Geometrical Methods for Data Analysis II: Van Hove Plots

- Separation of Beam and Target Fragmentation Regions
- The Van Hove plot

  three-particle final states

  applications of Van Hove plots

Combining Van Hove and Dalitz Plots

- Van Hove Plots for 4-body decays

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Thanks to Vincent Mathieu

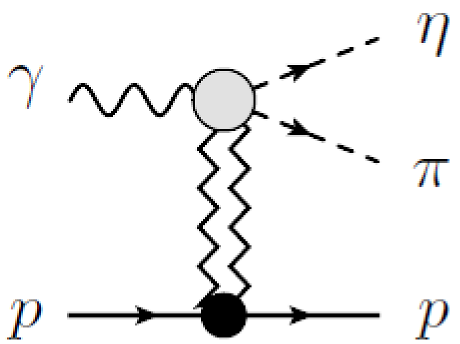
Leon Van Hove: 1924-1990

Belgian mathematical physicist
work on statistical physics, neutron scattering
statistical methods, multiparticle reactions

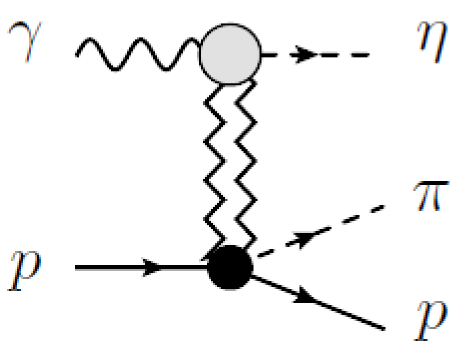
1961-1970, head of CERN Theory Division
1976-1980, Director-General, CERN
chaired scientific policy committee of ESA
established joint ESO/CERN symposium on
astronomy, cosmology & particle physics

Beam and Target Fragmentation

Consider the reaction: $\gamma + p \rightarrow \eta + \pi + p$
Want to distinguish between beam and target fragmentation



Beam fragmentation



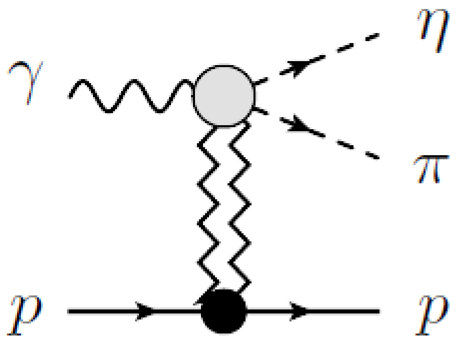
Target fragmentation

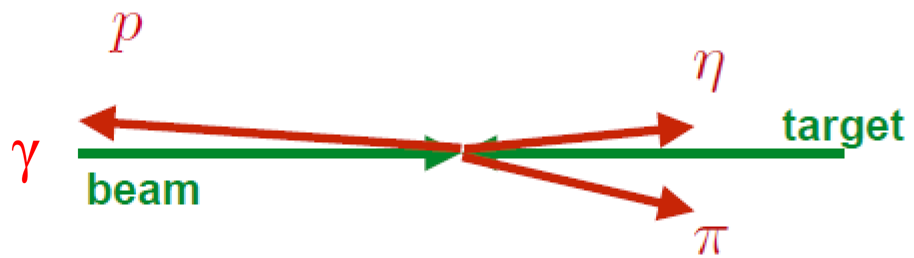
Wanted: a graphical approach to distinguish between the two regions

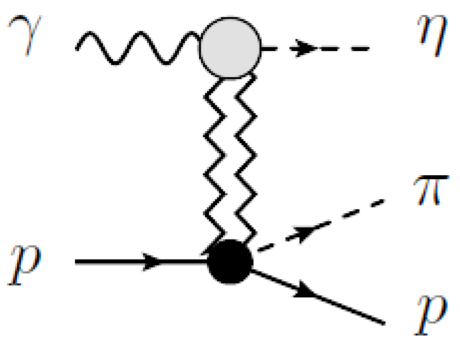
Beam vs. Target Fragmentation

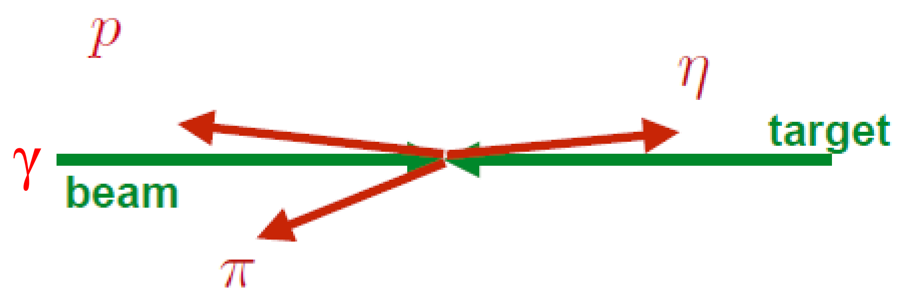
Example: the reaction $\gamma + p \rightarrow \eta + \pi + p$

Want to distinguish between beam (γ) and target (p) fragmentation









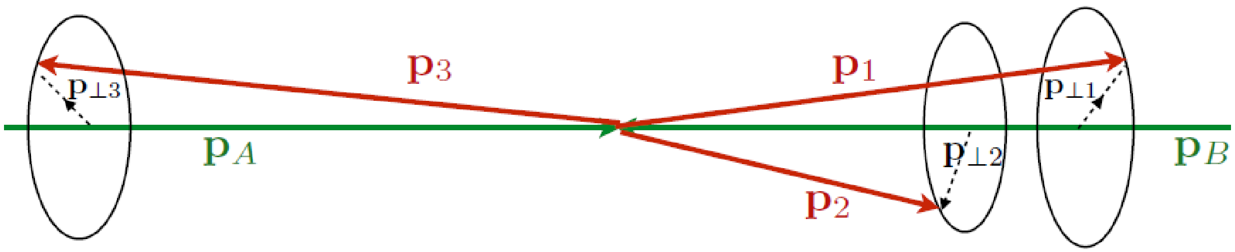
in CM frame, two regions can be separated

Van Hove's graphical method

PL **28B**, 429 (1969)

Consider process: A + B → 1 + 2 + 3, CM frame

q_i = longitudinal momentum of particle I, W = tot CM energy

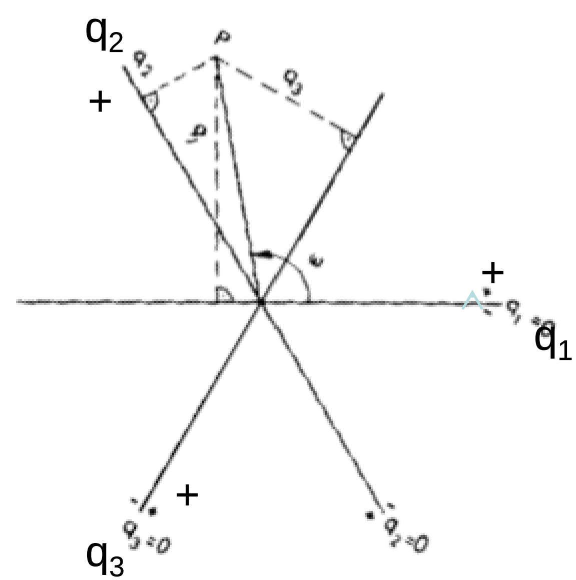


$$p_i = q_i + p_{\perp i} \ , \quad \sum_{i=1}^{N} q_i = 0 \ , \quad \sum_{i=1}^{N} |q_i| \leq W$$

