Peripheral Production of Resonances

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PERIPHERAL PRODUCTION OF RESONANCES

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ABSTRACT

We review the peripheral cross-sections of resonances that can$\underline{not}$ be produced by π-exchange. Explicitly we consider the four nonets of mesons n the L = 1 quark classification, $J^P = 0^+$ $\pi_N(980)$; 1^+, e.g., A_1, B; 2^+, e.g., A_2. We detail the constraints of SU_3, exchange degeneracy, factorization, pole extrapolation, Watson's theorem and duality as they have been gleaned from long and painful study of well-loved reactions. The controversial nature of the 0^+ and 1^+ nonets is related to a surprising suppression of their cross-sections (which are around 10 µb at 5 GeV/c) in non-diffractive processes. This is explained by a generalized vector meson photon analogy model recently proposed by Kislinger. We predict all differential cross-sections for these L = 1 quark states and show that these mesons are best studied in hypercharge exchange reactions, e.g., $\pi^- p \to Q^0 \Lambda$ and $K^- n \to A_1^- \Lambda$.

An explanation, using final state interaction theory, of the large background under the Q and A_1 observed in diffractive processes, suggests that quantum numbers of mesons are best studied in $\underline{non}$-diffractive reactions of the type mentioned above.

ON FANTASIES

ON DINOSAURS

ON PARTICLES THAT PIONS LOVE

According to the prophet, research is liken unto a hard and lonely journey through a torrid, unfriendly desert. From time to time, the barren trek is filled with ecstacy as a lush oasis appears; there our hero may lay down his administrative load and find hidden a piece of the cosmic jigsaw. After many years, with many toils and just the occasional ecstacy, our hero has pieced together a tiny bit of the cosmos. So with his precious knowledge, he finally returns

to the land of the living he left so long ago [1].......

In this paper, I would like to record such a trip, namely, a voyage through run-down oases - the homes of disreputable particles and inconsistent cross-sections. Places where even the tarnished gold of current theories shines like a waxing star.

The next section should have been a pedantic classification of various two-body reactions which would delineate the scope of this talk. However, it turned out so boring that I have put it in the first and only appendix. Here I just note that we will discuss reactions of the form:
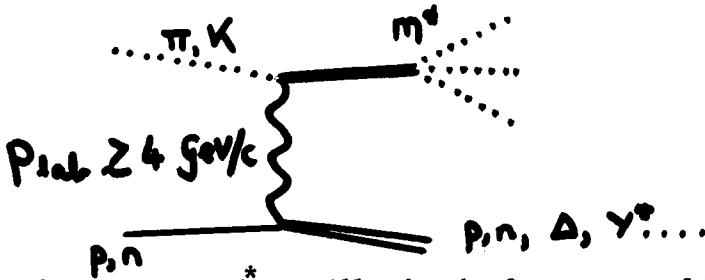
where for the resonances M^* we will take the four nonets of $L = 1$ quark states as the lowest-lying examples of controversial resonances [2] that cannot be produced by π-exchange. These are recorded in Table I. This table should be taken with the following grains of salt:

(i) States that can be produced by π-exchange and so are (comparatively) straightforward to study are shaded (see Section E_π of Appendix).

(ii) We tentatively identify the C meson seen in $\bar{p}p$ collisions [3] as the strange partner of the A_1. As we discuss in Section 3.5 under the $\pi^- p \to Q^0 \Lambda$ reaction, this is not a watertight assignment. For instance, it may be in the B nonet and further the two Q's - denoted hereafter Q_{A_1} and Q_B - may mix [4].

(iii) The $I = 0$, 1^+ mesons are only represented by the D meson seen in $\bar{p}p$ interactions. The second $I = 0$ A_1 partner can be consistently identified with either the E(1422) or the rather shaky M(953) [5]. There is essentially no experimental information on h or h' production.

Table I: L = 1 Quark States

isospin \ J^P	0^+ Nonet	1^+ "B" Nonet	1^+ "A_1" Nonet	2^+ Nonet
I = 1	$\pi_N(980)$	B(1235)	$A_1(1070)$	$A_2(1310)$
Strange I = ½	κ ($\approx$ 1250)	$Q_B(1300 \rightarrow 1400)$	Q_{A_1} = C(?) (1240 $\rightarrow$ 1290)	K_N^* (1420)
I = 0 singlet/octet mixing (?)	ε ($\approx$ 750)	h (?)	D (1285)	$f^0(1260)$
	s^* ($\approx$ 1000)	h' (?)	D' = E(1422) ? or M(953) ?	f'(1514)

Theoretically the situation is confused by the different mixing schemes. First, we can have "magic" mixing as exemplified by the ω and ϕ; in this case, we write the I = 0 particles M^* (magic $-\omega$) and M^* (magic $-\phi$) with an obvious notation. (M^* = D or h.) Alternatively we can have essentially no mixing as exemplified by the η and η'; in this case, we write M^* (octet) and M^* (singlet).

Duality schemes predict h, h' to have magic mixing and D, D' to be unmixed [6]. The naive quark model predicts exactly the opposite [7]; here we can only consider all possibilities.

(iv) In Table II, we list the dominant decay modes of the relevant 1^+ and 0^+ resonances. Further we also give either the observed value or SU_3 prediction for the partial widths. Note that the A_1 nonet predictions might be thought a little hazy as they are normalized to an A_1 width derived (presumably incorrectly - see Section 2.2) from diffraction data. However, similar results are given using the quark model to relate the A_1 to the B couplings [7,8]. In 3.5, we briefly consider the changes produced by a higher A_1 mass as the current determination of the latter must also be considered doubtful.

Table II: 0^+, 1^+ Meson Decays

Particle	Mass GeV	Decay	Width MeV	Source of Width
π_N (980)	0.98	$\pi\eta$	40	(a)
B (1235)	1.235	$\pi\omega$	100	Expt
Q_B	1.380	$K\rho$ $K^*\pi$ $K\omega$	32 80 9	(b)
h(octet - ?)	1.01	$\pi\rho$	40	(b)
h(magic $-\omega$ - ?)	1.25	$\pi\rho$	330	(b)
h'(magic $-\phi$)	1.5	$\bar{K}K^* + K\bar{K}^*$	75	(b)
A_1	1.07	$\pi\rho$	140	"Expt"
Q_{A_1} = C	1.24	πK^*	50	(c)
D (1285)	1.285	$\eta\pi\pi$ $K\bar{K}\pi$	Γ_{tot} = 21 ± 10	Expt
D' (magic $-\phi$) = E (?)	1.422	$\bar{K}K^* + K\bar{K}^*$	50	(c)
D' (singlet)	any	$\bar{K}K^* + K\bar{K}^*$	0	(c)

Sources: (a)　SU_3 and $\epsilon \to \pi\pi$ = 300 MeV

　　　　　(b)　SU_3 and $B \to \pi\omega$ = 100 MeV

　　　　　(c)　SU_3 and $A_1 \to \pi\rho$ = 140 MeV

The next section is shamelessly pedagogical. Indeed, it, too, almost suffered exile to a dusty appendix. In fact, it is an outline of theoretical weapons we can use to study our reactions. Explicitly we detail the constraints of SU_3, EXD (exchange-degeneracy), factorization, pole extrapolation, Watson's theorem (final state interactions) and the useful, if much abused, idea of duality. Most of these are well known but I emphasize that many popular applications of the Deck model (incorrectly justified using duality as either alternative or equivalent to a resonance description) are inconsistent with Watson's theorem. The latter follows rigorously from unitarity, and so rather than the Deck dinosaur, I suggest a fantasy[9] which is consistent with final state interactions and suggests resonance production should be

much cleaner in non-diffractive than diffractive processes (in agree-
ment with experiment).

In the third section, we analyze the experimental data on the
production of the 0^+, 1^+ and 2^+ states listed in Table I. The small
cross-sections observed for these processes is, a priori, very sur-
prising but has a natural explanation in a generalized vector meson
photon analogy model recently proposed by Kislinger[10]. The suppres-
sion of the 1^+ cross-sections (most of them are around 10 μb at
5 GeV/c and small t) predicted in Kislinger's model explains and
unifies (!) the controversial nature of these particles. (Does the A_1
resonance really exist?) We use SU_3, EXD and factorization to pre-
dict all cross-sections for these nonets and hopefully the curves in
Figs. 9-12 will be useful in planning experiments to elucidate the
properties of these particles. In particular, we can note that some
reactions (e.g., $K^- p \to Q^0 n$) are difficult because of large background
from much bigger π-exchange processes. However, there are others,
e.g., $\pi^- p \to Q^0 \Lambda$ [11], where there is no such background and one high
statistic experiment could at once settle the vexed question of the
properties and even existence of the 1^+ states predicted in the quark
model[2].

The final section has the customary pious conclusions but also
points out that many of the theoretical conclusions are rendered un-
necessarily vague by chronic inconsistencies in quoted experimental
cross-sections. This confusion stems mainly from different technical
assumptions (mass-cut?, t-cut?, background subtraction?, maximum like-
lihood fit?...). It would be nice if such data were recorded in a
way (e.g., the horrific (to some) spectre of a bubble chamber DST
bank) that new interest in a particular cross-section would allow a
unified treatment of the existing data. At present, one may only
smile wanly and hope a new experiment will analyze their data for
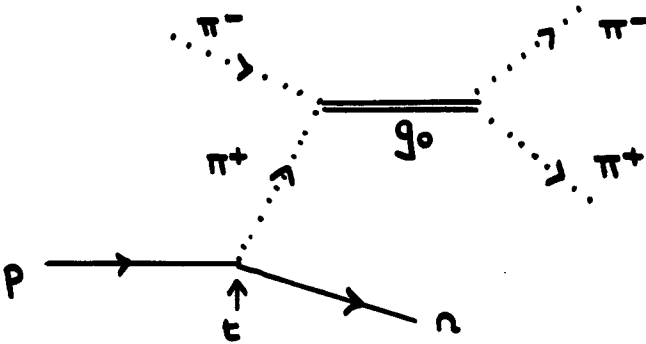your favorite resonance and/or in your favorite way.

2: WELL-LOVED FOLKLORE

Here we lay out the weapons to be used in fighting the grimy
ogre in Section 3. The order of presentation is less than logical.
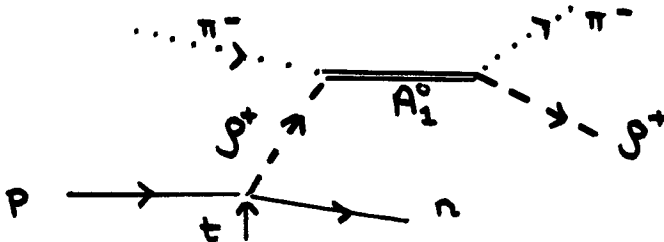
2.1: Pole-extrapolation

For π-exchange it is easy to estimate cross-sections:

e.g.,

$$\pi^- \qquad \pi^- \\ g_0 \\ \pi^+ \qquad \pi^+ \\ p \qquad t \qquad n$$

Thus the value is known at $t = m_\pi^2$ in terms of $\pi-\pi$ scattering and it is only a short (Chew-Low) extrapolation to the physical region ($t \leq 0$). To get the details right requires sophistication (e.g., the poor man's absorption model [12]) but rough estimates of the size of the cross-section present no difficulty.

For <u>vector and tensor exchange</u>, e.g.,

$$\pi^- \qquad \pi^- \\ \rho^+ \qquad A_1^0 \qquad \rho^+ \\ p \qquad t \qquad n$$

the situation is similar to the extent that the cross-section at the pole is again an absolutely normalizable elastic scattering (in this case $\pi^- \rho^+ \to \pi^- \rho^+$). However, the extrapolation is now much more difficult as one must pass from $t = m_\rho^2 \simeq 0.5$ (GeV/c)^2 to $t = 0$. It is believed that such an extrapolation is reasonable in the comparable

$$\pi^- \qquad \pi^0 \\ \rho^+ \\ p \qquad t \qquad n$$
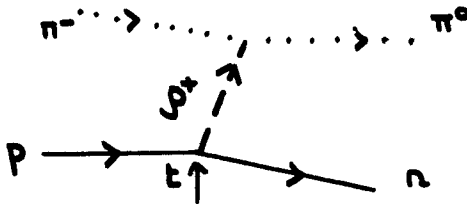
reaction; an exact conclusion being impossible due to our poor knowledge of the $\rho N\bar{N}$ coupling constant (for on-shell ρ's). Assuming this is the only difficulty, we can predict, e.g., $\pi^- p \to A_1^0 n$, by taking the

ratio of amplitudes for $\pi^- p \to A_1^0 n$ over $\pi^- p \to \pi^0 n$; this cancels the un-known $\rho \to N\bar{N}$ coupling. Thereby we relate:

$$\frac{d\sigma/dt\left[\pi^- p \to A_1^0 n\right]}{d\sigma/dt\left[\pi^- p \to \pi^0 n\right]} \quad \text{to} \quad \frac{\Gamma(A_1 \to \pi\rho)}{\Gamma(\rho \to \pi\pi)} \tag{1}$$

We can make two remarks on (1).

a) Clearly the same method will work for all 1^+ production by vector exchange. Again, for tensor exchange reactions (e.g., $\pi^- p \to B^0 n$ is pure A_2 exchange), we need <u>not</u> extrapolate all the way to the A_2 pole but rather use SU_3 and EXD (Section 2.4) to relate (in this example) $A_2 \to \pi B$ to $\omega \to \pi B$. So, in the general 1^+ reaction, we need only extrapolate to a vector meson pole in order to estimate the cross-section.

b) In Section 3, we show that (1) gravely overestimates the cross-sections for production of the A_1 and B nonets. So we need not worry about niceties in (1). For example, different choices of in-variant/helicity amplitudes in (1), can give a factor of 2 difference in the extrapolation prediction. However, this is an irrelevancy when faced by an order of magnitude discrepancy with experiment.
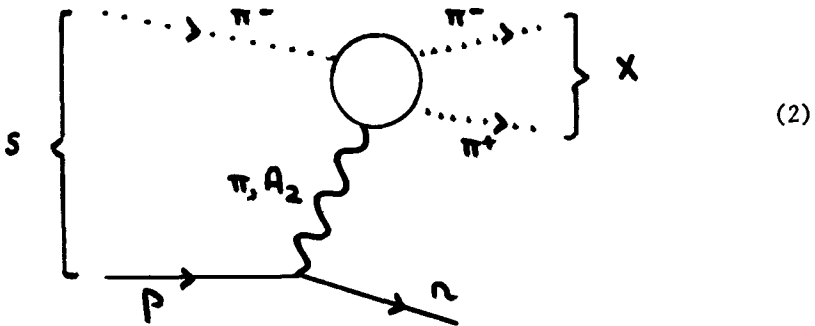
2.2: Watson's Theorem

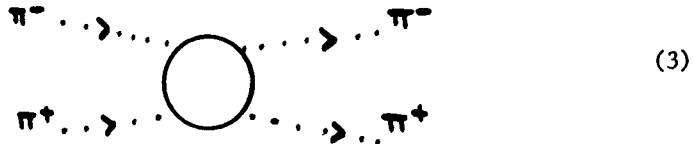a) "Theory" - This states that in <u>any</u> process, e.g.,
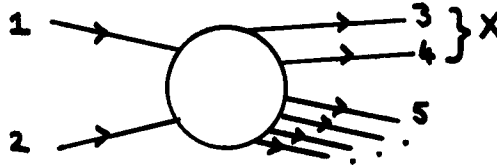
the amplitude for production of an eigenchannel 34 is proportional to $e^{i\delta}$ where δ is the phase shift for the same eigenchannel in
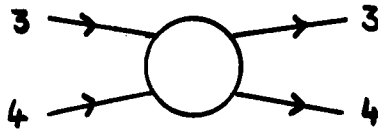
"elastic" scattering. For instance, in

$$(2)$$

for $m_{\pi\pi} \approx m_\rho$ and projecting out spin $J = 1$, we have only one eigen-channel – the $I = 1$, $J = 1$ $\pi\pi$ channel which is then proportional to $e^{i\delta}$ – the phase of the corresponding

$$(3)$$

amplitude. This is, of course, just the usual ρ Breit-Wigner phase. In (3), the (eigen) amplitude's phase is completely specified by $e^{i\delta}$. It is important to realize that this is not the case in (2) where one has additional sources of phases, e.g., Regge theory would give phases:

$$(\exp\left[-i\pi\alpha_{\pi,A_2}\right] + 1)/(2\mathrm{Sin}\ \pi\alpha_{\pi,A_2}) \qquad (4)$$

for the π,A_2 exchange contributions. One can still use the theorem if you realize that – lapsing into the language of my childhood training – every such imaginary part is associated with the discontinuity across a total (e.g., s in (2)) or sub- (e.g., X) energy cut. Then the Regge phase (4) is associated with the $\pi^- p$ and $\pi^+ n$ thresholds in (2) and has nothing to do with the $\pi^- \pi^+$ threshold. Granting this, one can state Watson's theorem as [13]:

>   Any sub-process in any scattering process is associated with a branch-cut in the corresponding sub-energy in the analytic function that is the scattering amplitude. This branch-cut generates an imaginary part whose phase is given by Watson's theorem.

So a proper application of Watson's theorem requires a complete parameterization of the scattering amplitude. This is clearly

impossible but we can state the following rule [14].

Watson's theorem never predicts absolute phases. **However, if a** **final sub-process has a rapidly varying – as a function of the sub-** **energy X – eigenphase** $e^{i\delta}$**, then the production amplitude will exhibit** **essentially the same phase variation in this eigenchannel.** The conditions of this rule are presumably satisfied for a resonant eigenphase.
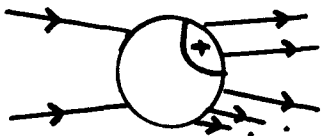
It is perhaps instructive to go through this in detail for a 2-particle sub-process. Then the discontinuity version of Watson's theorem is symbolically [13]:

$$(5)$$
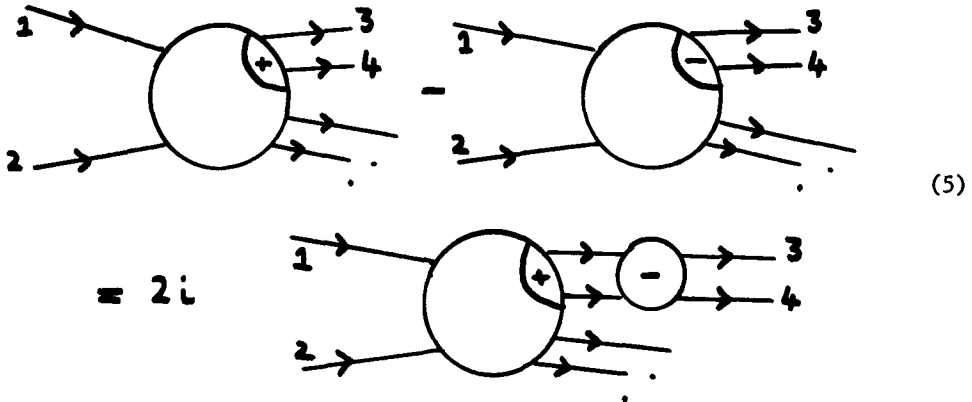
where the $\oplus$ signs denote one's position relative to the s_{34} threshold cut in the scattering amplitudes,, i.e.
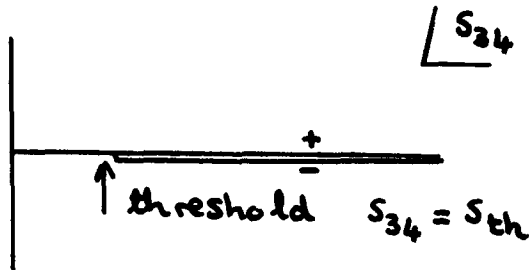
Such amplitudes have a square root branch point at $s_{34} = s_{th}$. So we write

$$= T = A + i\bar{B}$$

$$\text{where } \bar{B} = B\sqrt{s_{34} - s_{th}}$$

$$(6)$$

where neither A nor B are singular at the s_{34} threshold s_{th} ($s_{th} = 4m_{\pi}^2$ in (2)). Substituting (6) in (5) and writing the $2 \to 2$ eigen-

amplitude as $e^{i\delta}$ Sin δ gives

$$1/2i \left[(A + i\bar{B}) - (A - i\bar{B}) \right] = (A + i\bar{B})e^{i\delta} \text{ Sin } \delta$$

or putting $C = \bar{B}/\text{Sin } \delta$

$$(A + i\bar{B}) = e^{i\delta} C. \qquad (7)$$

Here C has <u>no</u> s_{34} singularity at $s_{34} = s_{th}$ but it does - like A and $\bar{B}$ - have thresholds and, hence, imaginary parts corresponding to all the other sub-energies. (7) is then the mathematical formulation of the rule we stated above.

    b) <u>Application to our reactions</u>

    Watson's theorem is powerful if (and only if) there are but a few open channels, and so but a few eigenstates. This is the case for <u>all</u> the resonances we are considering here. For instance, in
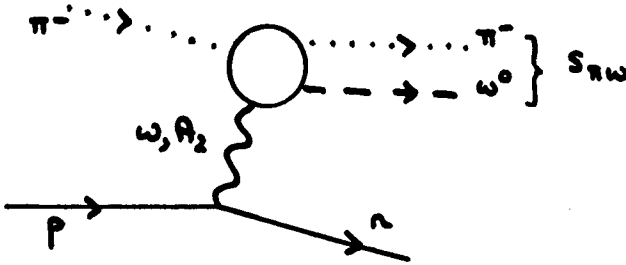
at $s_{\pi\omega} \approx M_B^2$, we deduce that the $J^P = 1^+ (\approx \text{s-wave})$ is $\propto e^{i\delta_B} = i$ at resonance where δ_B is usual Breit-Wigner phase shift for the B. Meanwhile, the $J^P = 1^+ (\approx \text{D-wave})$, $J^P = 0^-, 2^-$ states are $\propto \exp (\approx 0i)$ as these states are (presumed) non-resonant. Note that the "background" in a given eigenchannel <u>must</u> have the resonant phase; it need not have the resonance pole.

    c) <u>Fantasy</u>

    I would like to use the above formalism to divide high-energy peripheral scattering processes into two basic types:

$e^{i\delta}$ Sin δ Production

(i)   Direct channel formation - including photoproduction.
(ii) π-exchange processes at small t.
(iii) (?) Other non-diffractive processes at high energy and small t.

