)}$ have the expected simple pole behaviour, but $A_2^{(\pm,o)}$ looks different. Indeed it was in the A_2 amplitudes that gauge troubles occur. But a more detailed investigation by Ball[7)] has shown that the usual Mandelstam relation

$$A_i(s,t,u) = \text{pole terms} + \frac{1}{\pi}\int_{(M+m)^2}^{\infty} ds' \,\frac{\rho_s(s')}{s'-s} + \int_{(M+m)^2}^{\infty} \frac{1}{\pi}\, du' \,\frac{\rho_u(u')}{u'-u} + \frac{1}{\pi}\int_{4m^2}^{\infty} dt' \,\frac{\rho_t(t')}{t'-t} +$$

$$+ \frac{1}{\pi^2}\int ds'dt' \,\frac{\rho_{12}(s',t')}{(s'-s)(t'-t)} + \frac{1}{\pi^2}\int ds'du' \,\frac{\rho_{13}(s',u')}{(s'-s)(u'-u)} + \frac{1}{\pi^2}\int dt'du' \,\frac{\rho_{23}(u',t')}{(t'-t)(u'-u)} \, , \tag{66}$$

should be conjectured for i = 1, 3, 4 with pole terms given by Table XII. Each term in Eq. (66) also carries the isospin indices $(\pm,0)$. On the other hand $A_2^{(\pm,o)}$ should obey a similar relation with pole terms having only the more complicated form of Table XII and the single dispersion integrals should be dropped. As usual one has to add crossing relations the validity of which can be read off from Table XII for the pole terms:

$$F_1 = A_1 + (W-M)A_4 - \frac{t-1}{2(W-M)}(A_3 - A_4) = \frac{8\pi W}{W-M}\,\frac{\mathcal{F}_1}{(E_2+M)^{1/2}(E_1+M)^{1/2}}$$

$$F_2 = -A_1 + (W+M)A_4 - \frac{t-1}{2(W+M}(A_3 - A_4) = \frac{S\pi W}{W-M}\left(\frac{E_2+M}{E_1+M}\right)^{1/2}\mathcal{F}_2\,\frac{1}{q}$$

$$F_3 = (W-M)A_2 + A_3 - A_4 = \frac{8\pi W}{W-M}\,\frac{\mathcal{F}_3}{(E_2+M)^{1/2}(E_1+M)^{1/2}}\,\frac{1}{q}$$

$$F_4 = -(W+M)A_2 + A_3 - A_4 = \frac{8\pi W}{W-M}\left(\frac{E_2+M}{E_1+M}\right)^{1/2}\frac{4}{q^2}$$

Table XIII

The intermediate functions F_i resp. $\mathcal{F}_i$ in terms of A:
(the isospin indices (0,±) must be added to each symbol)

$$H^- = -\frac{1}{\sqrt{2}}\sin\Theta\cos\frac{\Theta}{2}(\mathcal{F}_3 + \mathcal{F}_4)$$

$$H^+ = -\sqrt{2}\sin\frac{\Theta}{2}(\mathcal{F}_1 + \mathcal{F}_2) + H^-$$

$$\Phi^+ = \frac{1}{\sqrt{2}}\sin\Theta\sin\frac{\Theta}{2}(\mathcal{F}_3 - \mathcal{F}_4)$$

$$\Phi^- = -\sqrt{2}\cos\frac{\Theta}{2}(\mathcal{F}_1 - \mathcal{F}_2) + \Phi^+$$

Table XIV

The connection between the $\mathcal{F}_i$ functions and the helicity amplitudes

These formulae differ somewhat from those given in Ref. 15 because of a slight difference in the polarization vectors $\vec{\epsilon}^{\,\lambda}$. We use $\vec{\epsilon}^{\,\lambda} = -\lambda/\sqrt{2}\,(\vec{\epsilon}_1 + \lambda\vec{\epsilon}_2)$ $(\lambda = \pm 1)$ where $\vec{\epsilon}_2$ lies in the plane defined by $\vec{k}$ and $\vec{q}$.

$$A_i^{(\pm,0)}(s,t,u) = \pm A_i^{(\pm,0)}(u,t,s) \tag{67}$$

where the upper sign holds for: 0,+ and i = 1, 3, 4 resp.: - and i = 2 and the lower sign holds for: 0,+ and i = 2 resp.: - and i = 1, 3, 4. A general proof of Eq. (67) can be given with the help of the general crossing relations and the odd behaviour of j_μ under C conjugation. By well-known methods[8] one deduces from Eq. (66) one-dimensional <u>dispersion relations</u>. For example, one has <u>for fixed t</u>:

$$A_i^{(\pm,o)}(s,t) = \text{pole terms} + \frac{1}{\pi}\int_{(M+m_\pi)^2}^{\infty} ds' \;\; \text{Im}\; A^{(\pm,o)}(s',t)\left[\frac{1}{s'-s} \pm \frac{1}{s'-u}\right] , \qquad (68)$$

where $u = 2\,M^2 + m_\pi^2 - s - t$.

A recent evaluation of these relations has been performed by W. Schmidt[58] for energy region around the first isobar. Here one normally (following Chew et al.[46]) approximates the imaginary part in Eq. (68) by the contribution of the magnetic dipole matrix element $M_{1+}^{3/2}$. This leads with help of Tables VII, XIII and XIV to

$$\text{Im}\; A_i^{\pm} = C^{\pm}(s)\;\text{Im}\; M_{1+}^{3/2}(s)\; f_i(s,t); \quad \text{Im}\; A_i^{0} = 0 \qquad (69)$$

with

$$f_1 = 3\;t - 1 + \omega(W+M); \quad f_2 = -3$$

$$f_3 = \tfrac{3}{2}(t-1)\frac{1}{W+M} + \omega - W - M; \quad f_4 = \tfrac{3}{2}(t-1)\frac{1}{W+M} + \omega + 2\;W + 2\;M \qquad (69a)$$

$$C^{\pm} = \tfrac{4}{3}\left(\frac{2}{t-1}\right)\frac{\pi}{qk}\,\frac{1}{[(W+M)^2-1]^{1/2}} \; . \qquad (69b)$$

For $M_{1+}^{3/2}$ occurring in Eq. (69) the approximation (48) was used and experimental phase shifts were taken to calculate $f_{1+}^{3/2}(s)$. The agreement with recent experimental results[59] on π^+ production is fairly good especially for large angles (see Fig. 17a and b). On the other hand, the Gourdin-Salin model[49] gives a somewhat better fit to the same data[60]. But one must keep in mind that the formulae (68) and (69) are different from the isobar model and do not contain any free parameter.

We should remark finally that improvement of the relation (48) has been discussed with dispersion relations[61] but the gained numerical results have been criticized[52]. In this treatment the possible influence of the exchange of a ρ meson (the ω does not contribute in π^+ production) has been neglected. This is in accord with A-parity arguments of Section IV. We discuss this problem in some more detail in the next section.

IX. THE ELECTROMAGNETIC COUPLING OF VECTOR MESONS*)

At the last High-Energy Conference at Dubna, Professor Baldin called the determination of the coupling strength between the photon, the pion, and the vector mesons (ρ,ω) "a problem of the day"[62]. Yet the results so far reported are rather conflicting. This section reviews the evidence for the magnitude of the coupling constants $g_{\gamma\pi\rho}$ and $g_{\gamma\pi\omega}$. We start with precise definition: the coupling between the photon, the pseudoscalar pion, and a vector particle can be written in a unique and gauge invariant way[63]:

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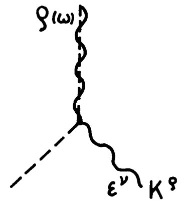
*) In preparing this section the author has made use of the material presented by H. Joos at DESY in December 1964.