$$q_1 = r \sin \omega \ ,$$

$$q_2 = r \sin(\omega - \frac{2\pi}{3}) \ ,$$

$$q_3 = r \sin(\omega - \frac{4\pi}{3})$$



3-body final state has 2 independent degrees of freedom

Define these as radius r and angle ω

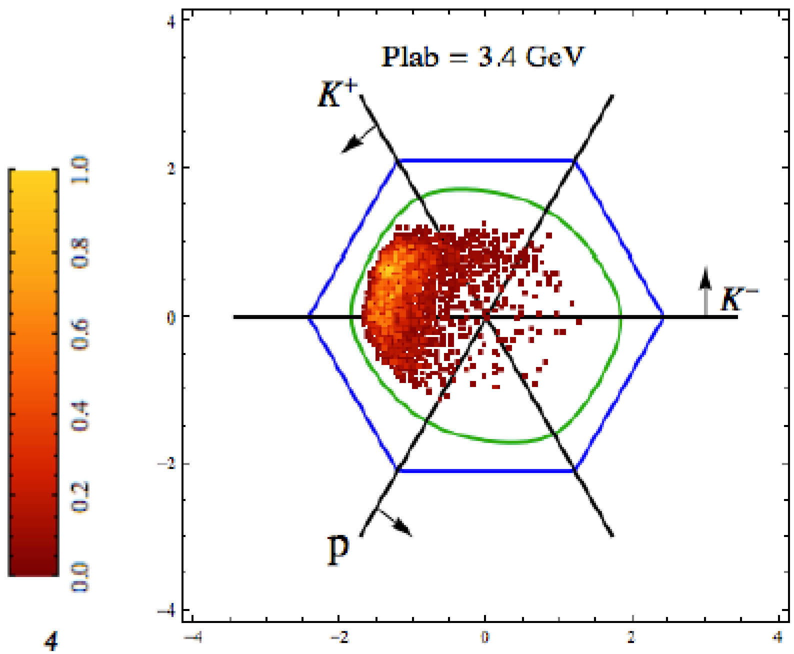
Define "positive, negative" for each q_i

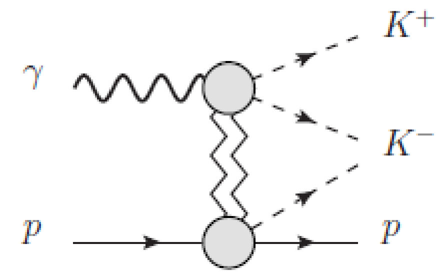
E conserv'n: allowed region is bounded

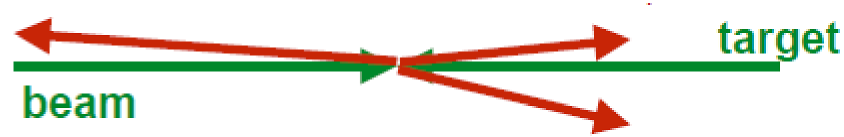
Example 1: Van Hove plot

Consider process: $\gamma + p \rightarrow K^- + K^+ + p$, CM frame

P_{lab} = 3.4 GeV







arrows denote direction of positive q_i (= in beam dir'n)

for massless particles, plot boundary = sides of hexagon

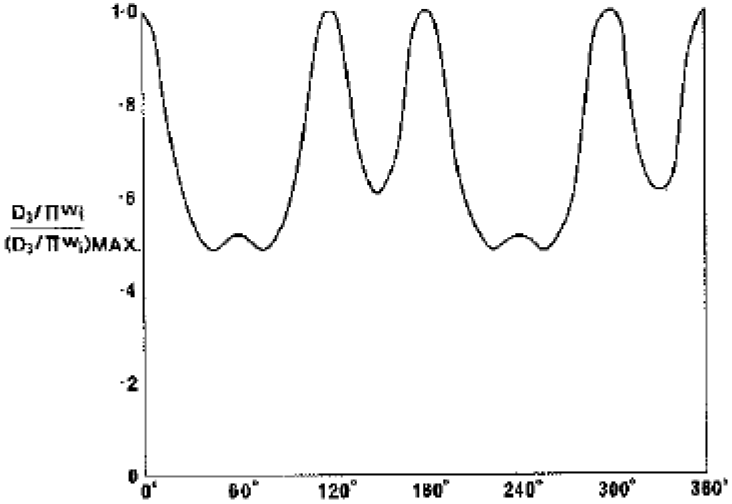
solid curve shows boundaries for physical masses

points on boundary = no transverse momentum

Details of Van Hove plots, 3-body Decays

Consider process: $\pi + p \rightarrow \pi + \pi + p$,

4 GeV



Dalitz Plot: if amplitude equal everywhere, DP is constant – equal phase-space density.
Not the case with VH plot!

$$W_i = \sqrt{q_i^2 + p_{\perp i}^2}$$

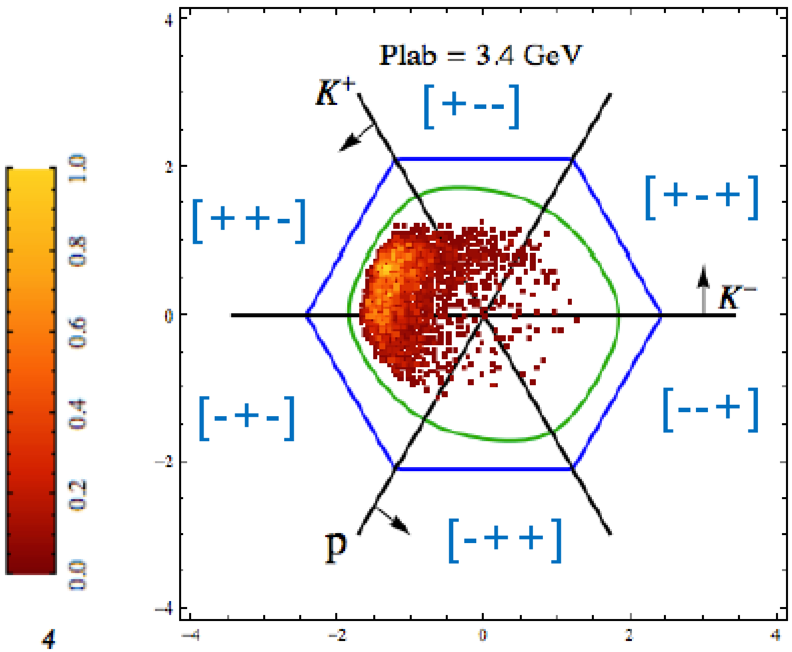
W_i smallest when $q_i = 0$.

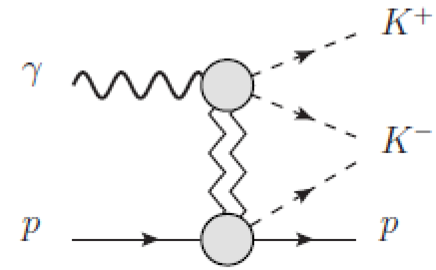
Phase space ~ $1/(W_1\ W_2\ W_3)$, W_i= cm energy particle i
Phase space enhancement in certain regions
Can give significant enhancement in some regions.
Can correct for enhancement by renormalizing amplitude.

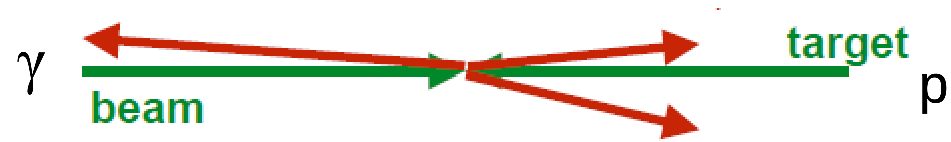
Interpretation of Van Hove plot

process: $\gamma + p \rightarrow K^- + K^+ + p$, CM frame

P_{lab} = 3.4 GeV







Label each sector by sign of q_i

($i=1$ K^-; $i=2$ K^+; $i=3$ p)

(+ = beam dir' n; -- = target dir' n)

vast majority of events have final proton "backwards" ($q_p < 0$)

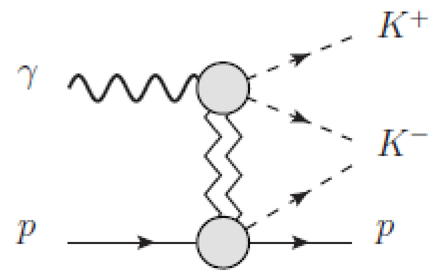
majority of events, kaons "beam fragmentation" region

