[24]W. Miller, Commun. Pure Appl. Math. 17, 527 (1964).
[25]W. Miller, *Lie Theory and Special Functions* (Academic, New York, 1968).
[26]J. D. Talman, *Special Functions: A Group Theoretic Approach* (Benjamin, New York, 1968).
[27]S. Ström, Arkiv Fys. 30, 267 (1965).
[28]M. Andrews and J. Gunson, J. Math. Phys. 5, 1391 (1964).
[29]J. Gunson, J. Math. Phys. 6, 852 (1965).
[30]A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton Univ. Press, Princeton, New Jersey, 1957).

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Gauge-Invariant Scaling Model of Current Interactions with Regge Behavior and Finite Fixed-Pole Sum Rules*

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A general nonperturbative model for the entire Compton amplitude which incorporates Bjorken scaling, gauge invariance, and Regge behavior is presented. We show that a covariantly regularized model based on the infinite-momentum-frame techniques of Drell, Levy, and Yan is equivalent to the manifestly covariant nonperturbative parton model of Landshoff, Polkinghorne, and Short. We also demonstrate that a general consequence of composite theories of hadrons with field-theoretic constituents which incorporates the above properties is the existence of a constant energy-independent and q^2-independent term in $T_1(\nu, q^2)$ (a "Kronecker delta" δ_{J0} term) and a $J=0$ fixed pole in $T_2(\nu, q^2)$. Sum rules for general Compton amplitudes are derived and a discussion of mass renormalization for electromagnetic self-energy corrections of hadrons is presented. We demonstrate that such sum rules are always finite, even in the presence of Regge behavior, when subtraction terms in the underlying parton-proton u-channel dispersion relation are taken into account. Analytic continuation in α is thus justified.

INTRODUCTION

Although many models of scaling behavior of the electromagnetic interactions have been proposed, the most compelling models continue to be those in which the hadronic matrix elements of the current behave as if the carriers of the current are elementary field-theoretic constituents. In addition to the general light-cone approach, specific dynamical models have been given by Drell, Levy, and Yan (DLY),[1] Landshoff, Polkinghorne, and Short (LPS),[2] and Drell and Lee.[3] One of the purposes of this paper is to show that a covariant regularized model based on the infinite-momentum techniques of DLY is equivalent to the LPS model, and displays many of the covariant features of the Bethe-Salpeter bound-state model of Drell and Lee.

One of the great virtues of the LPS model is that it naturally incorporates analytic Regge behavior of the scaling function $\nu W_2(x)$, reflecting the hadronic Regge behavior of the parton-proton amplitude. In this paper we present a related model, which is explicitly gauge-invariant, and allows a complete discussion of the entire Compton amplitude. The importance of Regge subtractions in the internal representation for the parton-proton amplitude is emphasized. The new model is defined in a linear operational fashion in terms of lowest-order calculations, and is eminently suitable for analyzing the interplay of fixed-pole, Regge behavior, current-algebra sum rules, and gauge invariance. One essential feature of the model is that all sum rules are automatically finite.

In this paper we also establish in detail the direct connection between scaling in local field theories and the presence of a polynomial-residual $J=0$ "fixed-pole" contribution to the Compton amplitude. The infinite-momentum-frame analysis is particularly useful for establishing the presence of this contribution in the case of fermion currents. The magnitude of the fixed pole is given by a finite integral over the deep-inelastic structure function $\nu W_2(x)$.[4,5]

The outline of this paper is as follows. In Sec. I we present an extremely simple derivation of the LPS nonperturbative covariant model based on time-ordered perturbation theory in an infinite-momentum reference frame. The Regge behavior of the scaling structure function is demonstrated. In Sec. II perturbation-theory calculations in scalar and fermion electrodynamics are presented which are particularly instructive for demonstrating the close correspondence between Bjorken scaling, $J=0$ fixed-pole behavior, and the requirements of gauge invariance. The necessity for covariant regularization is pointed out. Following this preparation we derive in Sec. III a finite gauge-invariant nonperturbative model, with general Regge behavior. A complete discussion and calculation of the analytic behavior of the Compton amplitude in this scaling theory is then presented. In Appendix A, a complete summary of fixed-pole sum rules for electromagnetic and weak processes is presented. In Appendix B, a general connection between the explicitly covariant and infinite-momentum techniques is discussed. Finally, in Appendix C, a discussion of mass renormalization for electromagnetic self-energy corrections of hadrons is presented.

I. THE NONPERTURBATIVE PARTON MODEL

In this section we present a simple derivation of the Landshoff, Polkinghorne, and Short covariant parton model,[2] based on time-ordered perturbation theory in an infinite-momentum reference frame.[6] The general procedure is to relate the analytic behavior of a parton-proton scattering amplitude to the proton structure function $\nu W_2(x)$. We first derive this relationship in lowest-order perturbation theory. The generalization to any order in perturbation theory will then be immediate. The term "parton" refers to the elementary carrier of the electromagnetic current within the hadrons: At time $t=0$ in the interaction picture, the current is a superposition of the free currents of these fields.

Thus, first consider the u-channel contribution to the parton-proton scattering amplitude from gluon exchange in lowest-order perturbation theory (see Fig. 1). All particles are taken as scalars.

It will be convenient to use the following Lorentz reference frame:

$$p=\left(P+\frac{M^2}{2P}\,,\vec{0}_\perp\,,P\right),$$
$$k'=\left(xP+\frac{\vec{k}_\perp{}^2+\mu_0{}^2}{2xP}\,,\vec{k}_\perp,xP\right), \tag{I.1}$$

where $0<x<1$, and we will eventually take $P\to\infty$. In terms of the variables $\vec{k}_\perp{}^2$ and x, we have

$$u\equiv(p-k')^2$$
$$=M^2+\mu_0{}^2-xM^2-\frac{\vec{k}_\perp{}^2+\mu_0{}^2}{x}+O\left(\frac{1}{P^2}\right),$$
$$u-\lambda^2=(1-x)\left(M^2-\frac{\vec{k}_\perp{}^2+\mu_0{}^2}{x}-\frac{\vec{k}_\perp{}^2+\lambda^2}{1-x}\right). \tag{I.2}$$

Using time-ordered perturbation theory, the time ordering of Fig. 1 gives

$$\mathfrak{M}_u=\frac{g^2}{(2\pi)^3}\frac{1}{2E_\lambda}\,\frac{1}{(E_p+E_k)-(E_k+E_{k'}+E_\lambda)+i\epsilon}$$
$$\xrightarrow[P\to\infty]{}\frac{g^2}{(2\pi)^3}\,\frac{1}{u-\lambda^2+i\epsilon}\,, \tag{I.3}$$

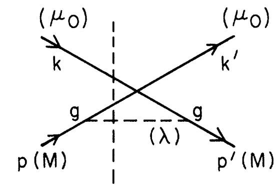

FIG. 1. Time-ordered perturbation-theory contribution to the parton-proton scattering amplitude. The parton line has mass μ_0.

in agreement with the covariant result since the contribution of the other time-ordering vanishes in order $1/P^2$.

To this same order in perturbation theory, the proton form factor at $q^2=0$ is (see Sec. II and Ref. 4)

$$1=F_1(0)$$
$$=Z_2+\frac{g^2}{(2\pi)^3}\int d^2k_\perp\int_0^1\frac{dx(1-x)}{2x}\,\frac{1}{(u-\lambda^2)^2}\,, \tag{I.4}$$

where Z_2 is the wave-function renormalization constant in second order. Here the surviving contribution for $P\to\infty$ comes from the time ordering shown in Fig. 2. The two energy denominators are the same as that appearing in $\mathfrak{M}_u$.

It is convenient to define a normalized distribution function $f(x)$:

$$F_1(0)=1=\int_0^1 f(x)dx\,, \tag{I.5}$$

with

$$f(x)=\int d^2k_\perp\frac{(1-x)}{2x}\,\frac{\mathfrak{M}_u}{(u-\lambda^2)}+Z_2\delta(1-x)\,. \tag{I.6}$$

In fact, as shown in Sec. II, we can identify the Bjorken scaling function $\nu W_2(x)=xf(x)$, $x=-q^2/2M\nu$. Note that $f(x)$ has one extra energy denominator beyond that of the parton-proton amplitude.

We now generalize the parton-proton amplitude to include the full complexities of a Reggeized hadronic amplitude. One can in fact show that if proton-proton scattering has Regge behavior, then

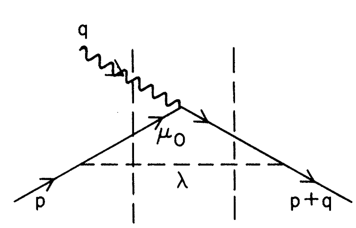

FIG. 2. Time-ordered perturbation theory contribution to the proton elastic form factor. This is the only time-ordering surviving at $P\to\infty$ in the reference frame defined in Eqs. (II.2) and (II.5).

one cannot avoid having Regge behavior in the parton-proton amplitude. We thus write

$$(2\pi)^3 \mathfrak{M}_u = \int \frac{\rho(m^2, \mu_0{}^2) dm^2}{u - m^2 + i\epsilon} - [\text{subtraction terms}], \tag{I.7}$$

where $\pi\rho$ is the imaginary part of the forward (anti) parton-proton scattering amplitude. The spectral-sum variable m^2 replaces λ^2 of the perturbation result. In the case of a Regge contribution, $\rho \propto (m^2)^\alpha$, $0 < \alpha < 1$, a subtraction term is of course required in order that the expression for $\mathfrak{M}_u$ be finite. As we shall see, the subtraction term does not enter the calculation of $\nu W_2(x)$ or the form factor, but it is essential in obtaining finite results for sum rules. The corresponding result for $f(x)$ is then

$$\frac{\nu W_2(x)}{x} = f(x) = \int d^2k_\perp \int dm^2 \frac{(1-x)}{2x} \frac{\rho(m^2, \mu^2)}{(u - m^2 + i\epsilon)^2} + Z_2 \delta(1-x). \tag{I.8}$$

Again $f(x)$ has one extra energy denominator beyond that of $\mathfrak{M}_u$. In the case of a composite hadron, one may take $Z_2 = 0$, corresponding to the absence of direct interactions of the photon with the proton.

As indicated in Eq. (I.8), one must in general take into account the off-shell dependence of the forward parton-proton amplitude when imbedded in the interior of a general amplitude. The usual Feynman off-shell variable μ^2 corresponds in time-ordered perturbation theory to the invariant four-momentum squared of the particle computed taking the energy component from energy conservation (in addition to the usual three-momentum conservation):

$$p - p_m = \left(xP + \frac{M^2(1-x) - m^2 - \vec{k}_\perp{}^2}{2(1-x)P}, \vec{k}_\perp, xP \right),$$

$$\mu^2 = (p - p_m)^2 = \frac{x(1-x)M^2 - xm^2 - \vec{k}_\perp{}^2}{1-x}, \tag{I.9}$$

or

$$\mu^2 - \mu_0{}^2 = \frac{x}{1-x}(u - m^2).$$

In general, this off-shell dependence ensures strong convergence of the $\vec{k}_\perp{}^2$ integrations, and the existence of Bjorken scaling—even in the case of spin-$\frac{1}{2}$ theories. The convergence occurs in the variable $\vec{k}_\perp{}^2/(1-x)$—corresponding to the covariant variable μ^2—rather than in $\vec{k}_\perp{}^2$ alone. In the Drell and Lee theory the convergence in μ^2 is a natural consequence of a bound-state Bethe-Salpeter model. In any case, it is a natural assumption for the off-shell behavior of a hadronic amplitude.

If we absorb into $\rho(m^2, \mu^2)$ two Feynman propagators, and define

$$\operatorname{Im} T(m^2, \mu^2) = \frac{\pi \rho(m^2, \mu^2)}{(\mu^2 - \mu_0{}^2)^2}, \tag{I.10}$$

then we obtain

$$f(x) = \frac{1}{\pi} \int d^2k_\perp \frac{x}{2(1-x)} \int dm^2 \operatorname{Im} T(m^2, \mu^2), \tag{I.11}$$

which is directly comparable to the LPS formula, Eq. (2.25). In our formula, one sums explicitly over both parton and antiparton, whereas LPS implicitly include antipartons via a crossed-channel contribution. An alternate identification of $f(x)$ in terms of composite wave functions of the proton defined in the infinite-momentum frame, and a discussion of the sufficient conditions for the Drell-Yan threshold theorem, are given in Ref. 5.

The general assumption of convergent off-shell behavior guarantees that the Bjorken scaling function $\nu W_2(x) = x f(x)$ comes solely from the "handbag" or "contiguous" diagrams shown in Fig. 3.

Perhaps the most important feature of the representation (I.11) is the natural way in which scaling-Regge behavior

$$f(x) \sim x^{-\alpha}, \quad x \to 0 \tag{I.12}$$

arises as a consequence of the hadronic Regge behavior of the parton-proton amplitude. For instance, if

$$\rho(m^2, \mu^2) = (m^2)^\alpha \beta(\mu^2) \tag{I.13}$$

one obtains (with $\zeta \equiv xm^2$)

$$f(x) = \tfrac{1}{2} x \int d^2k_\perp \int_0^\infty \frac{d\zeta\, \zeta^\alpha}{x^{\alpha+1}} \frac{1}{(1-x)} \frac{\beta(\mu^2)}{(\mu^2 - \mu_0{}^2)^2},$$

with

$$\mu^2 = \frac{x(1-x)M^2 - \zeta - \vec{k}_\perp{}^2}{1-x}. \tag{I.14}$$

Thus for small x,

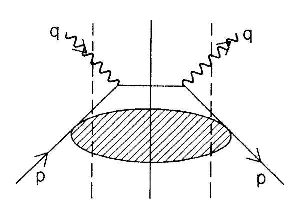

FIG. 3. The "handbag" or "$T^{(4)}$" contribution to the forward Compton amplitude.

$$f(x) \sim x^{-\alpha} \int d^2k_\perp \int_0^\infty d\zeta\, \zeta^\alpha \frac{\beta(-\zeta - \vec{k}_\perp{}^2)}{(\zeta + \vec{k}_\perp{}^2 + \mu_0{}^2)^2} \qquad \text{(I.15)}$$

and the integrals are convergent.

Thus, in this model, one includes hadronic interactions to all orders, and electromagnetic interactions to minimal order. In order to show explicitly how renormalization works and gauge invariance is satisfied in scaling models, we will present the results for the Compton amplitude to second order in g for scalar and covariantly regulated γ_5 spin-$\frac{1}{2}$ electrodynamics. The perturbation-theory calculations are done using time-ordered perturbation theory in an infinite-momentum frame since this provides the clearest treatment of the separation of the $J=0$ fixed pole (Kronecker δ) contribution and its relationship to Bjorken scaling. In Sec. III we construct a gauge-invariant nonperturbative model using a specific generalization of the lowest-order calculations. The derivations again result in the representation of Eq. (I.11) for $f(x)$, and illustrate the manner in which fixed-pole sum rules are made convergent in the presence of Regge behavior. In Appendix B, we give the direct connection between the explicitly covariant and infinite-momentum (or light-cone variable) techniques.

