Unitarization procedures applied to a Strongly Interacting EWSBS

Rafael L. Delgado

A.Dobado, M.J.Herrero, Felipe J.Llanes-Estrada and J.J.Sanz-Cillero,

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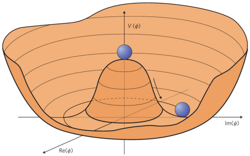
PRL **114** (2015) 221803, PRD **91** (2015) 075017, JHEP **1407** (2014) 149, JHEP **1402** (2014) 121 and J. Phys. G: Nucl. Part. Phys. **41** (2014) 025002

Empirical situation



doi:10.1038/nphys1874

- Electroweak symmetry breaking: $SU(2)_L \times SU(2)_R \to SU(2)_C$
- Three would-be Goldstone bosons ω.
- Equivalence theorem: for $s \gg 100\,\mathrm{GeV}$, Identify them with the longitudinal components of W and Z.
- A 125-126 GeV scalar "Higgs" resonance φ.

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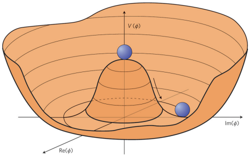

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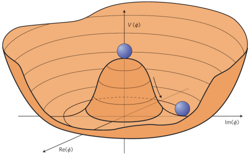

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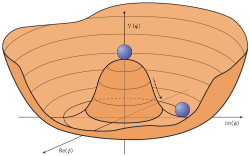

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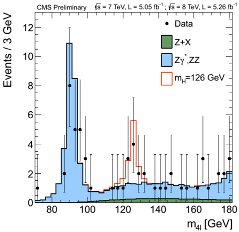

arXiv:1207.7235 [hep-ex]

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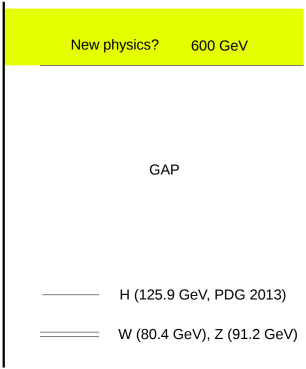
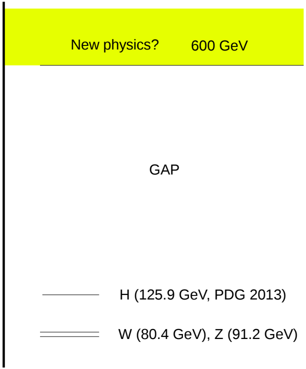


- IMPORTANT: No new physics!! *If there is any...*
- Four scalar light modes, a strong gap.
- Natural: further spontaneous symmetry breaking at $f > v = 246\,\mathrm{GeV}$?

Empirical situation



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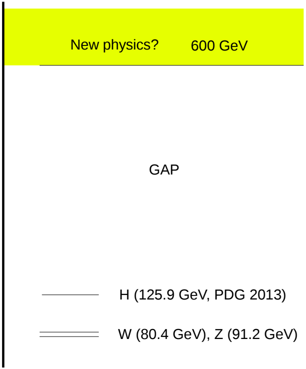

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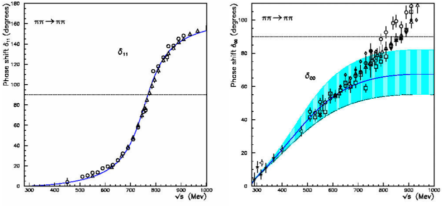
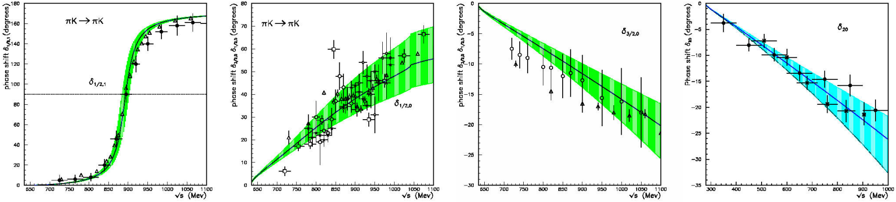
Effective Field Theory + Unitarity: similarity with low–energy (i.e.: hadronic) physics

Chiral Perturbation Theory plus Dispersion Relations.

Simultaneous description of $\pi\pi \rightarrow \pi\pi$ and $\pi K \pi K \rightarrow \pi K \pi K$ up to 800-1000 MeV including resonances.

Lowest order ChPT (WeinbergTheorems) and even one-loop computations are only valid at very low energies.





A. Dobado, J.R. Peláez

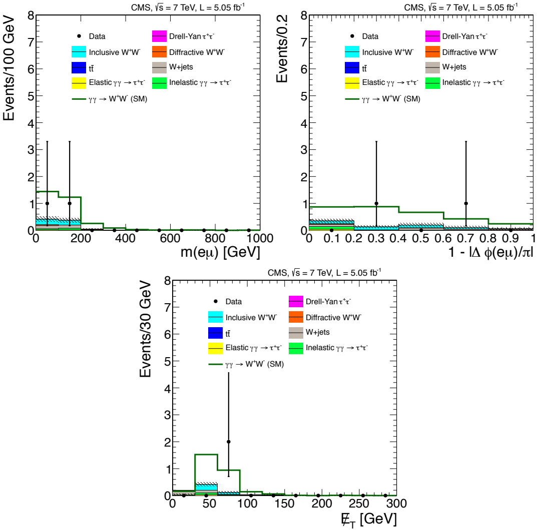
$W_L\, W_L$ scattering

We have no clue of what, how or if new physics...
Most general NLO Lagrangian for ω, h at low energy

