Lecture 1: Duality in Hadron Dynamics

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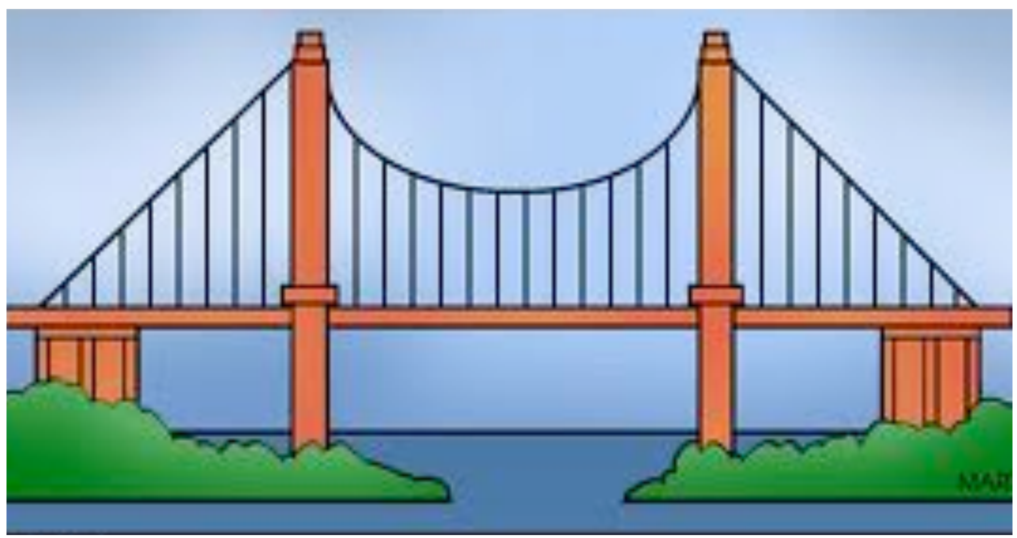
Hadrons as QCD Bound States

Data

Duality

Models



$\mathcal{L}_{QCD}$

Λ_{QCD}

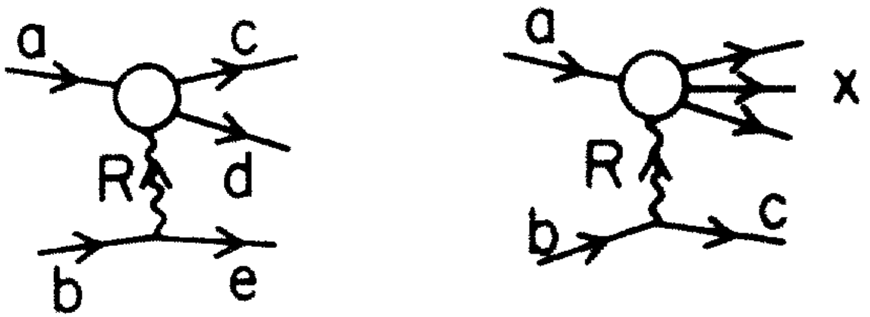
Lattice

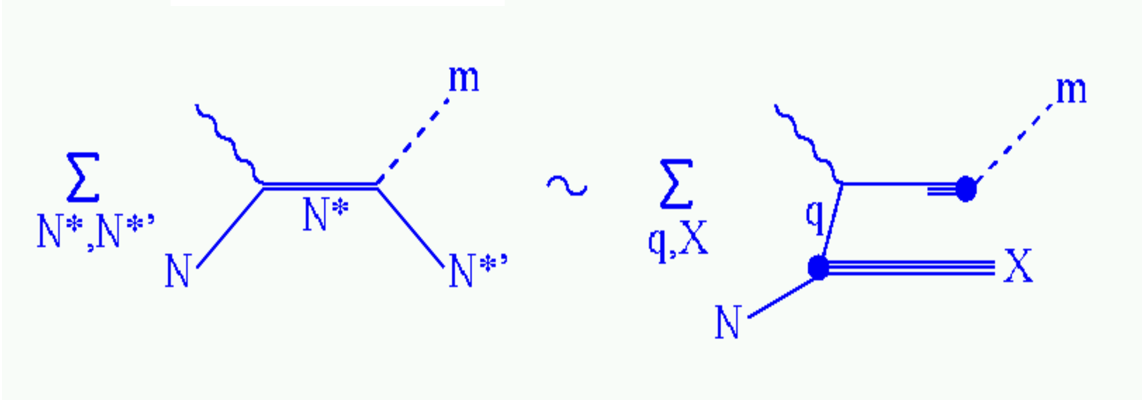
PQCD

A general principle of hadron interactions

Duality is observed in both soft and hard processes

- Hadron scattering: $\pi N \to \pi N, \ldots$
- Reggeon-hadron scattering
- $e^+e^- \to$ hadrons
- DIS $eN \to e\,X$
- Semi-inclusive $eN \to e\,h\,X$
- ...

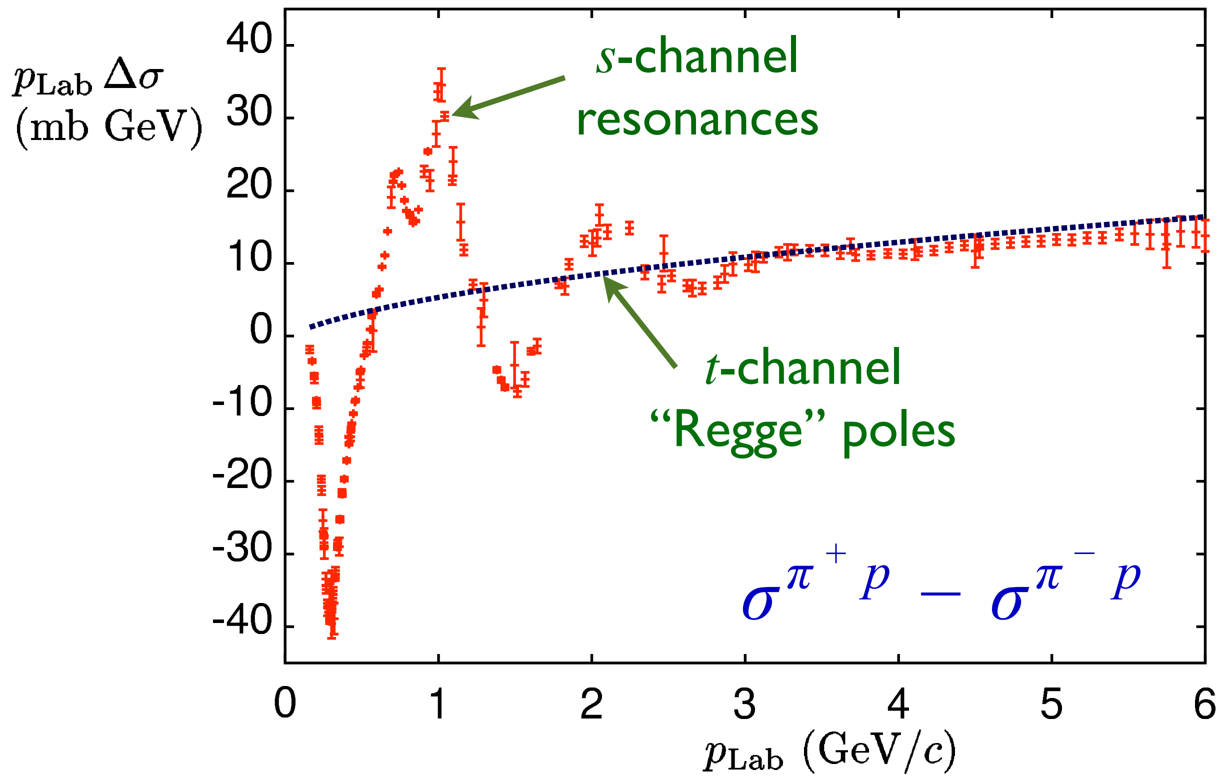




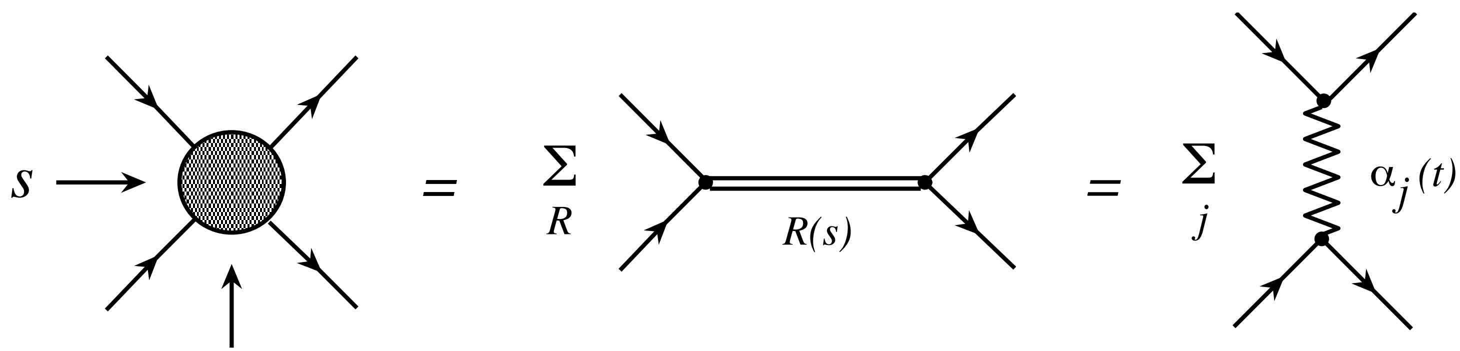
Duality implies that hadron and parton descriptions are equivalent.

It is a guideline in developing our understanding of hadron dynamics and of relativistic bound states.

Duality in hadron-hadron scattering



Lectures by D. Horn,...