II. COMPTON AMPLITUDE IN PERTURBATION THEORY

As an example of the techniques and utility of time-ordered perturbation theory (TOPT) and the infinite-momentum frame[6] we shall review the illustrative ϕ^3 field theory (scalar electrodynamics), where the proton is a composite of charged scalar and neutral scalar particles. Despite the simplicity of this model, many of the results of the subsequent sections are already exhibited within this example, especially the scaling behavior of the virtual Compton amplitude and the presence of a $J=0$ fixed singularity.

In the last part of this section we show the analogous results for a field theory in which a spin-$\frac{1}{2}$ proton is composed of a spin-$\frac{1}{2}$ charged particle and a neutral pseudoscalar (γ_5 vertex coupling). This example forms a bridge between the scalar electrodynamics model discussed in detail here and the γ_5 model of Drell, Levy, and Yan,[1] who concentrated on the scaling behavior of the νW_2. Our work (Sec. III) shows how covariant off-shell convergence factors required for Bjorken scaling can be introduced within such models while preserving gauge invariance. Again we shall utilize the infinite-momentum method, since this technique conveniently isolates the fixed-pole behavior of spinor theories.

It should be emphasized that the final results of the infinite-momentum method are covariant; time-ordered perturbation theory in the infinite-momentum frame is a rigorous alternative to the usual Feynman rules.[7]

A. Scalar Electrodynamics

We first calculate for later use the second-order wave-function renormalization constant $Z_2 = (1-B)^{-1} \cong 1 + B_{(2)}$. The methods are essentially those of Drell, Levy, Yan,[1] and Weinberg.[6] From Fig. 4 one obtains

$$B_{(2)} = \frac{-g^2}{(2\pi)^3} \frac{1}{2E_p} \int \frac{d^3k_1}{2\omega_{k_1}} \frac{d^3l_1}{2\omega_{l_1}} \frac{\delta^3(\vec{l}_1 + \vec{k}_1 - \vec{q})}{(E_p - \omega_{k_1} - \omega_{l_1})^2}. \qquad \text{(II.1)}$$

We parameterize the momenta as follows:

$$\vec{p} = \vec{P}, \quad \vec{k}_1 = x\vec{P} + \vec{k}_\perp, \quad \vec{l}_1 = (1-x)\vec{P} - \vec{k}_\perp,$$
$$\vec{k}_\perp \cdot \vec{P} = 0, \quad E_p = P + \frac{M^2}{2P}, \quad \omega_{k_1} = |x|P + \frac{\vec{k}_\perp{}^2 + \mu^2}{2|x|P}, \qquad \text{(II.2)}$$
$$\omega_{l_1} = |(1-x)|P + \frac{\vec{k}_\perp{}^2 + \lambda^2}{2|1-x|P}.$$

Then for $P \to \infty$,

$$B_{(2)} = \frac{-g^2}{(2\pi)^3} \int_0^1 dx \int \frac{d^2k_\perp}{2} \frac{x(1-x)}{D^2(\vec{k}_\perp, x)}, \qquad \text{(II.3)}$$

where

$$D(\vec{k}_\perp, x) = 2Px(1-x)(\omega_{k_1} + \omega_{l_1} - E_p)$$
$$= \vec{k}_\perp{}^2 + x\lambda^2 + (1-x)\mu^2 - x(1-x)M^2.$$

Note that the energy denominator $E_p - \omega_{k_1} - \omega_{l_1}$ is of order $(1/P)$ if the intermediate particles are moving forward relative to $\vec{P}$ (i.e., $0<x<1$), and of order P otherwise. In ϕ^3 theory there is no possibility of introducing compensating powers of P into the numerator (unlike spinor theory), and thus in the limit $P\to\infty$, only the region $0<x<1$ contributes.

The elastic form factor of a spin-0 particle, $F(q^2)$, is defined by

$$\langle p' | J_\mu(0) | p\rangle = \frac{1}{(2\pi)^3} \frac{1}{2E_{p'}} \frac{1}{2E_p} F(q^2)(p+p')_\mu. \qquad \text{(II.4)}$$

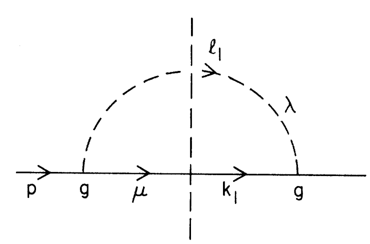

FIG. 4. Time-ordered perturbation-theory contribution to wave-function renormalization.

The diagrams which contribute to $F(q^2)$ through order g^2 in the scalar exchange interaction are shown in Fig. 5. Of the six time-ordered contributions to the order-g^2 Feynman amplitude, only the contribution of Fig. 5(b) survives for $P\to\infty$ if we choose the Lorentz frame such that

$$q=\left(\frac{q\cdot p}{P},\vec{q}_\perp,0\right), \tag{II.5}$$

where

$$q^2=-\vec{q}_\perp^{\,2}.$$

(For the elastic form factor, $2q\cdot p\equiv 2M\nu=-q^2$.) In each of the other diagrams at least one intermediate particle must be moving backwards by three-momentum conservation and may be neglected to order $1/P^2$. Using the parameterization

$$\begin{aligned}\vec{k}_2&=x\vec{P}+(\vec{k}_\perp+\vec{q}_\perp),\\ \omega_{k_2}&=xP+\frac{(\vec{k}_\perp+\vec{q}_\perp)^2+\mu^2}{2xP},\end{aligned} \tag{II.6}$$

$$\langle p'|J_\mu(0)|p\rangle=\frac{1}{2E_p}\frac{1}{2E_{p'}}\frac{g^2}{(2\pi)^3}\times\int\frac{d^2k_\perp dx\,P(2P)^2}{2\omega_{k_1}2\omega_{k_2}2\omega_{l_1}}\frac{(2k+q)_\mu}{AA'}, \tag{II.7}$$

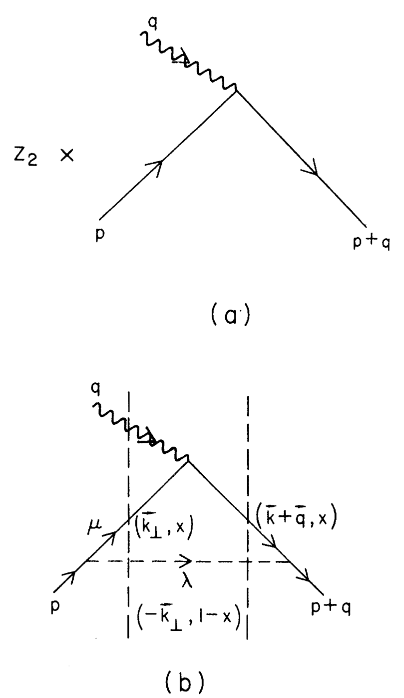

FIG. 5. Time-ordered perturbation theory contributions to the elastic form factor corresponding to Eq. (II.9).

where

$$\begin{aligned}A&=2P(E_p-\omega_{k_1}-\omega_{l_1})\\ &=\frac{-D(\vec{k}_\perp,x)}{x(1-x)},\\ A'&=2P(E_p+E_q-\omega_{k_2}-\omega_{l_1})\\ &=\frac{-D(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)}{x(1-x)}.\end{aligned} \tag{II.8}$$

By examining the $\mu=0$ components we obtain

$$F(q^2)=1+B_{(2)}+\frac{g^2}{2(2\pi)^3}\int_0^1dx\int d^2\vec{k}_\perp\frac{x(1-x)}{D(\vec{k}_\perp)D(\vec{k}_\perp+(1-x)\vec{q}_\perp)}. \tag{II.9}$$

As $\vec{q}_\perp\to 0$ (i.e., $q^2\to 0$) we see that $F(q^2)\to 1$. That is, $L_{(2)}$, the proper vertex contribution at $q^2=0$, is equal to $-B_{(2)}$. This, of course, is a requirement of any gauge-invariant theory or, equivalently, a consequence of the Ward identity. We define for future use the function $f(x)$, which to order g^2 is

$$f(x)=Z_2\delta(1-x)+\frac{g^2}{2(2\pi)^3}\int d^2k_\perp\frac{x(1-x)}{D^2(\vec{k}_\perp)} \tag{II.10}$$

so that $1=F(0)=\int_0^1 f(x)dx$. This is a special case of the results quoted in Sec. I. It is clear from the definition of the variable x that $f(x)$ is the fractional longitudinal momentum distribution function for the charged particle as seen in the infinite-momentum frame. In fact, by using time-ordered perturbation theory, it is clear that a normalized distribution function can be defined to any order in perturbation theory. In general a distribution function $f_a(x)$ can be defined for each type of charged constituent a within the hadron. Then

$$F(0)=\sum_{a,\bar{a}}\lambda_a\int_0^1dx\,f_a(x).$$

Turning now to the forward virtual Compton amplitude (spin averaged) we will calculate $T_1(q^2,\nu)$ and $T_2(q^2,\nu)$, where

$$T_{\mu\nu}=-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)T_1+\left(p_\mu-\frac{p\cdot qq_\mu}{q^2}\right)\left(p_\nu-\frac{p\cdot qq_\nu}{q^2}\right)\frac{T_2}{M^2}. \tag{II.11}$$

In the infinite-momentum frame previously defined [Eqs. (II.2) and (II.5)],

$$T_{00}\to\frac{P^2T_2}{M^2}, \tag{II.12}$$

while, if the componets $\mu,\nu=i$ are chosen orthogonal to $\vec{P}$ and $\vec{q}_\perp$, then

$$T_{ii} = -g_{ii}\, T_1 = T_1 \,. \tag{II.13}$$

Using the above definition of T_1 and T_2 we see that the Born contributions turn out to be

$$T_1^{\text{Born}} = +2, \quad T_2^{\text{Born}} = -\frac{8q^2M^2}{(2M\nu)^2 - q^4}\,, \tag{II.14}$$

and

$$\lim_{q^2\to 0} T_1^{\text{Born}} + \frac{\nu^2}{q^2}\, T_2^{\text{Born}} \to 0 \tag{II.15}$$

as required by analyticity of $T_{\mu\nu}$ in q^2.

To second order in g^2 the only surviving time-ordered diagrams contributing to the Compton amplitudes in the $P\to\infty$ limit are shown in Fig. 6. Using transverse components, the only contributions to T_1 arise from the "seagull"[(e) and (b)] and "handbag"[(c)] diagrams. The Born-type contributions (e) yield $Z_2 \times T_1^{\text{Born}}$. The seagull contribution (b) is calculated in close analogy with the form-factor calculation for $\vec{q}_\perp = 0$, the only difference being that the seagull "current" is proportional to $-2g_{\mu\nu}$ rather than $x2P$. Thus we immediately have

$$T_1^{\text{seagull}} = 2\int_0^1 \frac{f(x)}{x}\, dx \,. \tag{II.16}$$

We note that because of this analogy the dependence on t (of the Compton amplitude) for the seagull contribution is like that of the form factor. For diagrams (c), the outside two energy denominators are equal and proportional to $D(\vec{k}_\perp)$, whereas the middle ones are

$$\frac{1}{2P}\left[M^2 \pm 2M\nu - \frac{(\vec{k}_\perp \pm \vec{q}_\perp)^2 + \mu^2}{x} - \frac{\lambda^2 + k_\perp{}^2}{1-x}\right] \equiv \frac{-1}{2Px(1-x)}\, D'_\pm \,. \tag{II.17}$$

Including the contribution of (c) to T_1 we obtain

$$T_1(\nu, q^2) = 2\left\{ Z_2 + \frac{g^2}{2(2\pi)^3}\int d^2k_\perp \int_0^1 \frac{dx(1-x)}{D^2}\left[1 - \sum_{\pm\nu} \frac{2k_i{}^2(1-x)}{[\underline{D} + x(1-x)(\vec{q}_\perp{}^2 - 2M\nu) - i\epsilon]}\right]\right\}, \tag{II.18}$$

where k_i is the component of $\vec{k}_\perp$ perpendicular to $\vec{q}_\perp$, and we have defined

$$\underline{D} \equiv D(\vec{k}_\perp + (1-x)\vec{q}_\perp)\,, \quad D = D(\vec{k}_\perp).$$

For $q^2 = 0$, $\nu \to 0$, we can use the identity

$$\int d^2k_\perp \frac{4k_i{}^2(1-x)}{D^3} = \int d^2k_\perp \frac{2\vec{k}_\perp{}^2(1-x)}{D^3} = \int \frac{d^2k_\perp(1-x)}{D^2} \tag{II.19}$$

and the Ward identity $B_{(2)} = -L_{(2)}$ to verify the Thomson limit

$$\lim_{\nu\to 0} T_1(\nu, 0) = 2\,. \tag{II.20}$$

The integration by parts in $\vec{k}_\perp{}^2$ is essential here: If there had been an arbitrary cutoff in the $\vec{k}_\perp{}^2$ integration, then a surface term would be introduced and the low-energy theorem would fail.

At large energies only the seagull contributions of Figs. 6(b) and 6(f) survive because of the additional ν-dependent denominator in the handbag diagram. Thus

$$\lim_{\nu\to\infty} T_1(\nu, q^2) = 2\left[Z_2 + \frac{g^2}{2(2\pi)^3}\int d^2k_\perp \int \frac{dx(1-x)}{D^2}\right] = T_1^{\text{Born}} \int_0^1 dx \frac{f(x)}{x}\,. \tag{II.21}$$

This result will be generalized in later sections. Thus in the (coherent) impulse approximation ($\nu \gg$ binding energy), the Compton amplitude exhibits

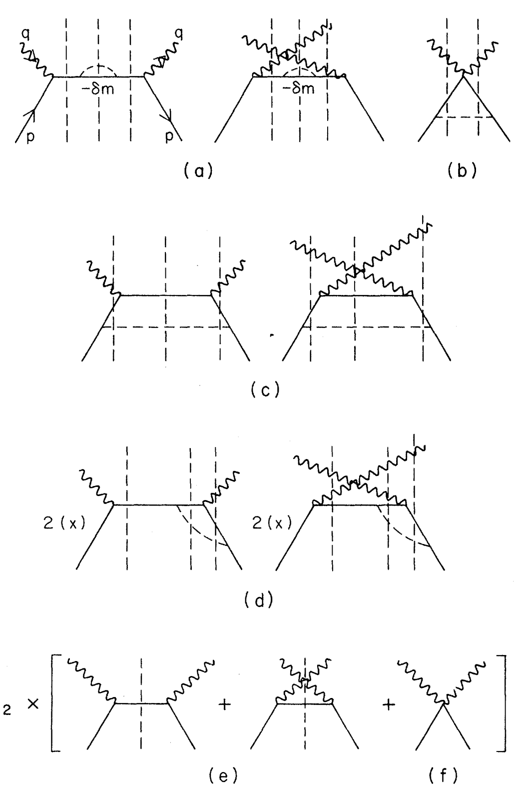

FIG. 6. Time-ordered perturbation theory contribution to the forward virtual Compton amplitude for spin-0 constituents. See Eq. (II.16) through (II.24).

an energy-independent, q^2-independent constant real term in the $g_{\mu\nu}$ amplitude. In complex J language, this is a $J=0$ Kronecker δ with t-independent energy dependence ν^0. We also note that the seagull contribution to the general Compton amplitude $T_{\mu\nu}(\nu, t, q_1^2, q_2^2)$ is independent of either proton mass at fixed t. The t dependence of this contribution is similar to that of the elastic form factor $F(t)$. These features all reflect the locality of the two-photon seagull interaction, and are consequences of the pointlike nature of the parton couplings.

