3. SVIE and some examples

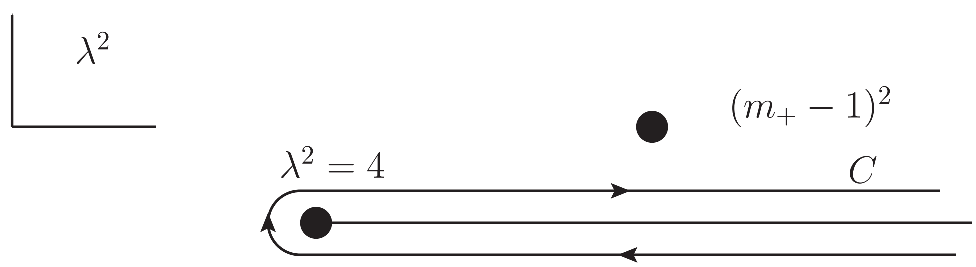
3.1 Single variable IE for Φ or ϕ

$$\Phi(s,m^2) = M(s) +$$

$$2M(s)\frac{1}{\pi}\int_4^\infty \frac{\rho'}{s'-s}\left\{\frac{1}{2}\int_{-1}^{1}\Phi(t(s',x))\mathrm{d}x\right\}$$

one step back.........

$$\Phi(t,m^2) = \frac{1}{2\pi \mathrm{i}}\int_C \frac{\Phi(\lambda^2,m^2)}{\lambda^2 - t}\,\mathrm{d}\lambda^2$$

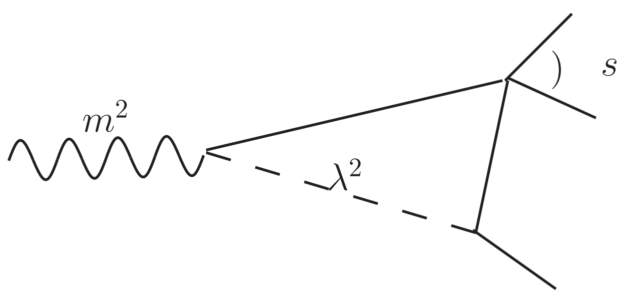


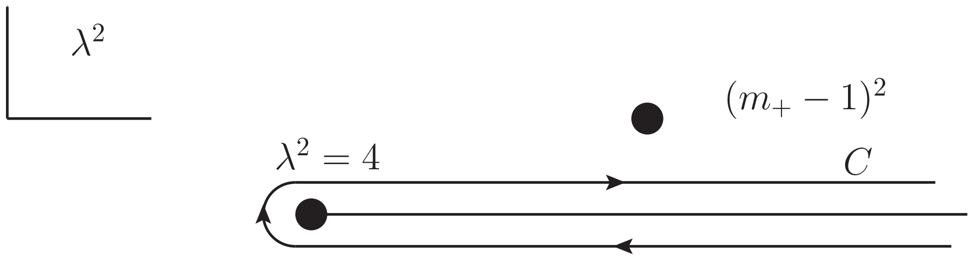
$$\text{int} = \frac{M(s)}{2\pi^2 \text{i}} \int_4^\infty \frac{\rho(s')}{s'-s} \int_{-1}^1 \text{d}x \int_C \frac{\Phi(\lambda^2, m^2)}{\lambda^2 - t(s', x)} \text{d}\lambda^2$$

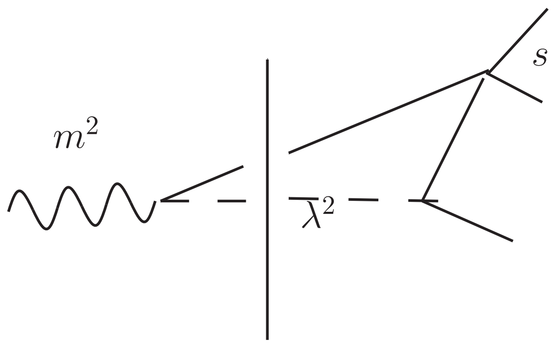
$$\to 2\frac{M(s)}{2\text{i}} \int_C \Phi(\lambda^2, m^2) f(s, \lambda^2, m^2) \text{d}\lambda^2$$

