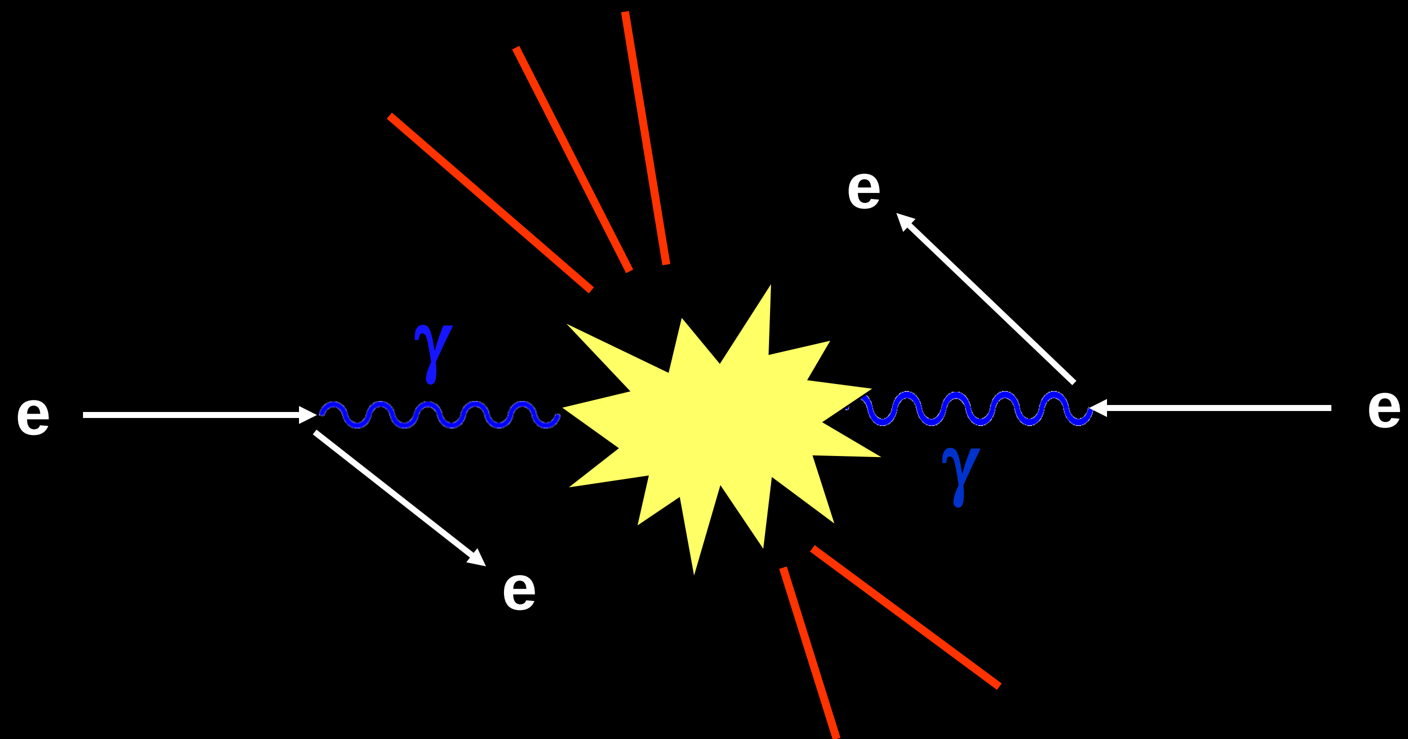
Light-by-light scattering: a paradigm for S-matrix methods

INDIANA UNIVERSITY

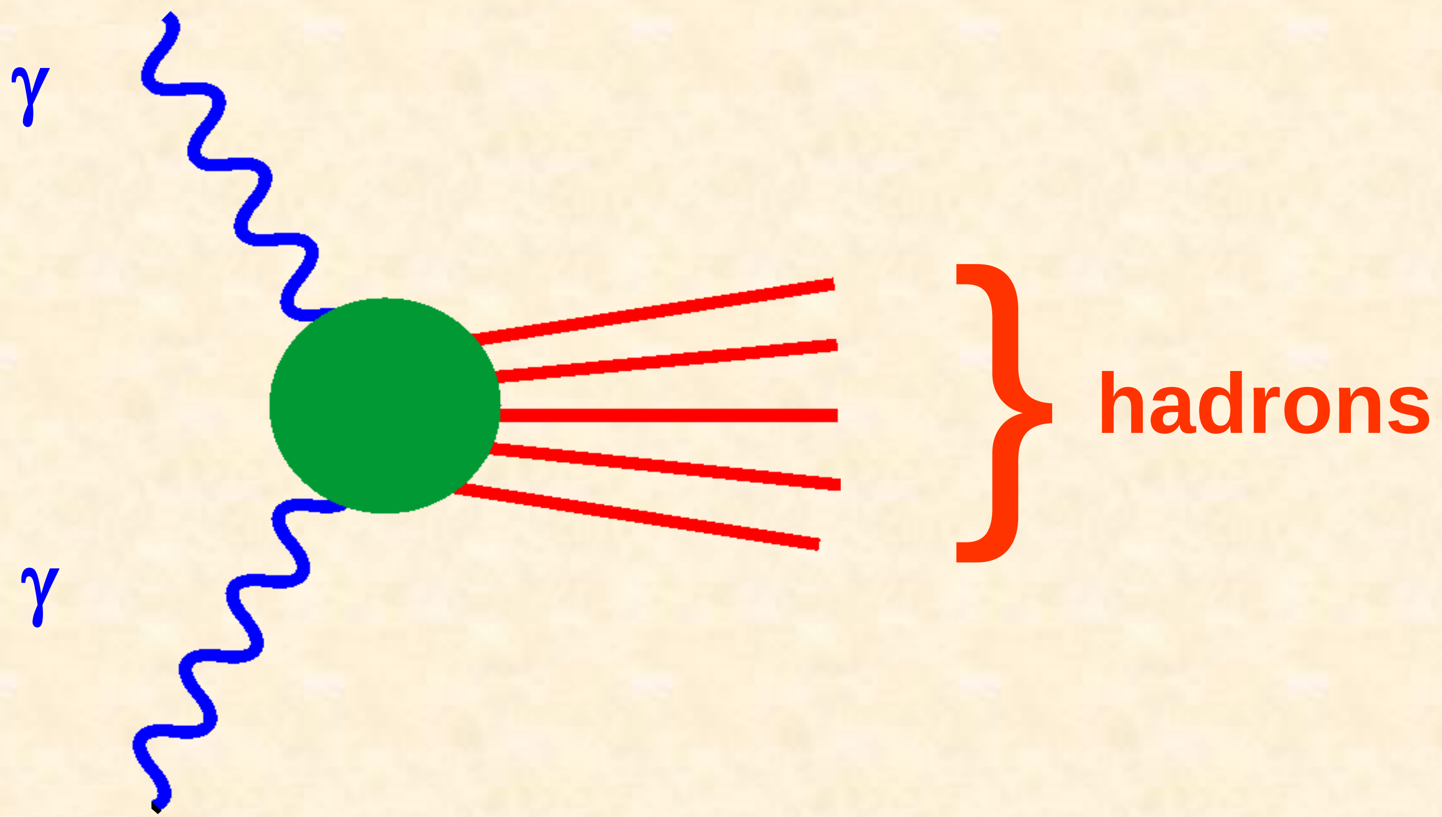
Tuesday June 9th, 2015

Michael Pennington

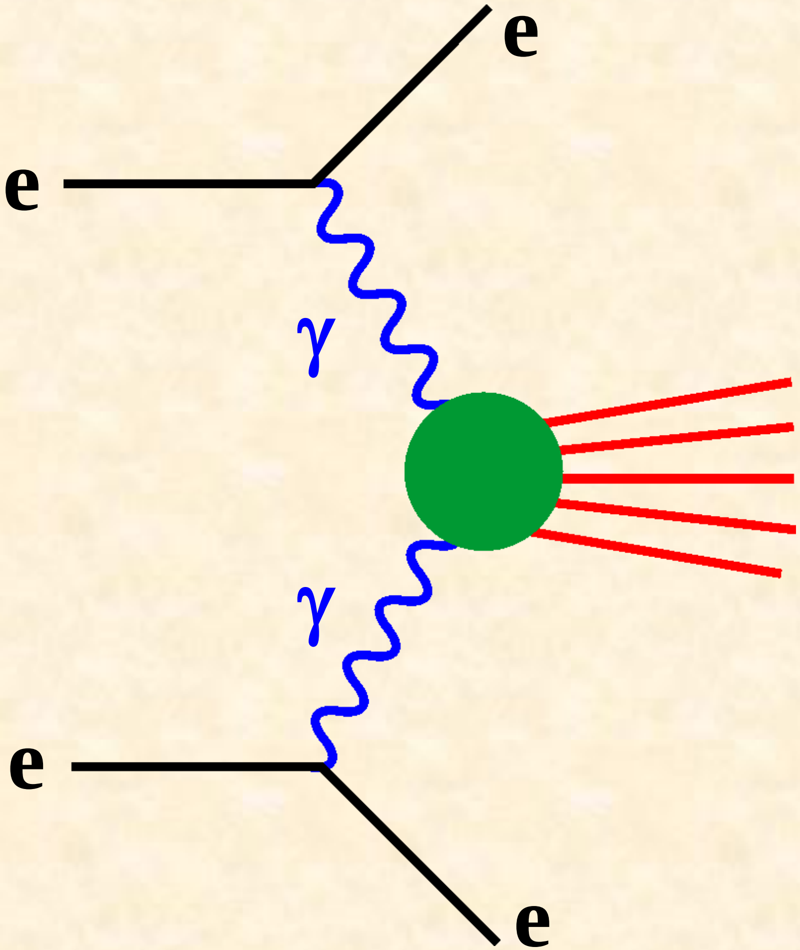
Light-by-light scattering:
a paradigm for S-matrix methods



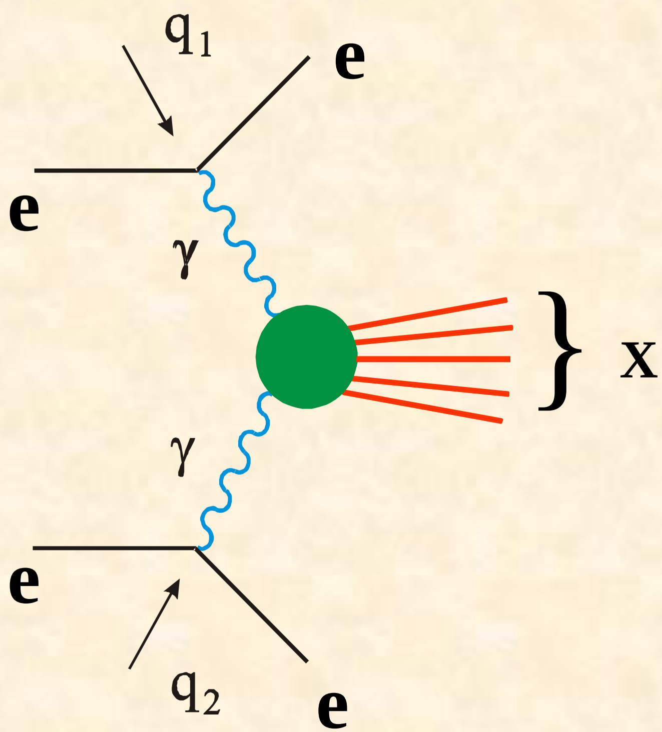
Two Photon Physics

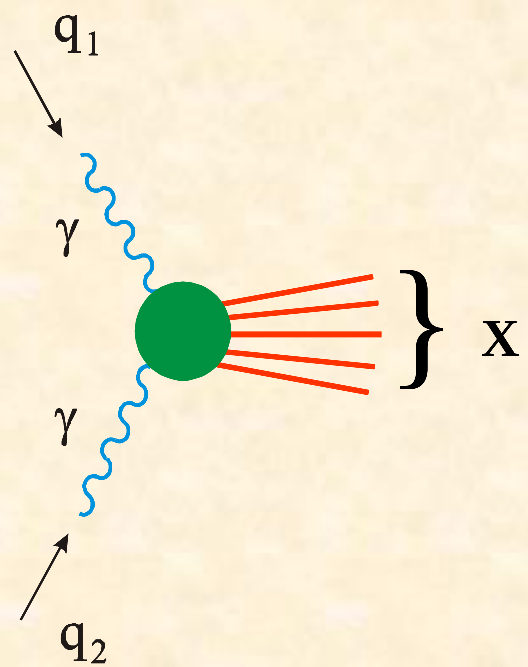


Two Photon Physics at e^+e^- colliders



Brodsky, Kinoshita & Terazawa





Brodsky, Kinoshita & Terazawa

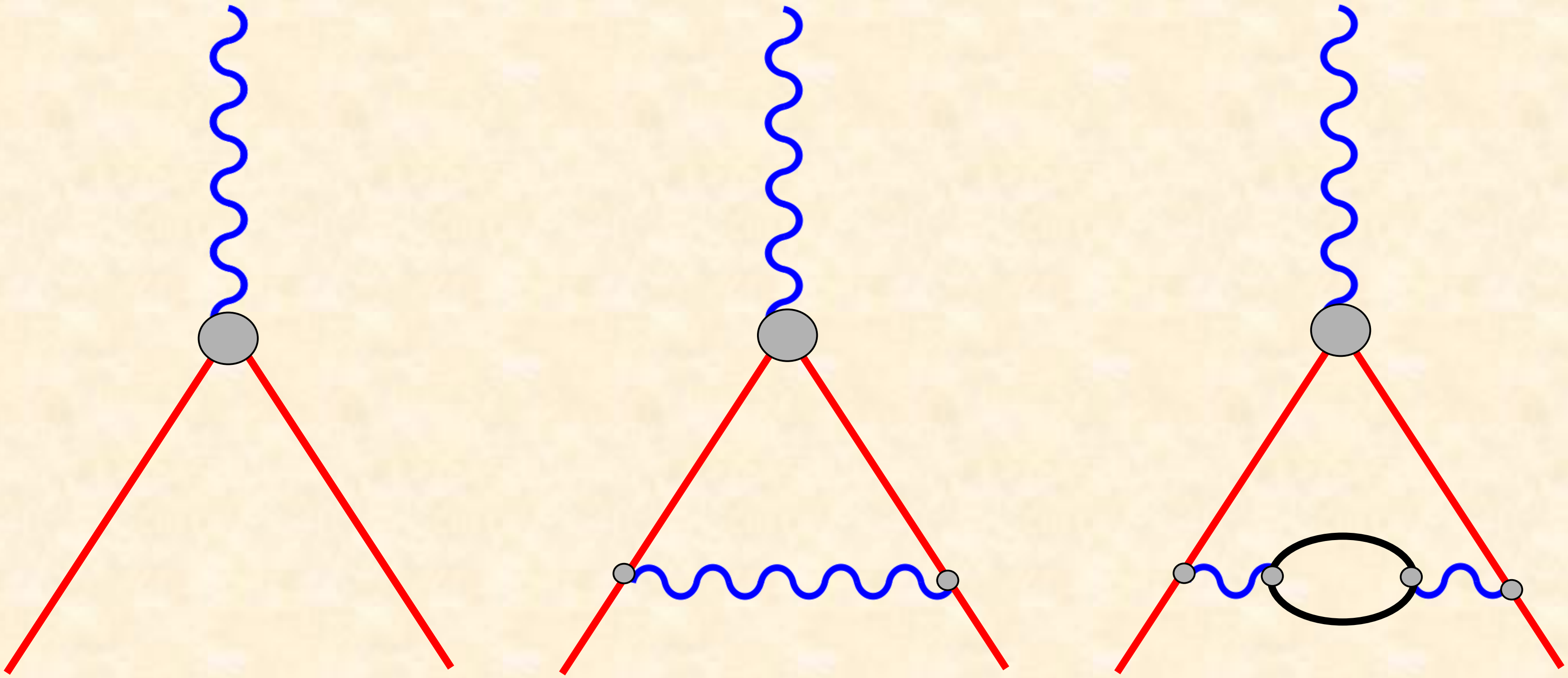
$$\sigma\,(e^+e^- \longrightarrow e^+e^-X) \;=\; \frac{\alpha^2}{2\pi^2}\,\boxed{ln^2 \frac{s}{4m_e^{\,2}}} \int \frac{dW^2}{W^2}\, f\!\left(\frac{W^2}{s}\right)\; \sigma\,(\gamma\gamma \longrightarrow X;\, W^2)$$

where $$f\,(x) \;=\; \frac{1}{2}\,(2+x)2\,ln\,(1/x) \;-\; (1-x)\,(3+x)$$

Light by Light

big uncertainty in $(g-2)_\mu$

g – 2 contributions



Dirac

Schwinger

$$a_\ell = (g\text{-}2)_\ell / 2$$

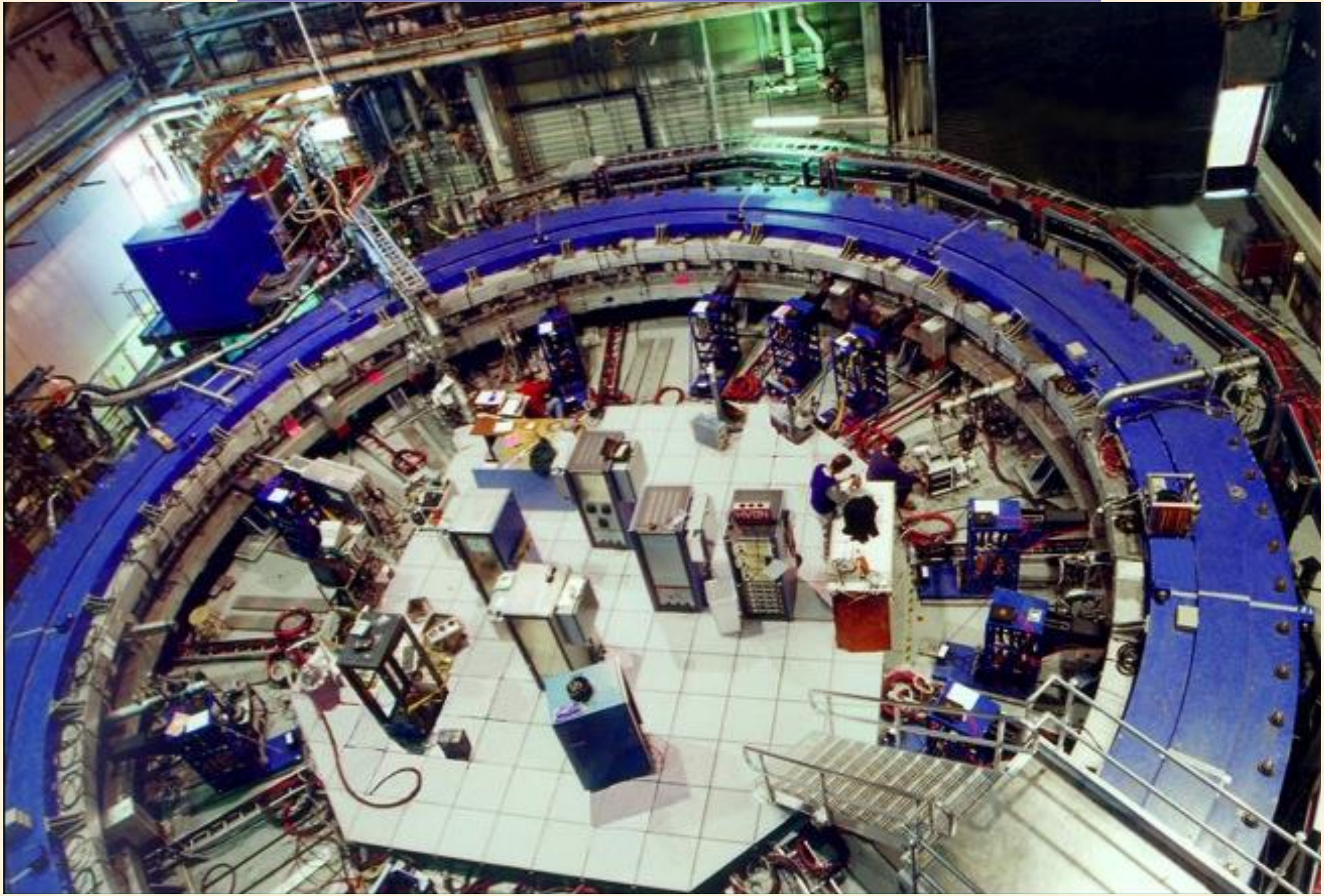
Mount Auburn Cemetery

$(g\text{-}2)_e$: Experiment v Standard Model

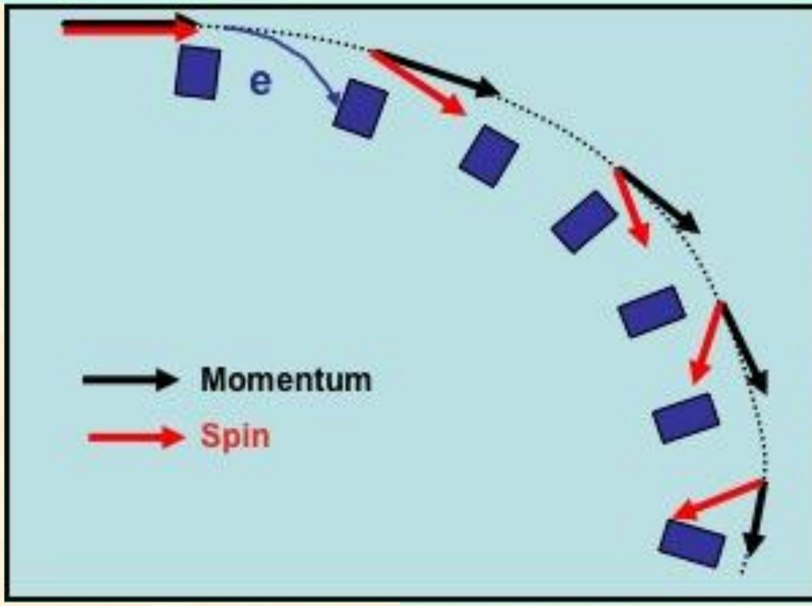
	a_e [10^{-11}]	Δa_e [10^{-11}]
QED $O(\alpha \rightarrow \alpha^5)$	115965218.007	0.007
Electroweak	0.003	0.001
Hadronic	0.168	0.02
Theory Total	115965218.178	0.02
Experiment	115965218.073	0.028

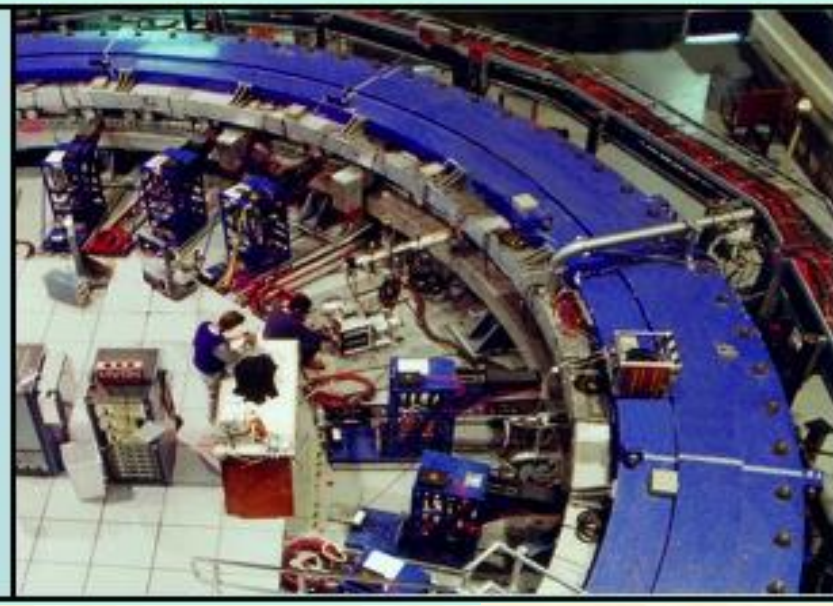
$1/\alpha$ (^{87}Rb) 137.035999049 (90)