The result of Eq. (II.21) is entirely analogous to that obtained for the Compton amplitude in atomic or nuclear physics for $\nu \gg BE$ (but below strong-interaction thresholds). It corresponds to Thomson scattering on the elementary constituents with effective mass $m_{\text{eff}}^{-1} = m_{\text{tot}}^{-1}\langle 1/x\rangle$. Further discussion may be found in Refs. 4 and 5.

The calculation of the amplitude T_2 (by examining T_{00}) proceeds in analogous fashion. The only new feature is that in the case of Fig. 6(a), one must perform mass renormalization. The net result for diagram (a) is

$$\frac{-1}{(2M\nu-\vec{q}_\perp^{\,2})^2}\,\frac{g^2}{2(2\pi)^3}\int d^2k_\perp \int_0^1 dx\left(\frac{1}{D'_+}-\frac{1}{D}\right)+(\nu\to-\nu); \tag{II.22}$$

the D^{-1} term is the mass-renormalization counterterm. Although the subtracted form is finite, the individual terms must be defined using a covariant regularization procedure—e.g., a Pauli-Villars negative-metric regulator. This allows us to replace $D^{-1}\to \underline{D}^{-1}$ in the mass subtraction term. This will be essential in obtaining the final forms. The other contributing figures are 6(c), 6(d), and 6(f). Figure 6(c) and the two diagrams of type (d) give

$$\frac{-g^2}{2(2\pi)^3}\int d^2k_\perp \int_0^1 dx\left[\frac{x^2(1-x)^2}{D^2D'_+}-\frac{2}{2M\nu-q_\perp^2}\,\frac{x(1-x)}{D'_+D}\right]+(\nu\to-\nu). \tag{II.23}$$

Adding in the Born contributions [Fig. 6(f)] and using the above replacement for the mass subtraction term, one obtains after rearrangement

$$\begin{aligned} T_2(\nu,q^2) &= \frac{4M^2}{2M\nu-q_\perp^2}\left[\frac{1}{1-B_{(2)}}+\frac{g^2}{2(2\pi)^3}\int d^2k_\perp\int_0^1 \frac{dx\,x(1-x)}{\underline{D}}\left(\frac{2}{D}-\frac{1}{\underline{D}}\right)\right] \\ &\quad\times \frac{-g^2}{2(2\pi)^3}4M^2\int d^2k_\perp \int_0^1 \frac{x^2(1-x)^2}{D'_+}\left(\frac{1}{D}-\frac{1}{\underline{D}}\right)^2+\nu\to-\nu. \end{aligned} \tag{II.24}$$

Of primary concern is the check of gauge invariance:

$$\lim_{q^2\to 0}\frac{\nu^2}{q^2}T_2+T_1=0. \tag{II.25}$$

For T_2 as $q_\perp^2\to 0$, one has from the first term of Eq. (II.24)

$$\frac{8q_\perp^2M^2}{4M^2\nu^4-q_\perp^4}\left[\frac{1}{1-B_{(2)}}+\frac{g^2}{16\pi^3}\int d^2k_\perp\int_0^1\frac{dx(1-x)x}{D^2}\right]+O(q_\perp^4)=\left[\frac{1}{1-B_{(2)}}+L_{(2)}\right]T_2^{\text{Born}}=\frac{-q^2}{\nu^2}T_1^{\text{Born}}. \tag{II.26}$$

The remainder reduces, using

$$\left[\frac{1}{D}-\frac{1}{\underline{D}}\right]^2=\frac{[2(1-x)\vec{k}_\perp\cdot\vec{q}_\perp]^2}{D^4}+O(q_\perp^4)\approx\frac{2(1-x)^2k_\perp^2q_\perp^2}{D^4}, \tag{II.27}$$

to

$$\frac{\nu^2}{q^2}T_2^{\text{non-Born}}=8M^2\nu^2\frac{g^2}{2(2\pi)^3}\int d^2k_\perp\int_0^1 dx\,\frac{x^2(1-x)^4 2k_\perp^2}{D^3[D^2-x^2(1-x)^2 4M^2\nu^2]}. \tag{II.28}$$

Writing T_1 as $T_1=T_1^{\text{Born}}+T_1^{\text{non-Born}}$ we have (at $q_\perp^2=0$)

$$\begin{aligned} T_1^{\text{non-Born}}(\vec{q}_\perp^{\,2}=0,\nu) &= 2\frac{g^2}{2(2\pi)^3}\int d^2k_\perp\int_0^1\frac{dx(1-x)}{D^2}\left[\frac{(1-x)}{D}2k_\perp^2-\sum_{\pm\nu}\frac{k_\perp^2(1-x)}{D+x(1-x)(-2M\nu)-i\epsilon}\right] \\ &= -2\frac{g^2}{2(2\pi)^3}\int d^2k_\perp\int_0^1\frac{dx(1-x)^4x^2 8M^2\nu^2x^2k_\perp^2}{D^3[D^2-4M^2\nu^2x^2(1-x)^2]}, \end{aligned} \tag{II.29}$$

which explicitly cancels $(\nu^2/q^2)T_2^{\text{non-Born}}$.

Of course, at $\nu \to 0$ only the Born contribution survives in T_1 and T_2 and the correct Thomson limit is obtained.

In the limit $\nu \to \infty$, $q_\perp^2$ finite, straightforward algebra yields

$$\lim_{\nu\to\infty} \nu T_2(q_\perp^2, \nu) = -\frac{q^2}{\nu} T_1^{\text{Born}} \int_0^1 \frac{dx}{x} f(x). \tag{II.30}$$

In complex-J-plane language this is a fixed pole with $\alpha = 0$ ($\nu T_2 \sim \nu^{\alpha-1}$), with residue linear in q^2.

Finally we consider the scaling region $\nu, \vec{q}_\perp^2 \to \infty$ with $\omega = 2M\nu/q_\perp^2$ fixed (Bjorken limit). The $k_\perp^2$ integral here is sufficiently convergent such that the standard limiting procedure is valid. We obtain

$$T_2(\nu, q_\perp^2) \xrightarrow[\text{Bj}]{} \frac{1}{1-B_{(2)}} \frac{4M^2}{2M\nu - q_\perp^2 + i\epsilon} + \frac{g^2}{2(2\pi)^3} \times \int_0^1 d^2k_\perp \int \frac{dx\, x(1-x)}{D^2} \frac{4M^2}{(2M\nu - q_\perp^2/x + i\epsilon)} \tag{II.31}$$

so that defining

$$\nu W_2 \equiv \frac{-1}{2\pi M} \operatorname{Im} \nu T_2 , \tag{II.32}$$

we have

$$\nu W_2(\nu, q_\perp^2) \xrightarrow[\text{Bj}]{} \int_0^1 dx f(x) x \delta\left(x - \frac{1}{\omega}\right) = \frac{1}{\omega} f\left(\frac{1}{\omega}\right). \tag{II.33}$$

The results given above in terms of $f(x)$, i.e.,

$$F(0) = \int_0^1 f(x) dx, \quad \nu W_2\left(x = \frac{1}{\omega}\right) = x f(x) , \tag{II.34}$$

and

$$\begin{aligned} T_1^{\text{FP}} &= \frac{\nu^2}{q^2} T_2^{\text{FP}} \\ &= T_1^{\text{Born}} \int_0^1 \frac{dx}{x} f(x) \\ &= T_1^{\text{Born}} \int_0^1 \frac{\nu W_2(x) dx}{x^2} \end{aligned} \tag{II.35}$$

are clearly valid to any finite order in ϕ^3 perturbation theory. The scaling result only depends upon the convergence of the $k_\perp^2$ integrations. In higher order we may define $f(x)$ from the infinite-momentum-frame time-ordered perturbation calculation of the form factor. All the matrix elements of interest are variations on the vertex operator in x space, i.e.,

(a) $x\lambda_a$ for the form factor,

(b) $2\lambda_a^2$ for the fixed pole in T_1,

(c) $x^2\lambda_a^2 \delta\left(x - \frac{1}{\omega}\right)$ for $\nu W_2(\omega)$

(λ_a = charge of parton a, all charges to be summed over); an effective local operator can clearly be derived in any case in which the impulse approximation applies between the times of emission and absorption of a given constituent, e.g., arbitrary currents acting at lightlike separation. $f(x)$ gives the hadronic emission matrix element for each constituent over which the above effective operators must be integrated, and as such is the unifying link between a large number of theoretically interesting quantities.

The renormalization procedure may be carried out to any finite order in perturbation theory in a straightforward fashion. The explicit occurrences of the wave-function renormalization factors Z_2^p and Z_2^a are shown in Fig. 7, for any of the above three effective operators. The wave-function renormalization of the parton propagating between the photons cancels for νW_2 in the scaling region.[1] We define the renormalized parton-proton scattering amplitude as usual as

$$T_R = Z_2^a Z_2^p T_{\text{unrenormalized}} . \tag{II.36}$$

Thus from Fig. 7(b) we see that $f(x)$ implicitly contains a factor Z_2^a when expressed in terms of renormalized parton-hadron scattering amplitude T_R [see Eq. (I.11)]. Thus $Z_2^a \neq 0$ for a scaling theory, and the parton constituents cannot be composite.[2]

We now turn to the analogous results for spin-$\frac{1}{2}$ perturbation theory, in which the proton consists of a spin-$\frac{1}{2}$ charged particle of mass μ and a neutral pseudoscalar of mass λ. We may compute the F_1 form factor trivially using the "good" $\mu = 0$ component of the current, and the same choice of Lorentz frame used above [(II.2) and (II.5)]. Then only one time-ordered diagram contributes and we obtain as in DLY[1]

$$F_1(0) = 1 = \int_0^1 f(x) dx , \tag{II.37a}$$

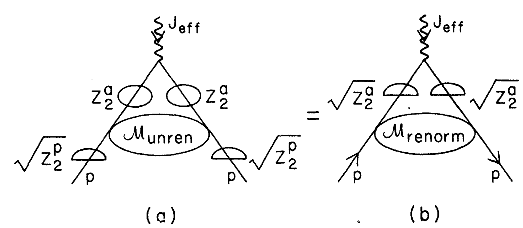

FIG. 7. Renormalization of vertex operators in the parton model. The occurrence of wave-function renormalization factors Z_2 and $\sqrt{Z_2}$ are represented by full and half circles, respectively.

where

$$f(x)=\frac{g^2}{2(2\pi)^3}\int\frac{d^2k_\perp x(1-x)\{[\vec{k}_\perp^{\,2}+(\mu-xM)^2]/x\}}{D^2(k_\perp,x)}\,. \tag{II.37b}$$

The denominator factor $D(k_\perp, x)$ is defined as before. The numerator bracket is the factor $2(p\cdot p_1-M\nu)$, which comes from the spin-averaged pseudoscalar coupling. When divergent, the $d\vec{k}_\perp^{\,2}$ integration will be temporarily defined in this and all subsequent formulas of this section by means of simple covariant regularization—either by a spectral condition on the mass λ^2 or specifically Pauli-Villars negative-metric subtraction in this mass. In the nonperturbative model discussed in Sec. III, this unphysical but gauge-invariant regularization is replaced by the assumption of strong off-shell convergence in the parton-proton scattering amplitude.

For the purpose of gauge invariance checks we will only need to calculate T_1 for $q^2=0$ and arbitrary ν. In Fig. 7 we show the contributing graphs to $T_{ii}(\nu,0)$ in order g^2 and the corresponding time-ordered graphs which survive in the limit $P\to\infty$. Here i refers to any component perpendicular to the 0 and $\vec{P}$ directions. Because of the Gordon identity, a graph in which a photon is attached to an external leg will not contribute unless it also connects to a backward-moving spinor. As first emphasized by DLY, intermediate states with backward-moving fermions can contribute in the $P\to\infty$ limit, since the numerator algebra can compensate for the two powers of P of the "bad" denominators. [A method for automatically including the contribution of Z graphs is given in Ref. 7. Here it is useful to exhibit them explicitly.] The complete result for $T_1(\nu,0)$ is

$$T_1(\nu,0)=\frac{g^2}{2(2\pi)^3}\int d^2k_\perp\int_0^1 dx\left\{\frac{x}{D_+}-(2)\frac{2}{D_+}-(2)\frac{2(1-x)\vec{k}_\perp^{\,2}}{DD_+}+2(1-x)^2\frac{\vec{k}_\perp^{\,2}S}{D^2D_+}+\frac{1}{xD_+}+\frac{(1-x)S}{D^2}+\frac{2(1-x)}{x}\frac{\vec{k}_\perp^{\,2}}{DD_+}\right\}+(\nu\to-\nu)+Z_2T_1^{\rm Born}\,, \tag{II.38}$$

where

$$D_+=D(\vec{k}_\perp^{\,2},x)+2M\nu x(1-x) \tag{II.39}$$

and

$$S=\frac{\vec{k}_\perp^{\,2}+(\mu-xM)^2}{x}\,. \tag{II.40}$$

The terms in the curly brackets for T_1 correspond to the contributions of (S_1), $(2)(V_1)$, $(2)(V_2)$, (H_1), (H_2), (H_3), (H_4+H_5), respectively, as shown in Fig. 8.

As in the ϕ^3 case, integration by parts in $\vec{k}_\perp^{\,2}$ is crucial in ascertaining that

$$\lim_{\nu\to\infty} T_1(\nu,0)=2,$$

the Thomson limit. For $\nu\to\infty$, only the Z contribution H_3 survives. This is true as well for $q^2\neq 0$, since the numerator traces do not depend on ν, but only on $\vec{q}_\perp$; it is thus not possible to compensate for the denominators which increase with ν. It is this feature which makes time-ordered perturbation theory so useful for extracting the energy-independent contribution to T_1. We thus obtain

$$\lim_{\nu\to\infty} T_1(\nu,q^2)=T_1^{\rm Born}\int_0^1\frac{f(x)}{x}\,dx\,. \tag{II.41}$$

Note that Z-graph H_3 takes the place of the seagull contribution of scalar electrodynamics. Because of the effective local coupling of both the Z-graph and the seagull contributions, this energy-independent contribution to the virtual Compton amplitude $T_{\mu\nu}(q_1^2, q_2^2,\nu,t)$ is independent of either photon mass q_1^2, q_2^2 at fixed t. Experimental implications of this remarkable behavior have been discussed in Ref. 4.

