Dispersive approach, Unitarity, Pion vector form factor

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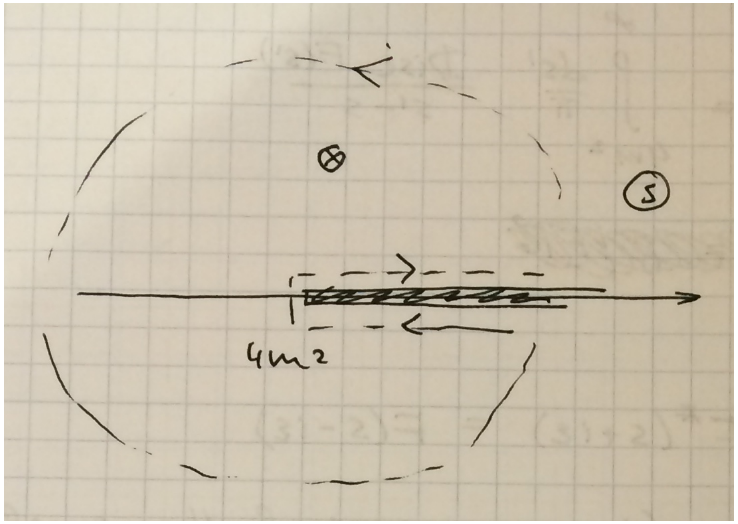
Dispersion relation



Cauchy Formula (analyticity)

$$F(s) = \frac{1}{2\,i} \int_{4m^2}^{\infty} \frac{ds'}{\pi} \frac{\mathrm{Disc}\, F(s')}{s' - s}$$

Reflection principle

$$F^*(s + i\epsilon) = F(s - i\epsilon)$$

Relation to unitarity

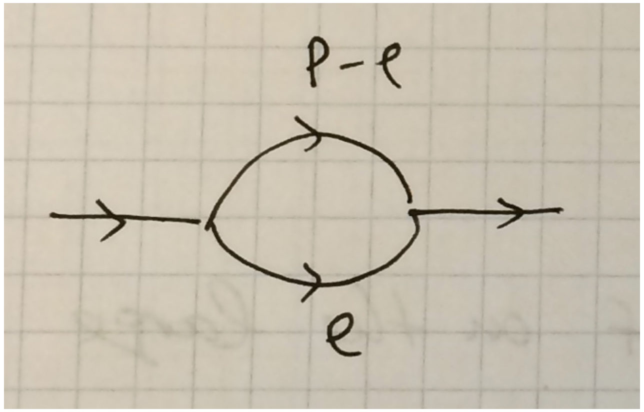
$$\mathrm{Disc}\, F(s) = F(s + i\epsilon) - F(s - i\epsilon) = 2i\, \mathrm{Im}\, F(s)$$

Calculation of Discontinuity

Cutkosky (cutting) rule

$$\frac{1}{p^2 - m^2 + i\epsilon} \to (-2\pi i)\,\delta(p^2 - m^2)$$

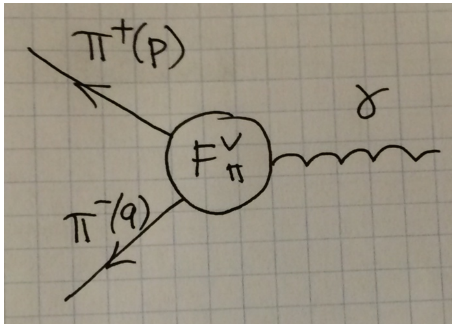
Example



$$\text{Disc}\, M = \frac{i}{8\pi}\sqrt{1 - \frac{4m^2}{s}} = \frac{i}{8\pi}\rho(s)$$