$e^{i\delta}$ Production

(iv)   Diffractive processes.
(v)   (?) Low and medium energy non-diffractive processes.

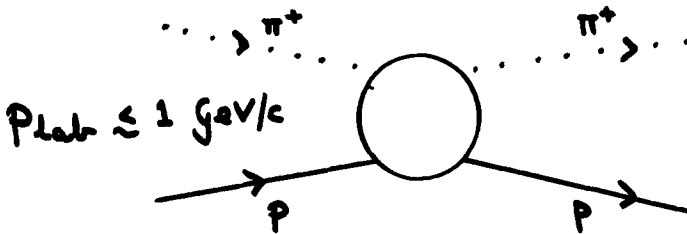(vi)   (?) Electroproduction - High q^2.

(vii) (?) All reactions at high -t.

To understand the distinction, consider some examples.

(i)   <u>Direct Channel</u>

We can write (5) as

$$T - T^* = 2iTT_2^* \qquad (8)$$

where T is the multiparticle and T_2, the $T_2^* \to 2$ amplitude. It is thus a <u>linear</u> (in T) unitarity relation. At low energies, say,


we may put $T = T_2$ whence unitarity becomes a <u>non-linear</u> constraint which implies

$$T_2 \propto e^{i\delta} \sin \delta \qquad (9)$$

where the "extra" Sin δ factor comes from the non-linearity of the unitary equation.

The amplitude (9) peaks at resonance ($\delta = 90°$) and vanishes when $\delta = 0$. Correspondingly, a Breit-Wigner parameterization is valid and resonances are produced very cleanly at low incident energy for there are only a few competing eigenchannels.

(iv) $e^{i\delta}$: <u>Diffraction Production</u>

e.g.,
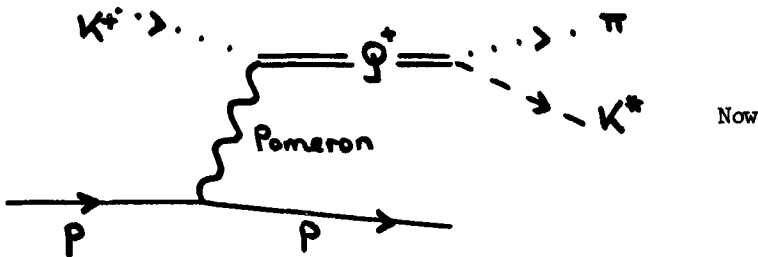
we have the general multiparticle amplitude T and we can no longer say

$T \alpha \; e^{i\delta}$ Sin δ but only that $T \alpha \; e^{i\delta}$. We deduce that there will be no
large peak at resonance ($\delta = 90^{\circ}$) [15] and also that the cross-section
can be non-zero even if $\delta = 0$ (or 180°). It follows that it is in-
valid to use a Breit-Wigner parameterization in such processes. For
instance, the latter implies zero cross-section when $\delta = 0$, a predic-
tion which is manifestly wrong. In any case, dinosaurs apart, we
deduce resonances will be produced with large background.

    (ii) $\underline{e^{i\delta}\text{ Sin }\delta:\quad \pi\text{-exchange}}$
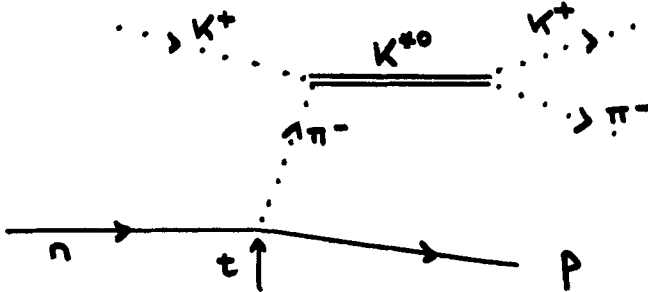

is proportional to $e^{i\delta}$ Sin δ, rigorously at $t = m_{\pi}^{2}$, and will remain so
for small t (in the physical region) as pole extrapolation is valid
for π-exchange [12]. Thus resonances produced by π-exchange will be
clean and accurately described by a Breit-Wigner.

    These points are illustrated in Fig. 1 which compares the
$e^{i\delta}\; K^{+}p \rightarrow (K\pi\pi)^{+}p$ diffractive reaction [16] with the $e^{i\delta}$ Sin δ π-exchange
process $K^{+}n \rightarrow (K^{+}\pi^{-})p$ [17]. Manifestly, the resonances are much cleaner
in the second case [18].

    (iii) $\underline{e^{i\delta}\text{Sin }\delta \;(?):\quad \text{Other Non-diffractive Processes}}$

    The case of the vector and tensor exchange processes [19], of
interest for Section 3, is less clear. If pole extrapolation is even
qualitatively right, then Regge exchange is proportional in, say,
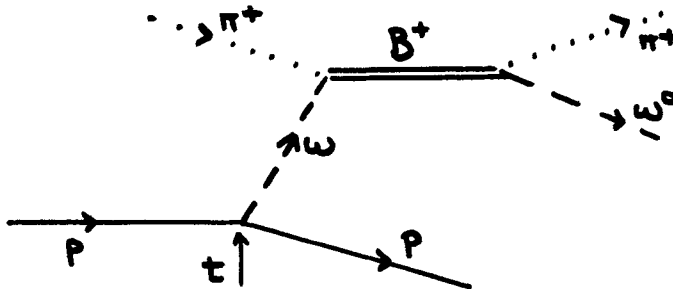
to beautiful $e^{i\delta}$ Sin δ $\pi\omega$ elastic scattering at $t = m_{\omega}^{2}$. My
guess is that this will extrapolate [20], and such reactions will
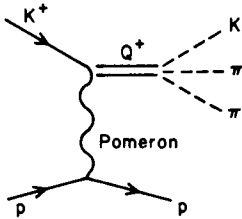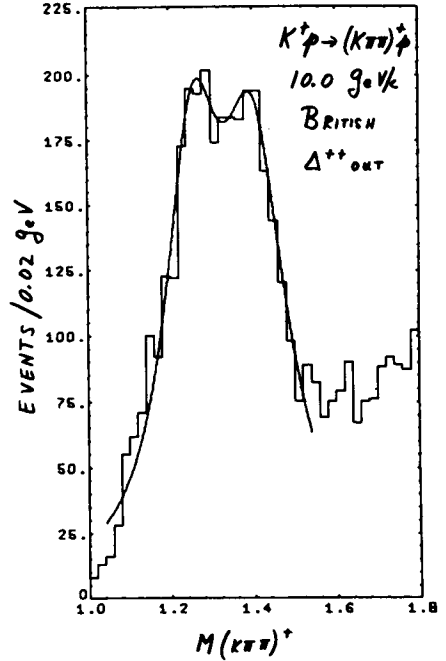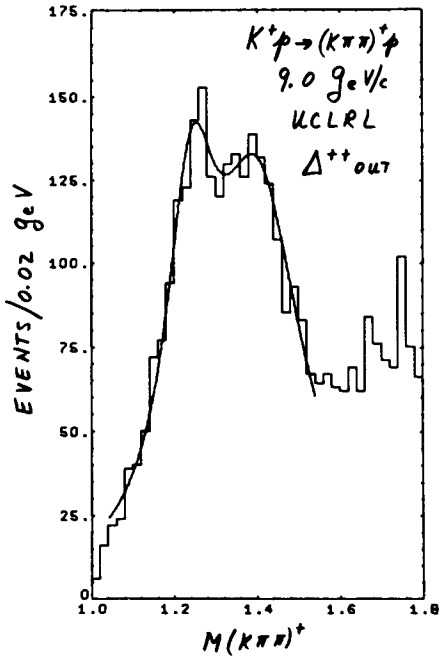
Fig. 1(a): A typical $e^{i\delta}$ diffractive production process (Ref. 16).
            The resonances (if present at all) have large
            background.
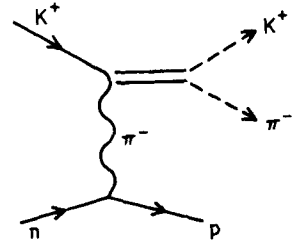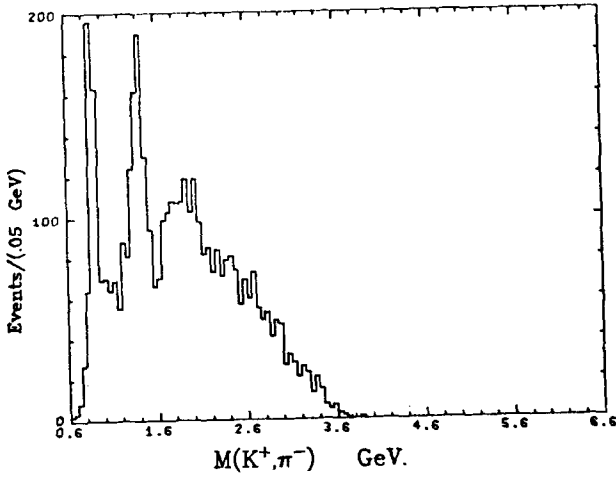
$K^+ n \rightarrow (K^+ \pi^-) p : 12$ GeV/c

$e^{i\delta}$ Sin δ Production?

Fig. 1(b):   A typical $e^{i\delta}$ Sinδ π-exchange process (Ref. 17).
             Resonance production is obviously present and
             very clean.

indeed be $e^{i\delta}$ Sin δ for small t. However, the low cross-sections make it impossible to find any conclusive experimental evidence either way.

We note that the Pomeron processes have no pole to extrapolate to..... [21] so we cannot find the missing Sin δ factor.

Let us speculate:

(v) $\underline{e^{i\delta}: \text{ Medium Energy Non-diffractive Reactions}}$

At $P_{lab} \lesssim 5$ GeV/c, there will be some contribution, say to A_1 production of the form

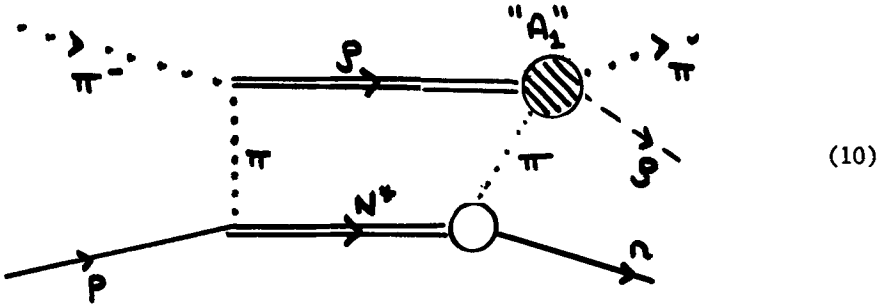

(10)

where the "re-scattering" ◉ (in 10) will give $e^{i\delta}$ not $e^{i\delta}$ Sin δ production. Theoretically, such a box diagram is a Regge-regge cut and it falls fast with energy ($\alpha_{cut} \approx -1$ in this case). Thus one might deduce that non-diffractive resonance production will become cleaner as energy increases and $e^{i\delta}$ Sin δ Regge Dreamland is approached.

(vi) Photon Processes

Consider photo or electro production, say, $\gamma N \to \pi N$ where the photon has mass q^2. At $q^2 = 0$, vector dominance (VDM) is roughly right and our amplitude is proportional to $\rho N \to \pi N$ which is a nice $e^{i\delta}$ Sin δ formation reaction of the type discussed in (i) [22]. However, for larger $-q^2$, VDM is inapplicable and so one would expect $e^{i\delta}$ production. We deduce that resonances should have increasing background as $-q^2$ increases.

Similar remarks can be made about inelastic electron scattering which essentially measures $\sigma_{tot}(\gamma p \to \gamma p)$. At low γp mass, unitarity may be used to relate this to sum of single photon processes ($\gamma N \to \pi N, \pi\pi N$). Again one predicts less background at low q^2 due to the effects of VDM non-linear unitarity giving "$e^{i\delta}$ Sin δ" resonances. This prediction appears consistent with the current data [23].

A comparable situation presumably exists in hadronic reactions. At large $-t(\approx 1 \text{ GeV(c)}^2)$, the arguments for $e^{i\delta}$ Sin δ production quite probably break down. Thus one enters a new regime of scattering

in which, liken unto diffraction scattering, resonances sit on large background. Unfortunately, as I have often lamented [12, 24], there is no serious experimental study of this important domain.

2.3: <u>Duality</u>

We will use duality in 2.4 to conclude that, say, $K^+n \to Q^0p$ is real at high energies with EXD ρ, A_2 exchange. This is very reasonable.....

However, one could <u>also</u> use it to relate:

(11)

or

(12)
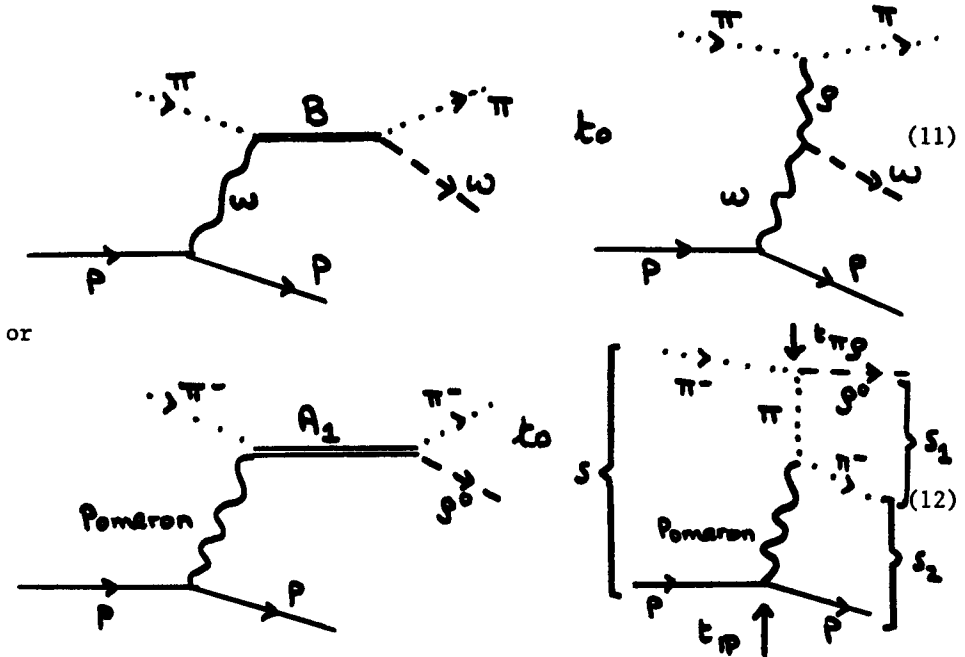
However, the $\pi\omega$ and $\pi\rho$ sub-energies are clearly too low in these cases for duality to be useful. It is reasonable to use it for relating

(13)
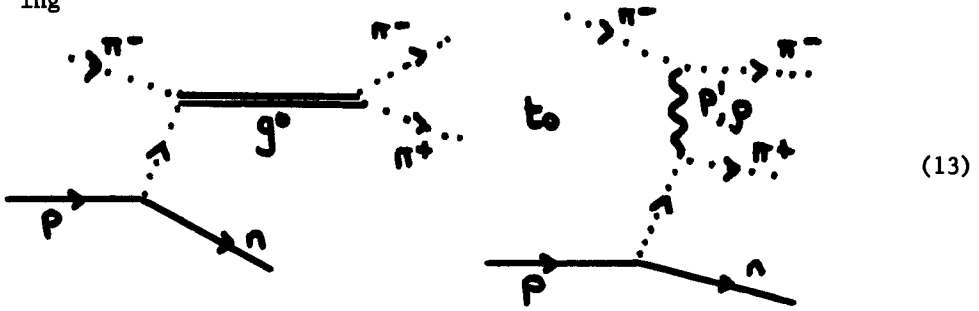
where there are several spin states present in the $\pi\pi$ system. Rather than duality, one can use for (11) and (12) Watson's theorem which as we discussed in 2.2 is valid precisely in the low sub-energy, small

number of eigenstate situations where duality is at its weakest. The deduction for our discussion in Section 3 is that I believe it makes little sense to consider exchange (ρ in (11), π in (12)) explanations of data at these low sub-energies.

It is perhaps useful to make some remarks on (12) - the Deck model Dinosaur:

(i) I once believed the Deck model was a reasonable description of diffraction because of the π pole in

However, this is not correct. Thus, at $t_{\pi P} = t_{Pomeron} = 0$, it is easy to see that the residue of the π-pole($t_{\pi\rho} = m_\pi^2$) vanishes. The easiest way to prove this is to note that the Deck amplitude is proportional to [25]
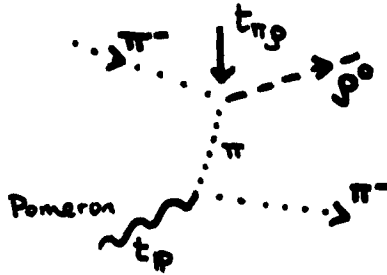
$$\frac{1}{t_{\pi\rho} - m_\pi^2} \cdot s_2^{(\alpha_{\pi P} = 1)} \tag{14}$$

where we mark s, s_1 and s_2 in (12).

But at $t_{\pi P} = 0$, we find

$$s_2 = \frac{m_\pi^2 - t_{\pi\rho}}{(s_1 - m_\pi^2)} \cdot s \tag{15}$$

Combining (14) and (15) we find no π-pole in the resultant amplitude. This has in fact been known for a long time [26] - it is the "successful" prediction of an s-wave A_1. However, it has not been emphasized that it implies zero π-pole residue.

(ii) Given this, I see no reason to prefer the π-exchange diagram over any other. Indeed, the three diagrams,
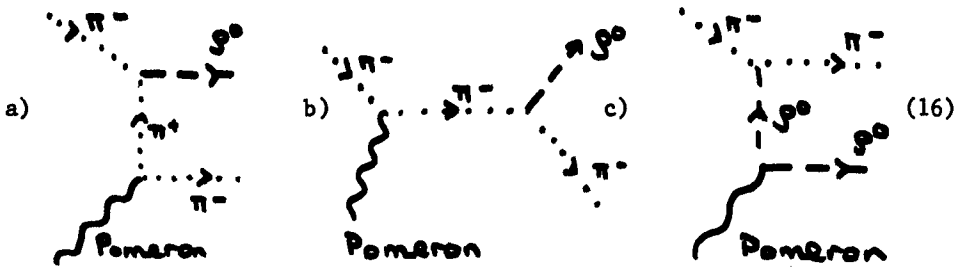
all give identical forms for the amplitude.   Now,

if we replace the Pomeron by a photon [27] – to which it is kine-
matically identical – then gauge invariance implies all three dia-
grams in (16) add up to give identically zero in the forward direction
(i.e., for real photons of zero mass). In this case, it would be di-
sastrous to take just 16(a) and ignore 16(b) and (c) [28].