We will now calculate the T_2 amplitude for all ν and q^2. It is easiest to evaluate T_2 by examining the $\mu=0$, $\nu=0$ component of $T_{\mu\nu}$, since the resulting currents cannot reverse the direction of a fermion line—the "good" current rule of DLY.[1] Thus only the time-ordered diagrams shown in Fig. 8 contribute. The contributing terms in the amplitude must be proportional to P^2, which results in a considerable simplification of the algebra. The result is

$$T_2(q^2,\nu)=\frac{g^2}{2(2\pi)^3}\int d^2k\int_0^1 dx\left\{\frac{(\vec{q}_\perp^{\,2}-2M\nu)(1-x)+\lambda^2-(M-\mu)^2}{D_+'H_+^2}-\frac{\lambda^2-(M-\mu)^2}{DH_+^2}+(2)\frac{[Sx(1-x)+\vec{k}_\perp\cdot\vec{q}_\perp(1-x)]}{DD_+'H_+}-\frac{x^2(1-x)^2S}{D^2D_+'}\right\}+(\nu\to-\nu,\vec{q}_\perp\to-\vec{q}_\perp)+Z_2T_2^{\rm Born}\,, \tag{II.42}$$

where

$$H_+ = 2M\nu - \vec{q}_\perp^{\,2} + i\epsilon,$$

$$D_+' = D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) + (\vec{q}_\perp^{\,2} - 2M\nu)x(1-x) - i\epsilon\,,$$

$$T_2^{\text{Born}} = \frac{4M^2}{H_+} + (\nu \to -\nu)\,. \tag{II.43}$$

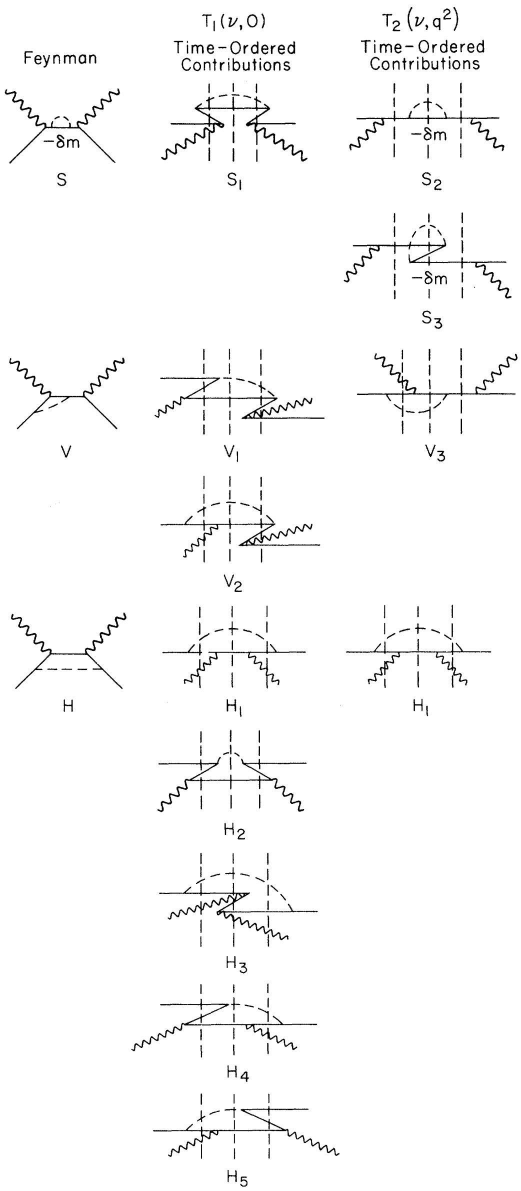

FIG. 8. Time-ordered perturbation theory contribution to the forward Compton amplitudes in the case of spin-$\frac{1}{2}$ constituents. The first column shows the total covariant Feynman amplitude. The corresponding time-orderings for $T_1(\nu, 0)$ and $T_2(\nu, q^2)$ surviving at $P\to\infty$ are shown in the second and third columns, respectively. [See Eqs. (II.38) and (II.42).] The Z-graph contribution H_3, the origin of the $J=0$ fixed singularity, reduces to a seagull-like contribution [see Fig. 6(b)] at $P\to\infty$.

The terms in the curly brackets for T_2 in (II.42) correspond to (S_2), (δm: S_2), (V_3), (H_1), respectively, as shown in Fig. 8.

Upon regularization, the δm subtraction for S_3 in fact cancels the S_3 contribution. As in the ϕ^3 calculation, this is made explicit by the ability to shift the $\vec{k}_\perp$ integration in S_3. After some algebra, one verifies the threshold constraint

$$\lim_{q^2\to 0}\; T_1(q^2, \nu) + \frac{q^2}{\nu^2} T_2(q^2, \nu) = 0,$$

and the Thomson limit. The $J=0$ fixed pole agrees with Eq. (II.35), with $f(x)$ defined from Eq. (II.37). Finally, the Bjorken scaling limit is

$$\lim_{\nu\to\infty,\,\omega = 2M\nu/\vec{q}_\perp^{\,2}\text{ fixed}} \nu W_2(\nu, q^2) = x f(x)\big|_{x=1/\omega}\,.$$

As stated before, these results in terms of $f(x)$ are more general than these specific perturbation-theory examples. When summed to all orders in perturbation theory, the fixed-pole sum rule may formally diverge at $x\sim 0$ due to Regge behavior $f(x)\sim x^{-\alpha}$, $0<\alpha<1$, but in actual fact, subtraction terms automatically arise which keep the sum rule finite. The mechanism for this and a full treatment of a nonperturbative model are given in Sec. III.

III. A FINITE GAUGE-INVARIANT NONPERTURBATIVE MODEL

In this section we consider a simple nonperturbative parton model for electromagnetic processes which has the following features:

(a) It is gauge-invariant by construction.

(b) It is explicitly covariant.

(c) It contains the off-shell suppression required to obtain scaling for deep-inelastic e-p scattering.

(d) It contains a proper treatment of Regge behavior in the parton model including the crucial role of subtraction terms.

(e) It yields a polynomial-residue fixed pole whose magnitude is given by a finite integral over the $\nu W_2(x)$ deep-inelastic structure function.

Further, the model can be generalized for any of the spin-dependent or spin-independent sum rules (see Appendix A). It can be employed as a theoretical laboratory for checking results based on light-cone dominance or parton-model intuition.

This model can be regarded as a gauge-invariant extension of the Landshoff-Polkinghorne-Short nonperturbative model.[2] The results can also be obtained from an infinite-momentum-frame OFPT (old-fashioned perturbation theory) approach, with covariant regularization.

The basic starting point for our model is a representation of the parton-proton forward scattering amplitude, Fig. 9(a), which is assumed to have the normal analytic features of a hadronic amplitude.

We write the off-shell forward amplitude as [$\mu^2=(p-k)^2, u=k^2$; see Fig. 9(a)]

$$T(u,\mu^2)=-\int\frac{d\mathfrak{m}^2 d\beta}{\mu^2-\beta}\left[\frac{\rho(\mathfrak{m}^2,\beta)}{u-\mathfrak{m}^2+i\epsilon}-\frac{\rho_R(\mathfrak{m}^2,\beta)}{-\mathfrak{m}^2}\right]. \tag{III.1}$$

It should be noted that the $d\beta$ and $d\mathfrak{m}^2$ integration contours can be taken as *complex* in the unphysical region. The general complex singularity structure indicated by perturbation theory for the parton-proton amplitude may then be accounted for if the contour integration is suitably defined.[8] An alternative method for treating the k^2 dependence is given in Appendix B.

Note that the total antiparton-proton cross section is proportional to ρ:

$$\sigma_{\bar{a}p}(s)\propto\frac{\rho(s,m_a{}^2)}{s}. \tag{III.2}$$

The subtraction term,[9] which only contributes to the real part of T, is necessary to ensure convergence of the representation (III.1), and will be crucial in the derivation of finite sum rules; it is required for that part of T which has Regge behavior

$$\rho^R(\mathfrak{m}^2,\beta)=\sum_{0<\alpha<1}(\mathfrak{m}^2)^\alpha\rho_\alpha(\beta). \tag{III.3}$$

In order to obtain the "softened" behavior necessary to derive scaling we take T to have off-shell damping in the variable μ^2. We thus assume that at least the first moment in β of $\rho^{NR}(m^2,\beta)$ and $\rho^R(\beta)$ vanishes. Such behavior is not unnatural for a hadronic amplitude, and it is a natural consequence of bound-state models for the target proton.[3]

The form (III.1) leads naturally to the following representation for the self-energy of the proton due to the emission and absorption of a parton [in general, one sums over all types of partons; see Fig. 9(b)]:

$$\Sigma(p)=+\int d\mathfrak{m}^2d\beta\int\frac{d^4k}{i}\,\frac{1}{(p-k)^2-m_a{}^2}\,\frac{1}{(p-k)^2-\beta}\left[\frac{\rho(\mathfrak{m}^2,\beta)}{u-\mathfrak{m}^2+i\epsilon}-\frac{\rho^R(\mathfrak{m}^2,\beta)}{-\mathfrak{m}^2}\right]. \tag{III.4}$$

We can generalize this further by using the general Källén-Lehmann representation for the parton propagator:

$$\frac{1}{(p-k)^2-m_a{}^2+i\epsilon}\to\int_0^\infty d\sigma\,\frac{1}{(p-k)^2-\sigma+i\epsilon}\,\tilde{\rho}(\sigma).$$

Then

$$\Sigma(p)\equiv I\int\frac{d^4k/i}{(p-k)^2-\Sigma_y}\left[\frac{\rho(\mathfrak{m}^2,\mu^2)}{k^2-\mathfrak{m}^2+i\epsilon}-\frac{\rho^R(\mathfrak{m}^2,\mu^2)}{-\mathfrak{m}^2}\right], \tag{III.6}$$

with

$$I\equiv\frac{1}{(2\pi)^4}\int d\mathfrak{m}^2d\beta d\sigma\tilde{\rho}(\sigma)\int_0^1 dy\left(\frac{d}{d\Sigma_y}\right), \tag{III.7}$$

where we have used a Feynman parameter to combine the denominators

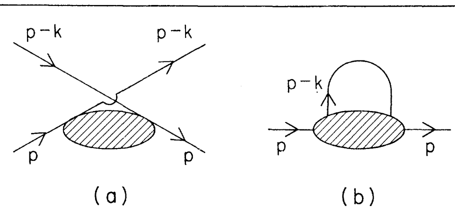

FIG. 9. (a) The forward parton-proton amplitude. (b) The corresponding contribution to the proton self-energy.

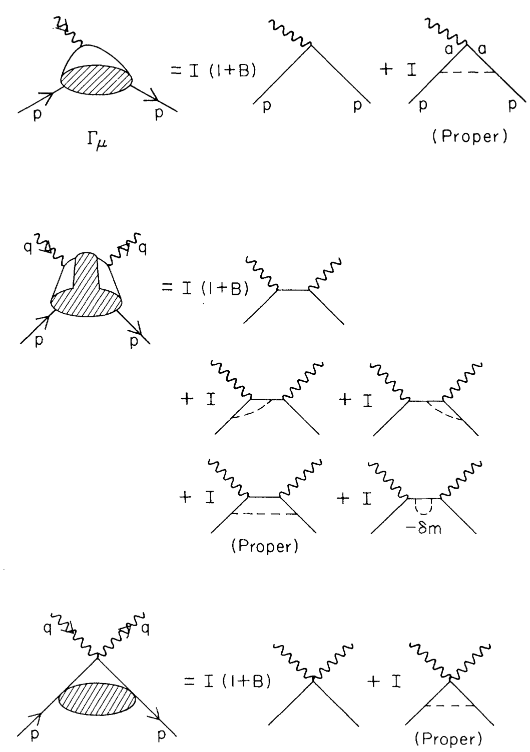

FIG. 10. Gauge-invariant nonperturbative model for the vertex and Compton amplitudes.

$$\frac{1}{(p-k)^2-\sigma}\,\frac{1}{(p-k)^2-\beta}=\int_0^1\frac{dy}{[(p-k)^2-\Sigma_y]^2}\,, \tag{III.8}$$

$$\Sigma_y=y\beta+(1-y)\sigma\,.$$

Note that the resulting nonperturbative theory is defined in a linear operational way on second-order perturbation theory results. Since we are starting with a finite expression for $\Sigma(p)$, the subsequent formulas we derive will all be well defined.

Starting from $\Sigma(p)$ we can use the Ward-Takahashi identities to derive gauge-invariant expressions for the form factor and the full Compton amplitude:

$$\begin{aligned}&\Sigma(p+q)-\Sigma(p)=-q^{\mu}\Gamma_{\mu}(p+q,p)\,,\\&\Gamma_{\mu}(p+q,p)-\Gamma_{\mu}(p,p-q)=-q^{\nu}T_{\mu\nu}\,.\end{aligned} \tag{III.9}$$

In each case this yields the proper amplitude. The full vertex and the full Compton amplitude illustrated in Fig. 10 also include the improper contributions. The case of a composite proton is $I(1+B)=0$, which eliminates the Born-like diagrams. This procedure yields the nontrivial, minimal gauge-invariant currents (i.e., terms which are not explicitly transverse). The results for the ρ term are thus identical to those obtained from the effective spectral sum of contributions corresponding to the elementary self-energy, vertex, and Compton diagrams of the usual Feynman perturbation theory. In the present case, the line carrying momentum $p^{\mu}-k^{\mu}$ is the only charged line. In the case of multiple charge, one applies the Ward identities to a basic self-energy diagram in which the external momentum is routed in proportion to the fraction of charge carried by that line. Thus, in the form-factor calculation, one sums the individual parton and antiparton contributions weighted by the parton charge.

Using Eq. (III.6) for $\Sigma(p)$ we obtain for the one-photon vertex

$$\Gamma^{\mu}=I\int\frac{d^4k}{i}\,\frac{(2p+q-2k)^{\mu}}{[(p+q-k)^2-\Sigma_y][(p-k)^2-\Sigma_y]}\left[\frac{\rho}{k^2-\mathrm{m}^2}-\frac{\rho^{\mathrm{R}}}{-\mathrm{m}^2}\right] \tag{III.10a}$$

$$=I\int(d^4k/i)\int_0^1dz\left[\int_0^1dx\,\frac{2(1-x)\{2xp+q[1-2z(1-x)]\}^{\mu}}{D^3(x,z)}\,\rho+\frac{(1-2z)q^{\mu}\rho^{\mathrm{R}}}{\mathrm{m}^2D^2(0,z)}\right]\,, \tag{III.10b}$$

where

$$D(x,z)=k_x'^{\,2}-x\mathrm{m}^2-(1-x)\Sigma_y+M^2x(1-x)+2\nu zx(1-x)+q^2z(1-x)[1-z(1-x)]+i\epsilon \tag{III.11}$$

and

$$k_x'=k-(1-x)p-z(1-x)q\,. \tag{III.12}$$

Since we are considering the on-shell vertex, $\nu\equiv p\cdot q=-q^2/2$. The odd terms in k' have been discarded in the numerator of (III.10b).

Since the denominators $D(0,z)$ and $D(x,z)$ are symmetric under $z\to1-z$ (for $\nu=-q^2/2$), the Regge subtraction term in (III.10) vanishes and the numerator vector in the surviving term is $x(2p+q)^{\mu}$. The stability condition $M<m+\Sigma_y$ is assumed. The form factor is thus

$$F(q^2)=-\pi^2I\int_0^1dz\int_0^1dx\frac{x(1-x)\rho}{[x\mathrm{m}^2+(1-x)\Sigma_y-x(1-x)M^2-q^2(1-x)^2z(1-z)]}\,. \tag{III.13}$$

We define the normalized distribution function $f(x)$ via

$$1=F(0)=\int_0^1dx\,f(x)\,, \tag{III.14a}$$

where

$$f(x)=-\pi^2I\frac{x(1-x)\rho}{x\mathrm{m}^2+(1-x)\Sigma_y-x(1-x)M^2}\,. \tag{III.14b}$$

For the Regge part of the spectral function ρ, we have

$$f^{\mathrm{R}}(x)=\frac{\pi^2}{(2\pi)^4}\sum_{\alpha}\int d\beta\,d\sigma\,\tilde{\rho}(\sigma)\rho_{\alpha}(\beta)\int_0^1dy\int d\mathrm{m}^2\frac{(1-x)^2x(\mathrm{m}^2)^{\alpha}}{[x\mathrm{m}^2+(1-x)\Sigma_y-x(1-x)M^2]^2}\,. \tag{III.15}$$

For $x\sim0$, the last integral is proportional to

$$\int_0^{\infty}d\mathrm{m}^2\frac{x(\mathrm{m}^2)^{\alpha}}{[x\mathrm{m}^2+\Sigma_y]^2}\propto\frac{1}{x^{\alpha}}\,. \tag{III.16}$$

Thus

$$f(x) \sim \sum_{1>\alpha>0} \gamma_\alpha x^{-\alpha} \quad (x \to 0), \tag{III.17}$$

where we have defined

$$\gamma_\alpha = \tilde{I} \int_0^\infty \frac{d\zeta\, \zeta^\alpha}{(\zeta + \Sigma_y)^2}, \tag{III.18}$$

and $\tilde{I}$ is the (β, σ, y) integral operator defined from the first line of (III.15).