$$\begin{aligned}\mathcal{L} &= \left[1 + 2a\frac{h}{v} + b\left(\frac{h}{v}\right)^2\right] \frac{\partial_\mu\omega^a\partial^\mu\omega^b}{2}\left(\delta^{ab} + \frac{\omega^a\omega^b}{v^2}\right) \\ &+ \frac{4a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b + \frac{4a_5}{v^4}\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b \\ &+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^a\partial^\nu\omega^a + \frac{2e}{v^4}\partial_\mu h\partial^\mu\omega^a\partial_\nu h\partial^\nu\omega^a \\ &+ \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2\end{aligned}$$

$\gamma\gamma$ scattering

- We also consider[1] the case of the $\gamma\gamma \to W_L^+ W_L^-$ and $\gamma\gamma \to Z_L Z_L$ scattering (unitarization is work in progress).
- Current efforts for measuring these channels (although only 2 events measured).
- Graphs from CMS, JHEP **07** (2013) 116.
- Wait for LHC Run–II and CMS–TOTEM.



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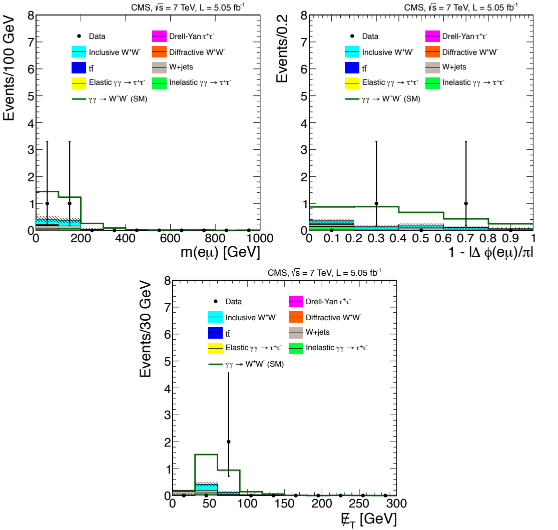

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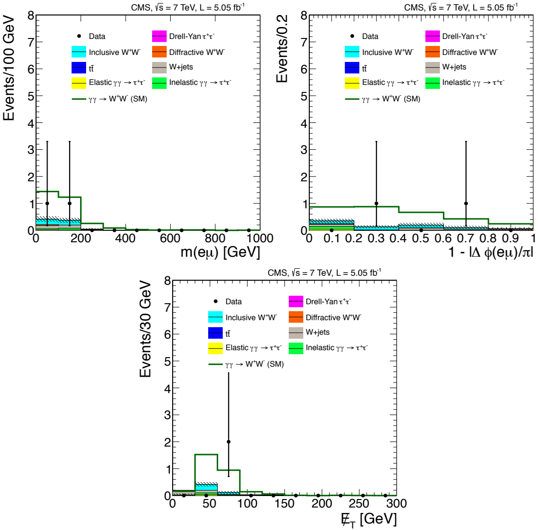

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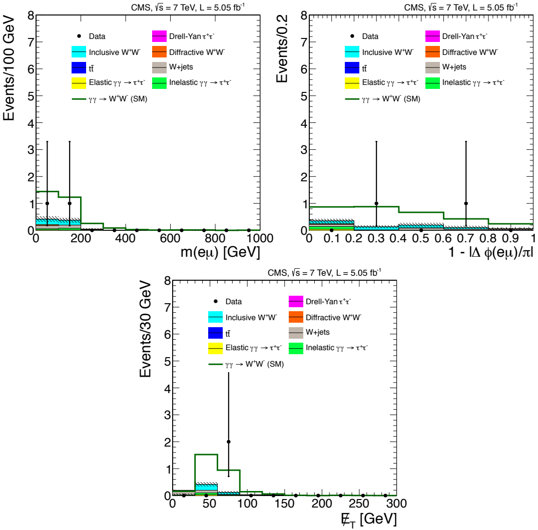

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Particular cases of the theory

- $a^2 = b = 1$, SM
- $a^2 = b = 0$, Higgsless ECL[2]
- $a^2 = 1 - \frac{v^2}{f^2}$, $b = 1 - \frac{2v^2}{f^2}$, $SO(5)/SO(4)$ MCHM[3]
- $a^2 = b = \frac{v^2}{\hat{f}^2}$, Dilaton[4]

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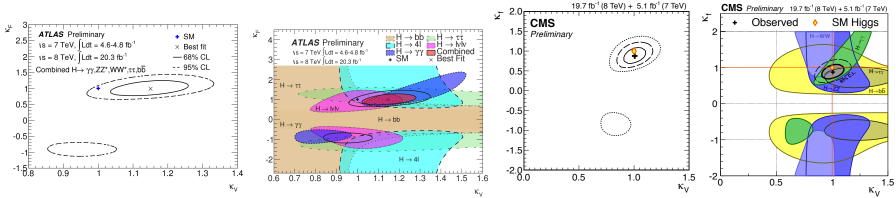
- As it would require measuring the coupling of two Higgses, there is no experimental bound over the value of b parameter[5] . Over a, at a confidence level of 2σ (95%),
  - CMS[6] .... $a \in (0.88, 1.15)$
  - ATLAS[7] .... $a \in (0.96, 1.34)$