BNL $(g\text{-}2)_\mu$ experiment



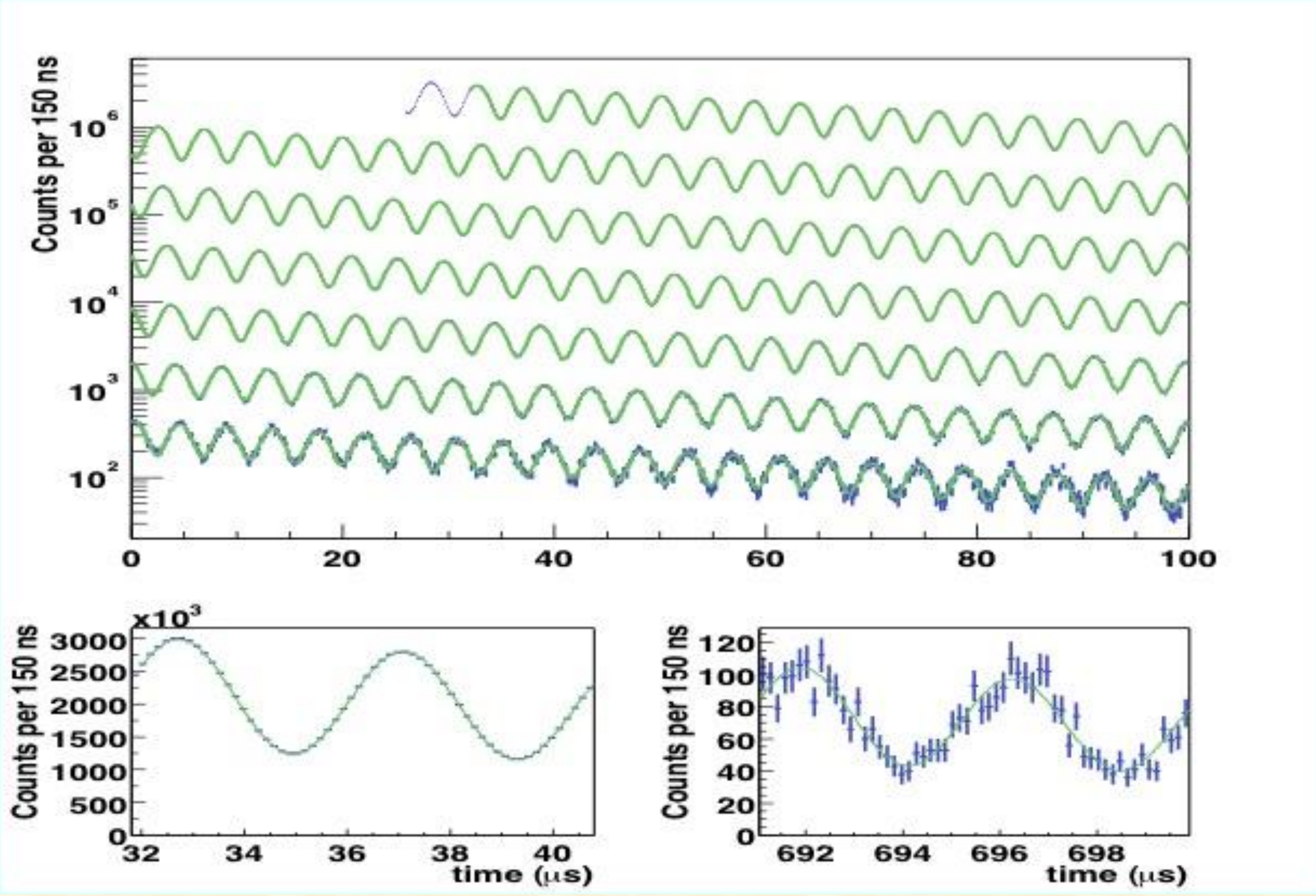
a_μ is proportional to the difference between the **spin precession** and the **rotation rate**





$$\Delta\omega = \omega_a = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

$$N(t) = N_0 e^{-t/\tau} [1 + A\cos(\omega_a t + \phi)]$$



$(g\text{-}2)_\mu$: Experiment v Standard Model

	a_μ [10^{-11}]	Δa_μ [10^{-11}]
QED $O(\alpha \rightarrow \alpha^5)$	116584718.95	0.04
Electroweak	156.0	1.0
Hadronic Vac Pol	6851	43
Hadronic LbL	116	40
Theory Total	116591839	59
Experiment	116592089	63

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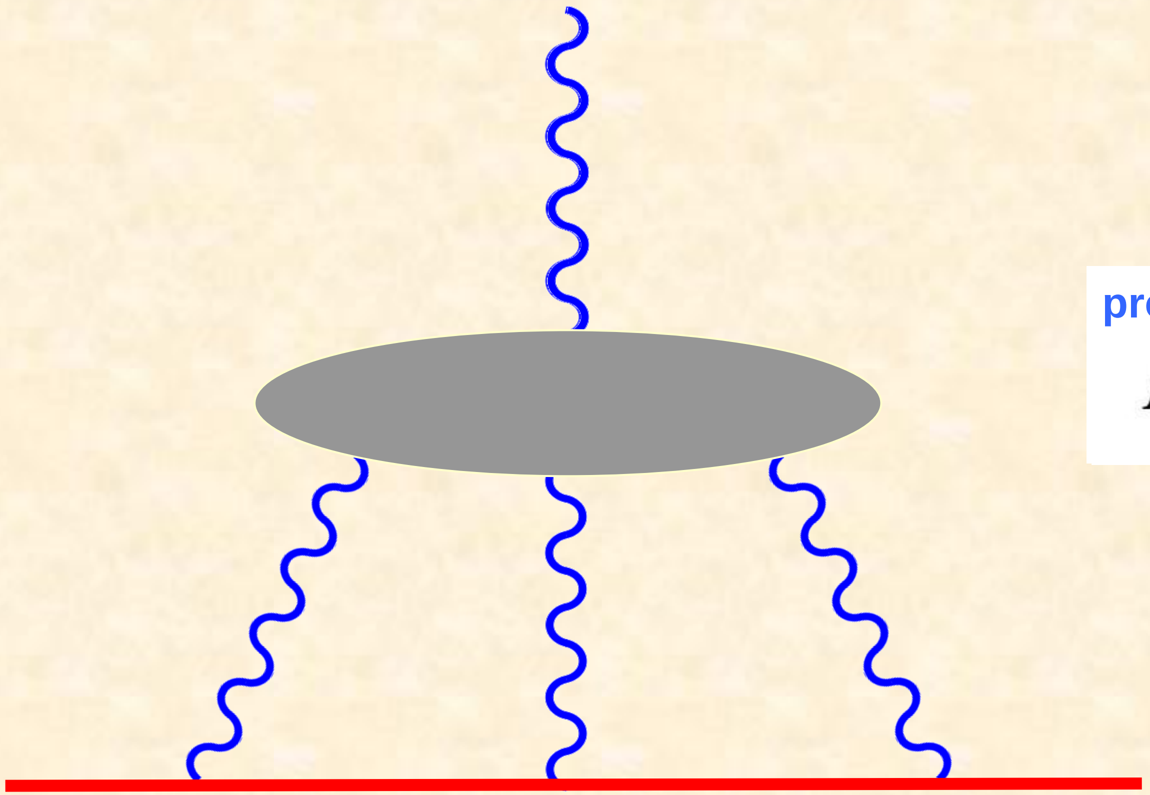
Light by Light

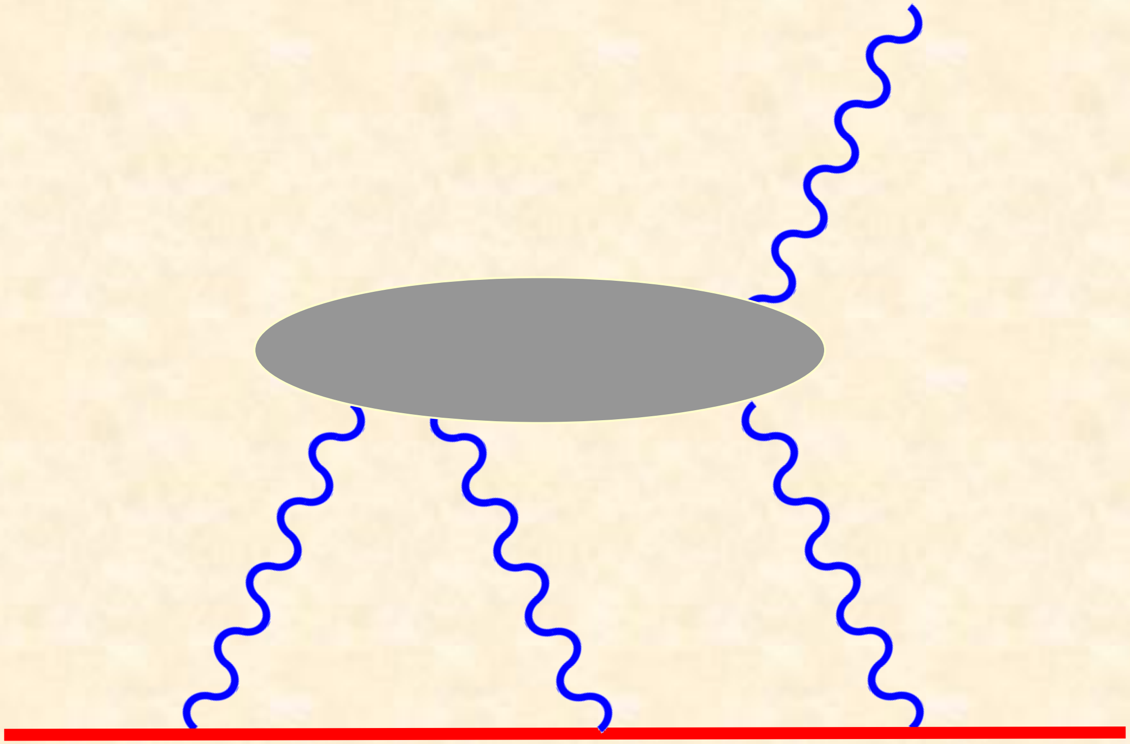


projector

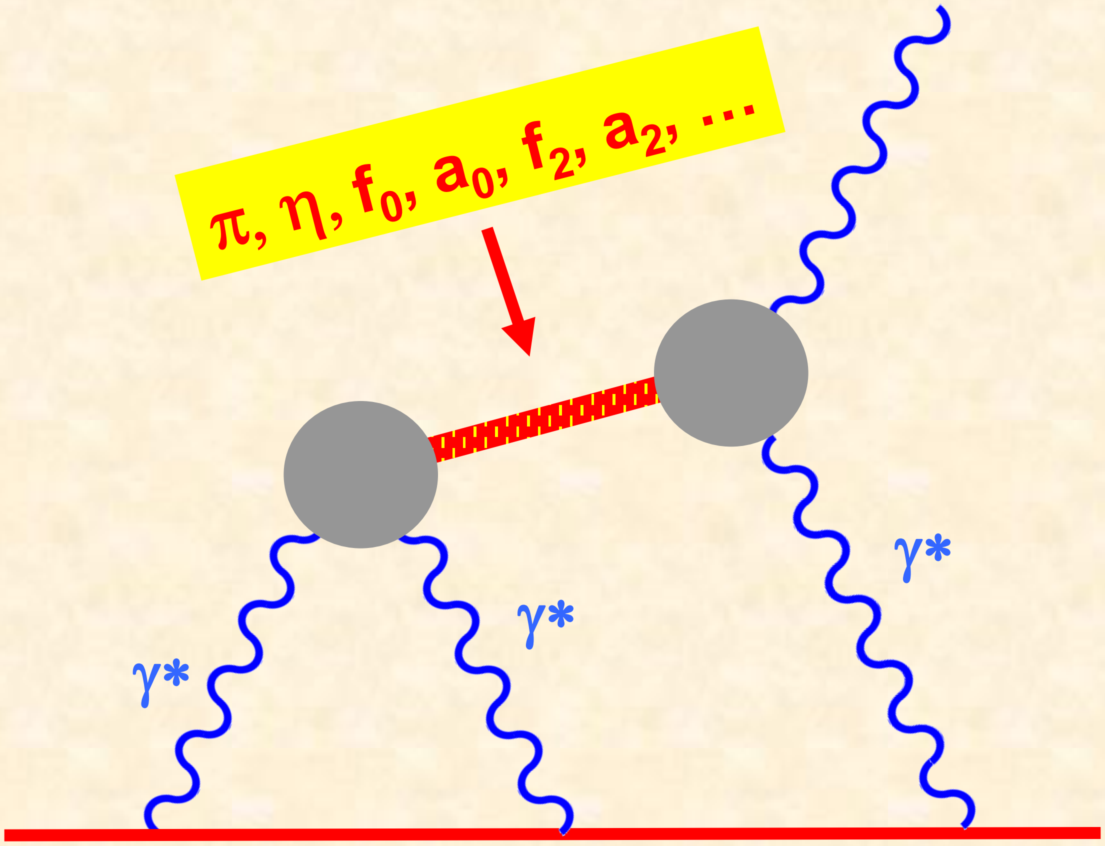
$$\Lambda^{\nu}(p',p)$$

$$a_\mu \quad = \quad \lim_{k\to 0} \, \mathrm{Tr}\left\{ (\not{p} + m)\, \Lambda^{\nu}(p',p)\, (\not{p}' + m)\, \Gamma_{\nu}(p',p) \right\}$$

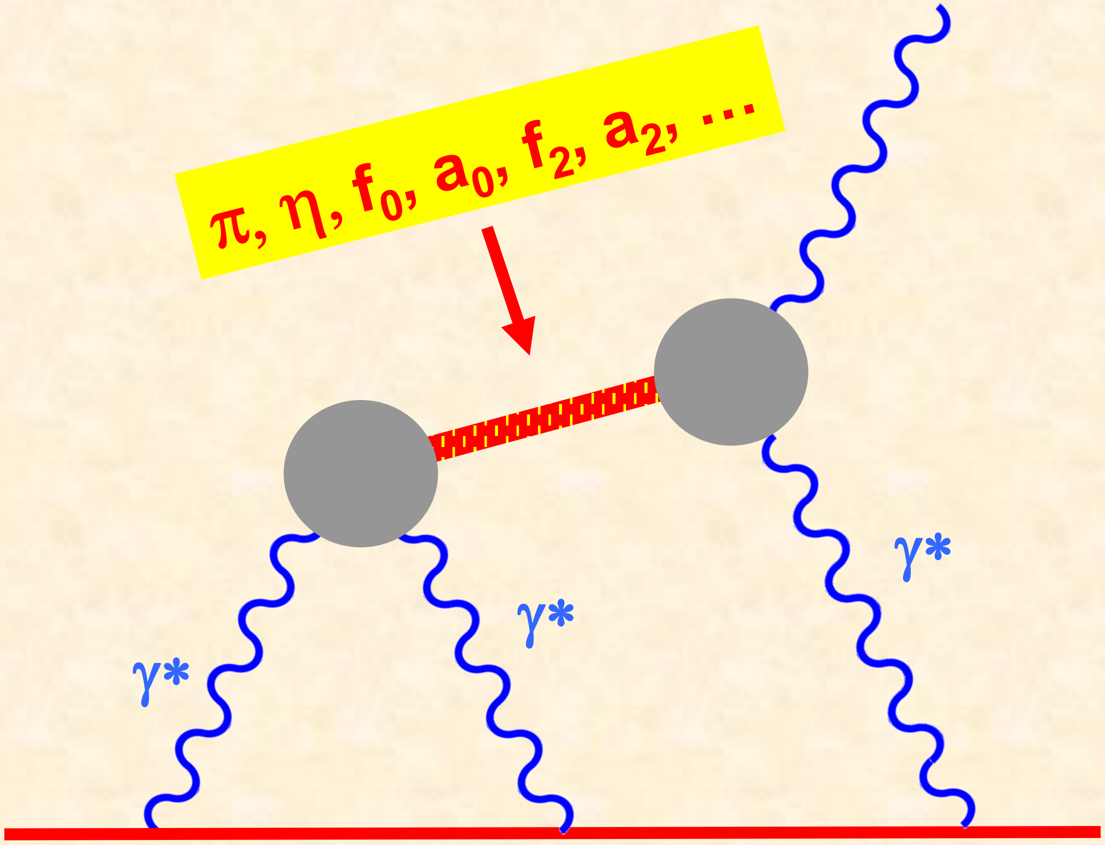
Light by Light



Light by Light

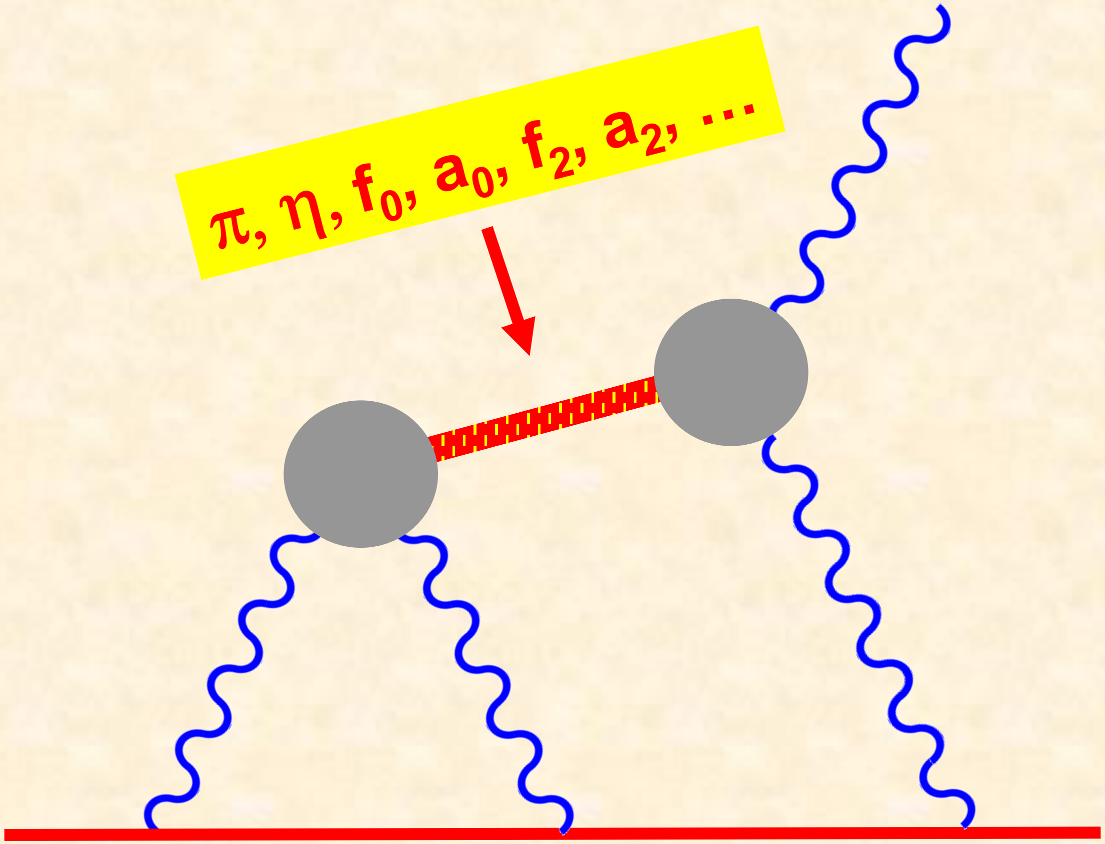


Light by Light

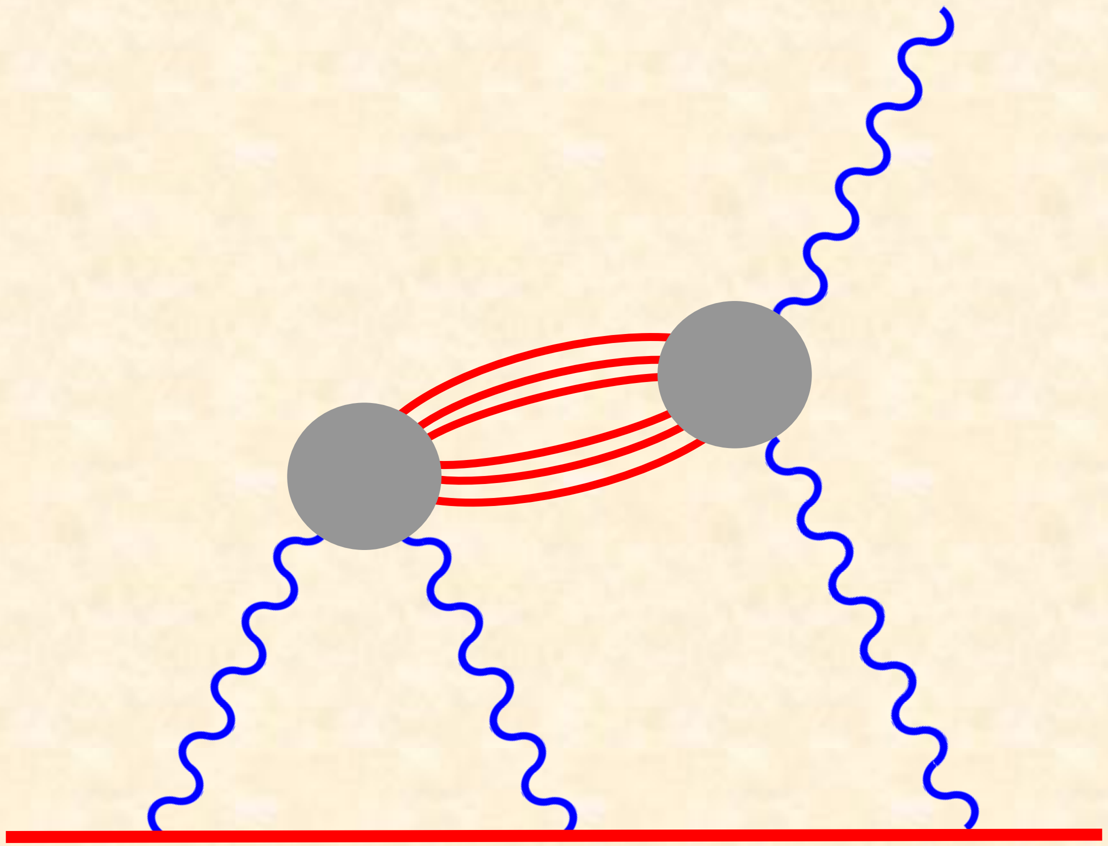


Contributions	BFP	HKS	KN	MV	PdRV	N/JN
π^0,η,η'	85 +- 13	82.7+-6.4	83+-12	114+-10	114+-13	99+-16
π,K loops	-10 +- 12	-4.5+-8.1			-19+-19	-19+-13
axial vectors	2.5+-1.0	1.7+-1.7		22+- 5	15+-10	22+-5
scalars	-6.5+- 2.0				-7+-7	-7+-2
quark loops	21 +- 3	9.7+-11.1			2.3+-	21+-3
Total	83 +-32	89.6+-15.4	80+-40	136+-25	105+-26	116+-39