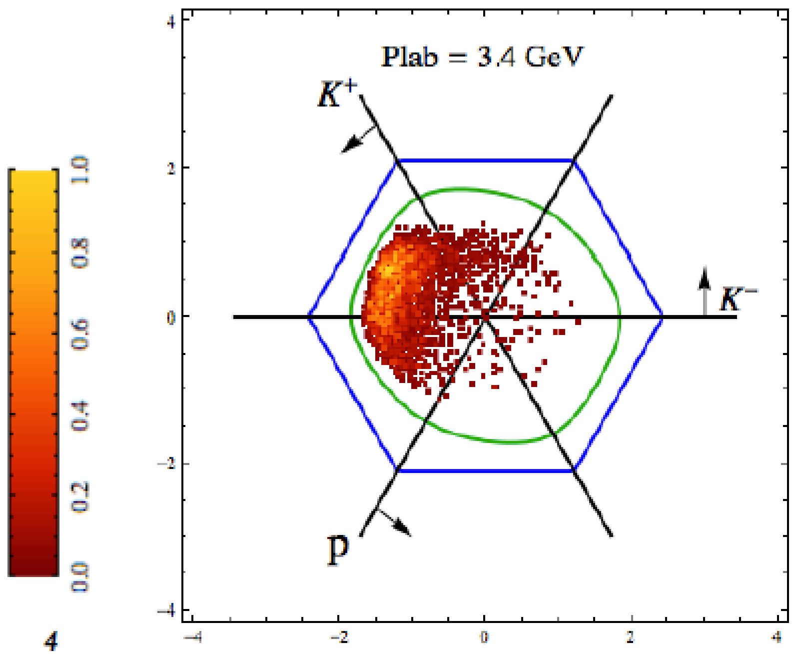
several events: one K in "target fragmentation" region

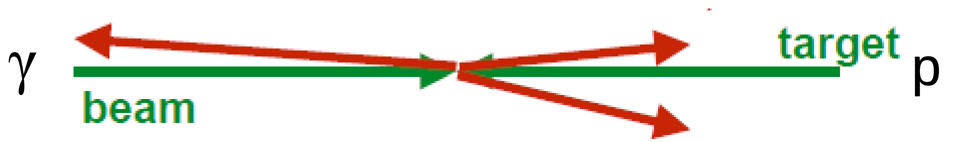
many events have significant transverse momentum

Interpretation of Van Hove plot

process: $\gamma + p \rightarrow K^- + K^+ + p$, CM frame
P_{lab} = 3.4 GeV





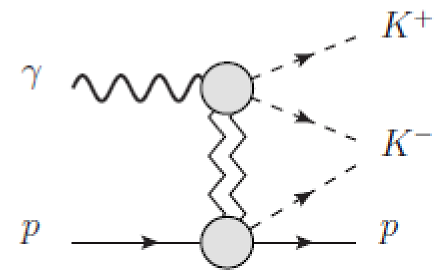


Q: in which sector of VH plot do you expect to see **ϕ resonance**? why?

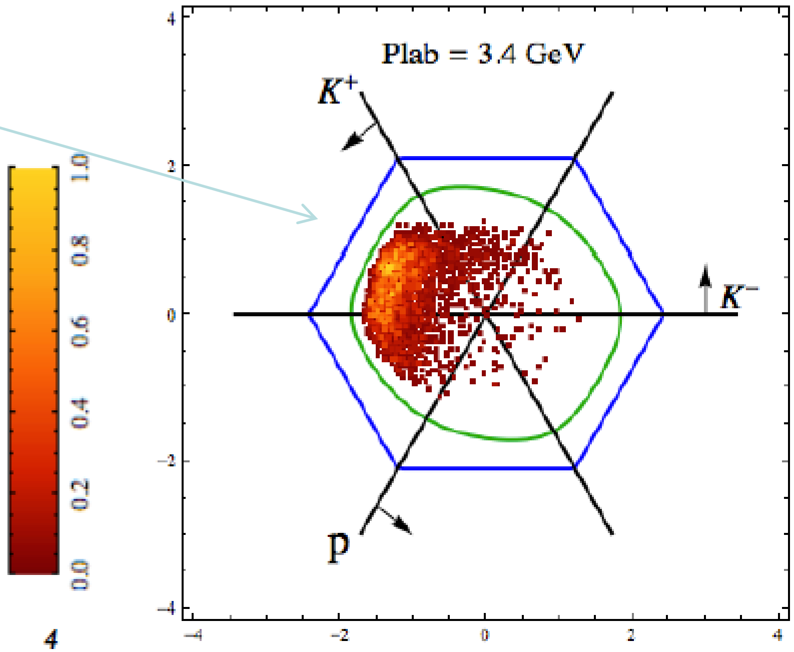
$$K^- = \bar{u}s\ ,\ K^+ = u\bar{s}\ ,\ p = uud\ ,\ \phi = s\bar{s}\ ,\ \Lambda = uds$$

Interpretation of Van Hove plot (2)

process: $\gamma + p \rightarrow K^- + K^+ + p$, CM frame
$P_{lab} = 3.4$ GeV



ϕ: K^+, K^- both in beam dir'n; Largest # of events in this sector

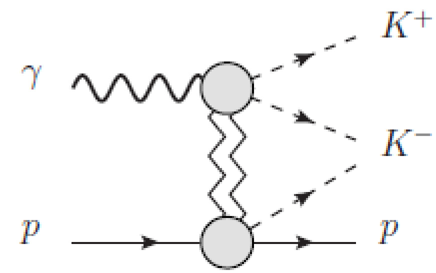


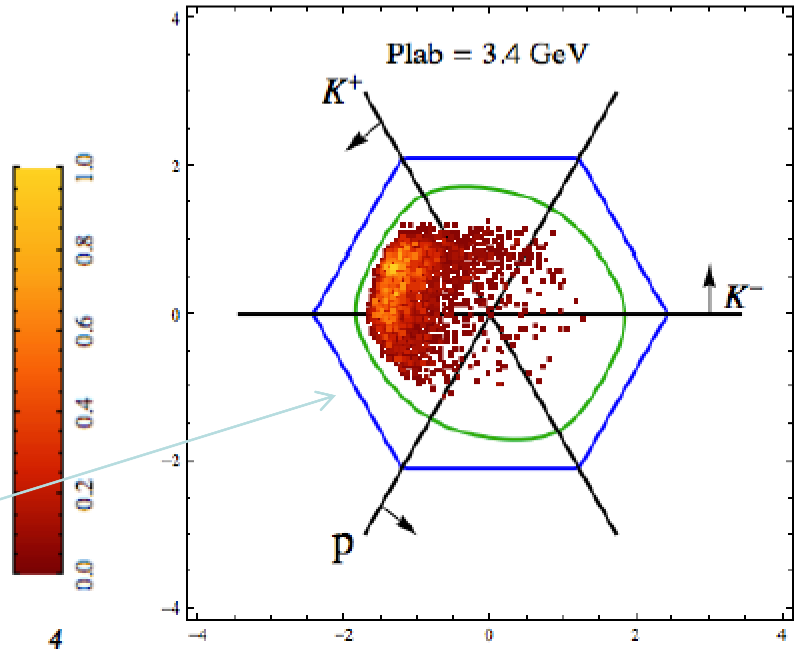
$$K^- = \bar{u}s \ , \ K^+ = u\bar{s} \ , \ p = uud \ , \ \phi = s\bar{s} \ , \ \Lambda = uds$$

Q: in which sector of VH plot do you expect to see **Λ resonance**? why?