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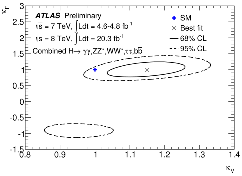

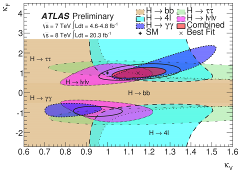

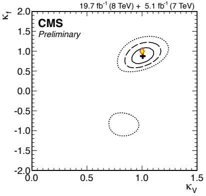

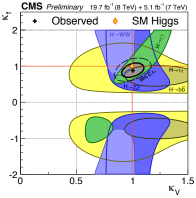

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Experimental bounds on low-energy constants

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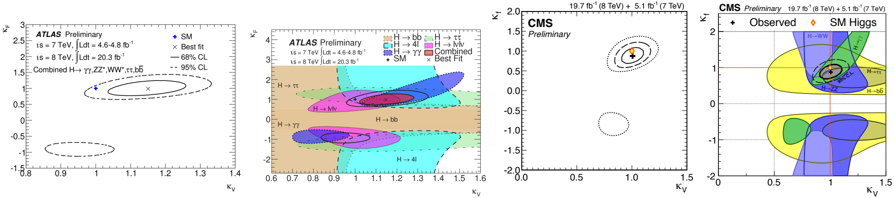

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Partial Waves

The form of the partial wave is

$$
\begin{aligned}
A_{IJ}(s) &= \frac{1}{64\pi}\int_{-1}^{1} d(\cos\theta) P_J(\cos\theta) A_I(s,t,u) \\
&= A_{IJ}^{(0)} + A_{IJ}^{(1)} + \dots
\end{aligned}
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Which will be decomposed as

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\begin{aligned}
A_{IJ}^{(0)} &= Ks \\
A_{IJ}^{(1)} &= \left(B(\mu) + D\log\frac{s}{\mu^2} + E\log\frac{s}{\mu^2} \right) s^2
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As $A_{IJ}(s)$ must be scale independent,

$$B(\mu) = B(\mu_0) + (D+E)\log\frac{\mu^2}{\mu_0^2}$$

Unitarization procedures

$$A^{IAM}(s) = \frac{[A^{(0)}(s)]^2}{A^{(0)}(s) - A^{(1)}(s)}$$

$$A^{N/D}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)} + \frac{1}{2}g(s)A_L(-s)}$$

$$A^{IK}(s) = \frac{A^{(0)}(s) + A_L(s)}{1 - \frac{A_R(s)}{A^{(0)}(s)} + g(s)A_L(s)}$$

$$A_0^K(s) = \frac{A_0(s)}{1 - iA_0(s)}$$

$$A_L(s) = \pi g(-s) D s^2$$

$$A_R(s) = \pi g(s) E s^2$$

$$g(s) = \frac{1}{\pi}\left(\frac{B(\mu)}{D+E} + \log\frac{-s}{\mu^2}\right)$$

PRD **91** (2015) 075017

Validity range of unitarization procedures

IJ	00	02	11	20	22
Method of choice	Any	N/D IK	IAM	Any	N/D IK

- The IAM method cannot be used when $A^{(0)} = 0$, because it would give a vanishing value.
- The N/D and the IK methods cannot be used if $D + E = 0$, because in this case computing $A_L(s)$ and $A_R(s)$ is not possible.
- The naive K-matrix method,

$$A_0^K(s) = \frac{A_0(s)}{1 - iA_0(s)},$$

fails because it is not analytical in the first Riemann sheet and, consequently, it is not a proper partial wave compatible with microcausality.

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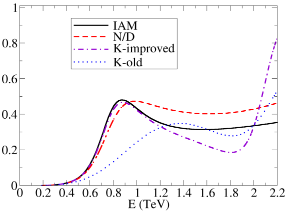
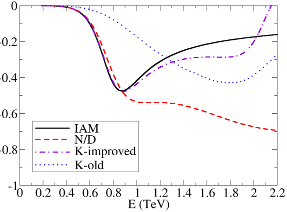
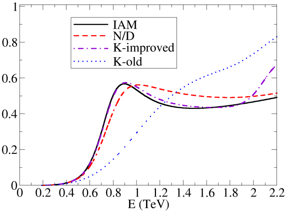
Scalar-isoscalar channels

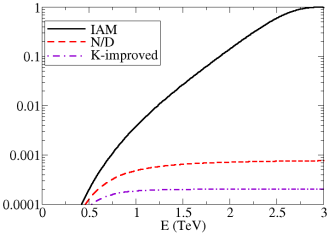


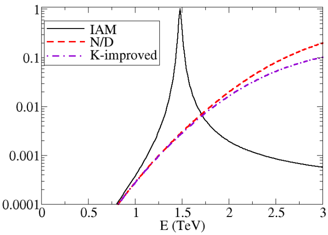




From left to right and top to bottom, elastic $\omega\omega$, elastic hh, and cross channel $\omega\omega \to hh$, for $a = 0.88$, $b = 3$, $\mu = 3\,\mathrm{TeV}$ and all NLO parameters set to 0.
PRL **114** (2015) 221803, PRD **91** (2015) 075017.

