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A NON PERTURBATIVE PARTON MODEL OF CURRENT INTERACTIONS

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Abstract: A formulation of the parton picture of current interactions is given without recourse to perturbation theory. In deep inelastic electron scattering the structure functions W_1 and νW_2 scale but the corresponding functions $\overline{W}_1$ and $\nu\overline{W}_2$ for electron-positron annihilation processes at high energy are likely to be divergent. Form factors for particles other than the partons themselves go to zero at infinite momentum transfer. The relation between the rate of decrease of the nucleon form factors and the behaviour of νW_2 near $\omega = 1$ is shown to depend upon further specific properties of parton-nucleon dynamics and so to be model dependent. Some similarities between the parton picture and a recently proposed Veneziano-like amplitude are discussed. A difference is that the parton picture permits the incorporation of the pomeron.

1. INTRODUCTION

The suggestion of Bjorken [1] that the amplitude νW_2 of electroproduction tends to a finite function in the deep inelastic limit has been illustrated in several theoretical frameworks, and appears to be supported by the experimental data [2]. It has also been proposed [3] that the corresponding amplitude $\nu \overline{W}_2$ of electron-positron annihilation has a similar behaviour at high energy, but this has less theoretical support and there is not yet sufficient data from storage-ring experiments to provide a check on it. A perturbation - theory model with a cut-off does make both [4] νW_2 and [5] $\nu\overline{W}_2$ have a finite asymptotic limit, but in a Veneziano-like model νW_2 remains finite [6], while $\nu\overline{W}_2$ may become divergent [7].

In this paper we examine the situation in a class of field-theory models where the electromagnetic current is coupled in a simple fashion to a parton field. We consider first the case where the parton corresponds to a scalar field ϕ and take the electromagnetic current to be

$$j^{\mu} = i\phi^{\dagger} \overleftrightarrow{\partial} \phi \, . \tag{1.1}$$

Secondly we make the parton correspond to a spin $\frac{1}{2}$ field ψ and take

$$j^{\mu} = \overline{\psi}\gamma^{\mu} \psi \, . \tag{1.2}$$

In each case the field is a Heisenberg field; we make no use of perturbation theory. Where there are several types of parton, the current will contain a number of terms like (1.1) or (1.2). A result of our work will be that in the deep inelastic limit the cross section is an incoherent sum of contributions from the various types of parton.

Expressions like (1.1) and (1.2) do not determine the current but need to be made more precise by stating restrictions on the fields ϕ and ψ. One way of doing this would, of course, be by the specification of a Lagrangian. However we do not adopt this approach, preferring to discuss a whole class of theories characterised by a dynamical postulate which leads to a finite νW_2 in the deep inelastic limit. This postulate is:

(i) Hadronic amplitudes having virtual partons as external particles go sufficiently rapidly to zero as the masses of the partons become large. We discuss this more precisely in sect. 2.

Further we fix the normalisation of the fields in (1.1) and (1.2) by the requirement:

(ii) The boson two-point function

$$\Delta'_F(k^2) = i \int d^4x \, e^{ikx} \, \langle 0 | T(\phi^\dagger(x)\,\phi(0)) | 0 \rangle \, , \tag{1.3}$$

becomes asymptotically equal to $(k^2)^{-1}$ at large k^2, and the fermion two-point function

$$S'_F(k) = i \int d^4x \, e^{ikx} \, \langle 0 | T(\overline{\psi}(x)\,\psi(0)) | 0 \rangle \, , \tag{1.4}$$

becomes asymptotically $\gamma k / k^2$. It is easy to show by an adaptation of the methods give in sect. 2 below, that this leads to canonical commutators for currents of type (1.1) and (1.2), when the commutator is defined by means of the BJL limit [8].

The postulate (ii) is a normal one [8, 9] but (i) is more restrictive. We know from the fact that a cut-off is needed in some perturbation-theory models [4] that it is easy to construct examples where (i) is not satisfied.

With condition (i) the amplitude νW_2 does tend to a finite asymptotic function in the deep inelastic limit but, as in the Veneziano-like models [6], we find that $\nu \overline{W}_2$ is generally expected to become divergent.

In the case (1.2) of the spin $\frac{1}{2}$ parton the electroproduction amplitude W_1 also remains finite at infinite energy, as predicted by Bjorken [1], in such a way that the electroproduction is dominated by contributions from transverse virtual photons, which seems largely to be the experimental situation [2]. In the case (1.1) of the scalar parton, W_1 goes to zero asymptotically.

In both the perturbation-theory [10] and the Veneziano-like [6] models there is a connection between the threshold behaviour of the asymptotic form of νW_2 and the behaviour of the proton elastic form factor at large momentum transfer. However the relationships are different in the two cases. This indication that such a connection does not have any deep fundamental origin is borne out in the present work, where we find that the threshold behaviour of νW_2 and the asymptotic behaviour of the form factor depend on different properties of the hadronic amplitudes.