"finite energy sum rules"

W. Melnitchouk (2010)
https://www.jlab.org/conferences/HiX2010/program.html

Igi (1962), Dolen, Horn, Schmidt (1968)

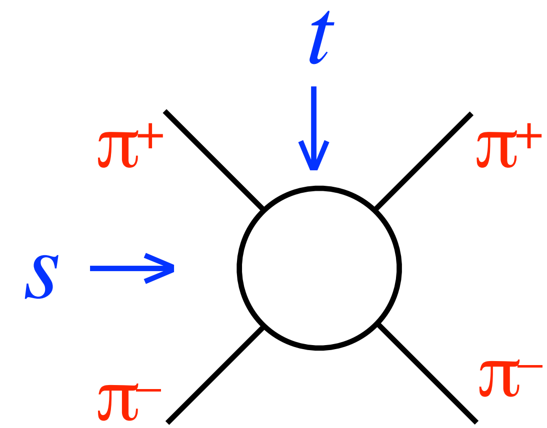
Analytic example: Dual amplitudes

Lectures by Szczepaniak

In 1968, Veneziano found a simple analytic function with many of the properties required for scattering amplitudes, including duality.
Lovelace applied this idea to the $\pi^+\pi^- \to \pi^+\pi^-$ scattering amplitude

$$A(\pi^+\pi^- \to \pi^+\pi^-) = \frac{\Gamma(1-\alpha_s)\,\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}$$

$$\alpha_s \equiv \alpha(s) = \tfrac{1}{2} + s \qquad (\alpha' \equiv 1)$$



The amplitude has poles at $\alpha = 1, 2, \ldots$: the $\rho, \omega, f, \ldots$ resonances.
The residues are polynomials of degree $\alpha = n$ in $\cos\theta = 1+2t/s$ $\quad (m_\pi=0)$

Thus the pole at $\alpha_s = n$ is a superposition of bound states with $J = 1, \ldots, n$

$$\lim_{s\to\infty} A(s,t) = \Gamma(1-\alpha_t)e^{-i\pi\alpha_t}s^{\alpha_t} \qquad \text{Regge behavior}$$

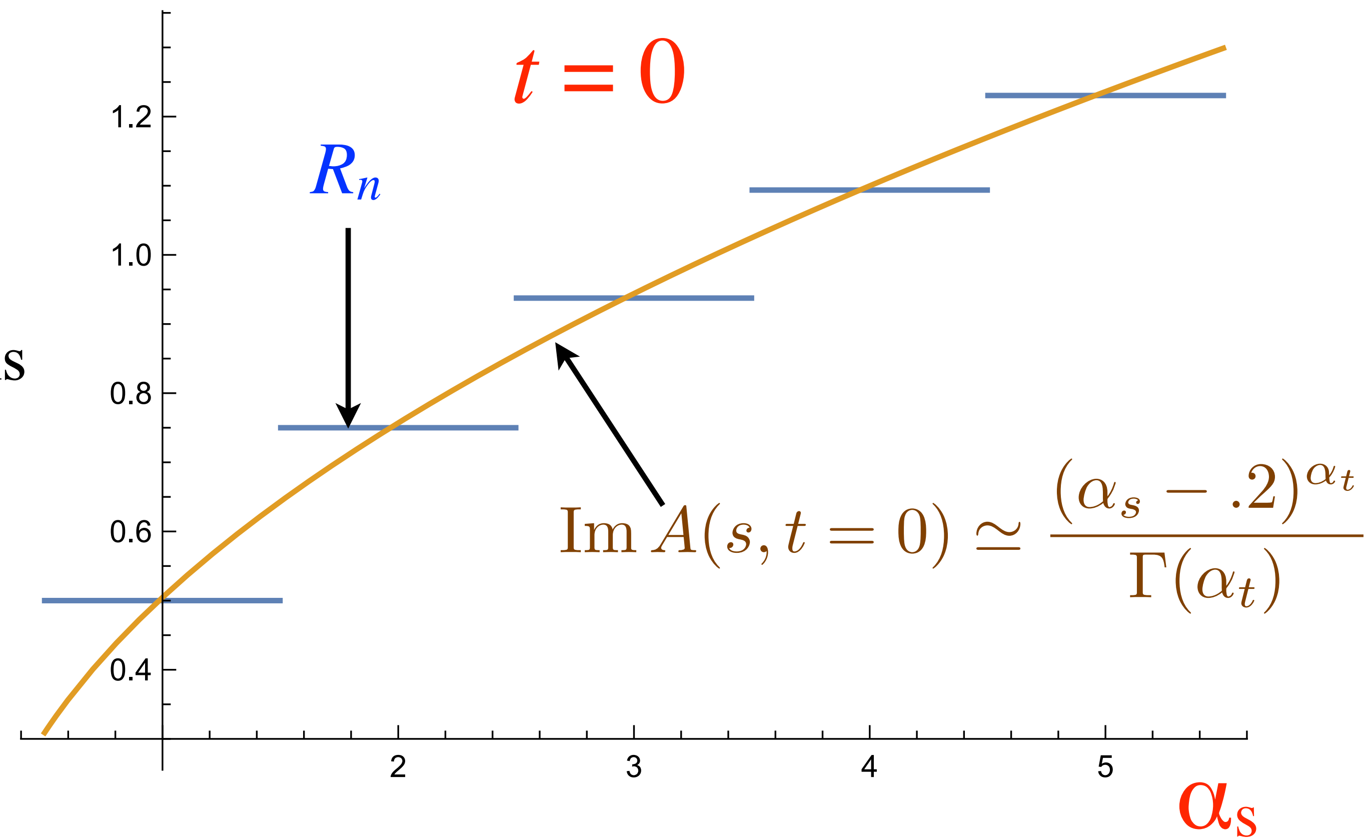
G. Veneziano, Nuovo Cim. **57A** (1968) 190
C. Lovelace, Phys. Lett. **28B** (1968) 264

The π+π- → π+π- dual amplitude A(s,t)

$$A(s,t) = \frac{R_n(\alpha_t)}{\alpha_s - n} + \ldots$$

Resonances vs Regge
in forward scattering

Resonance contributions
smeared over $\alpha_s \pm 0.5$

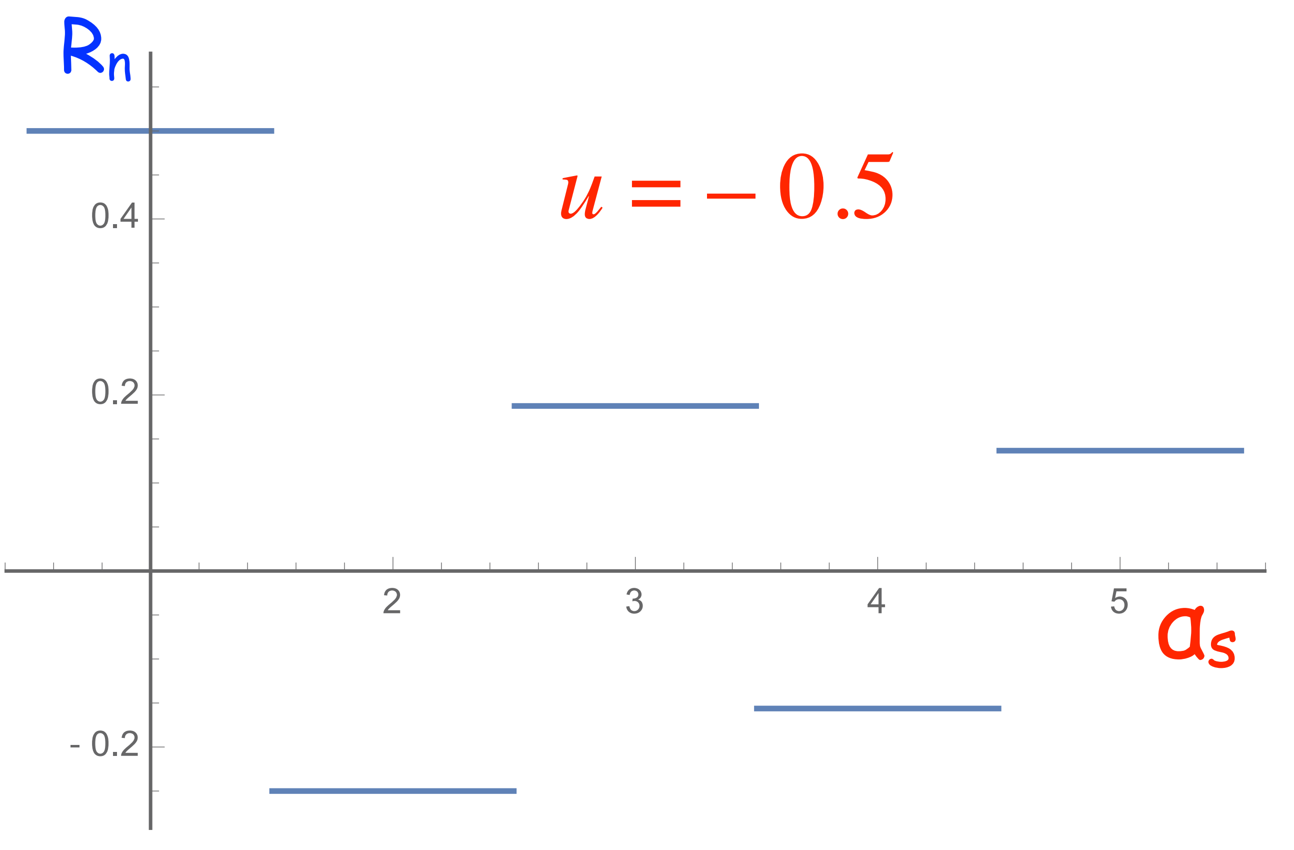


Backward scattering in π+π- → π+π-

The asymptotic dual amplitude is real in the backward (fixed *u*) direction.

An exchanged particle would have to be doubly charged (exotic).

Resonance contributions average to zero by alternating in sign ("superconvergence relation")

