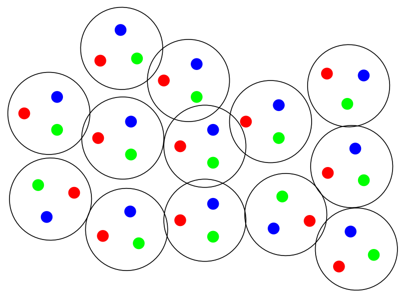
Quark Matter

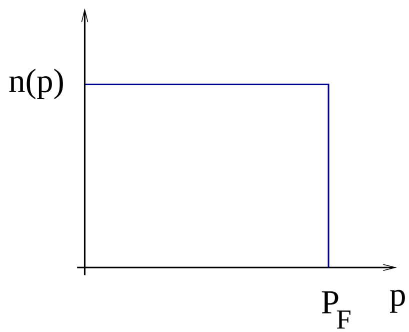
Very Dense Matter

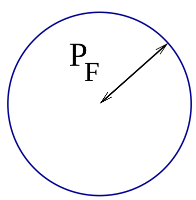
Consider baryon density $n_B \gg 1\,\mathrm{fm}^{-3}$



quarks expected to move freely

Ground state: cold quark matter (quark fermi liquid)





only quarks with $p \sim p_F$ scatter

$p_F \gg \Lambda_{QCD} \rightarrow$ coupling is weak

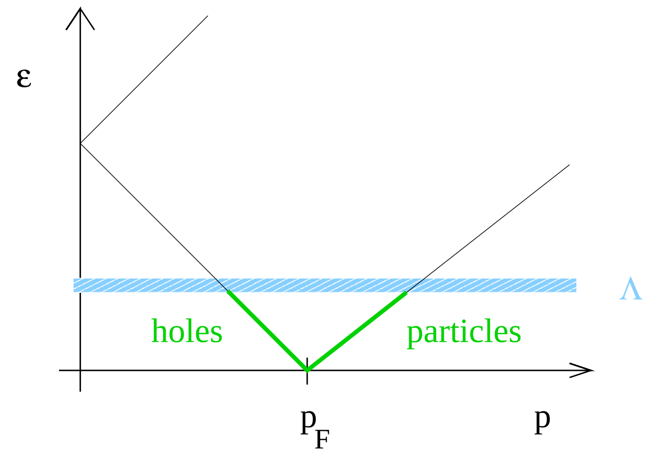
No chiral symmetry breaking, confinement, or dynamically generated masses

High Density Effective Theory

QCD lagrangian

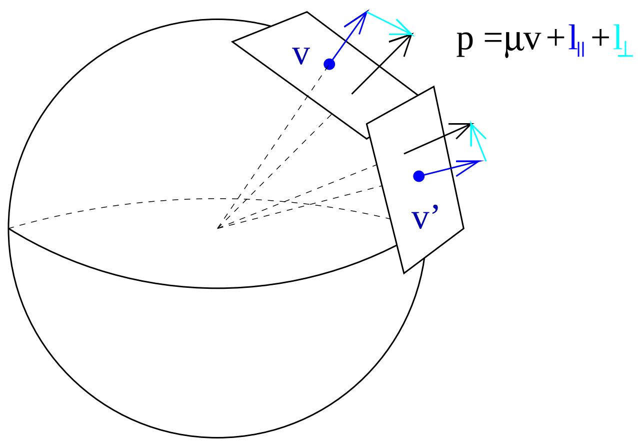
$$\mathcal{L} = \bar{\psi}\left(i \not{D} + \mu\gamma_0 - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

Quasi-particles (holes)



$$E_{\pm} = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

High Density Effective Theory, cont

Insert $\psi_{v\pm}$ in QCD lagrangian

$$\mathcal{L} = \sum_v \Big\{ \psi^\dagger_{v+} (iv\cdot D)\, \psi_{v+} + \psi^\dagger_{v-} (2\mu + i\bar{v}\cdot D)\, \psi_{v-}$$

$$+ \psi^\dagger_{v+} (i\not{D}_\perp)\, \psi_{v-} + \psi^\dagger_{v-} (i\not{D}_\perp)\, \psi_{v+} \Big\}$$