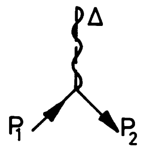
$$\frac{g_{\gamma\pi\rho}(\omega)}{m_\pi}\,\epsilon_{\mu\nu\rho\sigma}\,\epsilon^\nu\,K^\rho\,\Delta^\sigma \qquad (70)$$

where K^ρ, Δ^σ are the four momenta of the photon and the vector meson, respectively. The vector particle carries the index "μ". In terms of this constant the decay width of $\rho(\omega)$ into $\pi+\gamma$ is found to be

$$\Gamma\rho(\omega) \rightarrow \pi+\gamma = \frac{1}{24}\,\frac{g^2_{\gamma\pi\rho}(\omega)}{4\pi}\,m_\rho\left(1-(m_\pi/m_\rho)^2\right)^3 . \qquad (71)$$

Because of differences in conventional factors the best way to report values of the wanted quantity is by this width.

To calculate the influence of the exchange of a vector particle on the single pion production we need in addition the nucleon-$\rho(\omega)$ coupling which contains a Dirac and a Pauli-like term:



$$\left(g_{\rho,1}\,\gamma_\mu + g_{\rho,2}\,\frac{1}{2M}\,\sigma_{\mu\nu}\Delta^\nu\right) . \qquad (72)$$

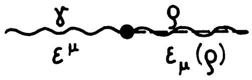
Using the well-known propagator of a vector particle

$$\frac{1}{m_\rho^2-t}\left(g_{\mu\nu}-\frac{\Delta_\mu\Delta_\nu}{m_\rho^2}\right) ,$$

one finds for the ρ-exchange diagram

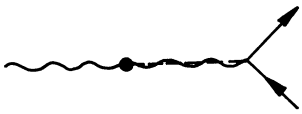
$$\frac{g_{\gamma\pi\rho}}{m_\pi}\,\frac{1}{m_\rho^2-t}\left[g_{\rho,1}\,M_4-\frac{g_{\rho,2}}{2M}\,(tM_1+M_2)\right]J_\alpha^{(o)} . \qquad (73)$$

Here we have used the invariants of Table XII. The occurrence of the isospin factor $J_\alpha^{(o)}$ expresses the fact that the ρ exchange only contributes to the isoscalar current (cp. Section III). For an ω exchange one has merely to change the index ρ into ω and to replace $J_\alpha^{(o)}$ by $J_\alpha^{(+)} = \delta_{\alpha 3}$. (Because of $I_\omega = 0$ no dependence on τ matrices arises.) In this treatment we have regarded the vector particles as stable particles. This can be done better with the aid of the Mandelstam representation[7]. Professor Höhler will discuss this subject in his seminar. The major change which comes about consists of an extra constant term to $A_1^{(o)}$ [the factor of M_1 in Eq. (73)]. Moreover a connection between the bracket in Eq. (73) and the nucleon form factor is established[64]. The physical basis of this connection can also be expressed in a simple model. By introducing a direct γ-ρ coupling



$$\frac{e}{2\gamma\rho} m_\rho^2 \epsilon^\mu \epsilon_\mu(\rho) \quad , \tag{74}$$

one gets a contribution to the isovector electromagnetic form factors through the diagram



$$\frac{e}{2\gamma\rho} \frac{m_\rho^2}{m_\rho^2 - t} \left(g_{\rho,1} \gamma_\mu + \frac{g_{\rho,2}}{2M} \sigma_{\mu\nu} \Delta^\nu \right) \cdot \tau_3 \tag{75}$$

where we have dropped the nucleon spinors. Originally one was inclined to identify Eq. (75) directly with the isovector form factors thus being led to

$$\frac{g_{\rho,2}}{g_{\rho,1}} = (K_p' - K_n') = 3.7 \ . \tag{76a}$$

If, on the other hand one makes use of a two-pole fit to the form factors and identifies Eq. (75) only with the ρ pole, recent results give[65]

$$\frac{g_{\rho,2}}{g_{\rho,1}} \approx 3 \ . \tag{76b}$$

These values agree with the result from an analysis of nucleon-nucleon scattering[66]

$$g_{\rho,1} = 3.26; \quad g_{\rho,2} = 12.1; \quad \frac{g_{\rho,2}}{g_{\rho,1}} = 3.7 \ . \tag{77}$$

Relying on this ratio, the contribution of ρ exchange to photoproduction contains only one open parameter

$$\Lambda = \frac{g_{\gamma\pi\rho}\, g_{\rho,1}}{8\pi\, m_\pi} \quad ,$$

which could be extracted from the experimental results on π^+ production if all other contributions are known. Unfortunately this supposition is not fulfilled. But the different authors[49,52,67] agree that Λ is small. To give an order of magnitude we quote the result by A.I. Lebedev[67]*)

$$\Gamma_{\rho \to \pi\gamma} < 0.1 \text{ MeV} \quad . \tag{78}$$

*) Knowing also the absolute value of the ρ N coupling [see Eq. (77)] one can calculate the radiative width of the ρ from Λ.

This upper bound contradicts strongly with a recent result from the investigation of the process[68]

$$\gamma + p \to \rho^0 + p \quad \text{at 2 GeV} .$$

Using the measured cross-section (13.2 μbarn) one finds with the one-pion exchange model where in addition to $\Gamma_{\rho \to \pi\gamma}$ only the well-known π N coupling enters

$$\Gamma_{\rho \to \pi\gamma} = 1.65 \text{ MeV} . \tag{79}$$

This value is somewhat larger than

$$\Gamma_{\rho \to \pi\gamma} = 0.5 \text{ MeV} ,$$

deduced from MacLeod et al.[69] from the two-pion production at lower energies (1 GeV).

Turning now to the ω coupling we expect from the A-parity argument given in Section III a larger value of $g_{\gamma\pi\omega}$. In fact for high energies[39] neutral pions are produced much more strongly than charged ones:

$$d\sigma^0 \approx (5 - 10)\, d\sigma^+ \quad \text{for about 2 GeV} .$$

Assuming that vector-meson exchange is responsible for this difference one has indeed

$$g_{\gamma\pi\rho} << g_{\gamma\pi\omega} .$$

It is clear from this short survey that much more information is wanted. For high energies one needs more detailed experimental results. For low energies the theoretical description of the "other" contribution should be improved[70].

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54) For a recent review of the one-pion exchange model, cp. J.D. Jackson, to appear in the Revs. Modern Phys.

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70) There is the possibility that the ρ-exchange in single-pion production is counterbalanced by some other contributions as suggested in Ref. 62).

Figure captions

Fig. 1 : Kinematics for photoproduction.

Fig. 2 : Resonances in the s-t plane.

Fig. 3 : The Born approximation for the pion photoproduction.

Fig. 4 : The meson trajectories contributing to photoproduction.

Fig. 5 : Total cross-section for pion production up to 1 GeV.

Fig. 6 : Kinematics and helicities for photoproduction.

Fig. 7a : Angular coefficients for neutral pion production.
7b

Fig. 8 : Polarization of recoil protons.

Fig. 9 : Differential cross-section for neutral pion production.

Fig. 10 : Excitation functions for neutral pion production up to 4 GeV.

Fig. 11 : Gourdin Salin fit to charged pion production.

Fig. 12 : Various contributions to the E_{0+} (electric dipole) amplitude.

Fig. 13 : The Drell diagram.

Fig. 14 : Drell process with isobar production.

Fig. 15 : The four diagrams for a gauge invariant treatment of the Drell process.