Our study of the elastic form factor leads to another conclusion: the form factor of an "ordinary" particle (meson or nucleon) goes to zero at large momentum transfer if and only if the partons are not themselves ordinary particles. Since the essential nature of a parton is that it interacts electromagnetically like a point particle this is to be expected.

2. SCALAR PARTONS

We are concerned with the matrix element

$$M^{\mu\nu} = i \int d^4x\, e^{iqx} \langle p | T(j^{\mu}(x) j^{\nu}(0)) | p \rangle \,, \tag{2.1}$$

where $|p\rangle$ is a proton state of momentum p, and it is understood that only the connected part of the matrix element is included and an average over proton polarisations is to be performed. Inserting (1.1) into (2.1) and using standard reduction formulae [11] leads to

$$M^{\mu\nu} = \frac{i}{(2\pi)^8} \int d^4k_1\, d^4k_2\, (2k_1 - q)^{\mu} (2k_2 - q)^{\nu}\, T_6(k_1 - q, k_1, k_2, k_2 - q) \,, \tag{2.2}$$

where T_6 is the six-point function that couples four partons to the two spin-averaged protons. This is shown diagrammatically in fig. 1a, where the arrows depict the direction of flow both of the labelled momentum and of positive charge.

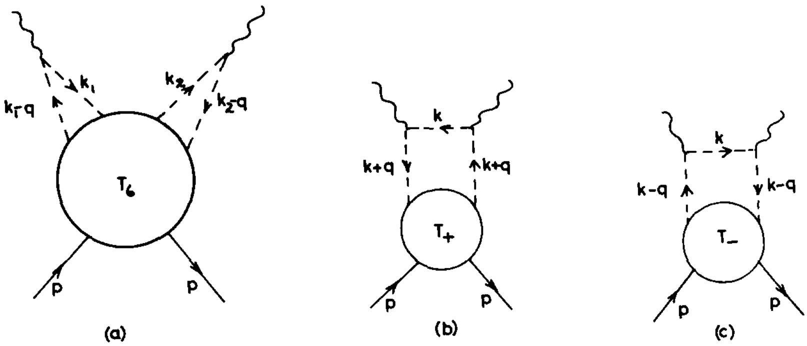

Fig. 1. (a) Diagrammatic representation of (2.2). The broken lines represent virtual partons. The arrows depict the direction of flow both of the labelled momentum and of positive charge. (b), (c) The contribution of the disconnected parts of T_6.

Note that T_6 is "non-amputated", it has poles in its parton legs at

$$(k_1 - q)^2 = m^2 \,, \qquad k_1^2 = m^2 \,, \qquad k_2^2 = m^2 \,, \qquad (k_2 - q)^2 = m^2 \,,$$

where m is the parton mass. It contains disconnected parts in which either the positive or the negative parton is a spectator [11]:

$$(2\pi)^4\, \delta^{(4)}(k_1 - k_2)\, \{T_+ \Delta_F'(k_1^2) + T_- \Delta_F'(\overline{k_1 - q}^2)\} \,. \tag{2.3}$$

Here $T_{\pm}$ are the amplitudes for the two-body forward scattering of positively and negatively charged off- mass-shell partons on the spin-averaged proton. The contributions from (2.3) to $M^{\mu\nu}$ are shown in figs. 1b and 1c. It is these contributions that will dominate in the deep inelastic limit. We show this in the appendix.

Our technique for exploring the deep inelastic limit is similar to that developed by Gribov [12] for the analysis of Regge cuts. (We do not expect the reader to have any previous knowledge of this technique). We begin by applying it to fig. 1c.

Write

$$k = xp + yq + \kappa \tag{2.4}$$

where the momentum κ is orthogonal to both p and q and so is spacelike ($\kappa^2 < 0$) and effectively two-dimensional. We may use x, y, κ as integration variables instead of k:

$$\mathrm{d}^4 k = J\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}^2\kappa \ . \tag{2.5}$$