(iii) What experimental evidence is there for the Deck model?
There can be essentially none for its really distinctive predictions –
for as we described above it has no distinctive features and all three
diagrams in (16) are kinematically identical. There are some predic-
tions of relative cross-section sizes... But a recent SLAC experi-
ment [29] has shown that it gives the wrong answer for $K^0 p \to Q^0 p$ v.
$\bar{K}^0 p \to \bar{Q}^0 p$ (Fig. 2). We can only conclude that the "success" of the
Deck model was based on its correct multiparticle kinematics [26] – a
virtue shared by many diagrams.

So finally, we turn our fancy towards more humdrum things.

2.4: SU_3, EXD, Factorization

(i) Denote the well-loved particles as follows:

    P : Pseudoscalar nonet π, ... η'
    V : Vector nonet ρ ... ϕ
    N : $\frac{1}{2}^+$ octet   p ... Λ
    D : $3/2^+$ decuplet $\Delta(1234)$ ... Ω^-

Denote the well-loved Regge exchanges:

    V : Vector nonet ρ ... ϕ
    T : Tensor nonet A_2 ... f'
    B : B nonet   B ... h'

(ii) Data on reactions like $\pi^- p \to \pi^0 n$, $\pi^+ p \to \pi^0 \Delta^{++}$, $\pi^- p \to K^0 \Lambda$,
$K_L^0 p \to K_S^0 p$ allow one to estimate [24,30] all the spin amplitudes for any
processes of the form
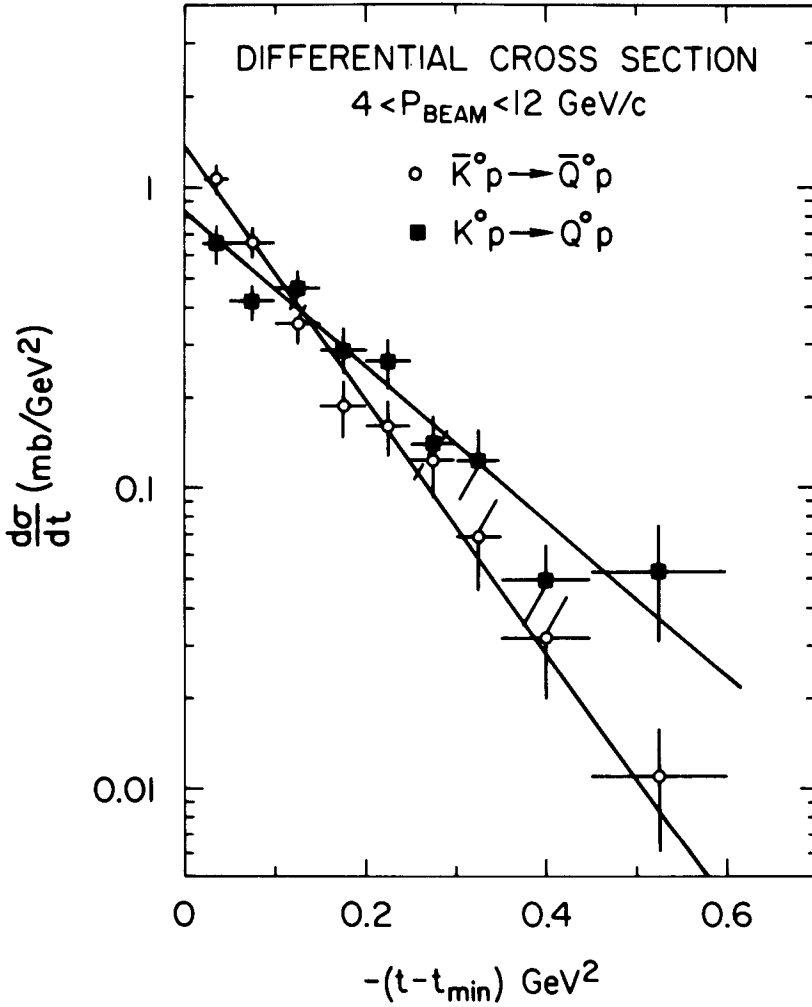
Fig. 2: Crossover at $t \approx -0.2$ $(GeV/c)^2$ between $K^0 p \to Q^0 p$ and $\bar{K}^0 p \to \bar{Q}^0 p$ (Ref. 29). The scale, <u>unlike</u> Fig. 5, is normalized for the decay $Q_0 \to K_S \pi^+ \pi^-$ only. The Deck model predicts Q^0 bigger than $\bar{Q}^0$ at $t' = 0$ – in contradiction to the data above.
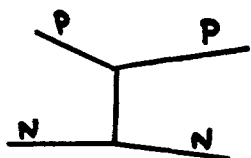
Normalizing one coupling
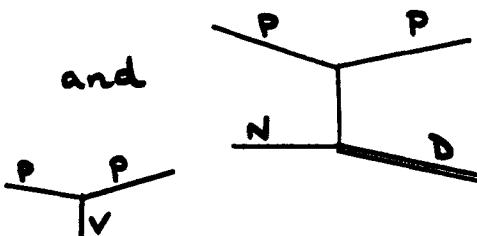
in whatever way we please, one can use factorization and EXD to find
all six Regge vertices.
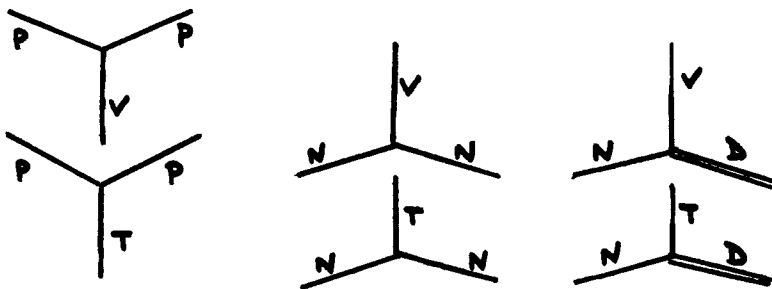
Each of the six couplings – on using SU_3 – leads to the couplings of
individual members of the multiplets. (This needs the known D/F
ratio for

)

(iii)   Further, $d\sigma/dt$ and density matrix element data on say
$\pi^- p \to \omega^0 n$, $K^- p \to \Lambda \omega^0$ allow one to separate the unnatural parity coup-
lings from the natural parity vertices discussed above, and so obtain

all the   amplitudes .

(iv)   Now given any one cross-section, e.g., $\pi^+ p \to B^+ p$ (ω, A_2 ex-
change) for a meson resonance reaction, we can clearly use (ii)   to
find – in this case – the coupling

Then SU_3 and EXD will give all such couplings
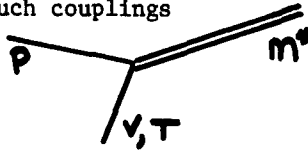
for meson resonances in a given multiplet. Coupling this with the general $N\bar{N}$, $N\bar{D}$ vertex, we deduce: <u>Given $d\sigma/dt$ for one meson resonance in a multiplet, we can at once predict all cross-sections of the form $PN \to M^* N$, $PN \to M^* D$.</u>

As always some caveats are necessary:

(v) If the M^*'s form an unmixed nonet (as do the $\pi...\eta'$), then it is illegal to use the quark model rules (i.e., no disconnected quark diagrams) to calculate the singlet cross-sections. This sin overestimates the singlet cross-section by a factor of 4 in the case of the $\pi...\eta'$ nonet [31]. In our application, we don't really know what any mixing angles are; so we shall forget this difficulty and, for definiteness, use the simple quark rules.

(vi) One can estimate, say, the coupling
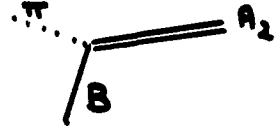
using EXD and the known

coupling. This is the tensor analogue of the π-B exchange degeneracy argument to relate ρ and ω production [32]. EXD is not perfect in the latter case, and one might assume a similar breaking for the tensor production. Then, symbolically, we have:
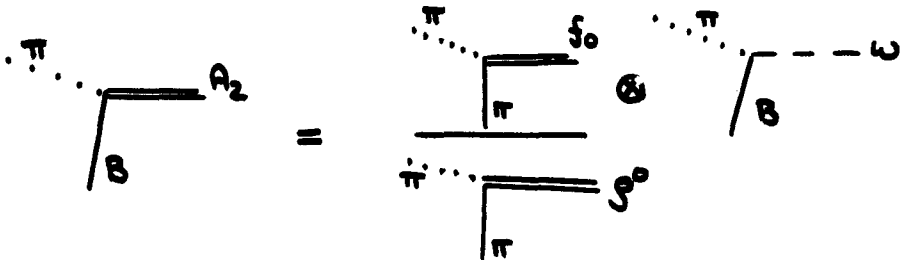
We shall use this broken EXD method to estimate A_2, $\pi_N(980)$ and $\phi_N(1680)$ production by B exchange. (In the $\pi_N(980)$ case, EXD relates

$\pi_N(980)$ to $\epsilon(\approx 750)$).
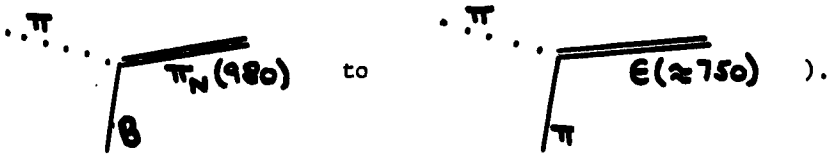
(vii) Absorption will modify some of the above results. All our results involve taking ratios and so some absorption effects cancel. However, the ratios involve different spin amplitudes with "Born-terms" of different t-dependencies. So, as the absorption depends on these effects, it certainly should not cancel completely. However, nobody can calculate the desired corrections [24] and so we shall completely ignore them hereafter.

3: PRODUCTION OF THE L = 1 QUARK MODEL STATES

3.1: Strategy

According to the discussion 2.4, one need only isolate the cross-section for the production of one member of any nonet to be able to use SU_3 to predict all others. Our knowledge of well-loved reactions enables us to guess the most favorable processes. Thus,

and
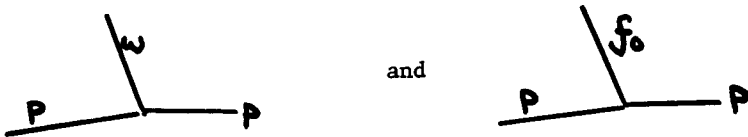
have large $N\bar{N}$ spin non-flip amplitudes. Thus, they correspond to large cross-sections.

or
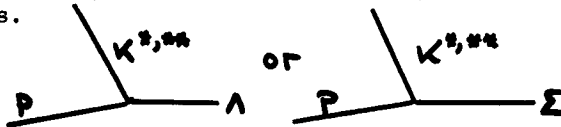
is also mainly non-flip but its size is suppressed; however, this is the usual strange particle production suppression. So $K^{*,**}$ exchange may be expected to produce resonances with a smaller cross-section than ω or f^0 exchange but with a similar t-dependence and a comparable signal/ background. On the other hand,

and

are
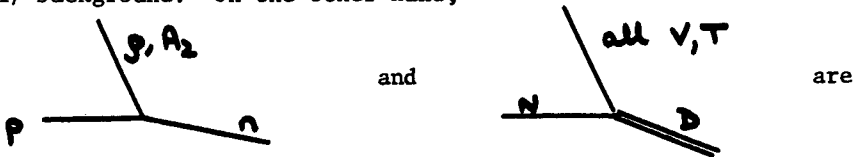
dominantly spin-flip and give rather measly small and flat cross-sections.

It is an important confirmation of theory, that the 1^+ and 2^+ mesons have been seen best in precisely those ω, f^0, $K^{*,**}$ exchange reactions predicted to be most favorable. In fact, the $K^{*,**}$ exchange

data is less certain simply because of the low statistics implied by the universally small cross-sections for associated production.

Let us look at some specific examples. However, first, a moment of sadness. Throughout this section, there will be no such thing as a clean prediction - unsullied by theoretical or experimental caveats. Rather, time after time, we must swat at monstrous factors of two as, devil-horned and dirty, they cling and infest the cross-sections of our desire. We just state the difficulties and record our compromises.

3.2: 2^+ Production

Consider the processes $\pi^\pm p \rightarrow A_2^\pm p$ around 5 GeV/c.

(i) There is a large f^0 exchange contribution [34,35]. Any such vector or tensor exchange must be helicity flip at the πA_2 vertex; parity forbids the coupling πA_2(helicity 0) $\rightarrow$ V,T. This prediction is confirmed by the density matrix elements observed for A_2 production at higher energy [33].

(ii) Helicity zero A_2's can be produced by I = 1, B exchange - this amplitude is now spin-flip at the $N\bar{N}$ vertex and so both f^0 and B exchange predict A_2 production to vanish in the forward direction.

One can estimate B exchange using EXD (cf., Section 2.4) but rather one eliminates both B and ρ exchange (cf., Fig. 9, the latter is the smaller of the two) by forming the I = 0 exchange combination.

$$d\sigma_0/dt = \tfrac{1}{2} \, d\sigma/dt \, \{\pi^+ p \rightarrow A_2^+ p \; + \; \pi^- p \rightarrow A_2^- p - 2(\pi^+ n \rightarrow A_2^0 p)\} \qquad (17)$$

(iii) Equation (17) has been discussed by Rosner [34], but unfortunately currently quoted experimental cross-sections are simply inconsistent [34,36]. So in Fig. 3, we mark a range of experimental differential cross-sections. This is compared with the absolute theoretical prediction given by the pole extrapolation model (i.e., the observed $\Gamma(A_2) \rightarrow \pi\rho$ plus SU_3 and EXD gives coupling $\pi A_2 \rightarrow$ Regge f_0 at $t = m_\rho^2$).

The qualitative agreement between theory and experiment in Fig. 3 appears to me rather impressive. The pole extrapolation gives both roughly the right absolute magnitude and a reasonable ($e^{\approx 3t}$) t dependence. It's interesting to note that this reaction is dominated - theoretically and experimentally - by a single spin-flip amplitude. Such an amplitude is known [24] from study of well-loved $\pi^- p \rightarrow (\pi,\eta)n$, etc., to be the (only?) case where simple Regge theory gives good predictions. So it was probably to be expected that pole extrapolation (which implicitly assumes perfect Regge theory) would work.

3.3: B Production

Consider $\pi^+ p \rightarrow B^+ p$ around 5 GeV/c. Experimentally [37-39] the situation is similar to A_2 production. Thus, the quoted cross-sections
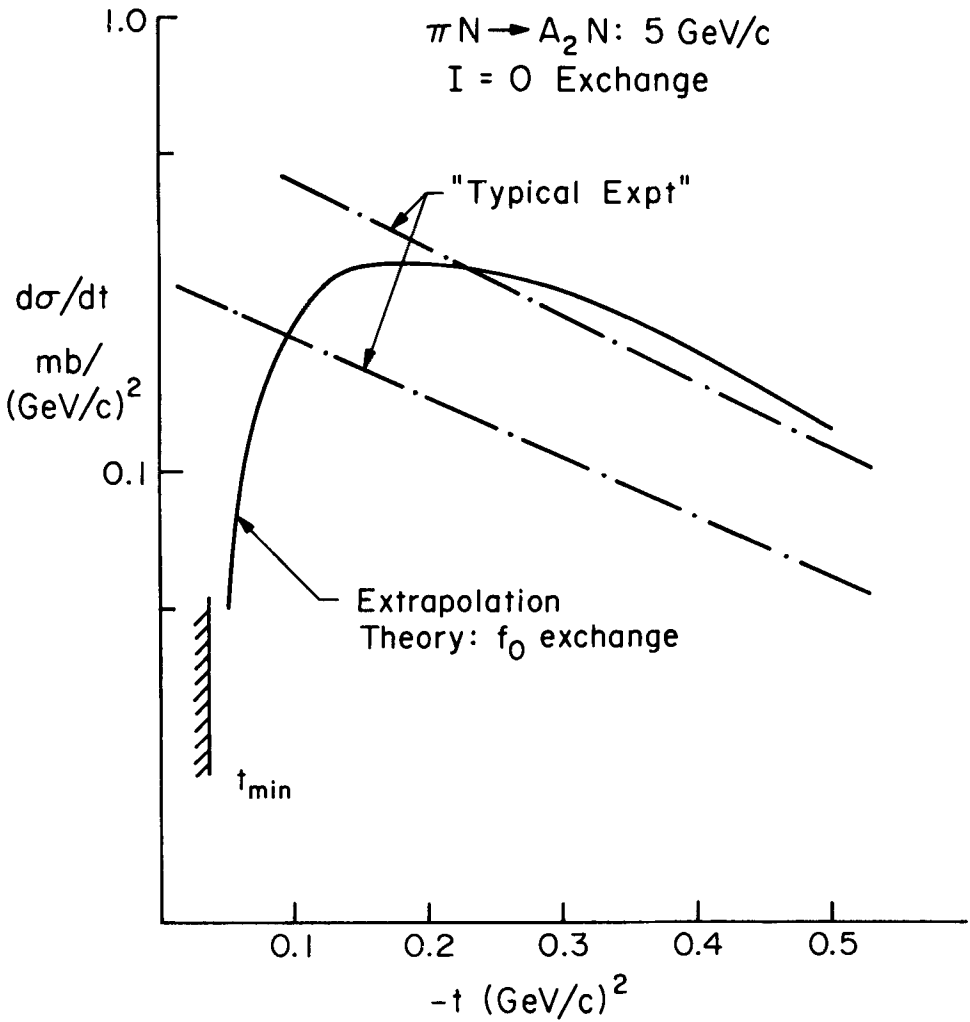
Fig. 3: I = 0 exchange πN → A₂N at 5 GeV/c (Eqn. (17)). Shown
are the absolute theoretical prediction and a range of
experimental values (Refs. 34 & 36).

again vary within a factor of 2 (!) but qualitatively the size and shape of $\pi^+ p \to B^+ p$ is similar to $\pi N \to A_2 N$ (I = 0 exchange) [40]. Theoretically the situation is quite different; again, we have a large (as defined in 3.1) amplitude: this time ω exchange. However, parity no longer forbids the πB (helicity 0) ω coupling and we expect a dominantly spin non-flip amplitude rather than the spin-flip which controlled A_2 exchange. This should give a large and steep (i.e., exp($\approx 8t$)) $d\sigma/dt$. Indeed, given $\Gamma(B \to \rho\omega) \gtrsim \Gamma(A_2 \to \pi\rho)$, pole extrapolation implies qualitatively

$$d\sigma/dt \; (\pi N \to BN) \approx m_\rho^2/t \; d\sigma/dt \; (\pi N \to A_2 N) \qquad (18)$$

This is made quantitative in Fig. 4 which compares the precise pole extrapolation prediction with experiment. As anticipated above, the theory is a complete disaster - being a factor of 30 too big in the forward direction. Figure 4 marks the πB spin-flip (i.e., B helicity $\pm$ 1) part of the theoretical cross-section. It agrees much better in both shape and size with the data. Combining this with the similarity of the A_2 and B cross-sections, we conclude that the latter is largely spin-flip. It follows that it is a matter of theoretical urgency to understand what has happened to the B helicity zero amplitude.

3.4: Kislinger's Model

In Kislinger's model [10], the vector meson Regge pole couplings are proportional to those of the photon. In the case of

(e.g., $\rho \to N\bar{\Delta}$),
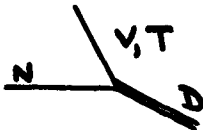
this is well known to give the successful prediction of the Stodolsky-Sakurai [41] decay distribution for the $3/2^+$ decuplet. Unfortunately, this latter prediction may also be obtained in the simple quark model [24,41]. In the past [24], I have preferred the quark derivation as this model has many other successes for the spin structure in PN $\to$ VD. However, the quark model has never been extended to the production of L $\geq$ 1 states (as are the A_1, A_2 and B states of interest now) It is thus particularly striking that Kislinger's photon analogy predicts that in all processes of the form PN $\to$ M*N, PN*, M*N*, the spin non-flip coupling, at either PM* or NN* vertex, vanishes at t = 0 for vector and tensor exchange. This is precisely what the $\pi p \to Bp$ data indicated and so experiment is in amazing qualitative agreement with Kislinger's model. This model predicts the non-flip zero simply because it is present in the photon couplings. The latter is easy to prove in general but, as an example, consider the vertex

G. FOX

Fig. 4: Comparison of $\pi^+ p \to B^+ p$ data (Refs. 37 & 38) scaled to 5 GeV/c and the ω, A_2 pole extrapolation model.

where a and b have spin 0^-. This can be written

$$T^\mu = X(q_a - q_b)^\mu + Y(q_a + q_b)^\mu \tag{19}$$

for invariant amplitudes X and Y. Gauge invariance, i.e.,

$$q_\gamma^{\ \mu} T_\mu = 0 : q_\gamma = q_a - q_b \tag{20}$$

implies

$$Y(m_a^2 - m_b^2) + Xt = 0 \tag{21}$$

i.e., for inelastic scattering, $m_a \neq m_b$, Y vanishes at $t = 0$ $(t = q_\gamma^2)$.