Integrate out ψ_{v-} at tree level

$$\psi_{v-} = \frac{1}{2\mu + i\bar{v}\cdot D}(i\not{D}_\perp)\psi_{v+}$$

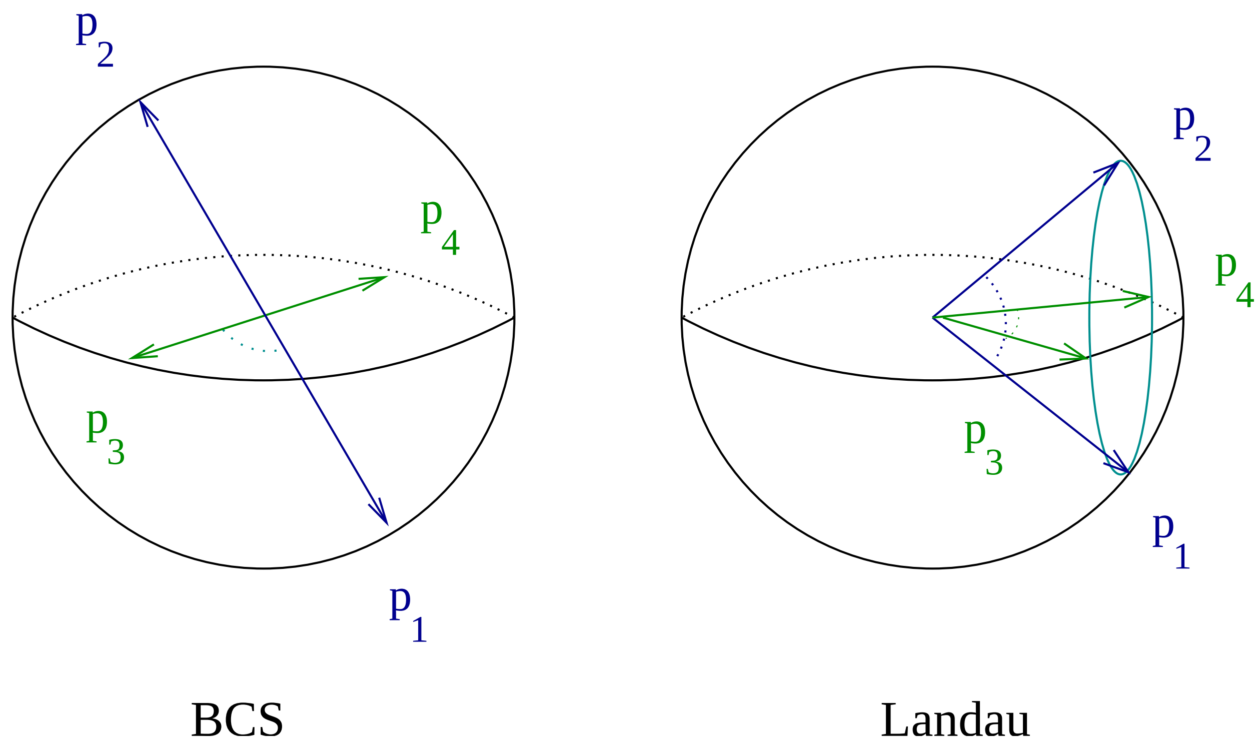
Effective lagrangian for ψ_{v+}

$$\mathcal{L} = \sum_v \psi^\dagger_v \left(iv\cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \ldots$$

Four Quark Operators

quark-quark scattering

$$(v_1, v_2) \to (v_3, v_4)$$

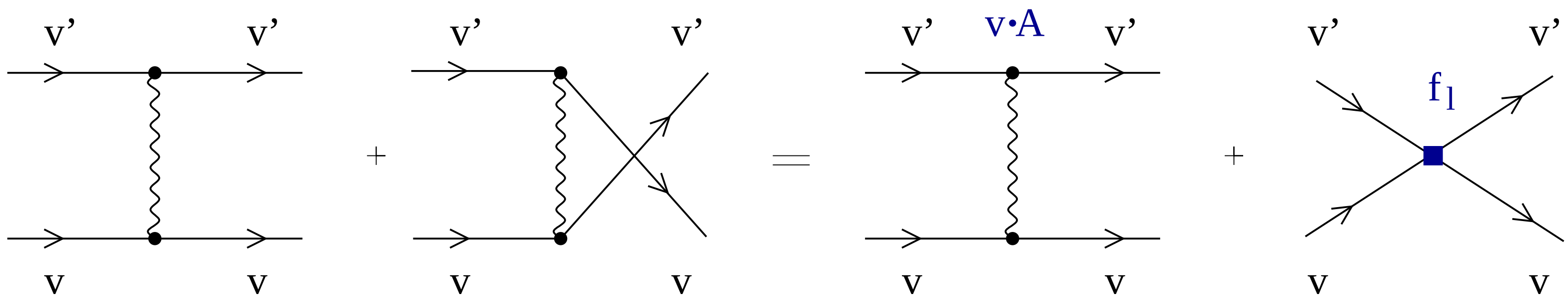


$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma\Gamma'} R_l^{\Gamma\Gamma'}(\vec{v} \cdot \vec{v}') \Big(\psi_v \Gamma \psi_{-v} \Big) \Big(\psi^\dagger_{v'} \Gamma' \psi^\dagger_{-v'} \Big),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma\Gamma'}(\phi) R_l^{\Gamma\Gamma'}(\vec{v} \cdot \vec{v}') \Big(\psi_v \Gamma \psi_{v'} \Big) \Big(\psi^\dagger_{\tilde{v}} \Gamma' \psi^\dagger_{\tilde{v}'} \Big)$$

Four Fermion Operators: Matching

Match scattering amplitudes on Fermi surface: forward scattering



Color-flavor-spin symmetric terms

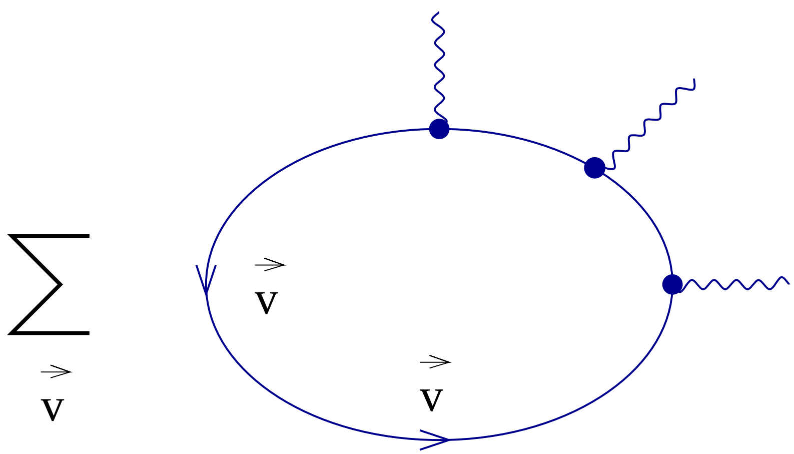
$$f_0^s = \frac{C_F}{4N_cN_f}\frac{g^2}{p_F^2}, \qquad f_i^s = 0 \ \ (i > 1)$$

Power Counting

Naive power counting

$$\mathcal{L} = \hat{\mathcal{L}}\left(\psi, \psi^\dagger, \frac{D_{||}}{\mu}, \frac{D_\perp}{\mu}, \frac{\bar{D}_{||}}{\mu}, \frac{m}{\mu}\right)$$

Problem: hard loops (large $N_{\vec{v}}$ graphs)



$$\frac{1}{2\pi}\sum_{\vec{v}}\int \frac{d^2 l_\perp}{(2\pi)^2} = \frac{\mu^2}{2\pi^2}\int \frac{d\Omega}{4\pi}.$$

Have to sum large $N_{\vec{v}}$ graphs

Effective Theory for $l \sim g\mu$

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^b_{\mu\beta}$$

Transverse gauge boson propagator

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\frac{\pi}{2} m^2 \frac{k_0}{|\vec{k}|}},$$

Scaling of gluon momenta

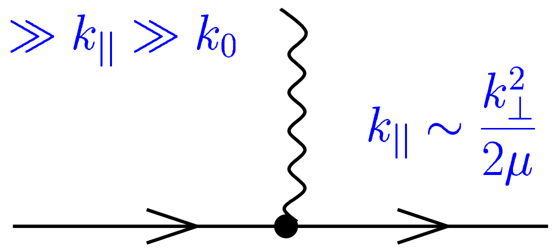
$$|\vec{k}| \sim k_0^{1/3} m^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Non-Fermi Liquid Effective Theory