In the deep inelastic limit

$$\begin{aligned} \nu &= p \cdot q \to \infty \ , \\ \omega &= \frac{-2\nu}{q^2} \quad \text{fixed} \ , \end{aligned} \tag{2.6}$$

the Jacobian has the asymptotic form

$$J \sim \nu \ . \tag{2.7}$$

The contribution of fig. 1c to $T^{\mu\nu}$ is

$$\frac{i}{(2\pi)^4} \int J\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}^2\kappa\ [2xp + (2y-1)\, q + 2\kappa]^{\mu}$$

$$[2xp + (2y-1)\, q + 2\kappa]^{\nu}\, \Delta_{\mathrm{F}}'(k^2)\, T_{-}(s', \mu^2) \ , \tag{2.8}$$

where

$$\begin{aligned} s' &= (p+q-k)^2 = (x-1)^2 M^2 + (y-1)^2 q^2 + 2(x-1)(y-1)\nu + \kappa^2 , \\ \mu^2 &= (q-k)^2 = x^2 M^2 + (y-1)^2 q^2 + 2x(y-1)\nu + \kappa^2 , \\ k^2 &= x^2 M^2 + y^2 q^2 + 2xy\nu + \kappa^2 \ , \end{aligned} \tag{2.9}$$

with M the nucleon mass. If we expand (2.8) in terms of the basis tensors

$$p^{\mu} p^{\nu} \ , \qquad p^{\mu} q^{\nu} + q^{\mu} p^{\nu} \ , \qquad q^{\mu} q^{\nu} \ , \qquad g^{\mu\nu} \ ,$$

the coefficient of $p^{\mu} p^{\nu}$ is

$$\frac{i}{(2\pi)^4} \int J\, \mathrm{d}x\, \mathrm{d}y\, \mathrm{d}^2\kappa\, \mathrm{d}\sigma\, \frac{\rho(\sigma)}{k^2 - \sigma} \left[4x^2 - \frac{2\kappa^2}{M^2 - \nu^2/q^2} \right] T_{-}(s', \mu^2) \ , \tag{2.10}$$

where we have introduced the Lehmann representation [8]

$$\Delta_F'(k^2) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{k^2 - \sigma} , \tag{2.11}$$

where according to postulate (ii) of sect. 1,

$$\int_0^\infty d\sigma \rho(\sigma) = 1 . \tag{2.12}$$

In deriving (2.10) we have made use of the relation, holding for the integrand, that $\kappa^\mu \kappa^\nu$ may be replaced by

$$\tfrac{1}{2}\kappa^2 \left[g^{\mu\nu} - \frac{p^\mu p^\nu}{M^2 - \nu^2/q^2} - \frac{q^\mu q^\nu}{q^2 - \nu^2/M^2} + (p^\mu q^\nu + q^\mu p^\nu) \frac{\nu}{M^2 q^2 - \nu^2} \right] . \tag{2.13}$$

The expression in square brackets in (2.13) is the unit matrix in the two-dimensional subspace orthogonal to p and q.

The imaginary part of (2.10) is the contribution of fig. 1c to W_2/M. The coefficient of $g^{\mu\nu}$, whose imaginary part is the contribution to MW_1, is obtained by replacing the square bracket in (2.10) by $2\kappa^2$.

Make the change of variable

$$y = y' - \frac{xM^2}{2\nu} , \tag{2.14}$$

so that in the deep inelastic limit (2.6)

$$\mu^2 = 2\nu(y' - 1) \left(x - \frac{y' - 1}{\omega} \right) + \frac{2xM^2}{\omega} (y' - 1) + \kappa^2 . \tag{2.15}$$

According to postulate (i) of sect. 1, the dominant contribution to the integral arises from that part of the integration in which μ^2 remains finite as $\nu \to \infty$. Hence we expect it to come from either of the regions

$$y' \approx 1 ,$$
$$y' \approx 1 + \omega x . \tag{2.16}$$

We dispose of the second possibility in the appendix, where we also deal with the possibility that y' is outside both of the regions (2.16), but that κ^2 is large in such a way that μ^2 remains finite.

Thus we introduce instead of y' the new variables $\bar{y}$:

$$y' = 1 + \frac{\bar{y}}{2\nu} , \tag{2.17}$$

and take the limit under the integration in (2.10). With (2.7) and (2.12) this gives

$$\frac{i}{\nu (2\pi)^4} \int dx\, d\bar{y}\, d^2\kappa \frac{x^2\, T_-(s', \mu^2)}{x - \omega^{-1}} ,$$
$$s' = (x - 1)(\bar{y} - M^2) + \kappa^2 ,$$
$$\mu^2 = x\bar{y} + \kappa^2 . \tag{2.18}$$

It is possible in the light of this argument to clarify postulate (i) somewhat. Outside the region (2.16) both μ^2 and s' will tend to infinity like ν. In order for this contribution to be less important than that given by (2.18) we require that in this limit $T_- \sim \nu^{-\gamma}$, $\gamma > 1$. A similar assumption is made in the Gribov reggeon calculus [12].