As we shall see below, the structure function $\nu W_2(x)$ is given by $xf(x)$. The Regge behavior of $f(x)$, which follows from the Regge behavior of the parton-proton total cross section, does not disturb the convergence of the x integration for $F(q^2)$. In the more general case of multiple internal charges, it is possible for the distribution function $f_a(x)$ for an individual parton to have Pomeron behavior. However, even-charge-conjugation contributions such as the Pomeron cancel in the summation over parton-antiparton types for the form factor

$$F(0) = \int_0^1 \sum_a \lambda_a f_a(x) dx . \tag{III.19}$$

The form factor does not receive any contribution from the Regge subtraction in $\Sigma(p)$.

Using the second Ward-Takahashi identity for the forward Compton amplitude

$$-q^\nu T_{\mu\nu} = [\Gamma_\mu(p+q, p) - \Gamma_\mu(p,p)] + [\Gamma_\mu(p,p) - \Gamma_\mu(p, p-q)], \tag{III.20}$$

we obtain the "uncrossed" diagram and half the "seagull" contribution from the first bracket of (III.20):

$$T^{(1)}_{\mu\nu} = I \int \frac{d^4k}{i} \left(\frac{\rho}{k^2 - \mathrm{m}^2} + \frac{\rho^{\mathrm{R}}}{\mathrm{m}^2} \right) \left\{ \frac{(2p+q-2k)^\mu (2p+q-2k)^\nu}{[(p+q-k)^2 - \Sigma_y][(p-k)^2 - \Sigma_y]^2} - \frac{g_{\mu\nu}}{[(p-k)^2 - \Sigma_y]^2} \right\} \tag{III.21a}$$

$$= I \int (d^4k/i) \int_0^1 dx \int_0^1 dz \left(\frac{\{2xp + q[1 - 2z(1-x)] - 2k'\}^\mu \{2xp + q[1-2z(1-x)] - 2k'\}^\nu 6(1-z)(1-x)^2 \rho}{D^4(x,z)} \right.$$
$$\left. + \frac{[q(1-2z) - 2k']^\mu [q(1-2z) - 2k']^\nu 2(1-z)\rho^{\mathrm{R}}}{D^3(0,z)\mathrm{m}^2} - \frac{g_{\mu\nu} 2(1-x)\rho}{D^3(x,0)} - \frac{g_{\mu\nu}\rho^{\mathrm{R}}}{D^2(0,0)\mathrm{m}^2} \right). \tag{III.21b}$$

In each term k'_μ is chosen to diagonalize the denominator D, defined in Eq. (III.11). The remaining contribution $T^{(2)}_{\mu\nu}$ is obtained from the substitution $q \to -q$, $\nu \to -\nu$.

As usual we define T_1 and T_2 from Eq. (II.11). The structure function $W_2(\nu, q^2) = (2\pi M)^{-1} \mathrm{Im}\, T_2(\nu, q^2)$ is obtained in the scaling region from the proper contribution $T^{(1)}_{\mu\nu}$ alone, and may be isolated from the $p_\mu p_\nu$ coefficient in Eq. (III.21b). Thus

$$\lim_{\substack{\nu \to \infty \\ -2\nu/q^2 = \omega \text{ fixed}}} \frac{\nu}{M} W_2(\nu, q^2) = \lim_{\mathrm{Bj}} \frac{4\pi^2}{2\pi} I \int_0^1 dx \int_0^1 dz\, x^2(1-x)^2(1-z)\rho\nu$$
$$\times \mathrm{Im}\{2\nu z x(1-x) + q^2 z(1-x)[1 - z(1-x)] - x\mathrm{m}^2 - (1-x)\Sigma_y + x(1-x)M^2 + i\epsilon\}^{-2} \tag{III.22a}$$

$$= -\pi^2 \lim_{\mathrm{Bj}} I \int_0^1 dx \frac{x^2(1-x) 2\nu\delta(2\nu x(1-x) + q^2(1-x))\rho}{x\mathrm{m}^2 + (1-x)\Sigma_y - x(1-x)M^2} \tag{III.22b}$$

$$= xf(x)|_{x=-q^2/2\nu} . \tag{III.22c}$$

The surviving term in the Bjorken limit is obtained by integrating Eq. (III.22a) once by parts in z. Only the surface term at $z=0$ contributes for $\nu \to \infty$. The existence of the scaling limit is guaranteed by the "softening" conditions on the spectral functions. Again, the Regge subtraction term does not contribute.

The invariant amplitude $T_1(\nu, q^2)$ may be isolated from the coefficient of the $g_{\mu\nu}$ terms which only occur in the proper part of the Compton amplitude. We obtain (after angular averaging in k')

$$T_1(\nu, q^2) = I \int (d^4k/i) \int_0^1 dx \int_0^1 dz \left[\frac{6(1-z)(1-x)^2 \rho k'^2}{D^4(x,z)} + \frac{2(1-z)\rho^{\mathrm{R}} k'^2}{\mathrm{m}^2 D^3(0,z)} - \frac{2(1-x)\rho}{D^3(x,0)} - \frac{\rho^{\mathrm{R}}}{\mathrm{m}^2 D^2(0,0)} \right] + (\nu \to -\nu) . \tag{III.23}$$

At this point we can check the low-energy theorem

$$\lim_{\nu\to 0} T_1(\nu,0)=(2)I\pi^2\int_0^1 dx\,\frac{(1-x)^2\rho-(1-x)\rho}{[x\mathfrak{m}^2+(1-x)\Sigma_y-x(1-x)M^2]}=2\int_0^1 dx f(x)=2\ , \tag{III.24}$$

which is the correct Thomson limit for the covariant normalization used here. A more complicated proof can be derived for the case of multiple charges.

We next investigate the high-energy limit of $T_1(q^2,\nu)$ at fixed q^2. From Eq. (III.23) we have

$$T_1(\nu,q^2)=-\tilde{I}\pi^2\int_0^1 dx\int_0^1 dz\left[\left(\frac{2(1-z)(1-x)^3\rho}{d^2(x,z)}-\frac{2(1-z)\rho^{\mathrm{R}}}{\mathfrak{m}^2 d(0,z)}\right)-\left(\frac{(1-x)^2\rho}{d^2(x,0)}-\frac{\rho^{\mathrm{R}}}{\mathfrak{m}^2 d(0,0)}\right)\right]+(\nu\to-\nu), \tag{III.25}$$

where

$$d(x,z)=x\mathfrak{m}^2+(1-x)\Sigma_y-x(1-x)M^2-2\nu z x(1-x)-q^2z(1-x)(1-z(1-x)) \tag{III.26}$$

and $\tilde{I}$ is the same as I without the $d/d\Sigma_y$ differentiation. We have grouped together uncrossed-diagram contributions and the seagull-diagram contributions ($z=0$ terms). The $d(0,0)$ term (which arises from the seagull Regge subtraction) may be rewritten as

$$-(2)\pi^2\tilde{I}\frac{\rho^{\mathrm{R}}}{\mathfrak{m}^2\Sigma_y}=-2\pi^2\tilde{I}\int_0^\infty\frac{\rho^{\mathrm{R}}dx}{(x\mathfrak{m}^2+\Sigma_y)^2}\ . \tag{III.27}$$

Thus the contribution to $T_1(\nu,q^2)$ from the seagull diagrams [second parenthesis of Eq. (III.25)] is

$$T_1^{\text{seagull}}(\nu,q^2)=2\int_0^\infty\frac{dx}{x}\tilde{f}(x)=2\left[\int_0^1\frac{dx}{x}\tilde{f}(x)-\sum_{0<\alpha<1}\frac{1}{\alpha}\gamma_\alpha\right], \tag{III.28}$$

where

$$\tilde{f}(x)=\theta(1-x)f(x)-\pi^2\tilde{I}\frac{\rho^{\mathrm{R}}}{(x\mathfrak{m}^2+\Sigma_y)^2}=\theta(1-x)f(x)-\sum_\alpha\gamma_\alpha x^{-\alpha}\ , \tag{III.29}$$

with γ_α as defined in Eq. (III.17). Thus the seagull diagrams yield a finite energy-independent, q^2-independent contribution to the T_1 amplitudes. Note that the subtraction term is crucial for the finiteness of the seagull contribution in the presence of Regge behavior. For $q^2=0$, the effects of the subtraction term actually cancel out in the total contribution for $T_1(\nu,q^2)$. An even simpler derivation of the δ_{J0} sum rule for $T_1(\nu,0)$ can then be given. See Ref. 4.

The remaining contribution to T_1 from the uncrossed and crossed graphs has normal Regge behavior. The presence of the ρ^{R} subtraction terms for these contributions is crucial for obtaining a finite result. The Regge terms only arise from the ρ^{R} contribution and the leading behavior at $x\to 0$: We have

$$-\frac{\pi^2}{(2\pi)^4}\int d\beta\, d\sigma\, d\mathfrak{m}^2\tilde{\rho}(\sigma)\sum_\alpha\rho_\alpha(\beta)(\mathfrak{m}^2)^\alpha\int_0^1 dz\int_0^1 dy\, 2(1-z)\left[\int_0^1 dx\,\frac{(1-x)^3}{d^2(x,z)}-\frac{1}{\mathfrak{m}^2[\Sigma_y-q^2z(1-z)]}\right], \tag{III.30}$$

where (with $\zeta\equiv x\mathfrak{m}^2$),

$$\begin{aligned}\int_0^\infty d\mathfrak{m}^2(\mathfrak{m}^2)^\alpha[\ \ ]\underset{x\to 0}{\sim}\int_0^\infty & d\mathfrak{m}^2(\mathfrak{m}^2)^\alpha\\ &\times\int_0^\infty dx\left[\frac{\theta(1-x)}{[x\mathfrak{m}^2+\Sigma_y-2\nu z x-q^2z(1-z)]^2}-\frac{1}{[x\mathfrak{m}^2+\Sigma_y-q^2z(1-z)]^2}\right]\\ &=\int_0^\infty d\zeta\,\zeta^\alpha\int_0^\infty\frac{dx}{x^{\alpha+1}}\left[\frac{\theta(1-x)}{[\zeta+\Sigma_y-2\nu z x-q^2z(1-z)]^2}-\frac{1}{[\zeta+\Sigma_y-q^2z(1-z)]^2}\right].\end{aligned} \tag{III.31}$$

Using integration by parts on ζ, the surface terms vanish (for $0<\alpha<1$) and Eq. (III.31) becomes

$$\alpha\int_0^\infty d\zeta\,\zeta^{\alpha-1}\int_0^\infty\frac{dx}{x^{\alpha+1}}\left[\frac{\theta(1-x)}{[\zeta+\Sigma_y-2\nu z x-q^2z(1-z)-i\epsilon]}-\frac{1}{[\zeta+\Sigma_y-q^2z(1-z)-i\epsilon]}\right]. \tag{III.32}$$

For ν large we scale

$$x=x'\frac{[\Sigma_y+\zeta-q^2z(1-z)]}{2\nu z} \tag{III.33}$$

and obtain

$$T_1(\nu, q^2) = T_1^{\mathrm{R}}(\nu, q^2) + T_1^{\mathrm{seagull}}, \tag{III.34a}$$

where

$$T_1^{\mathrm{R}} = -\frac{2\pi^2}{(2\pi)^4}\int d\beta\, d\sigma \sum_\alpha \rho_\alpha(\beta)\tilde{\rho}(\sigma)\int_0^1 dy \int_0^1 dz\, 2(1-z)\int_0^\infty d\zeta \frac{\alpha\zeta^{\alpha-1}}{[\zeta+\Sigma_y - q^2 z(1-z)]^{\alpha+1}}(2\nu z)^\alpha \xi_\alpha \tag{III.34b}$$

and

$$\xi_\alpha = \int_0^\infty \frac{dx'(x')^{1-\alpha}}{1-x'^2-i\epsilon} = -1 - e^{-i\pi\alpha} \tag{III.34c}$$

is the correct signature factor for a crossing-even amplitude. This result for T_1 shows the scaling behavior of the Regge term in the variable $\omega = -2\nu/q^2$, which arises from the $z \sim 1/q^2$ region. In the case of our spinless example, $q^2 T_1$ scales in ω. This illustrates the general point that the handbag diagram contains Regge ν^α terms which vanish in the scaling region.

The subtraction terms, which are crucial for the finiteness of T_1, are not evident from the approach of Landshoff *et al.*,[2] but arise from a consistent gauge-invariant finite treatment of the electromagnetic amplitudes. These terms do not contribute to the form factors $F(t)$ or W_2, which arise from currents proportional to the external momentum p^μ, since the subtraction terms which involve $d(x,z)$ with $x=0$ correspond to contributions in which the external momenta do not flow through the charged line. The Feynman variable x appearing in the above equations may be identified with the fraction of the momentum carried by the charged particle in the infinite-momentum frame of the proton.

As is evident from the perturbation-theory example given in Sec. II, the result (III.28) for the q^2-independent ν-independent $J=0$ Kronecker δ term in the T_1 amplitude holds as well for the case of spin-$\frac{1}{2}$ charged particles.[10] This result for T_1, which we first presented in Ref. 4, is a compelling feature of the scaling parton model. We can also derive a result for the $J=0$ fixed pole in $T_2(\nu, q^2)$, first given by Cornwall, Corrigan, and Norton,[11] who used a scaling Deser-Gilbert-Sudarshan (DGS) representation.

We shall show below that

$$T_2^{\mathrm{FP}}(q^2, \nu) = -\frac{2q^2M^2}{\nu^2}\left[\int_0^1 \frac{\tilde{f}(x)}{x}dx - \sum_{0<\alpha<1}\frac{\gamma_\alpha}{\alpha}\right]. \tag{III.35}$$

Thus, using (III.35), we see that

$$-\frac{\nu^2}{M^2q^2}T_2^{\mathrm{FP}}(q^2, \nu) = T_1^{\mathrm{FP}}. \tag{III.36}$$

This result is equivalent to the assumption of a fixed pole whose residue is polynomial in q^2 (as conjectured by Cheng and Tung[12]), plus the assumption of scale independence.

