for some constants a, b, and c. A similar result is valid for A_{II}. The behavior exhibited in (95) agrees qualitatively with experimental results. Moreover, a form quite similar to (95) has been used by Hwa[5] to obtain very reasonable multiplicity distributions.

The numerical test of this model is sufficiently complicated so we shall defer it to another publication.

APPENDIX

We derive Eq. (5) in this appendix. Writing $q_i = x_i q_\perp$, and dropping the subscript x on k_{ix}, we can write the integral in matrix form as follows:

$$I(q_\perp, K) = \frac{1}{2\pi}\int d^n k\, e^{-i\lambda K} d\lambda \times \exp[-\tfrac{1}{2}kak - \tfrac{1}{2}(k-q)a(k-q) + i\overline{\lambda}\cdot k], \qquad \text{(A1)}$$

where $\overline{\lambda}$ is a vector all of whose components are equal to λ. Completing the square in k and carrying out the k integrations, we obtain

$$\begin{aligned} I(q_\perp, K) &= \text{const.}\int_{-\infty}^{\infty} d\lambda \exp[-i\lambda K + \tfrac{1}{2}(q + i\overline{\lambda}a^{-1}) \times a(q + ia^{-1}\overline{\lambda})] \\ &= \text{const.}\int_{-\infty}^{\infty} d\lambda \exp[-i\lambda K + \tfrac{1}{2}i\lambda Q - \tfrac{1}{4}A^{-1}\lambda^2 - \tfrac{1}{4}qaq], \end{aligned} \qquad \text{(A2)}$$

where

$$Q = \sum_{i=1}^{n} q_i$$

and

$$A^{-1} = \sum_{i,j=1}^{n} (a^{-1})_{ij}.$$

Completing the square in λ in (A2) and carrying out the λ integration, Eq. (5) emerges.

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Large-Angle Scattering and the Interchange Force*

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A composite model of hadrons is used to discuss high-energy, large-angle scattering for elastic and quasielastic reactions. Arguments are given that constituent interchange should be the dominant interaction at large angles (both $|t|$, $|u| \gg m^2$), rather than gluon exchange. Using the asymptotic behavior of the electromagnetic form factors of the hadrons, predictions are made for the energy and angular dependence of a large number of processes. Detailed numerical comparisons with experiment are given for several reactions. A general discussion of the qualitative behavior of large-angle quasi-two-body processes is given. It is shown that data in this region can determine the quantum numbers and wave functions of the constituents.

I. INTRODUCTION

If any aspect of hadron-hadron reactions can reflect the properties of a basic interaction at short distance, it must be the region of large angle, large momentum transfer. Certainly, coherent multiparticle effects dominate in the forward and backward directions, but such Regge contributions fall off extremely rapidly, perhaps exponentially, as t or u increases. The observed smooth, structureless, and approximate power-law behavior of the pp and πp scattering cross sections at large t and u could be reflecting the simplicity of the elementary forces within and between hadrons. If hadrons are composite,[1] which would provide a natural explanation of the scaling observed in deep-inelastic electron scattering,[2] then there are two fundamental types of interaction mechanisms which would control large-angle hadronic scattering: (A) a direct elementary interaction between individual constituents of the participating hadrons, such as depicted in Fig. 1(a), and (B) the interchange of common constituents between hadrons as shown in Fig. 1(b).

The central assumption and starting point of our theory of scattering in the *deep* region, which is defined to be large s, t, and u, is that eventually the interaction time is sufficiently brief so that only a single interaction or interchange is possible. In our view, this is the definition of the relativistic impulse approximation for hadron-hadron scattering. Of course, corrections to this result can and must be computed to extend the validity of our lowest-order predictions to non-asymptotic regions. Both types of interaction, (A) and (B), probe the short-distance region since they depend upon the large-transverse-momentum behavior of the wave functions describing the binding of the constituents.

Direct interactions of type (A) surely occur in nature, if not in strong interactions, then at least on the electromagnetic level.[3] However, the lack of large corrections to Bjorken scaling, the absence of an obvious $G_E{}^4(t)$ term in elastic proton-proton scattering,[4] and the strikingly different angular distributions of pp and $\bar{p}p$ argue for a small coupling of such interactions. Independently of whether or not the above type of direct interaction is present, *constituent interchange inevitably takes place in any composite model*.[5,6] Therefore it becomes an experimental question to determine for what processes and in what kinematical regime a particular basic interaction mechanism dominates. Since the asymptotic form of the interchange amplitude can be obtained in a manner independent of the exact nature of the binding interaction, it provides the most economical and

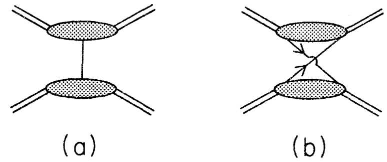

FIG. 1. The two basic types of interactions between hadrons: (a) gluon interchange and (b) constituent interchange.

simplest possible description of hadronic reactions in the deep scattering region.[7] In this paper, we shall explore the attractive possibility that constituent interchange dominates deep scattering and show that it is capable of correlating and describing many reactions for large s, t, and u, given only the asymptotic behavior of the electromagnetic form factors. Up to an over-all normalization constant, the asymptotic form for deep proton-proton scattering is predicted and appears to be in good agreement with experiment. Other processes are also treated. It should be stressed at this point that our purpose in this paper will be to develop a simple model of the hadrons and to confine our predictions to kinematic regions where the theory also is particularly simple. Extensions to the more general case will be given later.

In this paper, we will present a new covariant approach to the scattering of composite systems which is a rigorous alternative to the Bethe-Salpeter equation and greatly simplifies calculations of the interchange interaction. It is based on time-ordered perturbation theory in the infinite-momentum frame.[8] In this approach, the identification of the hadronic constituents with the carriers of the electromagnetic current is a natural assumption and has the following consequences:

(a) Bjorken scaling—the resulting treatment of inelastic electron scattering is equivalent to standard models.

(b) Drell-Yan relation—the connection between the form factor and the threshold behavior of the structure function[9] is automatically incorporated.

(c) Bloom-Gilman duality—in inelastic electron scattering, the resonances and the background fall off at the same rate in momentum transfer.[10]

(d) Electromagnetic fixed poles—the fixed poles in the Compton amplitude automatically have a polynomial residue.[11]

(e) Semihadronic fixed poles—$J=0$ fixed singularities in the amplitudes for photoproduction of composite hadrons are absent.

(f) Regge behavior—the requirements of binding and the existence of the interchange force allows the theory to develop Regge behavior in a manner which is fully consistent with conventional Regge theory at low momentum transfer. In addition, it describes the way in which Regge behavior is joined smoothly onto fixed-angle behavior.

In this paper, this new formalism is utilized to calculate several deep exclusive hadronic reactions. Its application to inclusive processes is presented in Ref. 6. It is amusing to note that our calculation of the interchange force is analogous to the computation of atom-atom scattering via electron interchange (rearrangement collision) with the neglect of higher-order electron-electron interactions.

It should be emphasized that constituent or parton interchange will occur, regardless of whether or not free asymptotic parton states exist. Johnson has argued that the absence of free asymptotic states may be consistent with certain field theories.[12] For our purposes, the noninteracting and virtual-parton states that are used may correspond simply to a combination of physical states with unit form factor. Thus, physical parton production need not actually be implied for inclusive processes. However, the identification of the basic constituents as the carriers of the electromagnetic current is necessary in order to relate deep scattering to the asymptotic behavior of the E and M form factors. Conversely, deep scattering may be used to predict the form factors of particles such as the π and K mesons.

If the choice of the quantum numbers of the constituents is made as in the valence quark model, then in some ways, one may regard the interchange theory as a dynamical realization of the duality diagrams of Harari and Rosner.[13] However, in the dynamical calculations presented here, we have no difficulty in treating pp and $\bar{p}p$ scattering and have no problems introducing spin into the theory. Interpolating formulas between the deep scattering and Regge regions are easily constructed and lead to asymptotic relations between Regge parameters which are *a priori* unrelated.[14] Our treatment of the deep region, and Feynman's picture of Regge behavior which arises from the interchange of wee partons[15] can form a unified and simple treatment of hadron-hadron scattering.

In the remainder of the Introduction, we will outline the main features of our theory which are then discussed in detail in the listed sections. In order to be able to calculate scattering in the deep region, an approach must be developed which will allow a convenient characterization of composite systems and a simple evaluation of their mutual scattering amplitude. In Sec. II, we shall discuss in detail a natural formalism for covariant bound-state computations which was briefly (and evidently inadequately) described and utilized in our recent papers.[5,6] The formalism uses time-ordered perturbation theory in an infinite-momentum frame of reference. In this formalism, pair creation by the current can be suppressed and the calculation of the electromagnetic form factor can be performed as illustrated in the time-ordered diagram of Fig. 2, where the photon interacts directly with the carriers of the electromagnetic current within the hadrons. All of the complications of the hadronic interactions reside in the multiparticle wave functions. The asymptotic behavior of the form factor $F(q^2)$ is then controlled by the asymptotic behavior in transverse momentum of one of the wave functions as illustrated in Fig. 3(a). The other wave function provides the convergence for the loop integral. Hence the form factor and the wave function have essentially the same asymptotic power-law behavior. In Sec. III the details of the form factor calculation are given in the spinless case and then in the more involved nucleon and pion cases that require the correct treatment of spin.

In Sec. IV, the basic interchange contribution to exclusive scattering, illustrated by the topologies shown in Fig. 3(b), is discussed in detail. This process can be computed in terms of the same wave functions used in the form-factor discussion. In the deep scattering domain, one wave

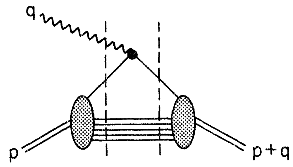

FIG. 2. The relevant time-ordered diagram for the form factor in a constituent model. Pair creation by the current is suppressed in the $P\rightarrow\infty$ limit for the frame (2.1).

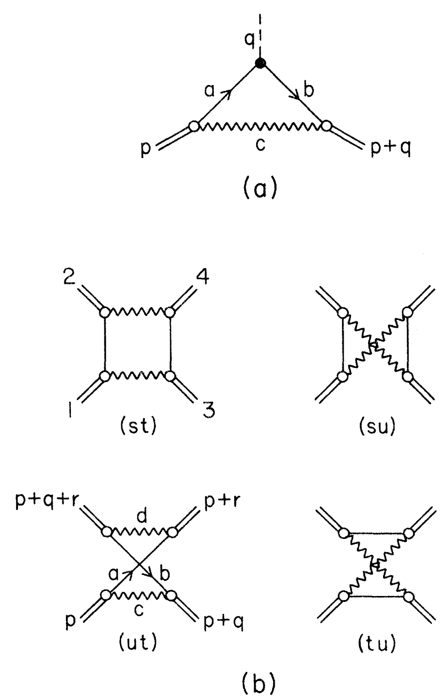

FIG. 3. The basic graph topologies for (a) the form factor and (b) hadron-hadron scattering.

function again supplies the convergence for the single-loop integration and each of the other three supply the asymptotic behavior of a form factor with an appropriate argument. The approximate structure of the (u, t) term in the invariant scattering amplitude for large s, t, and u turns out to be

$$M(s, t, u) \sim sF(s)F(t)F(u) . \tag{1.1}$$

Besides presenting the basic interchange calculation, in Sec. IV we also discuss a channel Hamiltonian formulation of rearrangement collisions which can considerably simplify certain calculations in time-ordered perturbation theory. In Sec. V, details are given on the additional effects of spin and crossing symmetry for a number of physical cases, including kaon-nucleon scattering, pion-nucleon scattering, nucleon-nucleon, and quasi-two-body reactions. In Sec. VI, the extension and continuation to nucleon-antinucleon elastic scattering and their annihilation to two mesons is discussed. In all cases, the interchange theory predicts an asymptotic fixed-angle cross section of the form

$$\frac{d\sigma}{dt} \sim s^{-N} R(\cos\theta), \tag{1.2}$$

where θ is the center-of-mass scattering angle.[16,17] The value of N and the function R are determined by the asymptotic falloff of the form factors of the hadrons involved in the reaction. Our theory then predicts a factorized form for the differential cross section in the deep region which provides a smooth interpolation between the structured forward and backward peaks.

II. RELATIVISTIC DESCRIPTION OF COMPOSITE STATES

As mentioned in the Introduction, the simplicity and elegance of the interchange theory discussed here rest in large part on the fact that a composite state is simply and concisely described in time-ordered perturbation theory in the infinite-momentum frame. As a preliminary to the true bound-state discussion let us first consider the form factor of a spin-zero hadron which is coupled to order g to a charged and neutral scalar constituent. The vertex can be computed from simple time-ordered perturbation theory (in the manner of Heitler) to order eg^2 from 3! time-ordered graphs. However, if we follow Weinberg[8] and Drell, Levy, and Yan,[8] and choose the reference frame such that

$$\begin{aligned} p_\mu &= ((M^2+P^2)^{1/2}, \vec{0}_\perp, P) \\ &\simeq (P + M^2/2P, \vec{0}_\perp, P), \\ q_\mu &= (q_0, \vec{q}_\perp, 0) \\ &\simeq \left(\frac{p\cdot q}{P}, \vec{q}_\perp, 0\right), \end{aligned} \tag{2.1}$$

with

$$(p+q)^2 = M^2 = M^2 + 2p\cdot q + q^2$$

and

$$q^2 = -q_\perp{}^2 + O(1/P^2),$$

then, in fact, only the single time ordering of Fig. 3(a) contributes in the limit $P\to\infty$. (The other five diagrams are suppressed by two powers of P due to the presence of backward-moving particles in an intermediate state.) The vertex contribution to the form factor from Fig. 3(a) is

$$\begin{aligned} &\langle p+q | J^\mu(0) | p\rangle \\ &\quad\equiv (2p+q)^\mu F(q^2) \\ &\quad= \frac{g^2}{(2\pi)^3}\int \frac{d^3p}{2E_1 2E_2 2E_1'} \\ &\qquad\times \frac{(p_1'+p_1)^\mu}{(E_p - E_1 - E_2)(E_{p+q} - E_1' - E_2)} . \end{aligned} \tag{2.2}$$

Parametrizing the intermediate (on-shell) mommenta as

$$\begin{aligned} p_1^\mu &= \left(|x|P + \frac{m^2+k_\perp{}^2}{2|x|P}, \vec{k}_\perp, xP\right), \\ p_1'^\mu &= \left(|x|P + \frac{(\vec{k}_\perp+\vec{q}_\perp)^2+m^2}{2|x|P}, \vec{k}_\perp+\vec{q}_\perp, xP\right), \\ p_2^\mu &= \left(|1-x|P + \frac{k_\perp{}^2+\lambda^2}{2|1-x|P}, -\vec{k}_\perp, (1-x)P\right), \end{aligned} \tag{2.3}$$

leads to energy denominators of the simple form

$$E_p - E_1 - E_2 = \begin{cases} \dfrac{1}{2P}\left[M^2 - \dfrac{k_\perp{}^2+m^2}{x} - \dfrac{k_\perp{}^2+\lambda^2}{1-x}\right] \equiv \dfrac{1}{2P}[M^2 - S(\vec{k}_\perp, x)], & 0<x<1 \\ O(P) \quad \text{otherwise} \end{cases} \tag{2.4a}$$

and

$$E_{p+q}-E_1'-E_2=\begin{cases}\dfrac{1}{2P}\left[M^2+2p\cdot q-\dfrac{(\vec{k}_\perp+\vec{q}_\perp)^2+m^2}{x}-\dfrac{k_\perp{}^2+\lambda^2}{1-x}\right]=\dfrac{1}{2P}[M^2-S(\vec{k}_\perp+(1-x)\vec{q},x)], & 0<x<1\\ O(P)\quad\text{otherwise}.\end{cases} \tag{2.4b}$$

Thus taking $P\to\infty$, we obtain the covariant result ($\vec{q}_\perp{}^2=-q^2$)

$$F(q^2)=\frac{1}{2(2\pi)^3}\int d^2k\int_0^1\frac{dx}{x(1-x)}\,\psi(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)\psi(\vec{k}_\perp,x), \tag{2.5}$$

where we have defined a two-particle wave function as

$$\psi(\vec{k}_\perp,x)=g[M^2-S(\vec{k}_\perp,x)+i\epsilon]^{-1}. \tag{2.6}$$

Note that the effective current (for $\mu=0$ or 3) is simply $2Px$.

