

Wave function is given by

$$\psi_n(x) = \psi_n(|\vec{p}|\alpha) \quad (1)$$

$\alpha$  is a parameter to be determined later.

$$\psi_n(x) = \sum_{k=0}^n a_{nk} x^{2k} e^{-x^2/2} \quad (2)$$

where

$$a_{nk} = \frac{(-1)^{k+n}}{k!} (2\pi\alpha)^{3/2} \left[ \frac{2(n!)}{\Gamma(n + \frac{3}{2})} \right]^{\frac{1}{2}} \frac{\Gamma(n + \frac{3}{2})}{(n-k)! \Gamma(k + \frac{3}{2})} \quad (3)$$

A check if you coded these functions right will be to calculate

$$\int \frac{d^3\vec{p}}{(2\pi)^3} \psi_n(|\vec{p}|\alpha) \psi_m(|\vec{p}|\alpha) = \begin{cases} 1, & \text{if } m = n \\ 0, & \text{if } m \neq n \end{cases} \quad (4)$$

for any  $\alpha > 0$ .