where

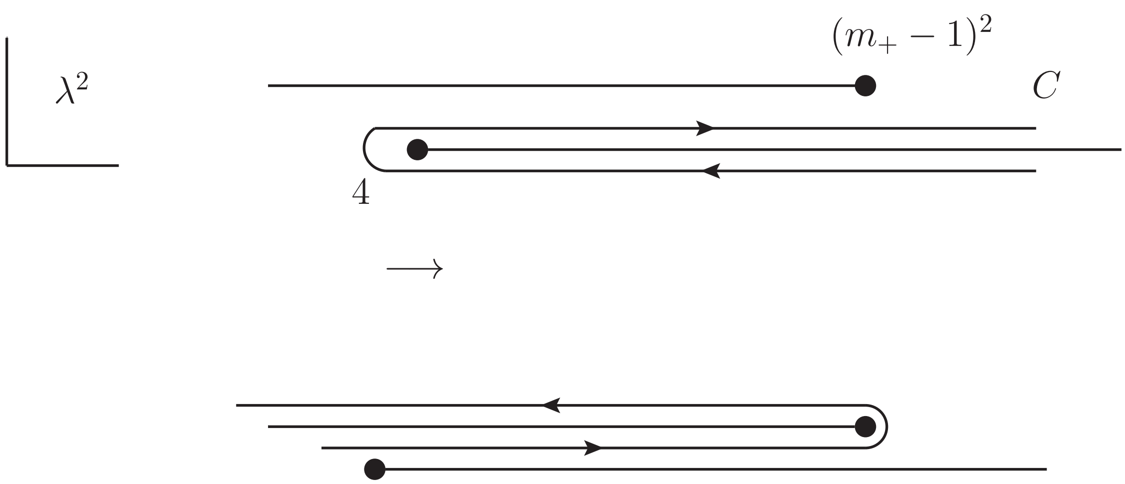
$$f(s, \lambda^2, m^2) = \frac{1}{\pi^2} \int_4^\infty \text{d}s' \frac{\rho(s')}{s'-s} \frac{1}{2} \int_{-1}^1 \frac{\text{d}x}{\lambda^2 - t(s', x)}$$

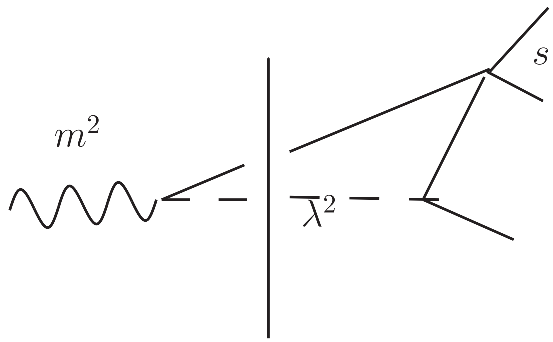






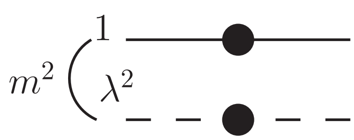
$$m^2 = (\lambda + 1)^2 \to \lambda^2 = (m - 1)^2$$



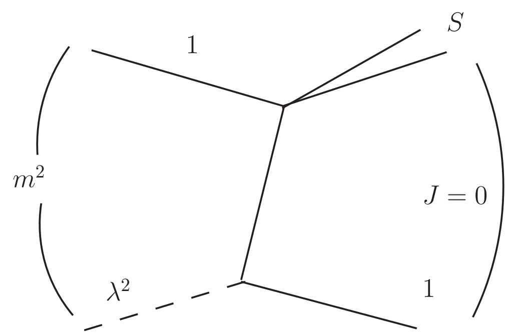


$$\mathrm{disc}_{\lambda^2=(m-1)^2} f(s,\lambda^2,m^2) = 2\mathrm{i}\Delta_1(\lambda^2,m^2,s)$$

$$\Delta_1 \propto \sigma(\lambda^2,m^2) Z(s,\lambda^2,m^2)$$



$\sigma(\lambda^2,m^2)$



$Z(\lambda^2,m^2,s)$

Z is a RPE process. It is singular on $\Gamma(s,\lambda^2,m^2) = 0$, i.e. on the boundary of the Dalitz plot. In D , develops imaginary part $\mathrm{i}\pi$.