Despite the simplicity of this example, we shall show in this section that the form of the result (2.5) is correct in general for a two-component wave function (spinless case), and is readily generalizable to the higher particle components of the state.

For the calculation of the asymptotic dependence of the form factor ($|\vec{q}_\perp|\to\infty$), one charged component is at large transverse momentum relative to the rest of the constituents. This is the important configuration that contributes to the asymptotic dependence of the form factor if the general n-point wave function has asymptotic inverse-power-law dependence. (In the case of *exponential* falloff, the large transverse momentum tends to divide among the constituents.) In this asymptotic region, we shall assume that after summing over all higher-particle-number states, the net effect is that the parton sees an effective "core" which acts as a single-particle state. It is evident that for a general bound system of two particles, a and c,

$$\psi(\vec{k}_\perp,x)=\phi(\vec{k}_\perp,x)[M^2-S(\vec{k}_\perp,x)+i\epsilon]^{-1}. \tag{2.7}$$

This displays the bound-state pole at

$$S=\frac{k_\perp{}^2+M_x{}^2}{x(1-x)}=(p_a+p_c)^2\Rightarrow M^2, \tag{2.8}$$

where

$$M_x{}^2=(1-x)M_a{}^2+xM_b{}^2$$

while allowing for additional falloff dependence in the momentum $\vec{k}_\perp$ transverse to the bound-state direction $\vec{P}$. Most generally, the dependence of the vertex factor $\phi(\vec{k}_\perp,x)$ can be expressed in terms of the covariant variables

$$M_c{}^2-(p-p_a)^2=\frac{k_\perp{}^2+M_x{}^2-x(1-x)M^2}{x}=(1-x)[S(\vec{k}_\perp,x)-M^2)] \tag{2.9a}$$

and

$$M_a{}^2-(p-p_c)^2=x[S(\vec{k}_\perp,x)-M^2]. \tag{2.9b}$$

The utilization of wave functions of the type (2.7) for the calculation of form factors and the interchange force is discussed in the next chapters. For these purposes, we must first discuss the manner in which the vertex function ϕ depends on the off-shell variables of Eq. (2.9). A simple example which exhibits the salient features is the case of spinless particles with a spinless gluon

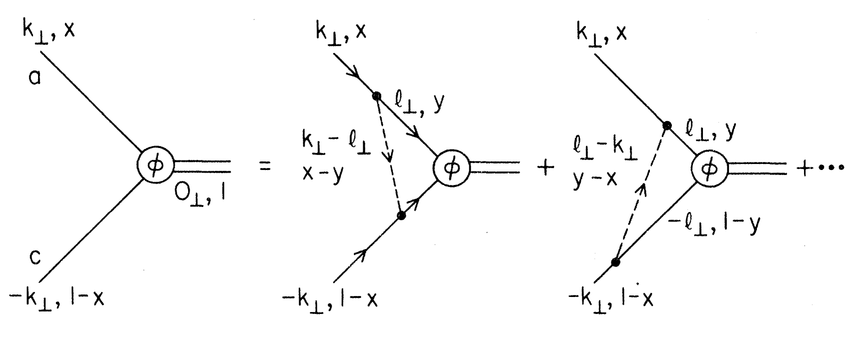

FIG. 4. An example of the lowest-order binding potential and the equation satisfied by the vertex function.

providing the binding force.

The equations for the vertex functions ϕ and the associated wave function ψ are depicted in Fig. 4. The dashed lines are the gluons, and the vertical dotted lines mark the time-ordered energy denominators. We choose a Lorentz reference frame such that

$$p^\mu = \left(P + \frac{M^2}{2P}, \vec{0}_\perp, P\right),$$
$$p_a^\mu = \left(xP + \frac{k_\perp^2 + m_a^2}{2xP}, \vec{k}_\perp, xP\right), \tag{2.10}$$
$$p_c^\mu = \left((1-x)P + \frac{k_\perp^2 + m_c^2}{2(1-x)P}, -\vec{k}_\perp, (1-x)P\right).$$

Since the $\vec{k}_\perp$ and x integrations are uniformly convergent, P can be taken to infinity after the matrix elements are written down. The four-vectors are chosen to conserve three-momentum, and the energy component is computed by applying the mass shell condition as is required in time-ordered perturbation theory.

The exchanged gluon of mass μ and coupling g, which is responsible for the binding, has its four-momentum parametrized as

$$\Pi^\mu = \left((y-x)P + \frac{(\vec{l}_\perp - \vec{k}_\perp)^2 + \mu^2}{2(y-x)P}, \vec{l}_\perp - \vec{k}_\perp, (y-x)P\right) \tag{2.11}$$

for $y > x$, corresponding to the first diagram on the right of Fig. 4, and a similar form holds for the second diagram with $y < x$. The equation for $\phi(k_\perp, x)$ takes the form

$$\phi(\vec{k}_\perp, x) = \frac{1}{2(2\pi)^3}\int d^2l \int_0^1 \frac{dy}{y(1-y)} \frac{\phi(\vec{l}_\perp, y)}{M^2 - S(l_\perp, y) + i\epsilon} V(\vec{l}_\perp, \vec{k}_\perp; x, y), \tag{2.12a}$$

where

$$|y-x|V = g^2\left\{\theta(y-x)\left[M^2 - \frac{\vec{l}_\perp^{\,2} + m_c^2}{1-y} - \frac{\vec{k}_\perp^{\,2} + m_a^2}{x} - \frac{(\vec{l}_\perp - \vec{k}_\perp)^2 + \mu^2}{y-x} + i\epsilon\right]^{-1} + \theta(x-y)\left[M^2 - \frac{\vec{l}_\perp^{\,2} + m_a^2}{y} - \frac{\vec{k}_\perp^{\,2} + m_c^2}{1-x} - \frac{(\vec{l}_\perp - \vec{k}_\perp)^2 + \mu^2}{x-y} + i\epsilon\right]^{-1}\right\}. \tag{2.12b}$$

Note that the normally complicated square-root phase space is linearized in the infinite-momentum-frame limit. Thus the equation for ϕ is

$$\phi(\vec{k}_\perp, x) \equiv [M^2 - S(\vec{k}_\perp, x) + i\epsilon]\psi(\vec{k}_\perp, x) = \frac{1}{2(2\pi)^3}\int d^2l \int_0^1 \frac{dy}{y(1-y)} V(\vec{l}_\perp, \vec{k}_\perp; y, x) \times \psi(\vec{l}_\perp, y). \tag{2.13}$$

This is a fully relativistic description of the bound state within the calculational rules of time-ordered perturbation theory; if V is given, then in principle ψ is completely determined.[18]

For the applications in this paper, we shall need to know that ψ is normalizable and to study its asymptotic form. A discussion of the nonrelativistic limit will be given elsewhere.

There are three limits of interest: $k_\perp^2$ large, $x \sim 1$, and $x \sim 0$. In each of these cases, it is easy to see that the right-hand side of Eq. (2.13) vanishes like S^{-1}. Thus the two covariant off-shell variables enter into the asymptotic dependence of ψ in a symmetric fashion. This unified limiting behavior then allows us to write the effective form for ψ in any of the above limits as

$$\psi(\vec{k}_\perp, x) \sim [S(\vec{k}_\perp, x)]^{-2} N(x), \tag{2.14}$$

where $N(x)$ is a relatively slowly varying function of x which does not vanish at the end points of x. Note that this discussion is appropriate to applications such as the form factors and parton-interchange exclusive scattering contributions where neither particle appears in the initial or final state. When one of the particles appears in the final state, the dependence for $x \sim 1$ and $x \sim 0$ may differ. See Ref. 6 for further discussions.

The inverse-square dependence in Eq. (2.14) is specific to the potential of Eq. (2.12). Since we will not need to commit ourselves to a specific binding interaction and indeed wish to avoid tying our results to a definite potential, our procedure will be to determine the power falloff of ψ in the variable S from the asymptotic form of the electromagnetic form factors. Then knowledge of the form factors will be sufficient to describe the asymptotic behavior of the exclusive cross section in the large t, u region.

If one wishes to normalize the wave function and to compute the relative normalizations of different reactions, then the wave function ψ must be described in more detail. The simplicity of Eq. (2.13) and its close resemblance to the nonrelativistic problem where one's intuition has already been developed, suggests many straightforward phenomenological models for the wave function. Perhaps the simplest is to set ψ equal to a relativistic "Hulthén" form:

$$\psi = (S - M^2)^{-1}(S - M_1^2)^{-1} N(x), \tag{2.15}$$

where $M_1^2 > M^2$ is a parameter characterizing the range of the force, and $N(x)$ is a smooth nonvanishing function of x.

Finally, we state the rule for establishing the correct wave function arguments in the more general case in which the bound state momentum is $z\vec{P} + \vec{q}_\perp$, coupling to constituents with momentum

$$p_a^\mu = \left(xzP + \frac{(\vec{k}_\perp + \vec{q}_\perp)^2 + m_a^2}{2xzP}, \vec{k}_\perp + \vec{q}_\perp, xzP \right),$$
$$p_b^\mu = \left((1-x)zP + \frac{k_\perp^2 + m_c^2}{2(1-x)zP}, -\vec{k}_\perp, (1-x)zP \right). \tag{2.16}$$

The wave function in this case is

$$\psi_{z\vec{P}+\vec{q}} = [M^2 - S(\vec{k}_\perp + (1-x)\vec{q}_\perp, x)]^{-1} \times \phi(\vec{k}_\perp + (1-x)\vec{q}_\perp, x). \tag{2.17}$$

This expresses the general rotation and translation properties of ψ. Note that the transverse-momentum argument $\vec{k}_\perp + (1-x)\vec{q}_\perp$ is that component of particle a's transverse momentum which is perpendicular to the direction of the incoming bound state.

If the constituents and bound state have spin, which definitely must be considered if one wishes to compare with experiment, then we shall assume that the leading term for large S factors in its spatial and spin dependence. Thus, for example, a pion coupling to two fermions can be described by a wave function which is the direct product of the standard Dirac spinors multiplied by the type of momentum space wave function discussed above.

In a complete description of a composite hadron, all the multiparticle components of the wave function must be specified. As mentioned earlier, it will be assumed that at high transverse momentum, a given constituent sees a combined coherent effect resulting from all of the other constituents; the latter can thus be treated dynamically as a single effective system or "core" of limited mass. This assumption allows us to describe the main features of the constituent wave functions in the regions of interest, and to readily link the interchange amplitude for elastic scattering to the asymptotic behavior of the form factor. In fact, the form (5.4) for the interchange amplitude is probably independent of the core assumption and is approximately true in a wide class of multiparticle models in which the wave function has approximately a power-law asymptotic dependence.

It is an interesting question as to whether the asymptotic behavior of the wave function is simply related to the most basic number of constituents. For example, the nucleon wave function will be shown to fall faster than that of the pion; this may be a reflection of the fact that (for $x \sim 1$) in the simplest models, the nucleon is a bound state of three quarks whereas the pion is composed of a quark pair. In any case, we can proceed by using effective two-body wave functions and defer until later, after more is known about their behavior and properties by comparing with experiment, a more unified treatment of the basic binding mechanism. Let us turn now to a discussion of the form factors of a composite system.

III. FORM FACTORS

In this section, we will give a detailed discussion of the electromagnetic form factors of the nucleons and mesons within the framework of the constituent and core approximation. Experimental knowledge of the asymptotic behavior of the form factors will yield information on the asymptotic behavior of the hadronic wave functions and vice versa. This discussion will also allow us to develop the type of analysis which is required in order to extract the leading asymptotic dependence in more complicated reactions. Many of the results which have been obtained from Bethe-Salpeter analyses will be immediately apparent here.

A. Scalar Particles

Recall that the matrix element of the current in the interaction picture is

$$\begin{aligned}(2p+q)^\mu F(q^2) &= \langle \psi_{p+q} | J^\mu(0) | \psi_p \rangle \\ &= \langle \psi_{p+q} | j^\mu(0) | \psi_p \rangle \\ &= \sum_a \lambda_a (2\pi)^{-3} \int \frac{d^3p}{2E_a 2E_c 2E_{a'}} (2E_{p+q}\psi_{p+q})(p_a + p_a')^\mu (2E_p \psi_p) + (\text{pair-creation states}).\end{aligned} \tag{3.1}$$

For $P\to\infty$, and the frame choice (2.1) for q^μ, the pair-creation states correspond to backward-moving particles in the wave functions and are suppressed by two powers of P in the amplitude. We thus obtain ($\vec{q}_\perp{}^2=-q^2$)

$$F(q^2)=\sum_a \lambda_a F_a(q^2),$$
$$F_a(q^2)=\frac{1}{2(2\pi)^3}\int d^2k_\perp\int_0^1 \frac{dx}{x(1-x)}\,\psi_a(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)\psi_a(\vec{k}_\perp,x), \tag{3.2}$$

by using the parametrization (2.3) and the rotation property (2.17). Note that the sum over a requires summation over both particles and antiparticles.