Interpretation of Van Hove plot (3)

process: $\gamma + p \rightarrow K^- + K^+ + p$, CM frame
$P_{lab} = 3.4$ GeV





Λ: p, K^- both negative; 2nd-most events in this sector

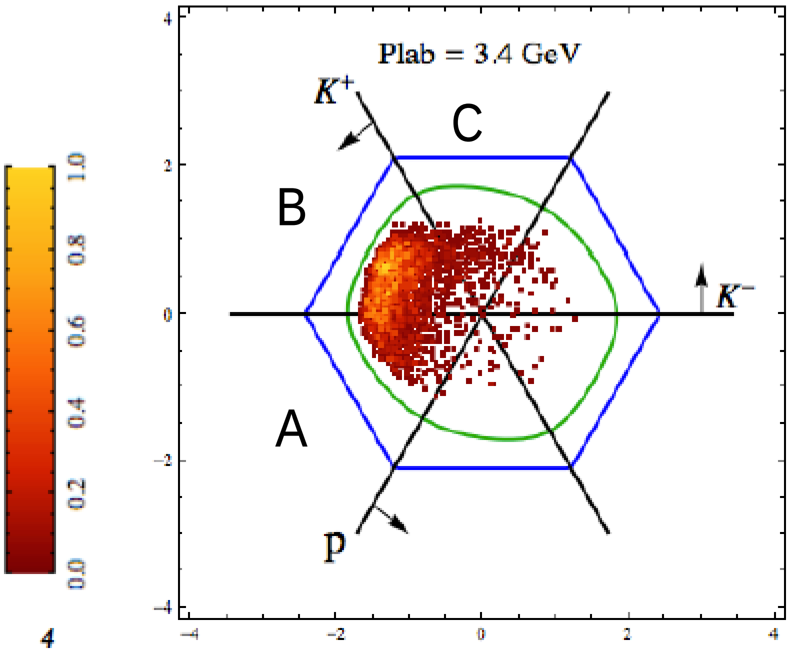
You see **Λ resonance** in lower L sector (p, K^- both negative)
Q: why isn't top center sector viable for Λ?

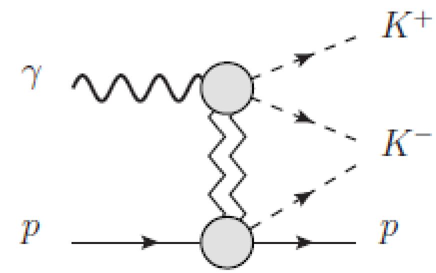
$$K^- = \bar{u}s\ ,\ K^+ = u\bar{s}\ ,\ p = uud\ ,\ \phi = s\bar{s}\ ,\ \Lambda = uds$$

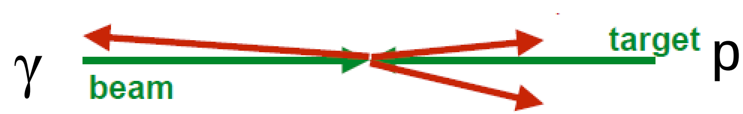
Various Sectors in Van Hove plot

process: $\gamma + p \rightarrow K^- + K^+ + p$, CM frame

P_{lab} = 3.4 GeV







sector **A:** K^-, p both in target dir'n: **Λ resonance**

sector **B:** K^+, K^- both in beam dir'n: **ϕ resonance**

sector **C:** K^+, p both in target dir'n: **pentaquark**

$$\mathbf{A}: \quad uud + \bar{u}s \Rightarrow uds = \Lambda$$

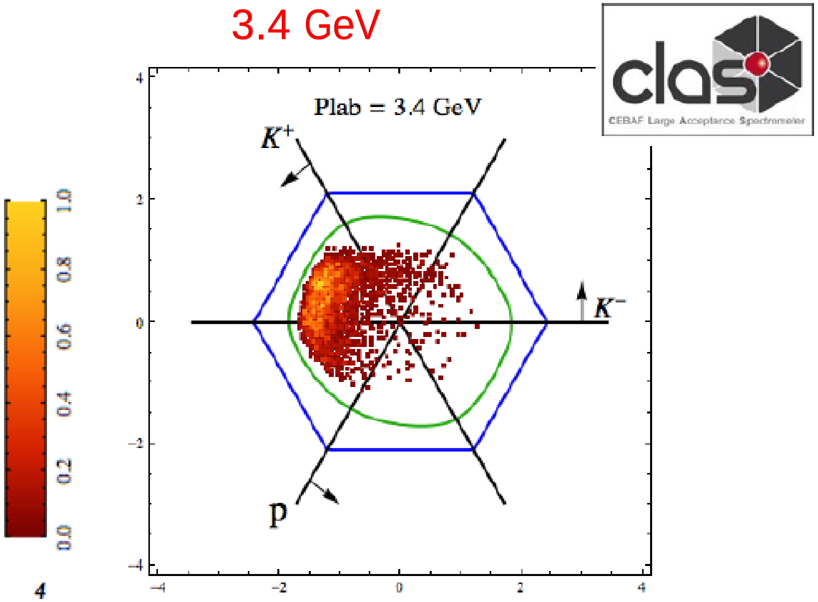
$$\mathbf{B}: \quad \bar{u}s + u\bar{s} \Rightarrow s\bar{s} = \phi$$

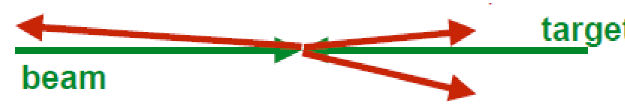
$$\mathbf{C}: \quad uud + u\bar{s} \Rightarrow \text{pentaquark}$$

Growth of Peripheral Reactions

$\gamma + p \rightarrow K^- + K^+ + p$

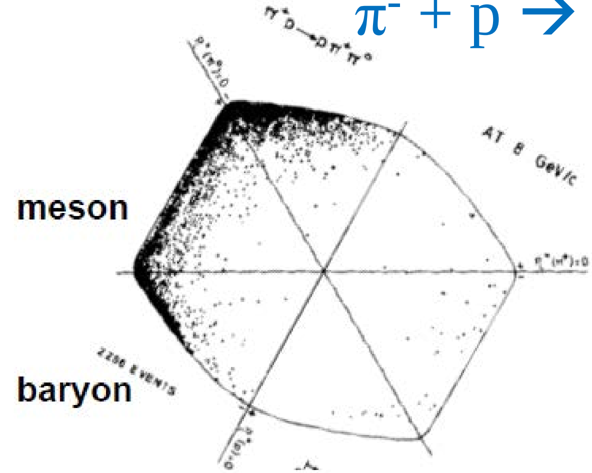
3.4 GeV

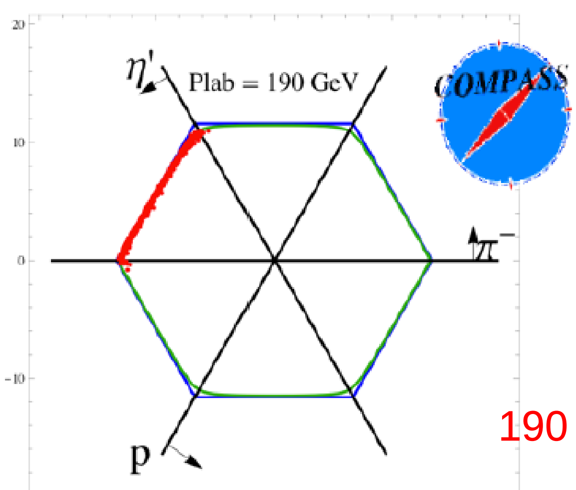




$\pi^- + p \rightarrow \pi^- + \pi^0 + p$

8 GeV





190 GeV

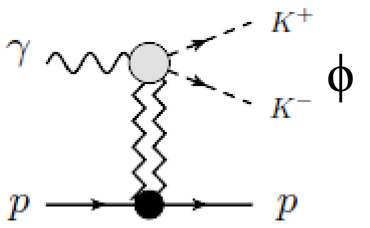
$\pi^- p \rightarrow \eta \pi^- p$

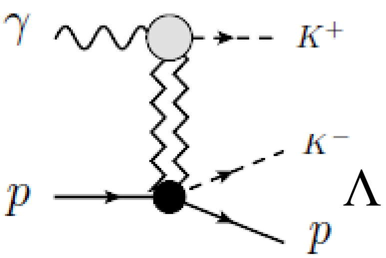
With increasing energy, transverse momentum rapidly decreases; already at 8 GeV (for some reactions) predominantly longitudinal