Fig. 16 : Gauge term corrections to the Drell diagram.

Fig. 17 : Excitation functions for charged pion production.

Fig. 18 : Angular distribution for charged pion production.

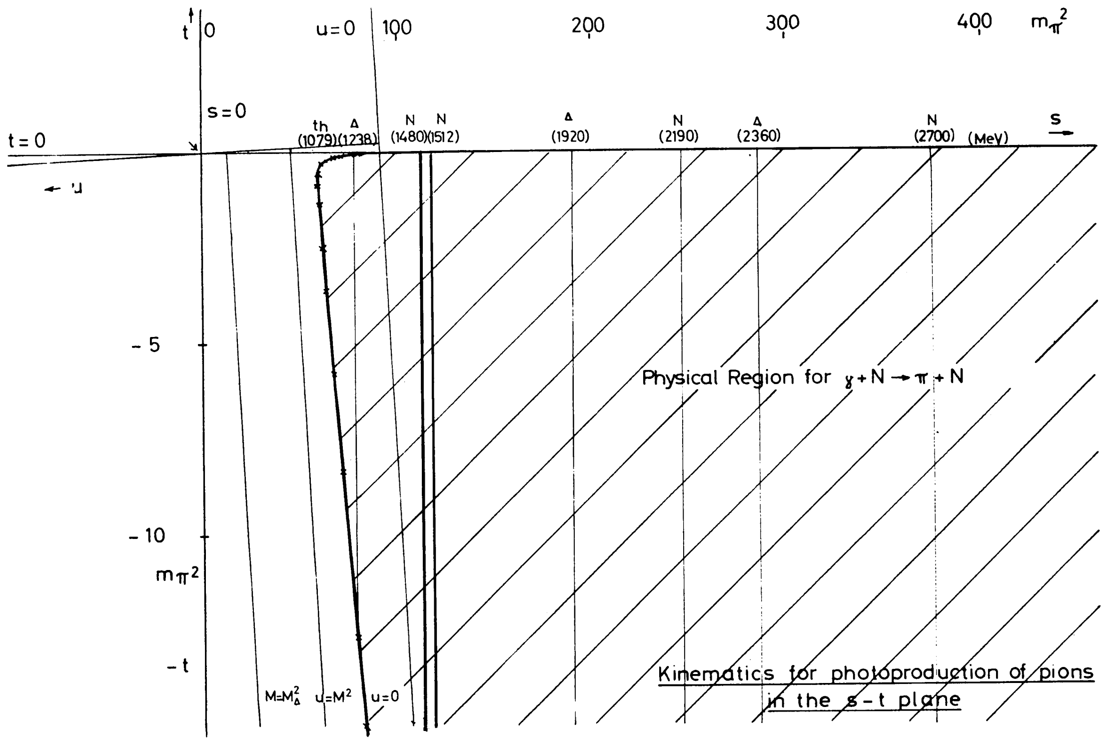

t
0
u=0
100
200
300
400
m_π^2
s=0
t=0
th (1079)
Δ (1238)
N (1480)
N (1512)
Δ (1920)
N (2190)
Δ (2360)
N (2700)
(MeV)
s
u
- 5
- 10
m_π^2
- t
Physical Region for γ + N → π + N
Kinematics for photoproduction of pions in the s - t plane
$M = M_\Delta^2$
$u = M^2$
u=0

Fig. 1

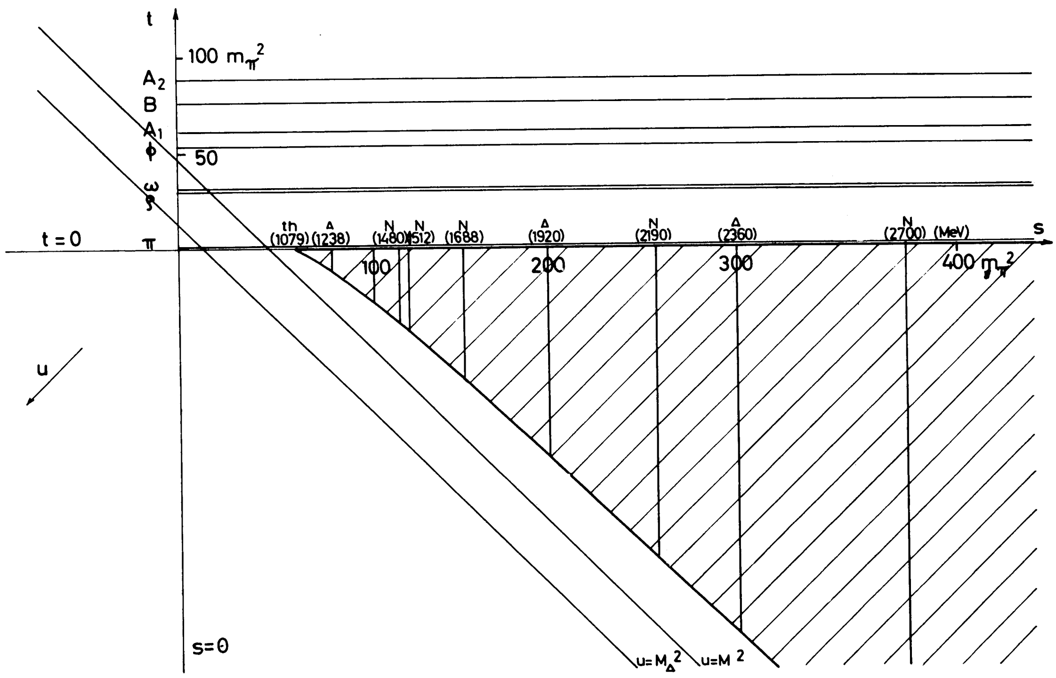
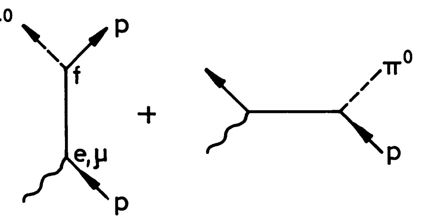

t
100 m_π^2
A_2
B
A_1
φ
50
ω
ρ
t=0
π
th (1079)
Δ (1238)
N (1480)
N (1512)
N (1688)
Δ (1920)
N (2190)
Δ (2360)
N (2700)
(MeV)
s
100
200
300
400 m_π^2
u
s=0
$u = M_\Delta^2$
$u = M^2$

Fig. 2

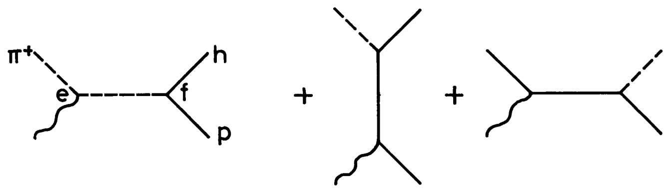
a) $\underline{\gamma + p \to \pi^0 + p}$



$$\frac{d\sigma}{d\Omega} = \left(ef\,\frac{M}{W}\right)^2 \frac{1}{E_2 + q\cos\Theta}\,\frac{q}{kW}\left\{(1+g_p')^2\,(\omega - q\cos\Theta)^2 + \right.$$

$$\left. + \frac{q^2}{2}\sin^2\Theta\left[g_p'^2\left(\frac{W}{M}\right)^2 - \frac{W}{K^2(E_2 + q\cos\Theta)}\right]\right\} .$$

b) $\underline{\gamma + p \to \pi^+ + n}$



$$\frac{d\sigma}{d\Omega} = 2\left(ef\,\frac{M}{W}\right)^2 \frac{q}{k}\left\{1 - \frac{q^2}{2k^2}\,\frac{\sin^2\Theta}{(\omega - q\cos\Theta)^2} - (g_p + g_n)\,\frac{\omega - q\cos\Theta}{W}\right.$$

$$\left. + \frac{1}{4W(E_2 + q\cos\Theta)}\left[(g_p' + g_n)^2\,(\omega - q\cos\Theta)^2 + (g_p' + g_n^2)\,\frac{q^2W^2}{M^2}\sin^2\Theta\right]\right\} .$$