One should note that there are two central assumptions made in deriving Eq. (3.1):

(1) the identification of the carriers of free electromagnetic current in

$$j^\mu(0)=:\sum_a \lambda_a\phi_a^\dagger\overleftrightarrow{\partial}_\mu\phi_a: \tag{3.3}$$

with the constituents of the hadrons, and

(2) the assumption that a sum over (parton + core) two-particle states, in effect, saturates the free-particle intermediate state sum required in (3.2).

The normalization condition on the wave functions from (3.2) is

$$\int_0^1 dx\, f_a(x)=1\quad(\text{all } a), \tag{3.4}$$

where

$$f_a(x)=\frac{1}{2(2\pi)^3}\int\frac{d^2k}{x(1-x)}|\psi_a(\vec{k}_\perp,x)|^2. \tag{3.5}$$

This is also the condition that $Z_2=0$ for a composite state. The function $f_a(x)$ is the normalized fractional longitudinal-momentum distribution function, which appears in deep-inelastic electron-proton scattering in the Bjorken limit where $x=Q^2/2m\nu$ and

$$\nu W_2(x)=\sum_a \lambda_a{}^2 x f_a(x). \tag{3.6}$$

We can easily check that the bound-state calculations are gauge-invariant:

$$q^\mu\langle\psi_{p+q}|j_\mu(0)|\psi_p\rangle=\frac{1}{2(2\pi)^3}\int d^2k\int_0^1\frac{dx}{x(1-x)}\,\psi(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)\,q\cdot(p_a+p_a')\psi(\vec{k}_\perp,x). \tag{3.7}$$

This vanishes since

$$q\cdot(p_a+p_a')=-q_\perp{}^2(1-x)-2\vec{k}_\perp\cdot\vec{q}_\perp \doteq -q\cdot(p_a+p_a') \tag{3.8}$$

vanishes upon symmetrization in $\vec{k}_\perp\to-\vec{k}_\perp-(1-x)\vec{q}_\perp$. The proof for the case of spin is just as simple.

Let us now determine the behavior of the form factor as the momentum transfer becomes asymptotic. Assuming that the wave function has power-law behavior for large S, $\psi\sim S^{-n}N(x)$ [see Eq. (2.14)], then for $q_\perp{}^2$ large for fixed x, there are two regions of the $\vec{k}_\perp$ integral which contribute: $\vec{k}_\perp\sim 0$ and $\vec{k}_\perp\sim-(1-x)\vec{q}_\perp$. These yield equal contributions to F_a:

$$F_a(q^2)\cong\lambda_a(\vec{q}_\perp{}^2)^{-n}\int_0^{1-O(m/|\vec{q}_\perp|)} dx\,x^{2n-1}(1-x)^{-1}N_1(x), \tag{3.9}$$

where

$$N_1(x)=\frac{N(x)}{(2\pi)^3}\int d^2k_\perp\psi(\vec{k}_\perp,x)[x(1-x)]^{-n} \tag{3.10}$$

is a finite and nonvanishing function of x since $\psi\sim[x(1-x)]^n$ at the end points. Thus we have immediately

$$F_a(q^2)\sim(\vec{q}_\perp{}^2)^{-n}\ln(\vec{q}_\perp{}^2/m^2) \tag{3.11}$$

in the asymptotic region. Note that the Drell-Yan relation[9]

$$\nu W_2(x)\sim(1-x)^{2n-1} \tag{3.12a}$$

for $x\to 1$ if

$$F(q^2)\sim(\vec{q}_\perp{}^2)^{-n}\quad(q_\perp{}^2\to\infty) \tag{3.12b}$$

is automatically satisfied (up to logarithmic modifications) when the wave function depends asymptotically on the symmetric variable S.

B. The Nucleon Form Factor

For the calculation of the nucleon electromagnetic form factors, one must include the full complexities of spin. We will assume that the carriers of the electromagnetic current are fermions. The matrix element of the electromagnetic vertex of the nucleon has the form (a sum over constituent charges is to be understood)

$$J_\nu=\frac{1}{2(2\pi)^3}\int d^2k\int_0^1 \frac{dx}{x^2(1-x)}\,\psi(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)\times\psi(\vec{k}_\perp,x)j_\nu\,, \quad (3.13)$$

where

$$j_\nu=\bar{u}(p+q)\Gamma'(m_b+\not{p}_b)\gamma_\nu(m_a+\not{p}_a)\Gamma u(p)\,. \quad (3.14)$$

We have used summation over internal spin to replace

$$\sum_\lambda u_\lambda(p_a)\bar{u}_\lambda(p_a)=m_a+\not{p}_a\,, \quad (3.15)$$

where

$$p_a{}^2=m_a{}^2=m_b{}^2\,. \quad (3.16)$$

The spinor matrix operators, Γ' and Γ, define the coupling of the core c and charged constituents a or b to the nucleon. Two cases will be of interest to us:

(1) Spin-0 core:

$$\Gamma'=\Gamma=1\,; \quad (3.17a)$$

(2) Spin-1 core:

$$\Gamma'\cdots\Gamma=\gamma_\alpha\cdots\gamma_\beta\left(-g^{\alpha\beta}+\frac{p_c^\alpha p_c^\beta}{m_c{}^2}\right)\,. \quad (3.17b)$$

These are perhaps the simplest choices for the spin coupling consistent with the quark model. The vector propagator for case (2) arises from the internal sum over spin states; by definition p_c is on the mass shell: $p_c{}^2=m_c{}^2$.

For the case of bound states, the backward-moving fermion states do not contribute to the current (3.13)—even for the case of so-called "bad" transverse current components. This is in contrast to perturbation theory [e.g., quantum electrodynamics (QED) and γ_5 coupling] in which the matrix elements of the bad currents (connecting oppositely-moving fermions) grow in P at the same rate as the backward-moving energy denominators.[15] For bound states, the extra suppression of the vertex functions ϕ eliminates all such contributions in the $P\to\infty$ limit. The absence of backward moving fermion contributions leads automatically to the absence of certain types of fixed-pole behavior in hadronic and photoproduction processes. (See Sec. IV.)

The absence of the backward-moving contributions permits Eq. (3.14) to be used for all components ν, and thus we may use the following projection operations to isolate the standard nucleon form factors, $G_E(q^2)$ and $G_M(q^2)$:

$$G_E=\frac{1}{8M^2}\sum_{\text{spins}}\bar{u}(p)\gamma\cdot\epsilon_L u(p+q)\epsilon_L\cdot J\,, \quad (3.18a)$$

where

$$\epsilon_L^\nu=(p+\tfrac{1}{2}q)^\nu(M^2+\vec{q}_\perp{}^2/4)^{-1/2}\,,\quad \epsilon_L{}^2=+1 \quad (3.18b)$$

and

$$G_M=\frac{1}{2\vec{q}_\perp{}^2}\sum_{\text{spins}}\bar{u}(p)\gamma\cdot\epsilon_T u(p+q)\epsilon_T\cdot J\,, \quad (3.19a)$$

where

$$\epsilon_T\cdot P=\epsilon_T\cdot q=0,\quad \epsilon_T{}^2=-1\,. \quad (3.19b)$$

Thus G_E and G_M are given by integrals over $\vec{k}_\perp$, and x of the type of Eq. (3.2) with the integrand multiplied by the projection operators applied to j_ν.

For a scalar core c, it turns out that the leading terms in the spin projections for large $\vec{q}_\perp{}^2$ cancel for G_M but not for G_E. Hence, G_E is larger by a factor of $(-q^2)$ in the asymptotic region. The experimental evidence is that G_M and G_E scale, that is, they are proportional to one another, and that $G_M\sim 1/(-q^2)^2$ for large $(-q^2)$, at least for $(-q^2)<4$ GeV^2. Since it seems unreasonable to suppose that G_E falls more slowly than G_M, we will assume that the scaling property—at least up to logarithms—is valid asymptotically.

The simplest spin choice for the core spin which reproduces $G_E\propto G_M$ scaling is spin one. Using the helicity conserving form given by Eq. (3.17b) for the coupling of the core, we obtain to leading order:

$$G_{E,M}(q^2)=\frac{1}{2(2\pi)^3}\int d^2k\int_0^1\frac{dx}{x^2(1-x)}\,\psi((1-x)\vec{q}_\perp,x)\times\psi(\vec{k}_\perp,x)j_{E,M}\,, \quad (3.20)$$

where

$$j_E=\tfrac{1}{2}q_\perp{}^2[(1-x)^2+O(k_\perp{}^2/x^2m_c{}^2)]+O(q_\perp^0)\,, \quad (3.21a)$$

$$j_M=-q_\perp{}^2[1-k_\perp{}^2/xM_c{}^2]+O(q_\perp^0)\,. \quad (3.21b)$$

The full nucleon form factor is then given by a sum over the charged spin-$\frac{1}{2}$ constituents. Both $\vec{k}_\perp$ small and $\vec{k}_\perp+(1-x)\vec{q}_\perp$ small contribute equally to the asymptotic behavior. Using the above equations and the standard form for $\psi\sim S^{-N}N(x)$, we obtain

$$G_{E,M}\sim(-q^2)^{-n+1}\ln(-q^2/m^2)\,. \quad (3.22)$$

This relation only holds asymptotically; the non-[illegible]ding terms are not in the same ratio, so the scaling law is only an approximate relation.

In order to yield dipolelike behavior at large q^2, the value of n should be near 3. Notice that the strong convergence of the $\vec{k}_\perp{}^2$ integration allows the above type of asymptotic analysis. Note also that for $n=3$, an additional contribution from spin-zero core exchange would modify only G_E to leading order, so that this freedom can be used to obtain the observed experimental ratio $G_M^{(p)}/G_E^{(p)} \sim \mu_p$.

If there were an intermediate or elementary vector particle which mediates the electromagnetic interaction such as in a vector-dominance theory, then the wave function ψ would fall off with at least one less power of S ($n \leq 2$). Later, when deep-elastic pp scattering is discussed and compared with experiment, it will be shown that such small values of n are ruled out.

C. The Pion Form Factor

Needless to say, unlike the nucleon case, the large-$\vec{q}_\perp{}^2$ behavior of the pion form factor is completely unknown. We shall show that the large-angle data for pion-nucleon elastic scattering does, in fact, demand that the pion form factor be close to that of a monopole, $F_\pi(q^2) \sim (-q^2)^{-1}$. Anticipating this prediction, we shall choose the pion wave function so as to produce this behavior. It will be interesting to see if pion-nucleon experiments at higher energies continue to be consistent with the simple monopole behavior.

We shall take the pion as a bound state of a spin-$\frac{1}{2}$ elementary constituent bound to a spin-$\frac{1}{2}$ core. The matrix element of the current is thus

$$(2p+q)^\nu F_\pi(q^2) = \frac{1}{2(2\pi)^3}\int d^2k \int_0^1 \frac{dx}{x^2(1-x)} \psi(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \times \psi(\vec{k}_\perp, x) T^\nu, \tag{3.23}$$

where

$$T^\nu = +\mathrm{Tr}[(m_b + \not{p}_b)\gamma^\nu(m_a+\not{p}_a)\gamma_5(m_c - \not{p}_c)\gamma_5]. \tag{3.24}$$

The form factor is easy to extract by choosing the $\nu = 0$ component and allowing $P \to \infty$:

$$\frac{T^0}{2P} = (1-x)^{-1}((k_\perp{}^2 + m_x{}^2) + \{[\vec{k}_\perp + (1-x)\vec{q}_\perp]^2 + m_x{}^2\} + 4xM_\pi m_c - (1-x)q_\perp{}^2). \tag{3.25}$$

Again the two regions of the $\vec{k}_\perp$ integration contribute equally. Unfortunately, the leading terms in $\vec{q}_\perp{}^2$ of T_0 cancel and the asymptotic behavior of F_π must be extracted with some care. We shall see that the required pion wave function must fall as $S^{-3/2}$. Using such a wave function and the identity

$$(AB)^{-3/2} = \frac{8}{\pi}\int_0^\infty dz\, z^{1/2}(A+Bz)^{-3}, \tag{3.26}$$

the $\vec{k}_\perp$ integration is easily carried out by combining denominators. Then (setting $m_a = m_c$ for convenience) the form factor becomes

$$F_\pi(q^2) \propto \int_0^1 dx \int_0^\infty dz\, z^{1/2}(1+z)^{-3}x(1-x) \times \left[1 + \frac{z(1-x)^2}{(1+z)^2}\frac{\vec{q}_\perp{}^2}{M_\pi{}^2}\right]^{-2}. \tag{3.27}$$

The symmetry of the integrand under the transformation $z \to z^{-1}$ can be used to show that the contribution for $z>1$ is equal to the contribution from $z<1$. After defining $w = z(1-x)^2\vec{q}_\perp{}^2/M_\pi{}^2$, and interchanging the w and x integrations, one then obtains the expected result, $F_\pi(q^2) \sim (-q^2)^{-1}$.

D. Transition Form Factors

It is an interesting matter at this juncture to discuss transition form factors for processes such as $\gamma + N \to N^*$, which are defined as the nondiagonal matrix elements of the current operator:

$$\langle \psi^*_{p+q} | J^\mu(0) | \psi_p \rangle \equiv F_{N^*-N}(q^2)\left[2p^\mu + \frac{q^2 + M^2 - M^{*2}}{q^2} q^\mu\right]. \tag{3.28}$$

The scalar case will be treated here, but the extension to the spinor problem does not change any of the conclusions. The form factor is given by

$$F_{N^*-N}(q^2) = \frac{1}{2(2\pi)^3}\int d^2k \int_0^1 \frac{dx}{x(1-x)} \psi_{N^*}(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \times \psi_N(\vec{k}_\perp, x). \tag{3.29}$$

If the excited state has nonzero angular momentum, the appropriate spin labels must be included, and the appropriate spherical harmonic dependence in ψ_{N^*} must be taken into account.