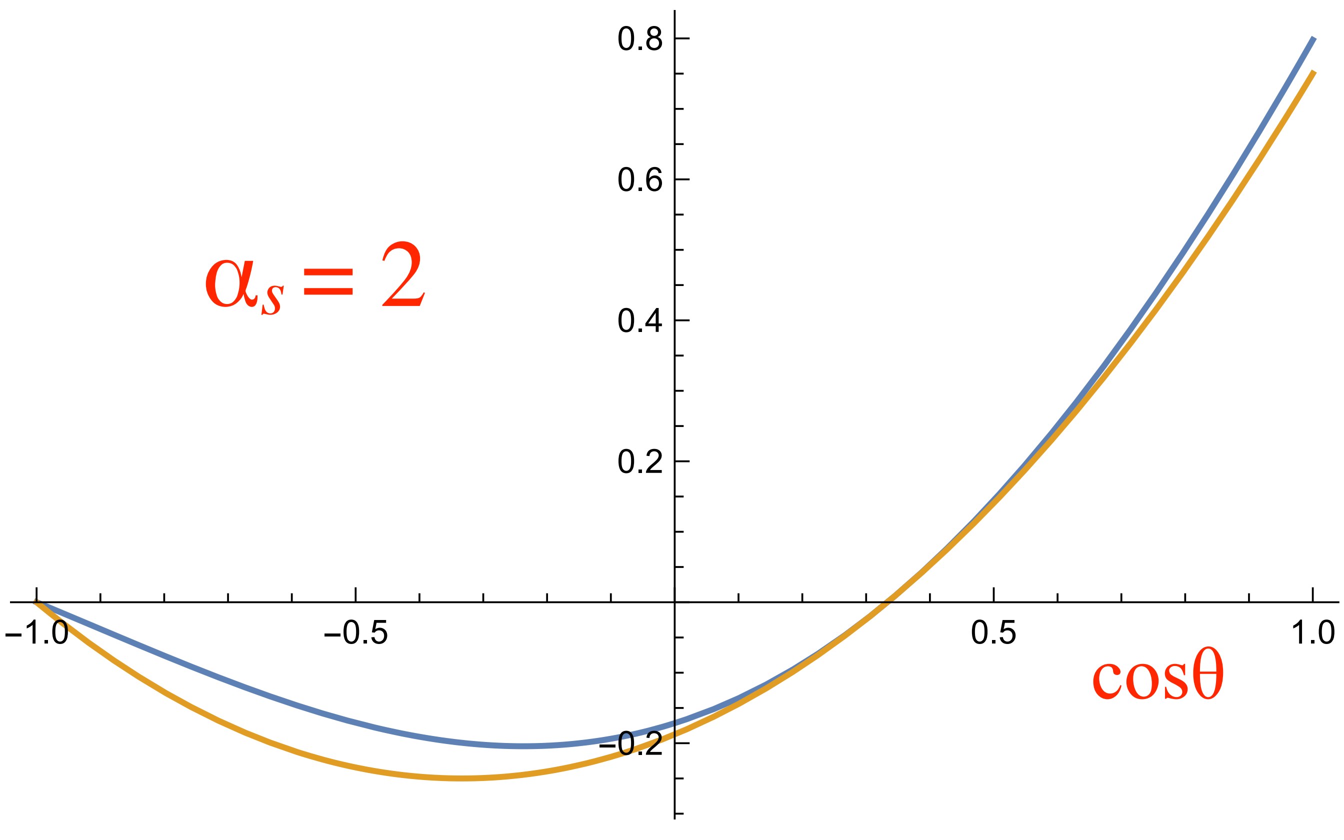


Angular distribution in π+π- → π+π- : α_s=2

$$A(s,t) = \frac{R_n(\alpha_t)}{\alpha_s - n} + \ldots$$

Regge: $\dfrac{\alpha_s^{\alpha_t}}{\Gamma(\alpha_t)}$

Residue: $R_n(\alpha_t)$



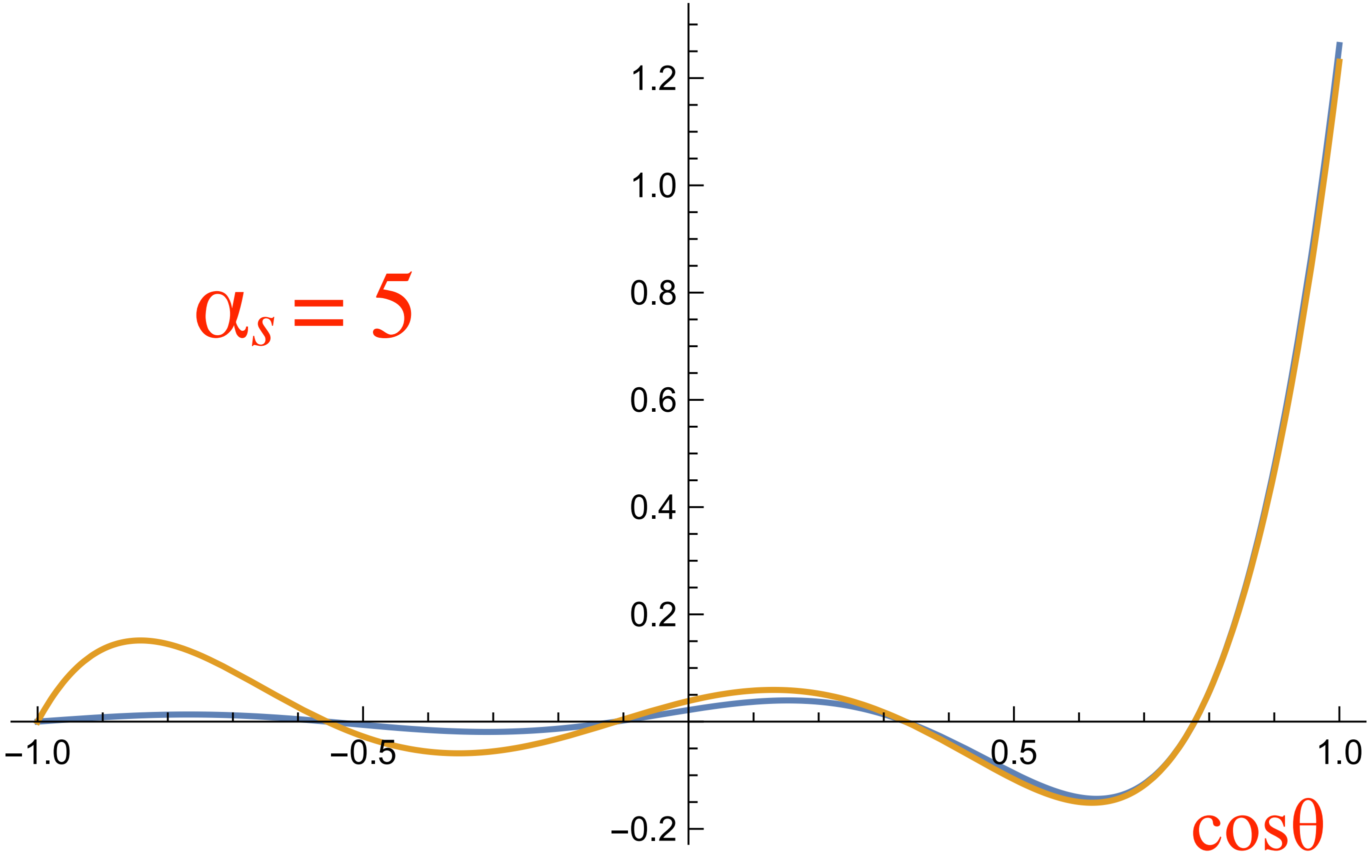
Angular distribution in π+π- → π+π- : α_s=5

$$A(s,t) = \frac{R_n(\alpha_t)}{\alpha_s - n} + \ldots$$

Regge: $\dfrac{\alpha_s^{\alpha_t}}{\Gamma(\alpha_t)}$

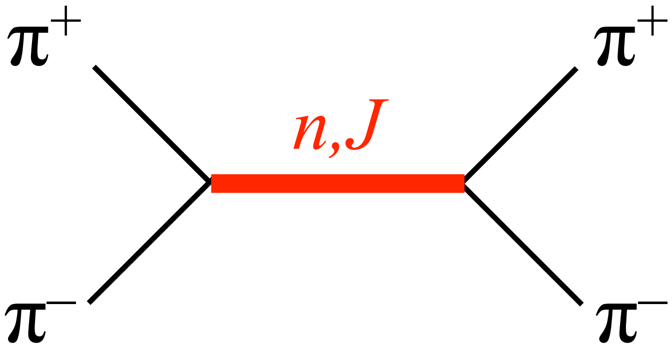
Residue: $R_n(\alpha_t)$

$\alpha_s = 5$



cosθ

Positivity in π+π- → R_n^J → π+π- : n,J=1,...,20



$$\propto \left(g_{\pi\pi}^{nJ}\right)^2 P_J(\cos\theta)$$

$g^2 > 0$: Positive coefficients!

By chance(?), all the 230 coefficients for $n,J \leq 20$ are in fact positive.

$n=1$ … $n=20$: $\left(g_{\pi\pi}^{nJ}\right)^2$

{0.25, 0.25}
{0, 0.38, 0.38}
{0.078, 0.078, 0.39, 0.39}
{0.019, 0.19, 0.17, 0.36, 0.36}
{0.053, 0.073, 0.26, 0.24, 0.31, 0.31}
{0.021, 0.14, 0.13, 0.29, 0.27, 0.25, 0.25}
{0.041, 0.068, 0.20, 0.19, 0.29, 0.28, 0.20, 0.20}
{0.021, 0.11, 0.12, 0.24, 0.23, 0.28, 0.27, 0.15, 0.15}
{0.035, 0.063, 0.17, 0.16, 0.26, 0.25, 0.25, 0.24, 0.12, 0.12}
{0.020, 0.096, 0.11, 0.21, 0.20, 0.27, 0.25, 0.22, 0.21, 0.089, 0.089}
{0.030, 0.059, 0.15, 0.15, 0.24, 0.22, 0.26, 0.25, 0.18, 0.18, 0.067, 0.067}
{0.019, 0.085, 0.098, 0.19, 0.18, 0.25, 0.24, 0.24, 0.23, 0.15, 0.15, 0.050, 0.050}
{0.027, 0.056, 0.13, 0.13, 0.22, 0.21, 0.25, 0.24, 0.22, 0.21, 0.12, 0.12, 0.037, 0.037}
{0.018, 0.077, 0.091, 0.17, 0.17, 0.24, 0.22, 0.25, 0.24, 0.19, 0.19, 0.10, 0.099, 0.027, 0.027}
{0.025, 0.053, 0.12, 0.12, 0.20, 0.19, 0.24, 0.23, 0.23, 0.22, 0.16, 0.16, 0.079, 0.079, 0.019, 0.019}
{0.017, 0.071, 0.086, 0.16, 0.15, 0.22, 0.21, 0.24, 0.23, 0.21, 0.21, 0.14, 0.14, 0.062, 0.062, 0.014, 0.014}
{0.023, 0.050, 0.11, 0.12, 0.19, 0.18, 0.23, 0.22, 0.24, 0.23, 0.19, 0.18, 0.12, 0.11, 0.048, 0.048, 0.010, 0.010}
{0.016, 0.065, 0.082, 0.15, 0.15, 0.21, 0.20, 0.24, 0.23, 0.22, 0.21, 0.17, 0.16, 0.096, 0.094, 0.037, 0.037, 0.0073, 0.0073}
{0.021, 0.048, 0.10, 0.11, 0.18, 0.17, 0.22, 0.21, 0.24, 0.23, 0.21, 0.20, 0.15, 0.14, 0.078, 0.077, 0.028, 0.028, 0.0053, 0.0053}
{0.016, 0.061, 0.078, 0.14, 0.14, 0.20, 0.19, 0.23, 0.22, 0.23, 0.22, 0.19, 0.18, 0.12, 0.12, 0.063, 0.062, 0.022, 0.021, 0.0038, 0.0038}