According to the usual ideas of analyticity, T_- has singularities in μ^2, a right hand cut in the variable s', and a left hand cut in the variable

$$u' = 2\mu^2 + 2M^2 - s' = (x+1)(\bar{y}+M^2) + \kappa^2 , \tag{2.19}$$

where in each case the singularity is slightly displaced below the positive real axis of the variable in question. If $|x| > 1$ all these singularities occur on the same side of the $\bar{y}$ contour of integration. This contour can therefore be completed by a semi-circle at ∞ in the opposite half plane to give a zero result. If $0 < x < 1$ the integration no longer gives zero but may be wrapped around the right hand s' cut. If $0 > x > -1$, the $\bar{y}$ integration may similarly be wrapped around the left hand u' cut. Thus $T_-(s', \mu^2)$ in (2.18) may be replaced by

$$2i\,[\,\theta(x)\,\theta(1-x)\,\mathrm{Im}\,T_{\mathrm{R}} + \theta(-x)\,\theta(1+x)\,\mathrm{Im}\,T_{\mathrm{L}}]\ , \tag{2.20}$$

where the suffixes R, L respectively denote the right and left hand parts of T_-. It will now be seen (fig. 2a) that the integral (2.18) is real for $|\omega| < 1$,

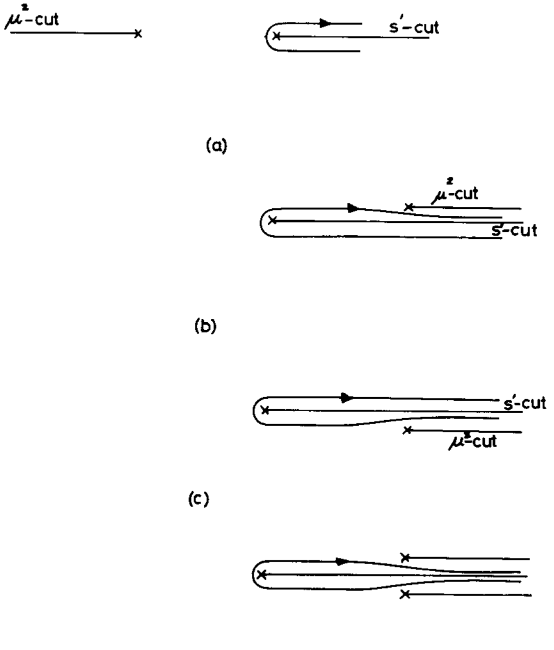

Fig. 2. (a) The integration contour in (2.21) drawn in the s' or $-y$ plane. (b), (c) The results of continuing (2.21) into the region $0 < \omega < 1$, passing to different sides of the possible singularity at $\omega = 1$. (d) The correct integration to give νW_2.

and that for ω outside this range it has an imaginary part obtained by replacing the denominator of the integrand by a delta function. In fact the range of values $\omega > 1$ is the kinematic region of interest in electroproduction, and the contribution to νW_2 is thus

$$-\frac{2}{(2\pi)^3\,\omega^2}\int \mathrm{d}\bar{y}\,\mathrm{d}^2\kappa\ \mathrm{Im}\,T_{\mathrm{R}}(s',\mu^2)\ ,$$
$$s' = (\omega^{-1}-1)(\bar{y}-M^2)+\kappa^2\ ,$$
$$\mu^2 = \omega^{-1}\,\bar{y}+\kappa^2\ . \tag{2.21}$$

The contribution from fig. 1b may be obtained from this by crossing, and is the same but with $\mathrm{Im}\,T_{\mathrm{R}}(s',\mu^2)$ replaced by $\mathrm{Im}\,T_{\mathrm{L}}(u',\mu^2)$ with

$$u' = (\omega^{-1}+1)(\bar{y}+M^2)+\kappa^2\ . \tag{2.22}$$

To discover the large ω behaviour of (2.21), change variable from $\bar{y}$ to $z = -\omega^{-1}\bar{y}$. Take the limit under the integral and insert for $\mathrm{Im}\,T_{\mathrm{R}}$ the large s' Regge behaviour

$$\mathrm{Im}\,T_{\mathrm{R}} \sim \beta(\mu^2)\,\nu'^{\alpha_0}\ ,$$
$$\nu' = \tfrac{1}{2}(s'-u')\ . \tag{2.23}$$

This gives the asymptotic form *

$$\frac{2}{(2\pi)^3}\,\omega^{\alpha_0-1}\int_0^\infty \mathrm{d}z\int \mathrm{d}^2\kappa\,\beta(\kappa^2-z)\,z^{\alpha_0}\ . \tag{2.24}$$

If the pomeron couples to the partons, $\alpha_0 = 1$.