The asymptotic dependence of F_{N^*-N} depends on the behavior of the N^* wave function. However, if this wave function falls off with the same power of S or faster than that of the nucleon, then the q^2 falloff of F_{N^*-N} is the *same as that of the elastic form factor* F_N. Note however, that for $\psi_{N^*} \sim S^{-n^*}$, $\psi \sim S^{-n}$, with $n^* > n$, the factor $\ln(-q^2)$ present in Eq. (3.11) for $F_N(q^2)$ is absent in $F_{N^*-N}(q^2)$. In the latter case, this is due to the fact that the large momentum transfer prefers not be routed through the N^* vertex: The leading behavior comes from the region $\vec{k}_\perp \sim -(1-x)\vec{q}_\perp$, and is determined by the nucleon wave function; the convergence of the

$k_\perp$ integration is controlled by the N^* wave function. It would seem very unnatural to expect that the wave functions of the excited states would fall slower than that of the nucleon. This would be the only way to break scaling between F_{N^*-N} and F_N.

The prediction that the transition form factors should scale at large momentum transfers with the nucleon form factor is quite striking. It is consistent with the data, but a definitive test at large momentum transfer is needed. This would be a simple yet crucial test of the composite theory. Now scattering processes will be discussed using this formalism.

IV. SCATTERING AND CHANNEL HAMILTONIANS

We now turn to the problem of calculating the elastic scattering of two composite systems.[19] We focus our attention on a covariant treatment of rearrangement collisions—the scattering due to the interchange of common constituents within the hadrons. The main assumption of our theory is that in the deep-elastic region, asymptotic s, t, and u, the time of interaction is insufficient for the interchange of more than one constituent. The problem then becomes analogous to that of electron rearrangement in atom-atom collisions, with the neglect of binding interactions between the interchanged electrons— and the neglect of corrections higher order in α from higher particle (photon, pair states) components of the atoms. In general, the hadronic problem requires a full multiparticle formulation. However, since in the deep scattering region, the wave function of the interchanged constituent is required in the region of high transverse momentum relative to the remaining constituents, it is reasonable to treat the latter system as a single particle or core, as was done in the infinite-momentum-frame form-factor calculations. Thus a two-particle approximation to the bound state is again relevant in this process. Note that the reference to a state of definite particle number applies specifically to an infinite-momentum reference frame. This is crucial if we are to relate the wave functions used in the interchange calculation to those used in the calculation of the form factor. This is because it is only in an infinite-momentum frame that the current interaction can be defined (for all $q^2 \leq 0$) in such a way as to not change particle number.

In the two-particle approximation, the calculation of a rearrangement collision is easily formulated within the framework of a multichannel Hamiltonian formalism. We recall first the infinite-momentum wave function equation describing the breakup of a bound state of mass M_B into its two constituents, a and c [for definitions of the momenta, see Eq. (2.10)]:

$$[M_B{}^2 - S(\vec{k}_\perp, x)]\psi_B(k_\perp, x) = [M_B{}^2 - K_a - K_c]\psi_B(k_\perp, x) = V_{ac}\psi_B\,, \tag{4.1}$$

where V_{ac} is an integral operator over $k_\perp$ and x, and K_a and K_c are the infinite-momentum-frame "kinetic energies"

$$K_a = (\vec{k}_\perp{}^2 + m_a{}^2)/x\,,\qquad K_c = (\vec{k}_\perp{}^2 + m_c{}^2)/(1-x)\,. \tag{4.2}$$

In general,

$$K_i = (\vec{k}_{\perp i}{}^2 + m_i{}^2)x_i{}^{-1}, \quad \sum_i x_i = 1\,. \tag{4.3}$$

We will consider the rearrangement collision defined by

$$\underset{(B)}{(a+c)} + \underset{(A)}{(b+d)} \to \underset{(C)}{(a+d)} + \underset{(D)}{(b+c)}\,. \tag{4.4}$$

For convenience, we define the external four vectors as [see Fig. 3(b)]:

$$P_B = p\,,\quad P_A = p+q+r\,,\quad P_C = p+q\,,\quad P_D = p+r\,, \tag{4.5}$$

so that $t = q^2$, $u = r^2$. The mass-shell conditions are

$$2q\cdot p = M_C{}^2 - M_B{}^2 - q^2\,,\qquad 2r\cdot p = M_D{}^2 - M_B{}^2 - r^2\,, \tag{4.6}$$

and

$$2q\cdot r = M_A{}^2 + M_B{}^2 - M_C{}^2 - M_D{}^2\,.$$

The most convenient infinite-momentum frame for this problem is defined by:

$$p^\mu = \left(P + \frac{M_B{}^2}{2P}\,,\vec{0}_\perp\,, P\right),\quad q^\mu = \left(\frac{q\cdot p}{P}\,,\vec{q}_\perp\,, 0\right),\quad r^\mu = \left(\frac{r\cdot p}{P}\,,\vec{r}_\perp\,, 0\right). \tag{4.7}$$

Thus we have the simple relations

$$t = q^2 = -\vec{q}_\perp{}^2\,,\qquad u = r^2 = -\vec{r}_\perp{}^2 \tag{4.8}$$

and the convenient orthogonality relation for elastic scattering

$$2\vec{q}_\perp\cdot\vec{r}_\perp = 0\,. \tag{4.9}$$

The initial noninteracting state is the direct product state $\psi_I = \psi_A\psi_B$ which satisfies

$$(E_I - K)\psi_I = V_I\psi_I \ , \tag{4.10}$$

where $V_I = V_{ac} + V_{bd}$ is the sum of the two binding potentials, and $K = K_a + K_b + K_c + K_d$ is the total kinetic energy. The initial infinite-momentum "energy" is

$$\begin{aligned} E_I &= M_A^2 + M_B^2 + (\vec{q}_\perp + \vec{r}_\perp)^2 \\ &= s - M_A^2 - M_B^2 \ . \end{aligned} \tag{4.11}$$

A similar equation holds for the final state ψ_F where only the potential $V_F = V_{ad} + V_{bc}$ is present.

The full interaction between the composite systems can be written in the form

$$V = V_I + V_F + W, \tag{4.12}$$

where W contains terms such as V_{ab}, V_{cd}, intrinsic multiparticle interactions, and the "optical" potentials which incorporate the effects such as absorption from higher multiparticle channels. The full wave function Ψ^+ satisfies the equation

$$[E - K]\Psi^+ = V\Psi^+ \ , \tag{4.13}$$

and

$$\begin{aligned} \Psi^+ &= \psi_I + (E - K - V_I)^{-1}(V - V_I)\Psi^+ \\ &= \psi_I + (E - K - V_F)^{-1} T_{FI}\psi_I \ . \end{aligned} \tag{4.14}$$

The transition operator is given by

$$T_{FI} = (V - V_I) + (V - V_F)(E - K - V)^{-1}(V - V_I) \ . \tag{4.15}$$

The required matrix element of the transition operator for the rearrangement process of interest is then

$$M_{FI} = \langle\psi_F | T_{FI} | \psi_I\rangle \ , \tag{4.16}$$

with $E \to E_I = E_F = s - M_A^2 - M_B^2$. Proceeding in the same manner, but constructing the full incoming wave function that is equal to ψ_F in the infinite future yields the alternative equation:

$$T_{FI} = (V - V_F) + (V - V_F)(E - K - V)^{-1}(V - V_I) \ . \tag{4.17}$$

In the lowest-order Born approximation, one therefore has the two alternative forms

$$\begin{aligned} M_{FI} &\cong \langle\psi_F | V - V_I | \psi_I\rangle \\ &= \langle\psi_F | V - V_F | \psi_I\rangle \ , \end{aligned} \tag{4.18}$$

which are indeed equal on the energy shell by virtue of the equations for ψ_I and ψ_F.

We now make the physical assumption that the main contribution to deep scattering from $V - V_I = V_F + W$ is due to the V_F term; that is, there are no explicit extraneous gluons present. The two terms W and V_F correspond precisely to the two contributions (A) and (B) discussed in the Introduction. Except for possible energy-independent absorption corrections contained in W, the higher Born contributions in Eq. (4.17) fall off more rapidly in s than the first Born contributions and will be neglected. The Pomeranchukon-dominated absorption corrections—while changing the normalization of the interchange contribution—do not change the energy or angular dependence obtained from the first Born calculation.

Thus, the appropriate form for the interchange transition amplitude is

$$\begin{aligned} M_{FI} &= \langle\psi_F | V_I | \psi_I\rangle \\ &= \langle\psi_F | V_F | \psi_I\rangle \ . \end{aligned} \tag{4.19}$$

Using the equations obeyed by ψ_I and ψ_F, we obtain

$$\begin{aligned} M_{FI} &= \langle\psi_F | E - K | \psi_I\rangle \\ &\equiv \langle\psi_F | \Delta | \psi_I\rangle \end{aligned} \tag{4.20}$$

$$= \frac{1}{2(2\pi)^3}\int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2}\, \Delta\psi_C(\vec{k}_\perp - x\vec{r}_\perp, x)\psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x)\psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x)\psi_B(\vec{k}_\perp, x) \ , \tag{4.21}$$

where

$$\begin{aligned} \Delta &= s - M_A^2 - M_B^2 - K_a - K_b - K_c - K_d \\ &= M_A^2 + M_B^2 - S_A(\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp, x) - S_B(\vec{k}_\perp, x) \\ &= M_C^2 + M_D^2 - S_C(\vec{k}_\perp - x\vec{r}_\perp, x) - S_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \ . \end{aligned} \tag{4.22}$$

This derivation makes it clear that in keeping only the interchange contribution, one is neglecting the parton-parton interaction relative to the parton-core, or binding, interaction.

It is interesting to see how this result arises in lowest-order perturbation theory. There are four surviving time-ordered contributions to the interchange amplitude which are illustrated in Fig. 5.

Defining the pointlike wave functions from the appropriate energy denominators, e.g., $\psi_B = g(K_B - K_a - K_c)^{-1}$, as in Sec. II, we have immediately

$$M_{FI} = \frac{1}{2(2\pi)^3} \int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2} \psi_C \psi_D \psi_A \psi_B \frac{1}{\Delta} [(K_A - K_b - K_d) + (K_B - K_a - K_c)][(K_D - K_b - K_c) + (K_C - K_d - K_a)], \tag{4.23}$$

which, if Eq. (4.22) for Δ is used, is precisely Eq. (4.21) for the pointlike case. Note that the common energy denominator Δ appears finally in the numerator. One can see from a comparison of this derivation with the channel Hamiltonian approach that Eq. (4.21) is in fact valid when ψ represents the wave function of a general two-component bound state. From the time-ordered perturbation approach, this is equivalent to treating the vertex functions $\phi = (M^2 - S)\psi$ as single time operators.

The generalization needed for the inclusion of spin is similar to that discussed for the electromagnetic form factors; detailed examples are given in Sec. V. We emphasize again the general feature that the bound states cannot couple to particles moving oppositely along P, as $P \to \infty$. This is because the spin couplings of the numerator which grow with P cannot compensate for the "bad" denominator as well as the extra suppression of the vertex functions, $\phi(P^2) \to 0$. In the case of the Compton amplitude in which photons couple pointwise to the fermion constituents, a z-graph [9] is certain to be present for the transverse currents. The resulting P^2/P^2 contribution has a local form similar to that of the "seagull" term $\phi^\dagger \phi A^2$ coupling in scalar electrodynamics, and leads to a term independent of energy and photon masses at fixed t in the Compton amplitude $T^{\mu\nu}$. Such a term corresponds to a δ_{J0} right-signature fixed singularity in $T_1(\nu, q^2)$. Further implications of this contribution have been discussed in Ref. 11.

The distinction between Compton scattering and photoproduction of composite hadrons (e.g., $\gamma + p \to \rho^0 + p$) is thus clear: The additional vertex suppression associated with the composite hadrons prevents any such z-graph contributions from surviving for $P \to \infty$, and the corresponding fixed singularity, i.e., energy-independent behavior, is not present. Thus, despite the fact that unitarity alone for lowest-order electromagnetic amplitudes does not rule out the presence of fixed poles of such fundamental origin, they are, in fact, excluded by the composite nature of the hadrons.

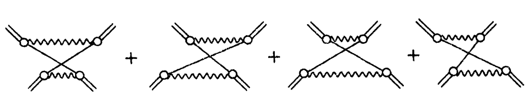

FIG. 5. The four time orderings that contribute to hadron-hadron scattering from constituent interchange.

V. ELASTIC SCATTERING IN THE DEEP REGION

This section will be concerned with extracting the asymptotic behavior in the deep scattering region (large s, t, and u) of the covariant scattering amplitude given in Eq. (4.21), resulting from the interchange of common constituents. Comparisons with experiment will also be presented.[20,21]

At fixed (c.m. angle, one can readily show that all but absorptive corrections to the single interchange process are either incorporated into the definition of the wave functions or give nonleading contributions in s. As discussed earlier, the absorptive corrections affect only the normalization of the interchange process since only the small $b \sim 0$ of the impact parameter profile $F(b)$ is probed—in this region $F(b)$ is slowly varying compared with that of the interchange contribution. A precise definition of the region of validity of the fully asymptotic form given below depends on masses and couplings, and can only be determined at this stage by comparison with experiment.

The behavior of the interchange amplitude, Eq. (4.21), is readily determined for asymptotic $t = -\vec{q}_\perp^{\,2}$, and $u = -\vec{r}_\perp^{\,2}$. Let us assume for convenience that hadron B has the most rapidly decreasing wave function at asymptotic transverse momenta. Then in the deep region, $\psi_B(\vec{k}_\perp, x)$ supplies the convergence of the transverse loop interaction, and only $\vec{k}_\perp \sim 0$ contributes in leading order in s. [If ψ_A has the same convergence as ψ_B, then there will be an equal contribution from the integration regions $\vec{k}_\perp \sim -(1-x)\vec{q}_\perp + x\vec{r}_\perp$, etc. In the case of $pp \to pp$, the four distinct regions of the k integration contribute equally.]