So $\rightarrow$ a phase additional to that of the isobar amplitude M

SVIE

$$\Phi(s,m^2) = M(s) +$$

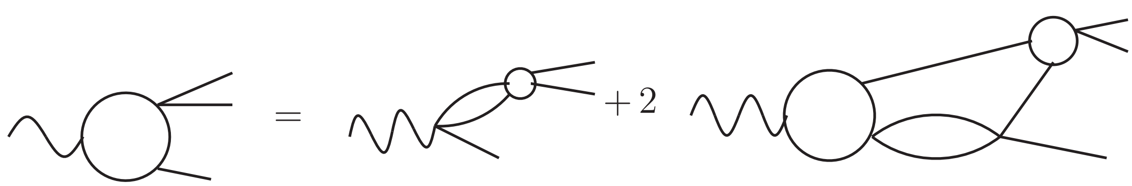
$$+2M(s)\int_{-\infty}^{(m-1)^2} \mathrm{d}\lambda^2 \Delta_1(\lambda^2, m^2, s)\Phi(\lambda^2, m^2)$$

$+$ other contributions from $\lambda^2 \leq 0$

Many channels: $\phi_{ij} = \delta_{ij} + \int K_{ik}\phi_{kj}\mathrm{d}\lambda^2$

Produced in j and scatters to i.

These correction factors only depend on the two-body Ms, and can be calculated once and for all, for use with programs fitting "isobar" production amplitudes $C_j(m^2)$



3.2 Some practical examples

A: 3π STEP 1: GENERALIZED ISOBAR EXPANSION

$$F(s,t,u,m^2) = \sum_J (2J+1) \sum_{\Lambda_1 l_1} \{$$
$$\mathcal{D}^J_{\Lambda_1 o}(2l_1+1) d^{l_1}_{\Lambda_1 0} C_1^{J\Lambda_1 l_1}(m^2)\Phi_1^{J\Lambda_1 l_1}(s,m^2)$$
$$+ \sum_{\Lambda_2 l_2} \ldots C^{J\Lambda_2 l_2}_{\Lambda_2 0}(m^2)\Phi_2^{J\Lambda_2 l_2}(t,m^2)$$
$$+ \sum_{\Lambda_3 l_3} \ldots C_3^{J\Lambda_3 l_3}(m^2)\Phi_3^{J\Lambda_3 l_3}(u,m^2)\}$$

Three complete sets, but each truncated

STEP 2: unitarity

$$\text{disc}_s \Phi_1^{J\Lambda_1 l_1}(s,m^2) = 2\mathrm{i}\rho M_1^{l_1 *}\Phi_1^{J\Lambda_1 l_1}(s,m^2) +$$
$$2\mathrm{i}\rho M_1^{l_1 *} \sum_{\Lambda_2 l_2} \frac{1}{2}\int_{-1}^{1} C^J_{\Lambda_1 l_1 \Lambda_2 l_2} \Phi_2^{J\Lambda_2 l_2}(t,m^2)\,\mathrm{d}\cos\theta_{12}$$

recoupling coefficient

+ similar contribution from Φ_3

$$\Phi_1^{J\Lambda_1 l_1}(s,m^2) = K.F._1^{J\Lambda_1 l_1} M_1^{l_1}(s)\phi_1^{J\Lambda_1 l_1}(s,m^2)$$

STEP 3:SVIE (Pasquier inversion - see notes)

e.g. $J^P = 1^-$ $\phi_\omega(s,m^2) = 1 +$

$$2\int_{-\infty}^{(m-1)^2} \mathrm{d}\lambda^2 \Delta_{1\omega}(\lambda^2,m^2,s)M(\lambda^2)\phi_\omega(\lambda^2,m^2)$$

plus contributions from $\lambda^2 \le 0$

All we need is $M(s)$!