The (renormalized) Born approximation for the pion photoproduction

$g_p' = g_p - 1 = 1.79; \quad g_n = -1.91 \qquad (\hbar = c = m_\pi = 1)$

Fig. 3

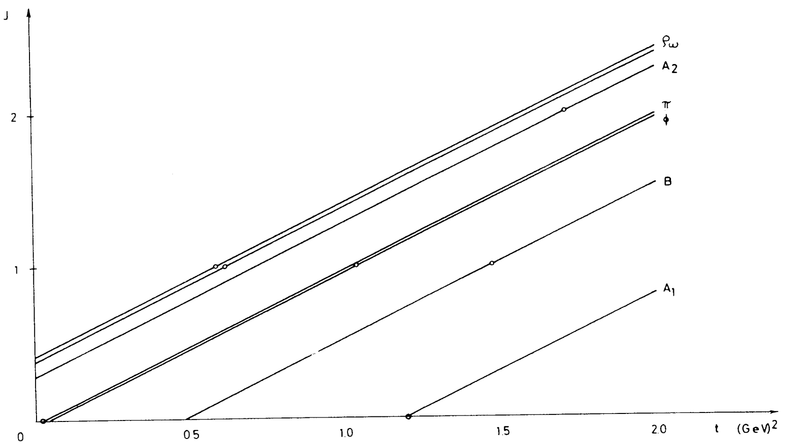


Fig. 4

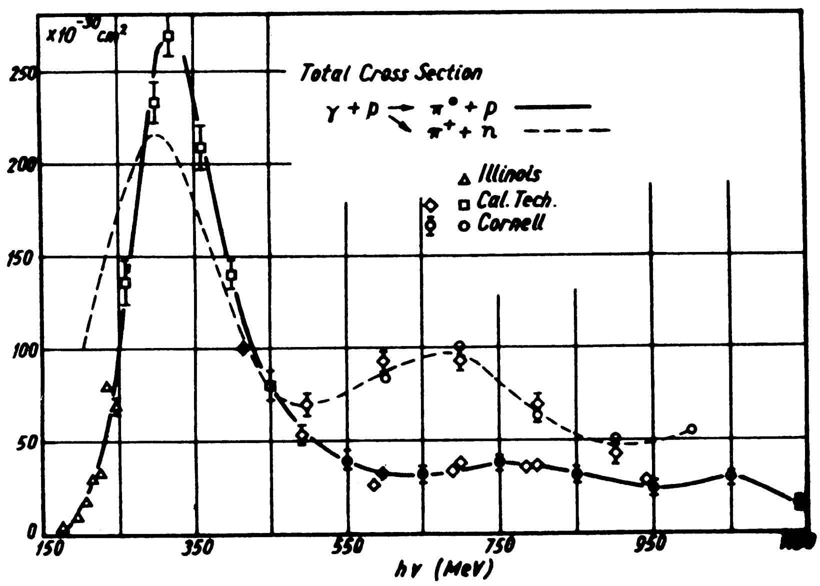


Fig. 5

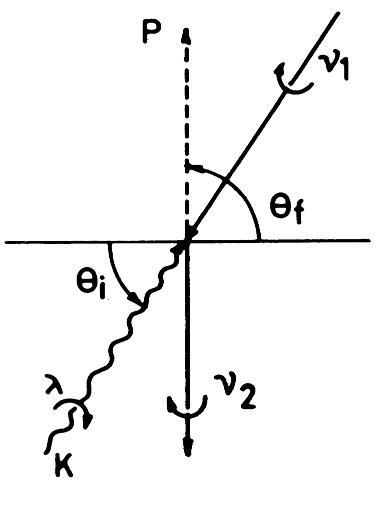


Fig. 6

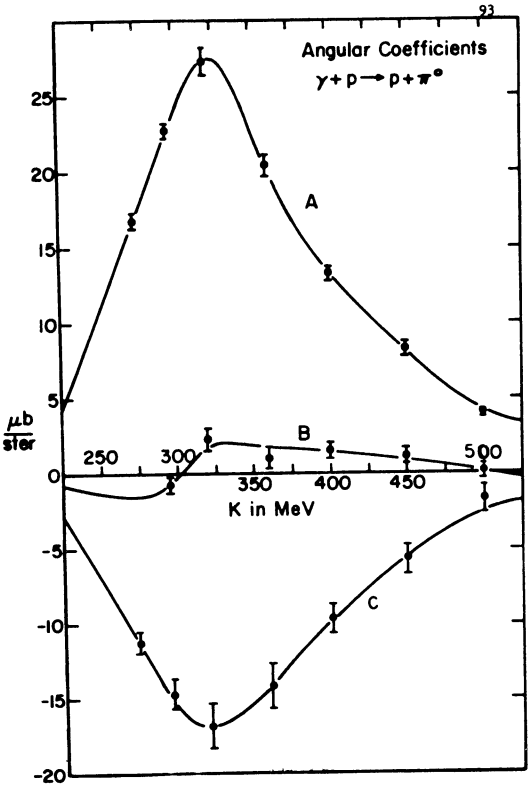

Angular Coefficients
γ+p → p+π°
A
B
C
μb/ster
K in MeV

Fig. 7 a