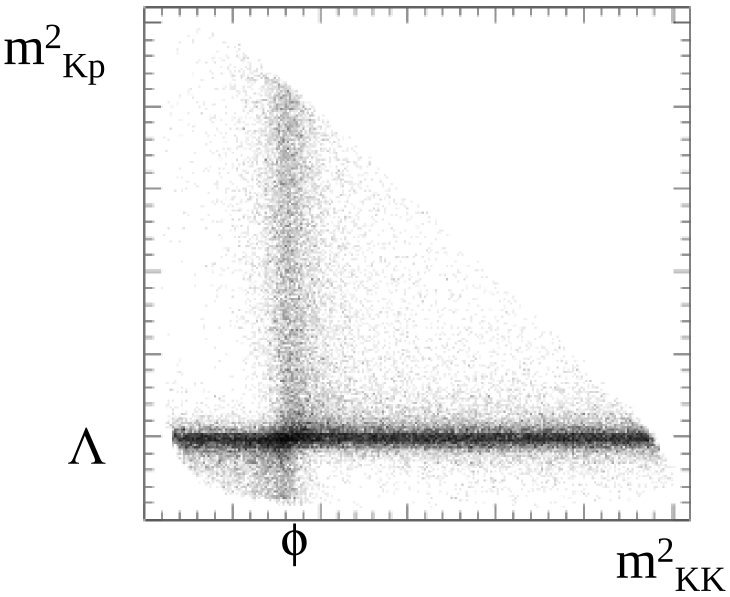
Example 2: Van Hove plot

Consider process: $\gamma + p \rightarrow K^{+} + K^{-} + p$

Will see both meson (ϕ) and baryon (Λ) resonances







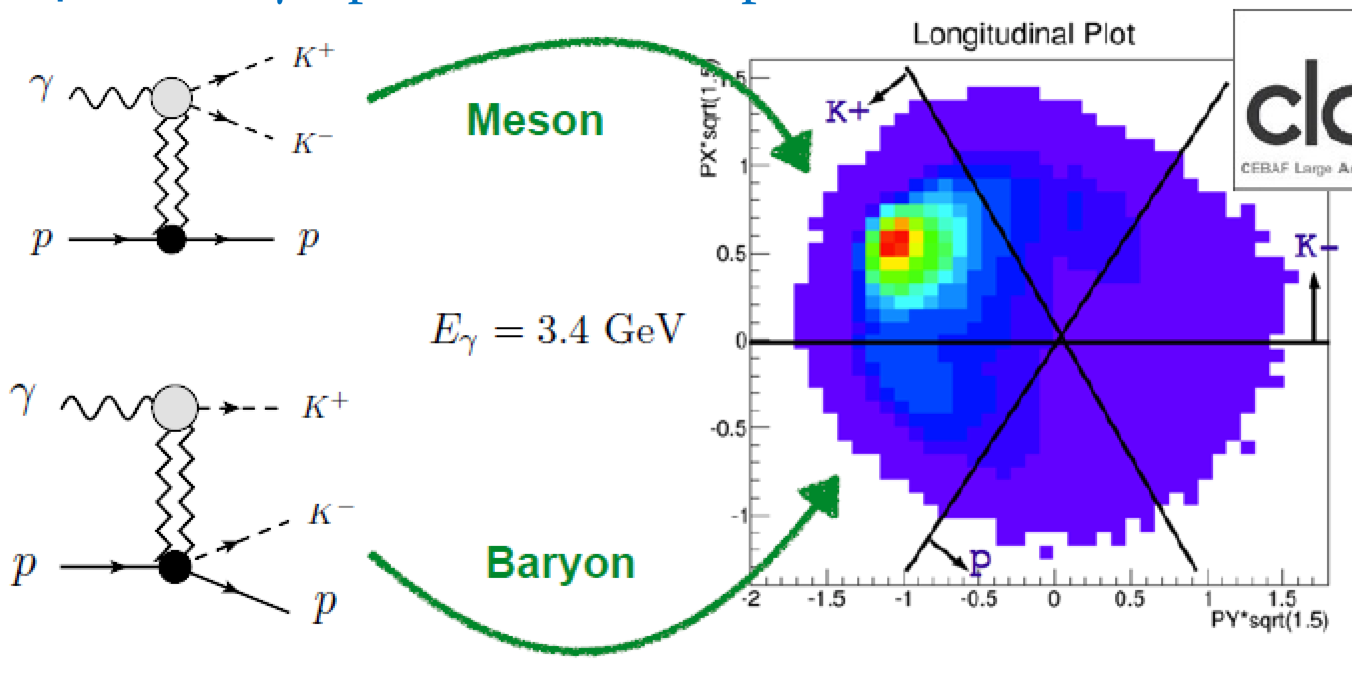
Separate beam, target fragmentation regions

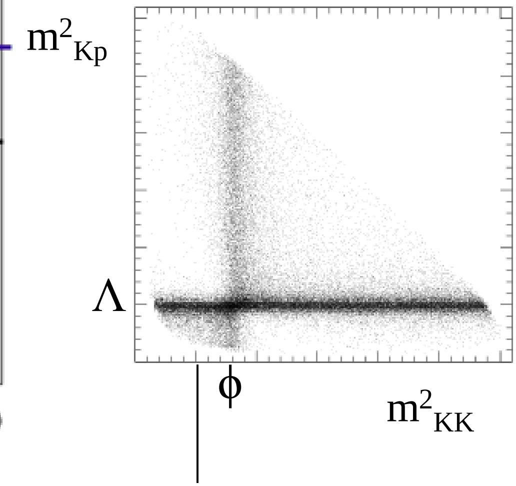
Expect significant interference between 2 resonances

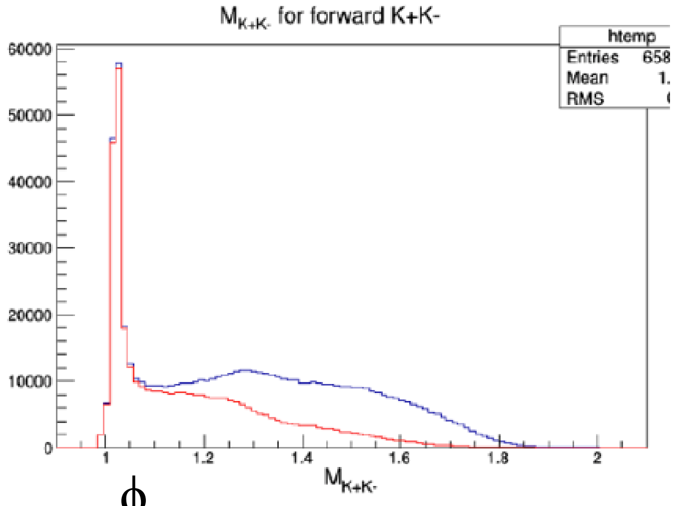
cuts in both Dalitz, van Hove plots can minimize interference