Light by Light



Light by Light



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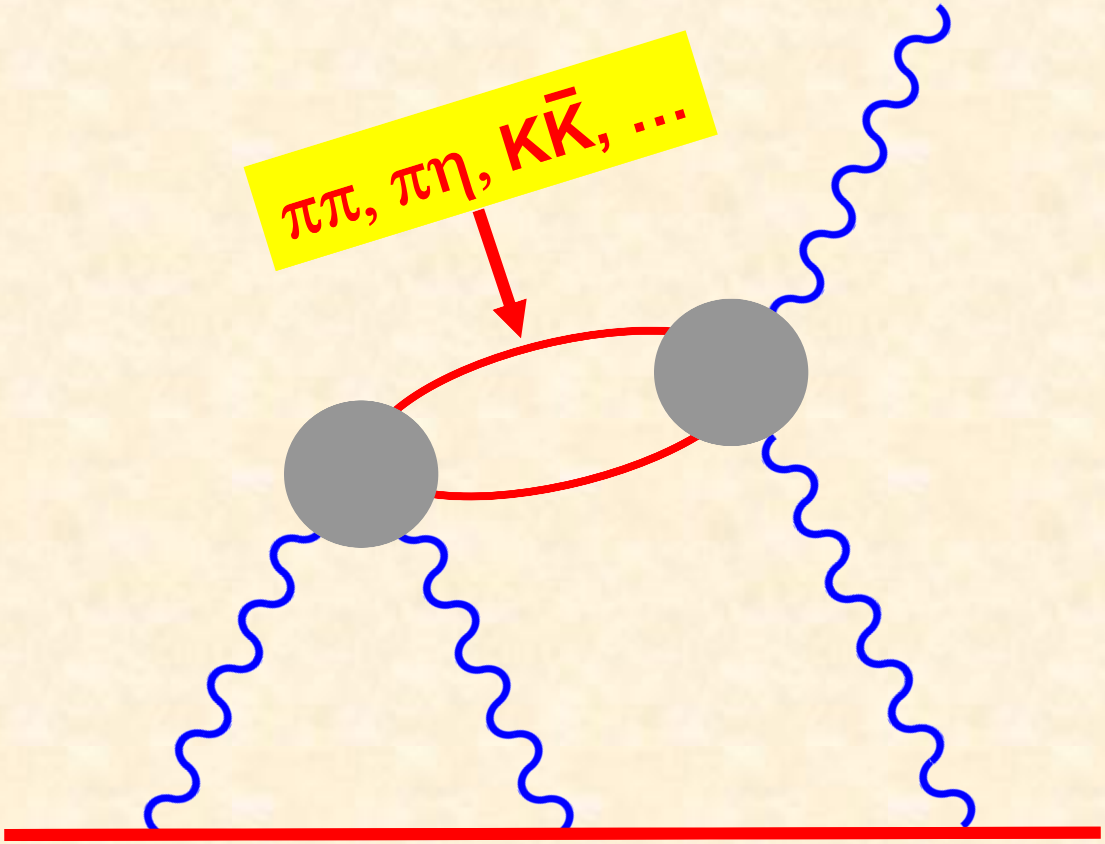

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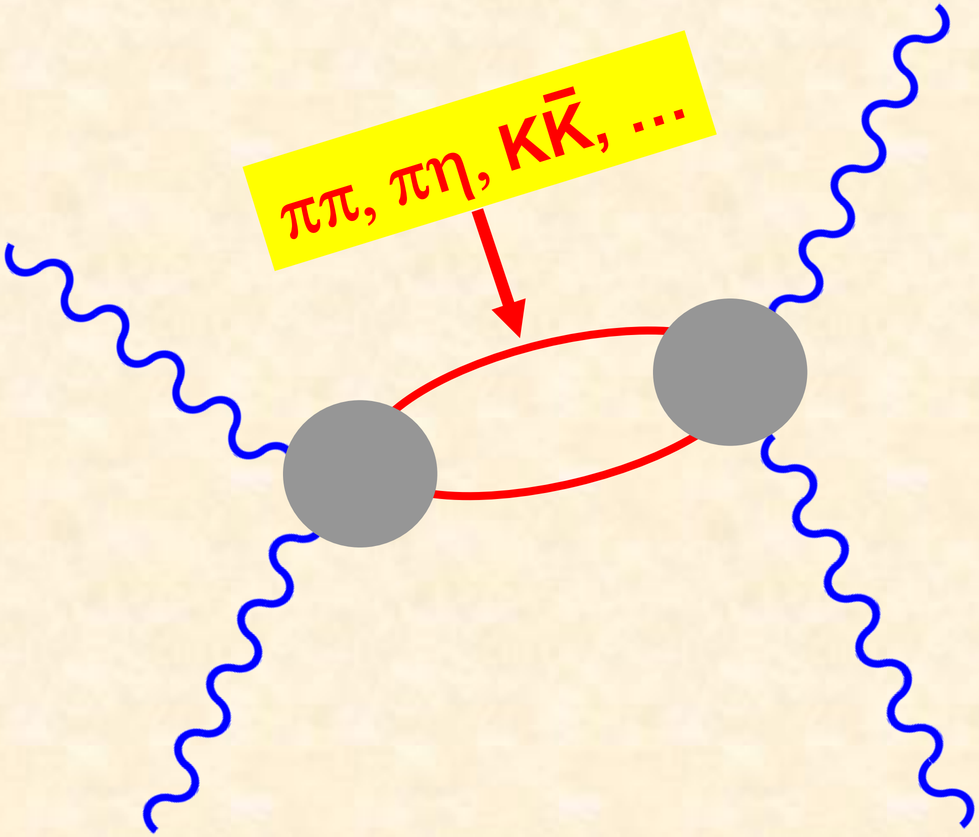
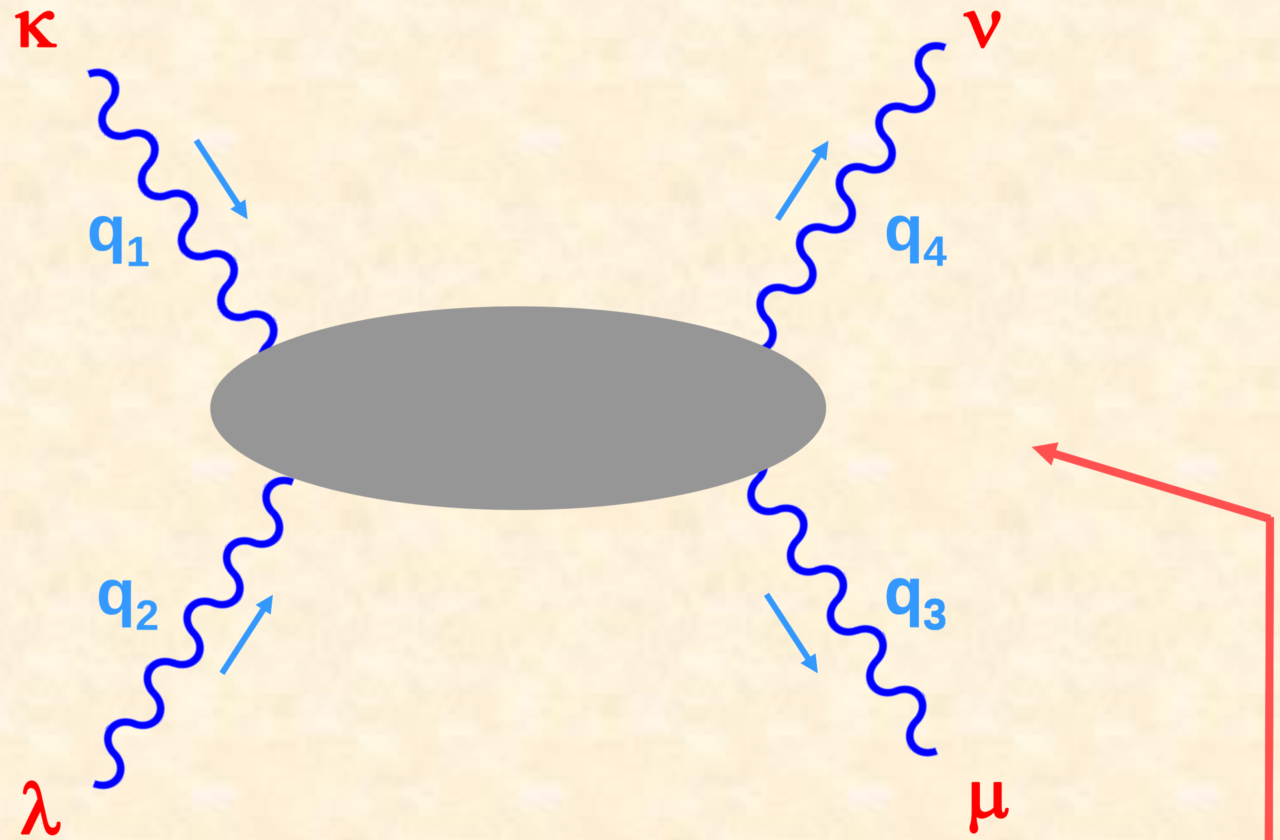
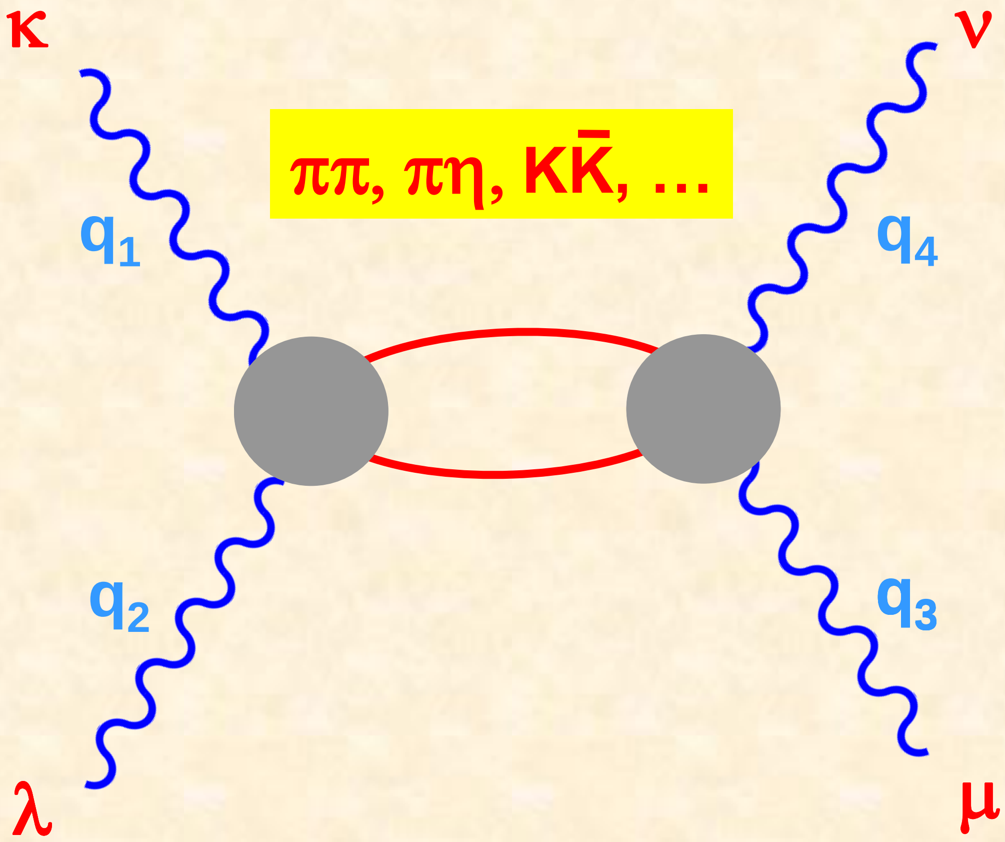


Light by Light

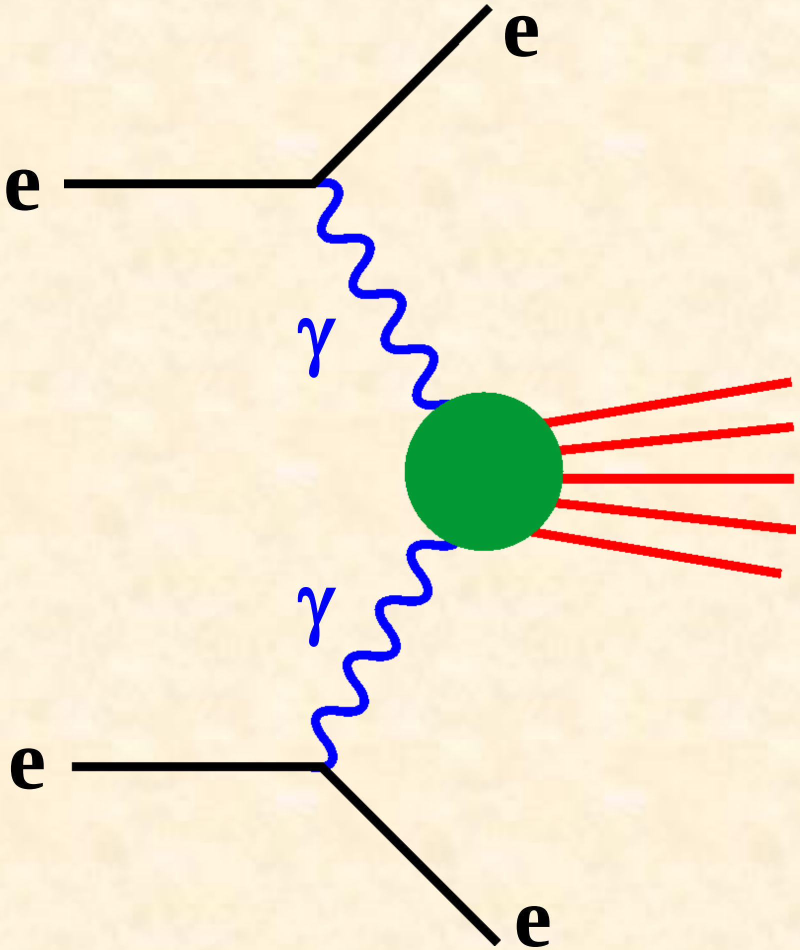


$$\Gamma_\nu = \frac{\alpha^3}{\pi} \int \frac{d^4q_1\, d^4q_2}{(2\pi^2)^2\, q_1^2\, q_2^2\, q_3^2}\, \frac{\gamma^\kappa\,(\not{p}' + \not{q}_1 + m)\,\gamma^\mu\,(\not{p} - \not{q}_2 + m)\,\gamma^\lambda}{((p'+q_1)^2 - m^2)\,((p-q_2)^2 - m^2)}\, \Pi_{\kappa\lambda\mu\nu}$$