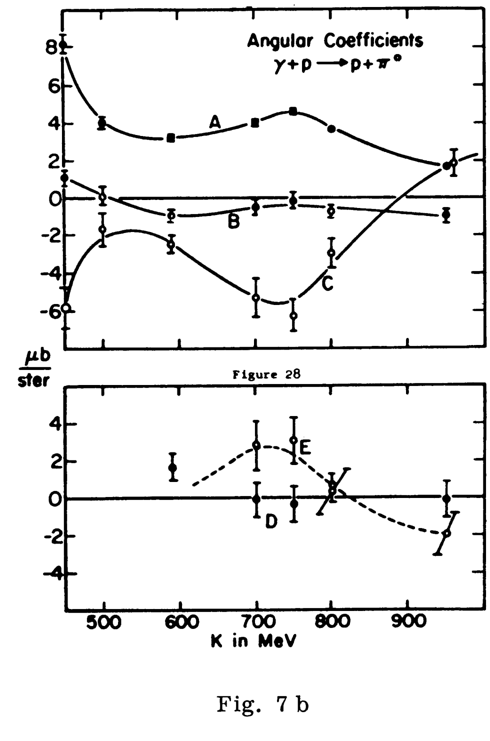
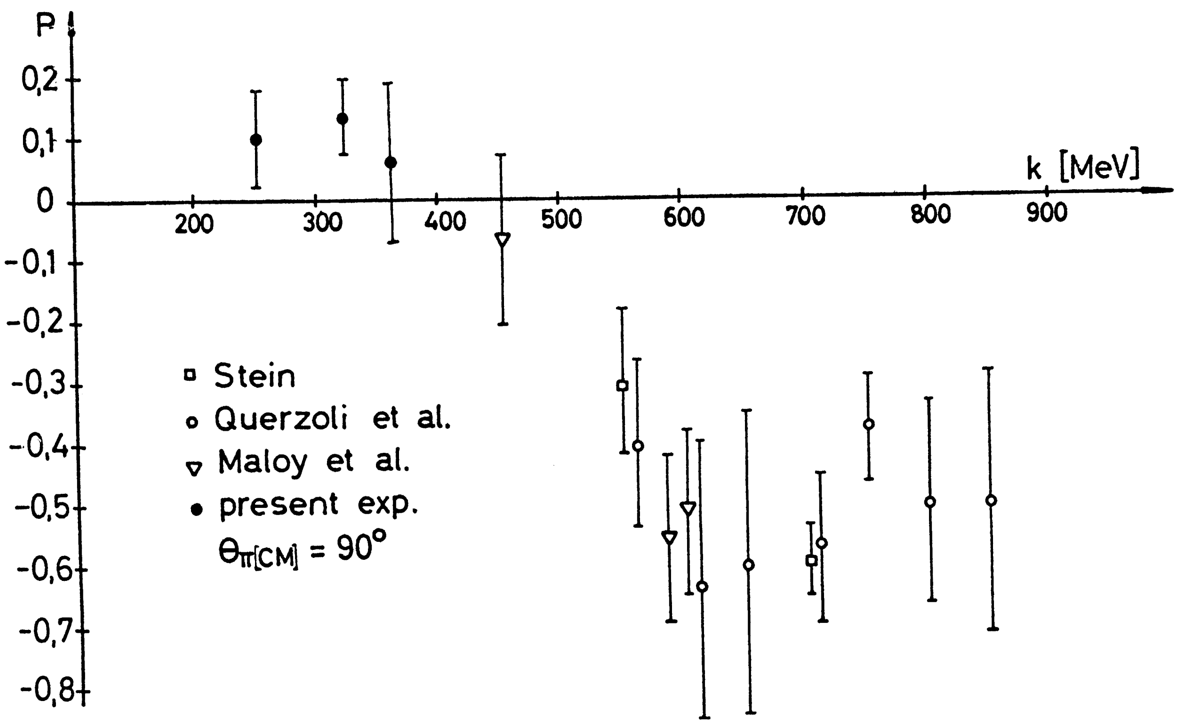
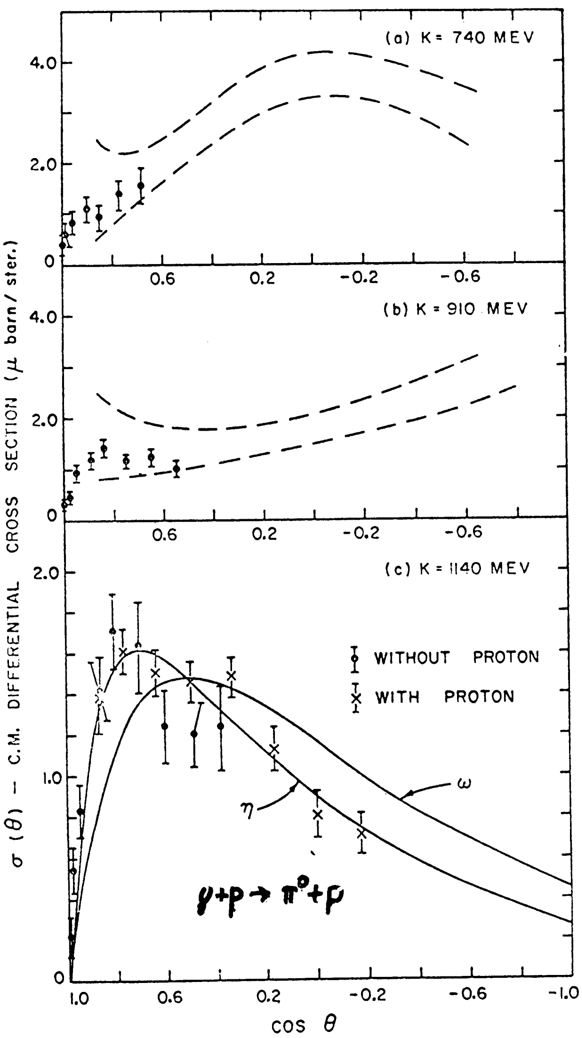

Angular Coefficients
γ+p → p+π°
A
B
C
μb/ster
Figure 28
E
D
K in MeV

Fig. 7 b



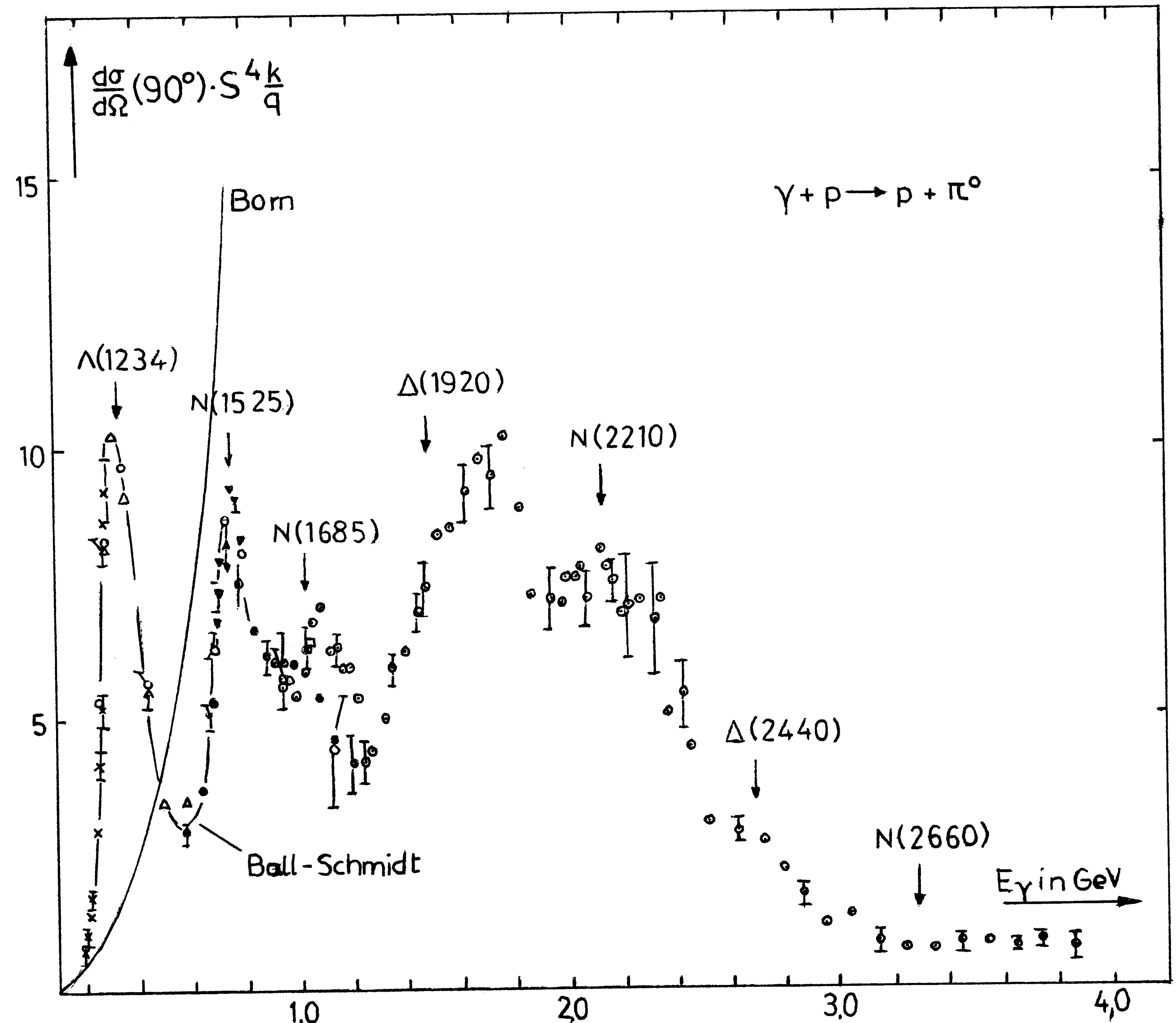
P
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
-0,6
-0,7
-0,8
200
300
400
500
600
700
800
900
k [MeV]
Stein
Querzoli et al.
Maloy et al.
present exp.
$\Theta_{\pi[CM]}$ = 90°