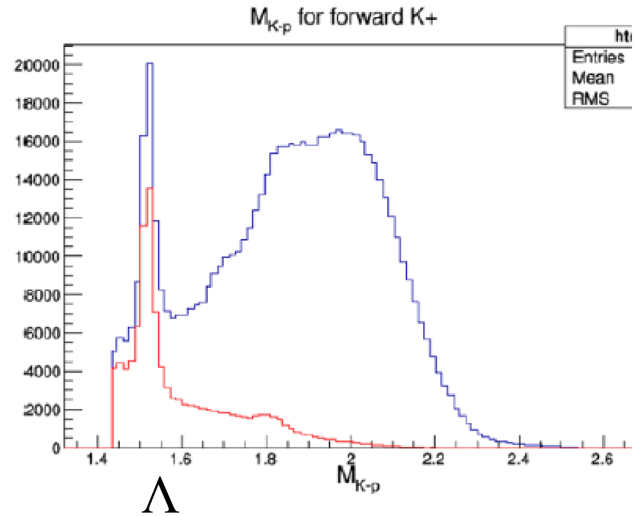
Cuts in Van Hove + Dalitz plots

process: $\gamma + p \rightarrow K^- + K^+ + p$ *simulated data*





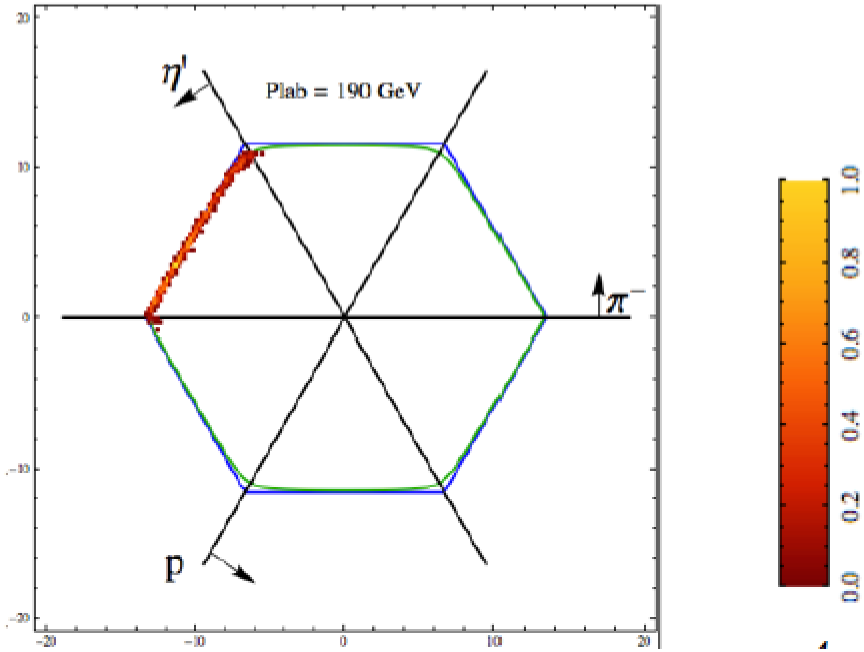


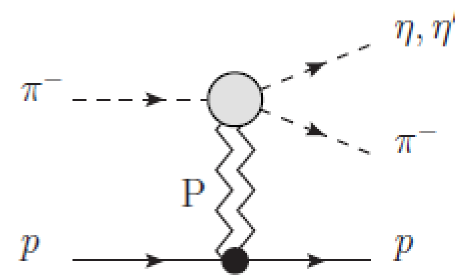


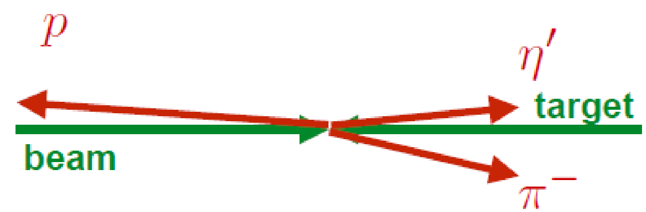
Taking VH sector cuts has potential to “clean up” resonances, especially Λ

Example 3: Van Hove plot

Consider process: $\pi^- + p \rightarrow \pi^- + \eta' + p$, CM frame
P_{lab} = 190 GeV

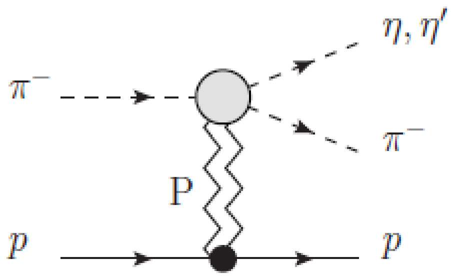


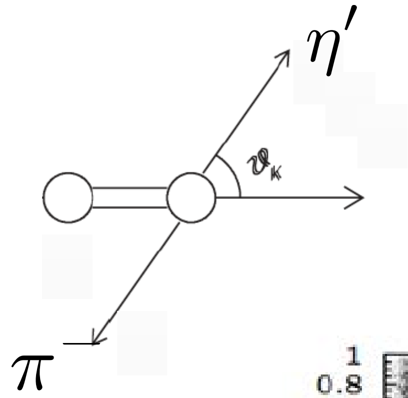




nearly all events in “beam fragmentation” region for mesons
very little transverse momentum for any events

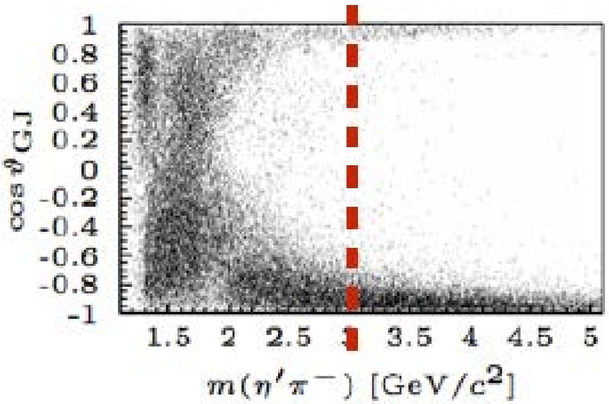
Unpacking the Van Hove plot





Analyze in Gottfried-Jackson frame: CM of mesons

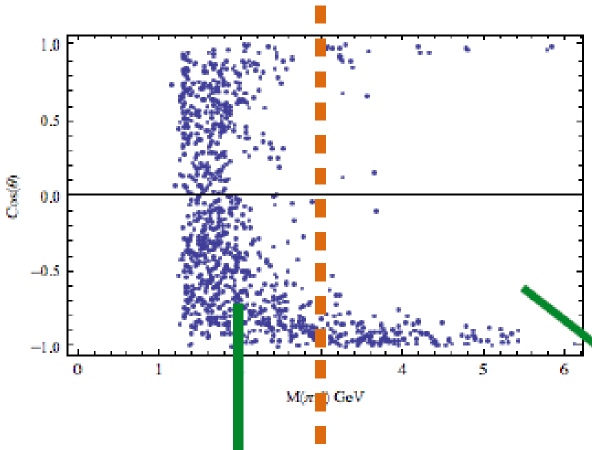
θ= angle of η’ to beam dir’n

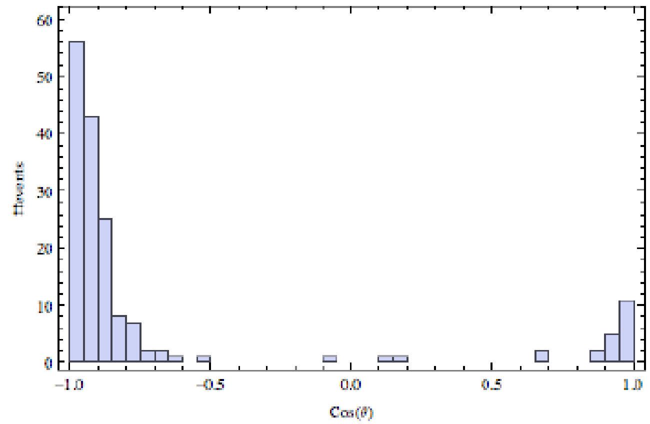


θ_{GJ}: low-mass ηπ pairs, relatively symmetric

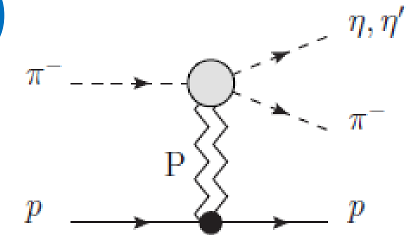
high-mass ηπ pairs, strongly asymmetric cos $\theta_{GJ} \sim -1$

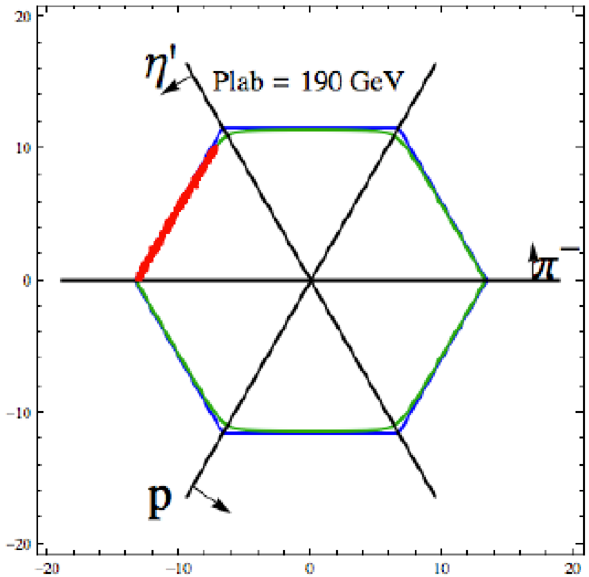
Unpacking the Van Hove plot (2)

