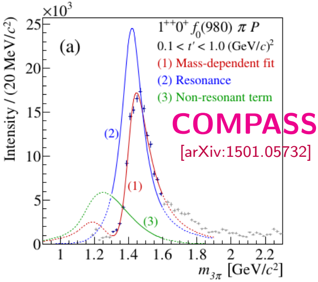
On the nature of $a_1(1420)$

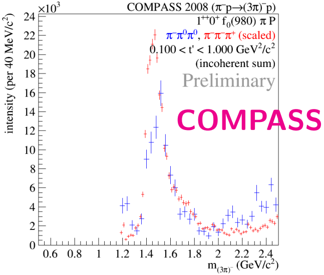
M. Mikhasenko, B. Ketzer, A. Sarantsev

HISKP, University of Bonn

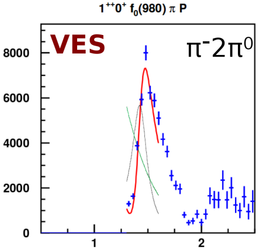
June 12, 2015

New a_1 - $1^{++}0^{+} f_0(980)\pi\ P-$wave

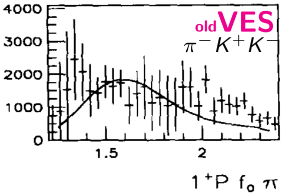




[PoS **Hadron2013**, 087 (2013)]



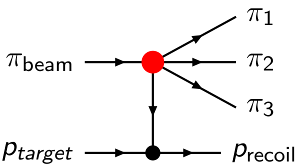
[PoS **Hadron2013**, 088 (2014)]

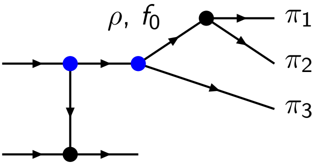


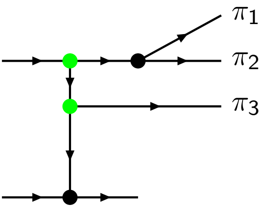
[Berdnikov *et al.*, Nuovo Cim. 107, 1941 (1994)]

Reactions at fixed-target experiments

- COMPASS (VES) is a fixed-target experiment.
- 190 GeV (29 GeV) pion beam.
- The recoil proton gives trigger (veto).
- Charged particles are observed by spectrometer, neutral ones are registered using calorimeters.

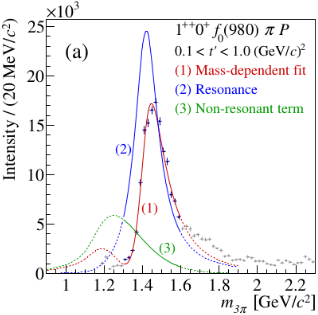


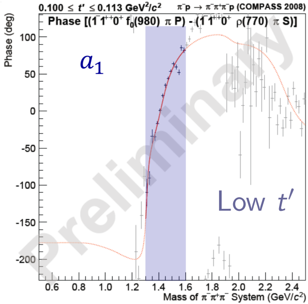


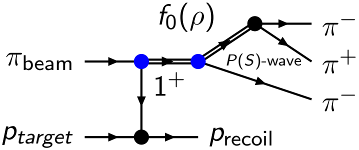


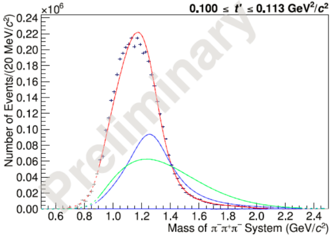
Resonance production and Deck-production amplitude.

1^{++}-waves in COMPASS: $f_0\pi$ P-wave and $\rho\pi$ S-wave

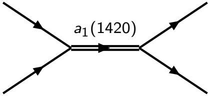








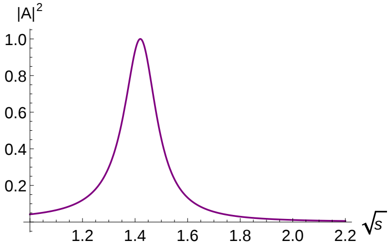
What is a resonance?