$$\text{Im}\, M = \frac{1}{16\pi}\rho(s)$$

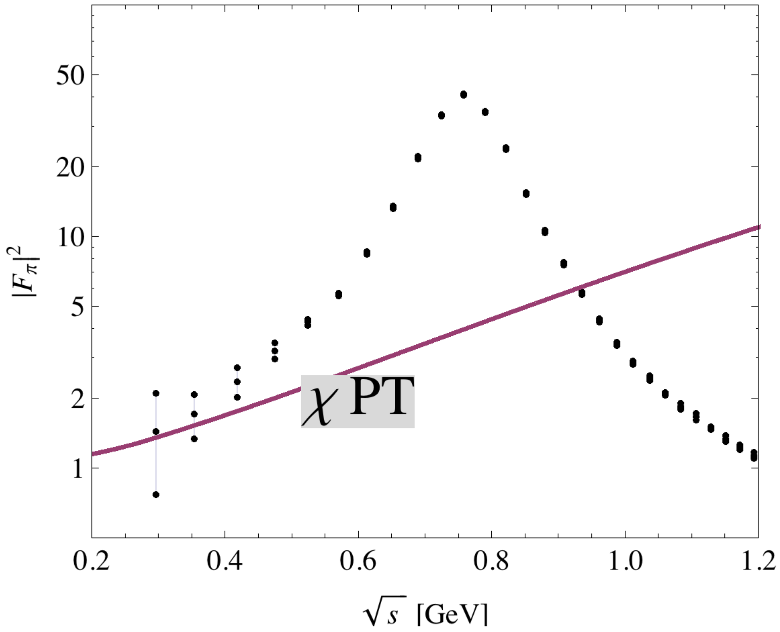
Pion vector form factor

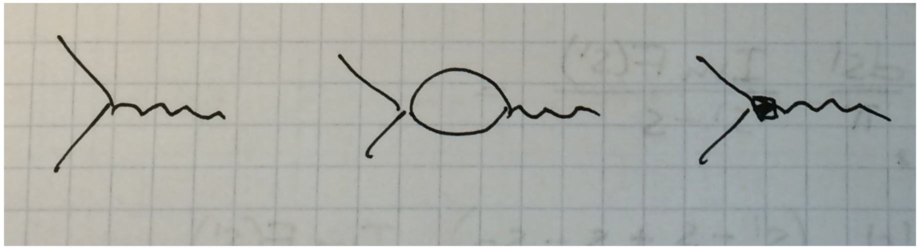


Transition from a photon into a pair of pions

$$\langle \pi^+(p)\pi^-(q)|J_\mu(0)|0\rangle = (p-q)_\mu\, F_\pi^V(s)$$

ChPT at low energy



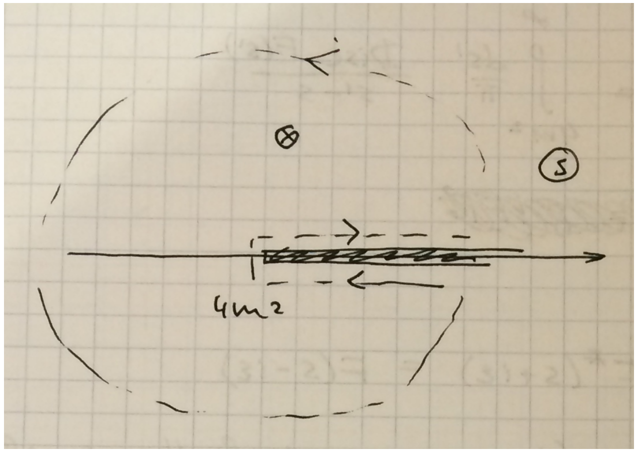


$$L \sim A_\mu\, \pi^+\, \partial^\mu \pi^- + \ldots$$

$$F_\pi^V(s) = 1 + \frac{1}{6}\frac{1}{4\pi f_\pi^2}(\bar{L}_6 - 1)\, s + \ldots$$