Vector-isovector channels

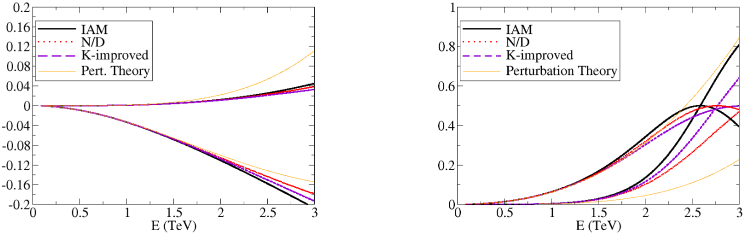




We have taken $a = 0.88$ and $b = 1.5$, but while for the left plot all the NLO parameters vanish, for the right plot we have taken $a_4 = 0.003$, known to yield an IAM resonance according to the Barcelona group, PRD **90** (2014) 015035.

PRD **91** (2015) 075017.

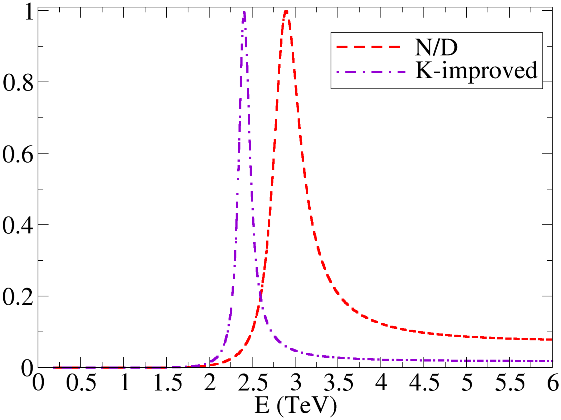
Scalar-isotensor channels ($IJ = 20$)



From left to right, $a = 0.88$, $a = 1.15$. We have taken $b = a^2$ and the NLO parameters set to zero. Both real and imaginary part shown. Real ones correspond to bottom lines at left and upper at low E at right.

PRD **91** (2015) 075017.

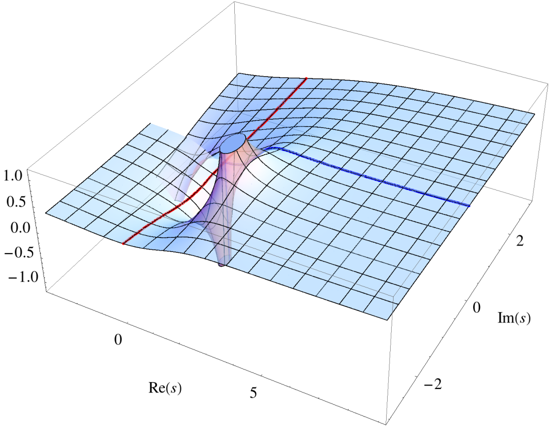
Isotensor-scalar channels ($IJ = 02$)



$a = 0.88$, $b = a^2$, $a_4 = -2a_5 = 3/(192\pi)$, all the other NLO param. set to zero.
PRD **91** (2015) 075017.

Resonance from $W_L W_L \to hh$

$a = 1$, $b = 2$, IAM,
elastic channel $W_L W_L \to W_L W_L$



Rafael L. Delgado,
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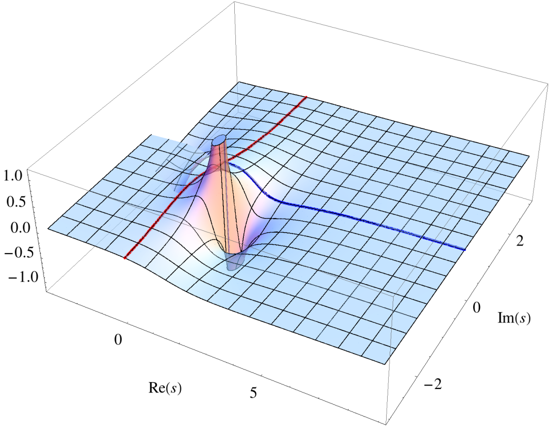
Felipe J. Llanes-Estrada,
Possible New Resonance from W_L W_L-hh Interchannel Coupling,

PRL **114** (2015) 221803

$a = 1$, $b = 2$, IAM,
inelastic channel $W_L W_L \to hh$



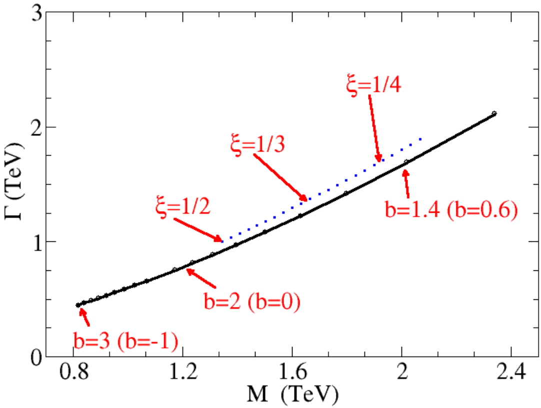
Rafael L. Delgado,
Antonio Dobado,
Felipe J. Llanes-Estrada,
Possible New Resonance from W_L W_L-hh Interchannel Coupling,