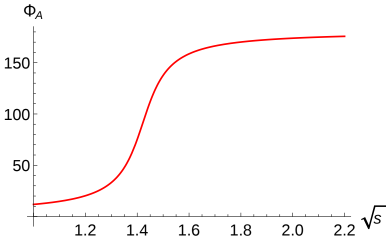
$A = \Bigg[$  $\Bigg] \sim \dfrac{1}{m^2 - s - i\, m\, \Gamma_{\text{tot}}}$

A resonance is a pole in the amplitude.

Manifestation:

- Bump in $\sqrt{s} = M_{12}$ with finite width.
- Phase motion. Only relative phase motion is observed.

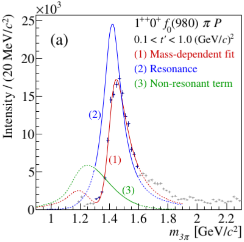


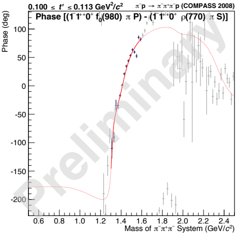


Is $a_1(1420)$ a resonance?

Might be, because

- narrow bump,
- sharp phase motion

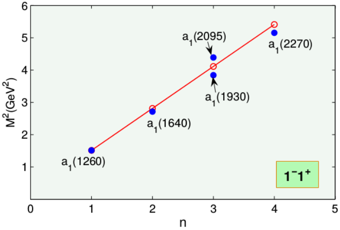




Issues to be clarified:

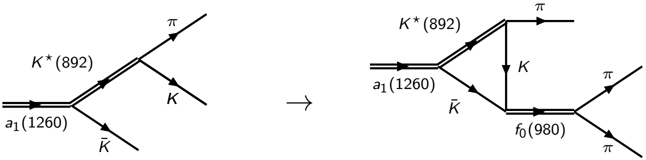
- $a_1(1420)$ doesn't fit to radial excitation trajectory as well as Regge-trajectory.
- Closeness of $a_1(1260)$.
- Width is too small compared to $a_1(1260)$.
- Strange coincidence of the mass to $K^\star(892)\bar{K}$ threshold, $\approx 1.380\,\mathrm{GeV/c^2}$.

[arXiv:1401.1134]



The interpretations of $a_1(1420)$

- 4-quark state candidate [Hua-Xing Chen *et al.*, arXiv:1503.02597], [Zhi-Gang Wang, arXiv:1401.1134].
- $K^\star K$ molecule (similar to XYZ interpretation)
- Dynamic effect of interference with Deck [Basdevant *et al.*, arXiv:1501.04643] .
- Triangle singularity [Mikhasenko *et al.*, Phys. Rev. D **91**, no. 9 (2015) 094015] :



final state rescattering of almost real particle. Logarithmic singularity in the amplitude of the processes:

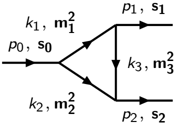
What is the triangle singularity?

Feynman approach

$$A(s_0, s_1, s_2) = g^3 \int \frac{d^4 k_1}{(2\pi)^4 i} \frac{1}{\Delta_1 \Delta_2 \Delta_3} = \frac{g^3}{16\pi^2} \int_0^1 \frac{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z}{D} \delta(1 - x - y - z),$$

$$\Delta_i = m_i^2 - k_i^2, \quad D = x\, m_1^2 + y\, m_2^2 + z\, m_3^2 - x\, y\, s_0 - z\, x\, s_1 - y\, z\, s_2.$$

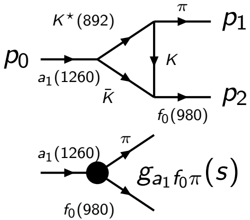
Positions of singularities are given by equations: [Landau, Nucl. Phys. **13**, 181 (1959)]

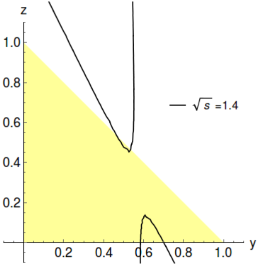


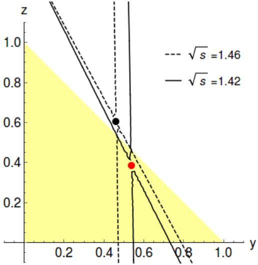
$$\begin{cases} k_i^2 = m_i^2, & i = 1 \ldots 3, \\ x\, k_{1\mu} - y\, k_{2\mu} + z\, k_{3\mu} = 0, & x, y, z \in [0, 1], \\ x + y + z = 1, & \end{cases}$$

- leading singularity: log. branching point, $A \sim \log(s - s_{\mathrm{sing}})$,
- square root branching points, $A \sim \sqrt{s - s_{\mathrm{sing}}}$ (normal thresholds).