X plays no role in the scattering at high energy (e.g., it is α q_γ^μ and so its contribution vanishes when dotted into the lower vertex $\gamma \rightarrow N\bar{N}$, $N\bar{B}^*$). Thus the non-flip $\gamma \rightarrow a\bar{b}$ amplitude is proportional to Y and hence t as claimed.

Finally we note that Kislinger's model has all the SU_3/EXD factorization properties discussed in 2.4. So no considerations based on this are affected. Also we should note that the SU_3 formulation means that we avoid the embarassing prediction that $\pi N \rightarrow K\Lambda$ vanishes at $t = 0$. (Putting $a = \pi$, $b = K$ in (21) would naively give this disaster.)

3.5: Production of the A_1 Nonet

This is a particularly tricky discussion because there appears to be a discrepancy between the diffractive and non-diffractive data. There are two important new sources of information on 1^+ diffraction. First, the Illinois partial wave analysis[42] of the 3π system in $\pi^- p \rightarrow (\pi^- \pi^- \pi^+)p$; second, the SLAC data[29] on $K^0 p \rightarrow Q^0 p$. I discuss these in turn.

(i) A_1 Diffraction Production

A very surprising feature of the 3π partial wave analysis[42] in

$$\pi^- p \rightarrow ("A_1", A_2 "A_3" \rightarrow 3\pi)p$$

is that the only phase variation seen is the circular rotation of the 2^+ phase relative to the 1^+ background in the A_2 region[43]. This is in nice agreement with a resonant A_2. Unfortunately, we decided in

2.2 that Watson's theorem implied that there could well be little
connection between resonance positions and bumps in the mass spectrum.
However, one "had to" observe relative phase-variations. In this
case, one should see the phase of the resonant 1^+ (S-wave) A_1 compared
to the 0^- background. One could advance five explanations for the
lack of observation of this effect.

   a) The A_1 does not exist - I reject this because of the impres-
sive evidence for its SU_3 partner: the D meson.

   b) The 0^- has a similar resonant phase variation to the A_1. I
reject this "daughter" possibility as being implausible at such
a low mass.

   c) The A_1 is <u>not</u> a "narrow" resonance at 1.07 GeV. Rather, it
is a broad effect centered at (presumably) higher mass. For in-
stance, if its mass was 1.285 GeV, one would expect a width of
230 MeV. This possibility has to be taken seriously because
<u>experimentally</u> - the mass of 1.07 GeV comes from a probably un-
justified (in "$e^{i\delta}$ processes" - see 2.2) association of reso-
nance position with a bump in the same diffraction data that
sees no phase variation. <u>Theoretically</u> the mass formulae based
on the quark model are, I believe, completely empirical and
could be adjusted for a new A_1 mass. There is the notorious pre-
diction $m_{A_1} = \sqrt{2}\, m_\rho$ which agreed miraculously with 1.07 GeV;
however, this should not be taken seriously.

   d) Final state interaction theory is wrong. For instance, our
rule in 2.2 stated that only "rapid" phase variations should be
detected. What total width corresponds to "rapid" variation?
An important check of these phases would be the experiment [44]
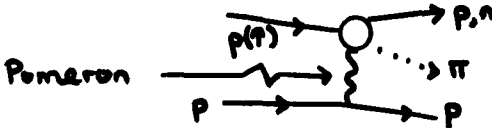
using the Argonne polarized proton beam and detecting the fast
$\pi^+ n$, $\pi^0 p$ system. This experiment has three advantages over the
πp reaction:

α) The two particle πp state is easier to handle than the 3π
    "A_1" decay - cf., my objection (e).

β) Polarization is more sensitive to phases than cross-section.

γ) The πp resonances, i.e., phases, are better known than
    their 3π counterparts.

   e) The data analysis is unreliable for a delicate interference
question like the A_1 phase. Against this possibility, we have
the analysis's successful isolation of the A_2 phase [43]. On the
other hand, we note that the simple ($\varepsilon\pi$, $\rho\pi$) quasi two-body de-
scription of the 3π system used in the analysis violates Watson's
theorem in the various 2π sub-channels. Possibly the phase
error in the 2π sub-systems obscures the overall 3π A_1 phase?

In conclusion, the 3π partial wave analysis - of which we have three reasonable interpretations, (c) to (e) above - casts serious doubts on the usual A_1 parameters. However, it doesn't directly tell us anything about non-diffractive production.

(ii) Q^0 Cross-over
        Let P be the diffractive and R the non-diffractive contribution to $K^0 p \rightarrow Q^0 p$. Then, according to folklore [45],

$$d\sigma/dt'(K^0 p \rightarrow Q^0 p) = |P|^2$$

$$d\sigma/dt'(\bar{K}^0 p \rightarrow \bar{Q}^0 p) = |P + R|^2 \qquad (22)$$

A SLAC group [2] has used these equations [29] to isolate R as

$$\mathrm{Im}R = \frac{d\sigma/dt'(\bar{Q}^0 p) - d\sigma/dt'(Q^0 p)}{2\sqrt{d\sigma/dt'(Q^0 p)}} \qquad (23)$$

Their data is shown in Fig. 2 and the value they extract from it for $|R|^2$, i.e., the non-diffractive contribution to $Kp \rightarrow Qp$, is shown in Fig. 5. The latter requires a lot of explanation.

The experimental data corresponds to a simple mass-cut $1.1 \lesssim M(K\pi\pi) \lesssim 1/5$ GeV and has been corrected for unseen decay modes. For this definition of the Q, the value one obtains for $|R|^2$ is too large to agree with two other methods of calculating it. First, we have Kislinger's model [10] which predicts that Q like B non-diffractive production should vanish at $t = 0$. So any non-flip amplitude (which is that measured by (23)) must be small. Second, we have the $\pi^- p \rightarrow Q^0 \Lambda$ data [11] to be discussed soon. This shows, in perfect agreement with Kislinger's model, a small flat $d\sigma/dt$ which is, in turn, inconsistent with a large non-flip amplitude. So we assume that most of the "mass-cut Q" observed in the K^0 experiment corresponds to $(e^{i\delta})$ background. Perusal of the mass-dependence of the similar $K^+ p \rightarrow Q^+ p$ data [16,47], suggests that with the Q_{A_1} parameters of Table II one could assign about $\frac{1}{4}$ of the SLAC cross-section to the true resonance Q_{A_1}. This has been done in Fig. 5.

Given the above, this figure now exhibits the usual diasgreement between experimental cross-sections and pole extrapolation. Without the mysterious factor of $\frac{1}{4}$, we would have agreed with pole extrapolation but not as discussed above with either Kislinger's model or $\pi^- p \rightarrow Q^0 \Lambda$.

Note that Fig. 5 suggests sizeable non-diffractive Q production at large $-t$ [46] (assuming this is given by the spin-flip part of the extrapolation theory). This should led to a break in $d\sigma/dt$ around $-t \approx 0.5$ $(GeV/c)^2$ at $P_{lab} \simeq 5$ GeV/c.

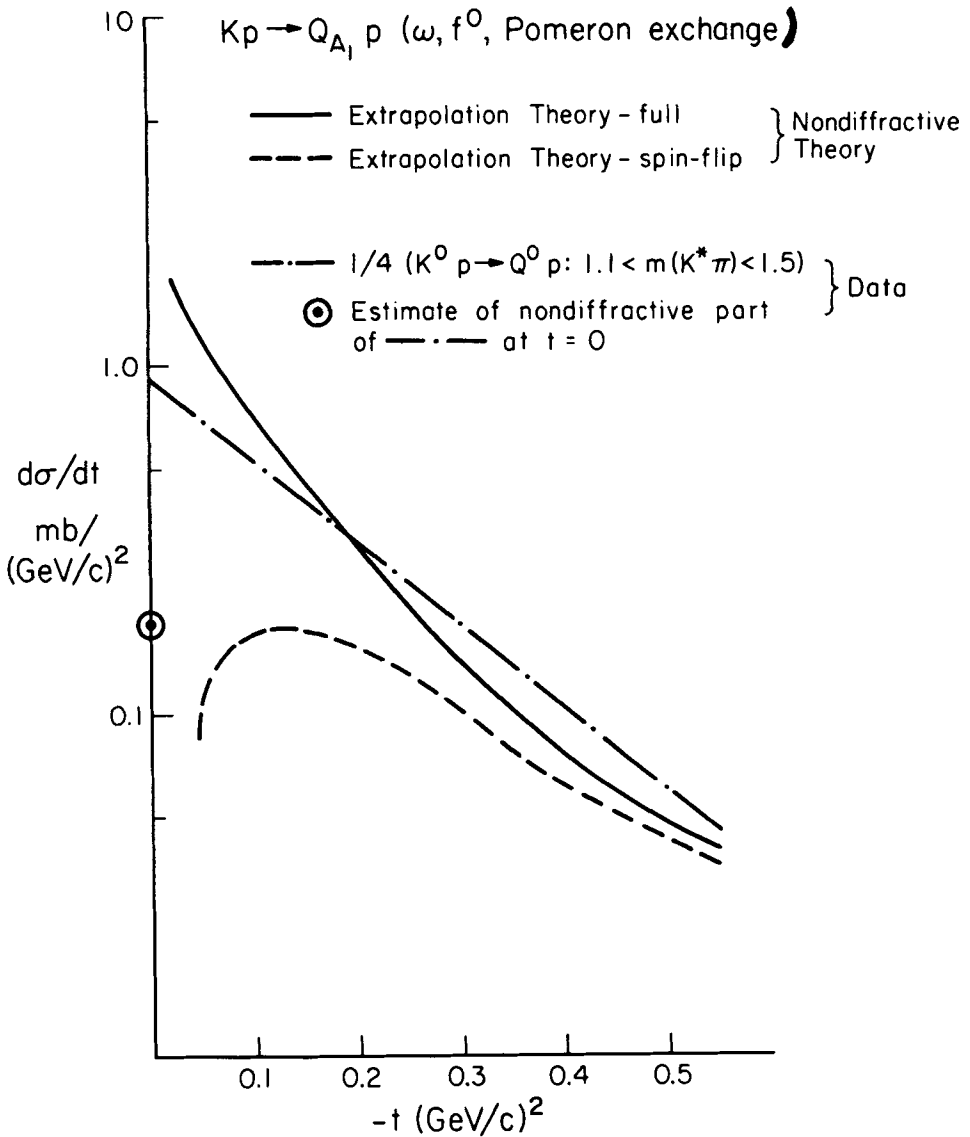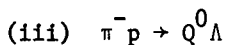Fig. 5: Experimental data (Ref. 29) – averaged over 4 ≲ P_lab ≲ 12 GeV/c – on the diffractive (—·—) and non-diffractive (⊙) parts of Kp → Qp. The extrapolation theory for non-diffractive part is calculated at 5 GeV/c.

(iii) $\pi^- p \to Q^0 \Lambda$

We now come to the most convincing evidence for non-diffractive production of the Q_{A_1} or Q_B. This comes from a study [11] of $\pi^- p \to (K\pi\pi)^0 \Lambda$ at 4.5 and 6 GeV/c. The $K\pi\pi$ mass distribution, shown in Fig. 6, indicates a sharp peak at $M(K\pi\pi) = 1.29$ and a broader enhancement around 1.4 GeV. There are three possible interpretations of this data.

(a) The first peak is identified with the C meson observed in $\bar{p}p$ annihilations around $M(K\pi\pi) = 1.24$ GeV[3]. The mass difference is attributed to background interference. This first Q is then Q_{A_1} while the enhancement at 1.4 GeV is a mixture of K^*_{1420} and a second relatively narrow Q_B resonance.

(b) We have the same identification of the first Q as the C meson but we assume it is Q_B. Then by analogy with our interpretation (c) of the Illinois A_1 analysis (3.5(i)), we assume the Q_{A_1} is much broader and hence indistinguishable from background. For instance, the mass assignment $M(Q_{A_1}) = 1.35$ GeV doubles the expected width and mixing may do other awful things.

(c) The 1.29 enhancement is Q_B while Q_{A_1} at lower mass (i.e., 1.24 GeV) has too small a cross-section to be seen.

Examination of the $d\sigma/dt'$ data for the two regions (I: $1.24 \leq M(K\pi\pi) \leq 1.34$, II: $1.34 \leq M(K\pi\pi) \leq 1.48$ GeV) rules out (c) but without ehough data to allow a partial wave analysis, one cannot distinguish (b) and (c). The data in Figs. 7 and 8 comes from a DST kindly sent me by Kwan Lai and Howard Gordon with normalization taken from the standard compilation [48]. For theory, we use SU_3 estimates of the type discussed in 2.4: that for $\pi^- p \to Q_B^0 \Lambda$ uses $\pi^+ p \to B^+ p$; that for $\pi^- p \to Q_{A_1}^0$ uses the SLAC $K^0 p \to Q^0 p$ crossover analysis [29] for normalization and the Kislinger model [10] for t dependence; finally for $\pi^- p \to K^*_{1420} \Lambda$ we use $\pi^\pm p \to A_2^\pm p$. This underestimates the K^*_{1420} cross-section by a factor of 2 because it omits a sizeable K and Q_B exchange contribution. This can be seen theoretically from the EXD analysis of 2.4 and experimentally [11] by comparison of $\pi^- p \to K^*_{1420} \Lambda$ and $\pi^- p \to K^*_{1420} \Sigma^0$. (The latter has no K_B exchange.)

Finally we can actually examine Figs. 7 and 8. First, we note the beautiful agreement between the t-dependence of theory and experiment. This flat t-behavior again supports Kislinger's idea that all such resonance production (Q_B, Q_{A_1}, K^*_{1420}) is mainly spin-flip. The normalization of the $\pi^- p \to Q_B^0 \Lambda$ and $K^*_{1420} \Lambda$ curves clearly allow either possibility (a) or (b). The $Q_{A_1}^0 \Lambda$ curve in Fig. 7 is in satisfactory accord with (a) when we consider the many arbitrary

$e^{i\delta}$ Sin δ Production ?
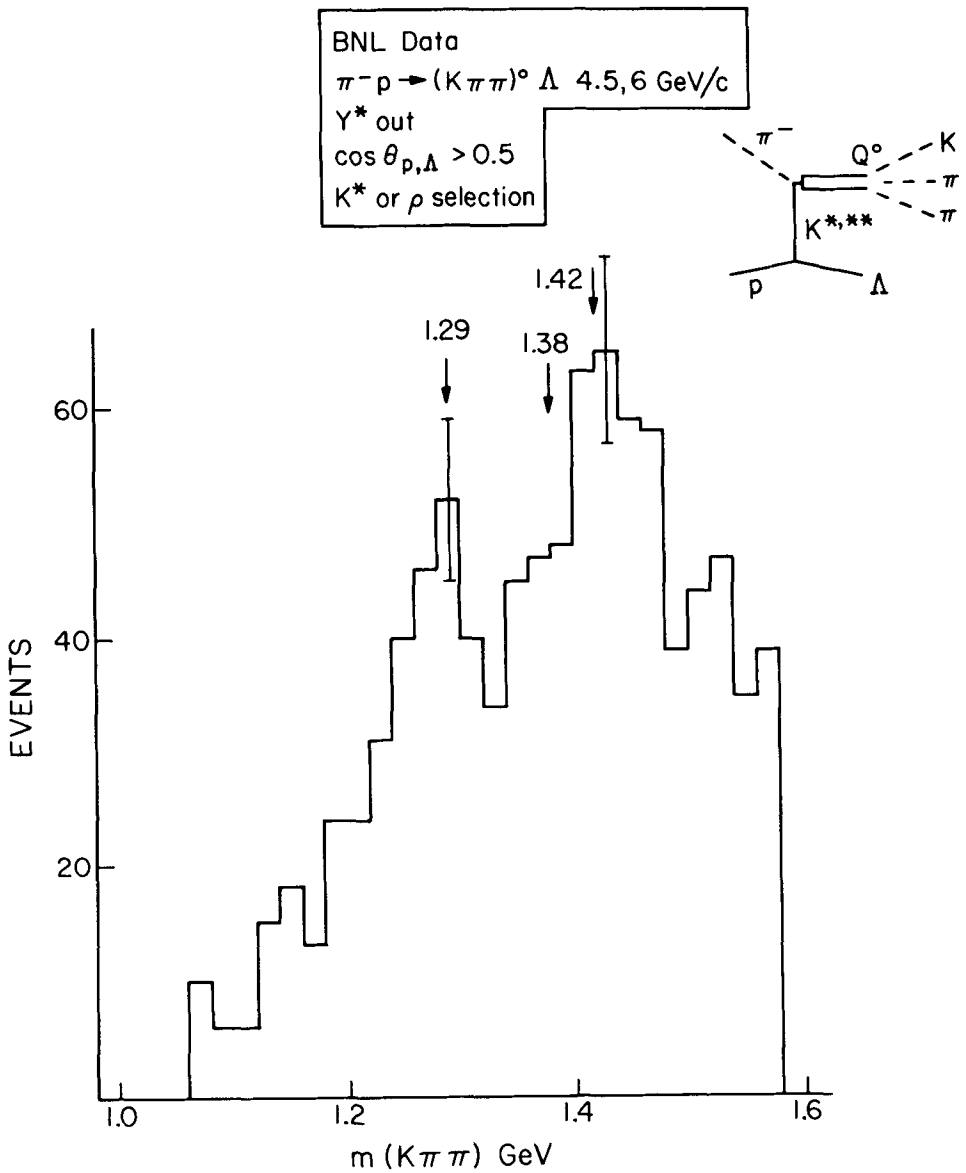
Fig. 6: K$\pi\pi$ mass distribution (Ref. 11) in $\pi^- p \rightarrow (K\pi\pi)^0 \Lambda$ at 4.5 and 6 GeV/c (momenta combined). This probably represents sum of Q_{A_1} (1.29), Q_B (1.38) and K^*_{1420} (1.42).

Fig. 7: "Lower Q": Experimental mass-cut data (Ref. 11) compared with SU₃ predictions for Q_{A_1} and Q_B.
Note: $t_{eff} = t' + t_{min}$ (m = 1.38).