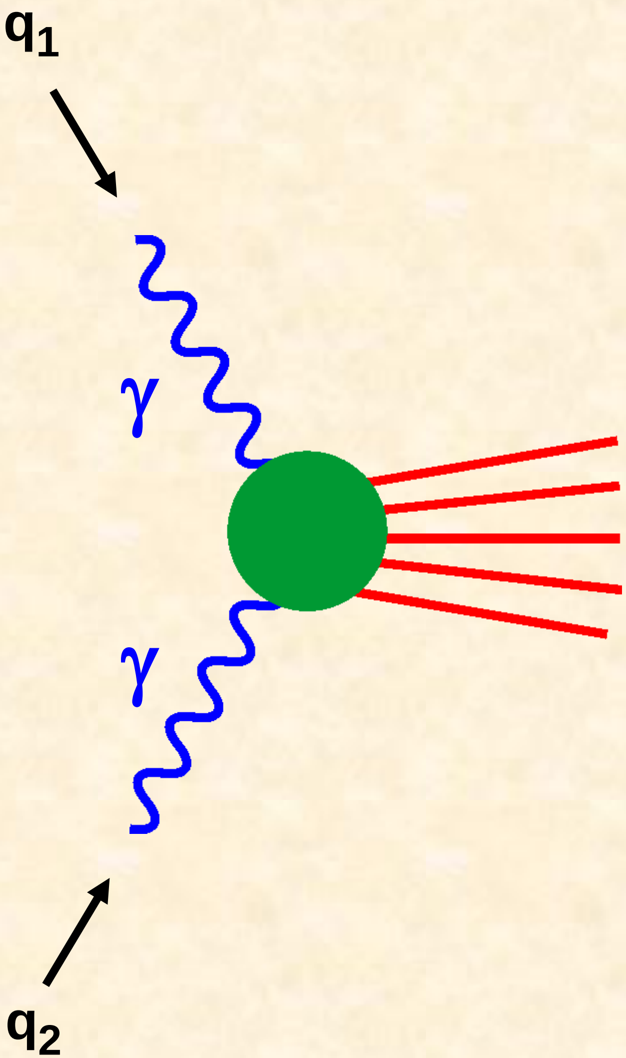
Light by Light



Two Photon Physics at e^+e^- colliders

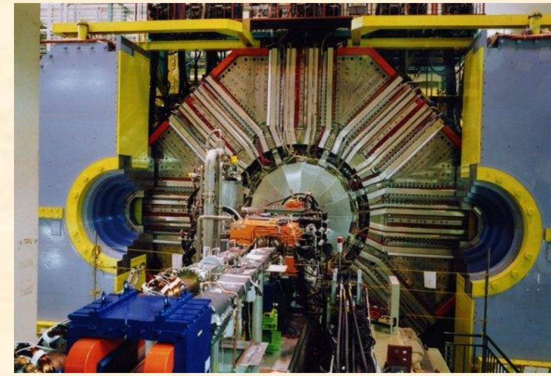


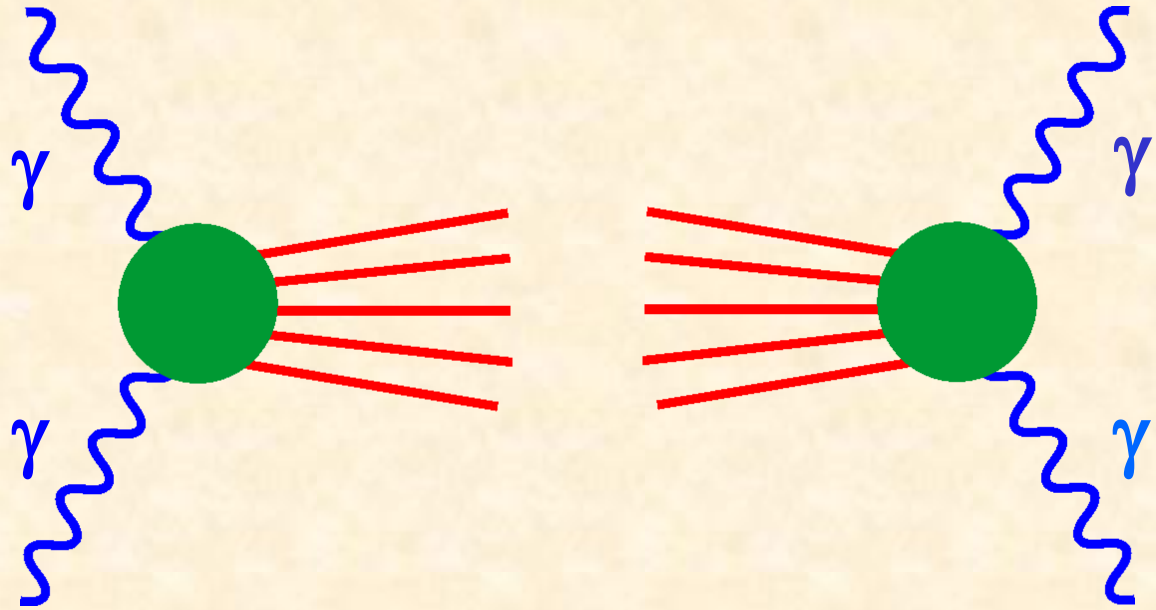
Brodsky, Kinoshita & Terazawa

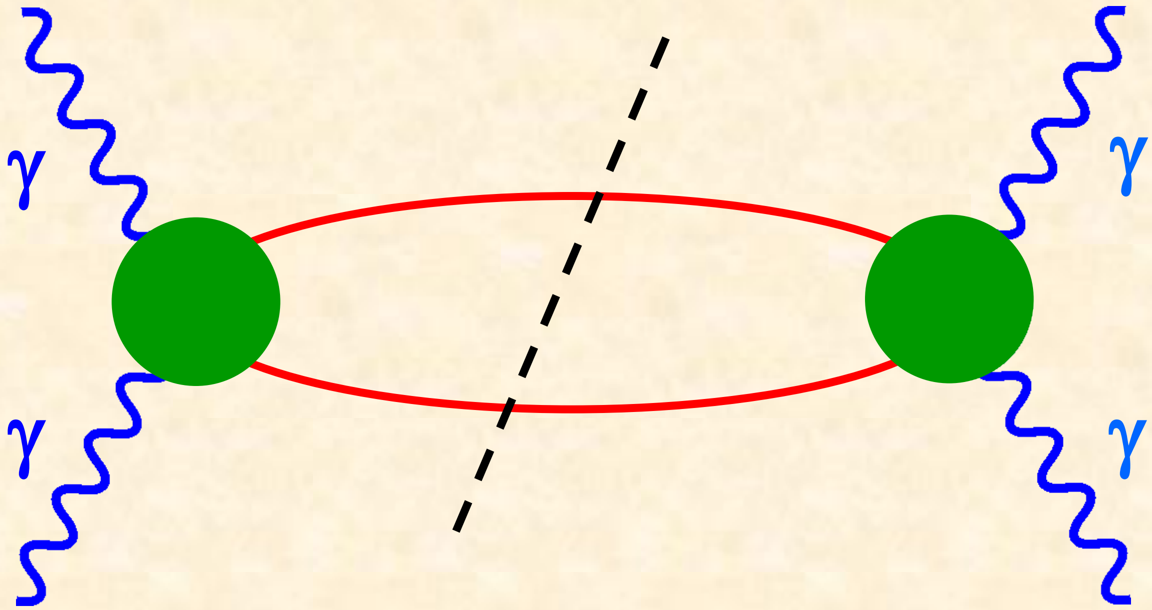


π^+	π^0	K^+	$\bar{K}^0$	π^0
π^-	π^0	K^-	K^0	η

BELLE

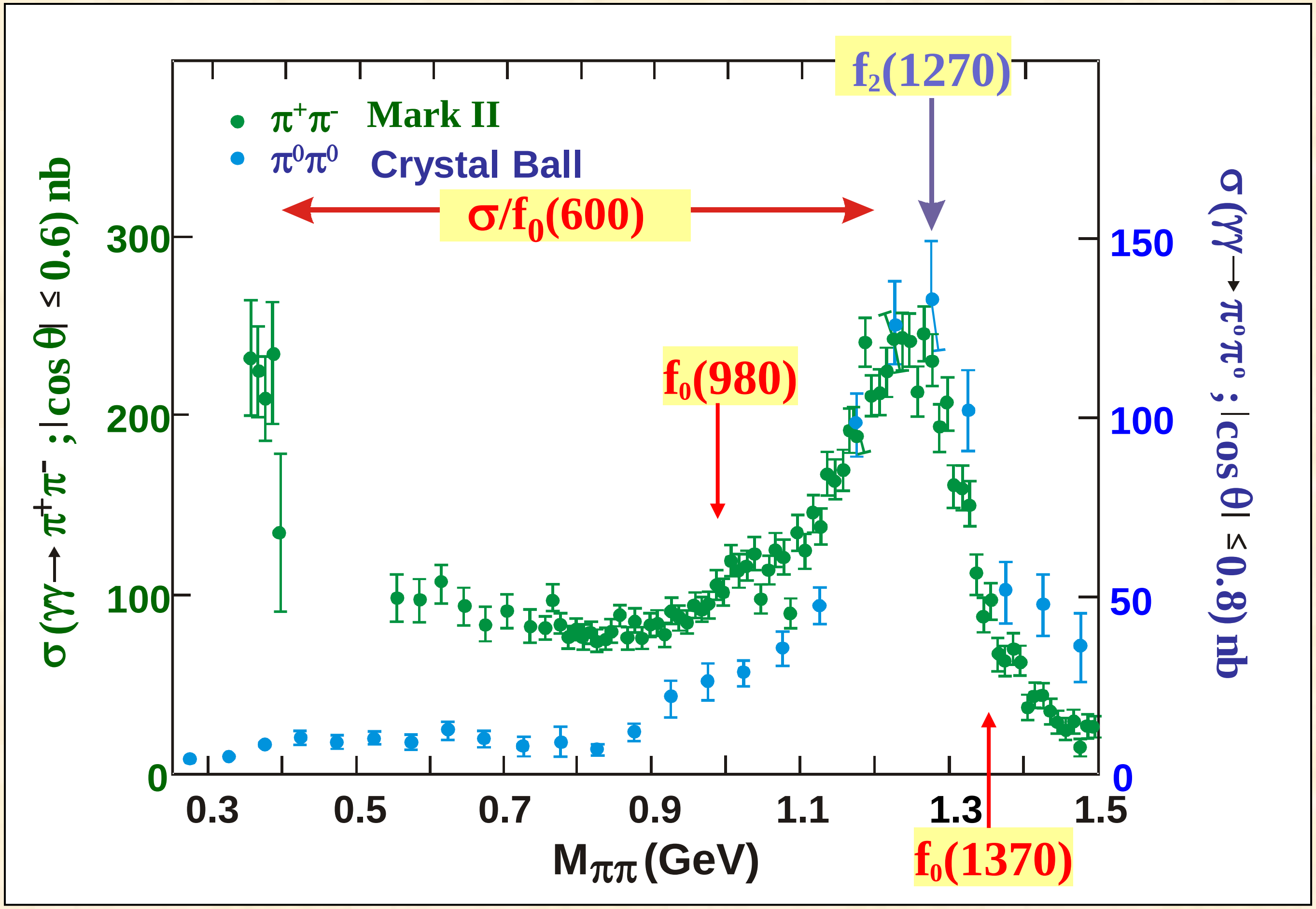



γ
γ
γ
γ



input discontinuity into dispersion relation for $\Pi_{\kappa\lambda\mu\nu}$

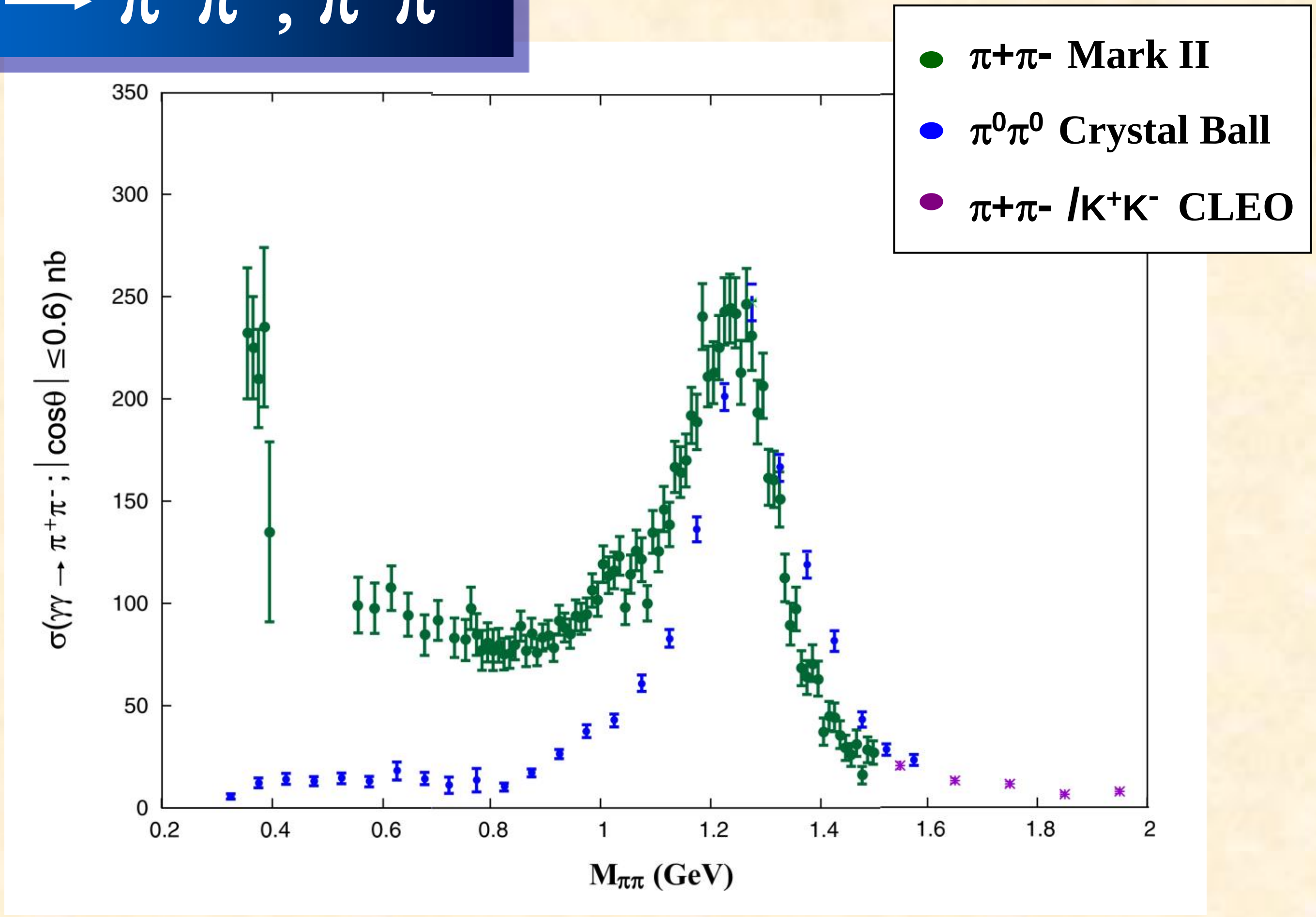
γγ couplings



– **Amplitude analysis**

- separate quantum numbers
- I, J, ….

$\gamma\gamma \longrightarrow \pi^+\pi^-, \pi^0\pi^0$



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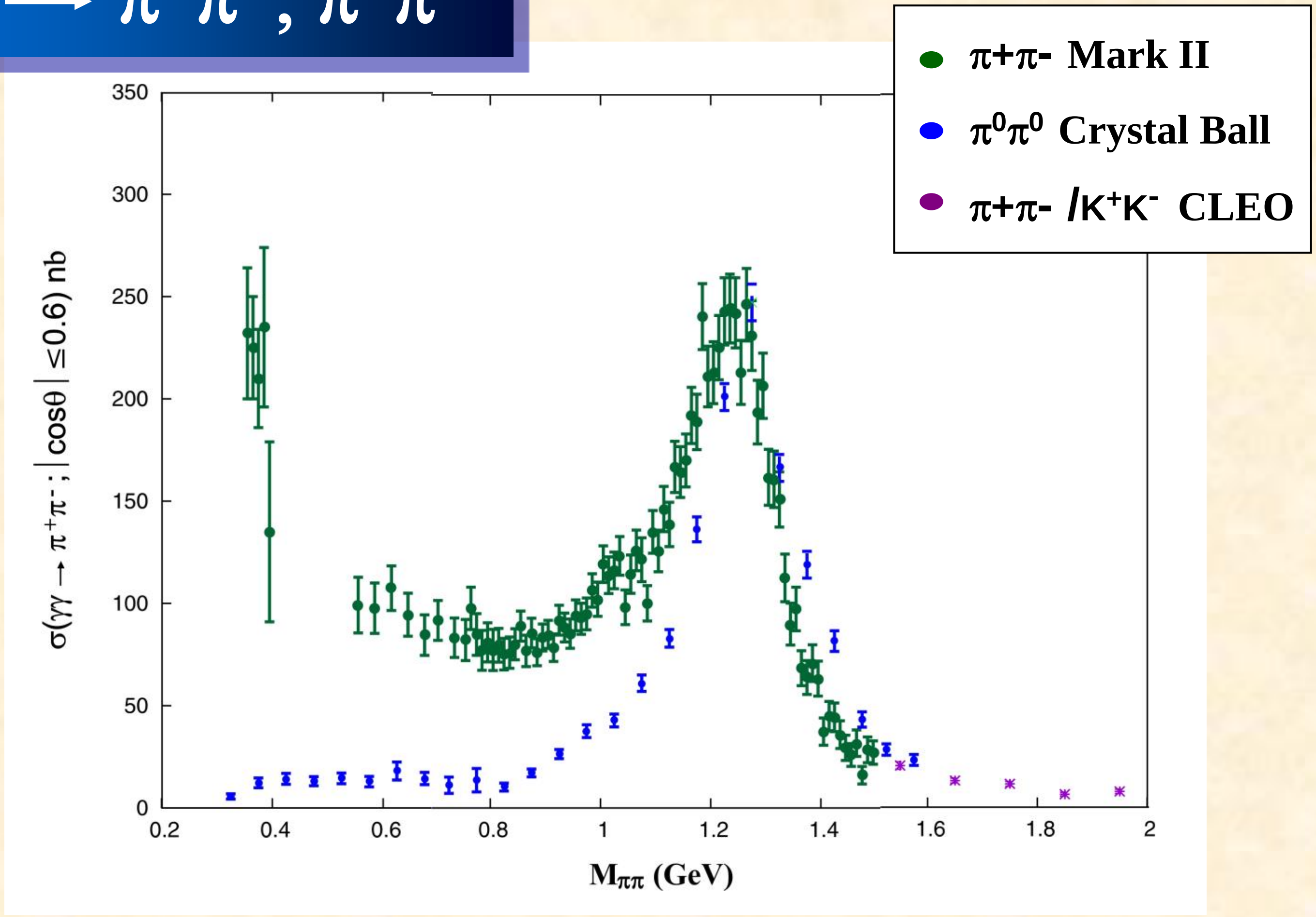

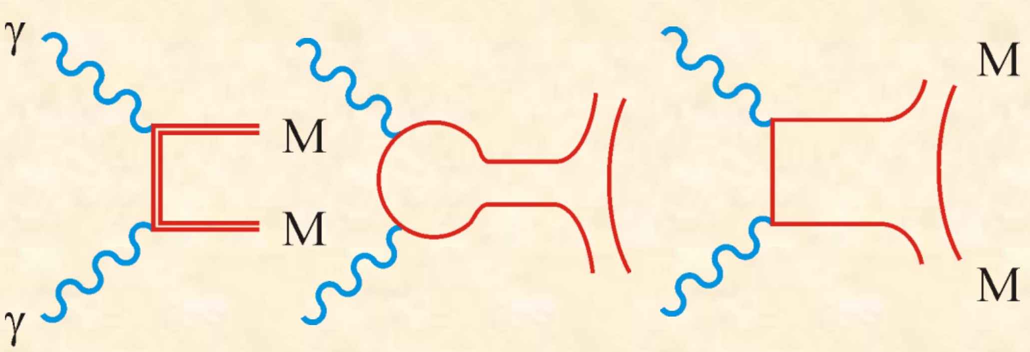

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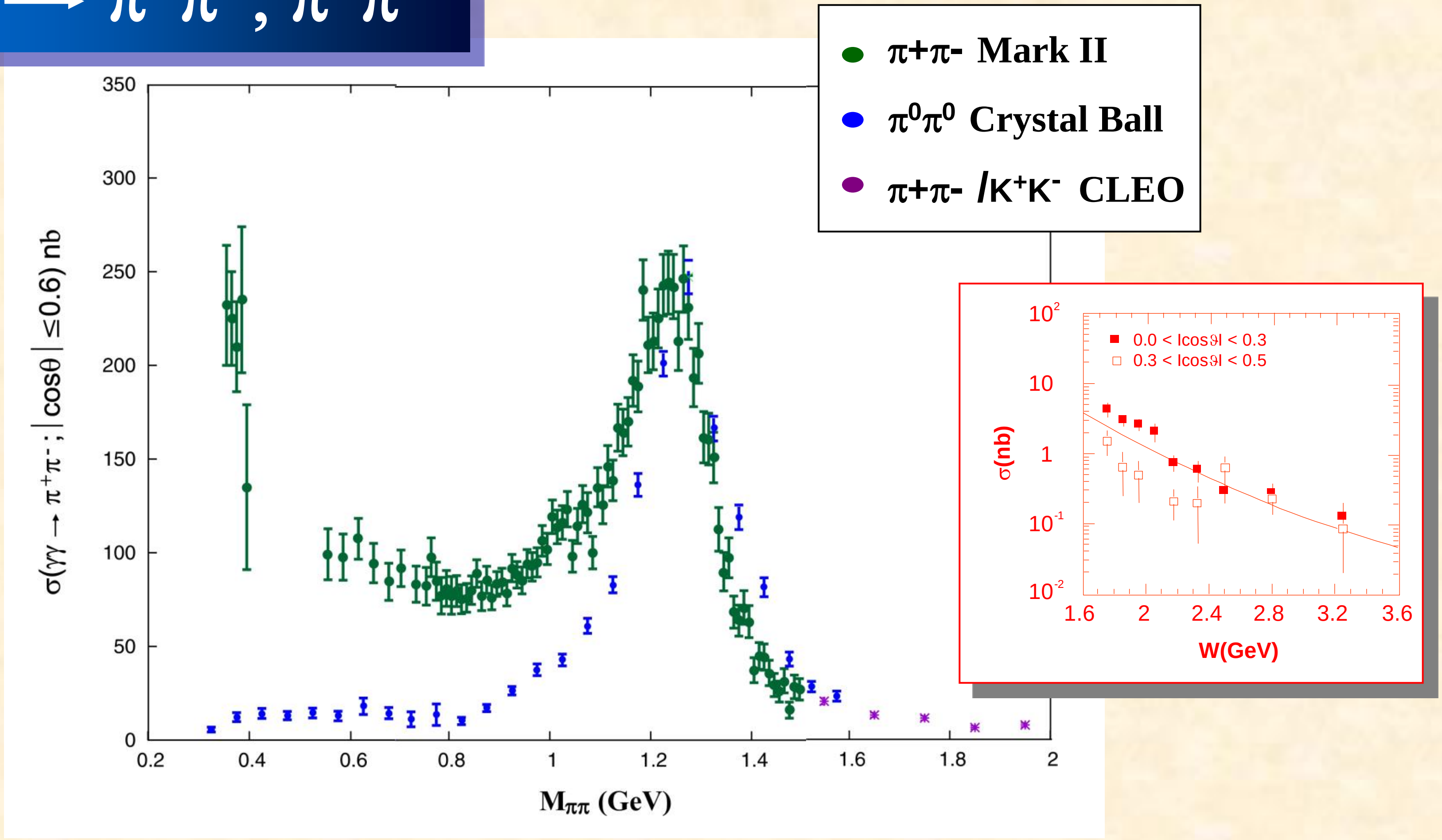
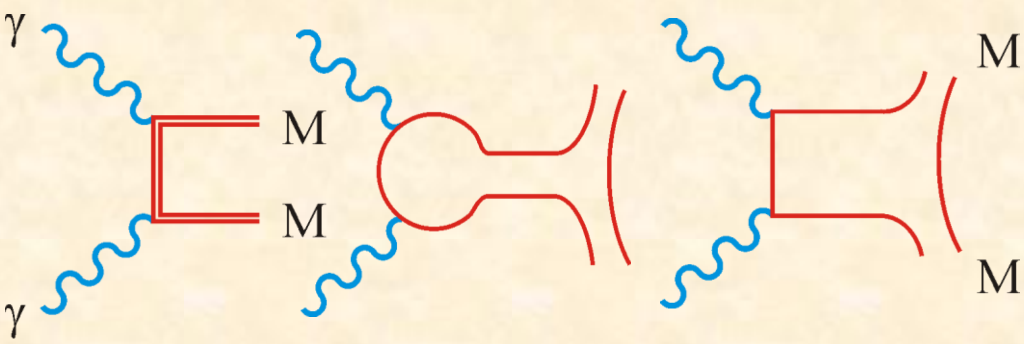
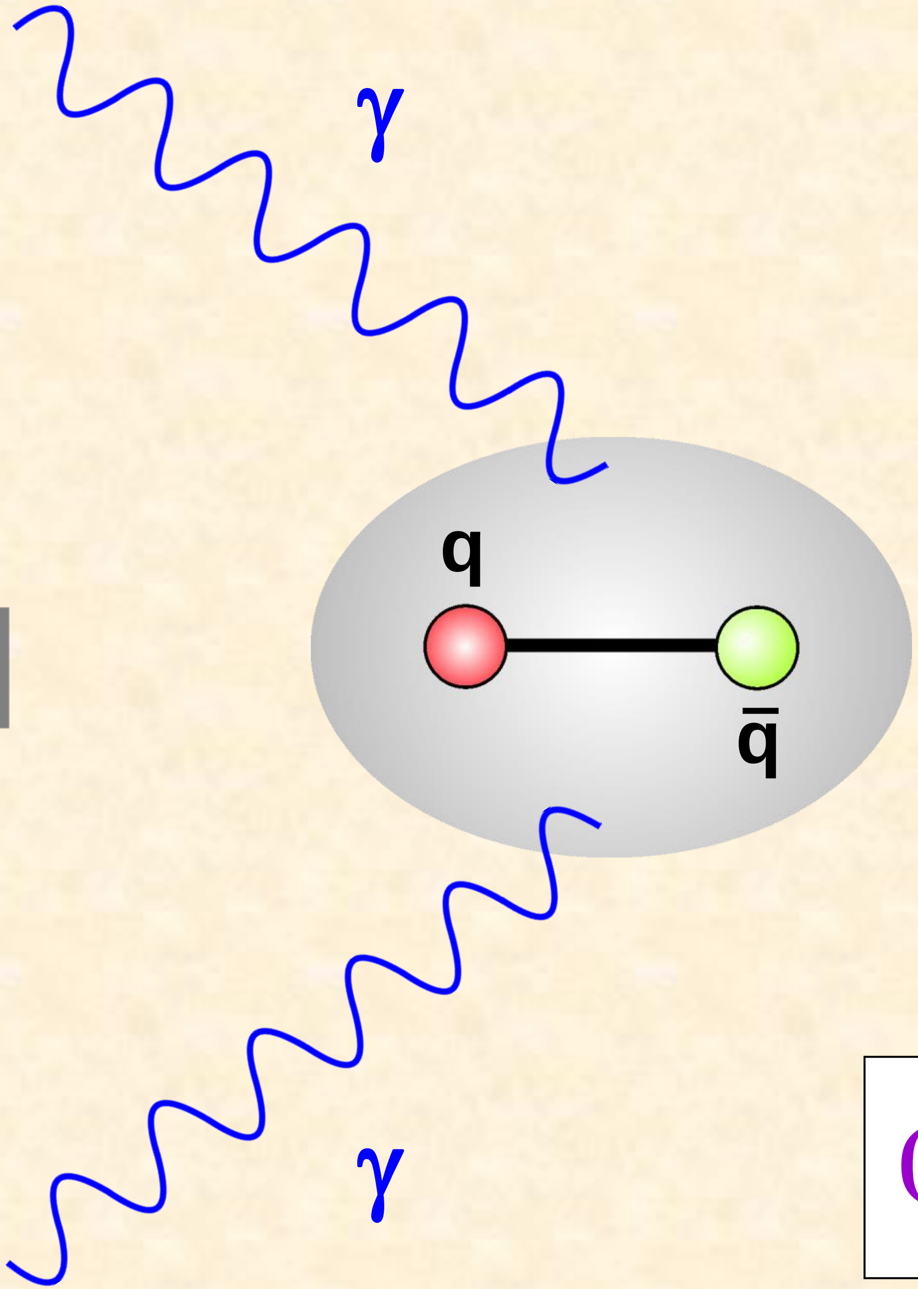




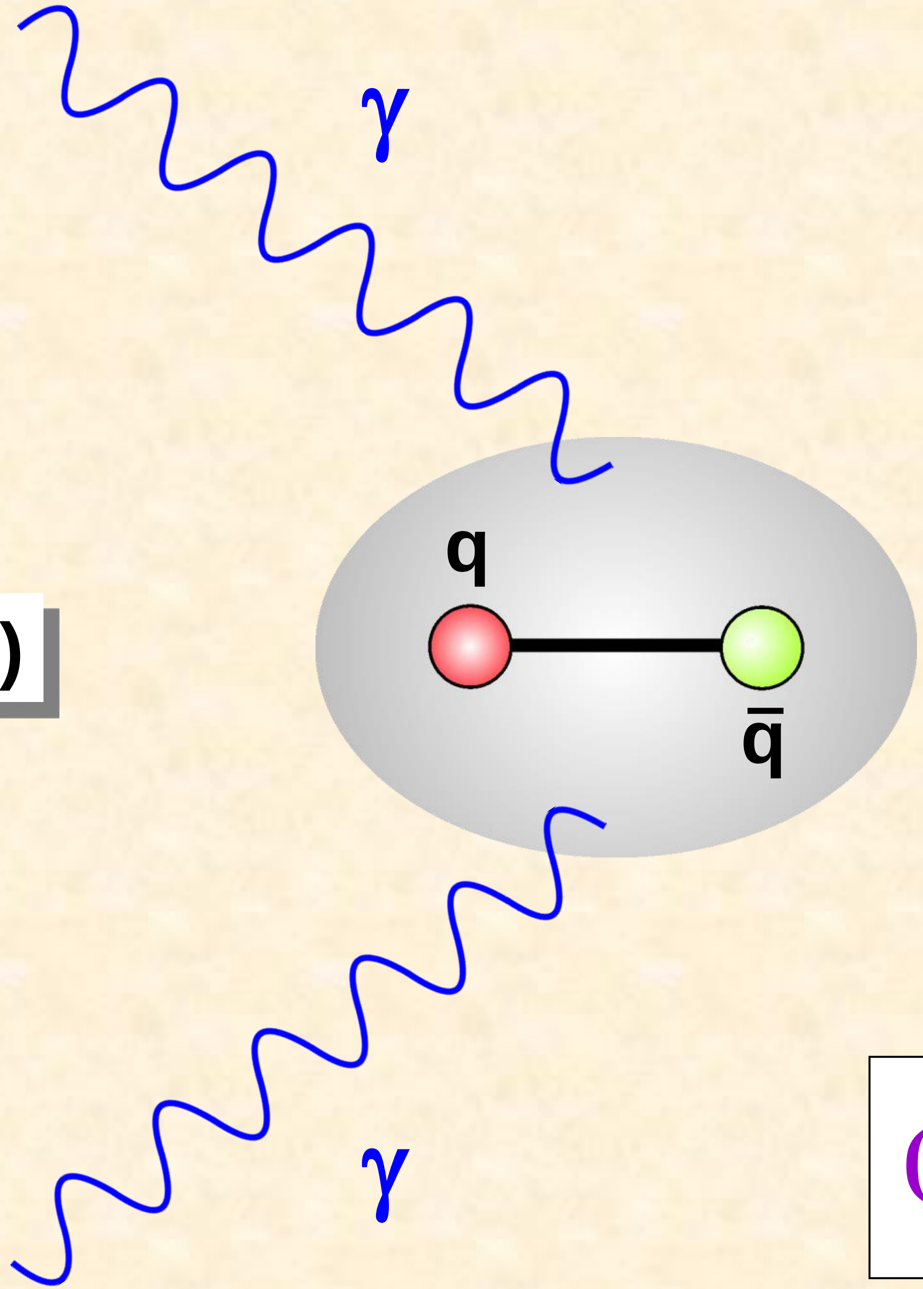
$q\bar{q}$



$|\Psi(0)|^2$

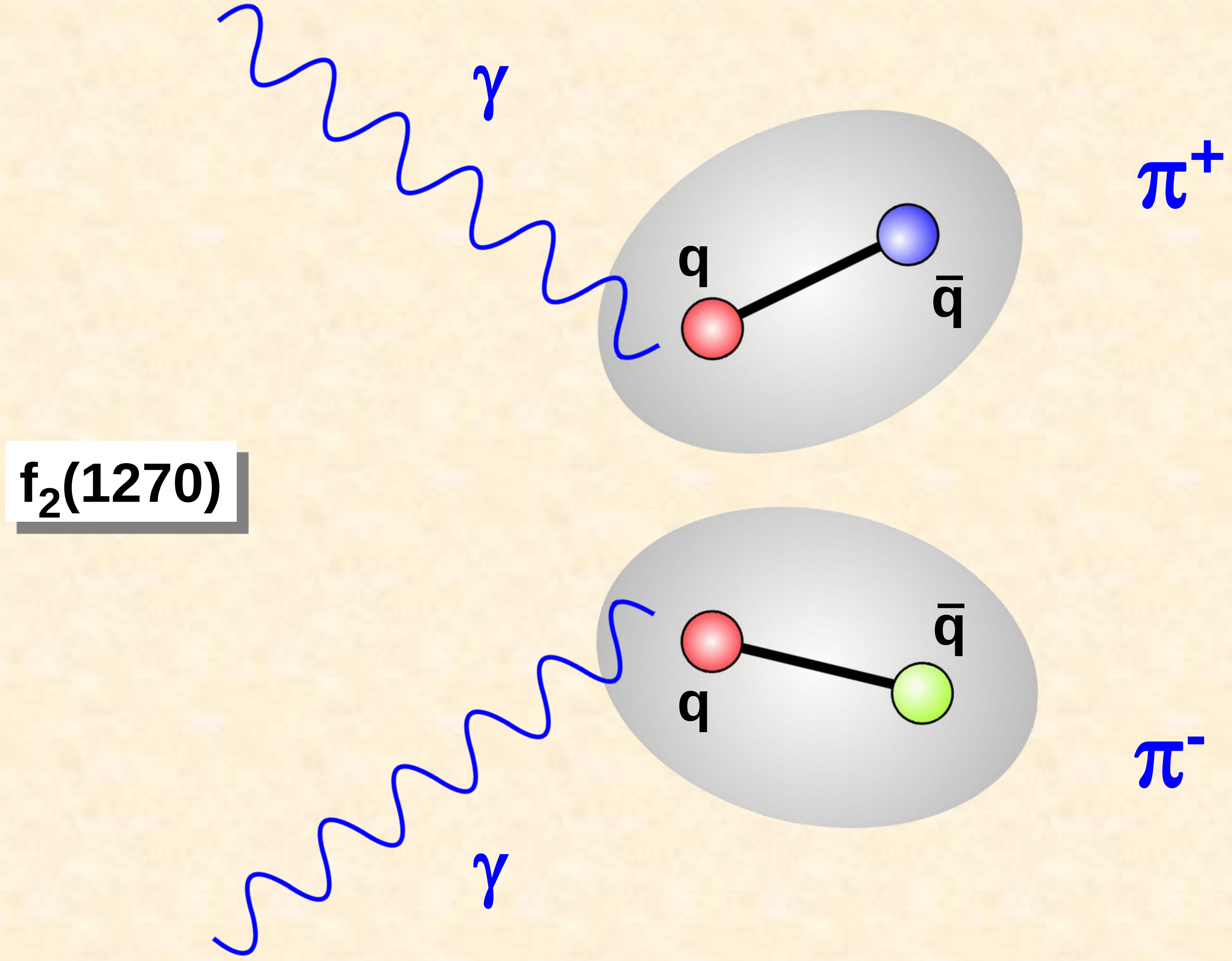
$$\left(\sum_q \langle e_q^2 \rangle\right)^2 \Pi_R$$