The derivation of the fixed pole in T_2 is in many ways more difficult than the derivation for the T_1 amplitude; for example, in the model described above, one must be certain to include the improper (proton-pole) diagrams shown in Fig. 10, as required by gauge invariance. In particular, the finiteness of the x integration for the fixed pole in T_2 requires inclusion of these contributions. In a true bound-state model the pole diagrams do not appear; in this case the $T^{(6)}$ diagrams (i.e., those which involve the connected six-point hadronic amplitude) are required to restore the full gauge invariance of the theory, and are necessary to obtain the correct result for the $J=0$ fixed pole in the T_2 amplitude.

There are also other methods available for constructing gauge-invariant amplitudes in composite models. For example, Scott[13] has used a ladder-approximation Bethe-Salpeter approach to construct, at least asymptotically, a gauge-invariant expression for exclusive electroproduction amplitudes at large transverse momentum. In the work of Drell and Lee,[3] the counterterms required for gauge invariance in inclusive electroproduction are constructed by demanding that the soft-photon theorems be satisfied in a minimal way. This method is used in Ref. 14 for constructing completely gauge-invariant exclusive electroproduction and Compton amplitudes in bound-state models. In any case, the subtraction terms required by Ward identities are present.

We first calculate an expansion for the self-energy terms in Fig. 10:

$$[\Sigma(p+q) - \Sigma(p)]\big|_{p^2=M^2} = I\int\frac{d^4k}{i}\left[\frac{\rho}{k^2-\mathfrak{m}^2+i\epsilon} - \frac{\rho^{\mathrm{R}}}{-\mathfrak{m}^2}\right]\left[\frac{1}{(p+q-k)^2-\Sigma_y} - \frac{1}{(p-k)^2-\Sigma_y}\right]. \tag{III.37}$$

The ρ^R term vanishes upon a shift of the (finite) k^μ integration. We then have

$$[\Sigma(p+q)-\Sigma(p)]|_{p^2=M^2}$$
$$=\tilde{I}\pi^2\int_0^1 dx(1-x)\rho\left[\frac{1}{d(x,1)}-\frac{1}{d(x,0)}\right]$$
$$=(2\nu+q^2)I\pi^2\int_0^1 dx\int_0^1 dz\frac{\rho x(1-x)}{h(x,z)}\ , \qquad \text{(III.38)}$$

where

$$h(x,z)=x\mathfrak{m}^2+(1-x)\Sigma_y-x(1-x)M^2 - (2\nu+q^2)zx(1-x)\ .$$

The vertex required in Fig. 10 is obtained from Eqs. (III.9) and (III.20). Again, the Regge subtraction term does not contribute. We thus obtain from the $p_\mu p_\nu/M^2$ coefficient

$$T_2(\nu,q^2)=\pi^2\mathfrak{m}^2 I\int_0^\infty dx\int_0^1 dz\,B(x,z)+(\nu\to-\nu)\ , \qquad \text{(III.39)}$$

where

$$B(x,z)=\theta(1-x)\rho\left[\frac{4x^2(1-x)^2(1-z)}{d^2(x,z)} - \frac{8x(1-x)}{(2\nu+q^2)d(x,z)} + \frac{4x(1-x)}{(2\nu+q^2)h(x,z)}\right], \qquad \text{(III.40)}$$

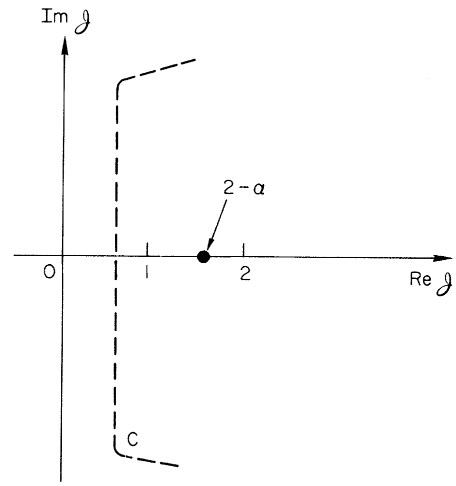

FIG. 11. Contour for the Mellin inversion formula, Eq. (III.44).

the three terms arising from the proper, vertex, and self-mass contributions, respectively.

It turns out to be convenient to isolate the leading ν behavior of T_2 using the Mellin transform technique. The contribution of the first term of $B(x,z)$ to the Mellin transform of T_2

$$\int_0^\infty d(2\nu)(2\nu)^{\mathcal{J}-1}\,T_2(\nu,q^2)\quad(\mathcal{J}>0) \qquad \text{(III.41)}$$

is

$$-2\pi^2M^2\tilde{I}\ \frac{\Gamma(\mathcal{J})\Gamma(3-\mathcal{J})}{\Gamma(3)}(-1)^{3-\mathcal{J}}\int_0^1 dx\int_0^1 dz\frac{4x^2(1-x)^3(1-z)\rho}{[x\mathfrak{m}^2+\Sigma_y(1-x)-x(1-x)M^2-q^2z(1-x)(1-z(1-x))]^{3-\mathcal{J}}[zx(1-x)]^{\mathcal{J}}}\ . \qquad \text{(III.42)}$$

To isolate the $J=0$ fixed-pole contribution we add and subtract the leading $x\to0$ integrand

$$\frac{4x^2(1-z)\rho^R}{[x\mathfrak{m}^2+\Sigma_y-q^2z(1-z)]^{3-\mathcal{J}}[zx]^{\mathcal{J}}}\ . \qquad \text{(III.43)}$$

Using the Mellin inversion formula

$$\int_C d\mathcal{J}F(\mathcal{J})(2\nu)^{-\mathcal{J}}\ , \qquad \text{(III.44)}$$

one may isolate the leading ν dependence by picking up the nearest $\mathcal{J}$-plane singularity of $F(\mathcal{J})$ to the right of the inversion contour C (see Fig. 11).

For the difference term the x integration is strongly convergent so that the leading ν dependence is obtained from the pole at $\mathcal{J}=2$ arising from a z integration of the form

$$\int_0^1\frac{dz}{z^{\mathcal{J}-1}}=\frac{1}{2-\mathcal{J}}\ . \qquad \text{(III.45)}$$

[Note that when the cross term $(\nu\to-\nu)$ is included, the resulting signature factor $(-1)^{\mathcal{J}+1}$ cancels the contribution of the $\mathcal{J}=1$ pole.] Expanding the denominator in powers of z gives

$$2\pi^2 M^2 \tilde{I}(-1)^{3-\mathcal{J}}\frac{\Gamma(\mathcal{J})\Gamma(3-\mathcal{J})}{\Gamma(3)}\int_0^1 dx\frac{1}{2-\mathcal{J}}$$
$$\times\left\{\frac{4x^2(1-x)^3\rho}{x^J(1-x)^{\mathcal{J}}}\left[\frac{(-1)}{[x\mathfrak{m}^2+\Sigma_y(1-x)-x(1-x)M^2]^{3-\mathcal{J}}}-\frac{(\mathcal{J}-3)q^2(1-x)}{[x\mathfrak{m}^2+\Sigma_y(1-x)-x(1-x)M^2]^{4-\mathcal{J}}}\right]\right.$$
$$\left.-\frac{4x^2\tilde{\rho}^{\mathrm{R}}}{x^{\mathcal{J}}}\left[\frac{(-1)}{[x\mathfrak{m}^2+\Sigma_y]^{3-\mathcal{J}}}-\frac{(\mathcal{J}-3)q^2}{[x\mathfrak{m}^2+\Sigma_y]^{4-\mathcal{J}}}\right]\right\} \qquad \text{(III.46)}$$

so that the residue of the pole at $\mathcal{J}=2$ gives a contribution to T_2:

$$\frac{-2\pi^2M^2\tilde{I}}{(2\nu)^2}\int_0^1 dx\left\{4(1-x)\rho\left[\frac{-1}{[x\mathfrak{m}^2+\Sigma_y(1-x)-x(1-x)M^2]}+\frac{q^2(1-x)}{[x\mathfrak{m}^2+\Sigma_y(1-x)-x(1-x)M^2]^2}\right]\right.$$
$$\left.-4\rho^{\mathrm{R}}\left[\frac{-1}{[x\mathfrak{m}^2+\Sigma_y]}+\frac{q^2}{[x\mathfrak{m}^2+\Sigma_y]^2}\right]\right\}. \qquad \text{(III.47)}$$

The corresponding calculation for the vertex and self-mass terms in $B(x,z)$ precisely cancels the q^2-independent terms in (III.47). The contribution of the remainder to the $J=0$ fixed pole is thus

$$\pi^2M^2I\frac{2q^2}{\nu^2}\int_0^1 dx\left[\frac{(1-x)\rho}{[x\mathfrak{m}^2+\Sigma_y(1-x)-x(1-x)M^2]}-\frac{\rho^{\mathrm{R}}}{[x\mathfrak{m}^2+\Sigma_y]}\right]. \qquad \text{(III.48)}$$

The final contribution to the $J=0$ fixed pole of T_2 is obtained from adding back in the Regge term (III.43). Exhibiting the $\mathfrak{m}^2$ dependence from

$$\rho^{\mathrm{R}}=\sum_\alpha \rho_\alpha(\beta)(\mathfrak{m}^2)^\alpha\ , \qquad \text{(III.49)}$$

we have (with $x\mathfrak{m}^2=\zeta$)

$$-2\pi^2M^2\frac{\Gamma(\mathcal{J})\Gamma(3-\mathcal{J})}{\Gamma(3)}\int_0^1 dz\int_0^1\frac{dx}{x^{\mathcal{J}-2}}\int\frac{dm^2 4(1-z)(\mathfrak{m}^2)^\alpha}{[x\mathfrak{m}^2+\Sigma_y-q^2z(1-z)]^{3-\mathcal{J}}(z)^{\mathcal{J}}}$$
$$=-2\pi^2M^2\frac{\Gamma(\mathcal{J})\Gamma(3-\mathcal{J})}{\Gamma(3)}\frac{1}{2-\mathcal{J}-\alpha}\int_0^1 dz\,4(1-z)\int_0^\infty\frac{d\zeta\,\zeta^\alpha}{[\zeta+\Sigma_y-q^2z(1-z)]^{3-\mathcal{J}}(z)^{\mathcal{J}}}\ . \qquad \text{(III.50)}$$

For the moment we will concentrate on the contribution of this term to the $J=0$ fixed pole. The required pole at $\mathcal{J}=2$ arises from the z^{1-J} terms in the z integration; specifically for $\mathcal{J}\sim 2$ we have

$$-2\pi^2M^2\frac{\Gamma(\mathcal{J})\Gamma(3-\mathcal{J})}{\Gamma(3)}\frac{1}{2-\mathcal{J}-\alpha}\frac{1}{2-\mathcal{J}}\int_0^\infty d\zeta\,\zeta^\alpha\left[\frac{-4}{[\zeta+\Sigma_y]^{3-\mathcal{J}}}+\frac{4q^2(3-\mathcal{J})}{[\zeta+\Sigma_y]^{4-\mathcal{J}}}\right]$$
$$=\frac{\pi^2M^2}{\alpha}\frac{1}{2-\mathcal{J}}\int_0^\infty d\zeta\,\zeta^\alpha\left[\frac{-4}{\zeta+\Sigma_y}+\frac{4q^2}{(\zeta+\Sigma_y)^2}\right]. \qquad \text{(III.51)}$$

As before, the q^2-independent term is canceled by similar contributions from the vertex and self-energy amplitudes. The linear q^2 term gives the fixed-pole contribution

$$-\pi^2M^2I\frac{2q^2}{\nu^2}\int_1^\infty\frac{\rho^{\mathrm{R}}dx}{[x\mathfrak{m}^2+\Sigma_y]}\ , \qquad \text{(III.52)}$$

which, combined with (III.48), is precisely Eq. (III.35).

Clearly T_2 also has terms $\nu^{\alpha-2}$ with Regge behavior arising from the explicit poles at $\mathcal{J}=2-\alpha$ in Eq. (III.50), as well as in the contributions of the vertex and self-energy amplitudes. We will verify that the Regge contributions of Eq. (III.50) are in fact "scaling Regge" contributions ($\nu T_2\sim\omega^{\alpha-1}$). The vertex and self-energy contributions vanish in the scaling limit.

For q^2 large, one may take $z=-\bar{z}/q^2$ and drop the z^2 term in the denominator of (III.50). One then obtains

$$-2\pi^2M^2I\frac{\Gamma(\mathcal{J})\Gamma(3-\mathcal{J})}{\Gamma(3)}\frac{1}{2-\mathcal{J}-\alpha}\int_0^\infty d\zeta\int_0^\infty\frac{d\bar{z}(-q^2)^{\mathcal{J}-1}\zeta^\alpha}{[\zeta+\Sigma_y+\bar{z}]^{3-\mathcal{J}}\bar{z}^{-\mathcal{J}}}$$
$$=-2\pi^2M^2I\frac{\Gamma(\mathcal{J})\Gamma(3-\mathcal{J})}{\Gamma(3)}\frac{1}{2-\mathcal{J}-\alpha}(-q^2)^{\mathcal{J}-1}\frac{\Gamma(\alpha+1)\Gamma(2-\mathcal{J}-\alpha)}{\Gamma(3-\mathcal{J})}\frac{1}{[\Sigma_y]^{1-\alpha}}\frac{\Gamma(1-\mathcal{J})\Gamma(1-\alpha)}{\Gamma(2-\mathcal{J}-\alpha)} \qquad \text{(III.53a)}$$
$$\underset{J\sim 2-\alpha}{\sim}-\pi^2M^2I\frac{\pi}{\sin\pi(2-\alpha)}\frac{\alpha\pi}{\sin\pi\alpha}\frac{1}{2-\mathcal{J}-\alpha}\left(\frac{-q^2}{\Sigma_y}\right)^{1-\alpha}, \qquad \text{(III.53b)}$$

which has exactly the q^2 dependence appropriate to scaling Regge behavior. Including the cross graph, the signature factor is $[(-1)^{2-\alpha}+1]$.

For $\alpha \to 1^-$, i.e., Pomeranchukon behavior, one obtains a finite contribution $\nu T_2 \sim \omega^0$, since the numerator zeros from (a) the signature factor and (b) the spectral contribution

$$\lim_{\alpha \to 1^-} I \frac{1}{(\Sigma_y)^{1-\alpha}} \sim (\alpha - 1) I \ln \Sigma_y \qquad \text{(III.54)}$$

are compensated by the denominator zeros: (a) $\sin\pi(2-\alpha)$ from the $z \sim 0$ integration and (b) $\sin\pi\alpha$.

This agrees with the proof of Landshoff and Polkinghorne[15] showing the survival of the Pomeranchukon contribution due to its coincidence with the fixed pole at $J = 1$.

IV. CONCLUSION

In this paper we have presented a general nonperturbative model for the Compton amplitude which incorporates Bjorken scaling, gauge invariance, and Regge behavior. In the case of the deep-inelastic electron scattering, the results agree with the Landshoff-Polkinghorne-Short[2] model and exhibit scaling Regge behavior. We have also given a particularly simple derivation of the LPS results for νW_2 using a covariantly-regularized infinite-momentum frame analysis.