$J = 0, \ldots, n$

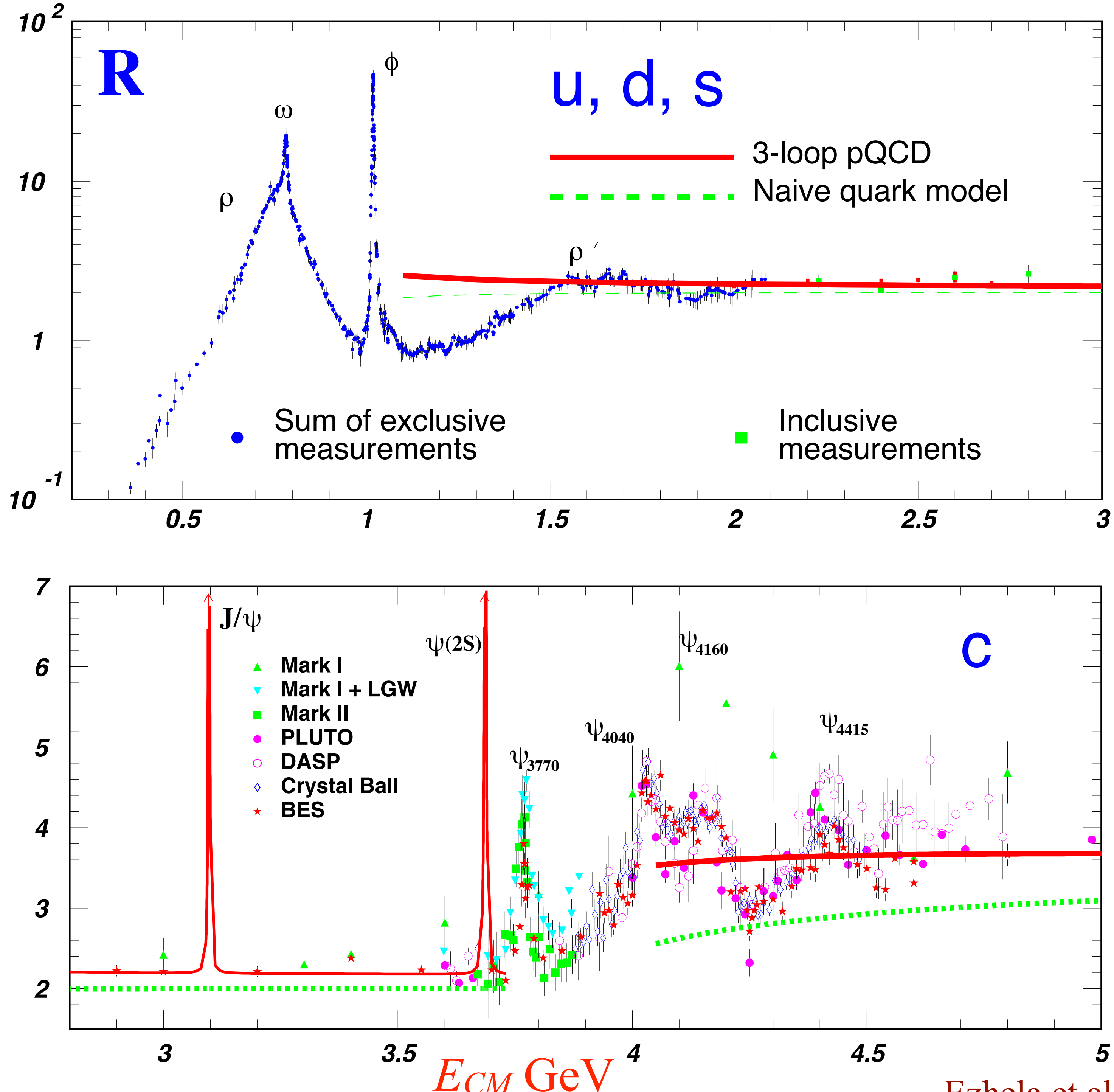
Exercise 1.1

Verify the positivity of the coefficients of $P_\ell(\cos\theta)$ in the residues of the poles at $\alpha_s = 1$ and $\alpha_s = 2$ of the dual amplitude

$$A(\pi^+\pi^- \to \pi^+\pi^-) = \frac{\Gamma(1-\alpha_s)\,\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}$$

Use $\alpha_s \equiv \alpha(s) = \frac{1}{2} + s$ and set $m_\pi = 0$.

Duality in e+e- → hadrons



Ezhela et al, hep-ph/0312114

Exercise 1.2

The J/ψ resonance contribution to $\sigma(e^+e^- \longrightarrow \text{hadrons})$, averaged over some interval ΔE in the CM energy, is by duality expected to equal the perturbative quark cross section $\sigma(e^+e^- \longrightarrow q\ qbar)$. Give an estimate of ΔE. Consider whether your result agrees roughly with what is expected.

The expression for the J/ψ contribution is

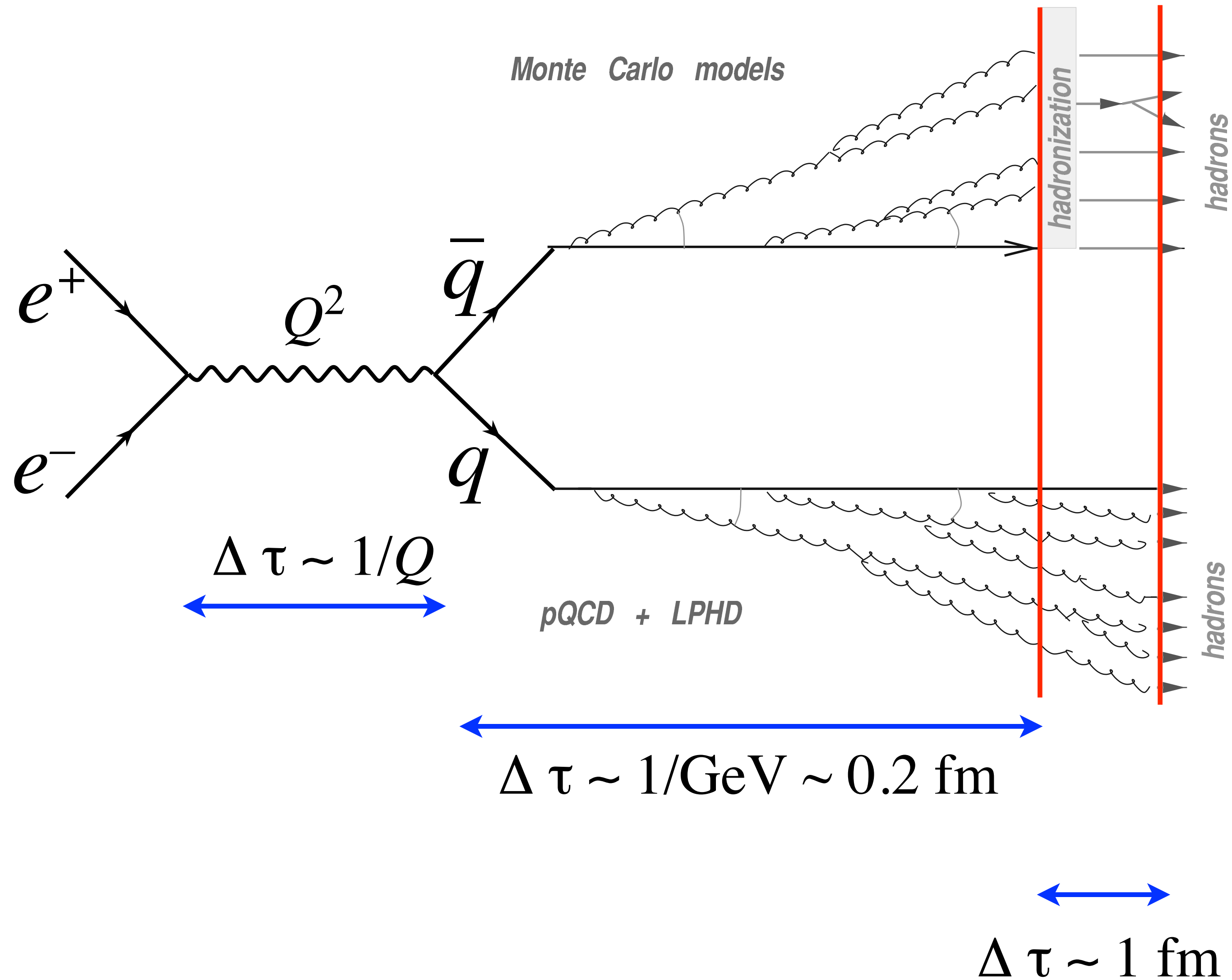
$$\int \sigma(e^+e^- \to J/\psi \to hadrons)\, dE_{CM} = \frac{6\pi^2\Gamma_e\Gamma_h}{M_{J/\psi}^2\Gamma_{tot}}$$

where Γ_e , Γ_h and Γ_{tot} are the J/ψ decay widths into e^+e^-, hadrons and the total width, respectively, are given by the PDG. For reference,

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{CM}^2}$$

Time evolution in e+e- → of hadrons

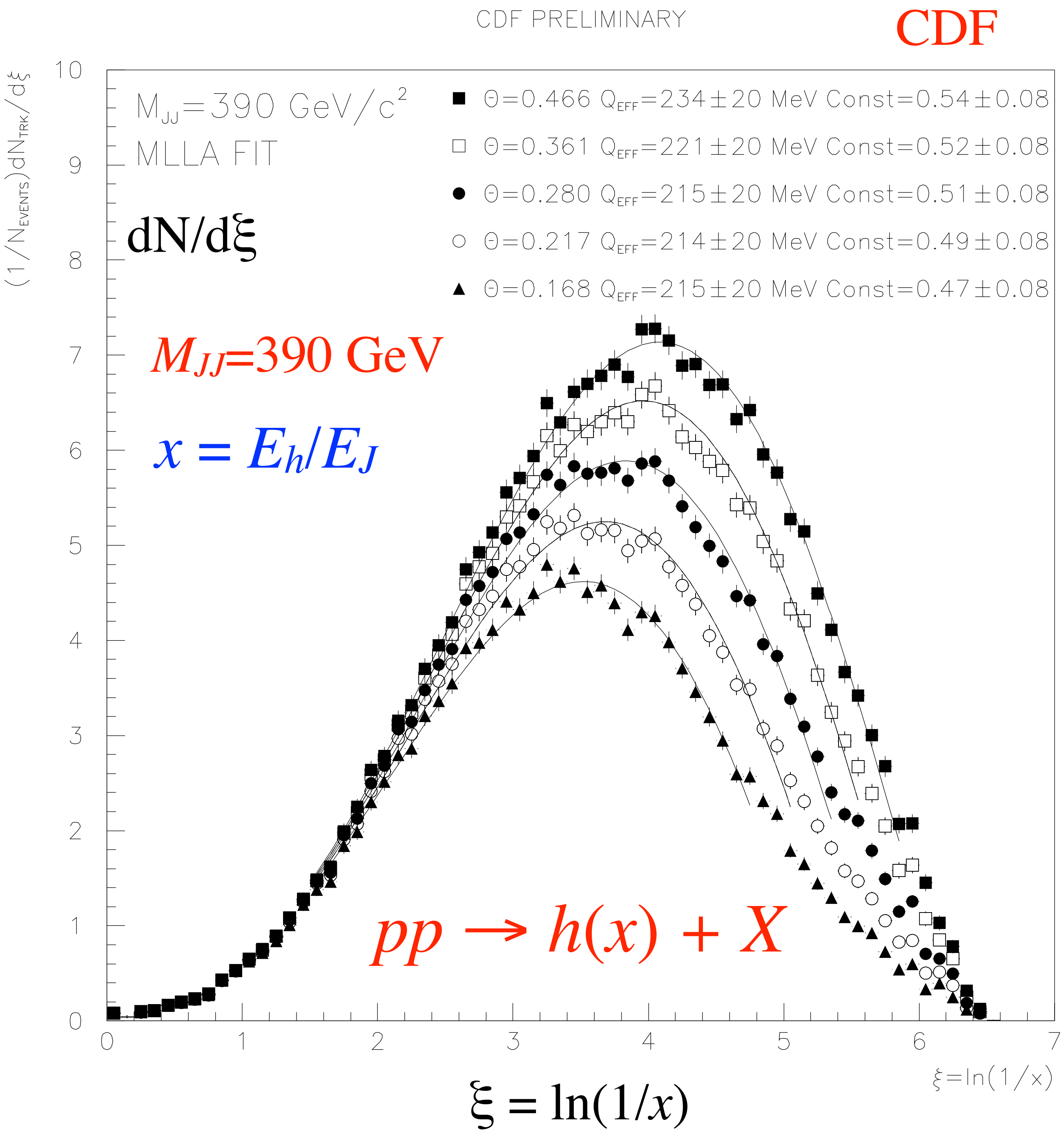
Final state evolves in (proper) time τ with decreasing virtuality and decreasing energy uncertainty ΔE



Evolution is unitary:
Measured cross section in energy interval $E_{CM} \pm \Delta E$ must average to (parton) cross section at $\tau \sim 1/\Delta E$

Local duality tells us that parton picture and PQCD are valid down to Q ~ 1 GeV

Hump-backed plateau



First confronted with theory in $e^+e^- \to h+X$.