$f_2(1270)$

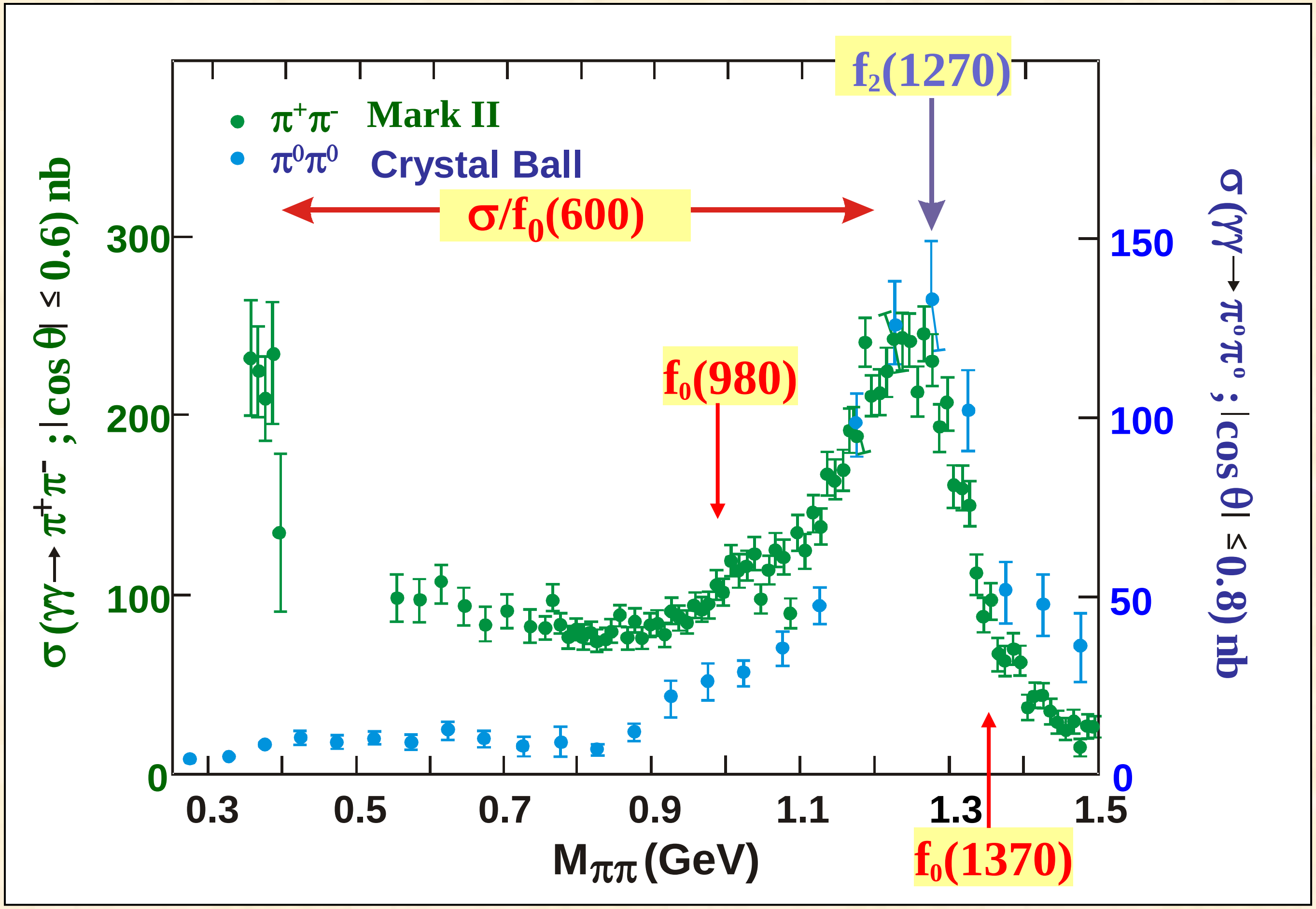


$|\Psi(0)|^2$

$$\left(\sum_q \langle e_q^2 \rangle\right)^2 \Pi_R$$


γ
π+
q
q̄
f2(1270)
q̄
q
π-
γ

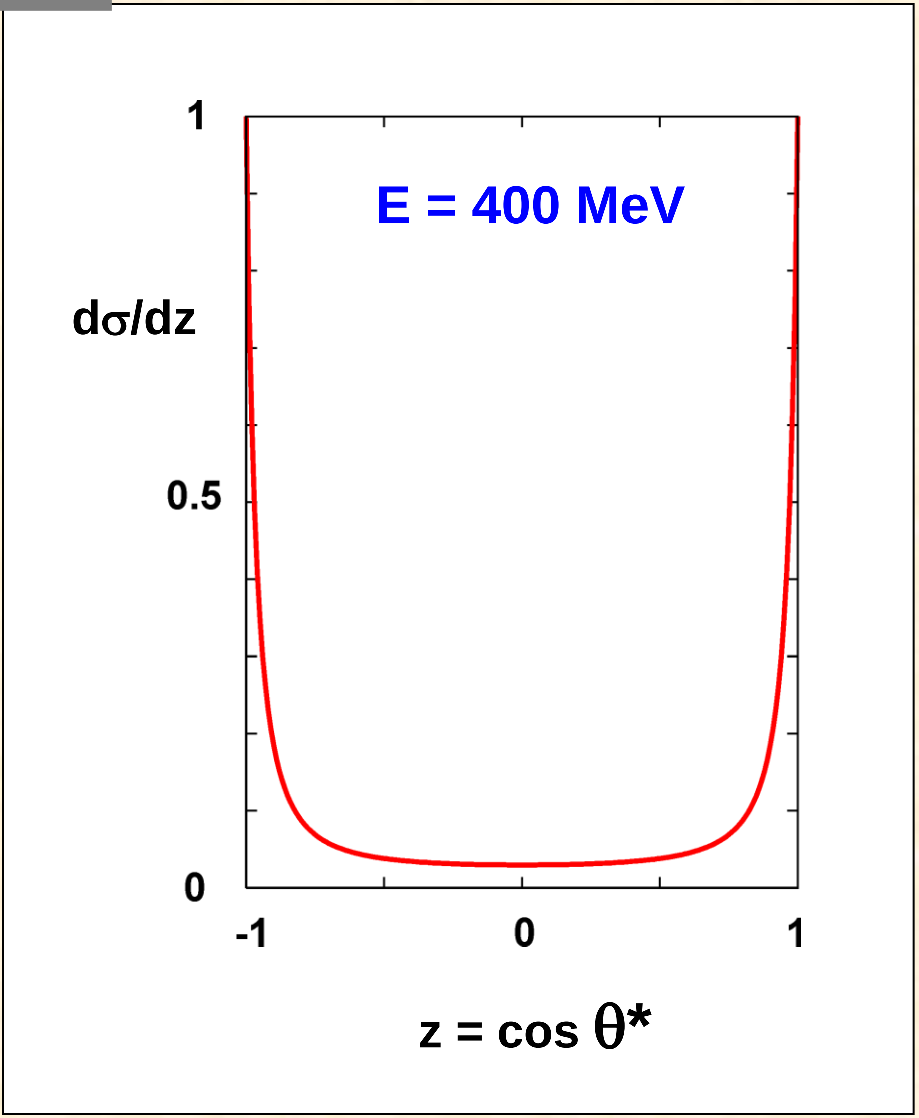
γγ couplings

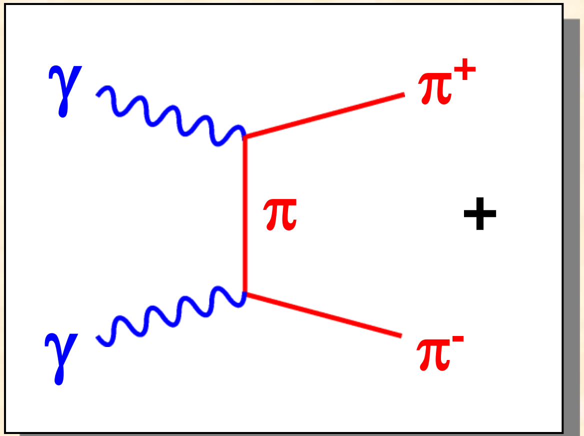


– **Amplitude analysis**

- separate quantum numbers
- I, J, ….

Angular distribution





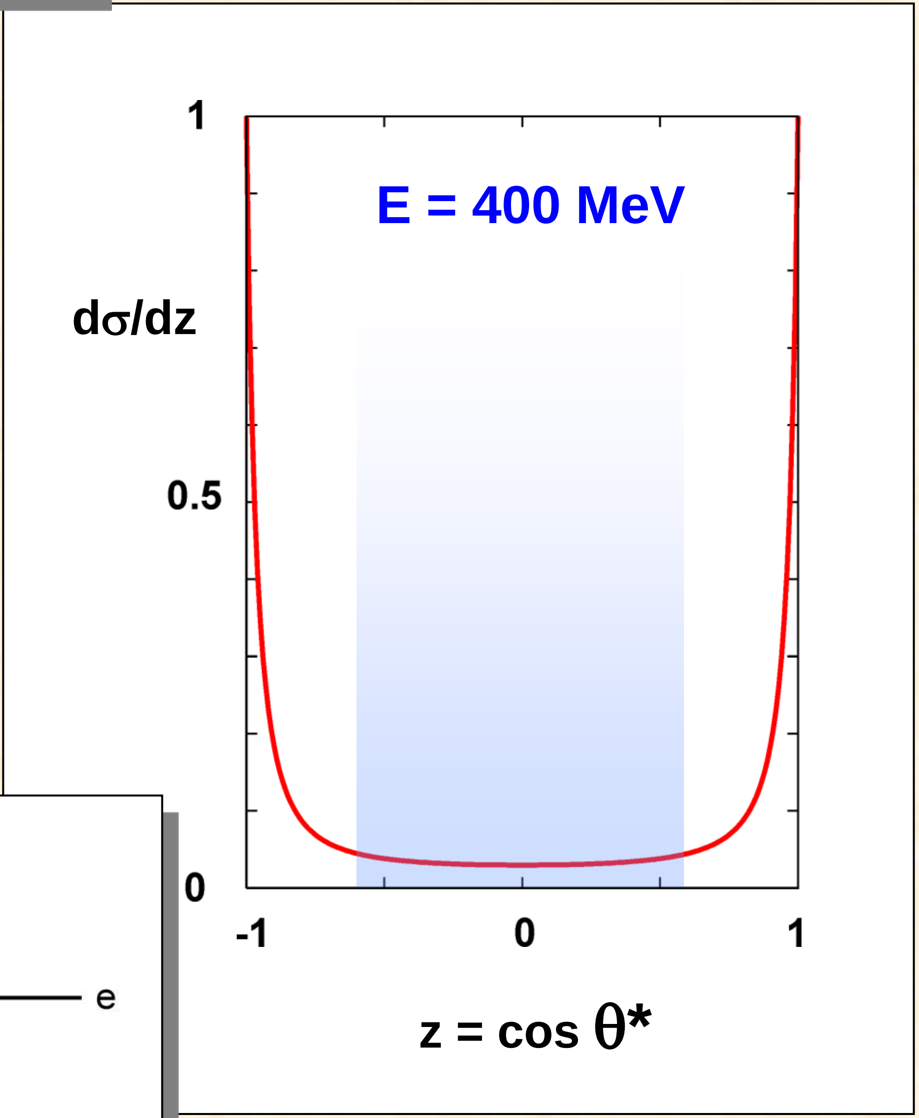
Angular distribution

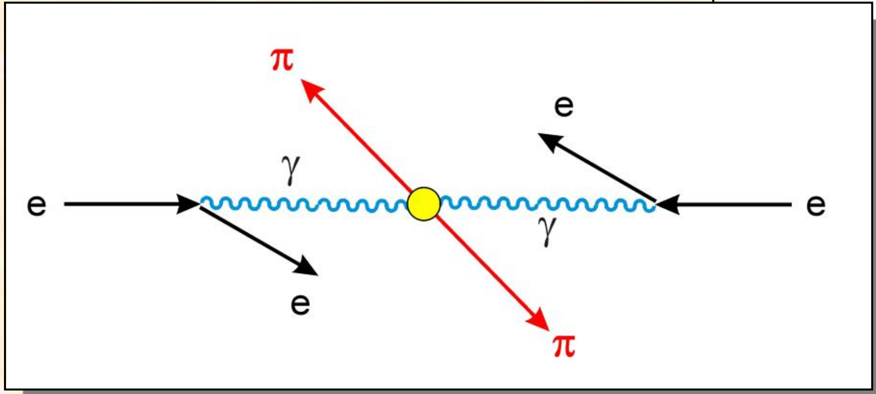
J_λ

S

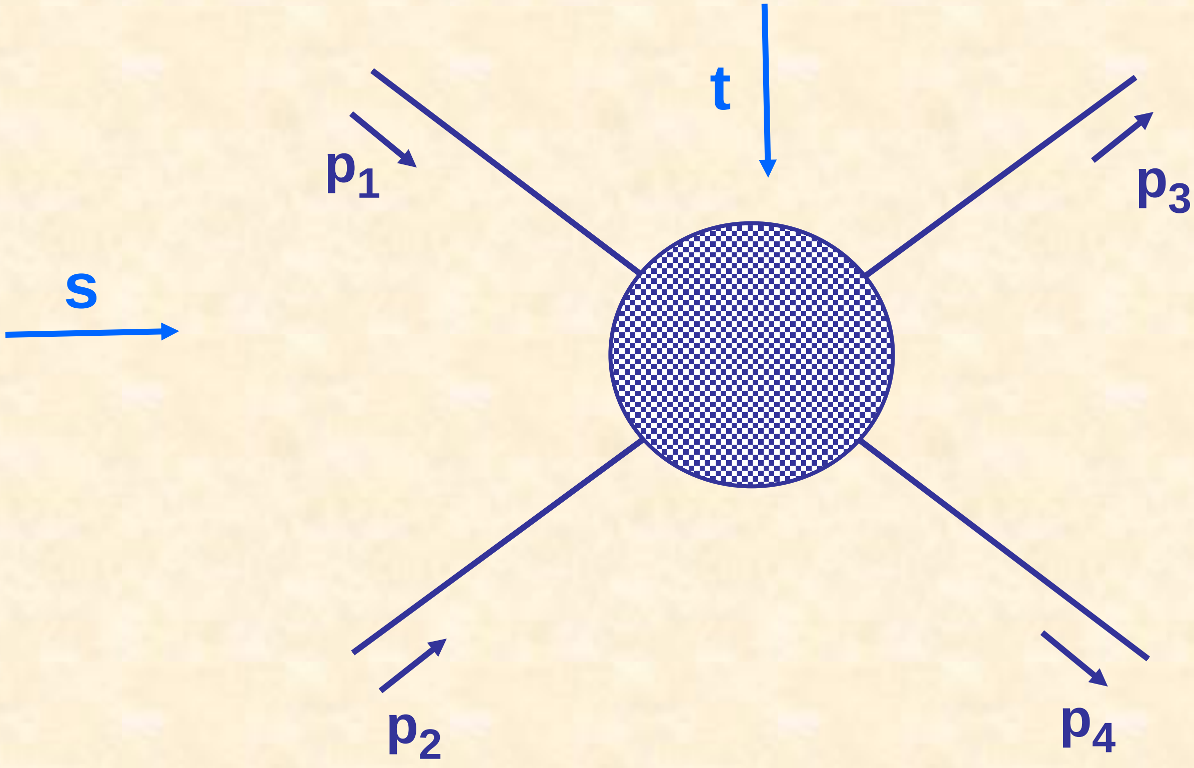
D_0, D_2,

...