The pinch singularity







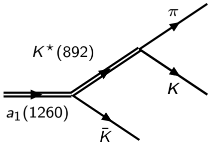
$$g_{a_1 f_0 \pi}(s|_{s=p_0^2}) = g^3 \int \frac{d^4 k_1}{(2\pi)^4 i} \frac{1}{\Delta_1 \Delta_2 \Delta_3} = \frac{g^3}{16\pi^2} \int_0^1 \mathrm{d}y \int_0^{1-y} \mathrm{d}z \, \frac{1}{D},$$

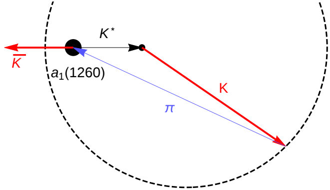
$$\Delta_i = m_i^2 - k_i^2 - i\epsilon, \quad D = (1-y-z)m_1^2 + y m_2^2 + z m_3^2 - y(1-y-z)p_0^2 - z(1-z-y)p_1^2 - yz p_2^2.$$

If Landau conditions [Nucl. Phys. **13**, 181 (1959)] are satisfied, $g_{a_1 f_0 \pi} \sim \log(s - s_0)$.

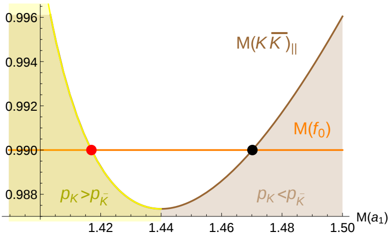
For a **box loop**, $A \sim (s - s0)^{-1/2}$, for **5-leg loop** $A \sim (s - s_0)^{-1}$ (pole).

Kinematical formulation of Landau equations





Imagine cascade reaction $a_1(1260) \to K^\star(892)\bar{K}$, then $K^\star \to K\pi$, and calculate invariant mass of K and $\bar{K}$ for the case when K is parallel to $\bar{K}$.



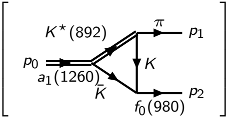
Particular form of Landau conditions:

- All particles in loop are on mass shell.
- The alignment of moments $\vec{p}_K \uparrow\uparrow \vec{p}_{\bar{K}}$.
- K is faster then $\bar{K}$.

Calculation of the rescattering: $a_1(1260) \to K^\star \bar{K} \to f_0 \pi$

For the realistic decay, we have expression in the numerator

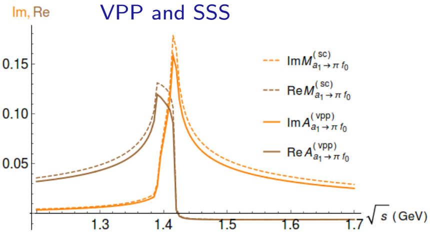
$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} =$

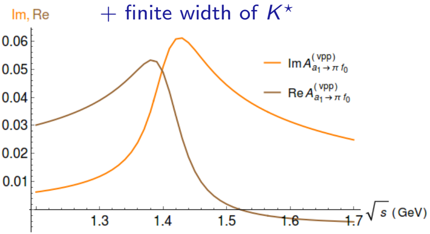


- Spin-Parity of particles.
- Width of $K^\star$

$$\mathbb{M}^{(\mathrm{vpp})}_{a_1 \to \pi f_0} = g^3 \int \frac{\mathrm{d}k_1^4}{(2\pi)^4\, i} \frac{\varepsilon_{0\mu}\left(g^{\mu\nu} - \frac{k_1^\mu k_1^\nu}{k_1^2}\right)(p_1 - k_3)_\nu}{(m_1^2 - k_1^2 - i\epsilon)(m_2^2 - (p_0 - k_1)^2 - i\epsilon)(m_3^2 - (k_1 - p_1)^2 - i\epsilon)} \sim \mathbb{M}^{(\mathrm{sc})}_{a_1 \to \pi f_0} |\vec{p}_\pi|.$$

If one fixes mass of f_0, i.e. $p_{f_0}^2 = m_{f_0}^2$, then only $p_0^2 = s$ is variable.