G. FOX

Fig. 8: "Higher Q + K*₁₄₂₀": Experimental mass-cut data (Ref. 11) compared with SU₃ predictions for Q_B and K*₁₄₂₀. The latter should be doubled to get all exchanges – see text.

assumptions in our interpretation of the SLAC Q_{A_1} data. Note that this curve does include the magic factor of $\frac{1}{4}$ discussed in 3.5(ii) and would have been far too large without it. Given the SLAC Q_{A_1} curve is not still miles too big, we can rule out possibility (c) because there is not enough "spare" cross-section at low $K\pi\pi$ masses.
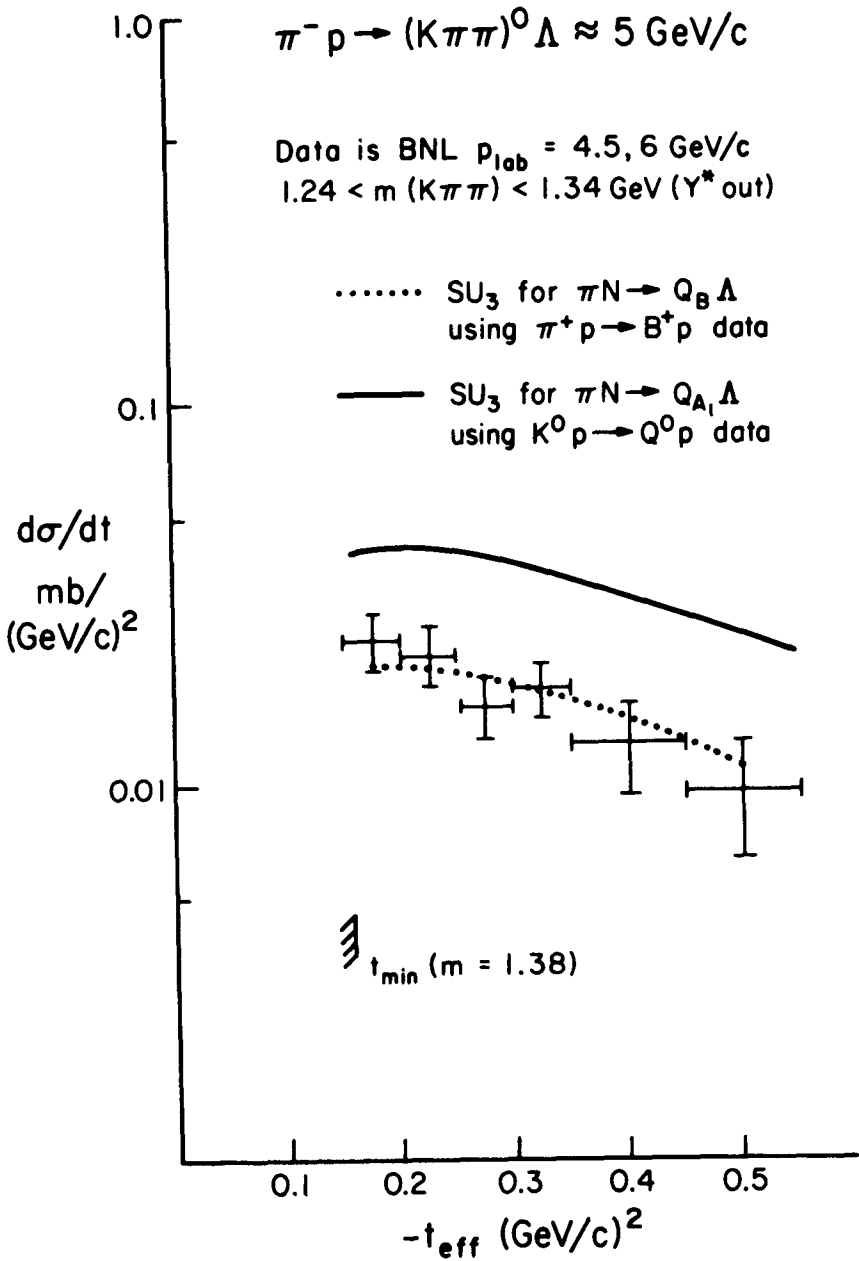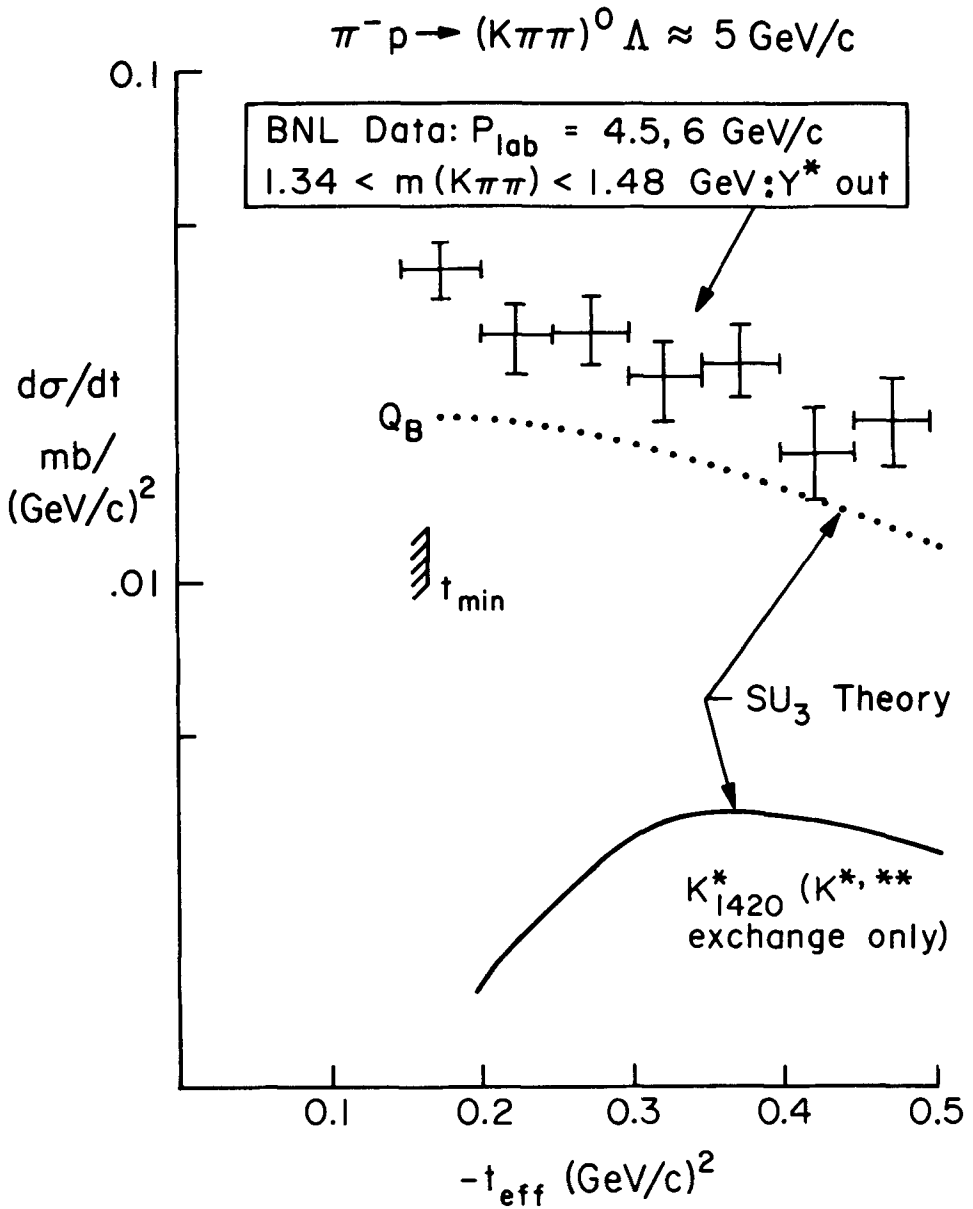
3.6: Predictions

Using $\pi^+p \to B^+p$ and $\pi^-p \to Q^0\Lambda$ (assuming the 1.29 GeV enhancement in Fig is Q_{A_1}), we can now, as described in 2.4, use SU_3 to predict the cross-sections for all members of the B and A_1 nonets. The results are given in Figs. 9 to 12 where we have also marked some typical $d\sigma/dt$ for well-loved reactions. The calculations are arbitrarily given for P_{lab} = 15 GeV/c; they can be scaled to your favorite momenta using the usual $P_{lab}^{2\alpha-2}$ ($\alpha \approx 0.5$ non-strange; $\alpha \approx 0.35$ strangeness exchange) scaling law. The only exceptions to this are first, the two B exchange reactions $\pi^-p \to \pi_N(980)n$ and $\pi^-p \to A_2^0n$. These scale more like P_{lab}^{-2} and the plotted cross-sections were calculated using the π-B EXD ideas described near the end of 2.4. Similarly for their strange partner [49] $K^-p \to \pi_N(980)\Lambda$ in Fig. 11(II) which is K and Q_B exchange, I have assumed P_{lab}^{-3} scaling.

As indicated in the introduction, the epithets magic-ω (sometimes abbreviated to magic), magic-ϕ, octet and singlet describe possible mixing schemes of the I = 0 D and h mesons. Further, we do not mark explicitly the $\pi^+p \to M^{*0}\Delta^{++}$ cross-sections (which are essentially identical to the analogous $\pi^-p \to M^{*0}n$ reaction) and $K^-p \to M^{*-}Y_{1385}^{*+}$, $\pi^+p \to M^{*+}Y_{1385}^{*+}$ which are rather small (see ratio [50] of $\pi^+p \to K^+\Sigma^+v$. $\pi^+p \to K^+Y_{1385}^{*+}$ in Fig. 10(I)).

3.7: Further Tests

Now we would like to test our predictions in Figs. 9–12 by comparison with various other reports of 1^+ production in the literature. These are generally not as statistically significant as the reactions in 3.3 and 3.5, but it is important simply to check that our theoretical predictions are no larger than current experimental upper limits.

(i) CEX Q: $K^+n \to K^0p$, $K^-p \to \bar{Q}^0n$

Charge exchange Q production has been looked for by Ferbel and collaborators [51] in a large sample of the world's data; they report no evidence for Q production and the only resonance seen is the K_{1420}^*. To interpret this result, I have taken one of the experiments [17] in their compilation where the normalization is readily available. So Fig. 13 shows the 12 GeV/c $K^+n \to (K^0\pi^+\pi^-)p$ reaction
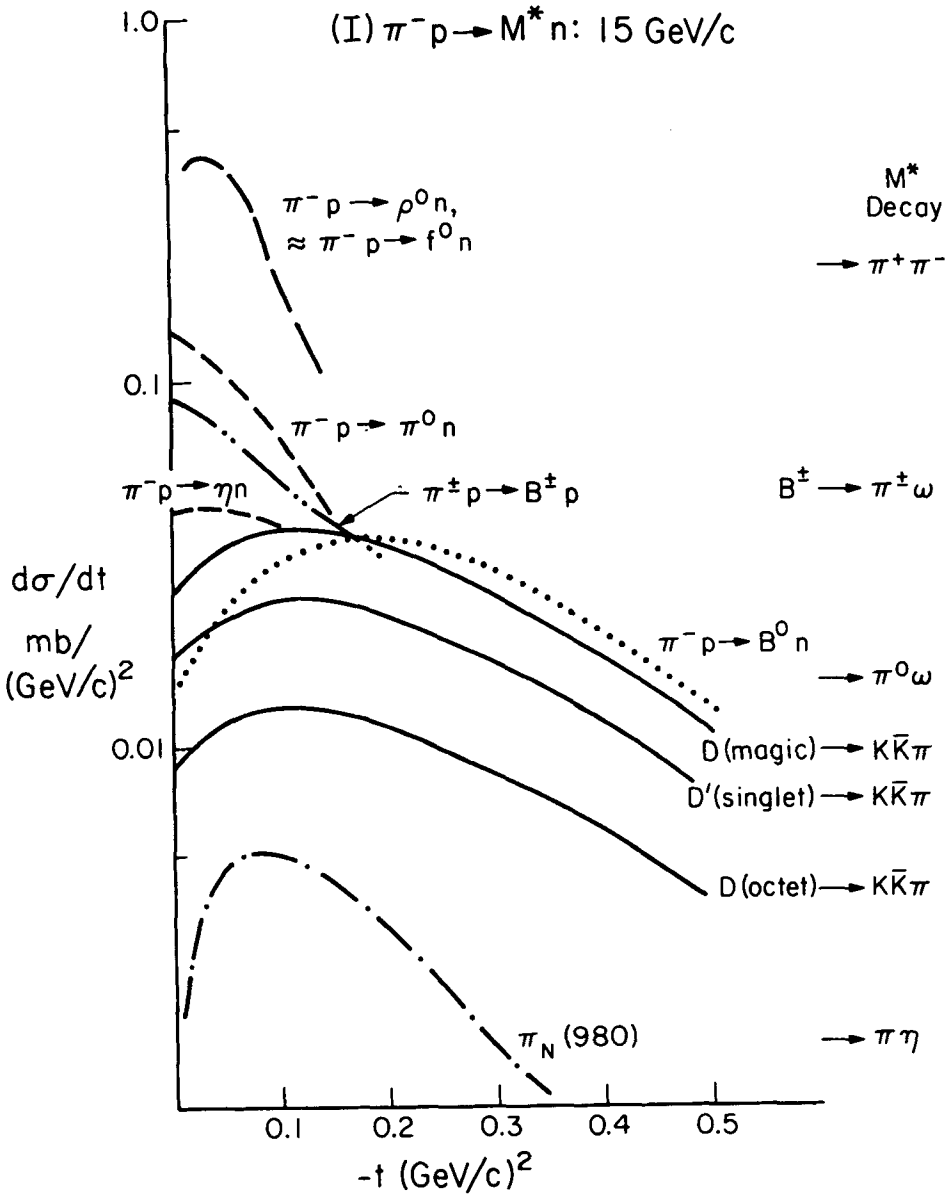
Fig. 9(I): SU_3 predictions described in Section 3.6 for some of the $\pi^- p \to M^* n$ reactions at 15 GeV/c. $\pi^+ p \to M^{*0} \Delta^{++}$ has identical $d\sigma/dt$ to the analogous $\pi^- p \to M^{*0} n$ reaction.
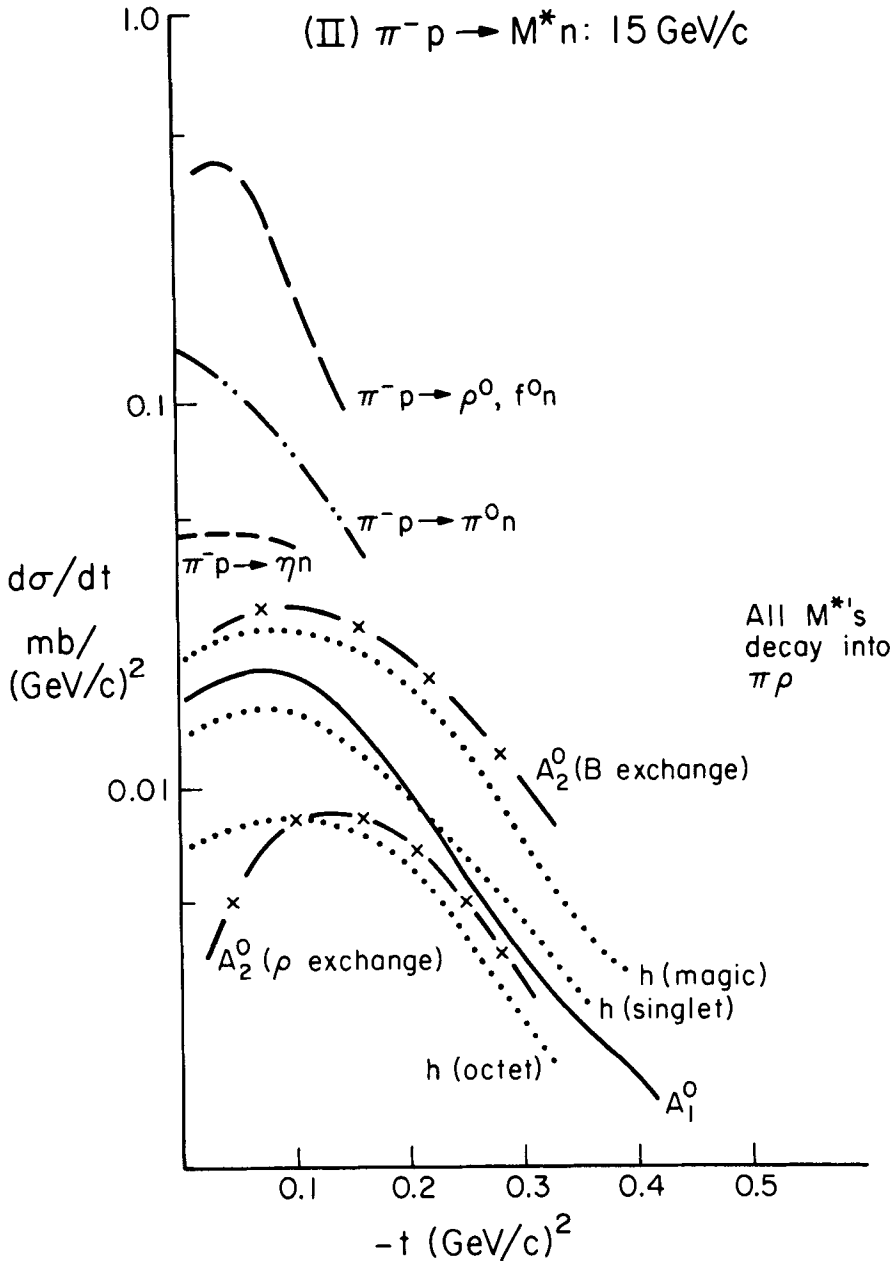
Fig. 9(II): SU$_3$ predictions described in Section 3.6 for $\pi^- p \to M^*$
($\to \pi\rho$)n at 15 GeV/c. $\pi^+ p \to M^{*0} \Delta^{++}$ has identical dσ/dt
to the analogous $\pi^- p \to M^{*0}$n reaction.

(I) $\pi^+ p \longrightarrow (M^{*+} \longrightarrow K\pi\pi) \Sigma^+$: 15 GeV/c
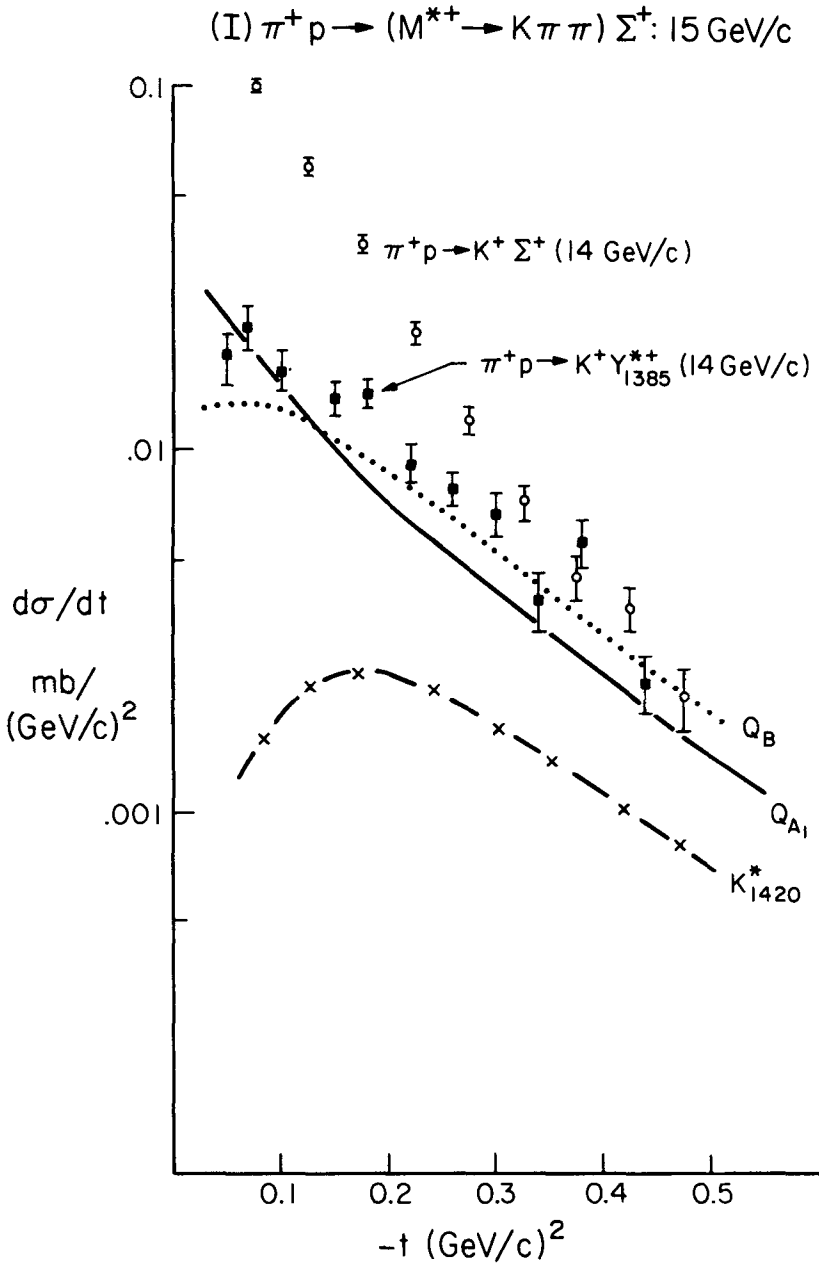
Fig. 10(I):   SU$_3$ predictions described in Section 3.6 for
$\pi^+ p \rightarrow M^{*+}\Sigma^+$ at 15 GeV/c.   The well-loved data
is from Ref. 50.