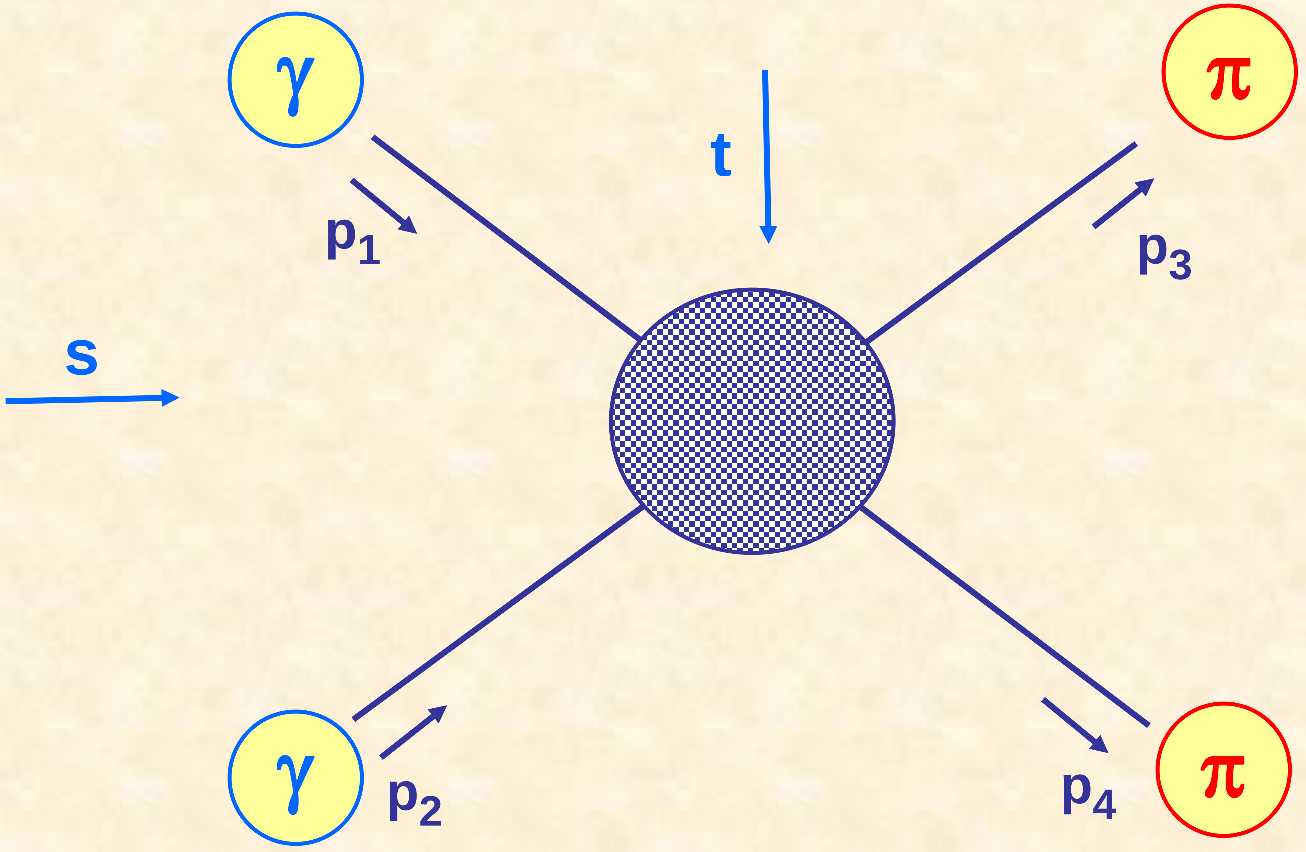




Relativistic kinematics



Relativistic kinematics

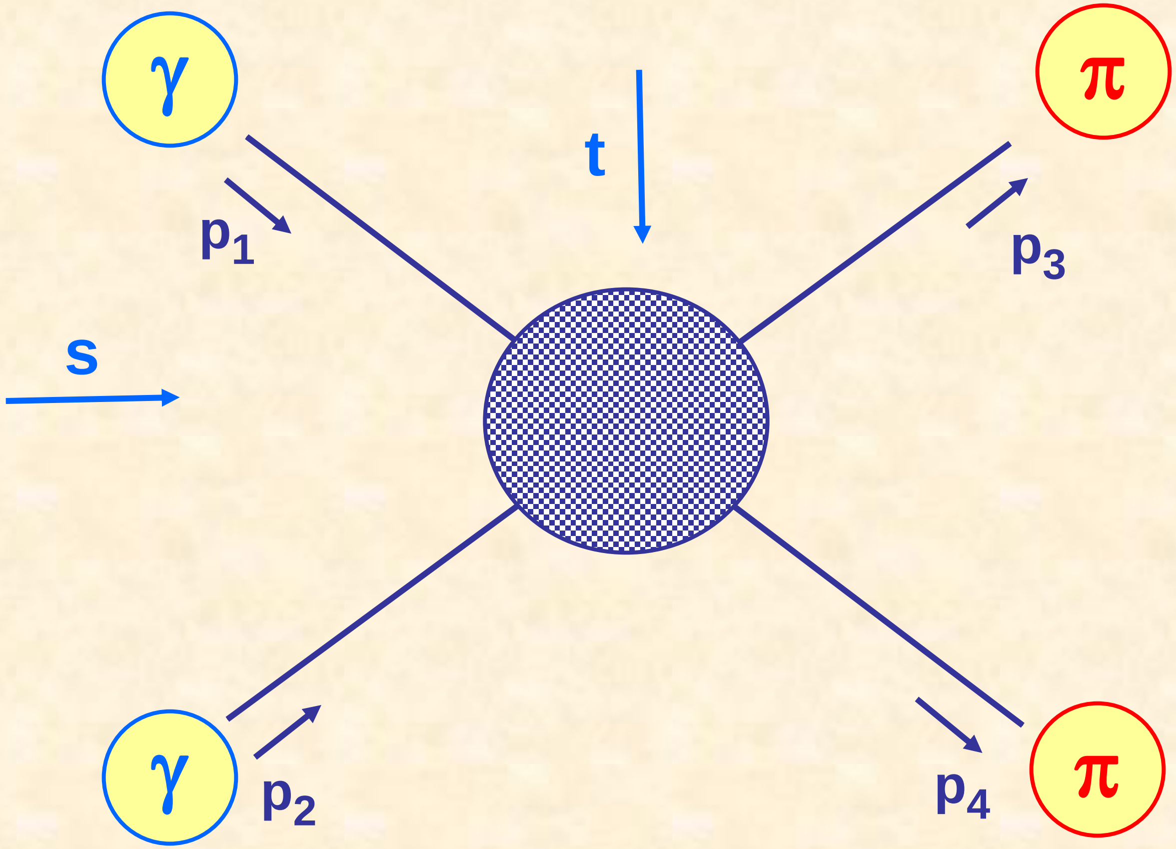


Relativistic kinematics

$s = (p_1 + p_2)^2$

$t = (p_1 - p_3)^2$

$u = (p_1 - p_4)^2$



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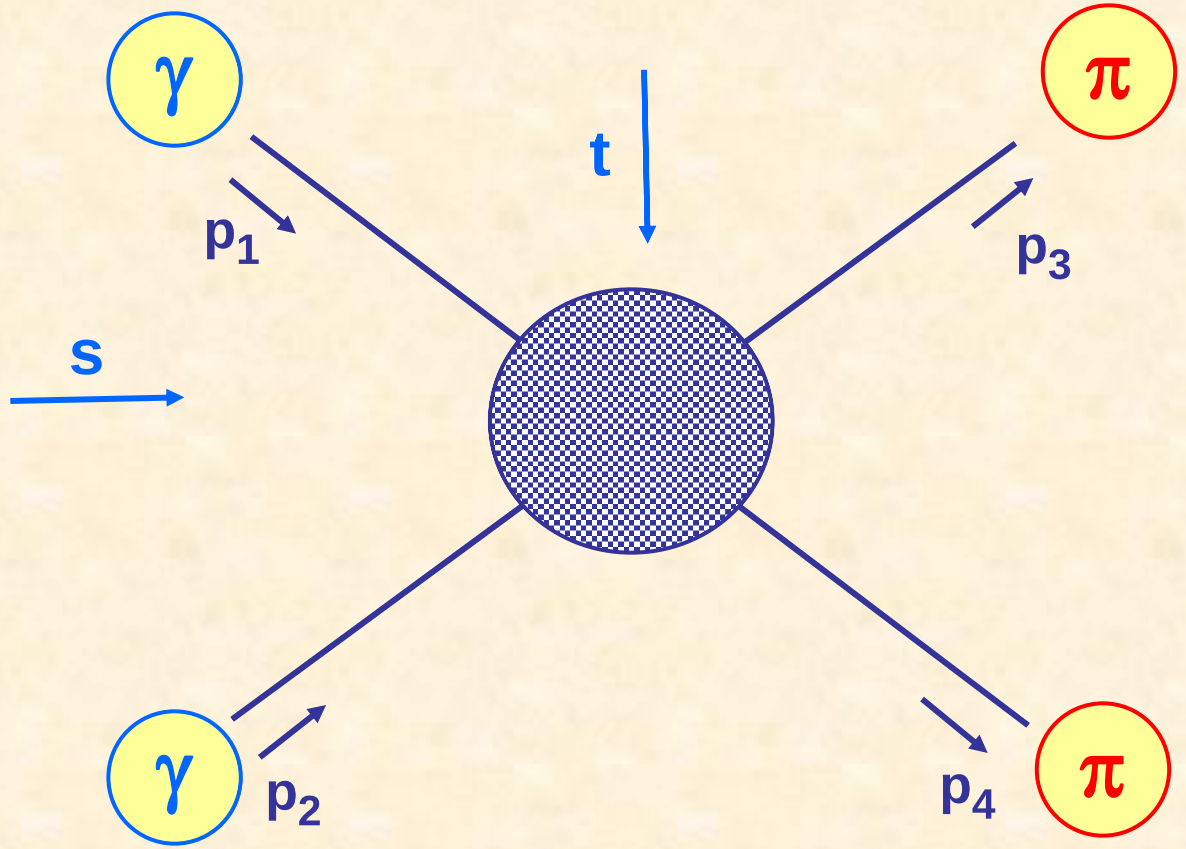
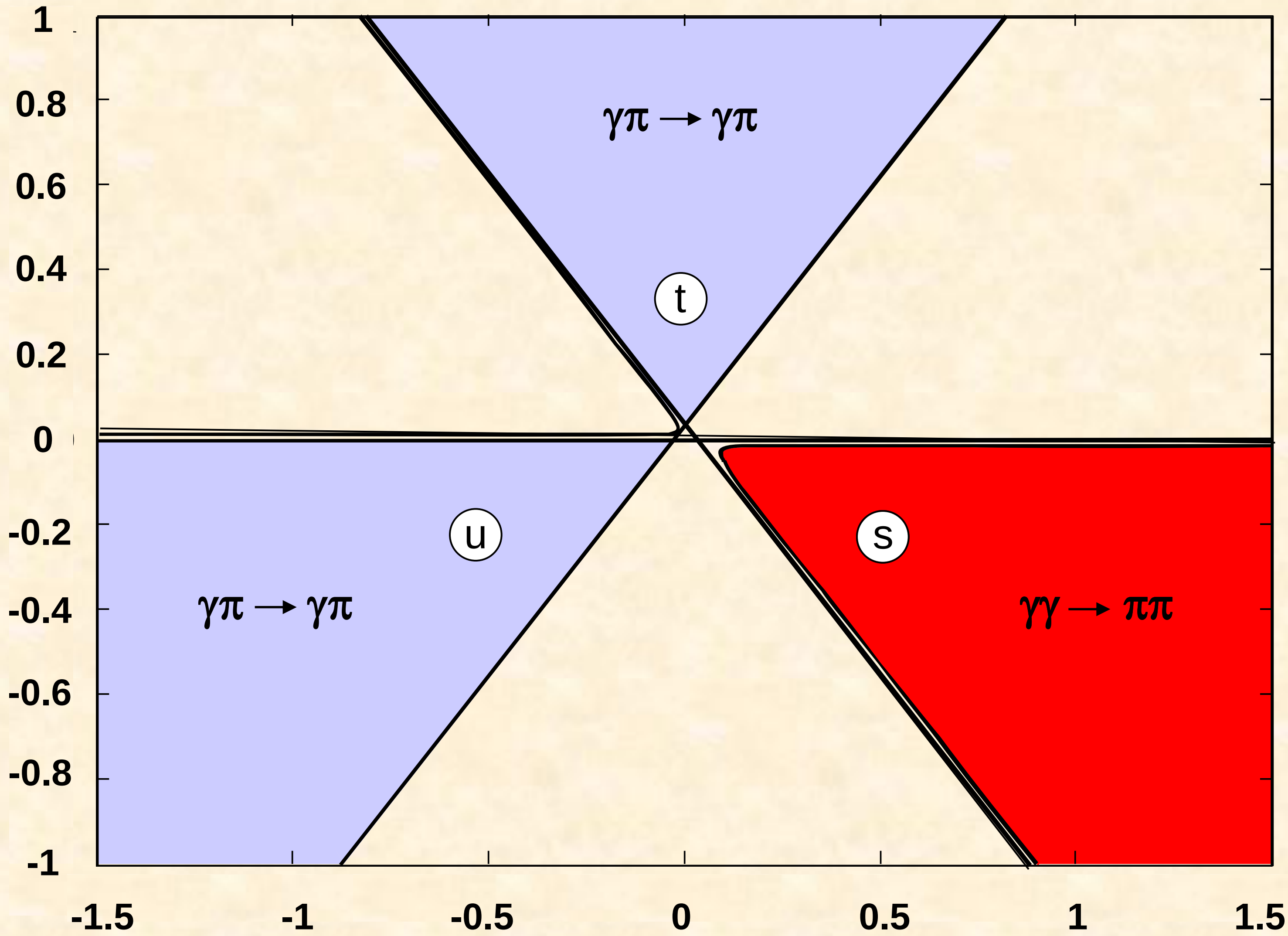
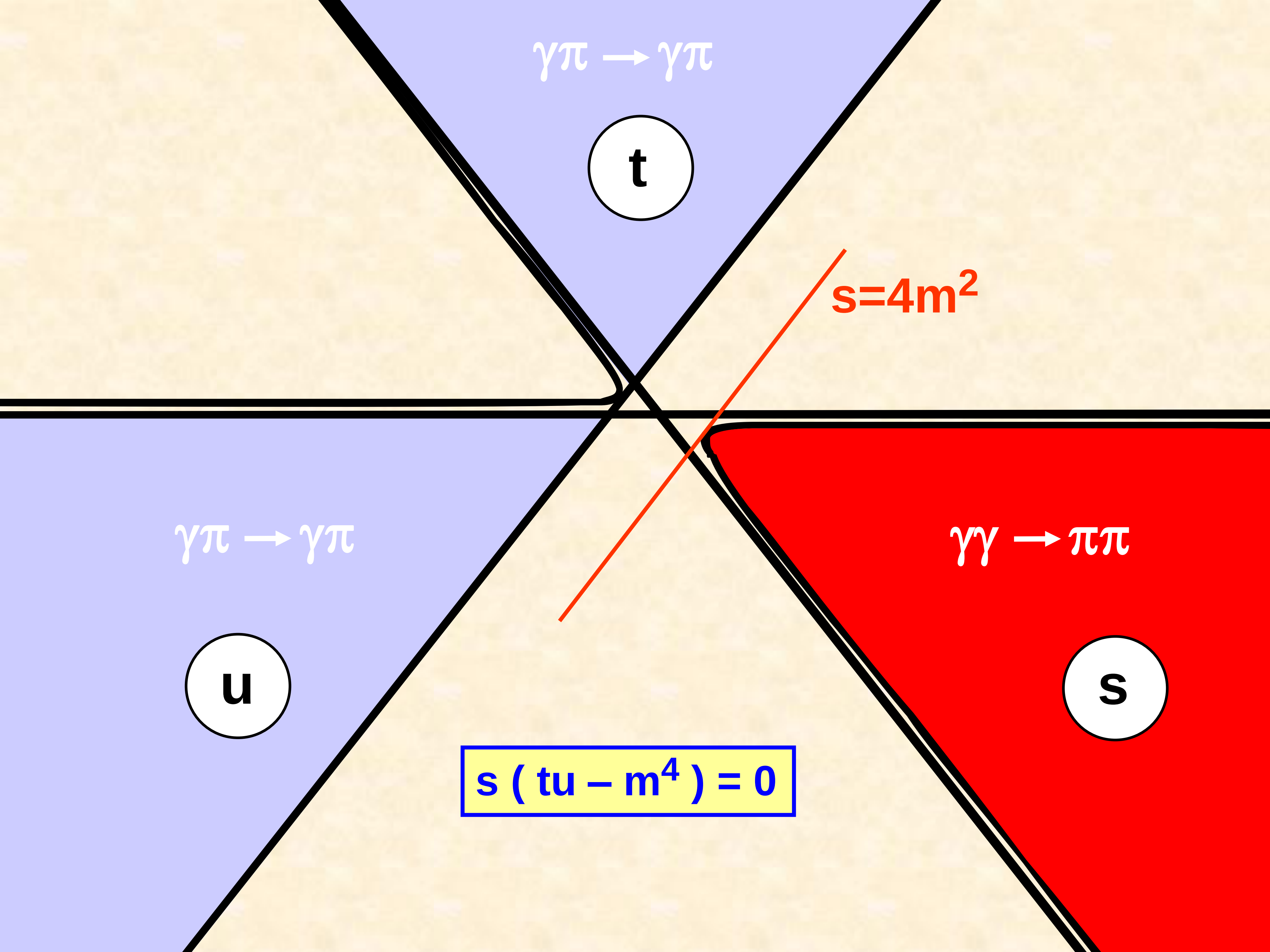


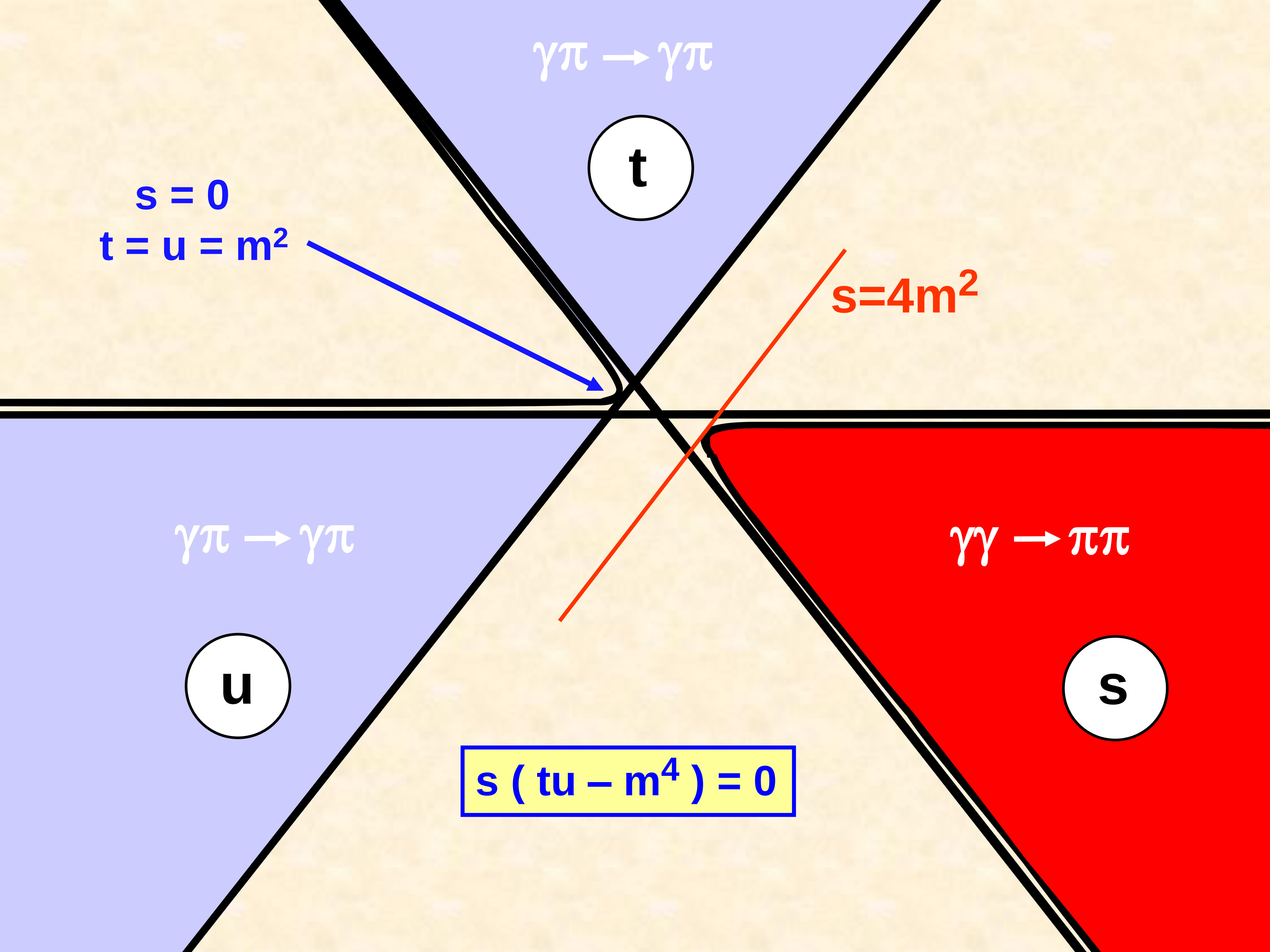
cm frame

$s = 4E^2, \quad t = m^2 - 2E^2 + 2Ek \cos\vartheta, \quad u = m^2 - 2E^2 - 2Ek \cos\vartheta$