Gluons carry large momenta $|\vec{k}| \gg |k_0|$. Quark dispersion relation

$$k_0 \simeq k_{||} + \frac{k_\perp^2}{2\mu}$$

quarks near FS: $k_{||} \ll k_\perp$



Scaling relations

$$k_\perp \sim m^{2/3} k_0^{1/3}, \qquad k_{||} \sim m^{4/3} k_0^{2/3}/\mu$$

Propagators

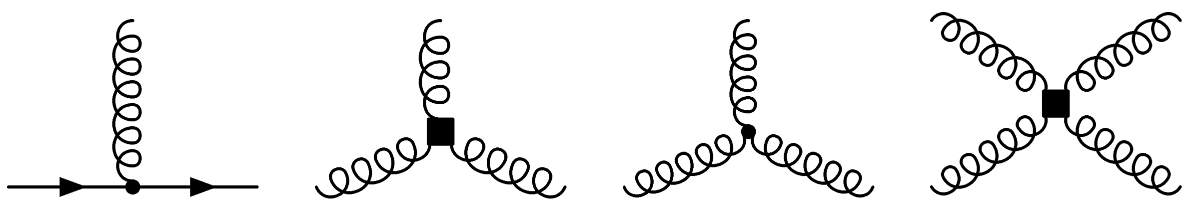
$$S_{\alpha\beta} = \frac{i\delta_{\alpha\beta}}{p_0 - p_{||} - \frac{p_\perp^2}{2\mu} + i\epsilon sgn(p_0)} \qquad D_{ij} = \frac{-i\delta_{ij}}{k_\perp^2 - i\frac{\pi}{2}m^2\frac{k_0}{k_\perp}},$$

Non-Fermi Liquid Expansion

Scale momenta $(k_0, k_{||}, k_\perp) \to (sk_0, s^{2/3}k_{||}, s^{1/3}k_\perp)$

$$[\psi] = 5/6 \qquad [A_i] = 5/6 \qquad [S] = [D] = 0$$

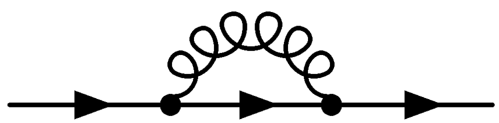
Scaling behavior of vertices



$s^{1/6}$ $\qquad s^{1/2}$ $\qquad s^{5/6}$ $\qquad s$

Systematic expansion in $\epsilon^{1/3} \equiv (\omega/m)^{1/3}$

Loop Corrections: Quark Self Energy



$$= g^2 C_F \int \frac{dk_0}{2\pi} \int \frac{dk_\perp^2}{(2\pi)^2} \frac{k_\perp}{k_\perp^3 + i\eta k_0}$$

$$\times \int \frac{dk_{||}}{2\pi} \frac{\Theta(p_0 + k_0)}{k_{||} + p_{||} - \frac{(k_\perp + p_\perp)^2}{2\mu} + i\epsilon}$$

Transverse momentum integral logarithmic

$$\int \frac{dk_\perp^3}{k_\perp^3 + i\eta k_0} \sim \log\left(\frac{\Lambda}{k_0}\right)$$

Quark self energy

$$\Sigma(p) = \frac{g^2}{9\pi^2} p_0 \log\left(\frac{\Lambda}{|p_0|}\right)$$

Quark Self Energy, cont

Higher order corrections?

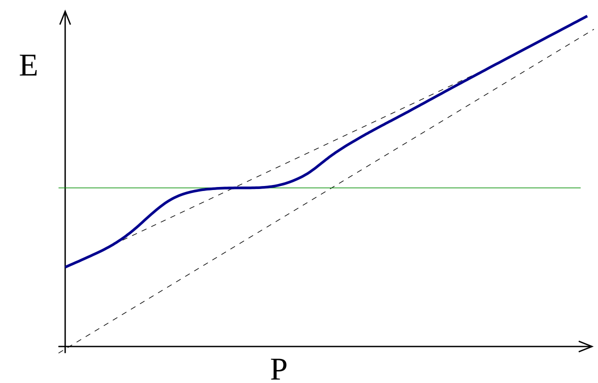
$$\Sigma(p) = \frac{g^2}{9\pi^2}\left(p_0 \log\left(\frac{2^{5/2}m}{\pi|p_0|}\right) + i\frac{\pi}{2}p_0\right) + O\left(\epsilon^{5/3}\right)$$