(II) $\pi^- p \rightarrow (M^{*0} \rightarrow K\pi\pi) \Lambda$: 15 GeV/c

$\pi^- p \rightarrow K^0 \Lambda$

$d\sigma/dt$

$\dfrac{mb/}{(GeV/c)^2}$

Q_{A_1} Q_B

K^*_{1420}

$-t$ (GeV/c)^2

Fig. 10(II) Fig. 13: $K\pi\pi$ mass distribution in $K^+ n \rightarrow (K^0 \pi^+ \pi^-)p$ at 12 GeV/c
(Ref. 17). This is discussed in Section 3.7(i).
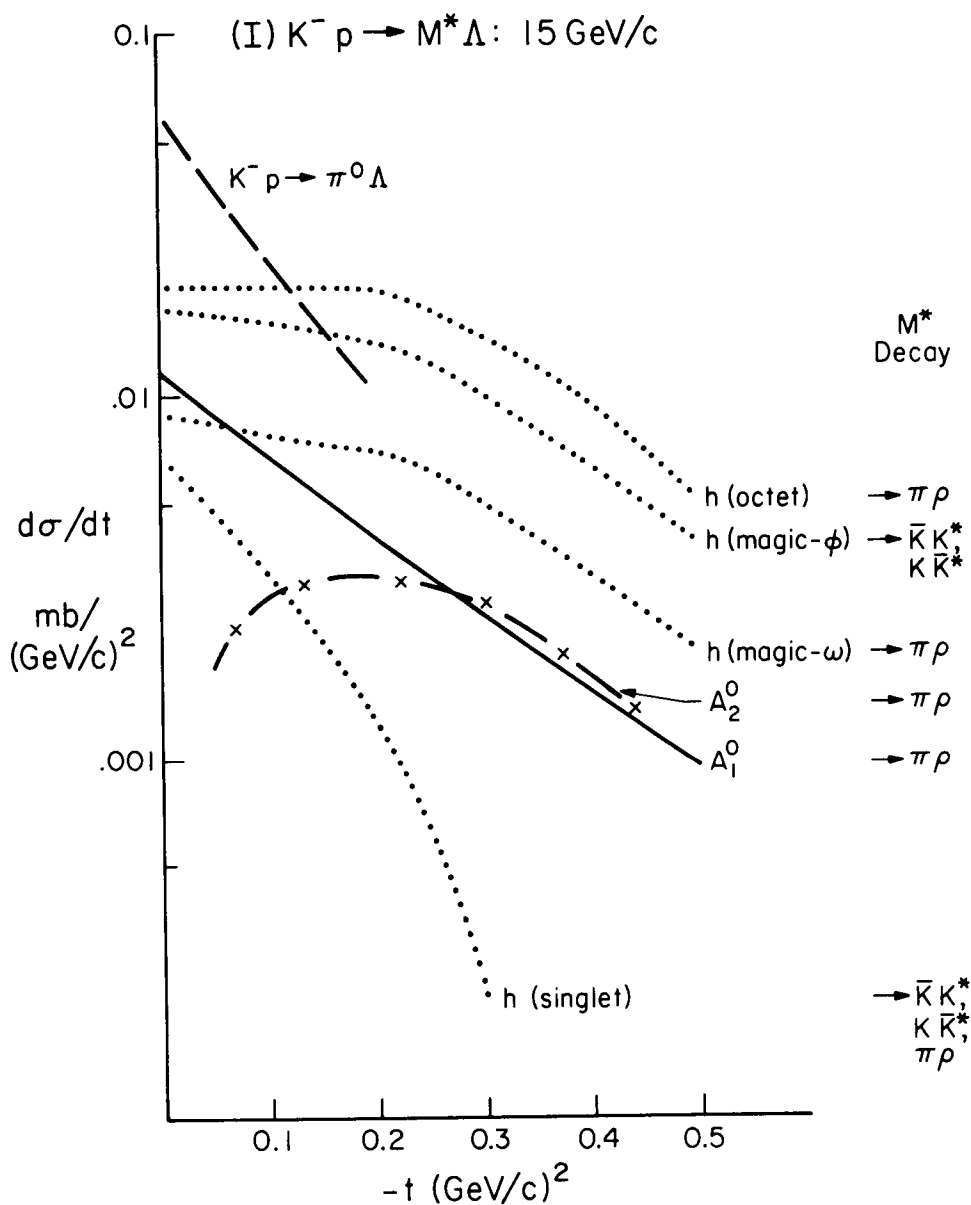
Fig. 11(I): SU_3 predictions described in Section 3.6 for some
$K^- p \to M^* \Lambda$ reactions at 15 GeV/c.
Note: $K^- n \to M^{*-} \Lambda = 2(K^- p \to M^{*0} \Lambda)$ for isospin-1 mesons.
The A_2 curve does not include K/Q_B exchange.
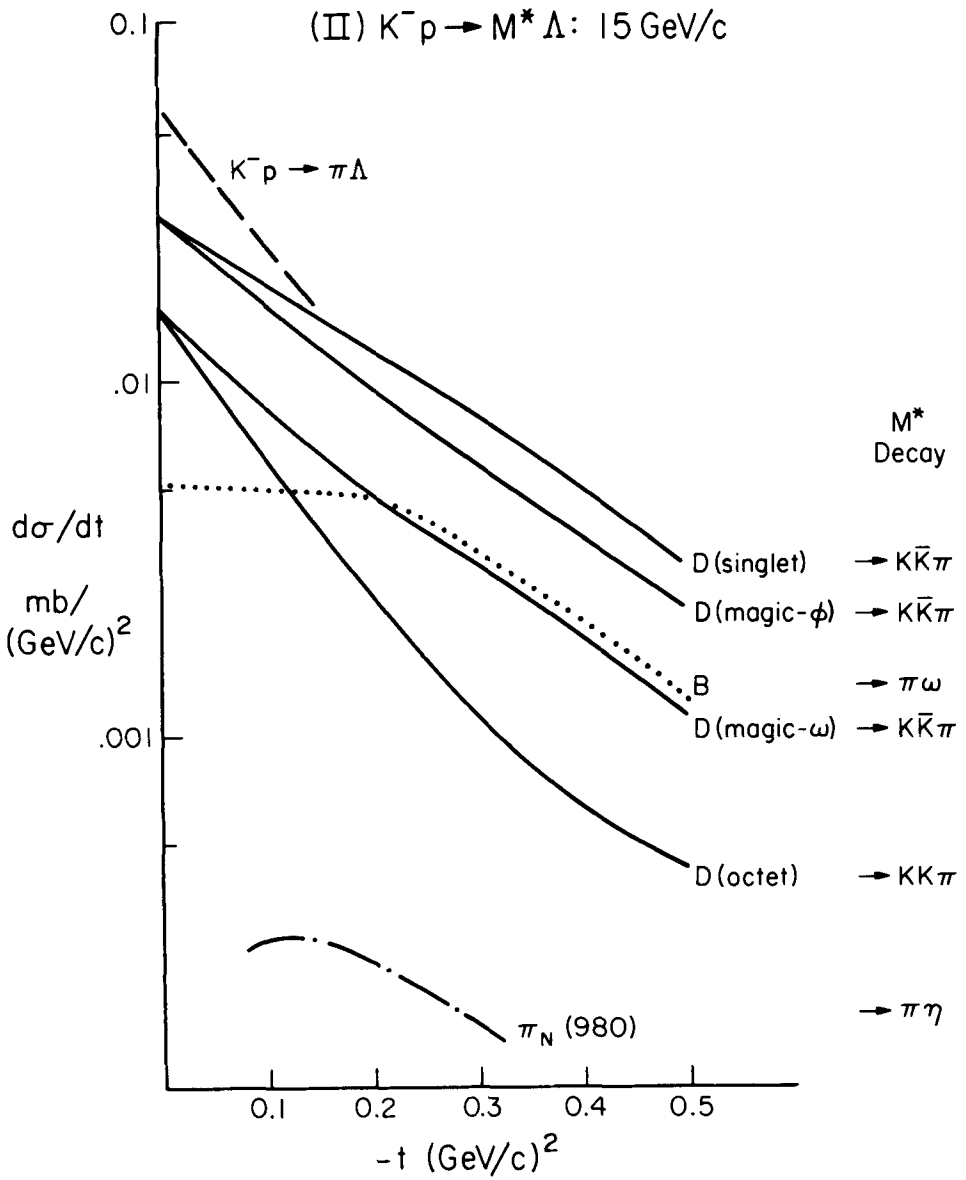
Fig. 11(II): SU$_3$ predictions described in Section 3.6 for some
K$^-$p → M*Λ reactions at 15 GeV/c. Note:
K$^-$n → M^{*-}Λ = 2(K$^-$p → M^{*0}Λ) for isospin-1 mesons.
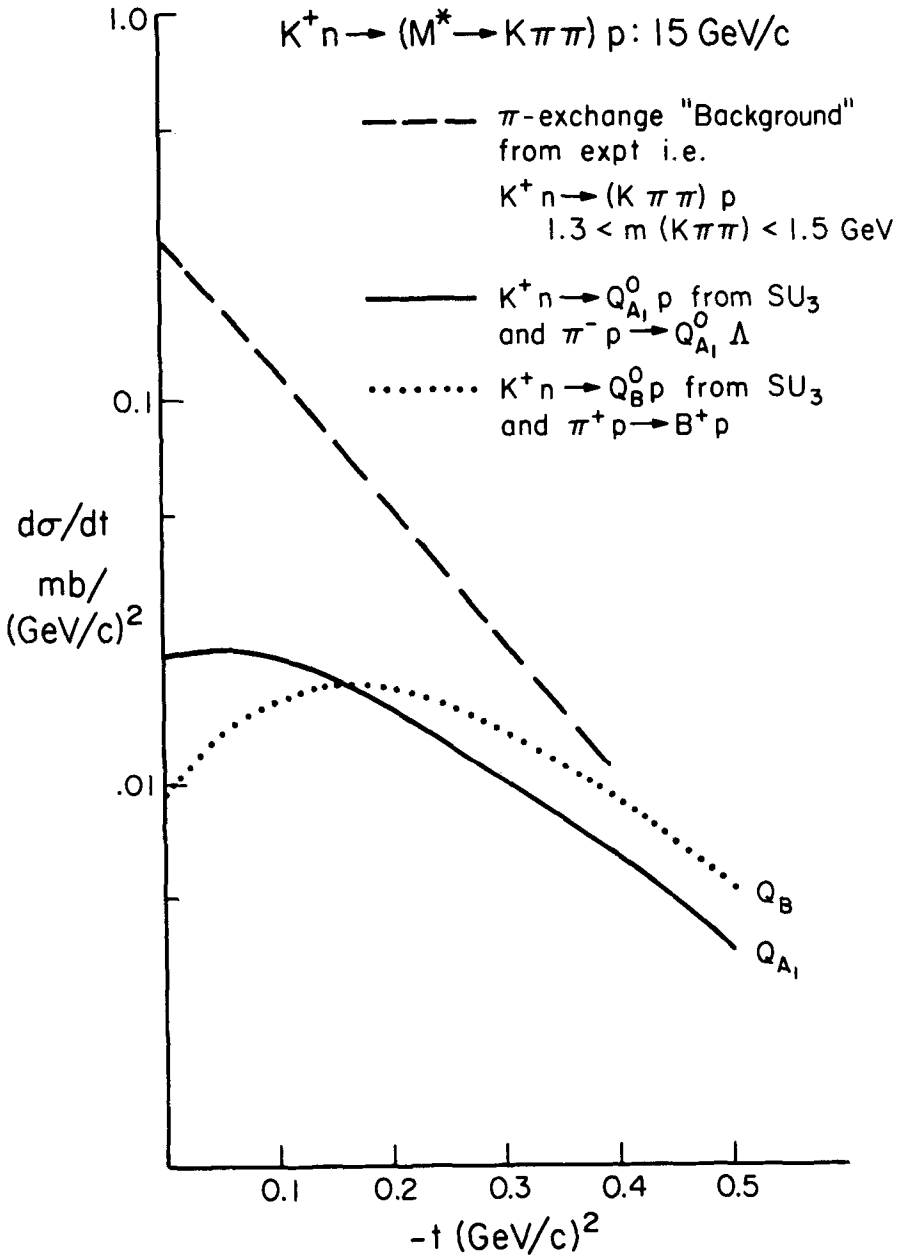
Fig. 12: SU$_3$ predictions described in Section 3.6 for K$^+$n → M^{*0}p
at 15 GeV/c. The π-exchange "background" is explained
in 3.7(i) and comes from Ref. 17, scaled to 15 GeV/c.
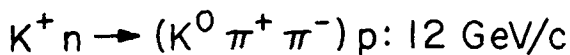
$$K^+ n \rightarrow (K^0 \pi^+ \pi^-) p : 12 \text{ GeV/c}$$

EXPT: $1.12 < m(Q) < 1.32 : \sigma \approx 8\mu b$

THEORY: $K^+ n \rightarrow (Q^0_{A_1} \rightarrow K^0 \pi^+ \pi^-) p : \sigma \approx 4\mu b$
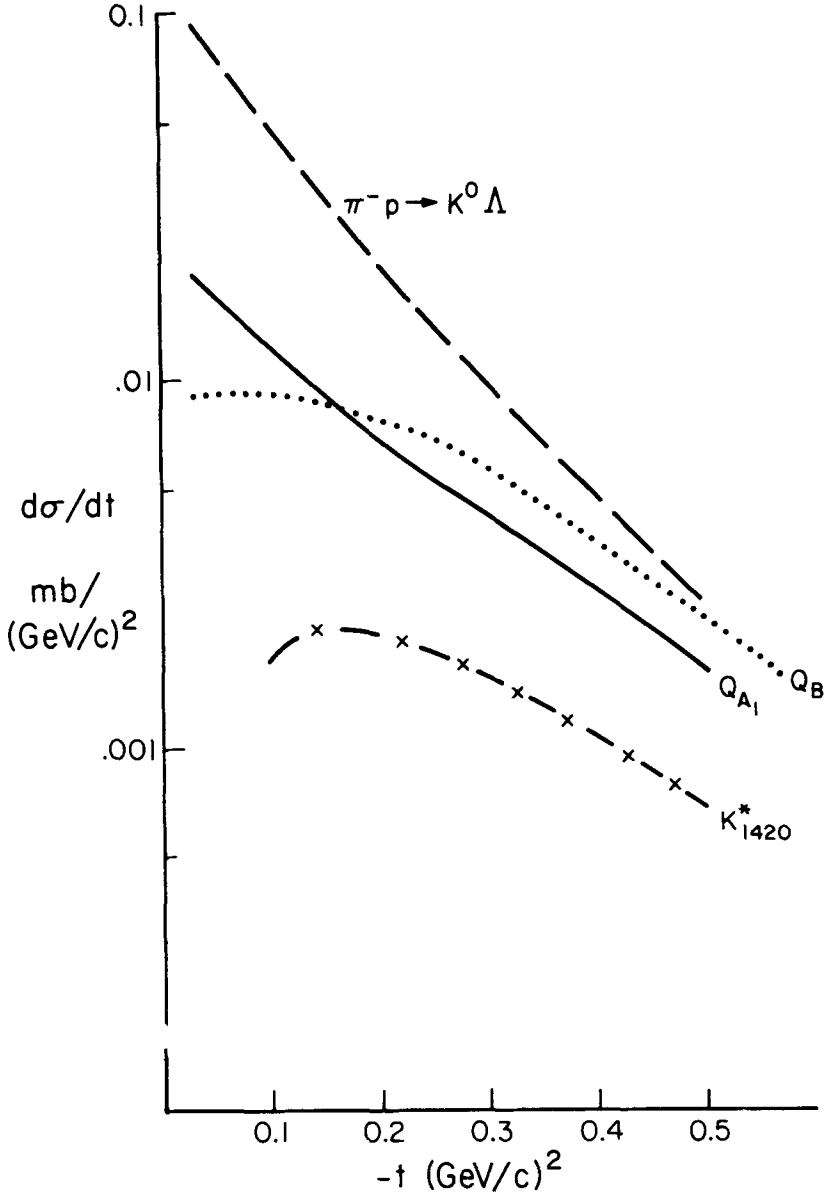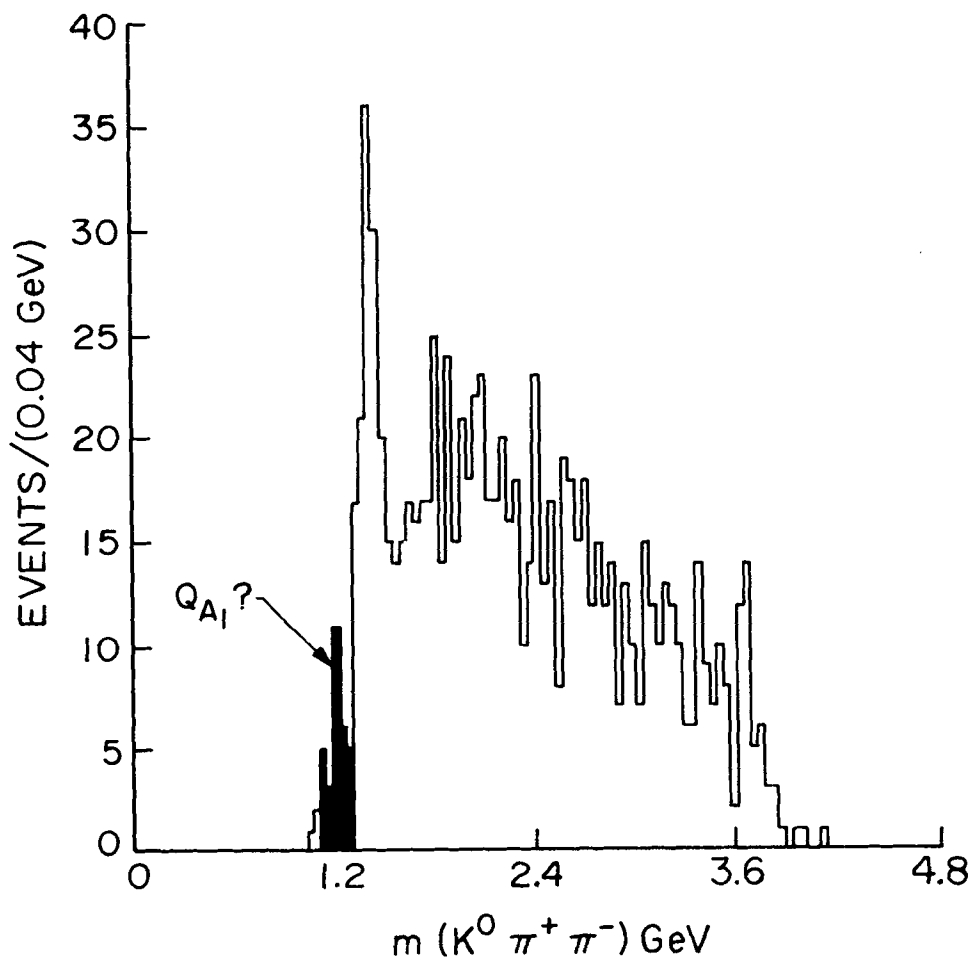

Fig. 13: $K\pi\pi$ mass distribution in $K^+ n \rightarrow (K^0 \pi^+ \pi^-)p$ at 12 GeV/c (Ref. 17). This is discussed in Section 3.7(i).

where there is no statistically significant Q signal below the K^*_{1420}.
However, this is no contradiction with theory, for the number of
events observed in the $Q^0_{A_1}$ mass-cut, $1.12 \leq M(K\pi\pi) \leq 1.32$ GeV, is
twice what we would expect from the resonance production $K^+n \to Q^0_{A_1}p$ [52]
(estimated using SU_3 and $\pi^-p \to Q^0_{A_1}\Lambda$). Moreover, $K^+n \to Q^0p$ is an ex-
ceedingly difficult place to detect the Q because of the large
π-exchange background. This was indicated in Fig. 12 for the K^*_{1420}
mass-cut, and the possibility of π-exchange explains why the
K^*_{1420}/Q_{A_1} ratio is so much larger in $K^+n \to (K\pi\pi)p$ than in
$\pi^-p \to (K\pi\pi)\Lambda$. (Compare Figs. 6 and 13.)

    (ii) <u>950 MeV Region</u>

A recent experiment [54] reported three narrow bosons, around
950 MeV, observed in $\pi^-p \to M^*n$ at 2.4 GeV/c and $t = t_{min}$. Theoreti-
cally, there is the uncontroversial narrow $0^-\eta'$ and the 40 MeV wide
$0^+ \pi_N(980)$. Further, Colglazier and Rosner have proposed [5] the 1^+ D'
of the A_1 nonet should be a narrow resonance in this mass region.
The theoretical estimates of these three cross-sections are given in
Fig. 14 - all are dominantly spin-flip amplitudes which suggests one
would do best to look for these particles a little away from the for-
ward direction. However, the theoretical estimates are not complete-
ly reliable for the incident momenta is rather low. Take the differ-
ence between the experimental and theoretical [31] η' values as some
estimate of a non-asymptotic correction (i.e., a small non-flip ampli-
tude); then the ratio of the theoretical curves η': $\pi_N(980)$: D' is

in embarassingly good agreement with the experimental ratio η': δ:
M(940). This cannot be taken seriously without further experimental
work; thus although the D' can consistently be identified with the
M(940), the experimentally narrow δ meson (quoted is $\Gamma_\delta < 5.9$ MeV) can
hardly be equated with the $\approx$ 40 MeV wide $\pi_N(980)$.