Fig. 8



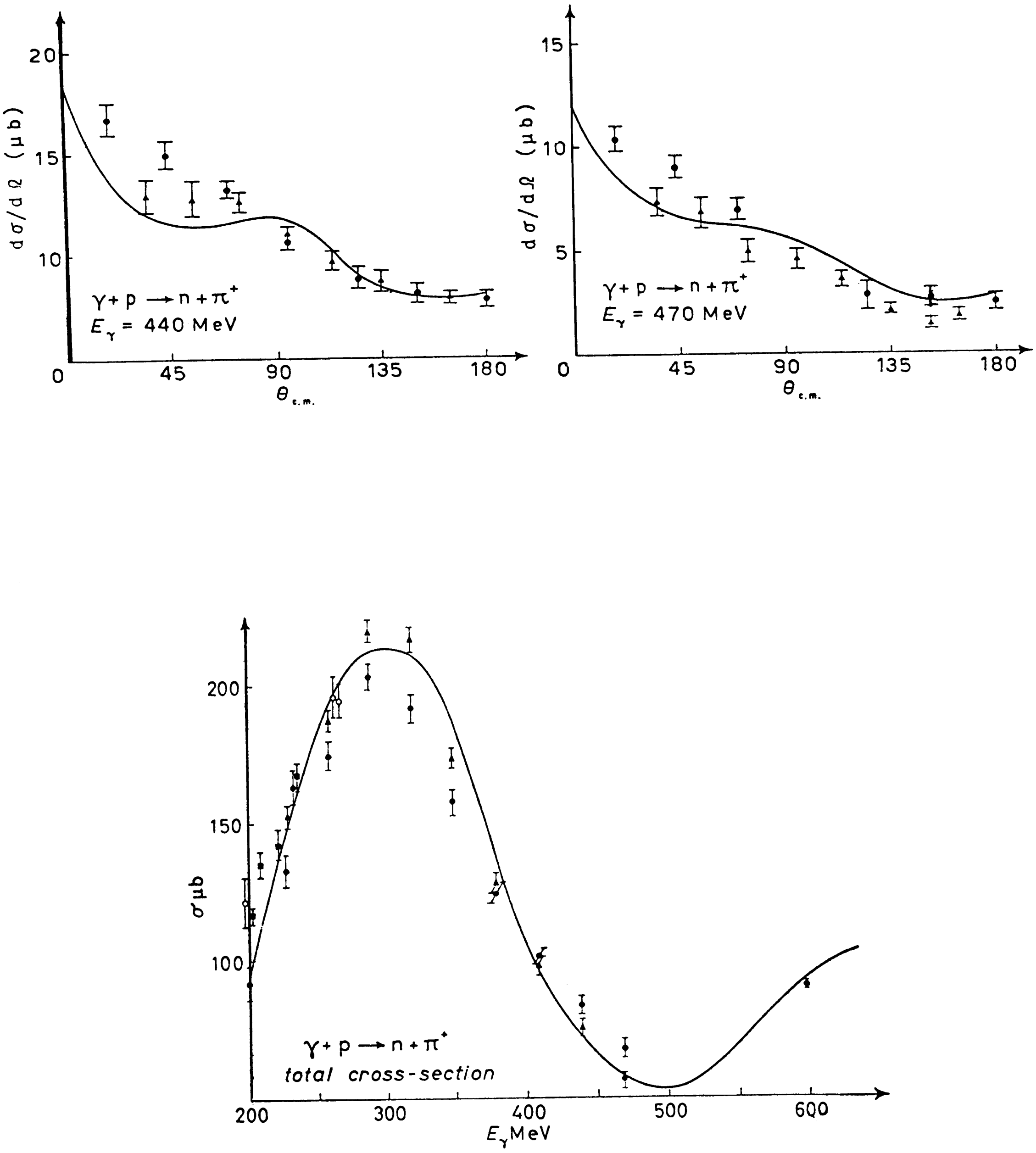
(a) K = 740 MEV
(b) K = 910 MEV
(c) K = 1140 MEV
σ (θ) – C.M. DIFFERENTIAL CROSS SECTION (μ barn / ster.)
WITHOUT PROTON
WITH PROTON
ω
η
γ+p → π°+p
cos θ

Fig. 9



Photoproduction of π^0 up to 4 GeV

Fig. 10



20
15
10
dσ/dΩ (μb)
γ+ p → n + π+
Eγ = 440 MeV
0
45
90
135
180
θc.m.
15
10
5
dσ/dΩ (μb)
γ+ p → n + π+
Eγ = 470 MeV
0
45
90
135
180
θc.m.
200
150
100
σ μb
γ+ p → n + π+
total cross-section
200
300
400
500
600
EγMeV