The asymptotic result can be expressed in the form

$$M(t,u) = -\int_0^1 \frac{dx}{x^2(1-x)^2} \left[\frac{(1-x)^2 q_\perp^{\,2} + x^2 r_\perp^{\,2}}{x(1-x)}\right] \times \psi_A((1-x)\vec{q}_\perp - x\vec{r}_\perp, x)\psi_C(-x\vec{r}_\perp, x) \times \psi_D((1-x)\vec{q}_\perp, x)[x(1-x)]^B \tilde{N}_B(x), \tag{5.1}$$

where

$$\tilde{N}_B(x) = \frac{1}{2(2\pi)^3}\int d^2k\,\psi_B(\vec{k}_\perp, x)[x(1-x)]^{-B} \tag{5.2}$$

is a smooth, finite function of x, and the quantity B is defined from the asymptotic form of the electromagnetic form factor of hadron B: $F_B(t) \sim (-t)^B$ (logarithmic factors in $F_I(t)$ are ignored). More generally, one could allow B to have a logarithmic dependence on t with obvious modifications of the final results. Using the asymptotic forms

$$\psi_L \sim S^{-L}N_L(x), \quad L = A, C, D \tag{5.3}$$

discussed in Sec. II, we immediately obtain

$$M(t,u) \cong sF_A(s)F_D(t)F_C(u)I(z) = s^{1-A-C-D}[\tfrac{1}{2}(1-z)]^{-D}[\tfrac{1}{2}(1+z)]^{-C}I(z), \tag{5.4}$$

where

$$z = \cos\theta_{\text{c.m.}}, \quad \tfrac{1}{2}(1-z) = -\frac{t}{s}, \quad \tfrac{1}{2}(1+z) = -\frac{u}{s}, \tag{5.5}$$

and we have defined

$$I(z) = -\int_0^1 dx\,\frac{\tilde{N}_B(x)N_A(x)N_C(x)N_D(x)}{[(1-x)^2(1-z)/2 + x^2(1+z)/2]^{A-1}} \times \frac{[x(1-x)]^{A+B+C+D-3}}{x^{2C}(1-x)^{2D}}. \tag{5.6}$$

In most of the applications, $I(z)$ is a relatively slowly-varying function of z, the x integrations being adequately protected for all values of z. If A, B, C, and D are such that the x integration is divergent, then logarithmic or stronger modifications in $\vec{q}_\perp^{\,2}$ or $\vec{r}_\perp^{\,2}$ will be present in $M(t,u)$.

The asymptotic form (5.4) can be contrasted with the Wu-Yang conjecture which involves only two form factors, both of which are functions of t (or u).

The 90° (c.m.) cross section in our theory has the simple power-law behavior

$$\lim_{s\to\infty}\frac{d\sigma^{(90°)}}{dt} = \frac{1}{16\pi^2 s^2}|M|^2 = s^{-2(A+C+D)}I(0)/16\pi^2, \tag{5.7}$$

which is completely determined by the power fall-off of the form factors. For the case of p-p scattering, $C = D = A \cong 2$, and

$$\frac{d\sigma^{(90°)}}{dt} \propto \frac{1}{s^{12}} \tag{5.8}$$

for large s. This prediction is not altered by spin effects [see Eq. (5.38)], and is in agreement with experiment (see Fig. 8).

Similarly in the case of πp elastic scattering, $A = C \cong 1$, $D \cong 2$, and

$$\frac{d\sigma^{(90°)}}{dt} \propto \frac{1}{s^8}, \tag{5.9}$$

which is again unaffected by the inclusion of spin. This is in excellent agreement with the data (see Fig. 6) for $s > 10$ GeV^2, and is the basis for our prediction that $F_\pi(t) \sim (-t)^{-1}$ for $-t > 5$ (GeV/c)^2.

Thus far we have simplified the interchange analysis by neglecting quantum number considerations. In actual fact, one must choose constituent models for the mesons and baryons which satisfy the required symmetry and conservation laws. For instance, the interchange contribution to pp scattering takes its simplest form in those models in which the proton wave function contains no antiquarks at high transverse momenta. In this case, only the interchange or (ut) topology [or (tu) by crossing symmetry] of Fig. 3(b) contributes. If antiquarks or more complicated core states were present in the region of interest, then "box" graphs of the (st) and (su) topology also must be included in the amplitude.

The contribution of the (st) topology is readily obtained from the real (ut) interchange amplitude by applying (su) crossing (after the angular part of the $d^2k_\perp$ integration has been performed). For example, applying (su) crossing to the results given in Eq. (5.2), one obtains for large s and t

$$M(t,s) = -(-t)^{-D}(-s-i\epsilon)^{-C} \times \int_0^1 dx\,\frac{\tilde{N}_B(x)N_A(x)N_D(x)N_C(x)}{[-(1-x)^2t - x^2s - i\epsilon]^{A-1}} \times \frac{[x(1-x)]^{A+B+C+D-3}}{x^{2C}(1-x)^{2D}}. \tag{5.10}$$

The small-x region of the integrand will, in general, contribute an imaginary part to the amplitude. However, in the physical cases of interest that will be discussed in detail, it is quite small in comparison to the real part.

A. Meson-Nucleon Scattering

The inclusion of the effects of spin in the interchange calculations is as straightforward as it is tedious. The nucleon will be taken as a composite state of a spin-$\frac{1}{2}$ (quark current) constituent bound to a core whose dominant spin is 1. Of course, spin-0 plus spin-1 would be the natural choices if the core is composed of two remaining quarks. The spin of the core is chosen to guarantee asymptotic G_E/G_M scaling (see Sec. III). The pion will be taken as a composite state of a spin-$\frac{1}{2}$ (quark current constituent) bound to a spin-$\frac{1}{2}$ core with quantum numbers of an antiquark. Thus both

(ut) and (st) diagrams contribute to this process [see Fig. 3(b)].

The pion-nucleon amplitude is written in the canonical form

$$M=\bar{u}(p+q)[-A+\gamma\cdot QB]u(p), \quad (5.11)$$

where

$$Q^\mu=(p+r)^\mu+(p+q+r)^\mu .$$

Projection operators which can be used to extract the invariant functions A and B are as follows: Define

$$P_1=\tfrac{1}{4}\sum_{\text{spins}} M\bar{u}(p)u(p+q) \quad (5.12a)$$

and

$$P_2=\tfrac{1}{4}\sum_{\text{spins}} M\bar{u}(p)\gamma\cdot\epsilon u(p+q), \quad (5.12b)$$

where

$$\epsilon\cdot(2p+q)=0 .$$

A suitable vector (in the infinite-momentum frame) is $\epsilon_\mu=(0,\vec{r}_\perp,0)$. Then

$$P_1=\tfrac{1}{2}\vec{q}_\perp^{\,2}(-A+MB)+\vec{r}_\perp^{\,2}MB, \quad (5.13a)$$

$$P_2=\tfrac{1}{2}\vec{q}_\perp^{\,2}\vec{\epsilon}\cdot\vec{r}_\perp B. \quad (5.13b)$$

Since the vector core c is coupled to the proton by a γ matrix, helicity should be conserved asymptotically and one expects and finds $A/sB\to 0$ as $s\to\infty$.

The necessary traces for the (ut) diagram in the deep scattering region are

$$P_1=\tfrac{1}{4}\mathrm{Tr}[\Gamma\not{p}i\gamma_5\not{d}i\gamma_5\not{a}\Gamma(\not{p}+M)(\not{p}+\not{q}+M)] \quad (5.14a)$$

and

$$P_2=\tfrac{1}{4}\mathrm{Tr}[\Gamma\not{p}i\gamma_5\not{d}i\gamma_5\not{a}\Gamma(\not{p}+M)\not{\epsilon}(\not{p}+\not{q}+M)], \quad (5.14b)$$

where $\Gamma\cdots\Gamma$ is the vector projection operator given in Eq. (3.17b). Mass terms in m_d, m_a, and m_b are neglected. If the traces in P_2 are performed, one obtains the leading order result

$$P_2=\tfrac{1}{2}\vec{q}_\perp^{\,2}\vec{r}^{\,2}\{-2\vec{q}_\perp^{\,2}[1-\vec{k}_\perp^{\,2}/(4xm_c^2)]\} . \quad (5.15)$$

By performing the trace for P_1, one discovers that A and B have the same asymptotic dependence (at fixed angle); this means that helicity is conserved and B dominates the differential cross section, i.e.,

$$\frac{d\sigma}{dt}\propto|B|^2\left(\frac{-u}{s}\right) , \quad (5.16)$$

where

$$B(t,u)=\frac{1}{2(2\pi)^3}\int d^2k\int_0^1\frac{dx}{x^2(1-x)^2}\Delta\psi_N(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)\psi_N(\vec{k}_\perp,x)\psi_\pi(\vec{k}_\perp+(1-x)\vec{q}_\perp-x\vec{r}_\perp,x)\psi_\pi(\vec{k}_\perp-x\vec{r}_\perp,x)$$
$$\times\left[-2q_\perp^{\,2}\left(1-\frac{k_\perp^{\,2}}{4xm_c^2}\right)\right]. \quad (5.17)$$

The asymptotic behavior of this amplitude in the deep scattering region, assuming a dipole nucleon behavior ($\psi_N\sim S^{-3}$) and a monopole pion form factor ($\psi_\pi\sim S^{-3/2}$), is

$$B(t,u)\cong(-t)^{-2}(-u)^{-3/2}\int_0^1 dx\tilde{N}(x)x^3[-(1-x)^2t-x^2u]^{-1/2}, \quad (5.18)$$

where

$$\tilde{N}(x)=N_N(x)N_\pi^{\,2}(x)\frac{2}{(2\pi)^3}\int d^2k_\perp\psi(\vec{k}_\perp,x)[x(1-x)]^{-3}\left(1-\frac{k_\perp^{\,2}}{4xm_c^2}\right) . \quad (5.19)$$

The (st) graph is achieved by continuing the variable u to $+(s+i\epsilon)$. The result is

$$B(t,s)\cong(-t)^{-2}(s+i\epsilon)^{-3/2}\int_0^1 dx\tilde{N}(x)x^3[+(1-x)^2t+x^2s+i\epsilon]^{-1/2}, \quad (5.20)$$

which has a small imaginary part for physical $t<0$.

If we adopt a definite quark wave function model for the pion and nucleon, then the required weightings of the (st) and (ut) amplitudes are determined. In general, the amplitudes which describe elastic and charge exchange scattering are of the form:

$$B(\pi^+p\to\pi^+p)=+\alpha B(t,u)+\beta B(t,s),$$
$$B(\pi^-p\to\pi^-p)=-\beta B(t,u)-\alpha B(t,s), \quad (5.21)$$
$$B(\pi^-p\to\pi^0n)=\frac{\beta+\alpha}{\sqrt{2}}[B(t,u)+B(t,s)].$$

The (su) topology would contribute in the simplest quark model, but the core routing should make it a small contribution. It will be neglected here. The asymptotic amplitude is then

$$B(\pi^+p)\cong s^{-4}[\tfrac{1}{2}(1-z)]^{-2}\int_0^1 dx\tilde{N}(x)x^3\{4\alpha(1+z)^{-3/2}[x^2(1+z)+(1-x)^2(1-z)]^{-1/2}$$
$$+\beta[x^2-(1-x)^2\tfrac{1}{2}(1-z)]^{-1/2}\}\,. \quad (5.22)$$

Let us now approximate this simple result to get even a simpler form. The β integral is almost independent of z for small z, and has a negligible imaginary part. The α integral is real, and its dependence on z is very close to $(1+z)^{-1/2}$. Furthermore, the coefficients of $+4\alpha$ and β are almost equal at $z=0$, as one can check by numerical integration. Thus a convenient form is

$$B(\pi^+p)\cong N_B s^{-4}[\tfrac{1}{2}(1-z)]^{-2}[4\alpha(1+z)^{-2}+\beta]\,, \quad (5.23)$$

which results in the differential cross sections

$$\frac{d\sigma}{dt}(\pi^+p)=\frac{\sigma_0}{s^8}\frac{(1+z)}{(1-z)^4}[4\alpha(1+z)^{-2}+\beta]^2 \quad (5.24)$$

and

$$\frac{d\sigma}{dt}(\pi^-p)=\frac{\sigma_0}{s^8}\frac{(1+z)}{(1-z)^4}[4\beta(1+z)^{-2}+\alpha]^2\,, \quad (5.25)$$

$$\frac{d\sigma}{dt}(\pi^-p\rightarrow\pi^0n)=\frac{\sigma_0}{s^8}\frac{(1+z)}{(1-z)^4}[\tfrac{1}{2}(\alpha+\beta)]^2 \times[4(1+z)^{-2}+1]^2\,, \quad (5.26)$$

where σ_0 is a constant. A comparison of the energy dependence at 90° is made with the data of Owen *et al.*[22,23] in Fig. 6. The angular dependence of the $\pi^-p\rightarrow\pi^-p$ reaction is compared with the same data for $\alpha=2$, $\beta=1$ in Fig. 7. (This corresponds to simple quark counting—two $\mathcal{P}$ quarks plus one $\mathfrak{N}$ quark interchange.[24]) The agreement is quite good even quite far from 90°, but clearly better data throughout the high-energy region is desired for a more crucial test. A search for the best values of α and β has not been made.

The ratios of the differential cross sections at 90° for these three processes are predicted in this leading approximation at large s to be

$$d\sigma(\pi^+p\rightarrow\pi^+p):d\sigma(\pi^-p\rightarrow\pi^-p):d\sigma(\pi^-p\rightarrow\pi^0n)$$
$$=(4\alpha+\beta)^2:(\alpha+4\beta)^2:\tfrac{25}{2}(\alpha+\beta)^2\,. \quad (5.27)$$

A particularly striking feature of this prediction is that once the ratio of $I=0$ and $I=1$ exchange is fixed, then the angular dependence in the deep region of all three processes is precisely determined.[25]

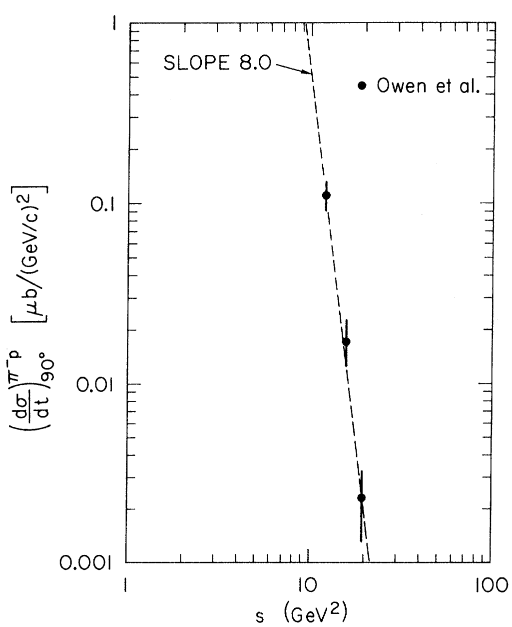

FIG. 6. The energy dependence of the 90° cross section for π^-p scattering.