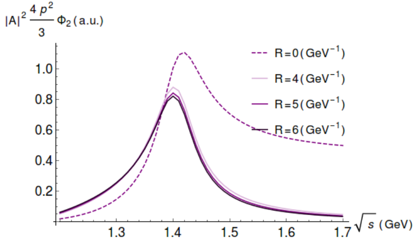


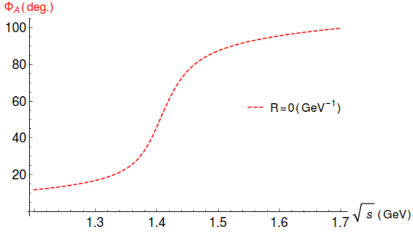
Further corrections to the amplitude

$K^\star(892) \to K\pi$, P-wave decay gives tail to the amplitude.
A left-hand singularity is introduced to correct the amplitude
$g_{K^\star K\pi} \to g_{K^\star K\pi} \times F(k_1)$. k_1 is $K^\star$ four-momentum.

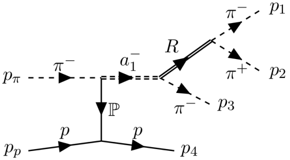
$$F(k_1) = \frac{M^2 - m_{K^\star}^2}{M^2 - k_1^2}, \quad M^2 = (m_\pi + m_K)^2 - \frac{4}{R^2} \; .$$

M is position of the left singularity, it corresponds to the size of $K^\star$:
$F \approx (1 + R^2|\vec{p_0}\,|^2)/(1 + R^2|\vec{p}\,|^2)$. $|\vec{p}|$ is $K^\star \to K\pi$ break up momentum.





Magnitude of the effect



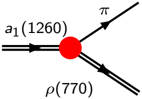
Following the experiment, we compare

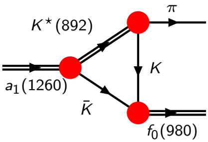
- main channel $a_1(1260) \to \rho(770)\pi$ S-wave decay,
- channel $a_1(1260) \to f_0(980)\pi$ only via kaon rescattering

Couplings: $a_1(1260) \to K^\star K, \rho\pi$, $K^\star \to K\pi$, $f_0 \to K\bar{K}$.

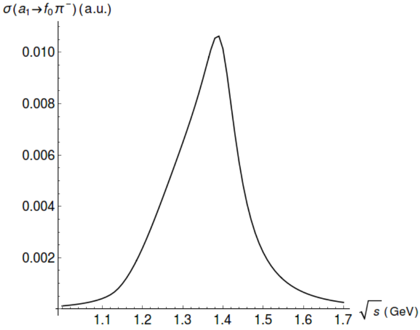
$\tau-$decay | $K^\star-$decay | General analysis of $\pi\pi, R_{K\bar{K}/\pi\pi}$

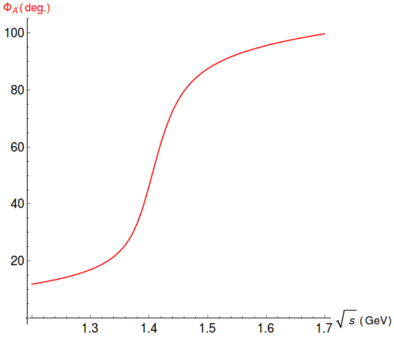
[Coan *et al.*, Phys.Rev.Lett. **92**, 232001 (2004)] [Garcia-Martin *et al.*, Phys.Rev.Lett. **107**, 072001 (2011)]





Result $a_1 \to f_0\pi$





- A narrow peak and sharp phase motion are obtained
- Peak-to-peak ratio $\sigma(a_1 \to f_0\pi)/\sigma(a_1 \to \rho\pi, S-\text{wave}) \approx 1\%$, close to experimental value.