As we have shown, a general consequence of composite theories of the hadrons, with field-theoretic constituents, which incorporate (a) Bjorken scaling (and thus "softened" off-shell behavior) and (b) gauge invariance, is the existence of a constant energy-independent q^2-independent term in $T_1(\nu, q^2)$ (a Kronecker δ: δ_{J0} term) and a $J=0$ fixed-pole term in $T_2(\nu, q^2)$. Contributions can be expressed in terms of $\nu W_2(x)$ as follows:

$$T_1^{\delta_{J0}}(q^2,\nu) = \frac{-q^2}{\nu^2} T_2^{J=0}(q^2,\nu) = 2\left[\int_0^1 \frac{dx}{x^2}\, \nu \tilde{W}_2(x) - \sum_\alpha \frac{1}{\alpha}\gamma_\alpha\right], \qquad \text{(IV.1)}$$

where

$$\nu W_2(x) \underset{x \to 0}{\sim} \sum_\alpha \gamma_\alpha x^{1-\alpha} \qquad \text{(IV.2)}$$

and

$$\nu \tilde{W}_2(x) \equiv \nu W_2(x) - \sum_\alpha \gamma_\alpha x^{1-\alpha}. \qquad \text{(IV.3)}$$

The sum rule for $T_2^{J=0}$ was first derived by Cornwall, Corrigan, and Norton.[11] The sum rule for $T_1^{\delta_{J0}}$ was derived for arbitrary parton spin in Ref. 4. More recently, the $T_1^{\delta_{J0}}$ result has been derived from the light-cone approach by Bander[16] and Frishman.[17] In addition, other authors[18] have confirmed our parton-model results. The extension to the nonforward Compton amplitude is given in Ref. 4. Applications to neutrino scattering and polarization measurements are discussed in Ref. 18 and Appendix A.

Notice that if the leading term in $\nu W_2(x)$ at $x \to 0$ has $\alpha < 0$, then, by integration, the right-hand side of the sum rule (IV.1) reduces correctly to

$$2\int_0^1 \frac{dx}{x^2}\, \nu W_2(x) \quad (\alpha < 0), \qquad \text{(IV.4)}$$

which is the result obtained directly in the parton model if there is no leading Regge behavior. Since (IV.1) and (IV.4) coincide for all $\mathrm{Re}(\alpha) < 0$, the result (IV.1) must be the unique analytic continuation of (IV.4) to positive α. The derivation given in Sec. III shows that this continuation in α is justified: The result (IV.1) is obtained automatically for $\alpha < 1$ when subtraction terms in the underlying parton-proton u-channel dispersion relation are taken into account. In general, all sum rules which are formally divergent at $x \sim 0$ due to leading Regge behavior may be rendered finite by analytic continuation in this manner. Further examples are given in Appendixes A and C and Ref. 18.

All of the derivations of the specific forms of the sum rule [Eq. (III.28)] assume normal Regge behavior of the underlying hadronic parton-proton forward scattering amplitude. In principle, it is possible that this amplitude could have a $J \cong 0$ Regge contribution at $t \to 0$. In this case, the portion of the Compton amplitude with $J \sim 0$ Regge behavior would be more complicated than that given in Eq. (III.28).[19] Nevertheless, the existence of an energy-independent photon mass-independent (at fixed t) contribution to the full Compton amplitude which derives from the elementary electromagnetic interactions is *not affected*. Since the J-plane position of the accidentally coincident Regge contribution is expected to depend on t, the fundamental terms, with energy dependence independent of t, can be isolated by direct measurements of the real part of the nonforward Compton amplitude.[4]

Physically, the q^2 independence of the δ_{J0} term in $T_1(q^2, \nu)$ is a direct consequence of the local space-time coincidence of the two current interactions. This is immediately apparent from the seagull contribution of the spin-0 currents, and is made explicit by the Z-graph contribution in the case of spin-$\frac{1}{2}$ currents. Such terms have dramatic and testable experimental consequences in Bethe-Heitler interference experiments and the 2γ annihilation processes measurable in $e^\pm e^-$ collisions.[4]

Finally, there is the important question of how these parton field-theoretic calculations can be of physical interest despite the fact that the elementary constituents are not seen in the final state. From one point of view, this model for the electromagnetic interactions of composite hadrons can be viewed as a theoretical laboratory which allows one to abstract the most fundamental features of local current interactions (including light-cone properties) without regard to the exact composition of the final state. Alternatively, if the physical deep-binding picture of Johnson and Drell[20] is relevant, then the calculations presented here could be valid when the free-particle states of the model are a good match to the near continuum closely-spaced levels of a bound-state model.

Note that in the case of the real part of the Compton amplitude, constituent production is not involved. In fact, if one believes in the existence of an elementary fundamental current within the hadron that is relevant to the calculation of elastic and inelastic form factors, then it is difficult to avoid the possibility of having two photons interact on the same current line. Thus inevitably one has contributions to virtual amplitudes, e.g., the real parts of T_1 and T_2, from local two-photon interactions, and the conclusions stated above must apply.

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APPENDIX A: FIXED POLES IN WEAK AND SPIN-DEPENDENT ELECTROMAGNETIC AMPLITUDES

In general, one expects all parton-model amplitudes with two or more currents to have fixed-pole behavior. Physically, the large four-momentum q^μ of the current is routed through the parton propagator in Fig. 3, rather than the supporting strongly-convergent parton-hadron scattering amplitude. The physical current scattering amplitude $T^{\mu\nu}$ thus reflects the elementary fixed-pole dependence of the parton-Born amplitudes. In this appendix we will give the parton theory fixed-poles for both neutrino inelastic scattering[21] and spin-dependent electroproduction.[18]

The spin-averaged virtual weak current scattering amplitude (with absorptive parts corresponding to the inelastic neutrino structure functions) has the form

$$\begin{aligned} T_{\mu\nu} &= \left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)T_1+\left(p_\mu-\frac{p\cdot q\, q_\mu}{q^2}\right)\left(p_\nu-\frac{p\cdot q\, q_\nu}{q^2}\right)\frac{T_2}{M^2}-i\,\frac{\epsilon_{\mu\nu\rho\sigma}}{2M^2}p^\rho q^\sigma T_3+\frac{q_\mu q_\nu}{M^2}T_4+\frac{(p_\mu q_\nu+p_\nu q_\mu)}{2M^2}T_5 \\ &= -g_{\mu\nu}\hat{T}_1+\frac{p_\mu p_\nu}{M^2}\hat{T}_2-i\frac{\epsilon_{\mu\nu\rho\sigma}}{2M^2}p^\rho q^\sigma \hat{T}_3+\frac{q_\mu q_\nu}{M^2}\hat{T}_4+\frac{(p_\mu q_\nu+p_\nu q_\mu)}{2M^2}\hat{T}_5\,. \end{aligned} \tag{A1}$$

The $\hat{T}_i(q^2, p\cdot q)$ are the kinematic-singularity-free amplitudes related to T_i by

$$\begin{aligned} &T_i=\hat{T}_i\ (i=1,2,3),\quad T_5=\hat{T}_5+\frac{2p\cdot q}{q^2}\hat{T}_2\,, \\ &T_4=\hat{T}_4-\frac{M^2\hat{T}_1}{q^2}-\frac{(p\cdot q)^2}{q^2}\hat{T}_2\,. \end{aligned} \tag{A2}$$

The numerator of the contributing spin-averaged parton amplitude is

$$\begin{aligned} t_{\mu\nu} &= \tfrac{1}{4}\mathrm{Tr}[(\not k+m)\gamma_\mu(1-\gamma_5)(\not k+\not q+m)\gamma_\nu(1-\gamma_5)] \\ &= 2\,[k_\mu(k+q)_\nu+k_\nu(k+q)_\mu-g_{\mu\nu}k\cdot(k+q) \\ &\quad -i\,\epsilon_{\mu\nu\rho\sigma}(k+q)^\rho k^\sigma]\ . \end{aligned} \tag{A3}$$

The parton momentum k^μ can be computed using the infinite-momentum method of Sec. II or the explicitly covariant method of Appendix B. The identification of terms of the same order in P then yields

$$\left.\begin{aligned} &\hat{T}_1=2xp\cdot q \\ &\hat{T}_2=4x^2M^2 \\ &\hat{T}_3=-4xM^2 \\ &\hat{T}_4=0 \\ &\hat{T}_5=4xM^2 \end{aligned}\right\}\times\int_0^1\frac{f(x)}{x}\frac{1}{2xp\cdot q+q^2+i\epsilon}\,dx \tag{A4}$$

plus an equal contribution with $p\cdot q\to -p\cdot q$, $\mu\to\nu$ obtained from the crossed amplitude.

Note that we have approximated the parton propagator

$$\frac{1}{(k+q)^2-m^2+i\epsilon}=\frac{1}{x\{M^2+2p\cdot q-[(\vec{k}_\perp+\vec{q}_\perp)^2+m^2]/x-(\vec{k}_\perp{}^2+\lambda^2)/(1-x)\}+i\epsilon}\to\frac{1}{2xp\cdot q+q^2+i\epsilon}\ . \tag{A5}$$

The leading fixed-pole behavior of the five invariant amplitudes is thus

$$\left.\begin{aligned} &\hat{T}_1^{\mathrm{FP}}=2\\ &\hat{T}_2^{\mathrm{FP}}=-\frac{2(q^2+a)M^2}{(p\cdot q)^2}\\ &\hat{T}_5^{\mathrm{FP}}=-\hat{T}_3^{\mathrm{FP}}=\frac{4M^2}{p\cdot q}\\ &\hat{T}_4^{\mathrm{FP}}=0\end{aligned}\right\}\times\left[\int_0^1 dx\frac{\tilde{f}(x)}{x}-\sum_\alpha\gamma_\alpha\frac{1}{\alpha}\right]. \tag{A6}$$

In the case of T_2, the direct and crossed amplitudes cancel in leading order; thus terms of order a/ν^2, where a is a model-dependent constant, can arise from corrections to the approximation (A5) as well as nonleading "$T^{(6)}$" diagrams. In some cases one can show that a, and hence

$$T_5^{\mathrm{FP}}=\hat{T}_5^{\mathrm{FP}}+\frac{2p\cdot q}{q^2}T_2^{\mathrm{FP}}=\frac{-4aM^2}{(p\cdot q)q^2}, \tag{A7}$$

as well as T_4^{FP}, vanishes, if, for example, a Ward identity for the weak current is satisfied, or if Z-graph contributions can be neglected at $P\to\infty$, as in composite models. This is discussed further in Ref. 22. As in the derivation of T_2^{FP} for the electromagnetic currents given in Sec. III, such a result depends on the cancellation of contributions from $T^{(4)}$ and $T^{(6)}$ diagrams.

As usual, the fixed-pole sum rules (A6) and (A7) must be summed over the types of parton constituents. The subtraction terms required for convergence in the case by Regge behavior can be obtained explicitly using the method of Sec. III or the analytic continuation method in the Conclusion. From Eq. (A4), we also obtain the scaling results ($\nu=p\cdot q/M$, $x=-q^2/2M_\nu$)

$$\hat{W}_1=\frac{f(x)}{2},\quad \frac{\nu}{M}\hat{W}_2=xf(x),\quad \frac{\nu}{M}\hat{W}_3=-f(x),\quad \frac{\nu}{M}\hat{W}_4=0,\quad \frac{\nu}{M}\hat{W}_5=f(x), \tag{A8}$$

and thus $\nu W_5=\nu W_4=0$.

Let us return to the electromagnetic case and examine the spin-dependent structure functions. Assuming the initial and final proton spin states are the same, we define

$$T_{\mu\nu}=\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)T_1+\left(p_\mu-\frac{p\cdot q\,q_\mu}{q^2}\right)\left(p_\nu-\frac{p\cdot q\,q_\nu}{q^2}\right)T_2+4Mi\,\epsilon_{\mu\nu\lambda\sigma}q^\lambda\left[\frac{s^\sigma G_1}{M^2}+\frac{(p\cdot q s^\sigma-s\cdot q p^\sigma)G_2}{M^4}\right], \tag{A9}$$

with $s^2=-1$, $s\cdot p=0$.

Let us assume the proton has positive helicity. Choosing the reference frame as in Sec. II, we can take the helicity vector s as

$$s_\mu^{(+)}=\frac{1}{M}\left(P,\vec{0}_\perp,P+\frac{M^2}{2P}\right),\quad P\to\infty; \tag{A10}$$

then $s\cdot p=0$, $s^2=-1$, $s\cdot q=p\cdot q/M$. The positive or negative spin vector for the on-mass-shell parton can be written as

$$w_\mu^\pm=\pm\left(xP,\vec{w}_\perp,xp+\frac{m^2+\vec{w}_\perp^{\,2}}{2xP}\right)\frac{1}{m}; \tag{A11}$$

then $w^2=-1$, and $w\cdot k=0$ if $\vec{w}_\perp\cdot\vec{k}_\perp=0$, $\vec{w}_\perp^{\,2}=\vec{k}_\perp^{\,2}$.

In general the parton-proton scattering amplitude has helicity-conserving and helicity-nonconserving contributions. Let us define h_+ (h_-) to be the amplitude for the emission of a positive- (negative-) helicity parton from the positive helicity proton. The numerator of the contribution (uncrossed) parton amplitude is then

$$\tfrac{1}{2}\mathrm{Tr}\left[(\not{k}+m)\gamma_\mu(\not{k}+\not{q}+m)\gamma_\nu\left\{h_+\left(\frac{1+\gamma_5\not{w}_+}{2}\right)+h_-\left(\frac{1+\gamma_5\not{w}_-}{2}\right)\right\}\right]=(h_+-h_-)\,im\,\epsilon_{\mu\nu\rho\sigma}q^\rho w_+^\sigma+\cdots. \tag{A12}$$

Matching this to $T_{\mu\nu}$ we obtain

$$G_2^{\mathrm{FP}}=0 \tag{A13}$$

and (taking $\mu=0$, $\nu=1$)

$$\frac{4G_1^{\mathrm{FP}}}{M^2}=\lim_{p\cdot q\to\infty}\int_0^1 dx[f_+(x)-f_-(x)]\times\left(\frac{1}{2xp\cdot q+q^2+i\epsilon}-\frac{1}{-2xp\cdot q+q^2+i\epsilon}\right) \tag{A14}$$

or

$$\frac{4p\cdot q}{M^2}G_1^{\mathrm{FP}}=\left[\int_0^1 dx\left(\frac{\tilde{f}_+(x)-\tilde{f}_-(x)}{x}\right)-\sum_{\alpha^+}\frac{\gamma_{\alpha^+}}{\alpha^+}+\sum_{\alpha^-}\frac{\gamma_{\alpha^-}}{\alpha^-}\right], \tag{A15}$$

where $f_\pm(x)$ is obtained from integrating $h_\pm$ over $d^2k_\perp$ and $f(x)=\frac{1}{2}[f_+(x)+f_-(x)]$. Since the amplitude $G_1(\nu,q^2)$ has leading behavior $\nu^{\alpha-1}$, the fixed-pole contribution is a Kronecker-δ singularity at $J=0$. The corresponding scaling structure function is

$$\frac{4p\cdot q}{M^2}\pi\,\mathrm{Im}G_1(x)=\frac{f_+(x)-f_-(x)}{2},\quad x=\frac{-q^2}{2p\cdot q}. \tag{A16}$$

The results (A15) and (A16) are to be summed over parton types weighted by square of the charge.