Scale determined by electric gluon exchange

No $p_0[\alpha_s \log(p_0)]^n$ terms

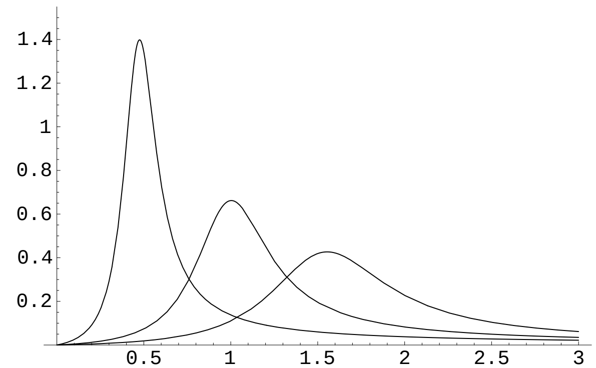
quasi-particle velocity vanishes as

$$v \sim \log(\Lambda/\omega)^{-1}$$

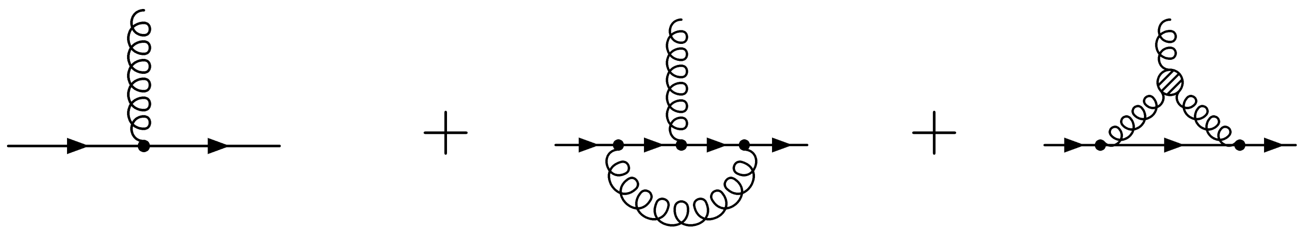


anomalous term in the specific heat

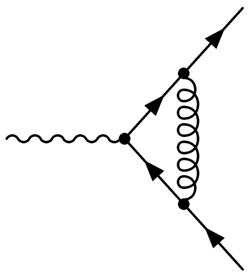
$$c_v \sim \gamma T \log(T)$$



Vertex Corrections, Migdal's Theorem

 $\sim \quad gv(1 + O(\epsilon^{1/3}))$

Can this fail? Yes, if external momenta fail to satisfy $k_\perp \gg k_0$



$$= g^2 e C_F v_\mu \int \frac{dk_0}{2\pi} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 + \frac{\pi}{2} m^2 \frac{k_0}{k_\perp}}$$

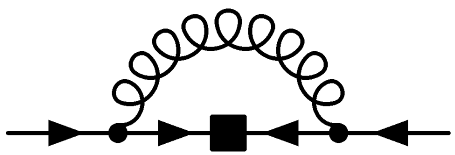
$$\times \int \frac{dk_{||}}{2\pi} \frac{1}{[p_{1,0} - k_{||} - \frac{k_\perp^2}{2\mu}][p_{2,0} - k_{||} - \frac{k_\perp^2}{2\mu}]}$$

Dominant terms in quark propagator cancel. Find

$$\Gamma_\mu(p_1, p_2) = \frac{eg^2}{9\pi^2} v_\mu \log\left(\frac{\Lambda}{p_0}\right).$$

Superconductivity

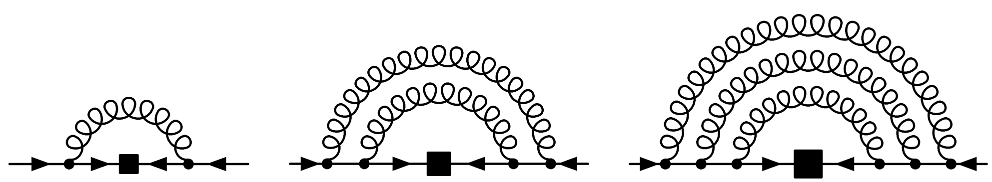
Same phenomenon occurs in anomalous self energy



$$= \frac{g^2}{18\pi^2}\int dq_0 \log\left(\frac{\Lambda_{BCS}}{|p_0-q_0|}\right)\frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta(q_0)^2}}$$

$\Lambda_{BCS} = 256\pi^4 g^{-5}\mu$ determined by electric exchanges

Have to sum all planar diagrams, non-planar suppressed by $\epsilon^{1/3}$



Solution at next-to-leading order (includes normal self energy)

$$\Delta_0 = 2\Lambda_{BCS}\exp\left(-\frac{\pi^2+4}{8}\right)\exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \qquad \Delta_0 \sim 50\,\mathrm{MeV}$$

CFL Phase

Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, \epsilon^{abI} \epsilon_{ijI}$$

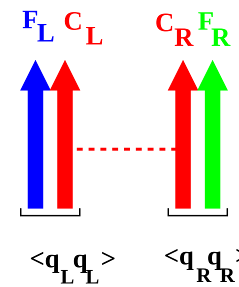
$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

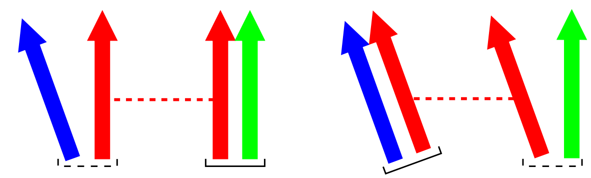
$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$

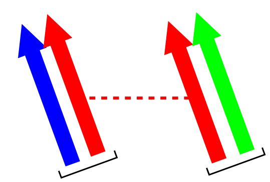
All quarks and gluons acquire a gap





Rotate left flavor

Compensate by rotating color



... have to rotate right flavor also !

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\} + (L\leftrightarrow R, X\leftrightarrow Y)$$

$$\psi_L \to L\psi_L C^T,\ X\to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = \mathbb{1}$$

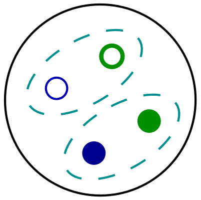
Quark loops generate a kinetic term for X, Y

$$\mathcal{L} = -\frac{f_\pi^2}{2}\left\{\mathrm{Tr}\left((X^\dagger D_0 X)^2 + (Y^\dagger D_0 Y)^2\right)\right\} + \ldots$$