    (iii) <u>h and D Production</u>
    D production has been studied in the reactions $\pi^+n \to D^0p$ [55,56]
and $\pi^+p \to D^0\Delta^{++}$ [57,58] which (theoretically) have equal cross-sections.
Perhaps the best data [55] is shown in Fig. 15. The t-distribution is
very flat and only one-third of the cross-section lies within
$0 \leq |t'| \leq .5$. Here the experimental cross-section for $D \to \delta\pi$ is
15 µb while my theoretical SU_3 estimate for D $\to$ all is 10 µb; this
agreement seems quite satisfactory. The theory is calculated for
pure octet assignment to the D; larger cross-sections might be ex-
pected for other mixing assignments. These values plus the very flat
$d\sigma/dt$, expected theoretically, are recorded in Fig. 16. Unfortunate-
ly, we cannot say very much about $D^0\Delta^{++}$ because no cross-sections [57,58]
are quoted for the small $|t'| \leq .5$ cut necessary to apply our peri-
pheral theory. Both $\pi^-p \to D^0n$ and $K^-n \to B^-\Lambda$ (see next section)
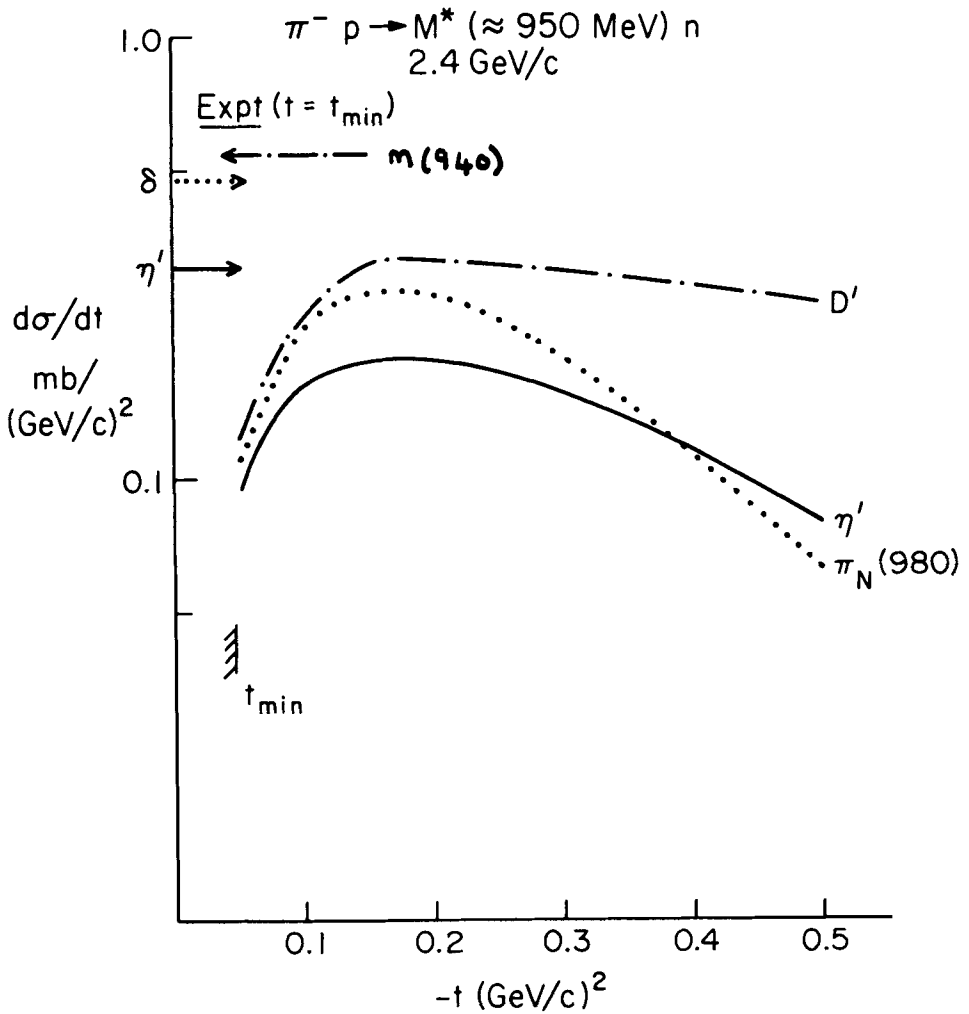
Fig. 14: Theoretical and experimental (Ref.54) narrow resonance
          production in the 950 MeV mass region and P_{lab} = 2.4 GeV/c.
          This is discussed in Section 3.7(ii).

$$\pi^+ n \longrightarrow (D^0 \longrightarrow \delta^\pm \pi^\mp) \, p$$

2.7 GeV/c

$$\sigma \text{ (all } t) = 44 \pm 8 \, \mu b$$

$$\sigma \text{ (} |t'| \leq .5) \approx 15 \, \mu b$$

Mass $(\eta \, \pi^+ \pi^-)$ GeV

Production Distribution

(b)
291 events

30

20

10

0

1.1   1.3   1.5

30

20

10

0

−1   +1

COS θ^*

▨ $= \eta \longrightarrow \pi^+ \pi^- \pi^0$

▢ $= \eta \longrightarrow MM$
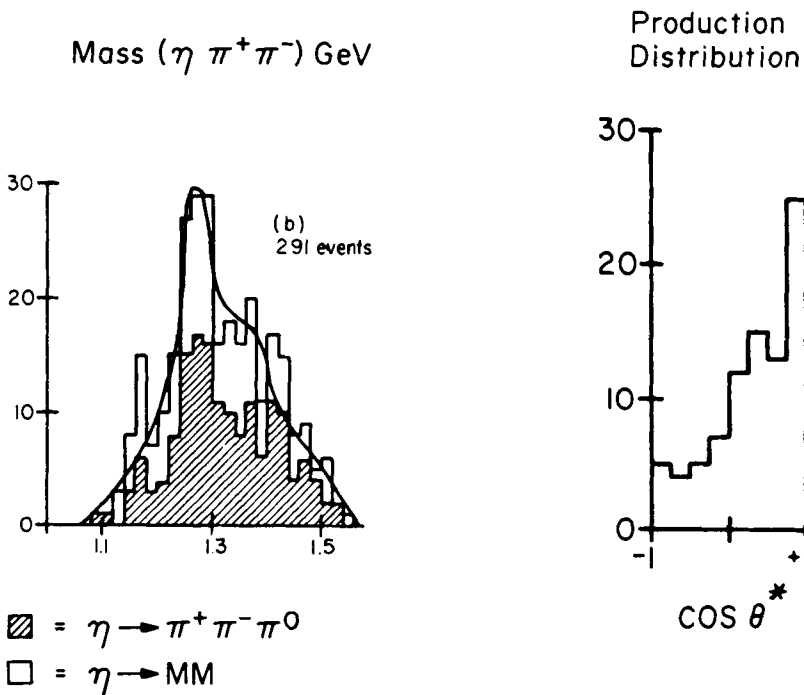
Fig. 15: $\pi^+ n \to (D \to \pi\delta)p$ at 2.7 GeV/c (Ref. 55).   See also Ref. 56 for $\pi^- p \to D^0 n$ at 2.5 → 4.2 GeV/c in the less favorable decay mode $D \to K\bar{K}\pi$.   This is discussed in Section 3.7(iii).

$$\pi^- p \rightarrow D^\circ n, \quad \pi^+ p \rightarrow D^\circ \Delta^{++}$$

Theory D :	magic	$\sigma \approx 30 \ \mu b$
at 5 GeV/c	octet	$\approx 10 \ \mu b$
	singlet	$\approx 20 \ \mu b$

Expt : $\quad \sigma \simeq 16 \pm 6 \quad D \rightarrow K\bar{K}\pi$ at 4 GeV/c

$\simeq 25 \pm 8 \quad D \rightarrow \delta\pi$ at 8 GeV/c

$$\pi^- p \rightarrow h^\circ n, \quad \pi^+ p \rightarrow h^\circ \Delta^{++} :$$

Theory h :	magic	$\sigma \approx 25 \ \mu b$
at 5 GeV/c	octet	$\approx 8 \ \mu b$
	singlet	$\approx 16 \ \mu b$

Controversial Expt $\sigma \approx 150 \ \mu b$ (4 GeV/c)
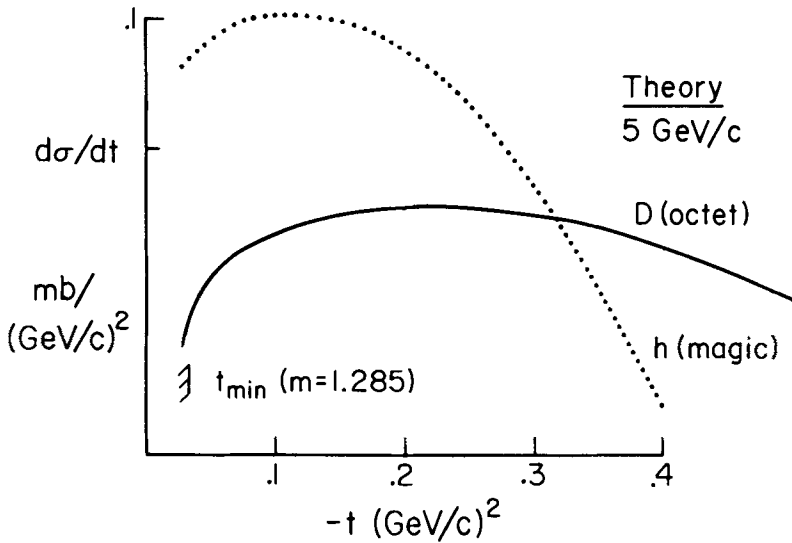
Fig. 16: h and D production at 5 GeV/c as discussed in Section 3.7(iii). t_{min} is marked for the $\pi^- p \rightarrow D^0 n$ reaction. D data is Refs. 56 &57. h data is Ref. 59.

suggest that the small t cross-section will be 2 to 3 times smaller than the overall quoted value. Within this large uncertainty, theory is consistent with experiment.

There is no convincing data on h production. There is a controversial claim [59] at 4 GeV/c, i.e., $\sigma(\pi^+ p \to h^0 \Delta^{++}) = 150$ μb which is greater than the "large" (in the sense of 3.1) reaction $\pi^+ p \to B^+ p$ with $\sigma = 100$ μb. I can only ignore this h cross-section which seems unreasonably large. Theoretically the difficulty with the h may simply be a large width ($M(h) \approx M(B)$ implies $\Gamma(h \to \pi\rho) = 330$ MeV – see Table II) and/or background from A_1^0 and A_2^0 which have the same decay. The h' – which if magically mixed would, like the ϕ and f', be produced only in $K^- p$ reactions (Fig. 11(I)) – may decay into $\overline{KK}^*$ and be narrower and cleaner than the h. It is important to look for this meson in those reactions which claim an f'; it could well have similar mass/decay modes/production mechanism and so be confused with the 2^+ f'.

(iv) $K^- n \to (A_1^-, B^-)\Lambda$
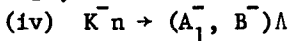
From the theory of 3.1, we expect these to be very favorable channels and fortunately this is borne out by experiment. The B^- is particularly satisfactory as it has no competing resonances in its $\pi^- \omega^0$ decay mode. The data [60,61] shown in Fig. 17 shows a clear B with large and amazingly flat cross-sections; even at 5 GeV/c only 1/3 of of the cross-section is within $0 \lesssim |t'| \leq .5$ (GeV/c)^2. For this cut we have an experimental [61] value of 8.5 μb compared with the theoretical estimate 9 μb. We have no test at 3 GeV/c because the t-dependence of the B^- cross-section [60] has yet to be published.

The A_1^- data [62], shown in Fig. 18, is much less clear – the presence of an A_2^- with the same decay produces too much background. Theoretically we expect a cross-section $K^- n \to A_1^- (\to \rho^0 \pi^-)\Lambda$ of 8 μb for the .5 t-cut. Experimentally the mass-cut $1.1 < M(\rho^0\pi^-) < 1.3$ GeV contains 14 μb on integration over all t.

3.8: ϕ_N (1680)

This was to have been a cosmic overview [63] on the production of yet higher mass mesons and baryons. However, being completely exhausted, I just note two pleasing examples, first, as described in the appendix, the best established of such resonances is the 3^- g which has an, as usual, large π-exchange cross-section. Now consider the ω-like member of this nonet which, somewhat curiously, is entitled the ϕ_N(1680). This is produced in $\pi^+ n \to \phi_N p$ by B and ρ exchange. We estimate the former by our super π-B corrected EXD trick as

$$\pi^+ n \to \phi_N p = (\pi^+ n \to \omega^0 p) \frac{\pi^+ n \to g^0 p}{\pi^+ n \to \rho^0 p} \qquad (24)$$

$K^-n \rightarrow \bar{B}\Lambda$

(I) 4.91 GeV/c

$K^-N \rightarrow \omega^o(783)\,\pi^-\Lambda$

(A)

EVENTS/0.1

-1.0   0.0   1.0

COS θ

EVENTS/0.050 GEV/C²

(B)

0.880   1.280   1.680   2.080

$M(\pi^-\pi^-\pi^+\pi^o)\,GEV/C^2$

Cross-sections

$|t'| < 0.5\,(GeV/c)^2 : \sigma = 8.5\,\mu b$
all t': $\sigma = 26 \pm 7\,\mu b$

(Mass-cut B:
corrected for $\omega \rightarrow 3\pi$
Branching ratio)

Mass-cut B: $1.13 \leq m(4\pi) \leq 1.33\,GeV$
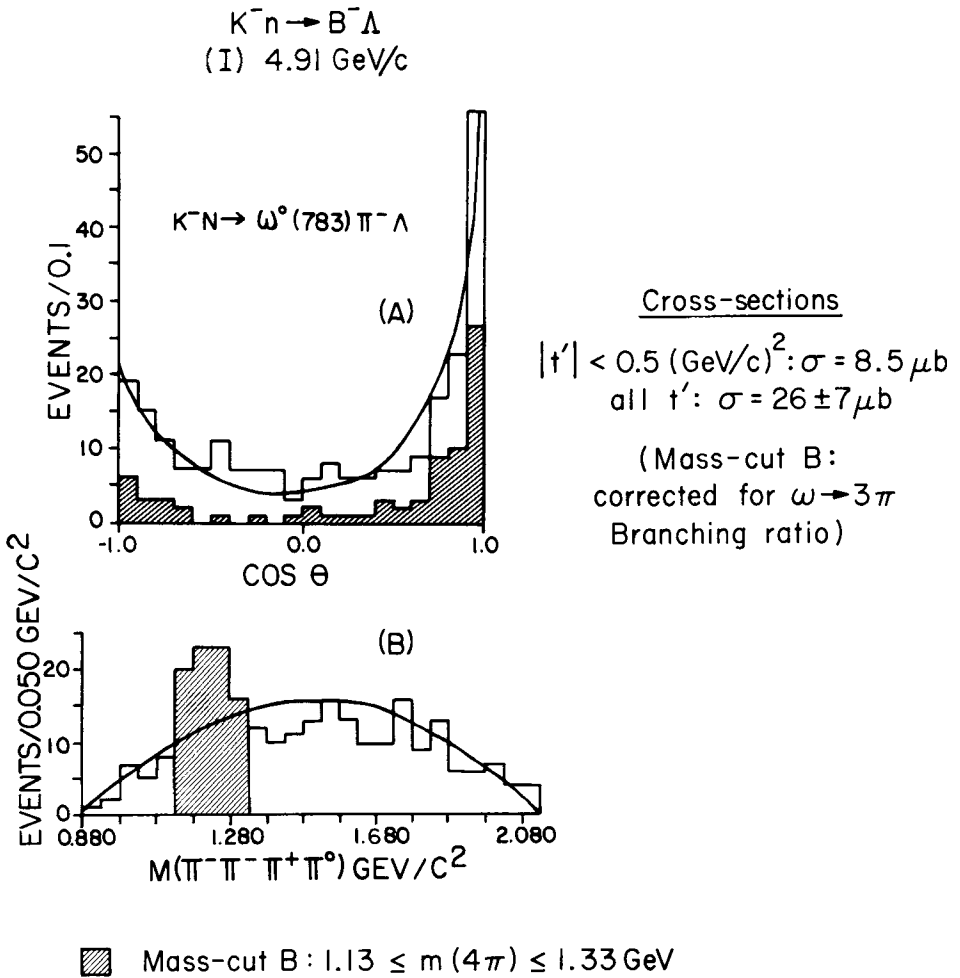
Fig. 17(I): $K^-n \rightarrow \bar{B}\Lambda$ at 4.91 GeV/c (Ref. 61). (A) cosΘ production;
(B) 4π mass distribution. <u>Both</u> plots select the ω
region: .74 $\leq$ m($\pi^+\pi^-\pi^o$) $\leq$ .84 GeV. This is discussed
in Section 3.7(iv).

$K^- n \rightarrow B^- \Lambda$

(II) 3 GeV/c
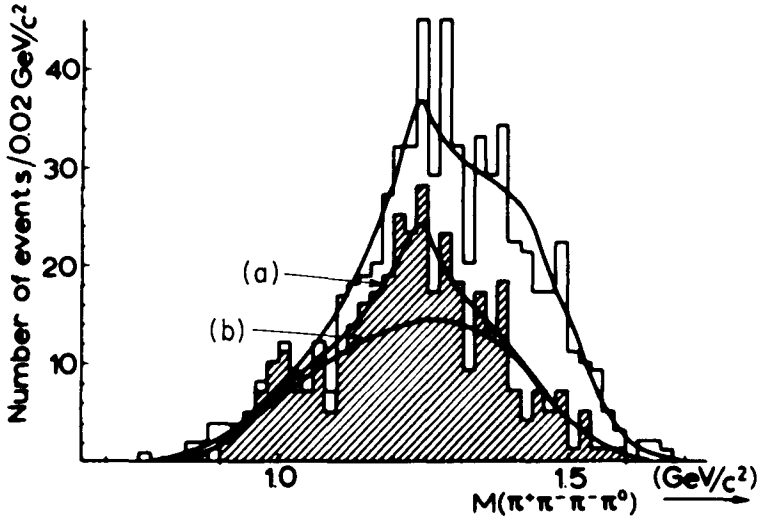
$\square$: total $K^- n \rightarrow \Lambda \pi^+ \pi^- \pi^- \pi^0$

$\boxtimes$: ω selection : $.72 \leq m(\pi^+ \pi^- \pi^0) \leq .86$ GeV

(a) Cosmic fit to $\boxtimes$ including B

(b) Non-B part of (a)

Cross-section : $\sigma(K^- n \rightarrow B^- \Lambda) = 94 \pm 25 \mu b$

Fig. 17(II): $K^- n \rightarrow B^- \Lambda$ at 3 GeV/c (Ref. 60). The curves are from minimum unlikelihood fits including many resonances and background. No information was published on the t-dependence of the cross-section. This is discussed in Section 3.7(iv).
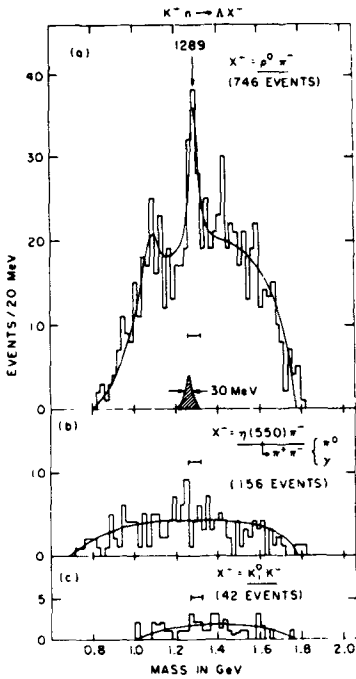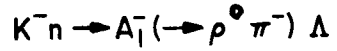
FIG. 2. (a) $\rho^0\pi^-$ mass distribution with events in $\Sigma(1385)^{\pm}$ region removed. A best fit to the data for two resonances and phase space is shown in the solid curve. Mass resolution in the "A_2" region is shown in the hatched area. (b) $\eta(550)\pi^-$ mass distribution with events in the $\Sigma(1385)^-$ region removed. The curve shows phase space. (c) $K^-K_1^0$ mass distribution. The curve again shows phase space.