Mandelstam Plane




γπ → γπ
t
s=4m²
γπ → γπ
u
γγ → ππ
s
s (tu – m⁴) = 0


γπ → γπ
t
s = 0
t = u = m²
s=4m²
γπ → γπ
γγ → ππ
u
s
s (tu – m⁴) = 0

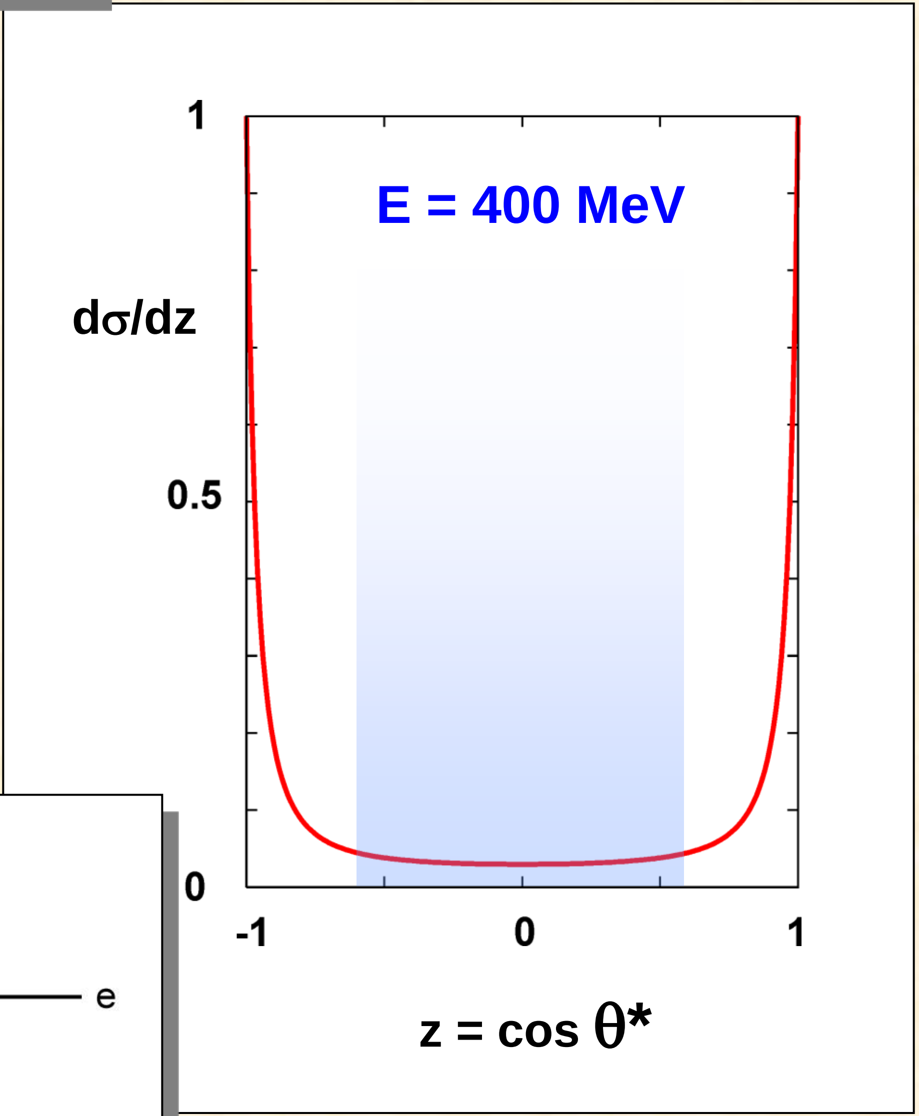
Angular distribution

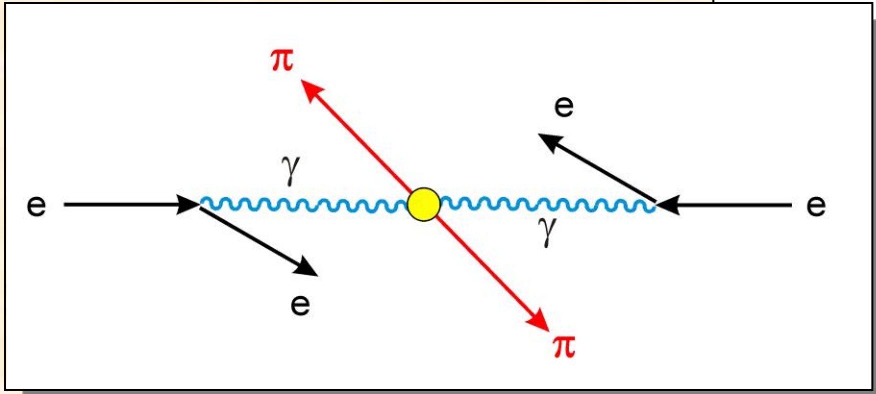
J_λ

S

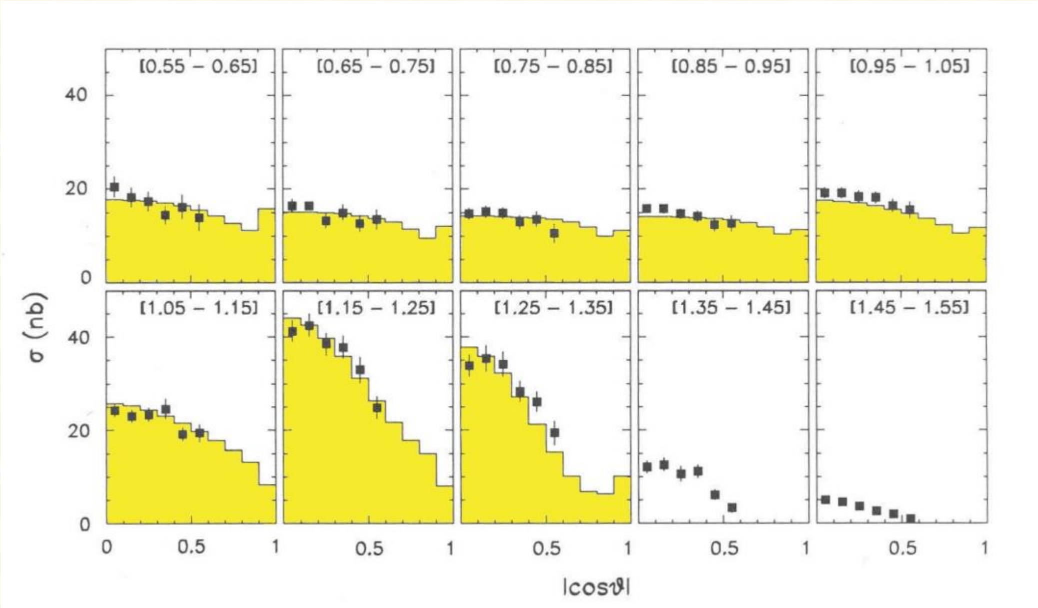
D_0, D_2,

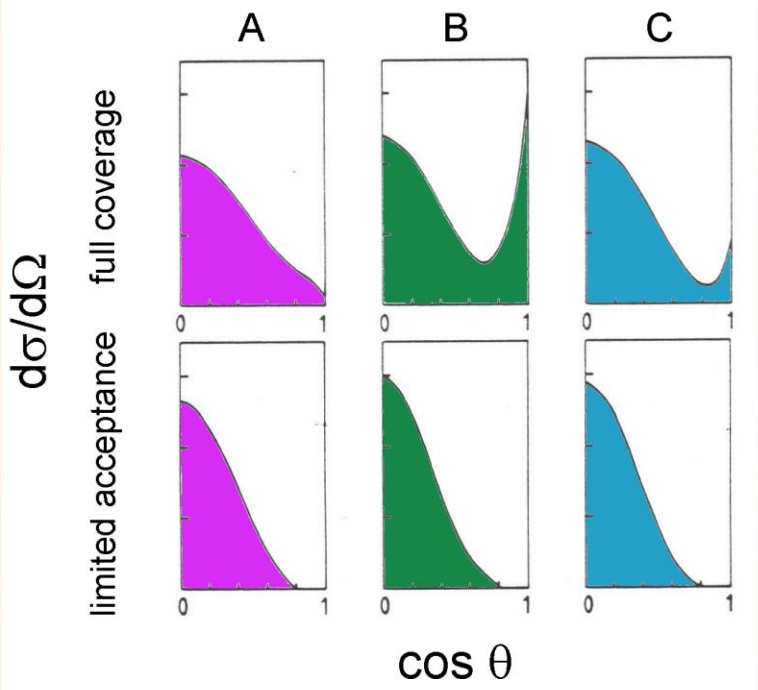
…

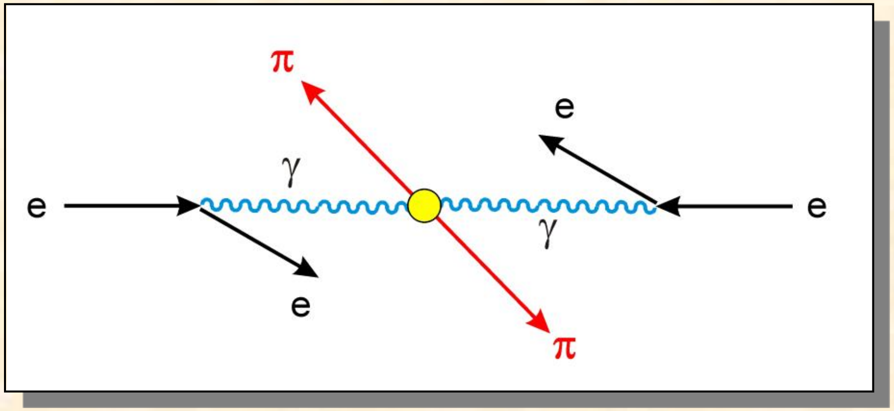




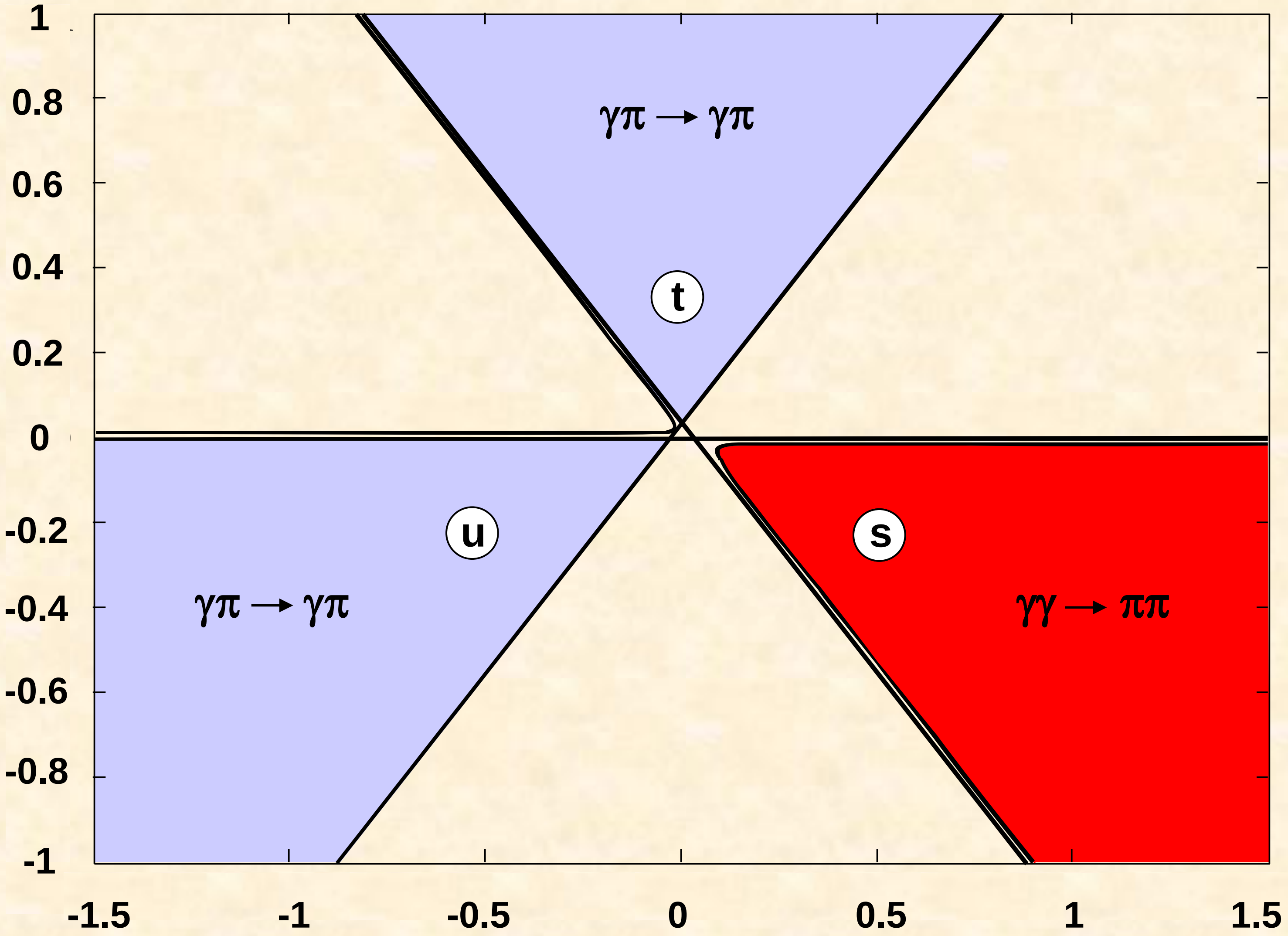
Angular distribution



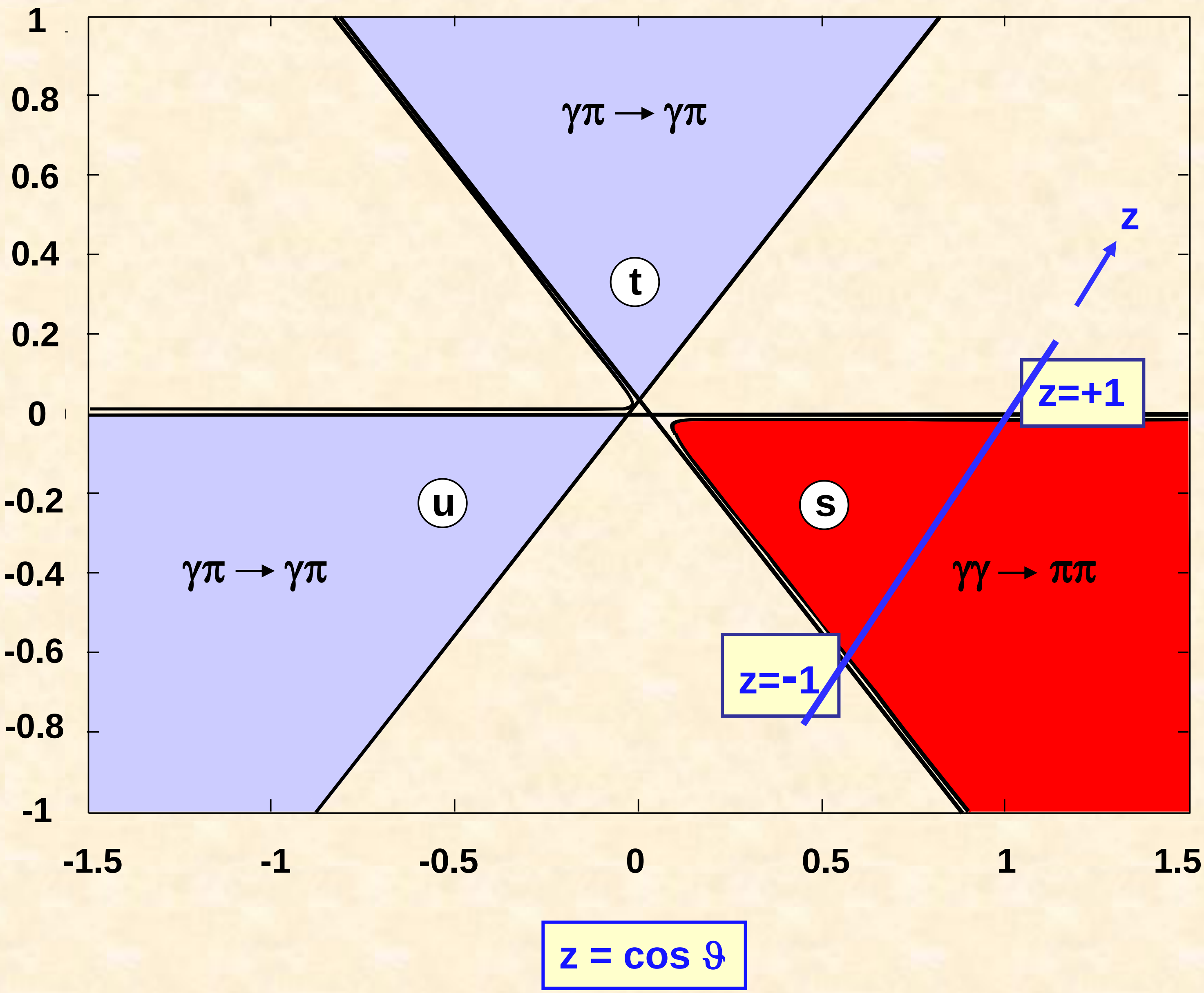




Mandelstam Plane



Mandelstam Plane



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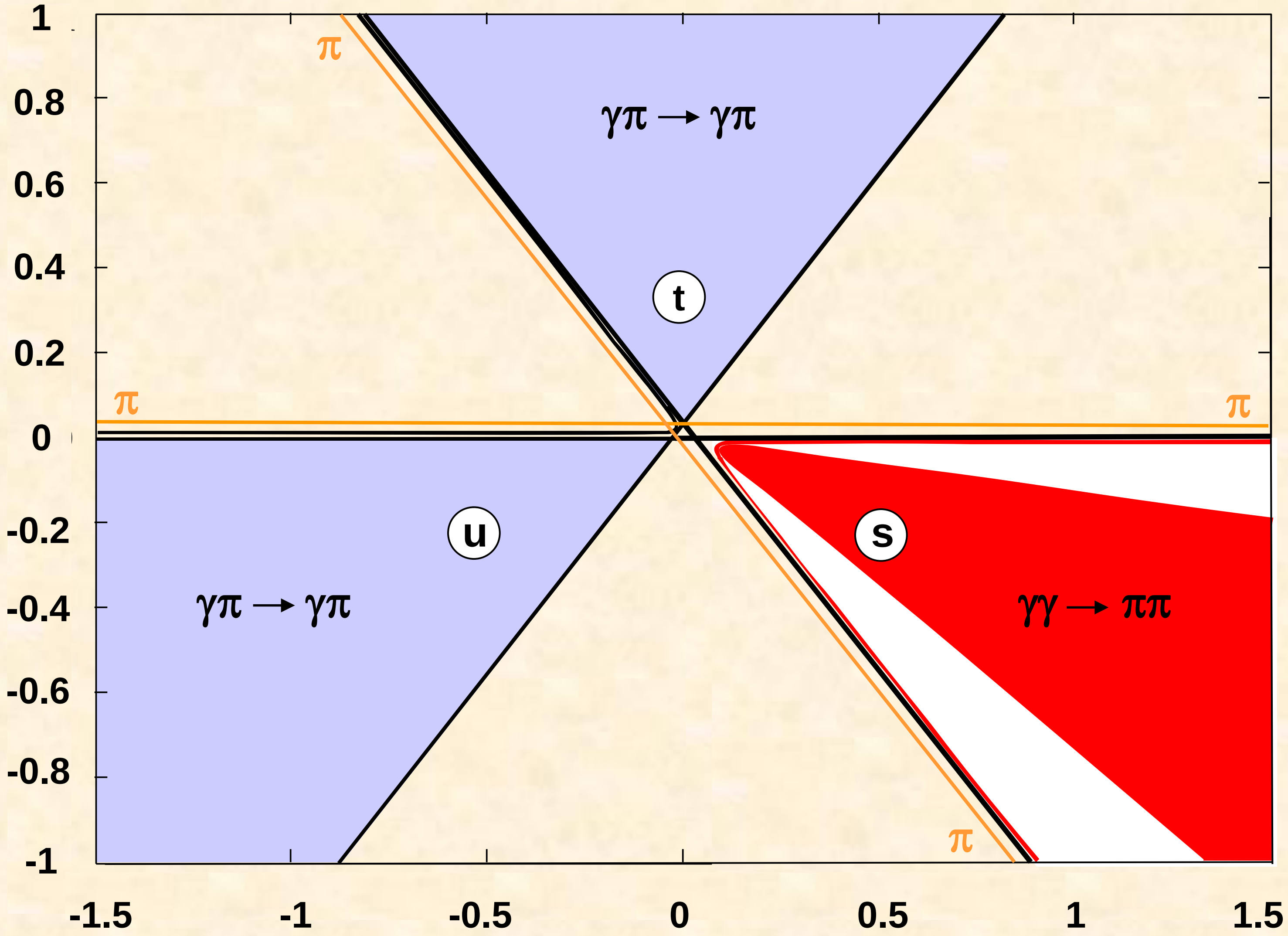
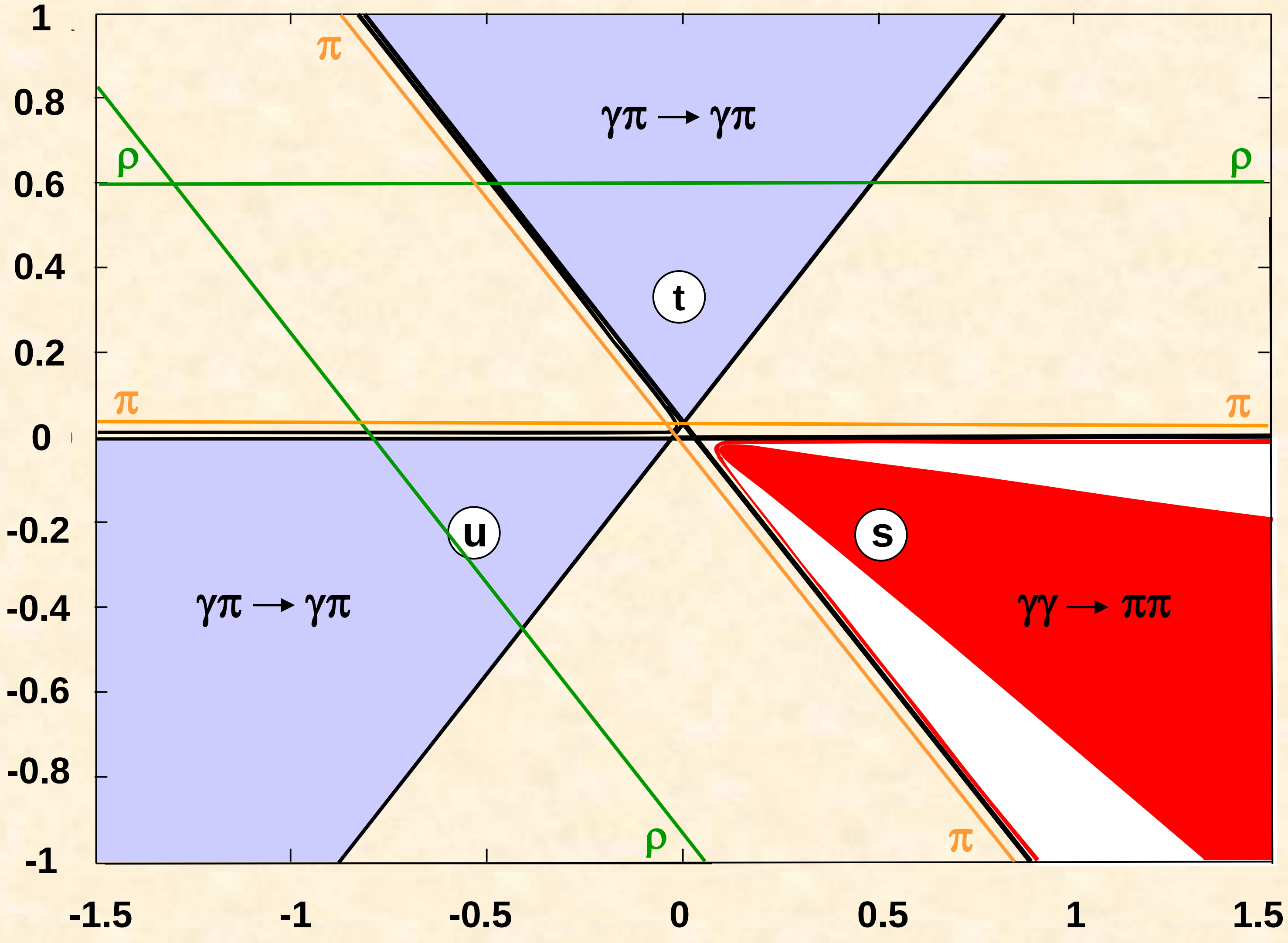
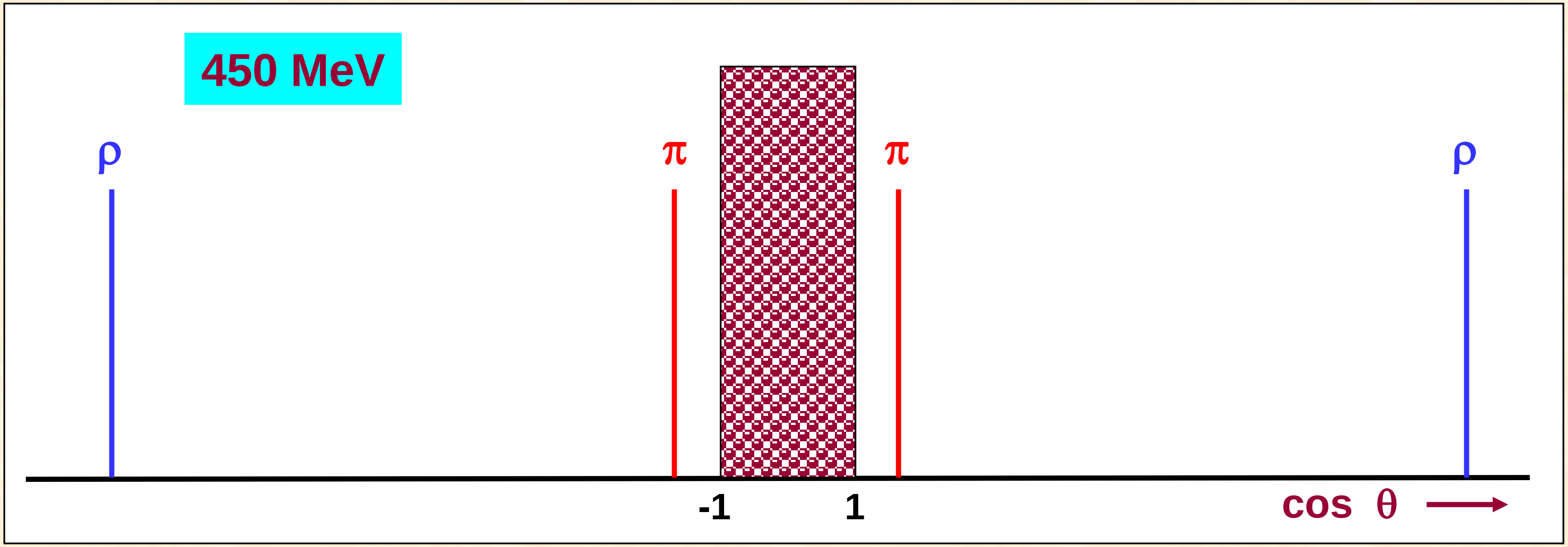
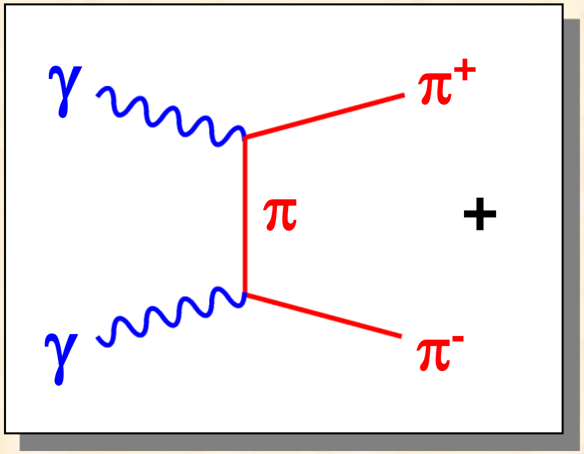
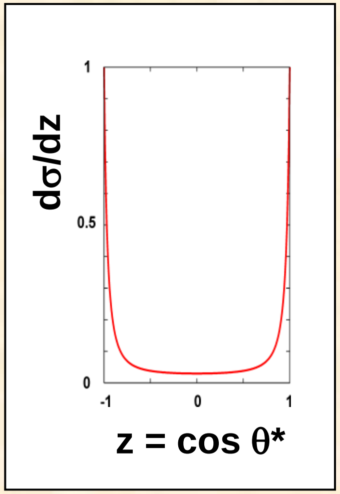