Fig. 11

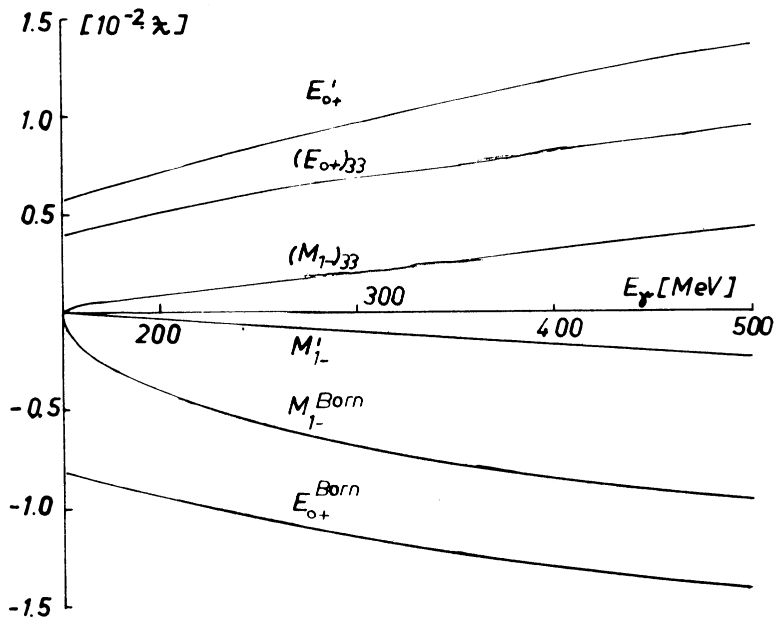


Fig. 12

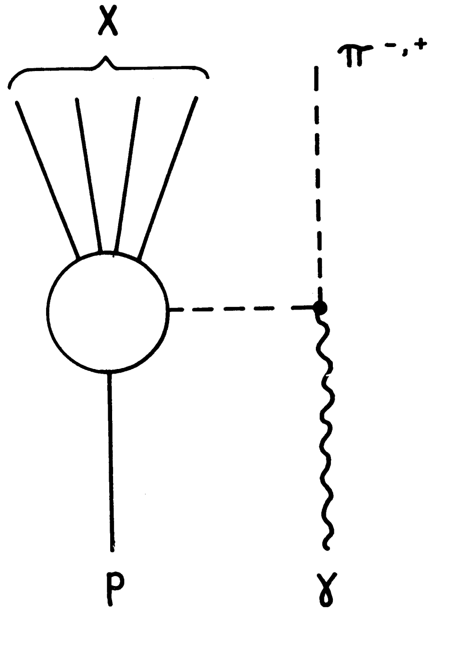


Fig. 13

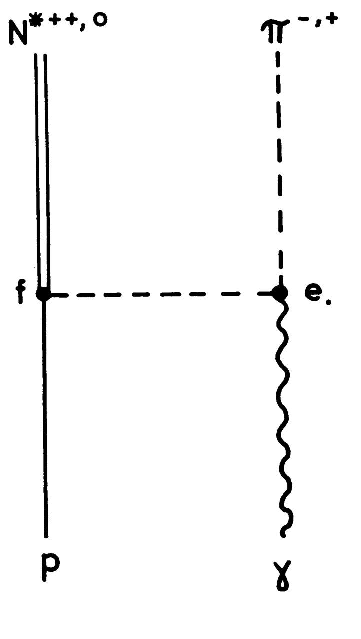


Fig. 14

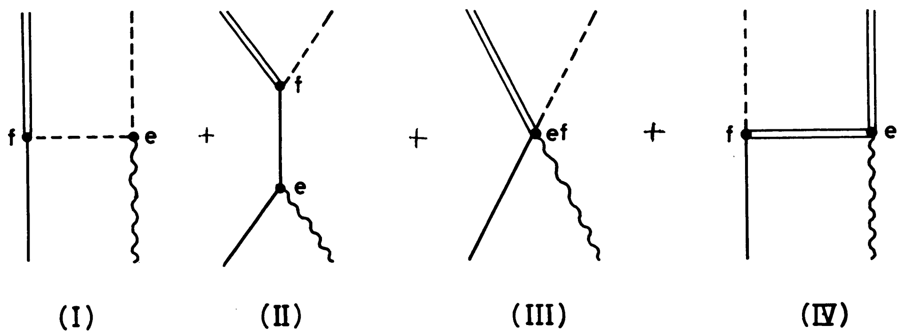

f
e
+
f
e
+
ef
+
f
e
(I)
(II)
(III)
(IV)

Fig. 15

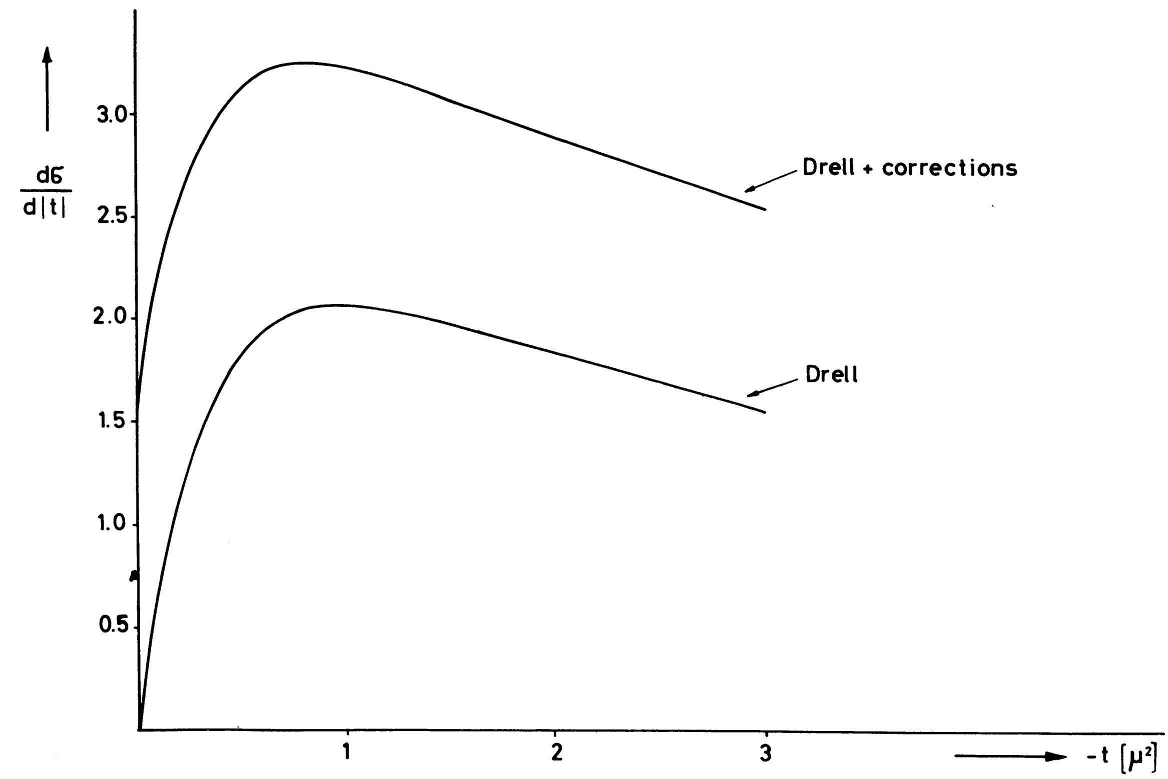

dσ/d|t|
3.0
2.5
2.0
1.5
1.0
0.5
Drell + corrections
Drell
1
2
3
-t [μ²]