PRL **114** (2015) 221803

Motion of the resonance mass and width

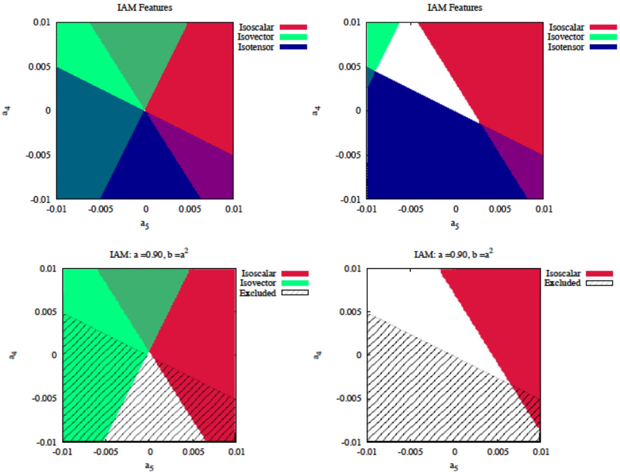
Dependence on b with $a^2 = 1$ fixed (upper curve) and for $a = 1\xi$ and $b = 12\xi$ with $\xi = v/f$ as in the MCHM (lower blue curve).

PRL **114** (2015) 221803



Resonances in $W_L W_L \to W_L W_L$ due to a_4 and a_5 paramet.

Espriu, Yencho, Mescia
PRD**88**, 055002
PRD**90**, 015035
At right, exclusion regions include resonances with $M_{S,V} < 600\,\mathrm{GeV}$.



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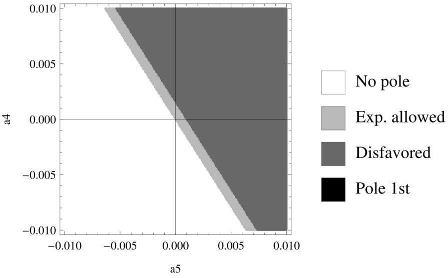
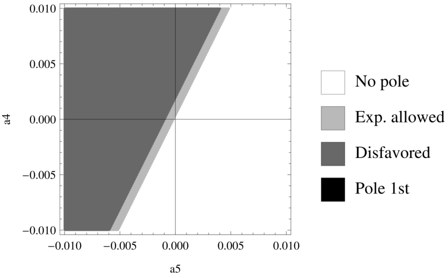
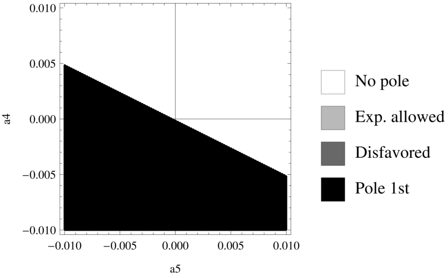
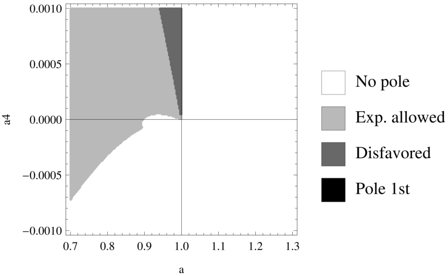


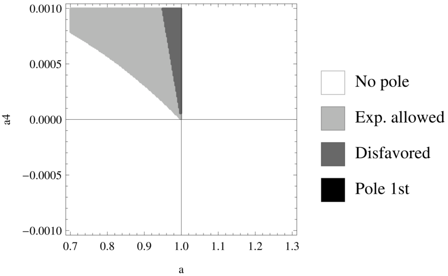


- $a = 0.90, \; b = a^2$
  PRD **91** (2015) 075017
- From left, clockwise,
  $IJ = 00, \; 11, \; 20$
- Excluding resonances
  $M_S < 700\,\mathrm{GeV}, \; M_V < 1.5\,\mathrm{TeV}$



Resonances in $W_L W_L \to W_L W_L$ due to a and a_4 parameters





- $b = a^2$
  PRD **91** (2015) 075017
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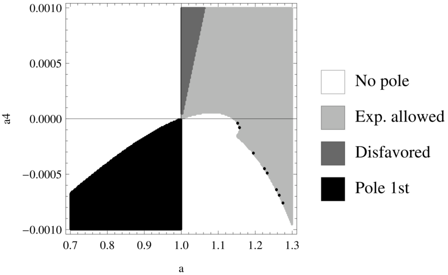
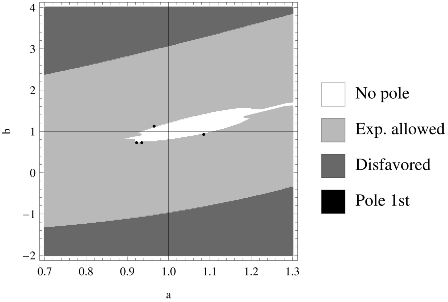
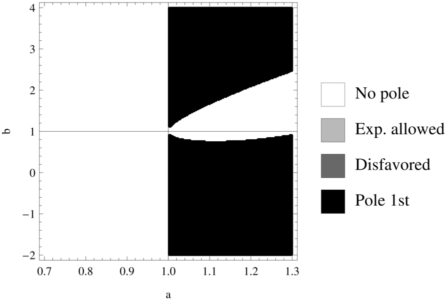
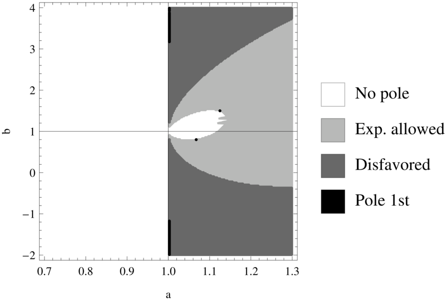