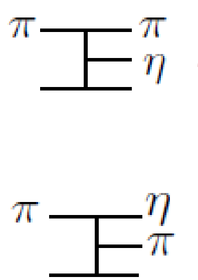


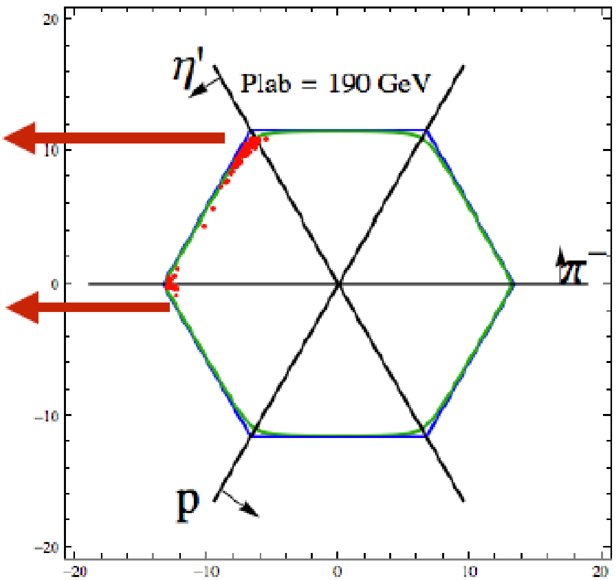


Gottfried-Jackson angle



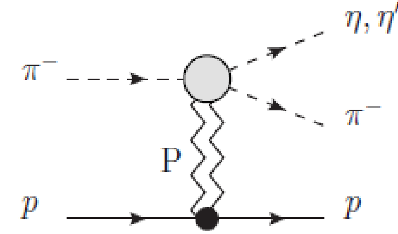


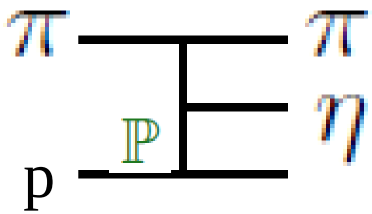




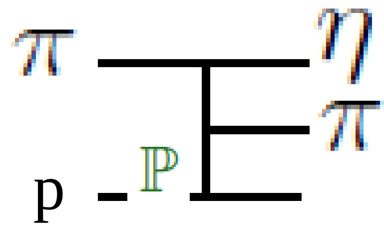
Hypothesis for preference for η’ to go “backward”?

Unpacking the Van Hove plot



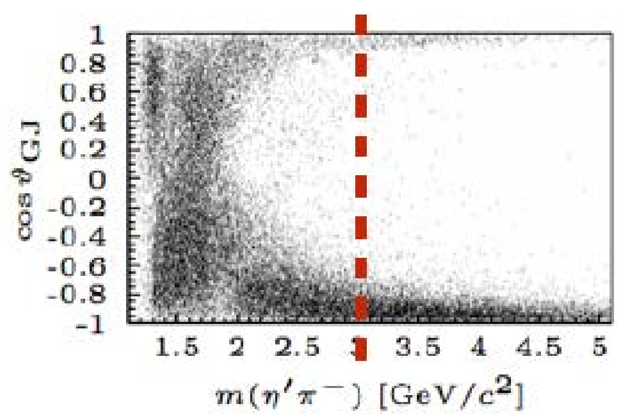


In Regge region, bottom vertex dominated by Pomeron; upper = $\mathbb{P}$,f



In Regge region, bottom vertex dominated by Pomeron; upper = a_2

But η’ couples very strongly to glue (anomaly); expect Pomeron coupling to dominate → θ_{GJ} ~ 180^o, as observed



Van Hove plots for 4-body Decays

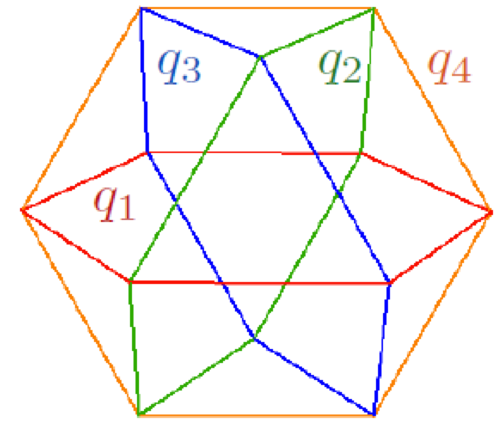
Consider process: A + B → 1 + 2 + 3 + 4, CM frame

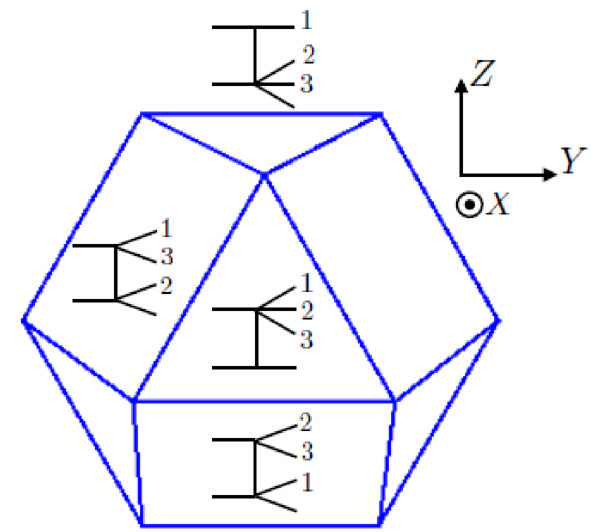
$$\sum_{i=1}^{4} q_i = 0 \ , \quad \sum_{i=1}^{4} |q_i| \leq W$$

$$X = \sqrt{\frac{3}{8}}\,(q_1 + q_2 + 2q_3)$$

$$Y = \sqrt{\frac{1}{8}}\,(q_1 + 3q_2)$$

$$Z = q_1$$





outer surface is **"cuboctahedron"**
(polygon with 14 faces)

triangles: 3 final particles in 1 direction

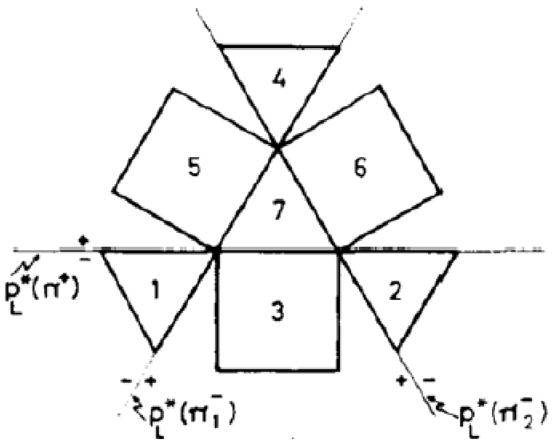
squares: 2 final particles in each direction

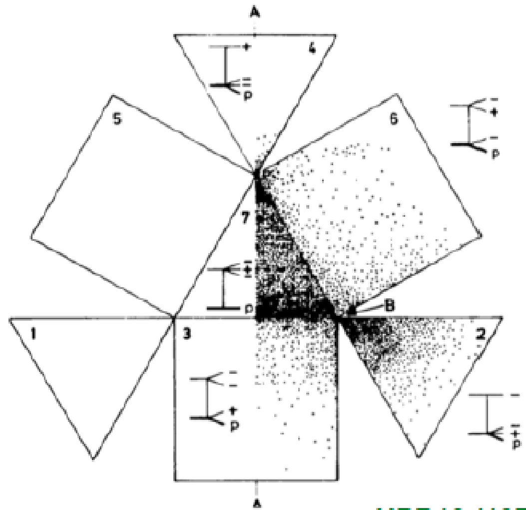
points on boundary: no transverse momentum

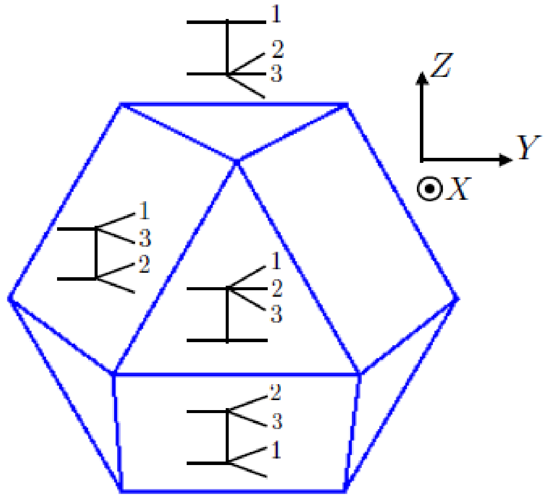
[visible faces: particle 4 in target direction]