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

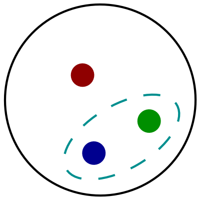
$$\Sigma = XY^\dagger$$

[8]+[1] GBs

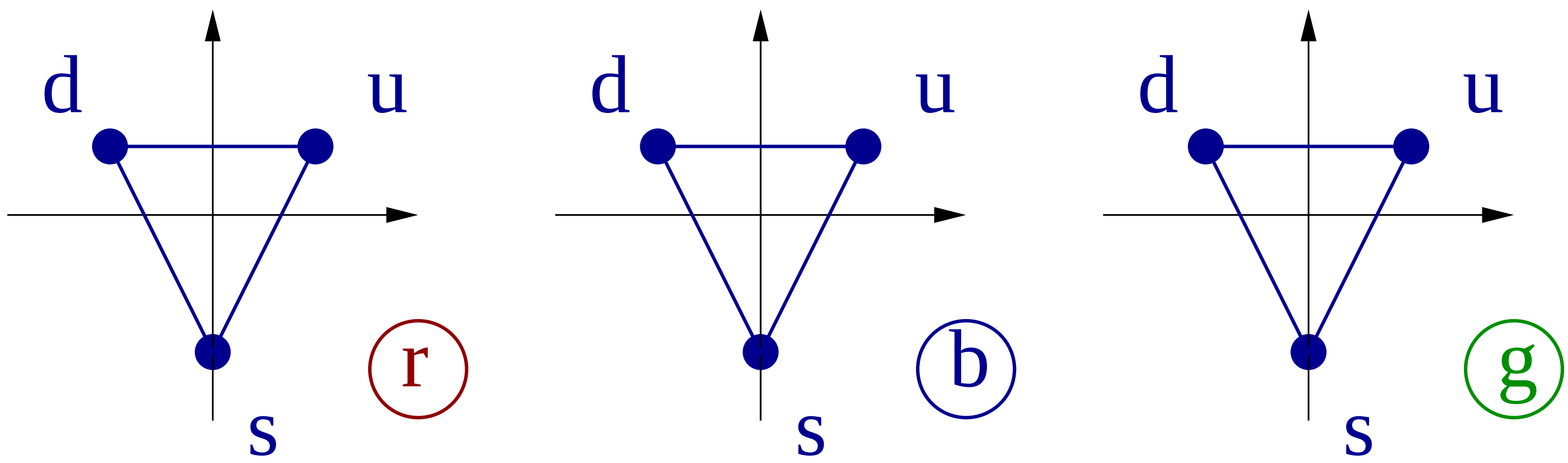


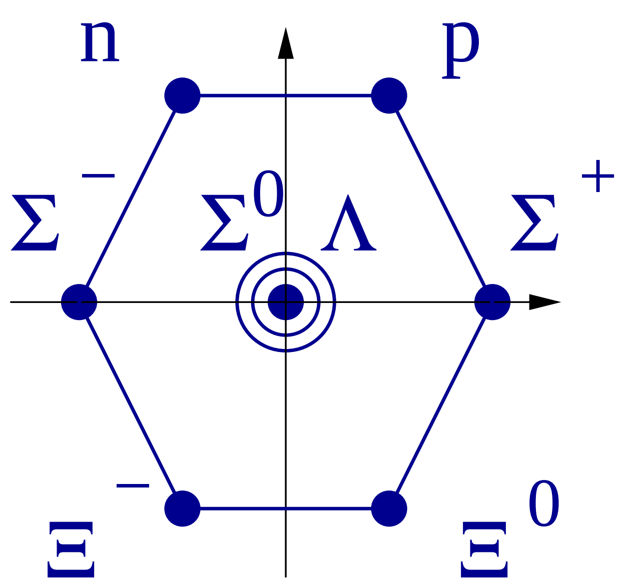
$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$

[8]+[1] Baryons



Quark Hadron Complementarity





Effective theory: CFL(B)χPTh

$$
\begin{aligned}
\mathcal{L} \; = \; & \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger\right) - v_\pi^2\mathrm{Tr}\left(\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)\right\} \\
& + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5\left\{\mathcal{A}_\mu, N\right\}\right) \\
& - F\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5\left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}
\end{aligned}
$$

with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

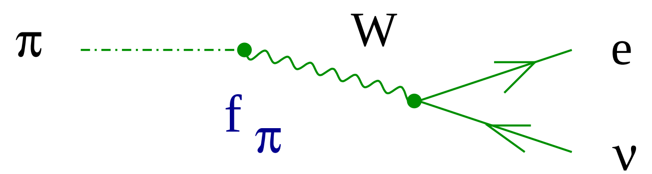
$$
\begin{aligned}
\mathcal{V}_\mu \; &= \; -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \\
\mathcal{A}_\mu \; &= \; -\frac{i}{2}\xi\left(\partial_\mu\Sigma^\dagger\right)\xi
\end{aligned}
$$

$$
f_\pi^2 = \frac{21 - 8\log 2}{18}\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad D = F = \frac{1}{2}
$$

Matching f_π

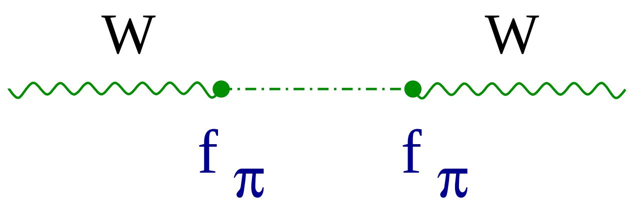
Compute f_π: Gauge $SU(3)_L \times SU(3)_R$ flavor symmetry

$$\nabla_\mu \Sigma = \partial_\mu \Sigma - i W_\mu^L \Sigma + i \Sigma W_\mu^R$$



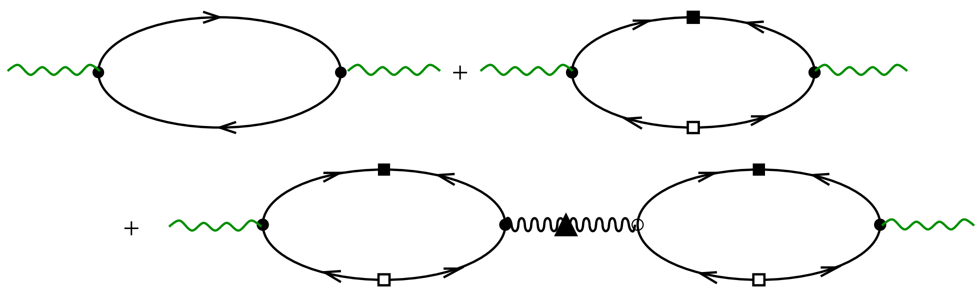
Higgs phenomenon

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left[(W_0^L - W_0^R)^2 \right] + \ldots$$



$$m_W^2 = f_\pi^2$$

Microscopic theory

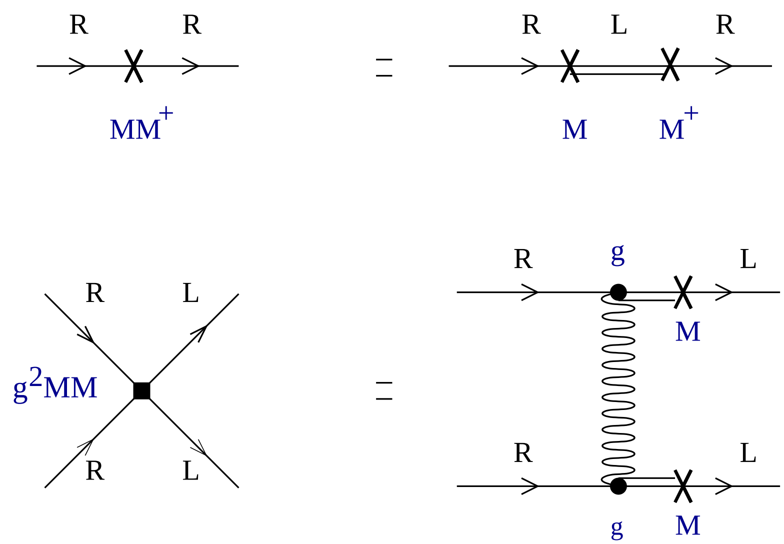


$$f_\pi^2 = \frac{21 - 8\log(2)}{18} \left(\frac{\mu^2}{2\pi^2} \right)$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu} \psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu} \psi_L$$