$$M(s) = \left[a + bq^2 + cq^4 + \frac{2q^3}{\pi\sqrt{s}} \ln\left(\frac{\sqrt{s}+\sqrt{s-4}}{\sqrt{s}-\sqrt{s-4}}\right)\right]^{-1}$$

where $s = 4 + 4q^2$. Log has Im part $-\pi$ for $s > 4$; parameters a, b determine ρ mass and width; parameter c controls asymptotic behaviour for large $s \ll 0$

Results:

IJRA and R J A Golding J. Phys. G 4, 43 (1978)

1. m^2-dependence (3-body resonance)

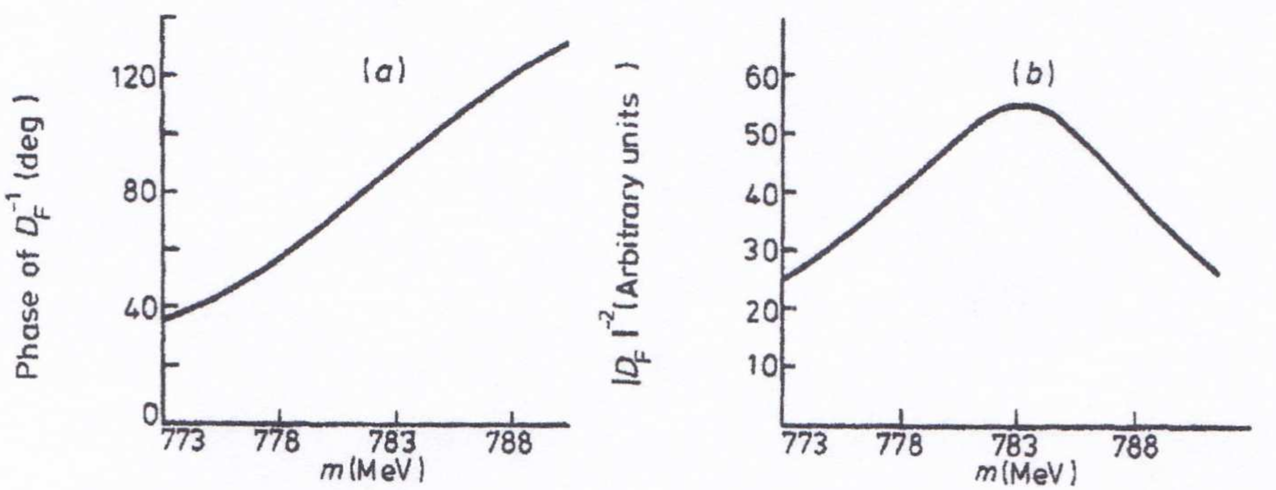


Figure 2. Phase (*a*) and square modulus (*b*) of D_F^{-1} for $m_\rho = 766$ MeV, $\Gamma_\rho = 133$ MeV and $c = 0{\cdot}01351$.

For physical m_ρ and Γ_ρ can generate resonance in m^2-variable for small positive c i.e. due to contribution from $\lambda^2 \ll 0$ region of integration (short range effect mimicking q$\bar{\text{q}}$). Can adjust c to get physical m_ω; predict width = 17 MeV (expt 8.5 MeV)

NOT claiming theory of ω! But does suggest this simple model has non-trivial 3-body dynamics too.....See next lecture....but (to anticipate) physical $3 \to 3$ amplitude has to be symmetric in $s \leftrightarrow \lambda^2$. This is the case for the RPE piece $\Delta_{1\omega}$ but not $\lambda^2 \leq 0$ kernels. Best strategy: cut λ^2 -integration off at $\lambda^2 = 0$. We don't believe this "dynamical" generation of ω, and calculations show that sub-energy variations (i.e. corrections to IM) dominated by physical region rescatterings.

2. Sub-energy dependence
Logarithmic singularity at $s = s_b$ is visible due to triangle graph with internal ρ meson. Slow m^2 variation. IM correction $\sim 20 - 30\%$ in magnitude, phase of some 20° generated at s-values near ρ resonance.