Examples

Selected examples:

- The nature of C-meson. [Achasov *et al.*, Phys.Lett. B207 (1988) 199]
  $\rho(1450) \to (K^\star \bar{K} \to \pi K \bar{K}) \to \pi\phi$
- $\eta(1405/1475)$ puzzle.
  $\eta \to (K^\star \bar{K} \to \pi K \bar{K}) \to \pi a_0(980), \pi f_0(980)$,
  $f_1(1420) \to (K^\star \bar{K} \to \pi K \bar{K}) \to \pi a_0(980), \pi f_0(980)$.
  [Jia-Jun Wu *et al.*, Phys.Rev.Lett. 108 (2012) 081803],
  [Xiao-Gang Wu *et al.*, Phys.Rev. D87 (2013) 1, 014023]
- Some of XYZ states. [Szczepaniak, arXiv:1501.01691]
  $Z_c(3900) - \pi \to (D^\star \bar{D} \to \pi D \bar{D}) \to J/\Psi\pi$
  $\Upsilon(5s) - \pi \to (B^\star \bar{B} \to \pi B \bar{B}) \to \Upsilon(1s)\pi$
- The nature of $a_1(1420)$. [Mikhasenko *et al.*, Phys. Rev. D **91**, no. 9 (2015) 094015]
  $a_1(1260) \to (K^\star \bar{K} \to \pi K \bar{K}) \to \pi f_0(980)$

Conclusions

- Huge data samples and advanced analysis tools make possible to see tiny features of strong interactions.
- $a_1(1420)$ is definitely interesting object to study.
- $a_1(1420)$ can be explained by triangle singularity, *resonance-like* effect of appearance of logarithmic branch point (not a pole) in amplitude caused by rescattering of kaons.
  - Sharp phase motion $\sim 90^\circ$ is obtained with respect to $a_1 \to \rho\pi$.
  - The magnitude of $\sigma(a_1 \to f_0\pi)$ is of the order of experimental value. Better knowledge of coupling is required.
- The full rescattering amplitude in presence of many inelastic channels can't be calculated, but difference to tree level can be noticed.
- We have now several hints that the triangle singularity plays important role in meson spectroscopy.

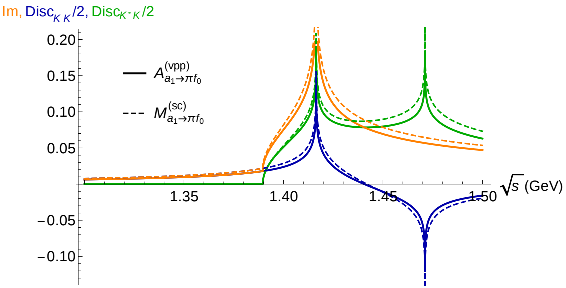
I thank Bernhard Ketzer and Andrey Sarantsev.

Thank you for attention

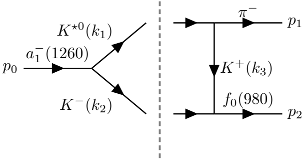
Backup

The imaginary part based on Cutkosky cutting rules

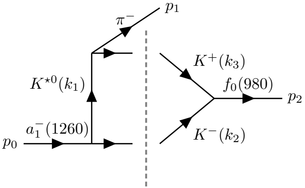
$$2\mathrm{Im}A = \mathrm{Disc}_{K\star\bar{K}}\,A + \mathrm{Disc}_{K\bar{K}}\,A$$



$\mathrm{Disc}_{K\star\bar{K}}\,A$:



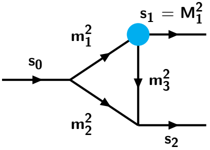
$\mathrm{Disc}_{K\bar{K}}\,A$:



The different kinematical regions, [Eden *et al.*, Cambridge Univ.Pr.(2002)]

Introduce normalized invariants

$$
\begin{aligned}
y_0 &= (s_0 - m_1^2 - m_2^2)/(2m_1 m_2),\\
y_1 &= (s_1 - m_1^2 - m_3^2)/(2m_1 m_3),\\
y_2 &= (s_2 - m_2^2 - m_3^2)/(2m_2 m_3)
\end{aligned}
$$



To simplify representation, we fix internal masses and $\mathbf{s_1}$.
Depending on value of $s_1(y_1)$, we have:

$|y_1| < 1$ "No decays": $M_1 < m_1 + m_3$, $m_1 < M_1 + m_3$, $m_3 < M_1 + m_1$.