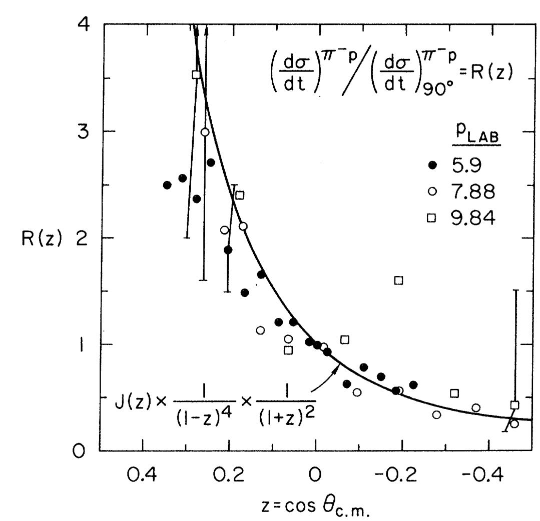

FIG. 7. The dependence of the π^-p differential cross section on z ($\cos\theta$) in the center-of-mass system. The lack of any strong energy dependence of the ratio should be noted. The solid line is our prediction given in Eq. (5.25) with $\alpha=2\beta$.

B. Kaon-Nucleon Scattering

If the valence quark assignments are assumed for the quantum numbers of the kaon and the nucleon, then their scattering amplitudes in the deep scattering region are extremely simple. Since the K^+ wave function only contains $\mathcal{P}$ quarks and $\bar{\lambda}$ quarks in the high-transverse-momentum region, then only the $\mathcal{P}$-quark (ut) interchange diagram contributes. This assumes that there are no high-momentum $\bar{\lambda}$ quarks in the proton wave function. Note that this does *not* exclude the presence of strange quarks and/or antiquarks in the quark sea at low momentum transfer. It then follows from crossing that $K^-p \to K^-p$ scattering in the deep region proceeds by $\bar{\mathcal{P}}$-quark transfer—only the (st) diagram can contribute. Furthermore

$$M(K_L p \to K_S p) = \tfrac{1}{2}[M(K^0 p \to K^0 p) - M(\bar{K}^0 p \to \bar{K}^0 p)], \quad (5.28)$$

and clearly

$$M(\bar{K}^0 p \to K^0 p) = 0, \quad (5.29)$$

where $K^0 p \to K^0 p$ proceeds by the (ut) interchange of an $\mathfrak{N}$ quark and $\bar{K}^0 p \to \bar{K}^0 p$ requires $\bar{\mathfrak{N}}$ quark (st) transfer. If the K-meson form factor scales with the pion form factor, one finds the deep-scattering predictions

$$\begin{aligned} &B(K^+ p \to K^+ p) = \alpha B(t,u), \\ &B(K^- p \to K^- p) = -\alpha B(t,s), \\ &B(K_L p \to K_S p) = \tfrac{1}{2}\beta[B(t,u) + B(t,s)], \end{aligned} \quad (5.30)$$

which, using the approximations of Eq. (5.23), become

$$\begin{aligned} &\frac{d\sigma}{dt}(K^+ p \to K^+ p) = \frac{\sigma_0}{s^8}\,\frac{1+z}{(1-z)^4}\,\frac{16\alpha^2}{(1+z)^4}, \\ &\frac{d\sigma}{dt}(K^- p \to K^- p) = \frac{\sigma_0}{s^8}\,\frac{1+z}{(1-z)^4}\,\alpha^2, \\ &\frac{d\sigma}{dt}(K_L p \to K_S p) = \frac{\sigma_0}{s^8}\,\frac{1+z}{(1-z)^4}\left[\frac{2}{(1+z)^2} + \tfrac{1}{2}\right]^2 \beta^2. \end{aligned} \quad (5.31)$$

The normalization constant σ_0 is the same as that of the πN reaction in the SU(3) limit.

The cleanest prediction is the s^{-8} behavior at fixed angle. However, the predicted angular dependences based on simple quark assignments are also very interesting. Except for the no-helicity-flip factor $(1+z)$, deep-elastic scattering of K^+p should be approximately symmetric about 90° rising towards small t and small u. The process $K^-p \to K^-p$, which depends on $B(s,t)$ only, should not have a backward peak, but should possess the same forward peaking as K^+p. Note that the odd-charge conjugation exchange reactions $\pi^- p \to \pi^0 n$ and $K_L p \to K_S p$ are predicted to have the same angular distribution and both should reach a minimum just beyond 90°.

The predicted ratio of $16/1/6.25\ (\beta/\alpha)^2$ for

$$d\sigma(K^+ p \to K^+ p)/d\sigma(K^- p \to K^- p)/d\sigma(K_L p \to K_S p)$$

at 90° is in qualitative agreement with the $P_{\rm lab} = 5$-GeV/c data[26] (although Regge exchange may still be important at this low energy). The steep asymmetric behavior of the 5-GeV/c $K^-p \to K^-p$ data and the relatively symmetric behavior of the 5-GeV/c K^+ data are in qualitative agreement with our theoretical predictions.

One sees that the parton interchange model together with the valence quark assignments, perhaps not surprisingly, connects smoothly with any possible forward or backward nonexotic Regge exchanges. The interchange force thus provides a smooth extrapolation between these peaks, falling as a power in s as the distance between the peaks increases. This is very suggestive that the interchange mechanism may provide the basic strong interaction. Then, if it is present in an amplitude, it can be built up at small t or u by virtual hadronic bremsstrahlung and gluon forces into a coherent Regge exchange. The implication of this picture would then be that Regge trajectories and residues at large momentum transfer must be such that they approach the amplitudes computed here. This possibility will be discussed in detail elsewhere.

It is interesting to note that the 5-GeV/c data of Chabaud *et al.*[26] suggest that K^+p scattering has more striking interference minima than K^-p. This is probably consistent with an interfering K^+p Regge contribution that is purely real, as suggested by exchange degeneracy which also predicts that K^-p would have a pure rotating Regge phase. This rotation can change the interference from destructive to constructive and vice versa. A thorough amplitude analysis of the data at large angles, and more data at larger energies, would clarify this situation and would be an interesting check of the interchange theory.

C. Nucleon-Nucleon Scattering

The calculation of nucleon-nucleon deep scattering is, in at least one respect, simpler than that of pion-nucleon deep scattering. If we assume that the proton wave function contains no antiquarks at high transverse momentum, then only the (ut) and (tu) topology contributes. The actual computation, however, is markedly more difficult. This is due to the necessity of including not only the half-unit spins of the nucleons and constituents, but also the unit spins of the nucleon cores (see Sec. III).

The calculations are especially difficult since the leading terms involve the $p^\alpha p^\beta$ part of the spin-1 projection operators.

Fortunately, the couplings of the cores conserve spin-$\frac{1}{2}$ helicity, and we can focus our attention on only two of the five invariant amplitudes (vector-vector and axial-vector–axial-vector) for the asymptotic calculations. Note that the helicities of nucleons in Fig. 3(b), A and D, and those of B and C are the same. Thus we write the resulting asymptotic amplitude in the form

$$\begin{aligned}\mathfrak{M} =& V(z)\bar{u}(p+q)\gamma_\mu u(p+q+r)\bar{u}(p+r)\gamma^\mu u(p)\\ &+A(z)\bar{u}(p+q)\gamma_5\gamma_\mu u(p+q+r)\\ &\times\bar{u}(p+r)\gamma_5\gamma^\mu u(p)\,. \end{aligned} \tag{5.32}$$

The contributions arising from the symmetrization of the initial or final nucleon is then easily obtained as

$$\begin{aligned}\overline{\mathfrak{M}} =& -V(-z)\bar{u}(p+q)\gamma_\mu u(p)\bar{u}(p+r)\gamma^\mu u(p+q+r)\\ &-A(-z)\bar{u}(p+q)\gamma_5\gamma_\mu u(p)\\ &\times\bar{u}(p+r)\gamma_5\gamma^\mu u(p+q+r)\,. \end{aligned}$$

The total amplitude is $\mathfrak{M}+\overline{\mathfrak{M}}$. The differential cross section is then proportional to

$$\begin{aligned}T =& \tfrac{1}{16}\sum_{\text{spins}}(\mathfrak{M}+\overline{\mathfrak{M}})(\mathfrak{M}+\overline{\mathfrak{M}})^\dagger\\ =& \tfrac{1}{2}s^2|V(z)+A(z)+V(-z)+A(-z)|^2\\ &+\tfrac{1}{2}t^2|V(z)-A(z)|^2+\tfrac{1}{2}u^2|V(-z)-A(-z)|^2\,. \end{aligned} \tag{5.33}$$

Isolation of the invariant amplitudes V and A may be achieved using the projection operators

$$\begin{aligned}\Gamma_V =& \tfrac{1}{16}\sum_{\text{spins}}\mathfrak{M}\bar{u}(p)\gamma_\nu u(p+r)\\ &\times\bar{u}(p+q+r)\gamma^\nu u(p+r)\\ =& \tfrac{1}{2}(s^2+t^2)V(z)+\tfrac{1}{2}(s^2-t^2)A(z) \end{aligned} \tag{5.34}$$

and

$$\begin{aligned}\Gamma_A =& \tfrac{1}{16}\sum_{\text{spins}}\mathfrak{M}\bar{u}(p)\gamma_5\gamma_\nu u(p+r)\\ &\times\bar{u}(p+q+r)\gamma_5\gamma^\nu u(p+r)\\ =& \tfrac{1}{2}(s^2+t^2)A(z)+\tfrac{1}{2}(s^2-t^2)V(z)\,. \end{aligned} \tag{5.35}$$

Thus,

$$V=\frac{1}{2s^2}(\Gamma_V+\Gamma_A)+\frac{1}{2t^2}(\Gamma_V-\Gamma_A)$$

and

$$A=\frac{1}{2s^2}(\Gamma_V+\Gamma_A)-\frac{1}{2t^2}(\Gamma_V-\Gamma_A)\,.$$

The interchange contribution is then written as

$$\mathfrak{M}=\frac{1}{2(2\pi)^3}\int d^2k\int_0^1\frac{dx}{x^2(1-x)^2}\;\psi_N(\vec{k}_\perp+(1-x)\vec{q}_\perp-x\vec{r}_\perp,x)\psi_N(\vec{k}_\perp,x)\psi_N(\vec{k}_\perp-x\vec{r}_\perp,x)\psi_N(\vec{k}_\perp+(1-x)\vec{q}_\perp,x)\mathfrak{L}\,, \tag{5.36}$$

with

$$\mathfrak{L}=\bar{u}(p+r)\gamma_\alpha(m_a+\not{p}_a)\gamma_\delta u(p)\bar{u}(p+q)\gamma_\beta(m_b+\not{p}_b)\gamma_\epsilon u(p+q+r)\left(g^{\alpha\epsilon}-\frac{p_d^\alpha p_d^\epsilon}{m_d^2}\right)\left(g^{\beta\delta}-\frac{p_c^\beta p_c^\delta}{m_c^2}\right)$$

and

$$m_a=m_b\,,\quad m_c=m_d\,.$$

The projections (5.34) and (5.36) were performed using the algebraic computation program REDUCE. We checked explicitly that the four possible origin shifts of the $\vec{k}_\perp$ integration yield the same answer in the asymptotic region. The $g^{\alpha\epsilon}g^{\beta\delta}$ terms do not contribute to the leading asymptotic behavior (by one power of s). The remaining terms have the same asymptotic behavior. For simplicity, we present the representative contribution from the $p_c^\beta p_c^\delta p_d^\alpha p_d^\epsilon/m_c^4$ term, from which we obtain

$$\Gamma_V\pm\Gamma_A=\frac{1}{2(2\pi)^3}\int d^3k\,k_\perp^2\frac{\psi_N(\vec{k}_\perp,x)}{x^3(1-x)^3}\frac{N_N^3(x)(1-x)^3x^3}{t^3u^3[x^2u+(1-x)^2t]^2}I_\pm\,, \tag{5.37}$$

where

$$
\begin{aligned}
I_+ = {} & t^3u[x^{-2}(1-x)^3 - x^{-2}(1-x)^2 - x^{-2}(1-x) + x^{-2} + x^{-1}(1-x)^2 - x^{-1}(1-x) + 1] \\
& + t^2u^2[x(1-x)^{-2} + 2x(1-x)^{-1} - x - 2x^{-2}(1-x)^{-1} + x^{-2}(1-x)^3 - 3x^{-2}(1-x)^2 + x^{-2}(1-x) \\
& \qquad + 3x^{-2} + x^{-1}(1-x)^{-2} - x^{-1}(1-x)^{-1} + 3x^{-1}(1-x)^2 - 6x^{-1}(1-x) \\
& \qquad + 3x^{-1} - 3(1-x)^{-1} + (1-x) + 1] \\
& - tu^3[x(1-x)^{-2} - 2x(1-x)^{-1} + x + x^{-1}(1-x)^{-2} - x^{-1}(1-x)^{-1} - 2x^{-1}(1-x)^2 + 5x^{-1}(1-x) \\
& \qquad - 3x^{-1} - (1-x)^{-1} - (1-x) + 2]
\end{aligned}
$$

and

$$
\begin{aligned}
I_- = {} & t^2u^2[x(1-x)^{-2} - 2x(1-x)^{-1} + x + x^{-1}(1-x)^{-2} - x^{-1}(1-x)^{-1} - 2x^{-1}(1-x)^2 + 5x^{-1}(1-x) \\
& \qquad - 3x^{-1} - (1-x)^{-1} - (1-x) + 2] \\
& + t^3u[x^{-2}(1-x)^3 - x^{-2}(1-x)^2 - x^{-2}(1-x) + x^{-2} + x^{-1}(1-x)^2 - x^{-1}(1-x) + 1] .
\end{aligned}
$$

The integral

$$\frac{1}{2(2\pi)^3}\int d^2k\, k_\perp{}^2\psi_N(\vec{k}_\perp, x)[x(1-x)]^{-3}N_N{}^3(x)$$

is a smooth function of x. Thus we obtain an asymptotic differential cross section of the form ($z = \cos\theta_{\text{c.m.}}$)

$$
\begin{aligned}
\frac{d\sigma}{dt} &= F_1{}^2(s)F_1{}^2(t)F_1{}^2(u)I(z) \\
&\simeq s^{-12}(1-z^2)^{-4}I(z) . \qquad (5.38)
\end{aligned}
$$