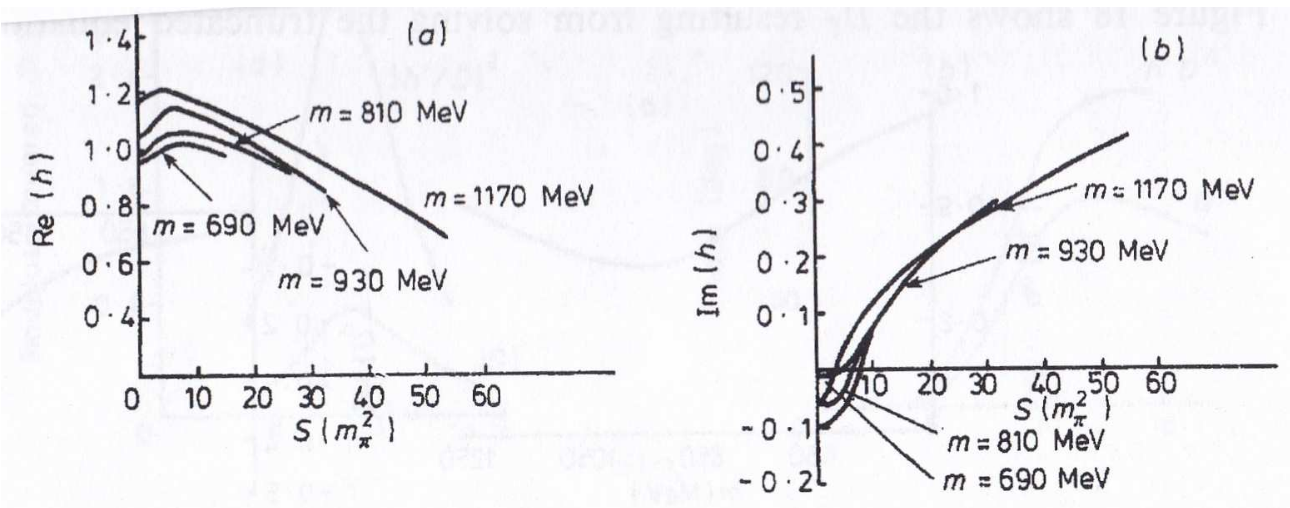


Figure 19. Real part (*a*) and imaginary part (*b*) of *h* calculated from the truncated equation for m_ρ = 766 MeV, Γ_ρ = 133 MeV, c = 0·0 and various values of *m*.

Other 3π calculations:

R. Pasquier Paris thesis (1973)

$J^P = 1^-(\omega), 1^+(a_1)\pi\rho\, L = 0, L = 2, \pi\epsilon\, L = 1$

K. R. Parker Oxford thesis (1979)

$J^P = 1^-, 1^+, 2^+$

J J Brehm Phys. Rev. D 23, 1194 (1981);

D 25, 3069 (1982) $J^P = 0^-, 1^+$

Substantial m^2-dependence in 1^+ case.

B: $\pi\pi N$

J. J. Brehm, Ann. Phys. 108, 454 (1977)

IJRA and JJB, Phys. Rev. D 17, 3072 (1978); D 20, 1119 , 1131 (1979)

$J^P = \frac{1}{2}^+, \frac{1}{2}^-, \frac{3}{2}^+, \frac{3}{2}^-$

πN isobars $S_{11}, S_{31}, P_{11}, P_{33}$

$\pi\pi$ isobars $l = 0, I = 0, 2; l = 1, I = 1$

Results:

* "It is most unlikely that unitarity corrections to the isobar model for $\pi N \to \pi\pi N$ will seriously modify m^2-channel resonance behaviour extracted via non-unitary IM fits"

***as the data then stood*

Nevertheless, some clear types of sub-energy variation:

(a) Logarithmic singularities (weak)

(b) In zero orbital angular momentum states, find "effective scattering length" behaviour

(c) For non-zero L find B-W "barrier" behaviour

IJRA and J J Brehm Phys Rev D 20 1131 (1979)

Item (b)