APPENDIX B

In this appendix we give a direct, general connection between the explicitly covariant and infinite-momentum (or light-cone variable) techniques.[23]

For illustration consider the covariant expression for the full vertex function shown in the first diagram of Fig. 10. For the spin-0 case we have

$$\Gamma^{\mu}=(2p+q)^{\mu}F(q^2)$$
$$=\int\frac{d^4k/i}{(2\pi)^4}\frac{(2k+q)^{\mu}\mathfrak{M}(k^2,(k+q)^2,u,t)}{[k^2-\mu_0{}^2+i\epsilon][(k+q)^2-\mu_0{}^2+i\epsilon]}, \quad \text{(B1)}$$

where $\mathfrak{M}$ is the full off-shell parton-proton amplitude with $u=(p-k)^2$ and $t=q^2$. As usual we write a u-channel dispersion relation

$$\mathfrak{M}=\frac{1}{\pi}\int\frac{\operatorname{Im}\mathfrak{M}(k^2,(k+q)^2,\mathfrak{m}^2,t)\,d\mathfrak{m}^2}{u-\mathfrak{m}^2+i\epsilon} \quad \text{(B2)}$$

modulo possible subtraction terms. We can parameterize the four-momenta as follows:

$$p=\left(P+\frac{M^2}{4P},\vec{0}_{\perp},P-\frac{M^2}{4P}\right),$$
$$q=\left(\frac{p\cdot q}{2P},\vec{q}_{\perp},-\frac{p\cdot q}{2P}\right)\quad q^2=-\vec{q}_{\perp}{}^2, \quad \text{(B3)}$$
$$k=\left(xP+\frac{\vec{k}_{\perp}{}^2+k^2}{4xP},\vec{k}_{\perp},xP-\frac{\vec{k}_{\perp}{}^2+k^2}{4xP}\right).$$

Notice that all invariants are independent of the parameter $P=\frac{1}{2}(p_0+p_3)$. Unlike the infinite-momentum calculation, P need not be large. For example, in the target rest frame, $P=M/2$; in general $\ln(2P/M)$ is the rapidity of p_{ν}.

The four degrees of freedom in k^{μ} are thus replaced by

$$\int\frac{d^4k/i}{(2\pi)^4}=\frac{1}{(2\pi)^3}\int d^2k_{\perp}\int_{-\infty}^{\infty}\frac{dx}{2|x|}\int_{-\infty}^{\infty}\frac{dk^2}{2\pi i}. \quad \text{(B4)}$$

The great merit of this parameterization (B3) is the simple factorization of the k^2 integration. For the calculation of Γ^{μ}, all the singularities in the k^2 plane necessarily lie in the lower half plane, except for the pole arising from

$$u-\mathfrak{m}^2+i\epsilon=(1-x)\left[M^2-\frac{k^2+\vec{k}_{\perp}{}^2}{x}-\frac{\mathfrak{m}^2+\vec{k}_{\perp}{}^2}{1-x}\right]+i\epsilon. \quad \text{(B5)}$$

Thus if $(1-x)/x$ is negative, the k^2 integration gives zero. On the other hand, for $0\leq x\leq 1$, we can close the contour in the upper half plane and obtain

$$F(q^2)=\frac{\Gamma^0+\Gamma^3}{2P}$$
$$=\frac{1}{\pi}\frac{1}{(2\pi)^3}\int d\mathfrak{m}^2\int_0^1 dx\,\frac{\operatorname{Im}\mathfrak{M}(k^2,(k+q)^2,\mathfrak{m}^2,t)|_{u=\mathfrak{m}^2}}{2x(1-x)[M^2-S][M^2-\tilde{S}]}, \quad \text{(B6)}$$

where

$$S=S(\vec{k}_{\perp}{}^2,x)=\frac{\vec{k}_{\perp}{}^2+\mu_0{}^2}{x}+\frac{\vec{k}_{\perp}{}^2+\mathfrak{m}^2}{1-x},$$
$$\tilde{S}=S(\vec{k}_{\perp}+(1-x)\vec{q}_{\perp},x), \quad \text{(B7)}$$

and at the pole $u=\mathfrak{m}^2$

$$k^2-\mu_0{}^2=x[M^2-S],$$
$$(k+q)^2-\mu_0{}^2=x[M^2-\tilde{S}]. \quad \text{(B8)}$$

This result reproduces the infinite-momentum TOPT results of Secs. I and II. The resulting expression for $f(x)$ is the LPS formula (I.11). Equation (B6) is a further generalization of the model-covariant calculations for $F(q^2)$ given in Eq. (III.13).

This method for relating covariant and infinite-momentum-frame calculations has general applicability; inclusion of spin factors is trivial. Notice that since $P=\frac{1}{2}(p_0+p_3)$ is an arbitrary parameter, we can, as usual, interpret the "light-cone" variable $x=(k_0+k_3)/2P$ as the fractional longitudinal momentum of the charged constituent in the frame in which p_3 becomes infinite. As we have stressed, any integrated result which diverges formally at $x\sim 0$ due to Regge behavior $\operatorname{Im}\mathfrak{M}\sim(\mathfrak{m}^2)^{\alpha}$ will be rendered finite when the subtraction terms in Eq. (B2) are considered; the final result can equivalently be obtained from analytic continuation in α.

APPENDIX C: MASS SHIFTS IN THE PARTON MODEL

The lowest-order shift in energy due to a change in the parton masses M_a can be obtained immediately in the parton model:

$$\delta E=\frac{M\,\delta M}{P}=\sum_a\left\langle\frac{M_a\,\delta M_a}{x_aP}\right\rangle,$$

i.e.,

$$\delta M^2=\sum_a\int_0^1 dx\,\frac{f_a(x)}{x}\,\delta M_a{}^2, \quad \text{(C1)}$$

which is the result obtained by Weisberger.[24] In fact, Eq. (C1) is undefined if $\nu W_2(x)$ has Regge behavior, and a more careful derivation must be given. For the case of scalar fields, the interaction energy density due to the mass shift

$$\mathcal{H}_I=\sum_a\phi_a^{\dagger}\,\delta M_a{}^2\,\phi_a \quad \text{(C2)}$$

is similar in operator structure to the electromagnetic seagull term

$$\mathcal{H}_I = -\sum_a e_a{}^2 \phi_a^\dagger(x)\phi_a(x)\vec{A}^2(x)\,. \tag{C3}$$

Thus we can use the analysis of Sec. III to show that the Regge subtraction terms necessary for the parton-proton amplitude representation lead to a subtracted form for the mass shift

$$\delta M^2 = \sum_a \int_0^\infty dx \frac{\tilde{f}_a(x)}{x}\,\delta M_a{}^2\,, \tag{C4}$$

where

$$\tilde{f}_a(x) = f_a(x)\,\theta(1-x) - \sum_{\alpha>0} x^{-\alpha}\gamma_\alpha \to 0 \text{ for } x\to 0 \tag{C5}$$

as in Eq. (III.28). The proof for spin-$\frac{1}{2}$ fields is similar: In this case we note the similarity of the Z-graph (effectively local) contributions of the $P\to\infty$ analysis

$$\mathcal{H}_I = \sum_a e_a{}^2 \frac{\overline{\psi}\gamma_i\gamma_0\gamma_j\psi A_i A_j}{x_a P} \tag{C6}$$

to the mass-shift interaction

$$\mathcal{H}_I = \sum_a \overline{\psi}\,\delta M_a \psi \tag{C7}$$

and we again obtain (C1). Note that if $\delta M_a{}^2 = e_a{}^2/e^2\,\delta M_0{}^2$, as is the case for the electromagnetic mass shifts of the partons, then we obtain

$$\delta M_a{}^2 = \frac{T_1^{\text{FP}}}{T_1^{\text{Born}}}\,\delta M_0{}^2 \tag{C8}$$

for the shift of the bound-state mass M due to electromagnetic mass shifts of the constituents and T_1^{FP} in the $J=0$ fixed pole (Kronecker delta: δ_{J0}) which can be obtained from Eq. (III.28). More generally we can obtain the mass shift due to other interactions (e.g., from the λ-$\mathcal{P}$ quark mass differences) and obtain the tadpole-model results for masses squared. These results also agree with the results obtained by Jaffe and Llewellyn Smith.[25] If the partons are isosinglets and isodoublets only as in the quark model, effects of the parton mass shift cancel in the $\pi^\pm$-π^0 mass difference but not in the n-p mass difference.

In general, the Cottingham formula for the electromagnetic mass shift includes the shift due to the electromagnetic self-energy δM_a of the field-theoretic constituents. The contribution is formally logarithmically (quadratically) divergent for spin-$\frac{1}{2}$(0) constituents, and thus the Cottingham formula will be divergent for $\Delta I<2$ mass differences such as n-p as long as scaling holds. As in the case of the leptons, higher-order electromagnetic or weak corrections presumably render the $\mathcal{N}$-$\mathcal{P}$ quark mass difference finite. Thus if the divergent piece of the Cottingham formula can be exactly identified with the self-mass divergences we obtain a finite result for the total mass shift:

$$\delta M_{\text{tot}}{}^2 = (\delta M^2)_{\text{R}} + (\delta M^2)_{\text{parton}}\,,$$

where $(\delta M^2)_{\text{R}}$ is the finite mass shift obtained from renormalizing the Cottingham formula via a subtraction term of the form (C4) with $(\delta M^2)_a$ given by the standard quantum-electrodynamics (QED) spin-$\frac{1}{2}$ or spin-0 result (covariant regularization in the photon mass is assumed), and $(\delta M^2)_{\text{parton}}$ is the shift in mass due to the physical finite mass shifts of the constituents, and may be computed from Eq. (C8). Thus, from this point of view, the n-p nucleon mass differences can never be computed from integrals over scaling contributions in the Cottingham formula without knowing the mass difference of the $\mathcal{N}$-quark and $\mathcal{P}$-quark constituent. This program for renormalization and further consequences of this point of view are discussed in Ref. 22.

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[1]S. D. Drell and T.-M. Yan, Ann. Phys. (N.Y.) 66, 555 (1971) and references therein.

[2]P. V. Landshoff, J. C. Polkinghorne, and R. Short, Nucl. Phys. B28, 225 (1971).

[3]S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972).

[4]S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D 5, 1384 (1972). The relation of the T_1 sum rule to atomic and nuclear physics problems is also discussed in this reference. S. J. Brodsky, F. E. Close, and J. F. Gunion, Phys. Rev. D 6, 177 (1972). This paper gives the phenomenological implications of a fixed singularity in the Compton amplitude.

[5]J. F. Gunion, S. Brodsky, and R. Blankenbecler, Phys. Lett. 39B, 649 (1972); Phys. Rev. D 8, 287 (1973).

[6]S. Weinberg, Phys. Rev. 150, 1313 (1966).

[7]S. J. Brodsky and R. Roskies, Phys. Lett. 41B, 517 (1972); S. J. Brodsky, R. Roskies, and R. Suaya, Phys. Rev. D (to be published).

[8]R. Blankenbecler, M. L. Goldberger, S. W. MacDowell, and S. B. Treiman, Phys. Rev. 123, 692 (1961). We wish to thank R. Blankenbecler for discussions on this point. A further extension of this method and the techniques of Appendix B for the convolution of two 4-point amplitudes is given by R. Blankenbecler, S. Brodsky, J. Gunion, and R. Savit [Phys. Rev. D (to be published)].

[9]Note that no additional subtraction terms enter. This is because the strong interaction parton-proton amplitude is assumed not to have any $a = 0$ fixed singularities itself. (This is the case for strong interaction amplitudes which obey the usual nonlinear unitarity equation. See also Ref. 5.) The subtraction term for the Regge piece is fixed by our convention in which the Regge contribution vanishes at $s = 0$. Other definitions are possible but do not alter the results.

[10]Note that the spin-0 parton seagull terms actually measure as well the magnitude of the operator Schwinger term. Since the distribution function of spin-0 partons is $2F_L(x)$, the magnitude of the Schwinger term S is given by

$$S = 4\int_0^\infty \frac{\tilde{F}_L(x)dx}{x},$$

with the Regge regulation defined as in (III.28). A detailed discussion of the Schwinger term is given in D. Broadhurst, J. F. Gunion, and R. L. Jaffe, Phys. Rev. D 8, 566 (1973).

[11]J. Cornwall, P. Corrigan, and R. Norton, Phys. Rev. Lett. 24, 1141 (1970); Phys. Rev. D 3, 536 (1971). These authors derive the correct expression for T_1^{FP} for purely spin-$\frac{1}{2}$ partons by assuming the Schwinger term to be 0. As we here show, the T_1^{FP} expression is completely independent of parton spin.

[12]T. P. Cheng and W. K. Tung, Phys. Rev. Lett. 24, 851 (1970).

[13]D. M. Scott, DAMPT report, 1973 (unpublished).

[14]R. Blankenbecler, S. Brodsky, and J. Gunion (in preparation).

[15]P. V. Landshoff and J. C. Polkinghorne, Phys. Rev. D 5, 2050 (1972).

[16]M. Bander, Phys. Rev. D 5, 3274 (1972).

[17]Y. Frishman, in *Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia, Ill., 1972,* edited by J. D. Jackson and A. Roberts (NAL, Batavia, Ill., 1973), Vol. 4, p. 119. See also K. Wilson, in *Proceedings of the 1971 International Symposium on Electron and Photon Interactions at High Energies,* edited by N. B. Mistry (Laboratory of Nuclear Studies, Cornell University, Ithaca, N. Y., 1972), p. 115.

[18]R. P. Hughes and H. Osborn, Nucl. Phys. B54, 603 (1973). These authors also consider the spin-dependent structure-function fixed poles discussed in Appendix A.

[19]See Ref. 5 for a complete discussion. The size of the fixed pole in on-shell Compton scattering assuming the absence of this last complication has been estimated in the case of proton targets by M. Damashek and F. J. Gilman, Phys. Rev. D 1, 1319 (1970); C. A. Dominguez, C. Ferro Fontan, and R. Suaya, Phys. Lett. 31B, 365 (1970). For neutron targets, the analysis has been performed by C. A. Dominguez, J. F. Gunion, and R. Suaya [Phys. Rev. D 6, 1404 (1972)] and for deuteron targets by J. F. Gunion and R. Suaya [*ibid.* 8, 156 (1973)].

[20]K. Johnson, Phys. Rev. D 6, 1101 (1972); S. D. Drell and K. Johnson, *ibid.* 6, 3248 (1972).

[21]D. Broadhurst, J. F. Gunion, and R. J. Jaffe (in preparation) discuss further sum rules and their consequences, which may be gleaned from the fixed-pole properties given in this Appendix or proved more directly by using parton-model techniques.

[22]J. F.Gunion, Phys. Rev. D 8, 517 (1973).

[23]We wish to thank Dr. M. Schmidt for suggesting to us the use of explicit k^2 integration to connect the infinite momentum and covariant approaches. See M. Schmidt (unpublished). The method used here is closely related to the Sudakov variable analysis of LPS, Ref. 2, and the method of S. Chang and S. Ma, Phys. Rev. 180, 1506 (1969); 188, 2385 (1969). See also Ref. 7 and T. M. Yan, Phys. Rev. D 7, 1780 (1973) and references cited therein.

[24]M. Weisberger, Phys. Rev. D 5, 2600 (1972).

[25]R. L. Jaffe and C. H. Llewellyn Smith, Phys. Rev. D 7, 2506 (1973), have also considered the problem of mass shifts in scaling models and have derived similar expressions. They assume the fixed-pole properties proved for parton models in this paper.