$$+ \frac{C}{\mu^2} (\psi_R^\dagger M \lambda^a \psi_L)(\psi_R^\dagger M \lambda^a \psi_L)$$



mass corrections to FL parameters $\hat{\mu}$ and $V^0_{RR;LL}$

Mass Terms: Match HDET to CFLχTh

Kinetic term: $\psi_L^\dagger X_L \psi_L + \psi_R^\dagger X_R \psi_R$

$$\begin{aligned} D_0 N &= \partial_0 N + i[\Gamma_0, N], \qquad \Gamma_0 = \mathcal{V}_0 + \frac{1}{2}\left(\xi X_R \xi^\dagger + \xi^\dagger X_L \xi\right) \\ \nabla_0 \Sigma &= \partial_0 \Sigma + i X_L \Sigma - i \Sigma X_R \end{aligned}$$

vector (axial) potentials

Contact term: $(\psi_R^\dagger M \psi_L)(\psi_R^\dagger M \psi_L)$

$$\mathcal{L} = \frac{3\Delta^2}{4\pi^2}\left\{[\mathrm{Tr}(M\Sigma)]^2 - \mathrm{Tr}(M\Sigma M\Sigma)\right\}$$

meson mass terms

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2}\mathrm{Tr}\left(X_L \Sigma X_R \Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

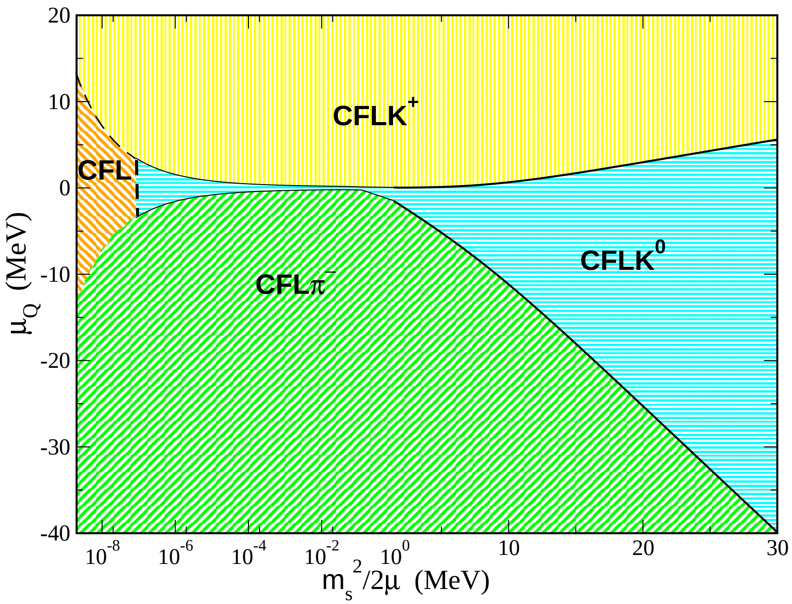
$$V(\Sigma_0) \equiv min$$

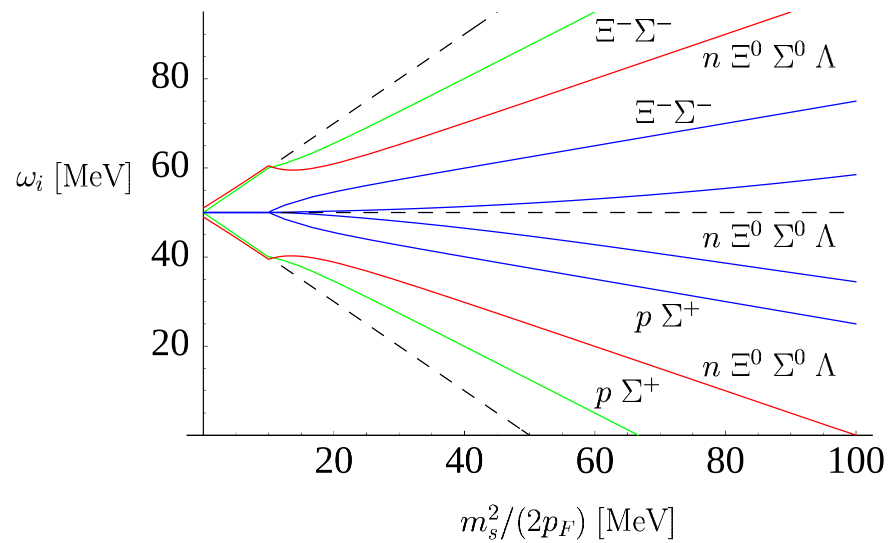
Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger \gamma_5 \rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi\frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger\frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum





meson condensation: CFLK

gapless modes (gCFLK)

stable?