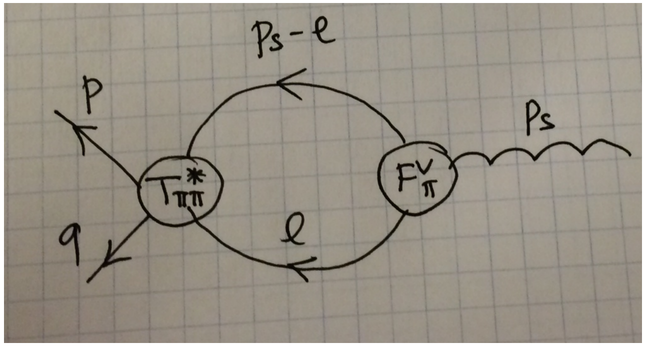
Pion vector form factor

Analyticity & Unitarity



$$\mathrm{Im}\, F^V_\pi(s) = \rho(s) t^*(s) F^V_\pi(s)$$

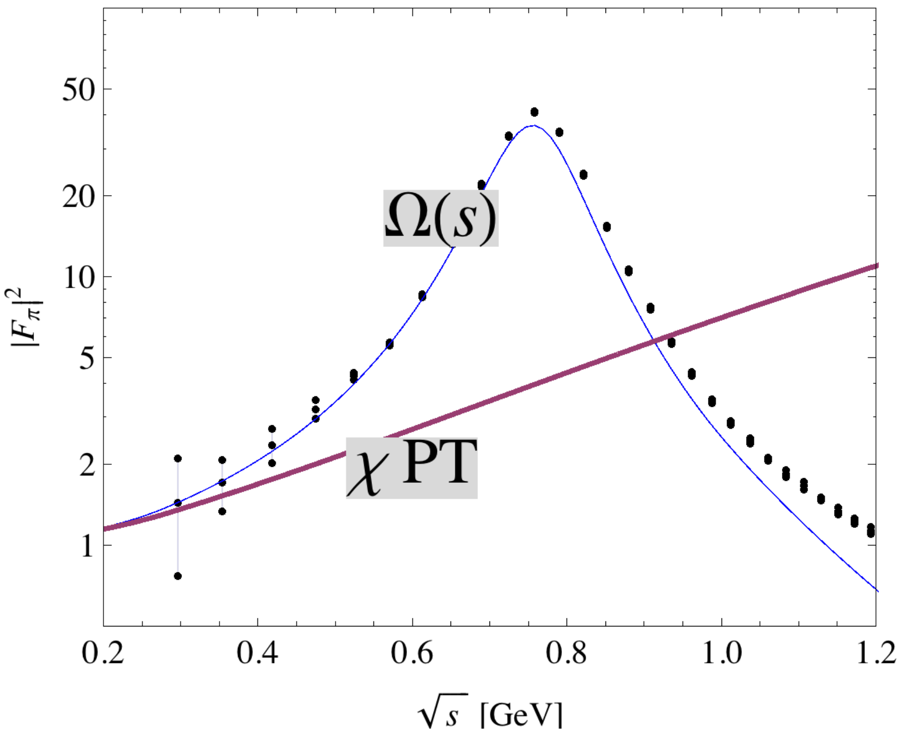
Watson theorem



$$\mathrm{Arg}\, F^V_\pi(s) = \delta(s)$$

Omnes function

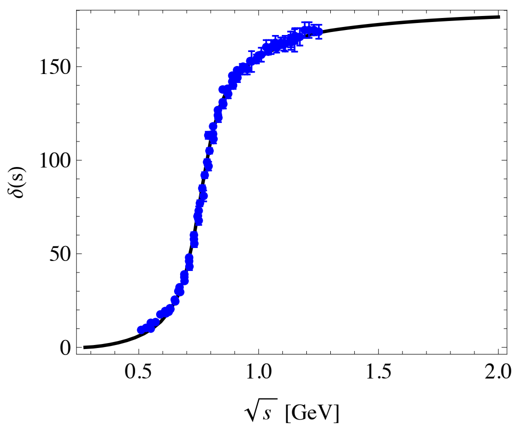
Analyticity & Unitarity



$$F_\pi^V(s) = P(s)\,\Omega(s)$$

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4m^2}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s}\right)$$

input



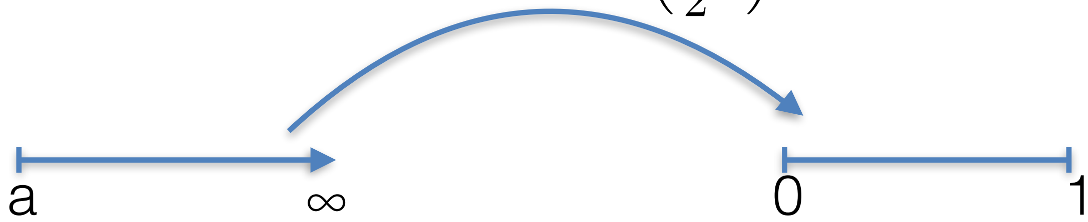
High energy behavior

$\delta(s) \to \alpha\,\pi$ then $\Omega(s) \to \dfrac{1}{s^\alpha}$

Numerical implementation

Tangent stretching:

$$y = a + C_{ex}\, tg\left(\frac{\pi}{2}x\right)$$



Example 2: $\int_2^{\infty} e^{-y^2}\, dy$

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<< NumericalDifferentialEquationAnalysis`

n = 10; Cext = 1;
wg = GaussianQuadratureWeights[n, 0, 1];
yn = 2 + Table[Cext * Tan[π / 2 * wg[[i, 1]]], {i, n}];
wgn = Table[wg[[i, 2]] (Cext π / 2) / (Cos[π / 2 wg[[i, 1]]] ^ 2), {i, n}];

Sum[Exp[-yn[[i]] ^ 2] wgn[[i]], {i, 1, n}]
Integrate[Exp[-y ^ 2], {y, 2., Infinity}]

0.00414553

0.00414553
```

Principle value integral

$$\int_{4m^2}^{\infty} \frac{ds'}{s'} \frac{f(s')}{s' - s - i\epsilon} = p.v. \int_{4m^2}^{\infty} \frac{ds'}{s'} \frac{f(s')}{s' - s} + i\pi \frac{f(s)}{s}$$

$$p.v. \int_{4m^2}^{\infty} \frac{ds'}{s'} \frac{f(s')}{s' - s} = \int_{4m^2}^{\infty} \frac{ds'}{s'} \frac{f(s') - f(s)}{s' - s} + \frac{f(s)}{s} \ln \frac{4\,m^2}{s - 4\,m^2}$$