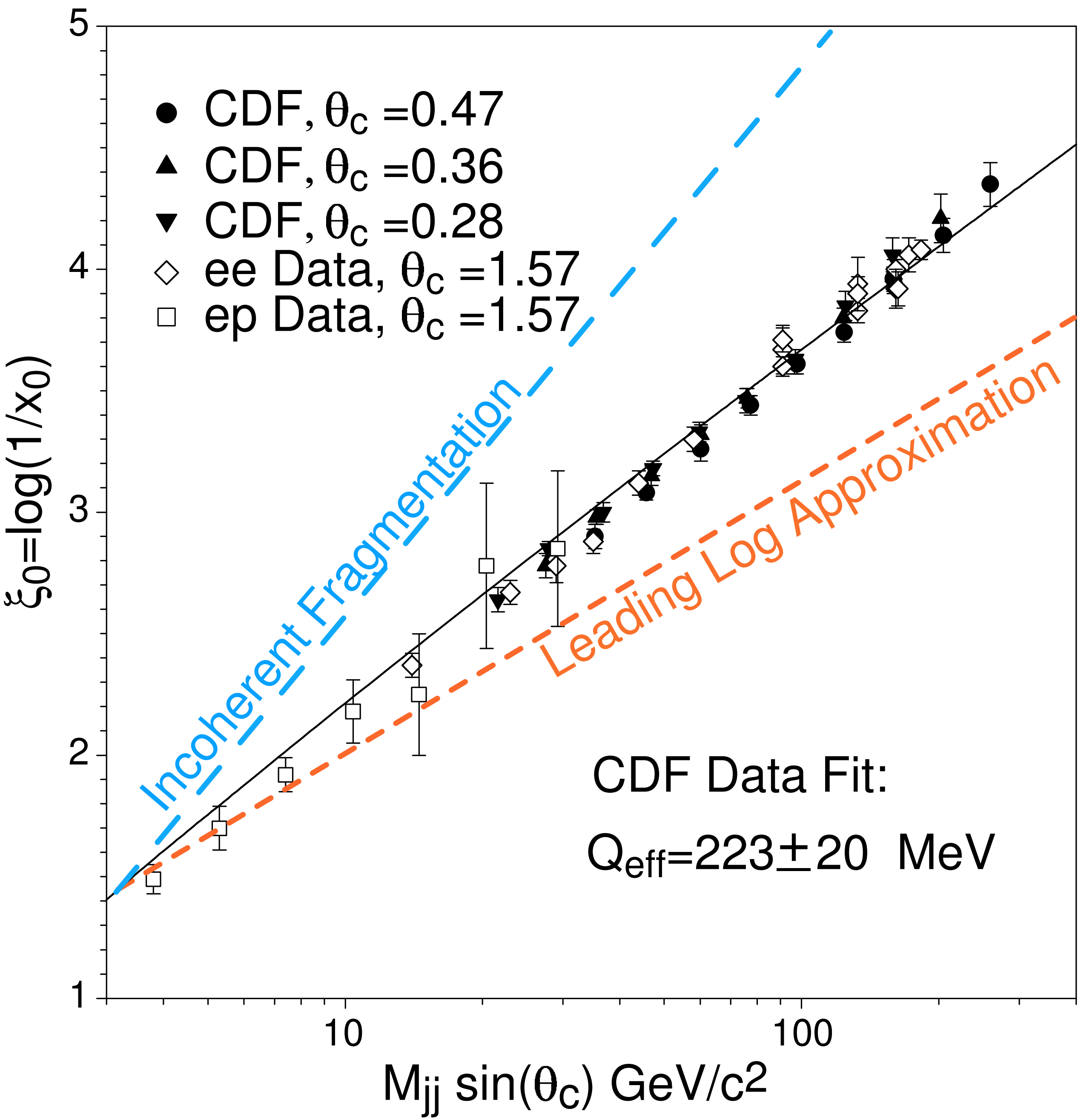
CDF (Tevatron)

$pp \to 2$ jets

Charged hadron yield as a function of $\ln(1/x)$ for different values of jet hardness, versus (MLLA) QCD prediction.

One free parameter – overall normalization (the number of final π's per extra gluon)

Hump (continued)



Position of the Hump as a function of

$Q = M_{jj} \sin \Theta_c$

(hardness of the jet)

is the parameter-free QCD prediction.

Yet another calculable – CIS – quantity.

Mark Universality:

same behaviour seen in e^+e^-, DIS (ep), hadron–hadron coll.

Soft Confinement

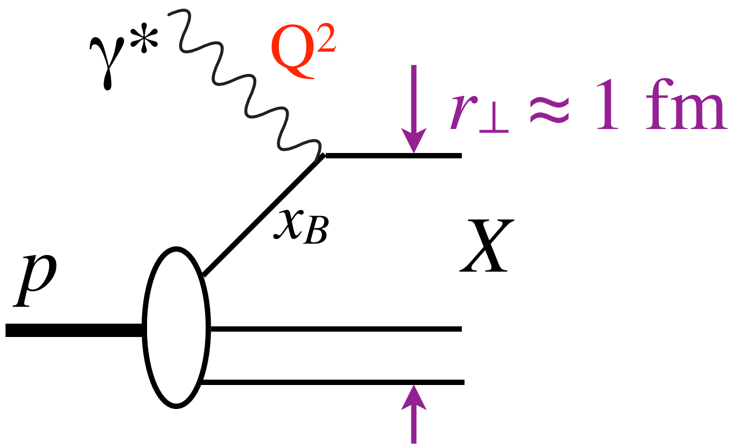
So, the *ratios* of particle flows between jets (intERjet radiophysics), as well as the *shape* of the inclusive energy spectra of secondary particles (intRAjet cascades) turn out to be formally calculable (CIS) quantities. Moreover, these perturbative QCD predictions actually work.
The strange thing is, these phenomena reveal themselves at present-day experiments via *hadrons* (pions) with *extremely small momenta* $k_\perp$, where we were expecting to hit the *non-perturbative domain* — large coupling $\alpha_s(k_\perp)$ — and potential failure of the quark–gluon language as such.

The fact that the underlying physics of colour is being impressed upon "junky" pions with 100–300 MeV momenta, could not be *a priori* expected. At the same time, it sends us a powerful message: confinement – transformation of quarks and gluons into hadrons – has a *non-violent* nature: there is no visible reshuffling of energy–momentum at the hadronization stage. Known under the name of the *Local Parton-Hadron Duality hypothesis* (LPHD), explaining this phenomenon remains *a challenge* for the future quantitative theory of colour confinement.

Inclusive vs Exclusive Hard Lepton Scattering

Inclusive DIS

$$\frac{d\sigma(ep \to eX)}{dQ^2 dx_B} \propto \sum_q f_q(x_B) \frac{e_q^2}{Q^4}$$



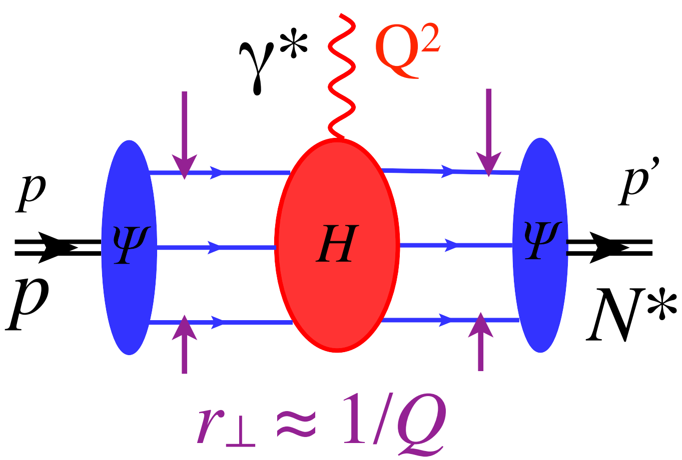
$f_q(x_B)$: Prob. to find q with $p_q = x_B\, p_N$

e_q^2 : Incoherent scattering on each q

$1/Q^4$: Dimensional analysis: Scaling

Exclusive $p \to N^*$

$$\frac{d\sigma(ep \to eN^*)}{dQ^2} \propto \Big[\sum_q e_q F^q_{p\to N^*}(Q^2)\Big]^2 \frac{1}{Q^4}$$