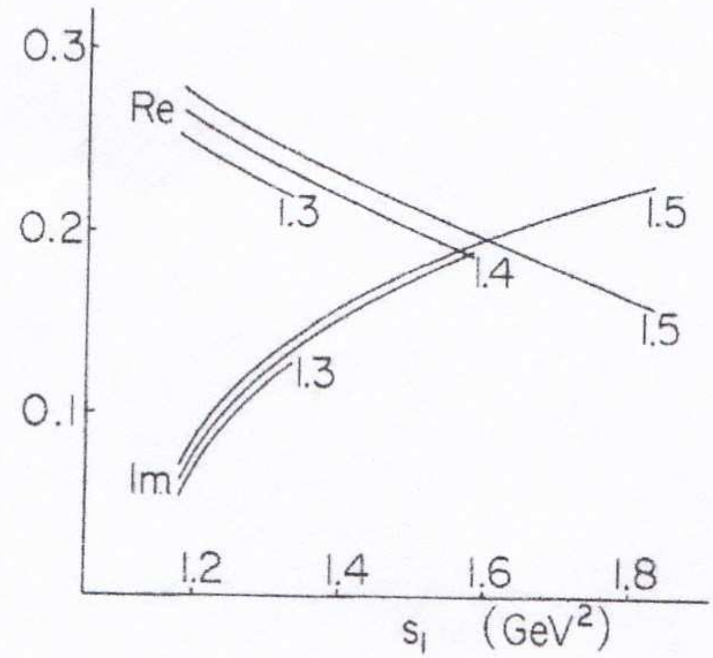


FIG. 11. Behavior of $I^{1/2^+}_{SS_1}$ for $W=1.3$, 1.4, and 1.5 GeV.

$$\pi\pi N \;\; J^P = \tfrac{1}{2}^+ \; : (\tfrac{1}{2}^-, \tfrac{1}{2}) \to (\tfrac{1}{2}^-, \tfrac{1}{2}), (\tfrac{1}{2}^-, \tfrac{3}{2})$$

$$S_1 \qquad\qquad S$$

Simple parametrization: $1/(1 - \mathrm{i}aq)$

$a \sim 0.5 - 1$fm

Item (c)

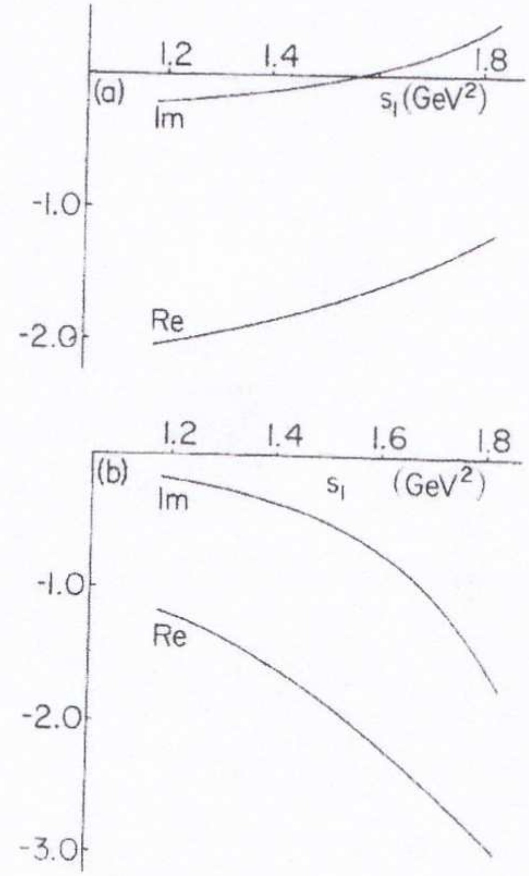


FIG. 5. Rescattering integrals for $W = 1.5$ GeV: (a) $I^{1/2+}_{\Delta\Delta}$ and (b) $I^{3/2+}_{\Delta\Delta}$.

simple parametrization: $e^{i\alpha}/[1+(pR)^L]^{\frac{1}{2}}$

$R \sim 1$fm c.f. B-W barrier factor