A numerical fit to $I(z)$ for small z yields

$$(1-z^2)^{-4}I(z) \sim (1-z^2)^{-5.2}I(0) . \qquad (5.39)$$

Thus the effect of spin is to increase the z dependence but not the over-all fixed-angle energy dependence. Note that we have chosen the asymptotic dependence of $\psi \sim S^{-3}$ to agree with an asymptopic dipole falloff of the nucleon form factors. As mentioned before, mass corrections within the dipole formula should be taken into account in any comparison with the data at nonasymptotic energies. In Fig. 8, we note that the energy dependence of the 90° cross section[27] changes from s^{-10} to s^{-12} as $t \cong -\frac{1}{2}s$ varies from $t = -6$ GeV^2 to -20 GeV^2. This shift in the power dependence cross section is consistent with a change from $(-t)^{-1.7}$ to $(-t)^{-2.0}$ in the power dependence of the F_1 form factor in this same region. Thus the asymptotic dependence of the 90° cross section is an extremely sensitive indicator of the rate of falloff of the elastic form factor. The consistency between six powers of the form factor and the 90° data seems to be remarkably good.[28]

Equation (5.37) also predicts an energy-independent form for the angular dependence of

$$
\begin{aligned}
\frac{d\sigma}{dt}\Big/\frac{d\sigma^{(90°)}}{dt} &= (1-z)^{-4}I(z)/I(0) \\
&\sim (1-z^2)^{-5.2} \qquad (5.40)
\end{aligned}
$$

in the deep asymptotic region. Our prediction is compared with experiment in Fig. 9. The angular dependence does seem to be nearly energy-inde-

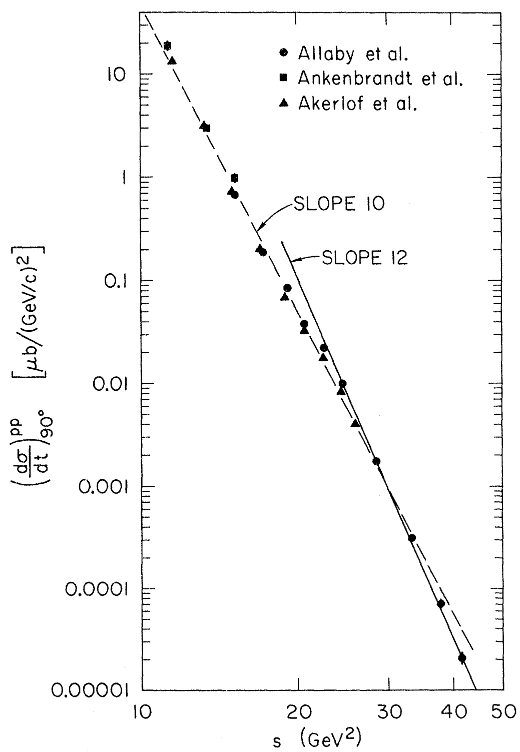

FIG. 8. The energy dependence of the 90° cross section for pp scattering.

pendent, and the predicted form seems to agree with experiment for $|t|$, $|u| > 5$ GeV^2. Note that the interchange theory gives a complete asymptotic prediction of both the energy and angular dependence; only the over-all normalization constant is undetermined.

We have carried out a fit to pp scattering data above 5 GeV/c. Only representative points were included for $|t| < 0.1$ (GeV/c)^2. The results are shown in Fig. 10. Although the data of Cocconi *et al.*[27] is not included, the fit agrees with this data as well. In order to describe the low-t region, we have included a Pomeranchukon-type description of the Regge region and dipole mass corrections to the ultimate constrained asymptotic interchange prediction of $d\sigma/dt \rightarrow s^{-12}(1-z^2)^{-5.2}I(0)$. We have fitted 431 data points with an χ^2/point of 1.5. The fit involves 8 parameters, 3 of which are associated with the normalization and dipole corrections to the interchange contribution and 5 of which are associated with the Pomeranchukon contribution. [The latter contribution is negligible for $|t| > 4$ (GeV/c)^2, and $p_{lab} < 30$ GeV/c.] Since the interchange contribution falls so rapidly in s at fixed t (like s^{-8}), it is of course extremely small and completely negligible at small t/s compared to the contribution of the Pomeranchukon at ISR (CERN Intersecting Storage Rings) energies ($s \gtrsim 900$ GeV^2). The parametrization of the Pomeranchukon contribution, which is determined from the low-energy data ($s < 50$ GeV^2) turns out to be consistent with the ISR measurements of $d\sigma/dt$.[29] Details will be presented elsewhere, but it should be remarked that since the interchange force will develop Regge behavior at small t, it will not vanish as rapidly as the zeroth-order theory used above. The fit can then be improved at intermediate t. Also, if the elastic form factor of the proton should fall more rapidly than $(-t)^{-2}$, then the predictions and the fit will need to be modified in the higher s and t range.

D. Annihilation Processes

Since the interchange theory is in principle a complete dynamical theory in the deep scattering region, and hence has analytic cross behavior, it is possible to continue the invariant amplitudes for the previously discussed processes in order to

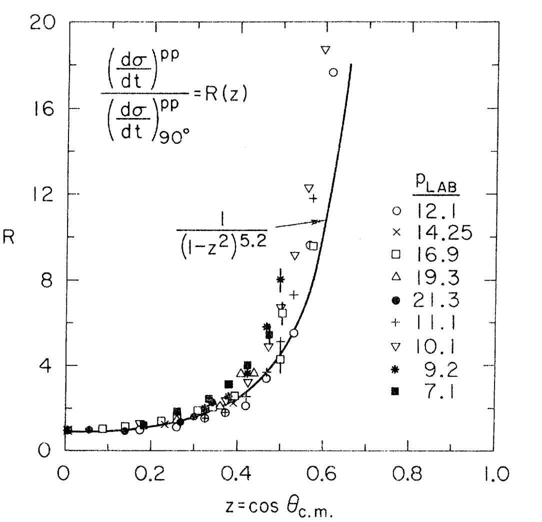

FIG. 9. The dependence of the pp differential cross section on z ($\cos\theta$) in the center-of-mass system. The lack of energy dependence is quite clear. The solid line is our asymptotic prediction given by Eq. (5.40). The data at 8.1 GeV/c has been omitted, but it would agree with this curve if the 90° point is not used for normalization.

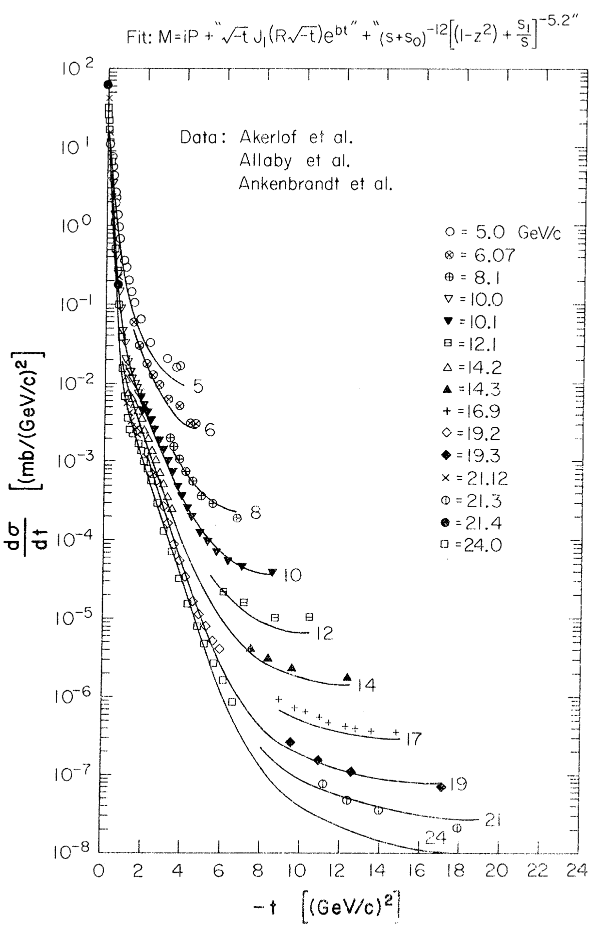

FIG. 10. A fit to the pp differential cross section when the forward Pomeranchukon exchange is included. The fit is constrained at large t, u to the asymptotic-interchange prediction $d\sigma/dt \rightarrow Cs^{-12}(1-z^2)^{-5.2}$.

obtain predictions for a variety of annihilation reactions. For example, the process $pp \to pp$ can be crossed to the process $p\bar{p} \to p\bar{p}$. Similarly the meson-nucleon scattering amplitudes can be crossed to yield predictions for $p\bar{p} \to \pi^+\pi^-$, K^+K^-, etc., in the deep scattering region where one again only needs to know the wave functions in the asymptotic region.

We begin by discussing the process $p\bar{p} \to \pi^-\pi^+$ which we obtain by $s \leftrightarrow t$ crossing from $\pi^-p \to \pi^-p$. Technically, one should begin with the exact description given by Eq. (5.22) for the invariant amplitude. By direct calculation, we have found that for $|z| \leq 0.5$, one may equally well use the approximate expression (5.23). The invariant amplitude becomes

$$B(\bar{p}p \to \pi^-\pi^+) \cong -\frac{N_B}{s^2}\left(\frac{\alpha}{t^2} + \frac{\beta}{u^2}\right), \tag{5.41}$$

where $t = (p_{\bar{p}} - p_{\pi^-})^2$, etc. Using this, one obtains the prediction

$$s^2\frac{d\sigma}{dt}(\bar{p}p \to \pi^-\pi^+) = \frac{\sigma_0(1-z^2)}{2s^6} \times [\alpha(1-z)^{-2} + \beta(1+z)^{-2}]^2. \tag{5.42}$$

Note that this can be obtained directly from $s \leftrightarrow t$ crossing of the approximate form of $s^2 d\sigma/dt$ given in Eq. (5.25). Thus spin does not complicate the crossing of these reactions, and the approximate expressions may be used reliably. Note that Eq. (5.42) predicts a fixed-angle cross section proportional to s^{-8} which is a characteristic of all the meson-baryon processes. Using $\alpha \sim 2\beta$ as found in $\pi^-p \to \pi^-p$ scattering, one expects a minimum in $d\sigma/dt$ just beyond 90°. The data [30] at 5 GeV/c are in good qualitative agreement with the interchange predictions, except, of course, for the very forward and backward direction where one expects baryon trajectory exchange to be important.

Proceeding in a similar manner, one can obtain a prediction for $\bar{p}p \to K^-K^+$ by $s \leftrightarrow t$ crossing from $K^-p \to K^-p$. Using Eq. (5.31), we obtain

$$s^2\frac{d\sigma}{dt}(\bar{p}p \to K^-K^+) = \frac{\sigma_0\alpha^2}{2s^6}\frac{(1+z)}{(1-z)^3} \tag{5.43}$$

and the ratio

$$\frac{d\sigma}{dt}(K^-p \to K^-p)\Big/\frac{d\sigma}{dt}(\bar{p}p \to K^-K^+) = 2(1-z)^{-1}. \tag{5.44}$$

The data [30] for $\bar{p}p \to K^-K^+$ do indeed indicate a sharp forward peaking and a very small, probably exotic, backward peak. The 90° ratio given above is in agreement with the ratio observed at 5 GeV/c.

Since continuation of our approximate forms for $s^2 d\sigma/dt$ should be quite reliable, we can obtain a prediction for $\bar{p}p \to \bar{p}p$ from the approximate deep-scattering formula for $pp \to pp$ [Eq. (5.40)]:

$$s^2\frac{d\sigma}{dt}(pp \to pp) \cong I(0)s^a(4tu)^{-L}$$

implies

$$s^2\frac{d\sigma}{dt}(\bar{p}p \to \bar{p}p) \cong I(0)|u|^a(4|t|s)^{-L},$$

where we expect that $a \sim 0.4$ and $L \sim 5.2$ in the lower-energy ranges. Thus pp is symmetric about 90° while $\bar{p}p$ should have only the forward peak. The ratio of the cross sections is

$$\frac{d\sigma}{dt}(pp)\Big/\frac{d\sigma}{dt}(\bar{p}p) = \left(\frac{s}{-u}\right)^{L+a} \cong \left(\frac{2}{1+z}\right)^{L+a}, \tag{5.45}$$

which at 90° is $\sim 2^{5.6} \sim 49$.

Experimentally, the 90° ratio at 5 GeV/c is near 100,[31] but of course the energy is too low to make a quantitative comparison meaningful. It is interesting, however, that such large ratios are predicted by the interchange theory—in simple Regge theories and Wu-Yang type theories, the ratio is of order unity.

E. Resonance Production

Once one has a basic understanding of elastic processes in the deep scattering region and thus of the meson and nucleon wave functions at large momentum transfer, useful information about the wave function of excited states and resonances may be obtained from their production cross section. We also note that measurements of polarization in such processes as $pp \to \Delta^{++}n$ can lead to a check on the interchange prediction (for the simple valence quark model) that the amplitude is purely real.

In the case of mesonic resonance production, e.g., $\pi + p \to \rho + p$, a comparison of the fixed-angle energy dependence of the cross section in the deep region with that for $\pi + p \to \pi + p$, is a sensitive probe of the asymptotic behavior of the ρ form factor. Since the ρ is presumably not elementary, $\pi + p \to \rho + p$ should fall faster with energy than $\gamma + p \to \pi + p$. The u dependence of the (ut) contribution to the $\pi + p \to \rho + p$ amplitude also reflects the asymptotic behavior of the wave function of the ρ. A systematic comparison of the above three processes and their analogs for other resonances would be of particular interest.

In the case of baryon-resonance production, such as $\pi + p \rightarrow \pi + N^*$, $K + p \rightarrow \pi + (\Lambda^*, \Sigma^*)$, and $p + p \rightarrow p + N^*$, it would be very surprising if the fixed-angle asymptotic energy dependence differed from that of the corresponding elastic processes, $\pi + p \rightarrow \pi + p$, $K + p$, and $p + p \rightarrow p + p$. Since presumably the $\gamma N^* N^*$ spin-averaged form factor (we phrase these results in terms of this form factor in order to make a statement independent of spins) falls at least as fast as the nucleon form factor, the situation is similar to that of the transition form factor (see Sec. III D); the extra convergence factors are simply absorbed in performing $\vec{k}_\perp$ integrations.