Resonances in $W_L W_L \to W_L W_L$ due to a and b parameters

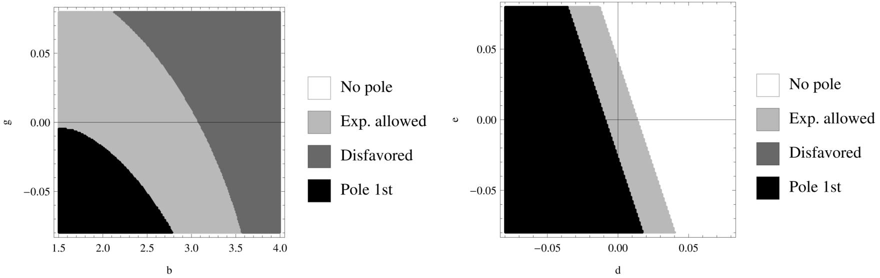




- PRL & PRD **91** (2015) 075017
- From left, clockwise, $IJ = 00,\ 11,\ 20$
- Excluding resonances $M_S < 700\,\mathrm{GeV}$, $M_V < 1.5\,\mathrm{TeV}$
- Constraint over b even without data about $W_L W_L \to hh$ and $hh \to hh$ scattering processes.



Resonances in $W_L W_L \to W_L W_L$ due to b, g, d and e parameters



Effective Theory, PRD **91** (2015) 075017, isoscalar channels ($I = J = 0$).

$\gamma\gamma$ scattering

- Two parameterizations have been considered (two effective Lagrangians obtained), giving the same results.
- One loop computation for the process $\gamma\gamma \to \omega_L^a \omega_L^b$.
- Siple result compared with the complexity of the computation.

$$
\begin{aligned}
\mathcal{M} &= ie^2(\epsilon_1^\mu \epsilon_2^\nu T_{\mu\nu}^{(1)})A(s,t,u) + ie^2(\epsilon_1^\mu \epsilon_2^\nu T_{\mu\nu}^{(2)})B(s,t,u) \\
T_{\mu\nu}^{(1)} &= \frac{s}{2}(\epsilon_1\epsilon_2) - (\epsilon_1 k_2)(\epsilon_2 k_1) \\
T_{\mu\nu}^{(2)} &= 2s(\epsilon_1\Delta)(\epsilon_2\Delta) - (t-u)^2(\epsilon_1\epsilon_2) \\
&\quad -2(t-u)[(\epsilon_1\Delta)(\epsilon_2 k_1) - (\epsilon_1 k_2)(\epsilon_2\Delta)] \\
\Delta^\mu &= p_1^\mu - p_2^\mu
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\begin{aligned}
M(\gamma\gamma \to zz)_{\rm LO} &= 0 \\
A(\gamma\gamma \to zz)_{\rm NLO} &= \frac{2ac_\gamma^r}{v^2} + \frac{(a^2-1)}{4\pi^2 v^2} \\
B(\gamma\gamma \to zz)_{\rm NLO} &= 0 \\
A(\gamma\gamma \to \omega^+\omega^-)_{\rm LO} &= 2sB(\gamma\gamma \to \omega^+\omega^-)_{\rm LO} = -\frac{1}{t} - \frac{1}{\mu} \\
A(\gamma\gamma \to \omega^+\omega^-)_{\rm NLO} &= \frac{8(a_1^r - a_2^r + a_3^r)}{v^2} + \frac{2ac_\gamma^r}{v^2} + \frac{(a^2-1)}{8\pi^2 v^2} \\
A(\gamma\gamma \to \omega^+\omega^-)_{\rm NLO} &= 0
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Work in progress

- Ref. JHEP**1407** (2014) 149 (scattering $\gamma\gamma \to \omega_L^+\omega_L^-$) only contains the 1–loop computation.
- The next steps will be...
  - computing $\omega\omega \to hh$ matrix element,
  - and performing the unitarization.
- Both for $\gamma\gamma$ and $\omega_L\omega_L$ scattering, we should
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- Besides, we are working on the $t\bar{t} \to \omega_L\omega_L$ channel.