Fig. 18: $K^-n \to A_1^-\Lambda$ at 3.9 GeV/c from Ref. 62. This is discussed in Section 3.7(iv).

(24) is easy to test because all four reactions were measured in the same experiment [64-67]. We should apply it to $d\sigma/dt$ separately for each t value. However, as g and ρ have the same (π-exchange) t-dependence, it is sufficient to note that the ϕ_N and ω are observed experimentally [64] to have the same shape for $d\sigma/dt$ (in agreement with (24) and check the overall normalization. This can be done using the production cross-sections in Table III plus the branching ratio [3]

$$\Gamma(g \to \pi^+\pi^-) = .4\ \Gamma_{tot}(g)$$

and a fifty percent increase in the g cross-section to take account of the non-zero t_{min} suppressing g relative to ρ production.

Table III: 6.95 GeV/c π^+n Cross-sections

Final State	Cross-section μb	Reference
$\phi_N(1680)$ $(\to \pi^+\pi^-\pi^0)p$	33.5 ± 9	64
g^0 $(\to \pi^+\pi^-)p$	39 ± 9	65
ω^0 $(\to$ all$)p$	86.4 ± 12.8	66
$\rho^0 p$	380 ± 80	67

(24) then predicts $\sigma[\pi^+n \to \phi_N(1680)p] = 33$ μb compared to the experimental value [64] for the $\pi^+\pi^-\pi^0$ decay mode of 33.5 ± 9 μb. As this is probably the dominant decay, theory and experiment are in very satisfactory agreement.

Actually we cheated somewhat as (24) should only be used for the B-part of the ω and ϕ_N cross-sections. However, we save such caveats for later [63].

4: Conclusions

These can be summarized as follows:
(i)   B nonet is alive and well; the SU_3 estimates are all consistent; the B itself is clearly seen in $K^-n \to B^-\Lambda$, $\pi^\pm p \to B^\pm p$.

(ii) A_1 nonet is alive; its parameters may change; the SU_3 estimates are again consistent for non-diffractive processes. The only unambiguous observation of a member of the A_1 nonet is in $\pi^-p \to Q^0_{A_1}\Lambda$. However, in this reaction, one could still assign the sighted Q to the B nonet.
Diffractive data prefers broad A_1?

(iii) **Cross-section's are suppressed**: the small non-flip ampli-
observed in these reactions is a challenge to theory. Kislinger's [10]
generalized photon-vector exchange analogy agrees with current experi-
ment.

(iv) **How do other higher resonance (meson/baryon) cross-sections
behave?** One could speculate that one may estimate <u>any</u> resonance pro-
duction as follows:
    a) π-exchange: use pole extrapolation - must work!
    b) B-exchange: "corrected EXD", e.g., equation (24) and sec-
        tion 2.4 - worked very well for $\phi_N(1680)$.

    c) V, T-exchange: calculate pole extrapolation (section 2.1)
        prediction and only take spin-flip part. Worked roughly
        for A_1, A_2 and B nonets.

These empirical rules should be useful in deciding if a proposed
experiment is of sufficient sensitivity to see a given resonance. A
rule for diffraction scattering (the only omission above) awaits
understanding of the lack of resonance structure in the current 3π
analysis [42].

(v) "$e^{i\delta}$ v. $e^{i\delta} \sin \delta$" Production (section 2.2): What are true
experimental systematics of resonance signal v. background in various
domains: Breit-Wigner fits are wrong in $e^{i\delta}$ processes, e.g., dif-
fraction scattering.

(vi) **Compile Data**: The discussion in section 3 was rendered
unnecessarily ambiguous by purely technical differences in resonance
definition, t-cuts, etc. It is essential, if such studies are ever to
be made quantitative, to record data in a form that a uniform treat-
ment of several different experiments may be made after the original
analysis is published. This is true for both bubble chamber data and
future spectrometer data.

(vii) **Cross-sections good for spectrometers**: Any spectrometer/
streamer chamber/triggered bubble chamber which can detect three or
more particle decays ($K^*\pi$, $\pi\rho$, $\omega\pi$, $\eta\pi$...) should be able to collect
high statistics on, say, the 1^+ mesons. The predicted cross-sections
are recorded in Figs. 9-12. Values for other higher resonances can
be estimated as in (iv) - they should be of comparable size but the
background will, of course, be larger.
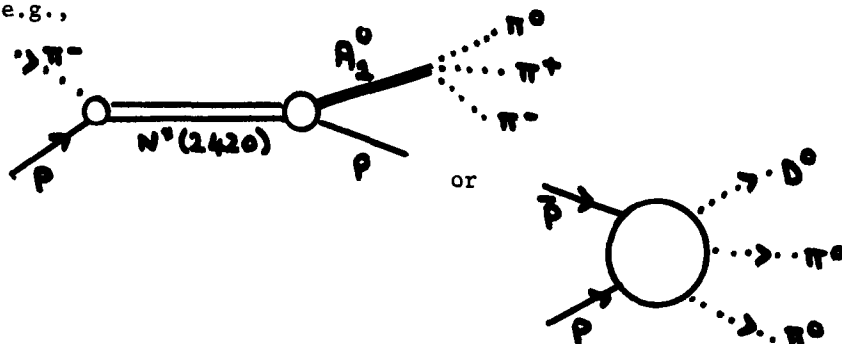
Acknowledgments

APPENDIX: Classification of Production Mechanism

Meson resonances can be produced in many different kinematic or dynamic regions. Specifically:

(A) Direct Channel
e.g.,

The $\bar{p}p$ annihilation data is, at present, perhaps the most convincing evidence [3] for the D and C mesons and hence of the whole A_1 nonet. However, the low incident momentum (giving the desired high cross-section) produces large background rendering it difficult both to see broad resonances and/or disentangle more than one state at a given mass by a detailed partial wave analysis. Further, I don't know of any useful conclusions as to the production mechanisms [68] so...

(B) High-energy Peripheral Production
e.g.,
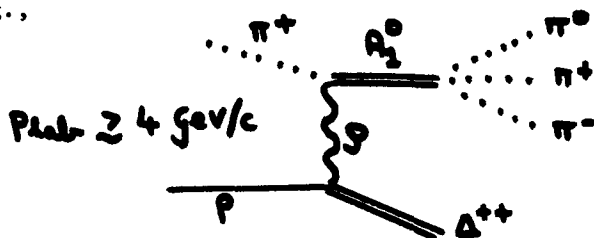
This type of reaction has the following advantages:

a) Resonances are cleanly produced with a background that (usually) decreases as energy increases.

b) Even if a resonance is not obvious in the mass spectrum one can do a partial analysis (of the $\pi^+\pi^-\pi^0$ system in the above diagram) to isolate the resonance.

c) The many new spectrometers should produce plenty of data of this type in the near future.

d) One can study the production mechanism (ρ exchange in the above diagram) using the (Regge) folklore gleaned from the study of similar exchanges in simpler reactions (e.g., $\pi^-p \to \pi^0n$). Thus, we can divide (B) into........

(B_1) Production of Well-loved Particles

e.g.,

or
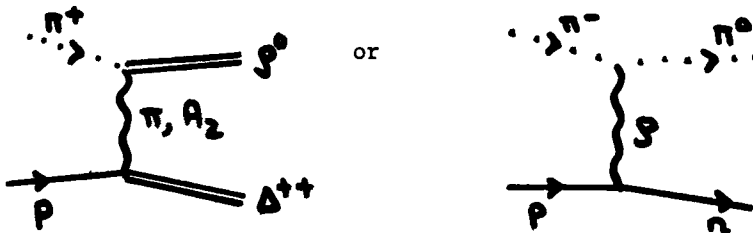
In these reactions the properties of produced states (mass, spin, parity...) are relatively well-known and $d\sigma/dt$, d.m.e. (density matrix element) data is very important for understanding dynamics of production mechanisms but not for studying the resonances themselves.

(B_2) Production of Controversial Particles

e.g.,
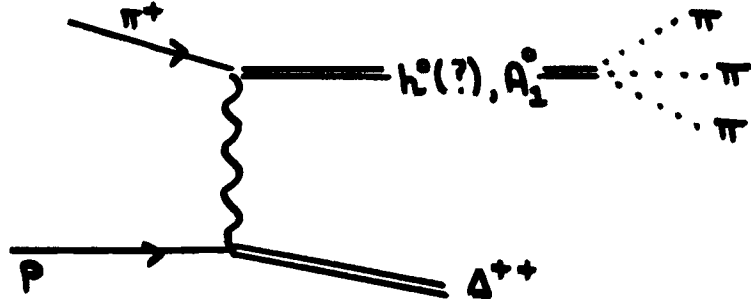
In this review, I use the lessons from studying the production mechanisms of (B_1) to systematize and predict the, at present, inconsistent and haphazardly measured cross-sections for the production of controversial particles. When I started this review, I thought that the main virtue of this study would be:

a)   Theoretical estimates of the size and nature of production mechanism can either suggest good reactions to study a particular resonance or decide whether a given claim was plausible by relating different observed cross-sections.

However, it had two further virtues.

b)   The small size of the production cross-sections of the 1^+ nonets provides an important constraint on theories.

c)   The same small size explains why it has been so hard to discover the 1^+ mesons in peripheral processes and shows that the current data provides no evidence against the existence of these states – so beloved of the quark model. As we shall see in section 3, there is, of course, little evidence <u>for</u> the same resonances. However, it is nice that the best hints for their production are in precisely those reactions the theory predicts to be most favorable, i.e., the "large"

(see 3.1) exchanges $\left[\pi^+ p \rightarrow B^+ p, \; \pi^- p \rightarrow Q^0 \Lambda, \; K^- n \rightarrow (A_1, B)\Lambda \right]$.

Finally we cannot only divide reactions according to the status of the produced resonance but also according to the nature of the exchanges.

E_π: π-exchange
_____
e.g.,
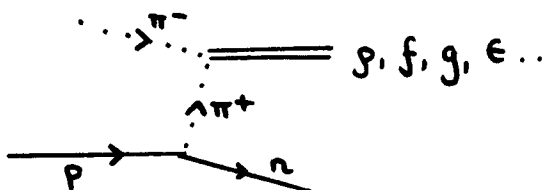
π-exchange processes have the following two characteristics:

a)   The sizes of cross-sections are both easy to estimate from the residue at the nearby π-pole and typically large in absolute value. For instance, at 5 GeV/c, we have $\sigma(\pi^- p \rightarrow \rho^0 n, \; f^0 n, \; \varepsilon^0 n) \approx 1$, .5 and .2 mb respectively.

b)   Again as demonstrated, both experimentally and theoretically in section 2.2, π-exchange processes are <u>certain</u> to exhibit, for small t, clean "$e^{i\delta} \sin \delta$" resonance production with the minimal background problems.

The high cross-section and clean production makes it quite obvious why the best established high mass bosons (i.e., the g(1680) and $K_N(1760)$ [3, 71] of spin 3^-) are seen in π-exchange processes (see section 3.8).

g)   The dynamics of the production mechanism is very interesting but I have reviewed it quite recently. The main features - good agreement at small -t with the zero parameter poor man's absorption model, P_{lab}^{-2} scaling at large -t - cannot be re-evaluated until the beautiful data from the new spectrometers on $\pi^- p \rightarrow \pi^+ \pi^- n$ has been analyzed [72].

E_{ND} Non-diffractive Meson (not π) exchange.
_____

These processes are the subject of this review. Typically they are described in the Regge language by the exchange of ρ, K^*..., A_2, K^{**}..., B, Q_B... nonets of vector, tensor and axial mesons. Compared with E_π processes

a)   Their cross-sections are typically 10 → 100 times smaller (i.e., from 5 μb for $K^- p \rightarrow \bar{Q}^0 n$ to 80 μb for the largest [40] ($\pi^\pm p \rightarrow A^\pm p$) at 5 GeV/c).

b)   It is not known whether in the terminology of section 2.2, they have clean $e^{i\delta} \sin \delta$ or the difficult $e^{i\delta}$ production.

c)   The small cross-sections implies that essentially no useful data is available for such processes except for the very first excited states. So as described in the introduction, we can only discuss the

L = 1 quark states. (See Table I for details.) Let us remember the
poor knowledge of π-exchange resonances in the higher quark multi-
plets where the conditions of observation are perhaps a factor of 10
better). We realize that one cannot seriously have expected current
data to tell us any reliable information on either violation of the
quark model systematics or confirmation of the $L \geq 2$ states, for non
π-exchange resonances. An interesting exception to this rule –
the B exchange $\phi_N(1680)$ – is discussed in section 3.8.

E_D Diffraction Production
_____

e.g.,
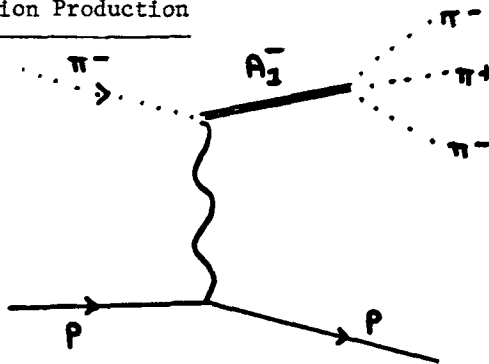
I will only discuss such processes to the extent that my final
state interaction fantasy, of 2.2, predicts such reactions will have
more "background" than non-diffractive resonance production experi-
ments.

E_B Baryon-Exchange
_____

e.g.,
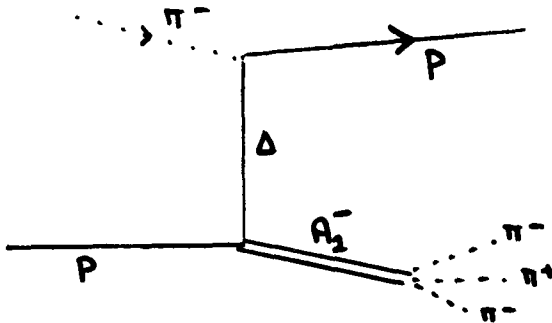
        This is an interesting set of reactions but our very poor under-
standing of baryon exchange even in the "simple" reactions πN → Nπ, [73]
precludes any non-trivial prediction. Recent work by Alex Firestone
on the K^+ world DST suggests comparable cross-sections for backward
$K^+p \to pK^+$ and $K^+p \to pQ^+$ of around 5 μb. Combining this with the sup-
pression of the forward non-diffractive Q production leads to the
curious circumstance of comparable cross-sections for forward and
backward charge exchange Q reactions. Of course, this lack of sup-
pression of the backward Q cross-section is expected in Kislinger's

model [10] where the photon analogy only constrains the forward data.

Thus there is both theoretical and experimental [73,74] evidence that backward scattering will be a fruitful field for studying the higher meson resonances.

REFERENCES AND FOOTNOTES

1.   This parable had a sad ending.......On his return, fellow
scientists greeted our hero with the primeval cry. "Good to see
you - glad to know you - have you been away?" Breathlessly our
hero told of his terrible journey but of its beautiful results.
        Before he could finish, their voices rose. "Hero! Just
the man - you can teach our children to dig the sand. You're
just the man - and build the castles high." Then, as one, the
children rushed upon our hero, trampling his precious load which,
crumpled and useless, was carried away in tears of sadness and
waves of neglect.......I can only hope this article will per-
suade people that a detailed search for small cross-section res-
onances is likely to be rewarding; otherwise the A_1, B, h...(and
who knows whatever higher particles?) will lie buried forever in
the sands of disillusionment.

2. J. L. Rosner, review talks at Caltech and Argonne (UCRL-20655
(1971) and ANL/HEP-7208 (1972) respectively) discusses the cur-
rent status of meson resonances.

3. Review of Particle Properties, Particle Data Group, Phys. Letters
39B, 1 (1972).

4. I will not discuss further, possible mixing between the Q_B and
and Q_{A_1} states in this paper (see Ref. 8). As experimentally,
the situation is so uncertain that even the number of Q's is un-
certain, such theoretical speculation appears to be icing a rot-
ten cake.

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(1971) classify the D' as an unmixed SU_3 singlet partner of the
A_1, and identify it with the M(953), reported in M. Aguilar-
Benitez et al., Phys. Rev. Letters 25, 1635 (1970).

6. J. Mandula, J. Weyers and G. Zweig, Ann. Rev. Nucl. Sci. 20,
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7. See, for instance, R. P. Feynman, M. Kislinger and F. Ravndal,
Phys. Rev. D3, 2706 (1971). Naively the B nonet is an orbital
excitation of the π nonet and so unmixed. The A_1 being an L = 1
excitation of the ρ should be a magic mixed nonet. However, the
duality arguments of Ref. 6 give the opposite result.

8. E. W. Colglazier and J. L. Rosner, Nucl. Phys. B27, 349 (1971).
See also E. W. Colglazier, Caltech thesis (1971, unpublished).

9. I would have called this fantasy a theory, but the latter is
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ment.

10. M. Kislinger, "Vector, Tensor and Pomeron Regge Couplings", Cal-
tech preprint CALT-68-341 (1971, unpublished).

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body Strange Particle Final States in π^-p Interactions at 4.5
and 6 GeV/c", Brookhaven preprint (1972).

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14. One can't predict sub-energy phase exactly because it is impos-
sible to vary one sub-energy and fix all the others. However,

current theory would predict that the latter phases vary much less rapidly than a resonant sub-energy phase. Hence, we derive the rule stated in the text.

15. Of course, there is a resonance pole on the "second sheet" which will lead to some peaking when $\delta = 90$...

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18. D. R. O. Morrison, Rapporteur's talk at the Kiev Conference (CERN D. Ph. II/PHYS 71-10) has a nice summary of diffractive data and emphasizes the difference between resonance production in diffractive and non-diffractive reactions.

19. B-exchange reactions are related by EXD to π-exchange. Thus, they are $e^{i\delta}$ Sin δ.

20. The difficulty with extrapolation reported in section 3 does not affect this conclusion. In the model of Kislinger the resonance production amplitude in the physical region is still proportional to the pole amplitude; there is just an extra factor of t which spoils formula (1) but not the $e^{i\delta}$ Sin δ nature of the production.

21. "f-f' dominance of the Pomeron" would relate the diffractive processes to an exchange diagram. This would then imply $e^{i\delta}$ Sin δ production for diffractive reactions. However, (see Gilman, this conference) this theory has other discrepancies with experiment and it is not yet clear this is a serious difficulty.

22. The argument is not quite complete as, for instance, the $\Delta(1234)$ is below threshold for $\rho N \rightarrow \pi N$. It is sufficient to use the (qualitative) quark model connection between $\rho N \rightarrow \pi N$ and $\pi N \rightarrow \pi N$ to decide that $\Delta(1234)$ is indeed "$e^{i\delta}$ Sin δ" in $\rho N \rightarrow \pi N$.

23. E. D. Bloom and F. J. Gilman, Phys. Rev. D4, 2901 (1971).

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33. K. J. Foley et al., Brookhaven preprint BNL-16714 (1972); G. Grayer et al., Phys. Letters 34B, 333 (1971).

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35. C. Michael and P. V. Ruuskanen, Phys. Letters 35B, 65 (1971).
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    $\pi^+ p \to B^+ p$).
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    (8 GeV/c, $\pi^+ p \to B^+ p$).
39. A. Shapiro, COO-1195-159, Illinois thesis (1969), "The $\pi^- p \omega$
    Channel in $\pi^- p$ Interactions at 5 GeV/c". G. Ascoli et al.,
    COO-1195-188 Illinois preprint (1970), "The Reaction $\pi^- p \to p\pi^- \omega^0$
    at 5 and 7.5 GeV/c". This $\pi^- p \to B^- p$ data has a similar t-depen-
    dence to $\pi^+ p \to B^+ p$ but is un-normalized.
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    Phys. Rev. 163, 1377 (1967). For $\pi^- p \to (D \to K^\pm K^0 \pi^\pm)n$, this refer-
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$$\sigma = 7 \pm 2 \ \mu b \quad : \quad P_{lab} = 2.5 \rightarrow 2.63$$
$$10 \pm 4 \ \mu b \quad : \quad P_{lab} = 2.9 \rightarrow 3.3$$
$$17 \pm 5 \ \mu b \quad : \quad P_{lab} = 3.8 \rightarrow 4.2$$

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