On the other hand, the behaviour of (2.21) near $\omega = 1$ depends on the form of T_{R} for large μ^2, at finite s'. Change the integration variable from $\bar{y}$ to s':

$$\frac{2}{(2\pi)^3}\,\frac{1}{\omega(\omega-1)}\int \mathrm{d}s'\,\mathrm{d}^2\kappa\ \mathrm{Im}\,T_{\mathrm{R}}(s',\mu^2)\ ,$$
$$\mu^2 = \frac{s'-\kappa^2}{1-\omega}+\frac{M^2}{\omega}+\kappa^2\ . \tag{2.25}$$

Suppose that for large μ^2

$$\mathrm{Im}\,T_{\mathrm{R}} \sim (\mu^2)^{-\gamma}\,f(s')\ . \tag{2.26}$$

(We have no reason to believe that such a simple behaviour is necessarily realistic.) Then on letting $\omega \to 1$ under the integral in (2.25) we obtain a result proportional to $(\omega-1)^{\gamma-1}$.

As has been remarked elsewhere [7], the fact that the value of γ may be such that this factor is not real for $\omega < 1$ provides a warning that it may not be possible to continue νW_2 into the region $0 < \omega < 1$ to obtain the corresponding amplitude $\nu\overline{W}_2$ describing electron-positron annihilation. The

* It should be remembered that the vector κ is spacelike, $\kappa^2 < 0$.

variables μ^2 and s' are linked together in the integration by the relation in (2.25). For $\omega > 1$ the singularities in the variable μ^2 are to the left of the s' cut, but this is no longer so for $0 < \omega < 1$. Thus an analytic continuation into this region results in the μ^2 singularities retreating to $-\infty$ and then reappearing from $+\infty$ in the s' plane, either on the upper side of the s' cut or on the lower, according to the route taken round $\omega = 1$ (figs. 2b and 2c). The difference between the two cases, if there is a difference once the integrations have been performed, corresponds [7] to the existence of singularities of the amplitude in the variable q^2. Neither continuation corresponds to the correct integral for $\nu\overline{W}_2$, which is obtained by evaluating the integral with the singularities in one of the two parton masses above the s' cut, and the other below (fig. 2d). As these μ^2 singularities are expected to occur in $\mathrm{Im}\, T_{\mathrm{R}}$ they will pinch the integration contour. Since we are dealing with a non-amputated amplitude the μ^2 singularities include a pole at the parton mass. The associated pinch may be expected to lead to a divergence of the integral. A similar divergence of $\nu\overline{W}_2$ is also, in general, a feature of Veneziano-like models [7].

In the scalar-parton model, the asymptotic form of MW_1 in the deep inelastic limit is ν^{-1} times an integral like (2.25), but with an additional factor $\omega^2\kappa^2$ in the integrand (the crossed term must, of course, also be added). Thus in this model W_1 is also $\mathrm{O}(\nu^{-1})$ in the deep inelastic limit. This does not agree with the experimental result, that $M^2 W_1 \sim \frac{1}{2}\omega\nu W_2$. To obtain this, we must use the spin $\frac{1}{2}$ parton model, corresponding to the current in (1.2), which is discussed in the next section.

3. SPIN $\frac{1}{2}$ PARTONS

In this case the contribution of fig. 1c to $T^{\mu\nu}$ is

$$\frac{i}{(2\pi)^4}\int J\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}^2\kappa\ \mathrm{Tr}\,\{\gamma^{\mu} S_{\mathrm{F}}'(k)\gamma^{\nu}\, T_{-}\}\ . \tag{3.1}$$

The parton-proton amplitude T_- is averaged over proton polarizations and so is a matrix in the parton spinor indices only. We write the propagator in the Lehmann representation

$$S_{\mathrm{F}}'(k) = \int_0^{\infty} \mathrm{d}\sigma \frac{\gamma k\,\rho_2(\sigma) + \rho_1(\sigma)}{k^2 - \sigma}\ , \tag{3.2}$$

where postulate (ii) of sect. 1 requires that

$$\int_0^{\infty} \mathrm{d}\sigma \rho_2(\sigma) = 1\ .$$

The technique for handling the spinor factors in (3.1) is the same as that devised by Gaisser and Polkinghorne [4] for use in analyses of perturbation theory. The trace expression is rearranged by means of a Fierz transformation in the form

$$\tfrac{1}{4}\sum_{i=1}^{5} T_-^i \Delta^{i\mu\nu} , \tag{3.3}$$

where

$$T_-^i = \mathrm{Tr}\{\Gamma_i T_-\} , \tag{3.4}$$

$$\Delta^{i\mu\nu} = \mathrm{Tr}\{\gamma^\mu S_F'(k)\gamma^\nu \Gamma_i\} , \tag{3.5}$$

with $\{\Gamma_i\}$ being the set of γ-matrices

$$\{1, \gamma^\alpha, \sigma^{\alpha\beta}, \gamma_5\gamma^\alpha, \gamma_5\} \tag{3.6}$$

suitably contracted in (3.3) over the indices α and β. The functions $\Delta^{i\mu\nu}$ are similar to those found in perturbation theory analyses [4] and we find that $\Delta^{5\mu\nu} = 0$ and $\Delta^{3\mu\nu}$ and $\Delta^{4\mu\nu}$ do not contribute to the total amplitude (obtained from the sum of contributions from 1b and 1c) since they are antisymmetric in μ, ν and the total amplitude must be symmetric. One readily finds that $\Delta^{1\mu\nu}$ is proportional to

$$\rho_1 g^{\mu\nu} , \tag{3.7}$$

and $\Delta^{2\mu\nu}$ is proportional to

$$\rho_2[g^{\alpha\mu}k^\nu + g^{\alpha\nu}k^\mu - g^{\mu\nu}k^\alpha] . \tag{3.8}$$