Fig. 16

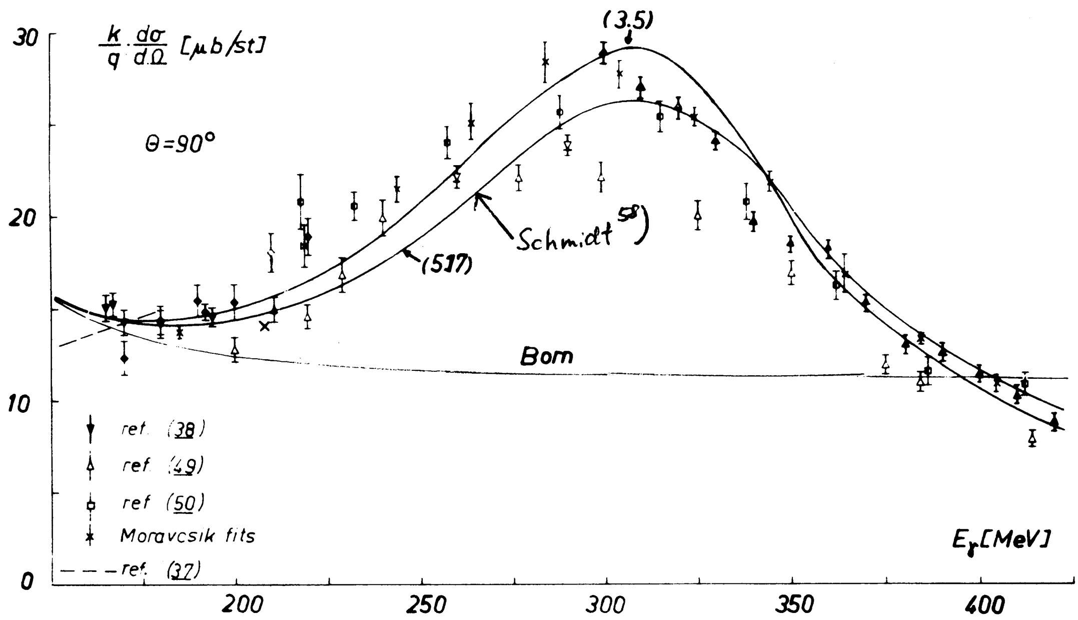


Fig. 17a

Fig. 17 a

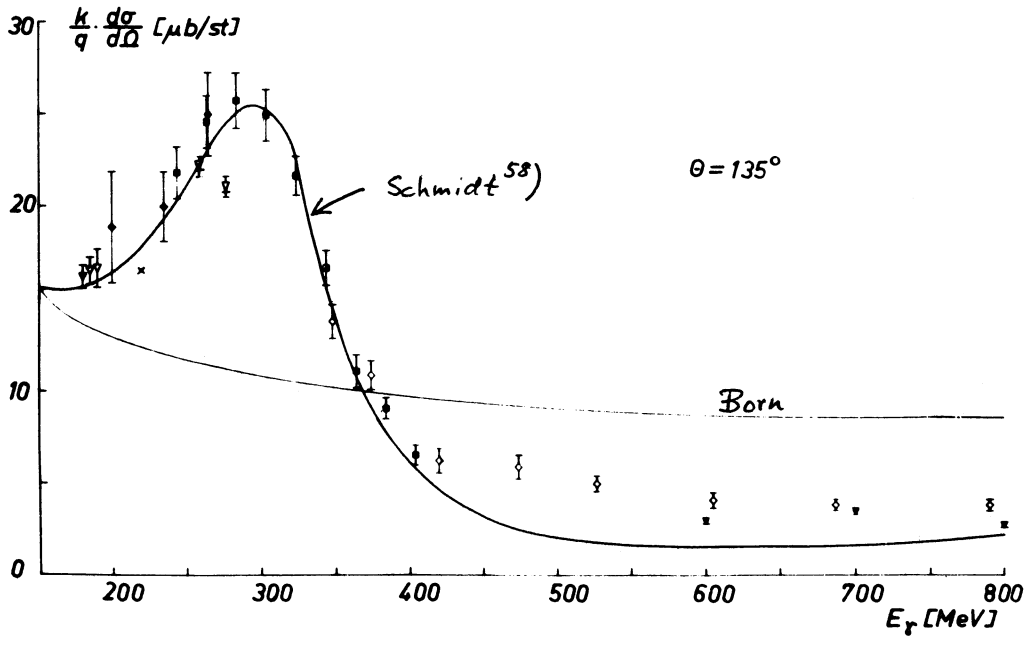


Fig. 17 b

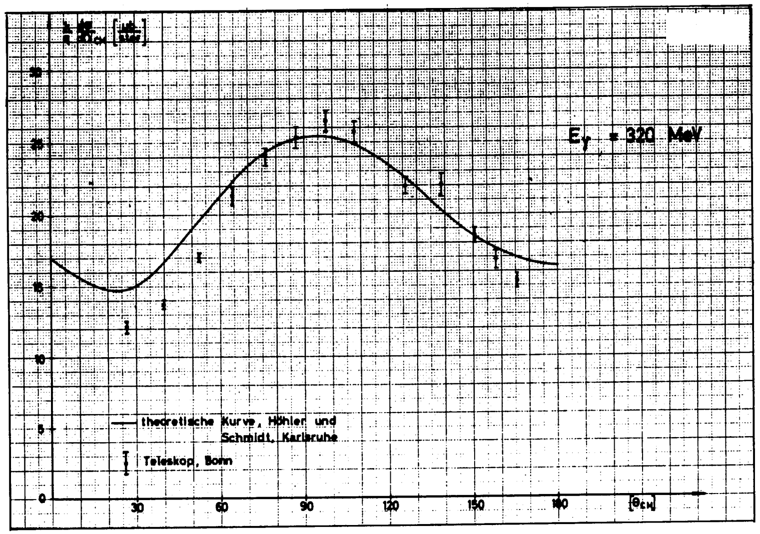


Fig. 18 a

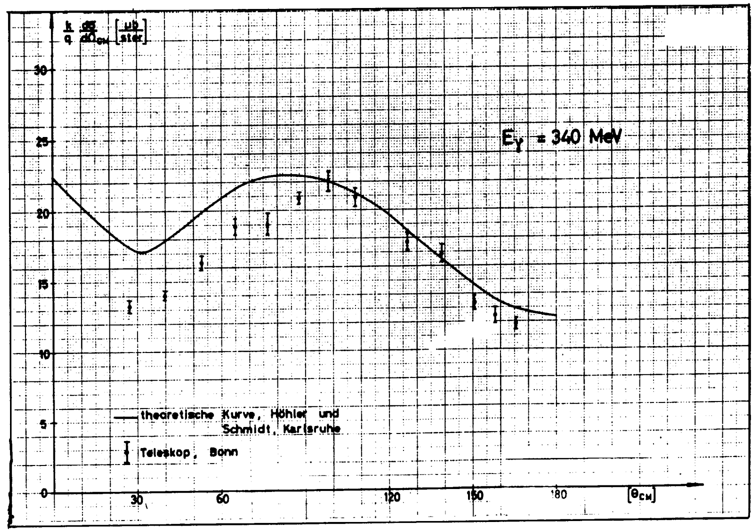


Fig. 18 b

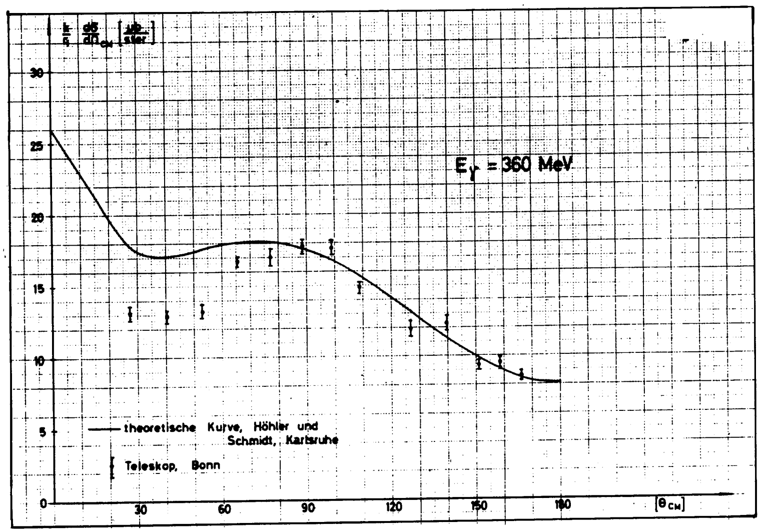


Fig. 18 c

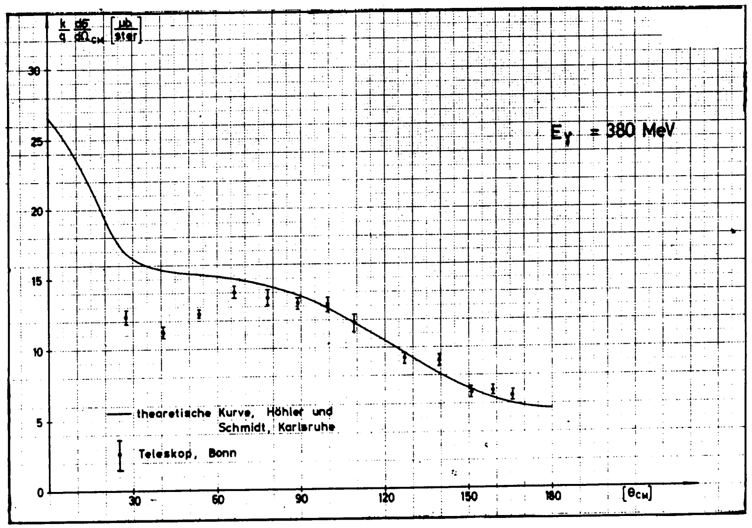


Fig. 18 d