Example, Van Hove plots, 4-body Decays

Consider process: $\pi^- + p \rightarrow \pi^+ + \pi^- + \pi^- + p$,
16 GeV, CERN PS



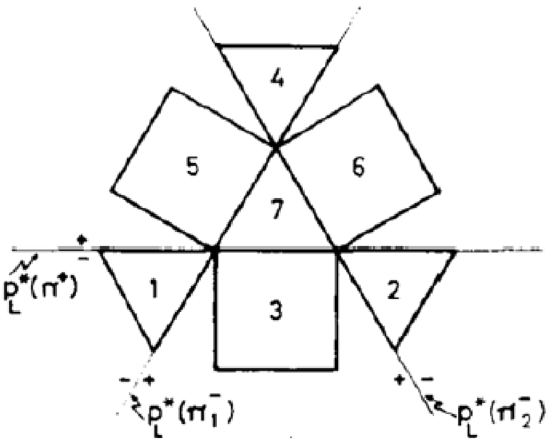


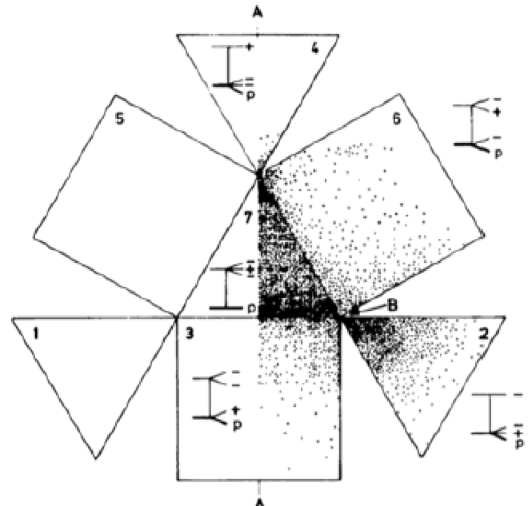


"unpack" outer surface (most pts near boundary, project onto boundary)
half where $q_p < 0$: 99.5% of data
2 identical π^-: need to reflect data
clear enhancement at certain geometries
p_L (π^+) is small.

Van Hove plots, 4-body Decays

Consider process: $\pi^- + p \rightarrow \pi^+ + \pi^- + \pi^- + p$,





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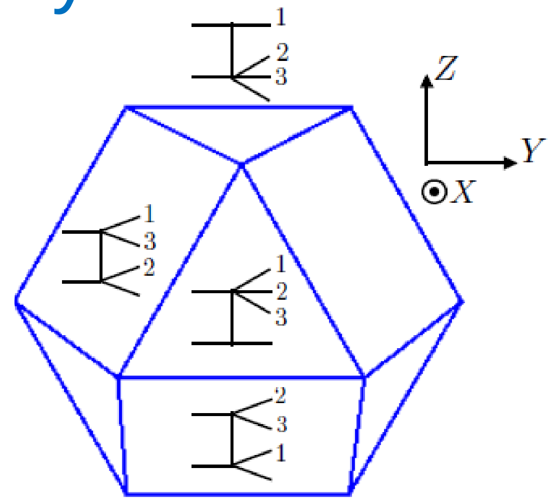


Phase space ~ $1/(W_1\ W_2\ W_3\ W_4)$, W_i= cm energy particle i

Phase space enhancement on lines, corners

Δ 4: π^+ forward, everything else backward

Δ 7: all pions forward, p backward

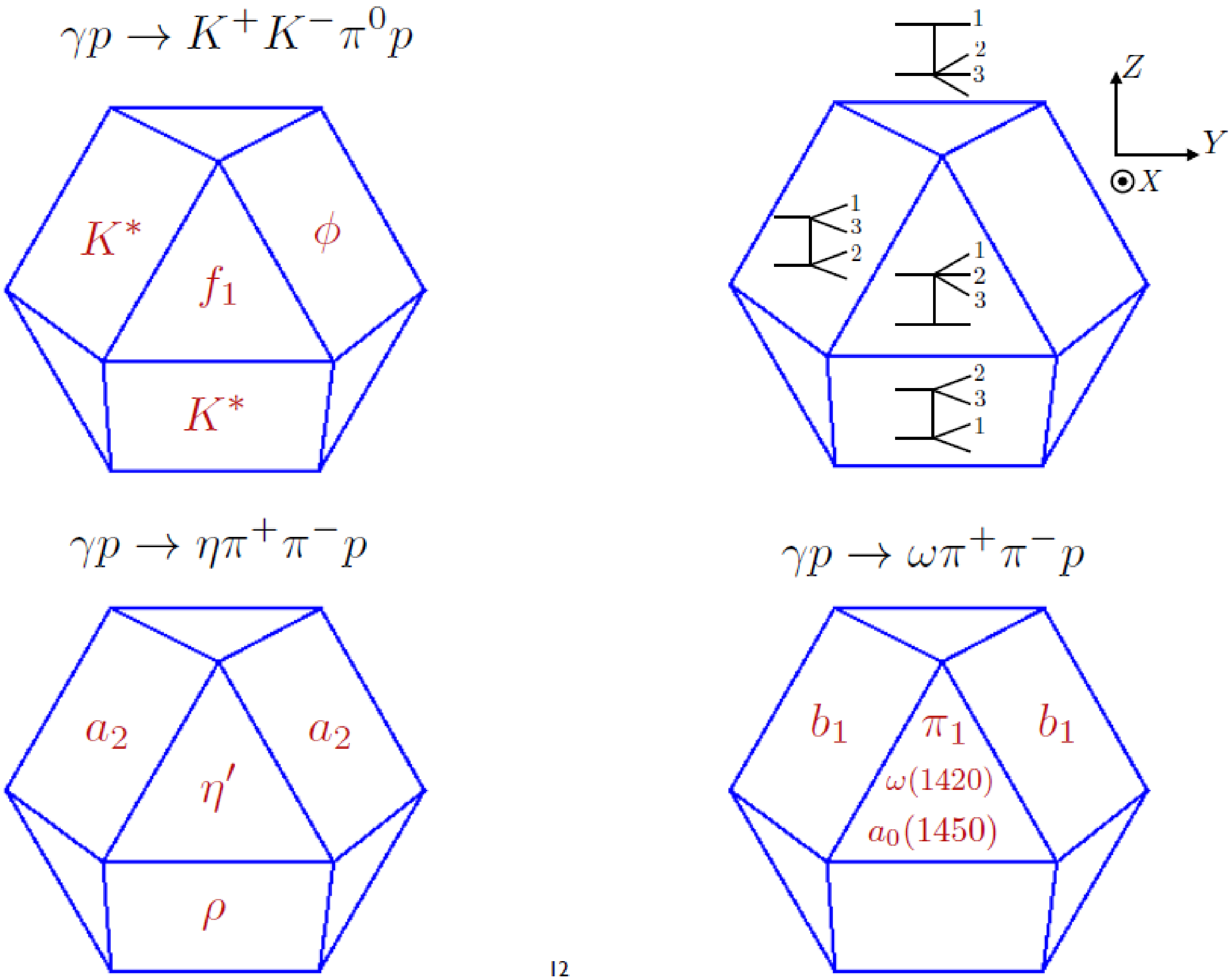
Δ 2: π^- forward, everything else backward

☐ 3: $(\pi^-\ \pi^-)$ forward, $(\pi^+\ p)$ backward

☐ 6: $(\pi^+\ \pi^-)$ forward, $(\pi^-\ p)$ backward

Reaction dominated by small number of configurations

Resonances in 4-body Photoproduction



identify sectors that will show resonance production

Conclusions:

Van Hove Plots: graphical display of 3-body decays
Separate beam and target regimes
Study correlations in longitudinal momenta between final particles

- Van Hove plots allow separation of overlapping resonances
- Can help make cuts to highlight certain processes
  help to identify particular physical processes

Can combine Dalitz, van Hove plots to maximize sensitivity

Van Hove Plot:

- Introduced ~ 1970, early use in reactions
- More or less forgotten until recently
- May again become useful tool in analyzing many-particle production