- The triangle singularity is known as **anomalous threshold**. It sits below the normal threshold for s_0, s_2.
- Appears in form-factors.

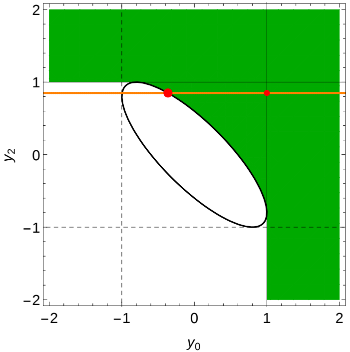
$y_1 < -1$ "Internal decay": $m_1 > M_1 + m_3$ or $m_3 > M_1 + m_1$.

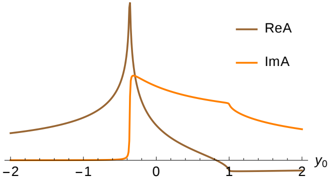
- Position is given by decay kinematics.
- Appears in decays.

$y_1 > 1$ "External decay": $M_1 > m_1 + m_3$.

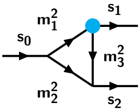
Is covered above, just if we choose another variable to fix.

"No decays" region: position of singularities





Invariant s_1 is fixed,
$|y_1| = |(s_1 - m_1^2 - m_3^2)/(2m_1 m_3)| < 1.$



- the position of leading log singularity is arc of ellipse,
- solid $y_0 = +1$, $y_2 = +1$ lines are normal thresholds: $(m_1 + m_2)^2$ for s_0 and $(m_2 + m_3)^2$ for s_2 (lower order singularities).
- Green area: $\mathrm{Im}A \neq 0$ (above threshold).

An example: deuteron size [Gribov lecture, Cambridge Univ.Pr.(2009)]

Non-relativistic quantum mechanics

Proton and neutron bound state,

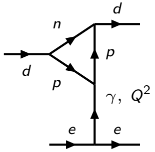
- binding energy: $\varepsilon = M_D - m_p - m_n$,
- ψ is the wave function, $\psi \sim e^{-r\sqrt{\varepsilon m}}$.
- Electron scattering amplitude: $f(q) = \frac{e^2}{q^2} F(q)$, $F(q)$ is a FF.
- $F(q)$ is given by charge density inside of the deuteron

$F(q) = \int d^3\mathbf{r}\, \psi^2(r)\, e^{i\mathbf{qr}} \approx 1 - \frac{q^2 R^2}{2} + O(q^4)$
R is electromagnetic radius,

$$R^2 \sim (\varepsilon m)^{-1}$$

Relativistic-theory framework

Radius is determined by t-channel singularities.



The singularities in Q^2:

- Normal threshold $Q_0^2 = (2m_p)^2$
- Anomalous threshold $Q_0^2 \approx 16\varepsilon m$

Further examples

Selected examples:

- Some Experimental Consequences of Triangle Singularities in Production Processes

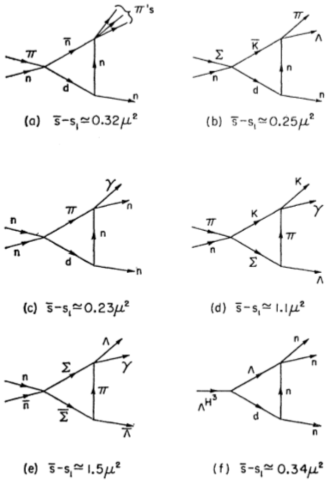
  [Landshoff *et al*, Phys. Rev. 127, 649 (1962)]

- The Deuteron as a composite two nucleon system

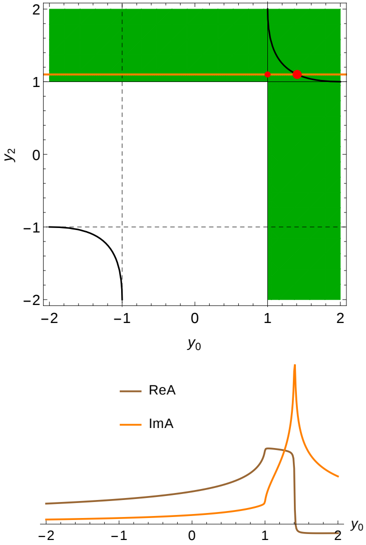
  [Anisovich *et al*, Nucl.Phys. A544 (1992) 747-792]