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Conclusions

- New scalar particle + mass gap
- New physics would very likely imply strong interactions, in elastic $W_L W_L$ and inelastic $\to hh$ scattering.
- For $a^2 = b \neq 1$, strong elastic interactions are expected for $W_L W_L$, and a second, broad scalar analogous to the σ in nuclear physics possibly appears. We identify a pole at $800\,\mathrm{GeV}$ or above in the second Riemann sheet very clearly, the question is whether it corresponds to a physical particle since it is so broad.
- Even if $a \simeq 1$, with small λ_i (higher powers of h), but we allow $b > a^2$, one can have strong dynamics resonating between the $W_L W_L$ and hh channels, likewise possibly generating a new scalar pole of the the scattering amplitude in the sub-TeV region.
- This fact allows to constrain b even in the absence of data about $W_L W_L \to hh$ and $hh \to hh$, just looking at the $W_L W_L$ scattering.
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- SM $\rightarrow$ unitarity.
- Higgsless model (now experimentally excluded) $\rightarrow$ unitarity violation in WW scattering $\rightarrow$ new physics.
- Higgs–like boson found $\rightarrow$ no unitarity violation?
- Not necesarily, with the present experimental bounds.
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Back Slides

I) IAM method

This method needs a NLO computation,

$$\tilde{t}^{\omega} = \frac{t_0^{\omega}}{1 - \frac{t_0^{\omega}}{t_1^{\omega}}},$$

where

$$t_1^{\omega} = s^2 \left(D \log\left[\frac{s}{\mu^2}\right] + E \log\left[\frac{-s}{\mu^2}\right] + (D+E) \log\left[\frac{\mu^2}{\mu_0^2}\right]\right)$$

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Check at tree level

We have checked[8], for the tree level case,

$$\mathcal{L} = \frac{1}{2}g(\varphi/f)\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab}+\frac{\omega^a\omega^b}{v^2-\omega^2}\right)$$
$$+\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\frac{1}{2}M_\varphi^2\varphi^2-\lambda_3\varphi^3-\lambda_4\varphi^4+\ldots$$
$$g(\varphi/f) = 1+\sum_{n=1}^{\infty}g_n\left(\frac{\varphi}{f}\right)^n = 1+2\alpha\frac{\varphi}{f}+\beta\left(\frac{\varphi}{f}\right)^2+..$$

where $a\equiv\alpha v/f$, $b=\beta v^2/f^2$, and so one, the concordance with the methods

[8] See J.Phys. G41 (2014) 025002.

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$$\mathcal{L} = \frac{1}{2}g(\varphi/f)\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab}+\frac{\omega^a\omega^b}{v^2-\omega^2}\right) + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}M_\varphi^2\varphi^2 - \lambda_3\varphi^3 - \lambda_4\varphi^4 + \ldots$$

$$g(\varphi/f) = 1+\sum_{n=1}^{\infty} g_n\left(\frac{\varphi}{f}\right)^n = 1+2\alpha\frac{\varphi}{f}+\beta\left(\frac{\varphi}{f}\right)^2+..$$

where $a \equiv \alpha v/f$, $b = \beta v^2/f^2$, and so one, the concordance with the methods

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II) K matrix

$$\tilde{T} = T(1 - J(s)T)^{-1}, \quad , J(s) = -\frac{1}{\pi}\log\left[\frac{-s}{\Lambda^2}\right],$$

so that, for $\tilde{t}_\omega$,

$$\tilde{t}_\omega = \frac{t_\omega - J(t_\omega t_\varphi - t_{\omega\varphi}^2)}{1 - J(t_\omega + t_\varphi) + J^2(t_\omega t_\varphi - t_{\omega\varphi}^2)},$$

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III)Large N

$N \to \infty$, with v^2/N fixed. The amplitude A_N to order $1/N$ is a Lippmann-Schwinger series,

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A_N &= A - A\frac{NI}{2}A + A\frac{NI}{2}A\frac{NI}{2}A - \ldots \\
I(s) &= \int \frac{d^4q}{(2\pi)^4}\frac{i}{q^2(q+p)^2} = \frac{1}{16\pi^2}\log\left[\frac{-s}{\Lambda^2}\right] = -\frac{1}{8\pi}J(s)
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Note: actually, $N = 3$. For the (iso)scalar partial wave (chiral limit, $I = J = 0$),

$$t_N^\omega(s) = \frac{t_0^\omega}{1 - Jt_0^\omega}$$

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IV) N/D

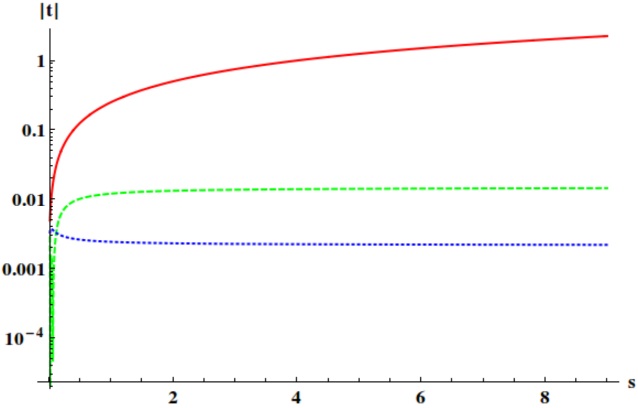
(elastic scattering at tree level only $\beta = \alpha^2$. See ref. J.Phys. G41 (2014) 025002). Ansatz

$$\tilde{t}^{\omega}(s) = \frac{N(s)}{D(s)},$$

where $N(s)$ has a left hand cut (and $\mathrm{Im}\, N(s > 0) = 0$)
$D(s)$ has a right hand cut (and $\Im D(s < 0) = 0$);