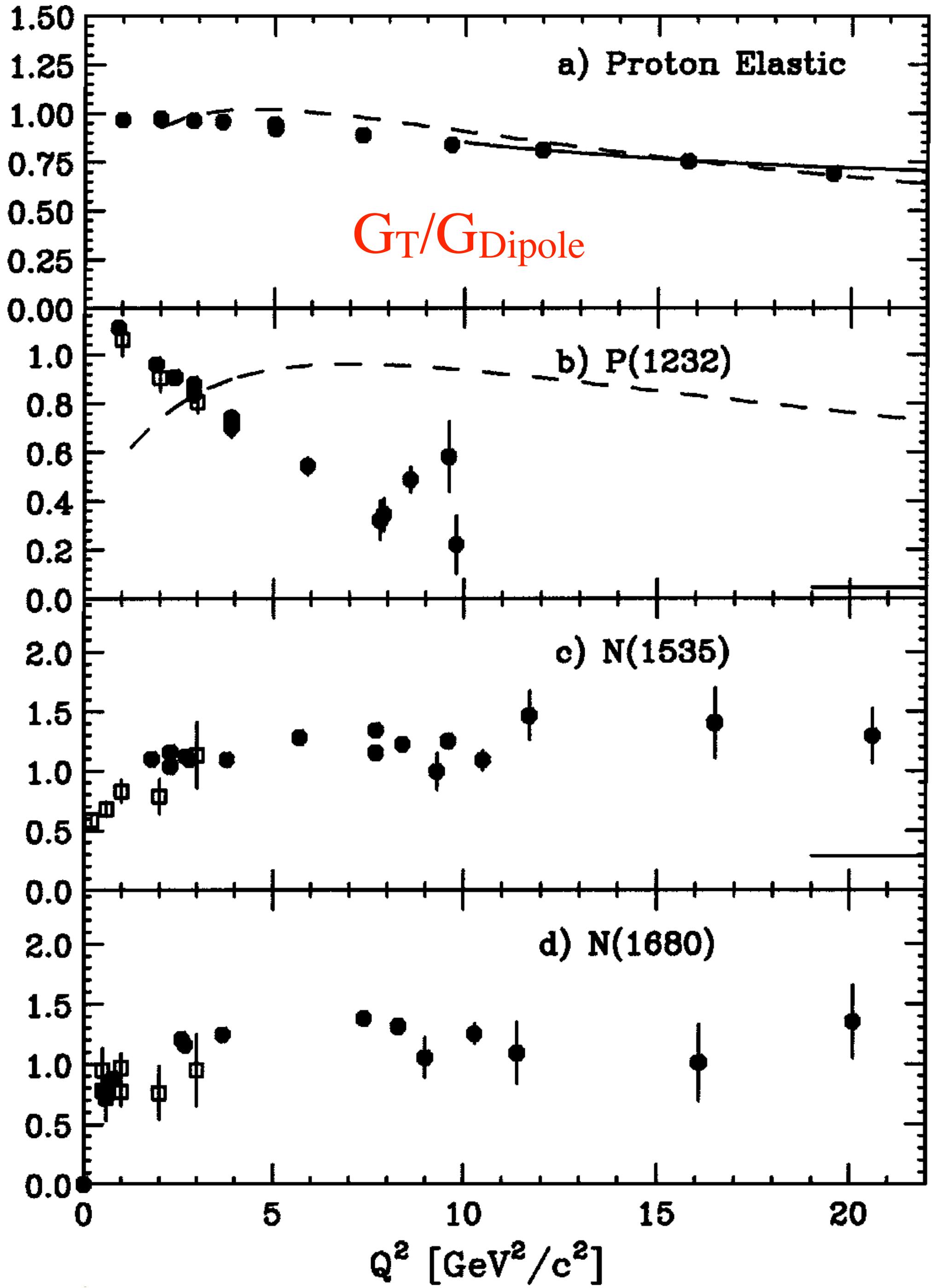
$F^q_{p\to N^*}(Q^2)$: $p \to N^*$ form factor for quark q
= 1 for pointlike p and N^*

$\sum_q [e_q \;\cdots]^2$ γ^* scatters *coherently* on quarks

$1/Q^4$: Dimension for poinlike hadrons

Q^2-dependence of form factors: Data

G. Sterman and P. Stoler, Annu. Rev. Nucl. Part Sci. **47** (1997) 193



Data on $p \to N^*$ FF's, scaled by Dipole FF:

$$G_{Dipole} \propto \left(\frac{1}{1+Q^2/0.71\ \mathrm{GeV}^2}\right)^2$$

To the extent that $F^q_{p\to N^*}(Q^2) \propto \dfrac{1}{Q^4}$

we get:

$$\frac{d\sigma(ep\to eN^*)}{dQ^2} \propto \left[\sum_q e_q F^q_{p\to N^*}(Q^2)\right]^2 \frac{1}{Q^4}$$

$$\propto \frac{1}{Q^{12}}$$, whereas for DIS:

$$\frac{d\sigma(ep\to eX)}{dQ^2 dx_B} \propto \sum_q f_q(x_B)\frac{e_q^2}{Q^4} \propto \frac{1}{Q^4}$$

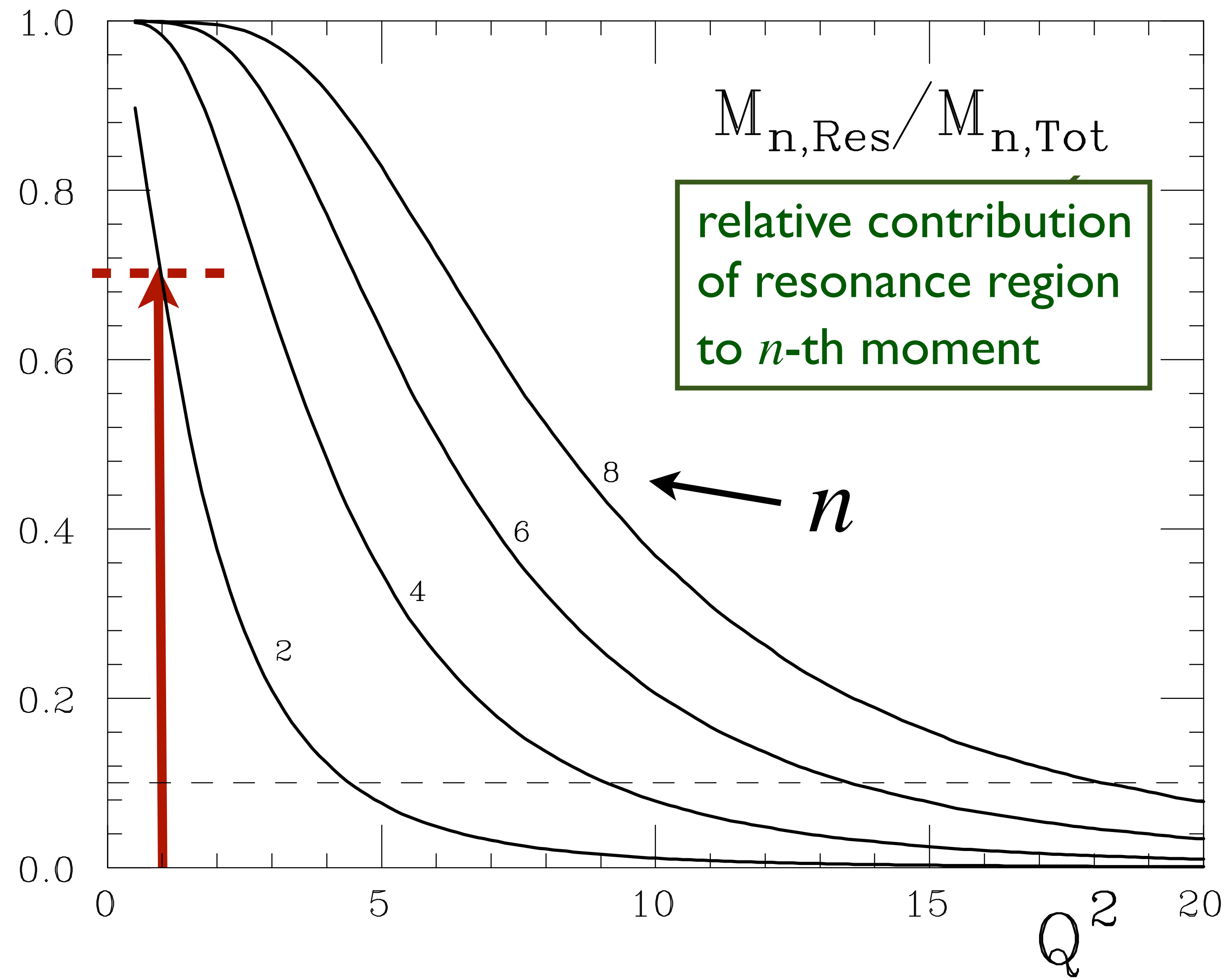
Dependence on the quark charges e_q

	DIS: $\sum_q e_q^2$	FF: $\sum_q e_q$
proton	$2\cdot\frac{4}{9}+\frac{1}{9}=1$	$2\cdot\frac{2}{3}-\frac{1}{3}=1$
neutron	$\frac{4}{9}+2\cdot\frac{1}{9}=\frac{2}{3}$	$\frac{2}{3}-2\cdot\frac{1}{3}=0$

Local duality between resonances (FF) and structure functions (DIS) cannot hold for both the proton and the neutron?

Resonances & higher twists

At $Q^2 = 1$ GeV^2, ~ 70% of lowest moment of $F_2(ep \to eX)$ comes from $W < 2$ GeV

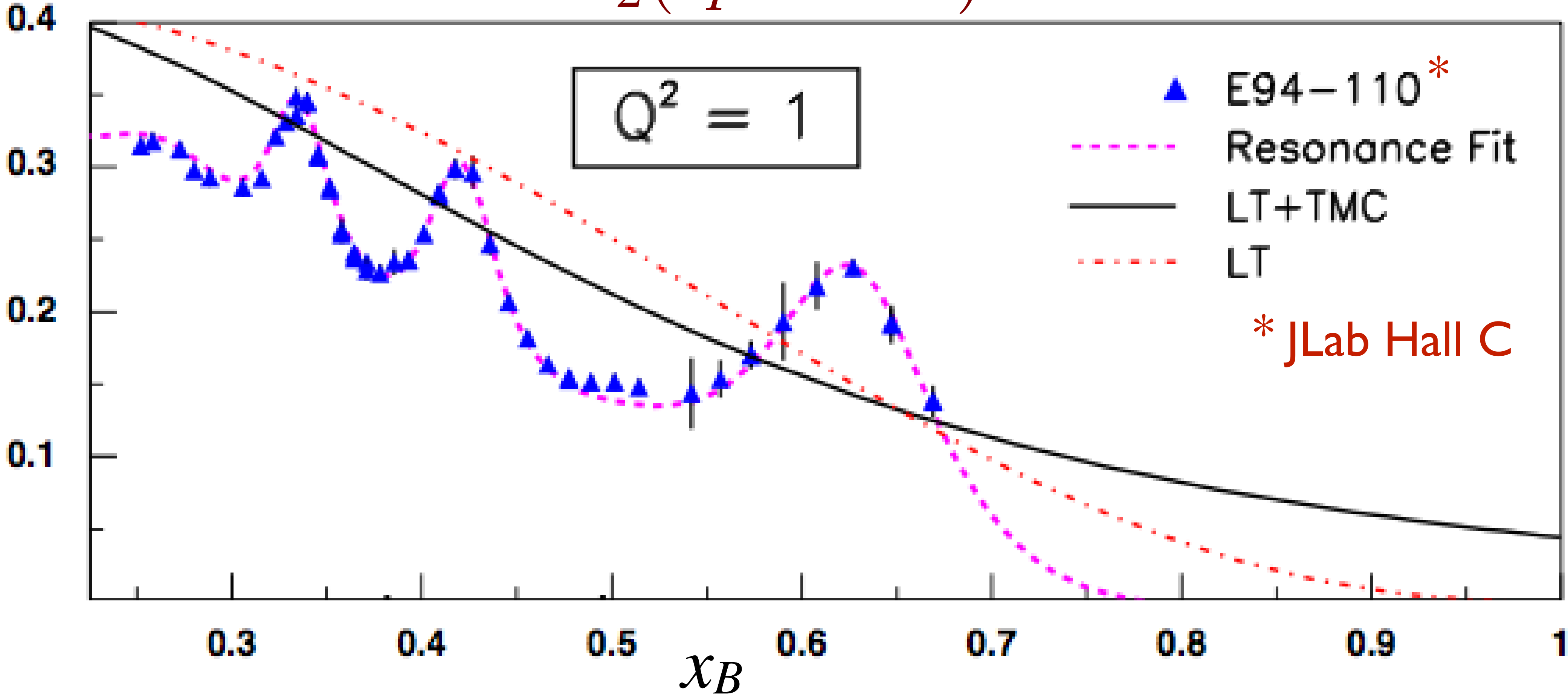


$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x, Q^2)$$

W. Melnitchouk (2010)

Bloom-Gilman Duality

$$F_2(ep \to eX)$$



Resonances average scaling (large Q^2) curve. This holds at all $Q^2 \geqq 1$ GeV^2