- Form-factors of meson decays

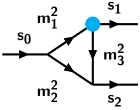
  [Melikhov, Phys.Rev. D53 (1996) 2460-2479]



"Internal decay" region: position of singularities



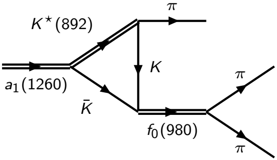
Invariant s_1 is fixed,
$y_1 = (s_1 - m_1^2 - m_3^2)/(2m_1 m_3) < -1.$



- position of leading log singularity is solid arc of hyperbola,
- solid $y_0 = +1$, $y_2 = +1$ lines are normal thresholds: $(m_1 + m_2)^2$ for s_0 and $(m_2 + m_3)^2$ for s_2.
- Green area: Im$A \neq 0$ (above threshold).

The triangle diagram

- condition of "internal decay" is satisfied: $m_{K^\star} > m_K + m_\pi$
- $a_1(1260)$ is rather wide, so s_0 can go above $K^\star \bar{K}$ threshold.
- f_0 sits at the $K\bar{K}$ threshold.

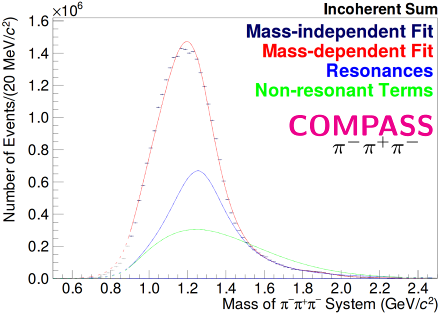


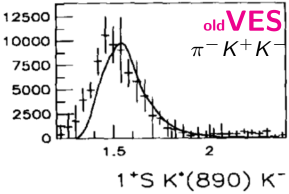
The triangle singularity can appear in the physical region. Whether it does is given by Landau equations.

Two isospin combinations contribute:

- $a_1^-(1260) \to K^{\star 0} K^- \to \pi^- K^+ K^- \to \pi f_0(980)$
- $a_1^-(1260) \to K^{\star -} K^0 \to \pi^- K^0 \bar{K}^0 \to \pi f_0(980)$

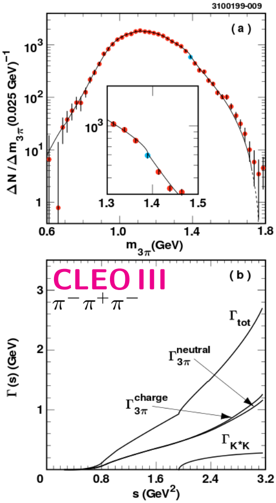
$a_1(1260)$ measurements





[Berdnikov *et al.*, Nuovo Cim. 107, 1941 (1994)]

[Aster *et al.*, Phys.Rev. D, **60**, 0120002 (1999)]



List of $a_1(1260)$ decays

TABLE III. Results of the nominal fit for the moduli $|\beta_i|$ and phases ϕ_{β_i} of the coefficients for the amplitudes listed in Eq. (6). The two errors shown are statistical and systematic, respectively. The branching fractions $\mathcal{B}$ are derived from the squared amplitudes (using the values of $|\beta_i|$), and are normalized to the total $\tau^- \to \nu_\tau \pi^- \pi^0 \pi^0$ rate. These do not sum to 100%, due to interference between the amplitudes.

		Signif.	$\lvert\beta_i\rvert$	ϕ_{β_i}/π	$\mathcal{B}$ fraction (%)
ρ	s-wave		1	0	68.11
$\rho(1450)$	s-wave	1.4σ	$0.12\pm0.09\pm0.03$	$0.99\pm0.25\pm0.04$	$0.30\pm0.64\pm0.17$
ρ	d-wave	5.0σ	$0.37\pm0.09\pm0.03$	$-0.15\pm0.10\pm0.03$	$0.36\pm0.17\pm0.06$
$\rho(1450)$	d-wave	3.1σ	$0.87\pm0.29\pm0.06$	$0.53\pm0.16\pm0.06$	$0.43\pm0.28\pm0.06$
$f_2(1270)$	p-wave	4.2σ	$0.71\pm0.16\pm0.05$	$0.56\pm0.10\pm0.03$	$0.14\pm0.06\pm0.02$
σ	p-wave	8.2σ	$2.10\pm0.27\pm0.09$	$0.23\pm0.03\pm0.02$	$16.18\pm3.85\pm1.28$
$f_0(1370)$	p-wave	5.4σ	$0.77\pm0.14\pm0.05$	$-0.54\pm0.06\pm0.02$	$4.29\pm2.29\pm0.73$