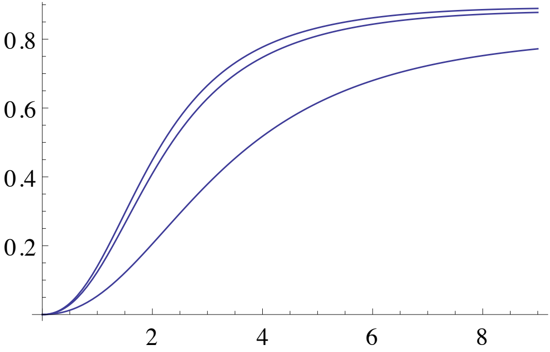
$$\begin{aligned} D(s) &= 1 - \frac{s}{\pi} \int_0^{\infty} \frac{ds' N(s')}{s'(s' - s - i\epsilon)} \\ N(s) &= \frac{s}{\pi} \int_{-\infty}^{0} \frac{ds' \,\mathrm{Im}\, N(s')}{s'(s' - s - i\epsilon)} \end{aligned}$$

Coupled channels, tree level amplitudes



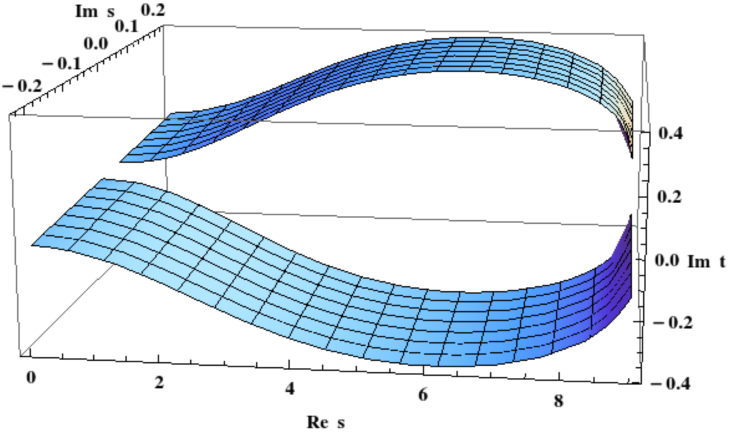
$f = 2v$, $\beta = \alpha^2 = 1$, $\lambda_3 = M_\varphi^2/f$, $\lambda_4 = M_\varphi^2/f^2$. OX axis: s in TeV^2.

Tree level, modulus of $\tilde{t}_\omega$, K matrix

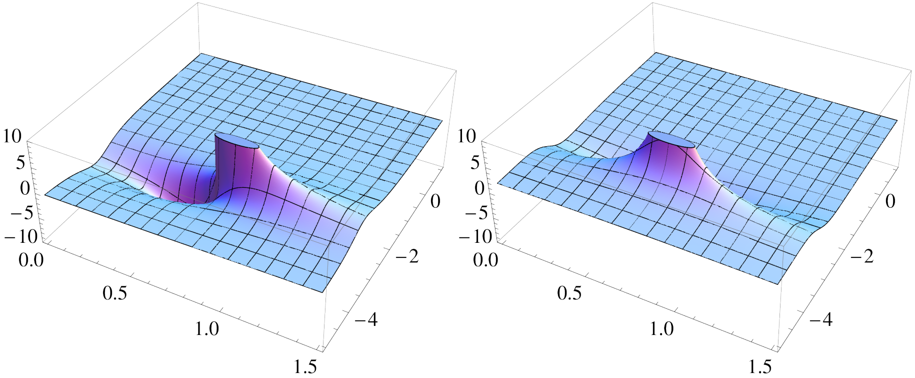


- All units in TeV.
- From top to bottom, $f = 1.2,\ 0.8,\ 0.4\,\mathrm{TeV}$
- $\Lambda = 3\,\mathrm{TeV}$
- $\mu = 100\,\mathrm{GeV}$

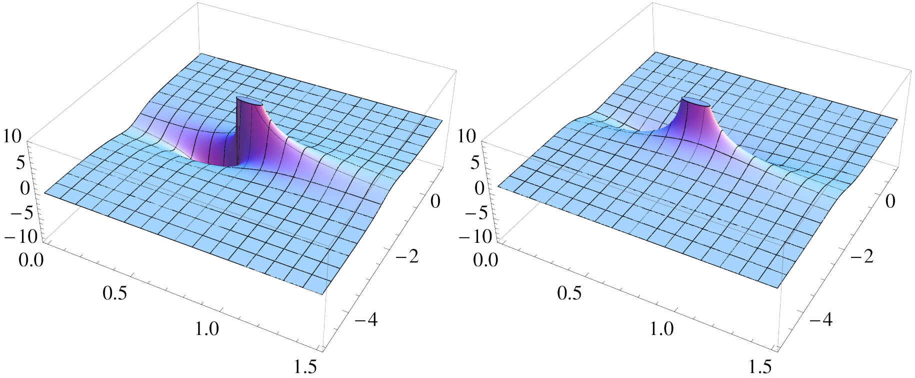
$\mathrm{Im}\, t_\omega$ in the N/D method, $f = 1\,\mathrm{TeV}$, $\beta = 1$, $m = 150\,\mathrm{GeV}$



Re t_ω and Im t_ω, large N, $f = 400\,\mathrm{GeV}$



$\mathrm{Re}\, t_\omega$ and $\mathrm{Im}\, t_\omega$, large N, $f = 4\,\mathrm{TeV}$



Tree level, motion of the pole position of t_ω K–matrix, $M_\phi = 125\,\mathrm{GeV}$, $f \in (250\,\mathrm{GeV}, 6\,\mathrm{TeV}))$