TMC = Target Mass Correction

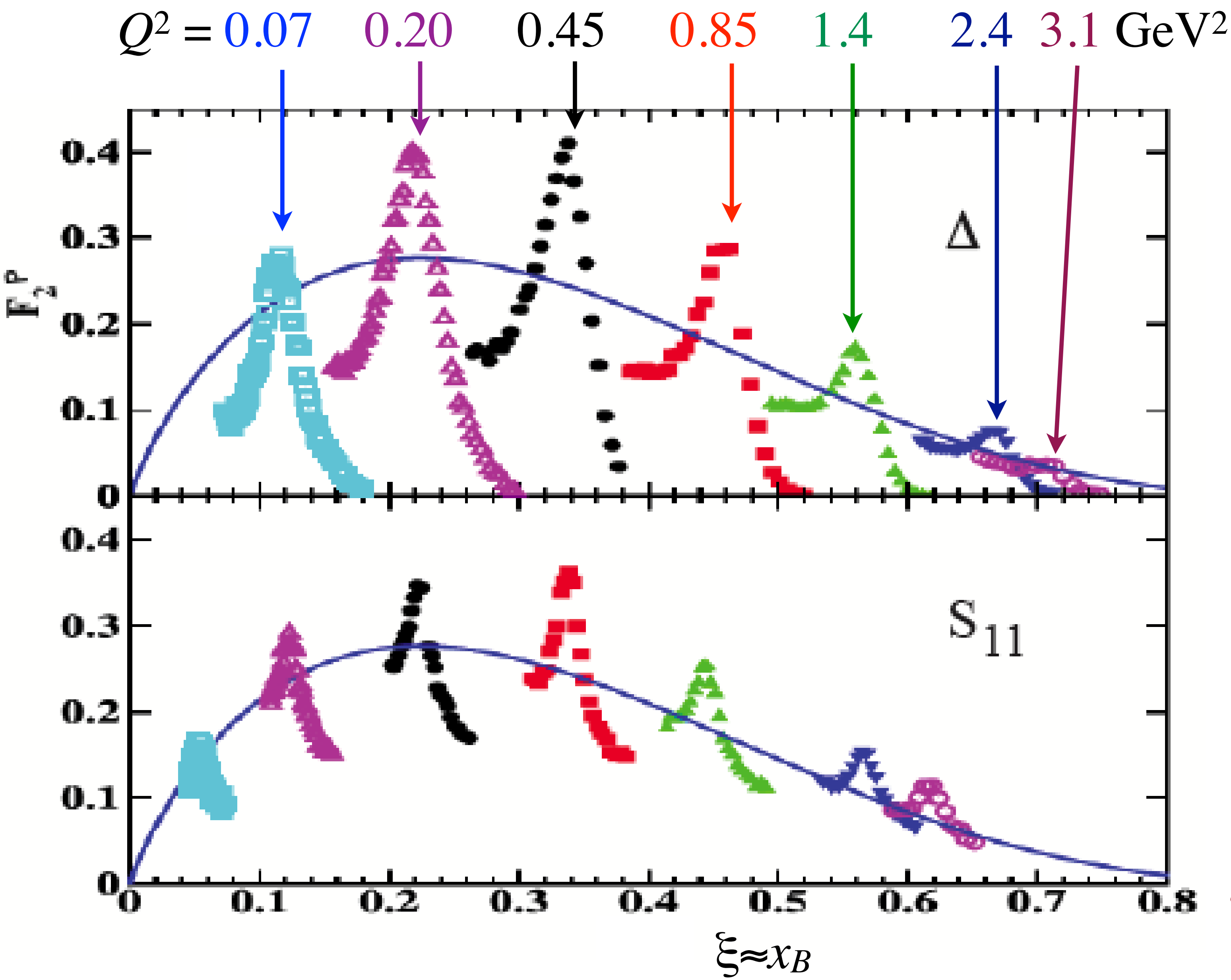
Bloom & Gilman (1970)

W. Melnitchouk (2010)

Resonances slide on the scaling curve

$$W^2 = M_{N^*}^2 = M_N^2 + \frac{(1-x_B)Q^2}{x_B}$$

Solid curve: Large Q^2



Jlab Hall C

C.S. Armstrong et al, PRD **63** (2005) 094008

$1.2 < W^2 < 1.9$ GeV²

"Δ"

$1.9 < W^2 < 2.5$ GeV²

"S_{11}"

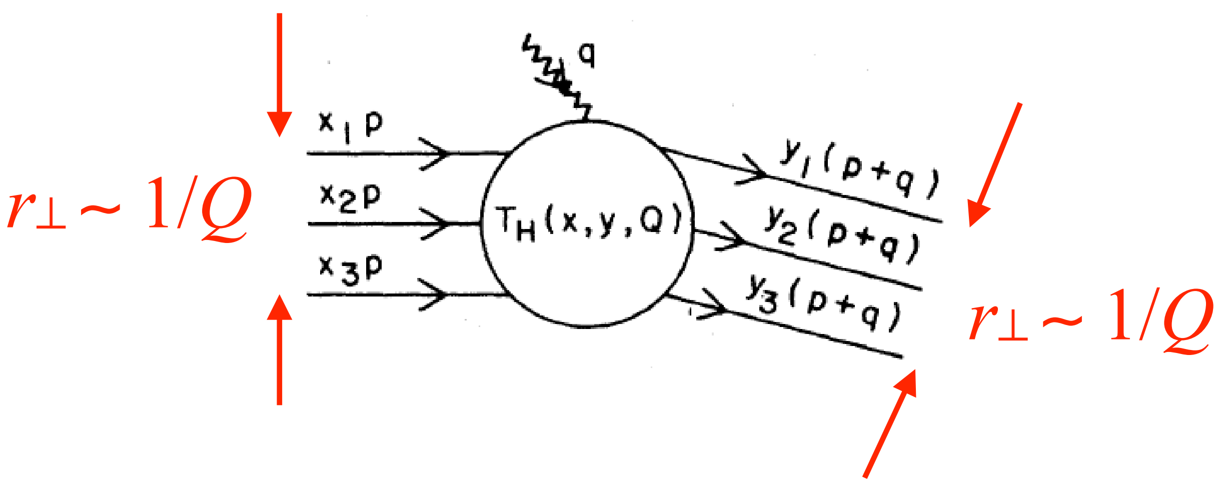
$$\xi = \frac{2x_B}{1+\sqrt{1+4M_p^2 x_B^2/Q^2}}$$

Exercise 1.3

The contribution to a hadron h form factor $F_h(Q^2)$ from a Fock state with p+1 constituents is expected to behave as

$$F_h(Q^2 \to \infty) \propto (1/Q^2)^p$$

Show that this behavior is implied by the requirement that all constituents of both the initial and final h should be within the virtual photon resolution $1/Q$ of each other in transverse space.



Exercise 1.4

The Drell-Yan-West relation links the power behaviors of hadron form factors $F_h(Q^2)$ measured in $e+h \longrightarrow e+h$ with the $F_2(x_B)$ structure function measured in $e+h \longrightarrow e+X$ as follows:

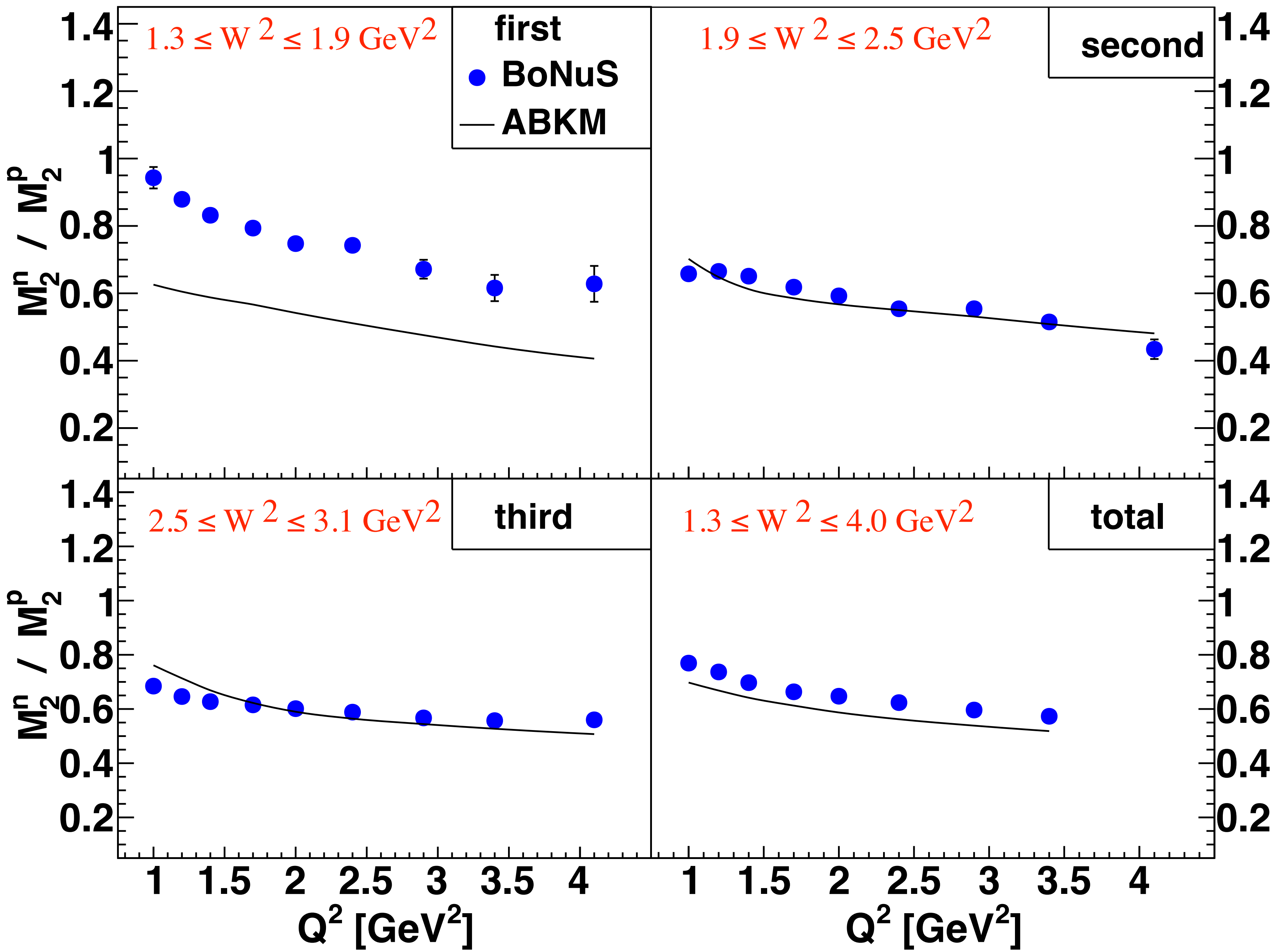
$$F_h(Q^2 \to \infty) \propto (1/Q^2)^p \quad \text{corresponds to} \quad F_2(x_B \to 1) \propto (1-x_B)^{2p-1}$$

In the duality limit where the mass of the hadronic system X is fixed we have $1 - x_B \propto 1/Q^2$. Since the inclusive cross section is

$$\frac{d\sigma}{dQ^2 dx_B} \sim F_2(x_B) \quad \text{we have} \quad \frac{d\sigma}{dQ^2} \sim F_2(x_B) d(1-x_B) \sim F_2(x_B)\frac{dM_X^2}{Q^2}$$

Show that this implies the DYW relation between hadron form factors at large Q^2 and the behavior of the structure function for $x_B \longrightarrow 1$.