Compare: Model Calculations

mode	$\tilde{Q}$	effective chemical potential	leading order	baryon mode
ru	0	$-\frac{2}{3}\mu_e + \frac{1}{2}\mu_3 + \frac{1}{3}\mu_8$	μ_0	
gd	0	$+\frac{1}{3}\mu_e - \frac{1}{2}\mu_3 + \frac{1}{3}\mu_8$	μ_0	$(\Lambda_0, \Lambda_8, \Sigma_0)$
bs	0	$+\frac{1}{3}\mu_e - \frac{2}{3}\mu_8 - \mu_s$	μ_0	
rd	-1	$+\frac{1}{3}\mu_e + \frac{1}{2}\mu_3 + \frac{1}{3}\mu_8$	μ_0	Σ^-
gu	1	$-\frac{2}{3}\mu_e - \frac{1}{2}\mu_3 + \frac{1}{3}\mu_8$	μ_0	Σ^+
rs	-1	$+\frac{1}{3}\mu_e + \frac{1}{2}\mu_3 + \frac{1}{3}\mu_8 - \mu_s$	$\mu_0 - \mu_s$	Ξ^-
bu	1	$-\frac{2}{3}\mu_e - \frac{2}{3}\mu_8$	$\mu_0 + \mu_s$	p
gs	0	$+\frac{1}{3}\mu_e - \frac{1}{2}\mu_3 + \frac{1}{3}\mu_8 - \mu_s$	$\mu_0 - \mu_s$	Ξ^0
bd	0	$+\frac{1}{3}\mu_e - \frac{2}{3}\mu_8$	$\mu_0 + \mu_s$	n

Instabilities

Consider meson current

$$\Sigma(x) = U_Y(x)\Sigma_K U_Y(x)^\dagger \qquad U_Y(x) = \exp(i\phi_K(x)\lambda_8)$$

$$\vec{\mathcal{V}}(x) = \frac{\vec{\nabla}\phi_K}{4}(-2\hat{I}_3 + 3\hat{Y}) \qquad \vec{\mathcal{A}}(x) = \vec{\nabla}\phi_K(e^{i\phi_K}\hat{u}^+ + e^{-i\phi_K}\hat{u}^-)$$

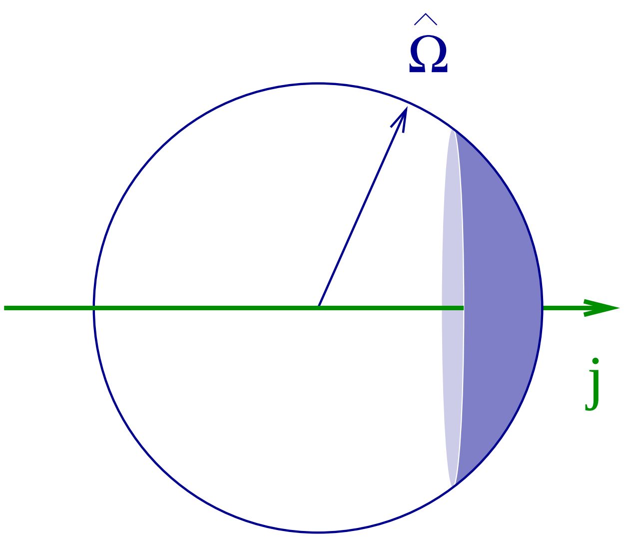
Gradient energy

$$\mathcal{E} = \frac{f_\pi^2}{2} v_\pi^2 \jmath_K^2 \qquad \vec{\jmath}_k = \vec{\nabla}\phi_K$$

Fermion spectrum

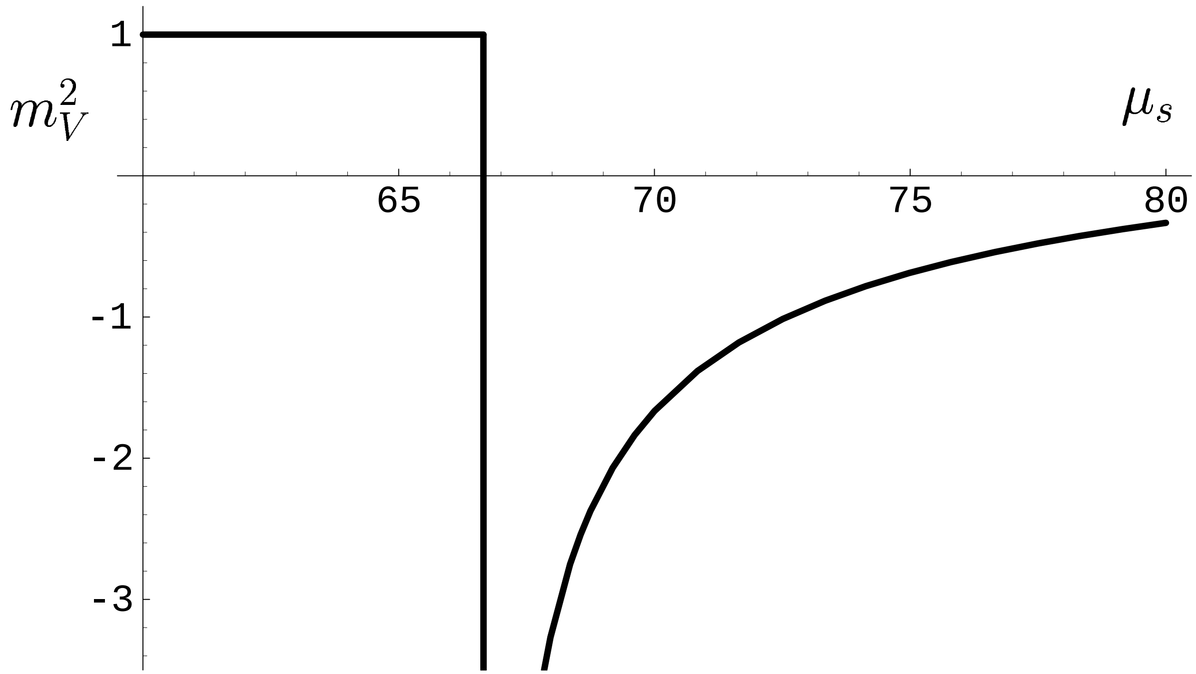
$$\omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K$$

$$\mathcal{E} = \frac{\mu^2}{2\pi^2}\int dl \int d\hat{\Omega}\; \omega_l \Theta(-\omega_l)$$



Stability lost

$$m_V^2 = \left.\frac{\partial^2 \mathcal{E}}{\partial \jmath^2}\right|_{\jmath=0}$$