Since T_- is a function of the vectors p, $k-q$, the corresponding T_-^i can be written

$$T_-^1 = T^1 , \tag{3.9}$$

$$T_{-\alpha}^2 = T_1 p_\alpha + T_2(k-q)_\alpha , \tag{3.10}$$

where T^1, T_1 and T_2 are invariant functions.

In order to identify W_1 one looks for the imaginary part of the coefficient of $g^{\mu\nu}$. Contributions to this coefficient arise from both Δ^1 and Δ^2. The latter contains a term with a factor ν formed from the contraction of the yq^α in k^α in (3.8) with the p_α present in (3.10). This term dominates in the deep inelastic limit and is the reason why W_1 itself has a non-zero limit. (Note that the ν formed from xp^α in (3.8) and $(y-1)q_\alpha$ in (3.10) is not effective since $(y-1) \sim \nu^{-1}$). Thus the dominant contribution to W_1 in the limit is obtained from

$$\frac{i}{(2\pi)^4}\int J\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}^2\kappa \int_0^\infty \mathrm{d}\sigma \frac{(T_1 + xT_2)\rho_2 y\nu}{k^2-\sigma} . \tag{3.11}$$

This is the analogue of equations of the type of (2.10) in sect. 2. The further analysis proceeds exactly as before, obtaining the leading contribution from the neighbourhood of $y \sim 1$. It is not necessary to repeat the details.

W_2 may be obtained by, for example, isolating the coefficient of $p^\mu q^\nu$ in (3.1). The imaginary part of this coefficient is $\frac{1}{2}\omega W_2/M^2$. The coefficient itself is just ν^{-1} times the expression (3.11). Therefore we find the expected relation

$$2M^2 W_1 = \nu\omega W_2 \;, \tag{3.12}$$

corresponding to $R = 0$ for spin $\frac{1}{2}$ partons. Of course W_2 could also be obtained by calculating the imaginary part of the coefficient of $p^\mu p^\nu$. One readily sees that the same answer is obtained. This is one of a number of similar simple checks on the gauge invariance of the theory.

4. THE ELASTIC FORM FACTOR

In this section we show that the partons are not protons. Nor are they any other particle whose elastic form factor goes to zero for large momentum transfer Δ^2.

Using again the reduction formulae of field theory, we find that with the current (1.2) the elastic form factor of the proton takes the form drawn in fig. 3a. If, and only if, the parton is a proton, the parton-proton amplitude contained within this diagram contains a disconnected part in the Δ^2 channel. This contributes to the form factor a term drawn in fig. 3b, which is independent of Δ^2. We show that the remaining part goes to zero at large Δ^2, so that only if fig. 3b is not present does the whole form factor go to zero.

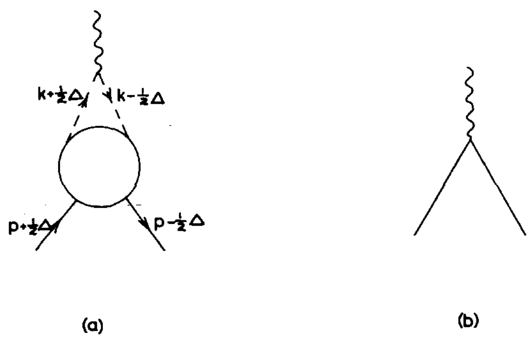

Fig. 3. (a) Diagrammatic representation of the proton elastic form factor. (b) The contribution of the disconnected part that occurs when the current couples directly to the proton.