The neutron to proton $M_2(Q^2)$ ratio: $\sum_q e_q^2$ test

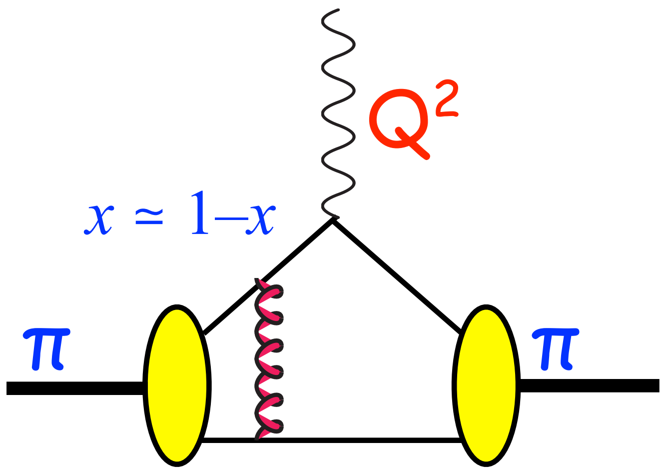


I. Niculescu, et al, 1501.02203

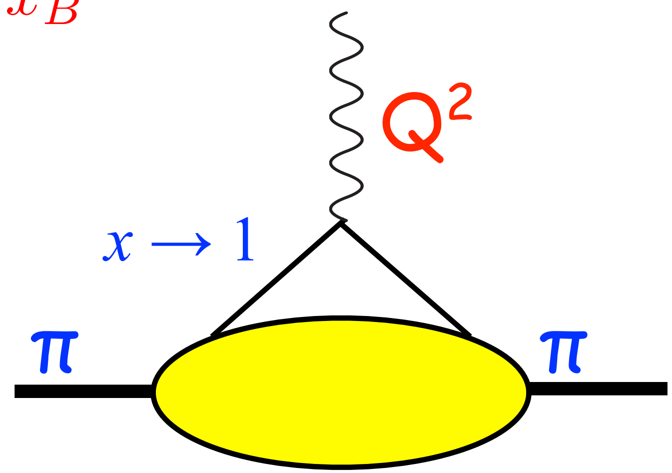
Form Factor Dynamics implied by duality

The quark scatters on a single quark, which carries $x_B \to 1$ of the momentum

$$M_{N^*}^2 - M_N^2 = \frac{(1-x_B)Q^2}{x_B}$$



$\Rightarrow$



γ* scatters coherently on all valence quarks

$$\sigma \propto \Big(\sum_q e_q\Big)^2$$

"Brodsky-Lepage" mechanism

γ* scatters on a single quark carrying nearly all momentum

$$\sigma \propto \sum_q e_q^2$$

"End-point" or "Feynman" mechanism
DIS dynamics

The dominance of the end-point mechanism is supported also by PQCD.

Applications of duality

Data at high x_B is kinematically constrained to low $W^2 = M_N^2 + \frac{(1-x_B)Q^2}{x_B}$

To the extent that the resonance region $W^2 \simeq M_{N^*}^2$ agrees, on average, with the scaling, high W^2 structure function, parton distributions can be determined for $x_B \to 1$.

Conversely, resonance parameters are determined from dispersion integrals, *e.g.*, in Light Cone Sum Rules

Anikin et al, arXiv:1505.05759

Examples:

- "EMC effect" in the nuclear structure functions shows up both in resonance and continuum data.
- The longitudinal structure function F_L
- $\gamma^* p$ helicity cross sections

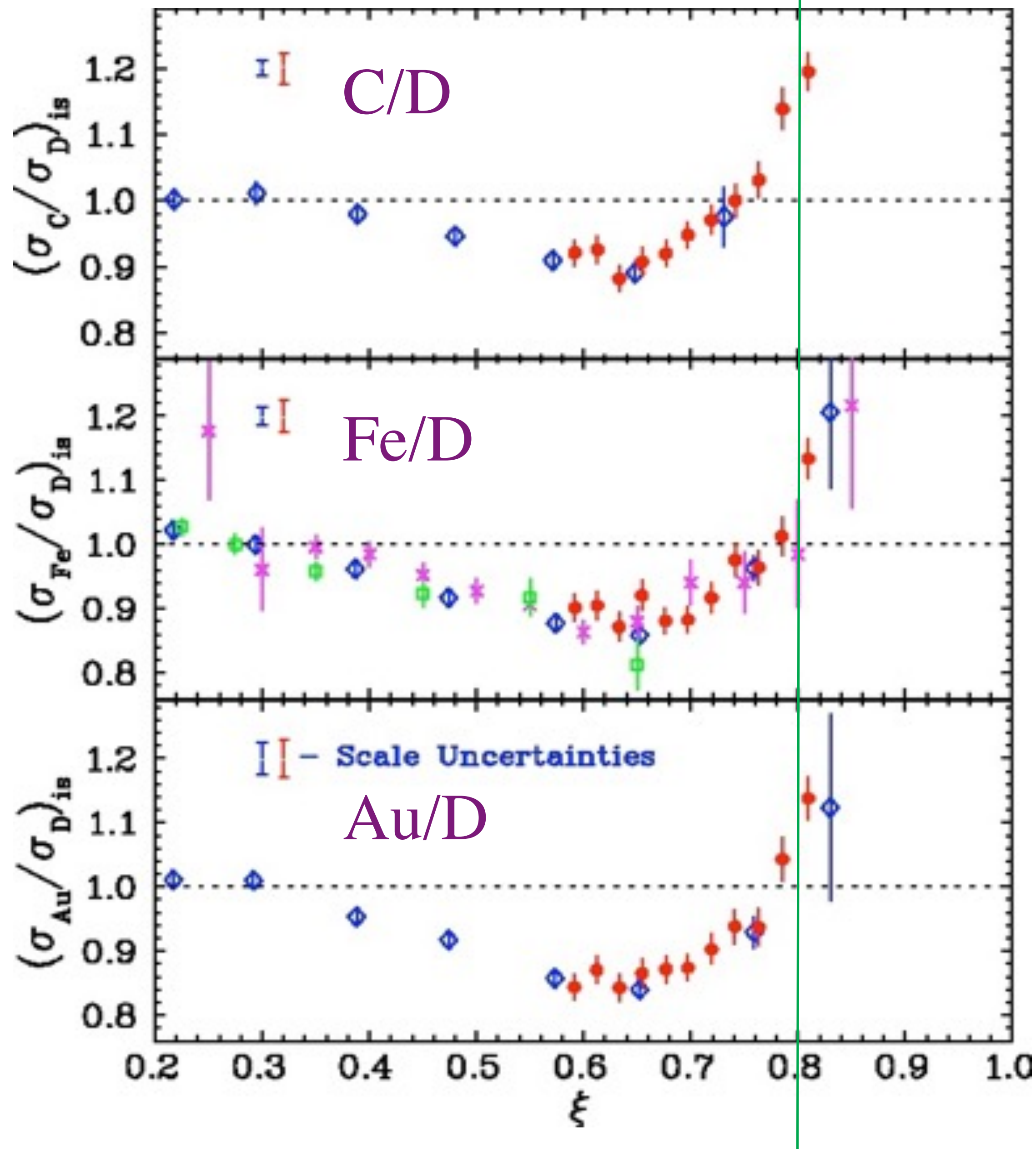
Duality and the EMC Effect

Red = resonance region data

Blue, purple, green = deep inelastic data from SLAC, EMC

Medium modifications to the structure functions ***are the same*** in the resonance region as in the DIS

Cross-over can be studied with new data

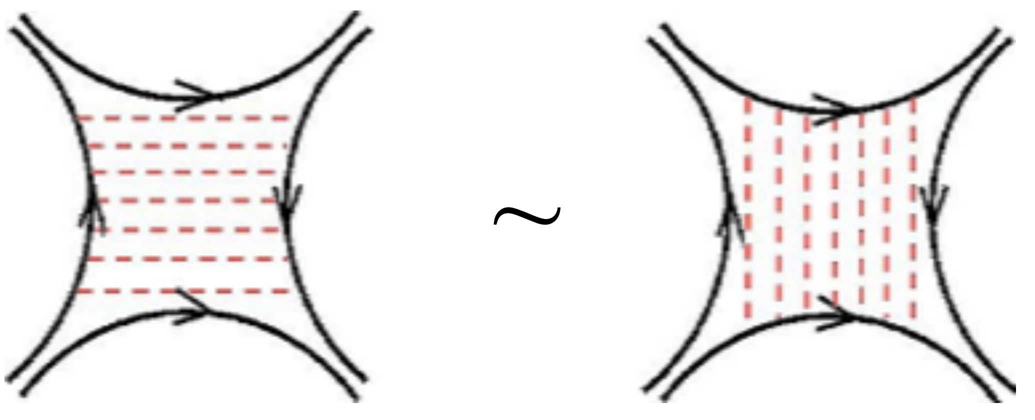


Cynthia Keppel (2005) *J. Arrington, et al., nucl-ex/0307012*

Implications of duality

- Resonances build scattering: the two must be considered together.
- The masses, spins and couplings of all bound states are related.
- Unitarity causes local adjustments (decay widths, thresholds,…)

- Hadrons are highly relativistic bound states: $\Delta M^2 \sim M^2$.
- One quark can carry nearly all momentum (form factors).
- It is important to consider the frame dependence of bound states.
- Dual diagrams are relevant.



Kopeliovich et al, arXiv:0811.2024