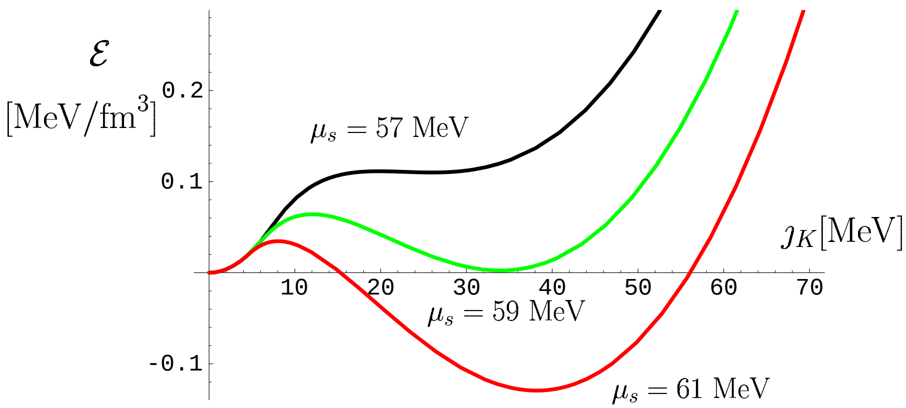


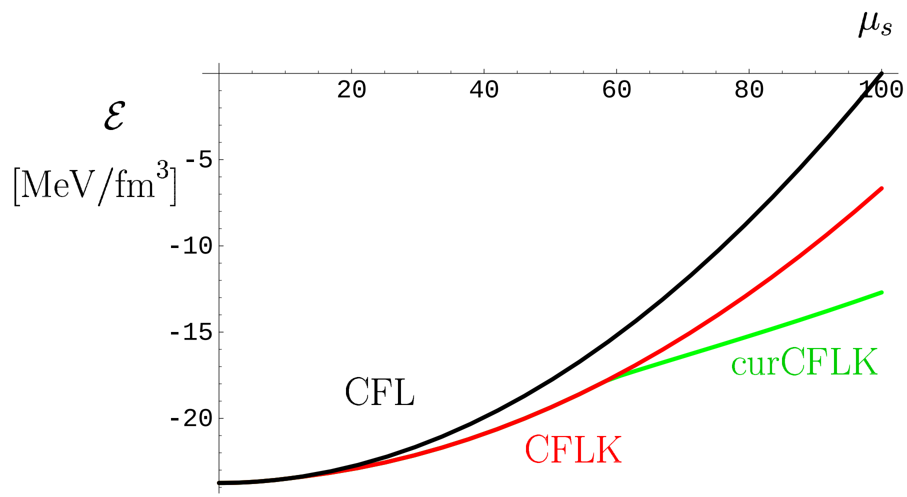
$$\mathcal{E} = C f_h(x) \qquad x = \frac{j_k}{a\Delta} \quad h = \frac{3\mu_s - 4\Delta}{a\Delta}$$

$$f_h(x) = x^2 - \frac{1}{x}\left[(h+x)^{5/2}\Theta(h+x) - (h-x)^{5/2}\Theta(h-x)\right]$$

see also: Son & Stephanov cond-mat/0507586, Kryjevski hep-ph/0508180

Energy Functional





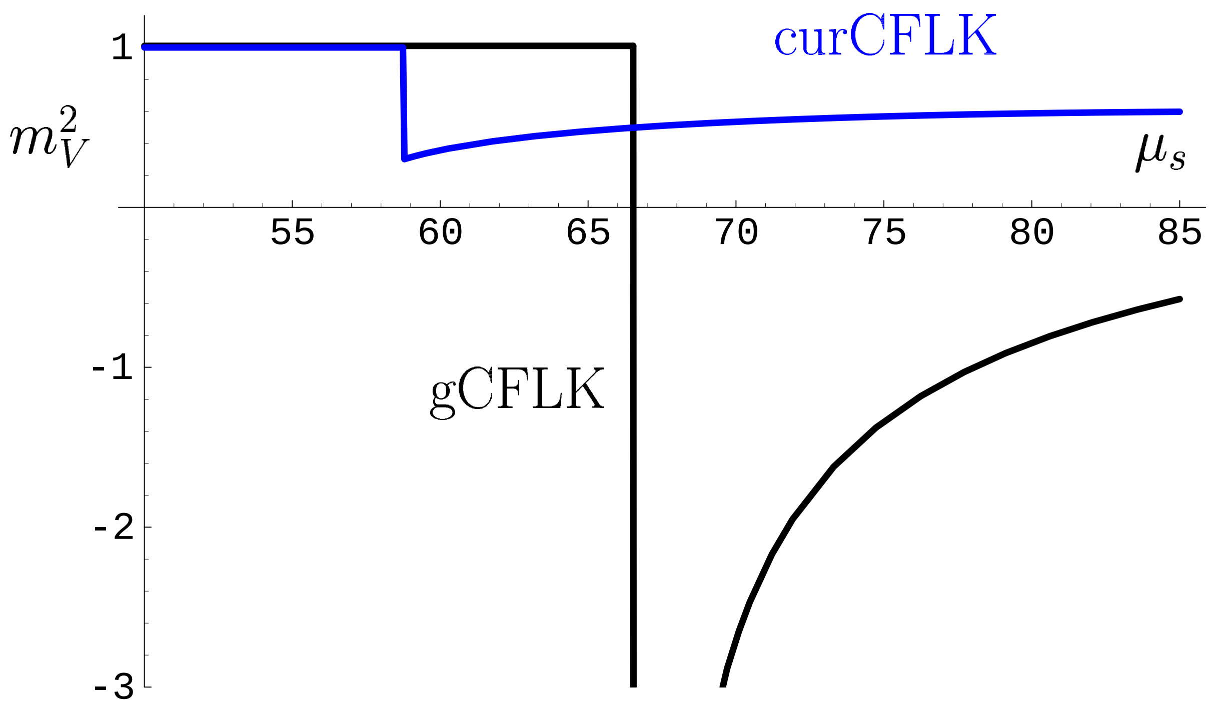
$$\left.\frac{3\mu_s - 4\Delta}{\Delta}\right|_{crit} = a h_{crit} \qquad h_{crit} = -0.067 \qquad a = \frac{2}{15^2 c_\pi^2 v_\pi^4}$$

[Figures include baryon current $j_B = \alpha_B/\alpha_K j_K$]

Stability found

$$m_V^2 = \left.\frac{\partial^2 \mathcal{E}}{\partial \jmath^2}\right|_{\jmath_0}$$

$$m_V^2 = \left.\frac{\partial^2 \mathcal{E}}{\partial \jmath^2}\right|_{\jmath=0}$$



$$\mathcal{E} = C f_h(x) \qquad x = \frac{j_k}{a\Delta} \quad h = \frac{3\mu_s - 4\Delta}{a\Delta}.$$

Phase Digram, $m_s \neq 0$