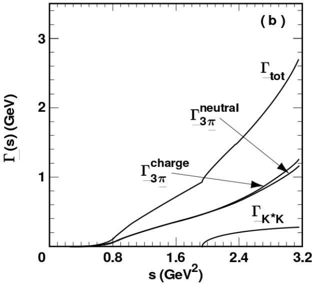
[Aster *et al.*, Phys.Rev. D, **60**, 0120002 (1999)]

a_1 decays, [Aster *et al.*, Phys.Rev. D, **60**, 0120002 (1999)]

The best knowledge is from τ decays (mass dependent analysis).
Short results: $m_{a_1} \approx 1.33\,\text{GeV}$, $\Gamma_{a_1}(m_{a_1}) \approx 814\,\text{GeV}$,
$\text{Br}(a_1 \to \rho\pi) \approx 60\%$, $\text{Br}(a_1 \to \sigma\pi) \approx 20\%$, $\text{Br}(a_1 \to f_0(1370)\pi) \approx 7\%$,
$\text{Br}(a_1 \to K^\star \bar{K}) \approx 2.2\%$.

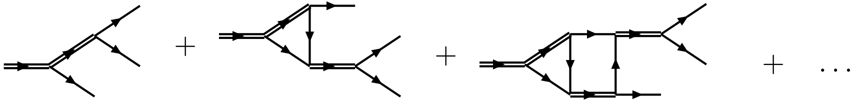
Correction for 'finite size of meson' is done by exponential suppression (not by Blatt-Weitskopf), $R = 1..2/GeV^2$ is strongly correlated with $\Gamma_{a_1}(m_{a_1})$.

- $a_1 \to \rho\pi$ S wave,
- $a_1 \to \sigma\pi$ P wave,
- $a_1 \to K^\star \bar{K}$ S wave.



Let final states re-scatter

Isobar model $+$ 1^{th}-rescattering $+$ 2^{th}-rescattering $+$ etc.



Possible isobars from $a_1(1260)$: $\rho(770)$, $f_0(500)$, $\rho(1450)$, $f_0(1370)$, $K^\star(892)$. And then all possible particles can be result of secondary interaction.

$K^\star \to K\pi$ P-wave decay

- P-wave. In our parametrization:

$$\Gamma = \frac{1}{2m_{K^\star}} \frac{1}{8\pi} \frac{2|\vec{k}|}{m_{K^\star}} \times g^2_{K^\star K\pi} \frac{4}{3} |\vec{k}^2|.$$

- $\Gamma_{K^\star \to K\pi} = 50.8\,\text{MeV}$. So, $g^2_{K^\star K\pi} = 31.2$.
- From isospin $\text{Br}(K^{\star -} \to K^0\pi^-)/\text{Br}(K^{*-} \to K^-\pi^0) = 2$.

$f_0(980)$ decays

- Flatte Flatte, Phys.Lett.(1976),
- Flatte-Like Achasov(2003), Bary(2005),

$$f_e \sim \frac{1}{m_{f_0}^2 - s - i(g_{\pi\pi}^2 \rho_{\pi\pi} + g_{KK}^2 \rho_{KK})/2}$$

$$f_e \sim \frac{1}{m_{f_0}^2 - s - i m_{f_0}(\bar{g}_{\pi\pi} k_\pi + \bar{g}_{KK} k_K)}$$

The problem with parameters of f_0 is scaling behaviour near $K\bar{K}$ threshold. It results in $R = g_{KK}^2/g_{\pi\pi}^2$ is known (more or less) $\approx 3\ldots4$, branching ratio $f_0 \to \pi\pi$ is ≈ 0.4, but g_{KK}, $g_{\pi\pi}$ variates.

$$\bar{g}_{\pi\pi} = \frac{g_{\pi\pi}^2}{8\pi m_{f_0}^2} = 0.1\ldots0.6$$