The data of Amaldi *et al.*[32] for $pp \rightarrow pN^*$ suggests that this process has the same energy and (large) angle dependence as does elastic scattering and supports the above predictions from the interchange model.

VI. CONCLUSION

In this paper we have proposed a unified treatment of scaling behavior in virtual electromagnetic processes, the asymptotic behavior of elastic form factors, and hadron reactions in the deep scattering region (s large, t/s, u/s fixed). The essential physical assumption which links these processes is the existence of charged constituent states within the composite hadron which have minimal electromagnetic interactions and elementary propagation properties at short distances. We should emphasize that the existence of free states for the constituents is not required. In physical terms, even if the constituents are bound by virtue of energy thresholds or selection rules, the interchange of common constituents still occurs in hadron-hadron scattering when the bound-state wave functions overlap. This is explicitly true for the (ut) (crossed) graph contribution [Fig. 3(b)] to the interchange process, since the amplitude is real and particle production is not implied. The absorptive contributions of the box graph (st) and (su) amplitudes are generally negligible in the calculation of exclusive cross sections in the large-angle region, and thus the results are essentially unchanged if there is some dynamical final-state modification of these absorptive parts.

Although our results are more general, it is of course appealing and probably compelling to identify the fundamental constituents of the hadrons, i.e., the structureless carriers of the electromagnetic current, with the quark representation of current algebra on the light cone. In its description of virtual electromagnetic processes, our theory shares features with the field-theoretic parton models of DLY, LPS, and Drell and Lee.[8] In these models scaling is guaranteed if the parton-proton amplitude has convergent off-shell behavior. In our model this condition is guaranteed by the convergence properties of the bound-state wave function. [This softening condition also guarantees the existence of fixed-pole behavior in the Compton amplitude as discussed in Ref. 11.]

Our theory is consistent with the general picture of Feynman in which hadronic scattering is a consequence of parton exchange. The theory developed here is based on the dominance of the interchange of only two basic constituents in exclusive scattering at fixed angle. In contrast, the Regge description of large-angle scattering requires an infinite coherent sum of Regge cut and pole contributions. In some models, this requires the exchange of an infinite number of dual model quarks.[17] If these different theories were to be equivalent, it would require that one of our exchanged (current) quarks be a superposition state involving an arbitrary number of dual model quarks.

One somewhat model-dependent assumption of our theory is the simplified treatment of the hadronic bound-state wave function. We have assumed that (1) when one constituent is at large momentum transfer relative to the rest, the state can be approximated as a two-particle state (for $P \rightarrow \infty$), and (2) whenever it is safe, we have taken this two-component wave function to be an inverse power-law function of the total energy variable $S = (p_a + p_b)^2$ only, rather than remaining the dependence in the individual "off-shell" variables $(p - p_a)^2$ and $(p - p_b)^2$. This analogous to suppressing the relative energy dependence of the Bethe-Salpeter wave function, and is justified for our applications to exclusive scattering, where it yields an invariant scattering amplitude. In cases in which one of the constituents appear formally in an asymptotic state, the appropriate offshell variable must be used for the dependence of the vertex function.

We have also made an additional important conjecture: The dominant constituents appearing at large transverse momentum in the $P \rightarrow \infty$ hadronic wave function are the simplest quark representation states (e.g., $q\bar{q}$ for the mesons and qqq for the baryons). This conjecture becomes reasonable if one notes that the complications of multiparticle states due to prior (virtual) hadronic bremsstrahlung (see Fig. 11) vanish rapidly in p_T. At low momentum transfer, this bremsstrahlung process yields states of arbitrary number and type and may be regarded as one origin of the wee parton or Regge spectrum in the structure function $\nu W_2(\omega)$ at large ω.[6] As pointed out by LPS,[8] the scaling Regge behavior of $\nu W_2 \sim \omega^{\alpha - 1}$ for $\omega \rightarrow \infty$ reflects the Regge behavior of the forward antiparton (quark) amplitude $M_{\bar{q}p} \sim s^{\alpha}$. In fact, Regge behavior of this amplitude inevitably arises from the same type of hadronic bremsstrahlung process depicted in Fig. 11.[33] Thus Regge behavior and duality in νW_2 and $M_{\bar{q}p}$ actually reflects the Regge behavior and duality of normal hadron-proton scattering amplitudes. For $\omega \sim 1$, the bremsstrahlung is suppressed and only the simplest quark states of the proton wave function at infinite momentum contribute. However, at large ω, the bremsstrahlung picture implies that the target hadron (with momentum $P \rightarrow \infty$) in inelastic e-p scattering is usually another hadron of lower momentum zP $(1 > z > 1/\omega)$ than the target proton itself. Thus sum rules involving νW_2 are only valid when the bremsstrahlung effects cancel—i.e., when the measured current commutator corresponds to a conserved charge. Examples of legitimate sum rules are those of Adler,[34] Gross and Llewellyn Smith,[35] and Brodsky, Gunion, and Jaffe.[36]

In the calculations of the interchange amplitude at large t and u, three of the four wave functions are evaluated at large relative transverse momentum, and thus only the simplest common quark interchange diagrams need to be considered. In the case of pp scattering, only the crossed diagrams contribute. In the case of the charged Kp amplitudes, only a single quark diagram contributes in each of the two cases. In pion-nucleon scattering, both quark interchange (tu) and antiquark transfer diagrams (st) are required. As we have seen, all of our predictions with these simplest of quark models are consistent with experiment, but one cannot rule out some admixture of more complicated quark states in the large transverse-momentum hadronic wave function.

In the calculations presented here, we have obtained the results for the (st) and (su) amplitudes from the appropriate analytic continuation of the simpler (tu) amplitude. We have checked, using the $P \rightarrow \infty$ method, that this crossing is valid for the case of point couplings in perturbation theory. In the case of bound-state wave functions, one must note that the assumption that the wave functions are determined by the single variable S is not valid for the direct computation of amplitudes with absorptive parts. Thus further development and the relaxation of the single variable S assumption will be required before one has a completely crossing-symmetric theory.

It would be useful to develop other calculational approaches to the interchange theory, especially within the explicit covariant formalism of Drell and Lee,[8] or the covariant Sudakov variable analysis of Landshoff, Polkinghorne, and Short.[37] We can also anticipate that many of our results can be obtained from a light-cone approach.

One important step in the interchange theory is to prove that the impulse approximation, as defined in the Introduction, is valid at large t and u. In fact this is guaranteed by the approximately power-law behavior of the consituent wave functions. The exchange or interchange of a higher number of constituents is not important in the asymptotic region, since large momentum transfer is preferentially carried by the minimum number of exchanged particles. (This is not the case if the wave functions have exponential falloff.) Diagrams such as those of Fig. 12 correspond to absorptive corrections and because of their range in impact space, they essentially change only the over-all normalization, and not the energy or angular dependence of the basic interchange amplitude. Diagrams such as Fig. 11 corresponding to hadronic bremsstrahlung corrections contribute to the Reggeization of the amplitude at small t and u but vanish exponentially away from the forward or backward direction. Thus despite the large magnitude of hadronic couplings, it is justifiable to compute the lowest-order interchange amplitude in the deep scattering region, and to obtain the asymptotic result (1.2) for the cross section at fixed angles.

It should be emphasized that only the asymptotic form of the cross section for $s \rightarrow \infty$ at fixed t/s, u/s is determined by the simple interchange amplitude. Higher-order diagrams and finite-mass corrections lead to modification of order m^2/t

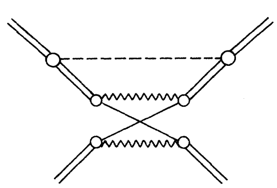

FIG. 11. A typical bremsstrahlung graph which leads to a Regge-type behavior at small momentum transfers.

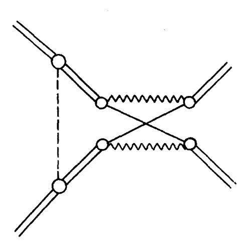

FIG. 12. Graph of a typical absorption correction which modifies the magnitude of the scattering amplitude.

and m^2/u. Thus the measured large-angle cross sections are expected to gradually approach the predicted asymptotic result in a manner similar to the approach of the elastic proton form factors to their true asymptotic behavior, but not to exhibit the quick onset of Bjorken scaling found in deep-inelastic electron-proton scattering. Fits to data must allow for these mass corrections. The next stage in the development of this theory must involve more precise determination of the properties of the binding interaction (Is it due to vector gluons, the physical states themselves, or other ?) and calculations of the corrections to the asymptotic formulas given here. Although power-law forms have been used in our examples, other similar analytic forms, e.g., $\ln F(t) \sim (a - b \ln t) \ln t$, are allowed with obvious modifications of the final results.

A simple and dramatic consequence of the interchange theory is a prediction for the asymptotic behavior of the effective trajectory at large t for an elastic process $A + B \to C + D$. Using the simplest model, the contribution of the (ut) amplitude at fixed, large $t = (P_A - P_C)^2$, is

$$M^{A+B \to C+D} \underset{u \to -\infty}{\sim} (-u)^{\alpha_{AC}(t)} \beta(t),$$

where, for consistency, the effective α must satisfy

$$\alpha_{AC}(t \to -\infty) = 1 - A - C$$
$$\cong \begin{cases} -3 \text{ for } pp \to pp \\ -1 \text{ for } \pi p \to \pi p \end{cases}.$$

We have defined the values of A and C from the asymptotic behavior of the elastic form factors

$$F_A(t) \underset{t \to \infty}{\sim} (-t)^{-A}, \quad F_C(t) \underset{t \to -\infty}{\sim} (-t)^{-C}.$$

The power-law dependence of $\beta(t)$ at large t may also be determined from the form factors of the particles involved in the reaction. Further consequences and comparisons with data are given in Ref. 14. We have also shown that this result for the effective trajectory also applies to the triple-Regge region of single-particle inclusive processes at large transverse momenta.[6]

A complete discussion of photon-induced (real and virtual) exclusive processes will be given elsewhere. Here we shall simply emphasize that, in general, the s dependence at fixed angle is always less steep than the corresponding vector-meson-induced processes because of the direct electromagnetic coupling of the photon to the elementary constituents. (All of the observed hadrons are assumed to be composite.) As we have discussed, a $J = 0$ fixed singularity—i.e., an $\epsilon_1^* \cdot \epsilon_2$ amplitude independent of energy and photon mass of fixed t with form factor dependence in t—is predicted in the Compton amplitude, but not in ρ photoproduction. Thus a dramatic breakdown of vector dominance is predicted in the deep scattering region. In general, the composite nature of the hadrons will eliminate the $P \to \infty$ contributions due to z-graphs, and hence fixed poles, from purely hadronic processes and photoproduction of hadrons.

The recent measurements of large-angle pion photoproduction of Anderson *et al.*[38] generally confirm our predictions for this process: The energy dependence at 90° is approximately s^{-7} as expected, and also the existence of a flat central region in the angular distribution is clear. A similar confirmation is provided by their measurements of the transition process $\gamma p \to \pi^- \Delta^{++}$. Since a detailed discussion of the features of exclusive and inclusive photoproduction and electroproduction processes will be given elsewhere, let us simply note that the spinless predictions for photomeson production yield the value $\alpha_{\text{eff}}(-\infty) = -\frac{1}{2}$. This would then provide a natural explanation for the rather strange behavior of the empirically observed effective α for this process which lies near zero. Thus the interchange contribution could easily be confused with a fixed pole (which has $\alpha_{\text{eff}} = 0$), particularly in the nonasymptotic t region.

All of the above processes should demonstrate the unity of the underlying physics of high-transverse-momentum and virtual-photon processes.

The simple physical picture of large-angle scattering afforded by the theory of the interchange force allows one to unify such diverse reactions as photon- and hadron-induced exclusive and inclusive processes. The general properties of the central angular distributions, their approximately power-law fall-off in energy, and the relation between their general shape and the quantum numbers of the interacting hadrons deserve further theoretical and experimental studies. Large-angle data covering a wide range of energies are particularly important for testing the interchange theory and for determining the details of the hadronic wave functions and the properties of their constituents.

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Unitary Multiperipheral Model with Diffractive Production

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A unitary model of multiparticle amplitudes with multiperipheral and diffractive production mechanisms is presented. The model has a bootstrap solution for the elastic amplitude of the form $\{(J-1)+[(J-1)^2-R_0t]^{1/2}\}^{-1}$, leading to constant total cross sections at high energies. Inelastic cross sections and multiplicity distributions may be predicted in qualitative agreement with experiment.

I. INTRODUCTION

It is generally accepted today that a realistic model of particle production in hadron collisions at high energies should contain both multiperipheral dynamics and diffractive fragmentation,[1] while at the same time satisfying the constraints imposed by the unitarity condition.[2] Unitary models of multiparticle amplitudes where particle production comes exclusively from multiperipheral mechanisms have been developed by several authors.[3] We present here a model in which the S matrix is unitary at high energies, but which includes both multiperipheral and diffractive production. Besides, a bootstrap solution exists which leads to a constant total cross section, in contrast with most previous models in which self-consistent solutions lead to $(\log s)^2$ behavior of the total cross section, thus saturating the Froissart bound.

The model is based on the eikonal approximation with the inclusion of the possibility of production of excited states of the external particles. The masses of these excited states are allowed to vary continuously within the limits imposed by the validity of the approximation. The excited states are allowed to decay into a nucleon and pions.

We thus have an infinite-channel model in the external "nucleons." In Sec. II we show how the inclusion of the possibility for excitation of the incoming particles leads to results similar to those obtained by Aviv, Sugar, and Blankenbecler (ASB),[3] but with the inclusion of an energy-dependent function $I(s)$, which is essentially an integral over the squares of the coupling functions for the excited "nucleons." The basic amplitude is specified by the diagram of Fig. 1.[4] All S-matrix elements are then shown to satisfy unitarity exactly at high energies. In Sec. III we show that there exists a bootstrap solution for the elastic amplitude which when combined with a restriction on the asymptotic behavior of the function $I(s)$ gives scattering amplitudes leading to constant total cross sections. We then compute inelastic cross sections and particle distributions, showing how a particular choice of coupling functions for the excited "nucleons" (which also determines their decay amplitudes) may lead to results in qualitative agreement with experiments.

Section IV contains a brief summary of the results and some concluding remarks.

II. THE MODEL

To define the kinematics, let us follow ASB and write

$$P_1 = m_1(\cosh y_1; 0, 0, \sinh y_1)\ ,$$