Label the momenta as in fig. 3a, and write

$$k = \alpha P + \beta\Delta + \chi \;, \tag{4.1}$$

where χ is a momentum orthogonal to both P and Δ. Since $(P \pm \frac{1}{2}\Delta)^2 = M^2$, the masses of the two partons are

$$\mu_\pm^2 = (k \pm \tfrac{1}{2}\Delta)^2 = \alpha^2 M^2 + \Delta^2[(\beta \pm \tfrac{1}{2})^2 - \tfrac{1}{4}\alpha^2] + \chi^2 \;. \tag{4.2}$$

According to postulate (i) of sect. 1, the dominant contribution to the form factor arises from finite values of these two masses as $\Delta^2 \to \infty$, that is from* $\beta \approx 0$ and $\alpha \approx \pm 1$. The contributions from these two regions are similar, being related by crossing the proton lines. Consider one of them for definiteness, and make the changes of variables

$$\alpha = 1 + \frac{\bar{\alpha}}{\Delta^2} ,$$

$$\beta = \frac{\bar{\beta}}{\Delta^2} . \tag{4.3}$$

The asymptotic behaviour of the form factor then takes the form

$$\int d\bar{\alpha}\, d\bar{\beta}\, d^2\chi\, g(\mu_+^2, \mu_-^2, \chi^2)\, (\Delta^2)^{\tilde{\alpha}(\chi^2)-1} ,$$

$$\mu_\pm^2 = M^2 + \chi^2 - \tfrac{1}{2}\bar{\alpha} \pm \bar{\beta} , \tag{4.4}$$

where $\tilde{\alpha}$ is the leading Regge trajectory of the parton-proton amplitude in the parton+proton channel and g is a Regge residue function. Because χ is spacelike, the exponent of Δ^2 is $\leqslant 0$ throughout the integration. Also, because the singularities in $\mu_\pm^2$ are all in Im $\mu_\pm^2 = 0$ - the $\bar{\alpha}$ integration can be completed by a semicircle at infinity in the upper half plane to give zero. That is, the form factor goes to zero for large Δ^2. How rapidly it goes to zero depends on how rapidly g goes to zero for large $\mu_\pm^2$. That is, the asymptotic behaviour of the form factor depends on the asymptotic behaviour of the parton-proton amplitude when both the parton mass and the momentum transfer become large. This is to be contrasted with the behaviour of νW_2 near $\omega = 1$ which, as we showed in sect. 2, depends on the behaviour of the parton-proton amplitude when the parton mass becomes large, but the other variables remain finite.

5. DISCUSSION

The model presented in this paper provides a parton, or point-particle, picture of deep inelastic scattering without the need to resort to perturbation theory. A result of our work is that, when there are several types of parton, the total contribution to the scaled structure functions is given by an incoherent sum of the contributions from the different types of parton. The point-like character of the partons is expressed by their electromagnetic form factors not tending to zero at infinite momentum transfer, so that the partons are certainly not nucleons. The basic spirit of the approach is similar to that used by West [13] in constructing his phenomenological model for the electromagnetic structure of the proton.

* This time we can exclude the possibility that $\mu_\pm^2$ remain bounded through χ becoming divergent, since this solution would make the energy variable of the hadron amplitude become large. This corresponds to a fixed angle limit and so the amplitude tends rapidly to zero.

In sects. 2 and 3 we discussed the forward-scattering current matrix elements, since it is their imaginary parts which give the structure functions of deep inelastic scattering. However it is easy to repeat the analysis with $t \neq 0$ and one finds that the amplitudes still scale. This would not be so for theories based on a simple phenomenological diffraction mechanism [14], but it is true in the Veneziano-like model [6].

Another feature common to the model of this paper and the Veneziano-like model is that there is a connection between the mechanism responsible for the scaling of νW_2 and the current-algebra fixed pole. This is most readily seen by considering the amplitude for the scattering of charged $I = 1$ vector currents, relevant to deep inelastic neutrino scattering. The analysis of the deep inelastic limit needs only trivial changes in the arguments of sects. 2 and 3. Once again the dominant contribution comes from the region $y \approx 1$. If one attempts to calculate the Regge limit ($\nu \to \infty$, q^2 fixed) in a similar way there are now two significant regions

$$x \approx 0 \;, \tag{5.1a}$$

$$y \approx 1 \;. \tag{5.1b}$$

A straightforward evaluation of their contributions shows that (5.1a) gives Regge behaviour

$$\sim \nu^{\alpha(t)-2} \tag{5.2a}$$

where $\alpha(t)$ is the Regge pole dominating the parton-proton amplitude, while (5.1b), the region giving also the deep inelastic contribution, yields the fixed pole behaviour

$$\sim -\frac{2F(t)}{\nu} \;, \tag{5.2b}$$

where $F(t)$ is the form factor of the $I = 1$, $I_3 = 0$ current. It can be shown that when $\alpha(t)$ is an odd-signature trajectory the Regge term in (5.2a) has a pole at $\alpha(t) = 1$, which cancels the corresponding pole in (5.2b). It was explained by Bronzan et al. [15], and verified by them in a perturbation-theory model, that this feature is necessary in order that the complete double-flip amplitude does not have a spin-one pole in the t-channel. A similar mechanism makes (5.2b) finite at $\alpha(t) = 1$ when $\alpha(t)$ has even signature; Abarbanel et al. [16] pointed out that this must be so in order that the pomeron couple to the double-flip amplitude.

Another way of exploring the connection between the scaling of νW_2 and the current-algebra fixed pole is by means of sum rules; we propose to discuss this elsewhere [17].

Finally, we note that there is one striking difference between the model of this paper and that provided by the Veneziano-like amplitudes. No dual theory can satisfactorily accommodate the pomeron, but it finds a natural place in the formalism discussed here.

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APPENDIX

We argue here that the contribution we have considered in the text is actually the dominant one in the deep inelastic limit.

Deal first with the second possibility in (2.15), by making the change of variable from y' to $\bar{y}'$, where

$$y' = 1 + x\omega + \bar{y}'/2\nu \,, \tag{A.1}$$

and taking the limit (2.6) under the integral. Then (2.10) becomes

$$\begin{aligned} &\frac{i}{\nu(2\pi)^4} \int \mathrm{d}x\, \mathrm{d}\bar{y}'\, \mathrm{d}^2\kappa\, \frac{x^2 T_-(s', \mu^2)}{x+\omega^{-1}} \,, \\ &\mu^2 = -x(\bar{y}' - 2xM^2) + \kappa^2 \,, \\ &s' = -2x\omega\nu \,. \end{aligned} \tag{A.2}$$

At first sight this is $O(\nu^{\alpha_0 - 1})$. However, the $\bar{y}'$ integration can be completed by a semicircle in the lower/upper half plane for $x \gtrless 0$ and so vanishes. How rapidly the contribution goes to zero depends on how rapidly T_- goes to zero at large μ^2 when s' is large.

Deal next with the possibility that κ^2 is large. Change variable from κ^2 to ξ, where

$$\kappa^2 = \frac{2\nu}{\omega}(y' - 1)^2 - 2\nu x(y' - 1) + \xi \,. \tag{A.3}$$

Then if x, y' are not in either of the regions (2.16), μ^2 will be finite in the deep inelastic limit only if ξ is finite. Since κ is spacelike, this can only happen where $[(y'-1)^2/\omega - x(y'-1)] < 0$. Given this, there is then no restriction on the range of values of ξ so we can integrate it from $-\infty$ to ∞ (in the non-finite part of this integration the properties of T_- implied by postulate (i) will produce a negligible contribution, but we may as well include it). Asymptotically, as $\nu \to \infty$, s' and k^2 become independent of ξ. Hence the ξ integration gives zero. Again, the rate at which the contribution goes to zero depends on the large μ^2 behaviour of T_- for large s'.

Again, consider the contribution from the connected part of T_6 in fig. 1a. Write

$$k_i = x_i p + y_i q + \kappa_i \,, \qquad i = 1, 2 \,, \tag{A.4}$$

where the κ_i are orthogonal to both p and q. According to postulate (i) the dominant contribution arises from finite values of

$$\begin{aligned} k_i^2 &= x_i^2 M^2 + y_i^2 q^2 + 2\nu x_i y_i + \kappa_i^2 \,, \\ (k_i - q)^2 &= x_i^2 M^2 + (y_i - 1)^2 q^2 + 2\nu x_i (y_i - 1) + \kappa_i^2 \,, \end{aligned} \tag{A.5}$$

in the deep inelastic limit. We dispose of the solution with non-finite κ_i^2 in the same way as before. This leaves

$$x_i \approx \frac{2y_i - 1}{\omega} ,$$

$$y_i \approx 0 \text{ or } 1 . \tag{A.6}$$

Treating these in the same way as before, we find that each possibility leads to a contribution to νW_2 that goes to zero at least as fast as ν^{-1}. (The Jacobians corresponding to (A.4) give a factor ν^2 but the changes of variables corresponding to the conditions (A.6) yield ν^{-4} giving a net behaviour for W_2 of ν^{-2}).

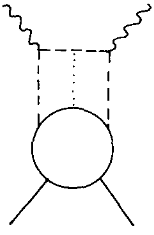

Fig. 4. A class of Feynman diagrams that give contributions to W_1 of order ν^{-1}. The dashed lines represent partons of spin $\frac{1}{2}$; the dotted line is a scalar gluon.

Similar arguments yield contributions to W_1 or order ν^{-2} also. For spin $\frac{1}{2}$ partons this may seem at first sight surprising since it is known [4] that Feynman diagrams of the form of fig. 4 give contributions to W_1 of order ν^{-1}. The explanation is simple. These contributions arise from regions where $k_1^2 \sim \nu$, $k_2^2 \sim \nu$, that is two of the parton masses are large. These particular Feynman diagrams only go to zero like ν^{-1} in this case. However these contributions are still negligible compared to those from fig. 1b and fig. 1c in the scaling limit.

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