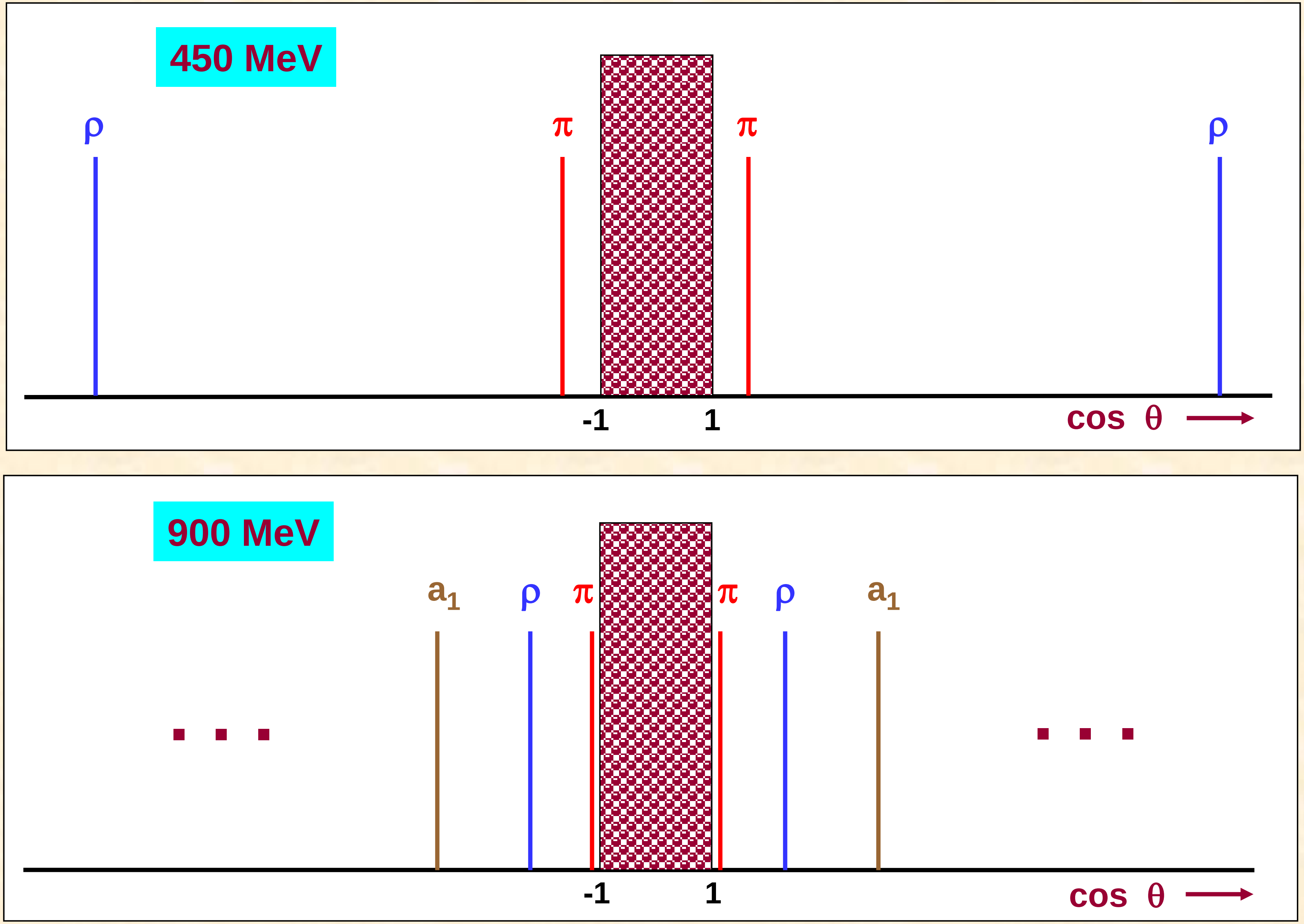
Mandelstam Plane



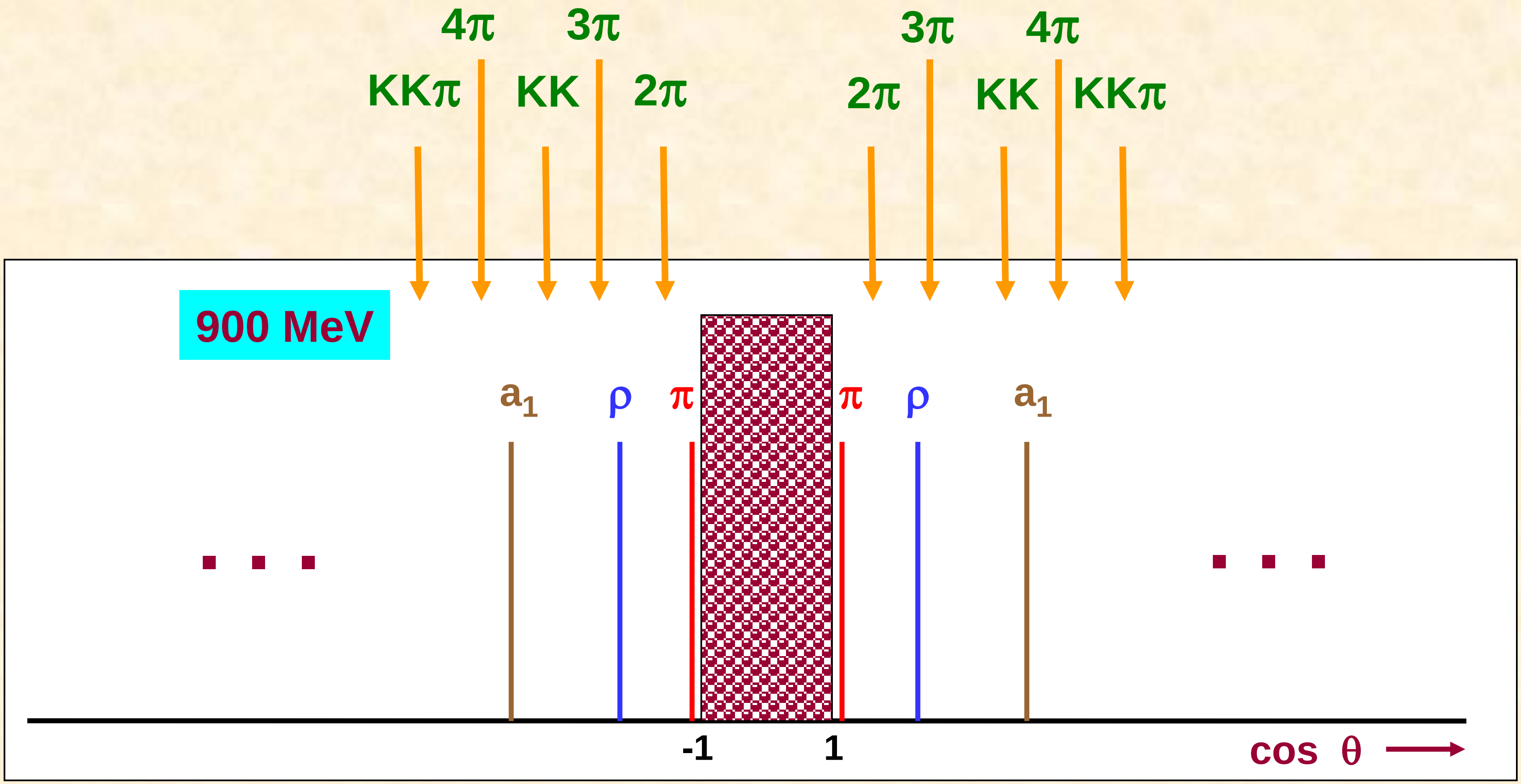

450 MeV
ρ
π
π
ρ
-1
1
cos θ

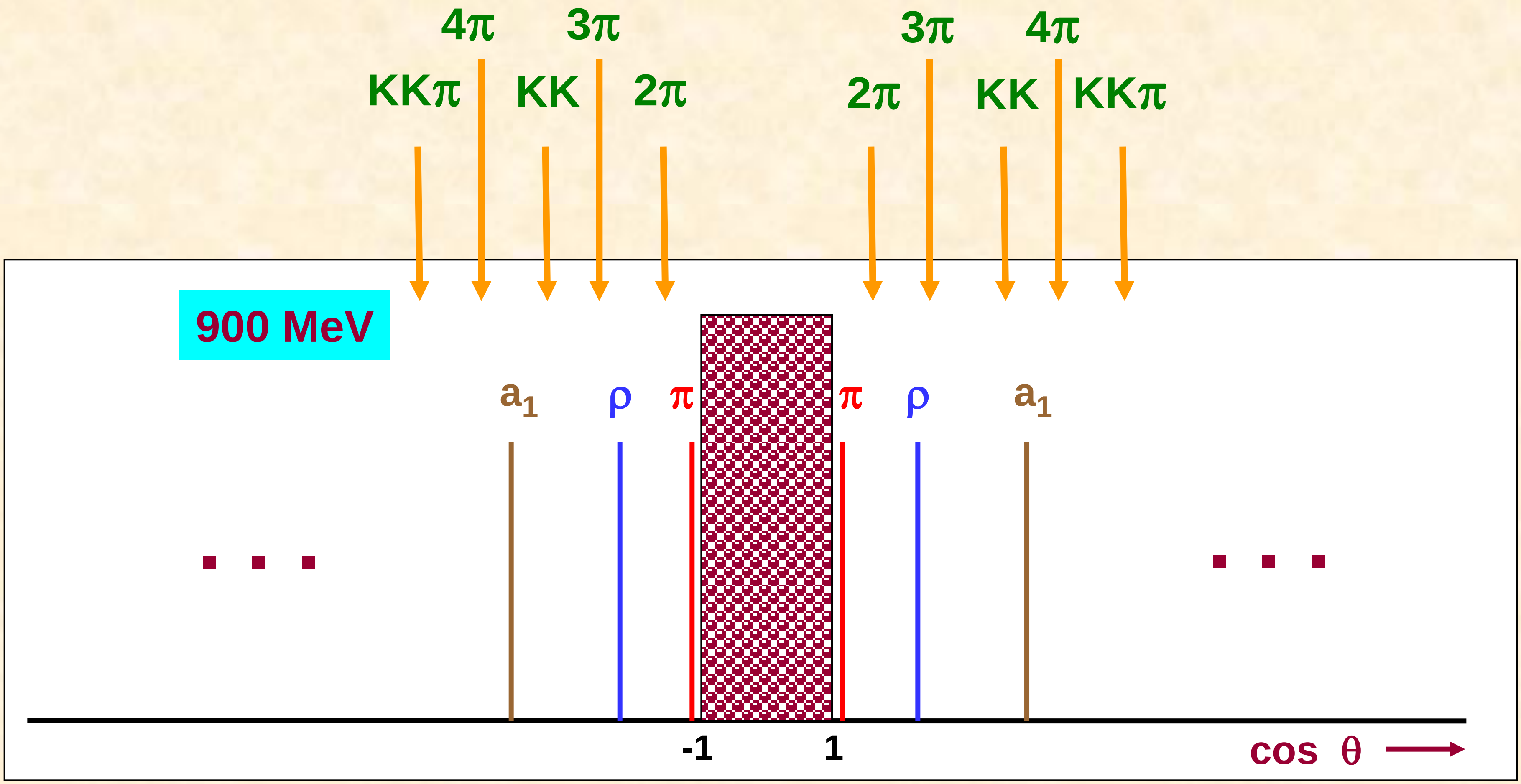

γ
π+
π
+
γ
π-


dσ/dz
1
0.5
0
-1
0
1
z = cos θ*



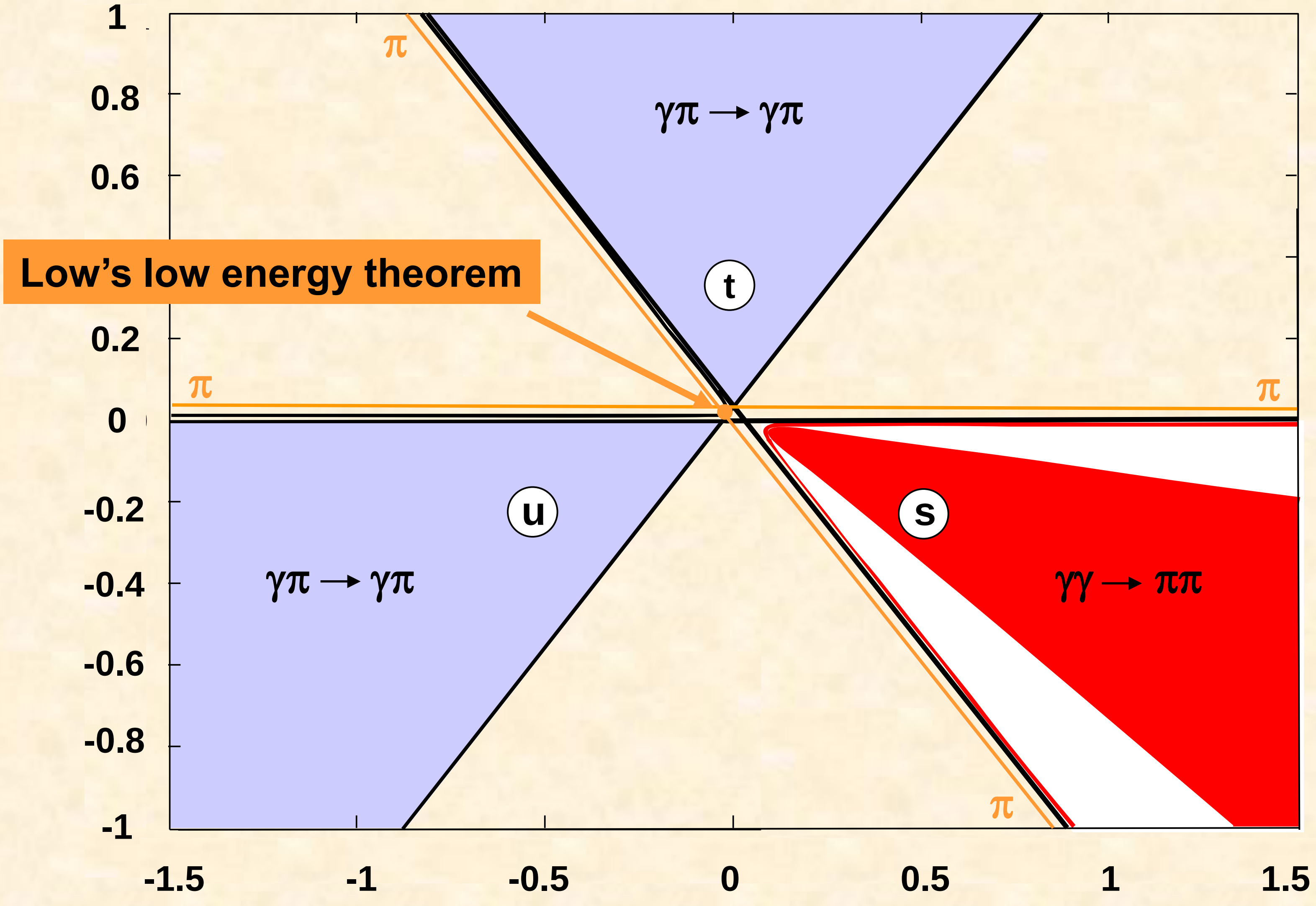
450 MeV
ρ
π
π
ρ
-1
1
cos θ
900 MeV
a1
ρ
π
π
ρ
a1
. . .
. . .
-1
1
cos θ


4π
3π
KKπ
KK
2π
2π
3π
4π
KK
KKπ
900 MeV
a1
ρ
π
π
ρ
a1
-1
1
cos θ

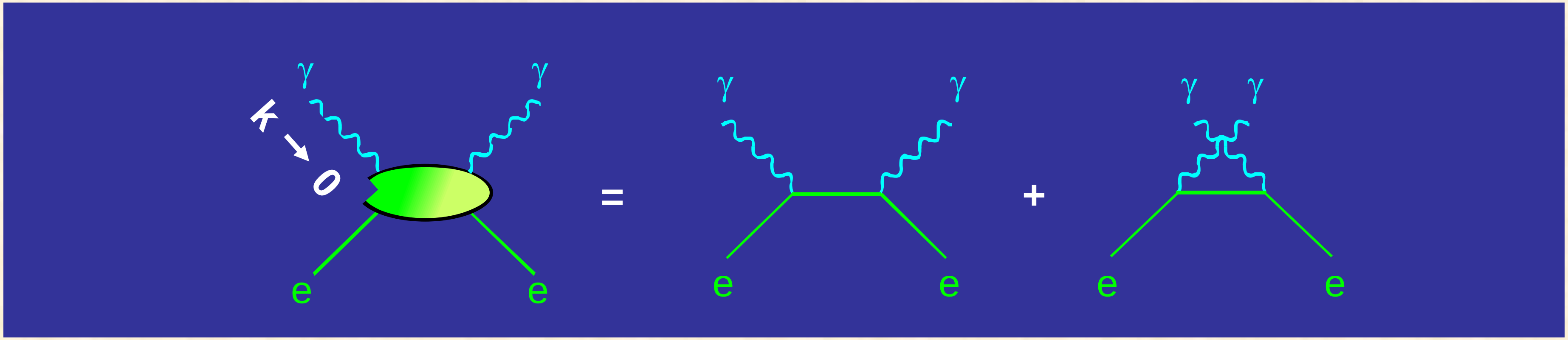


4π
3π
3π
4π
KKπ
KK
2π
2π
KK
KKπ
900 MeV
a1
ρ
π
π
ρ
a1
. . .
. . .
-1
1
cos θ

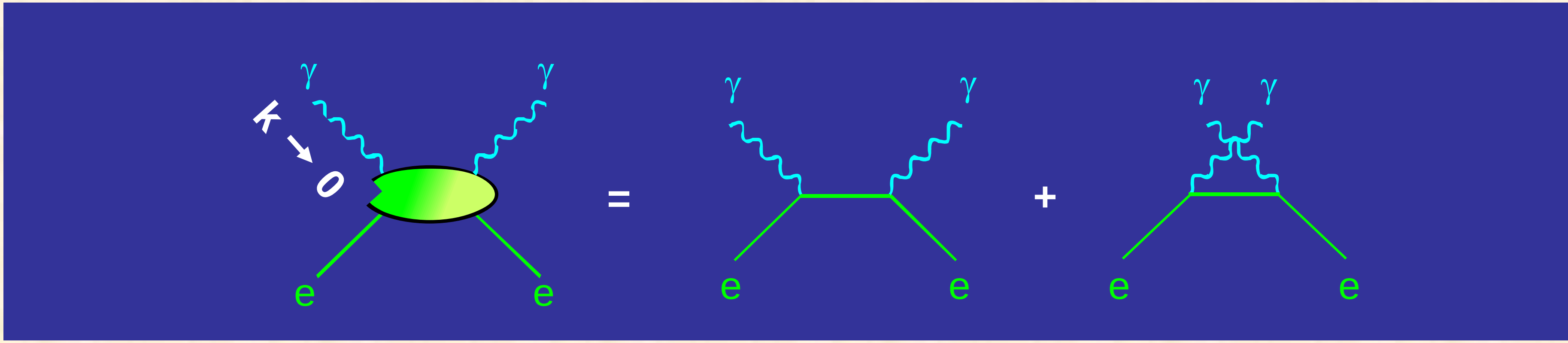
Mandelstam Plane

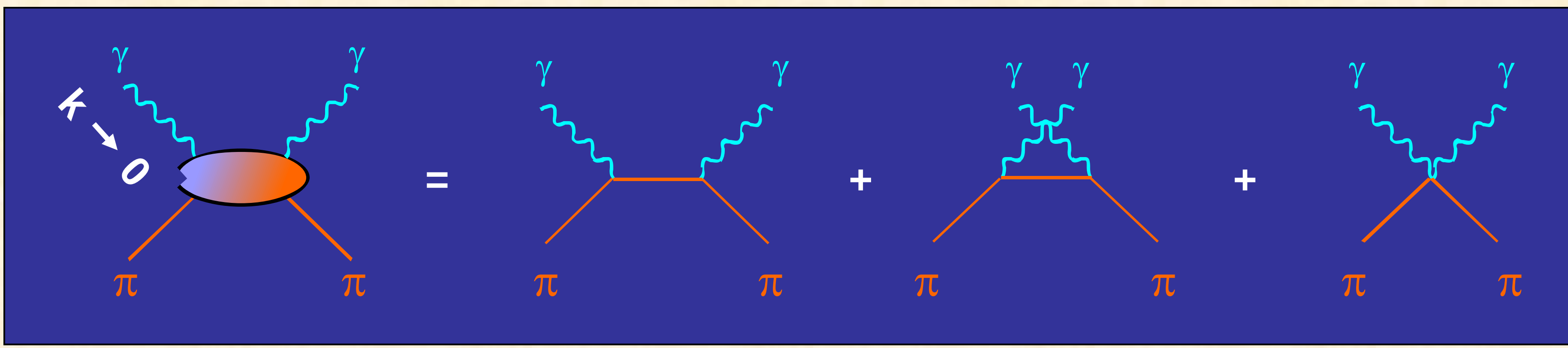


Low energy theorems

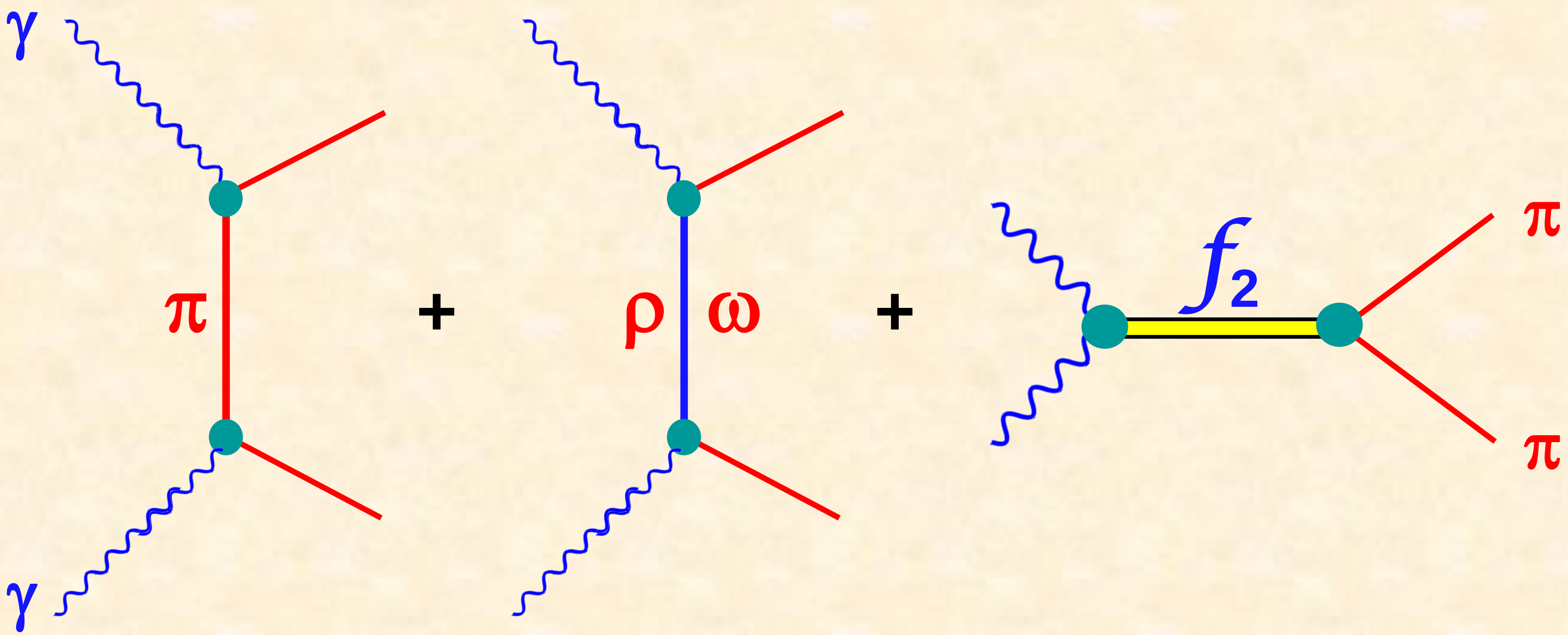


Low energy theorems





Is amplitude just :

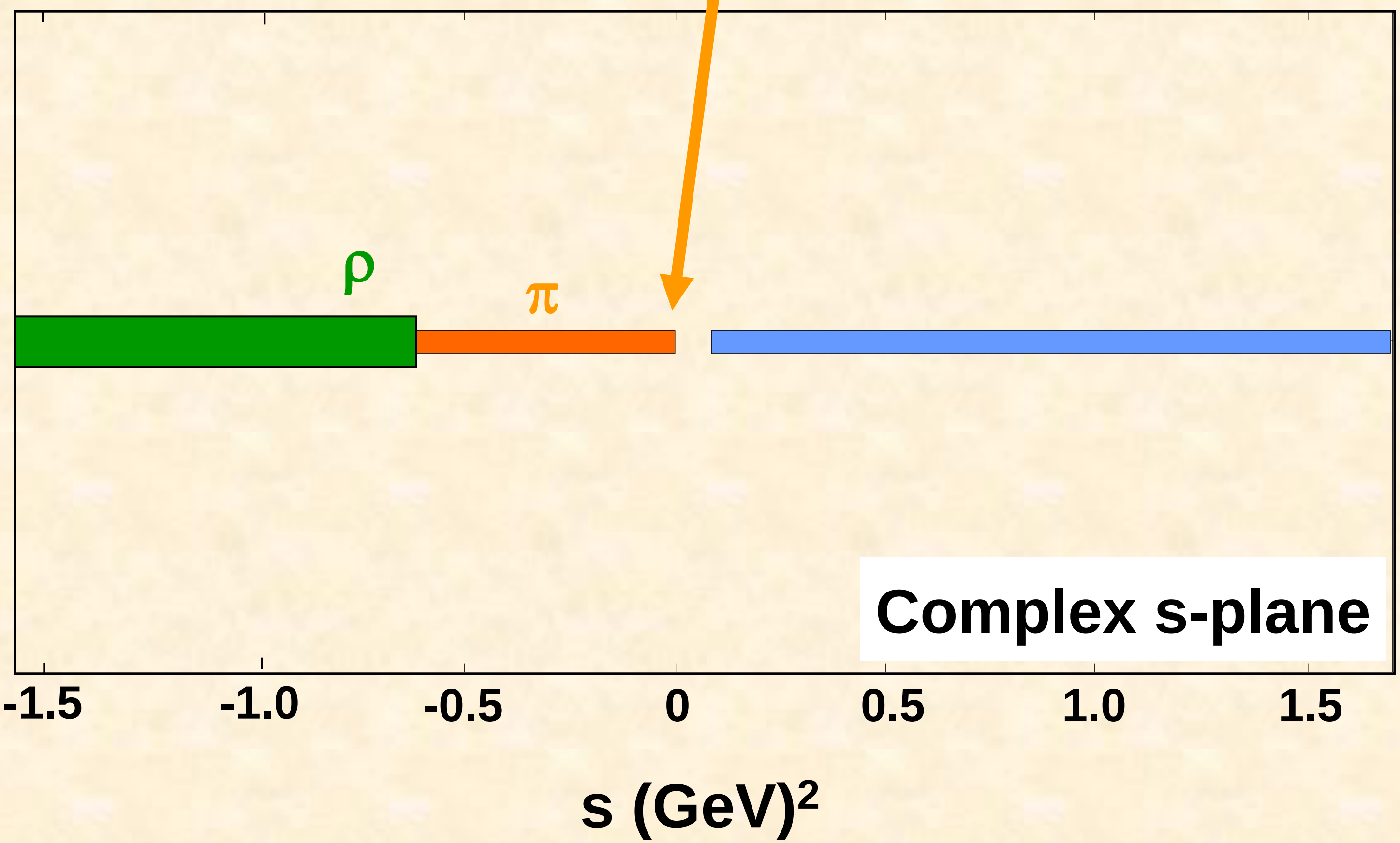


if not, why not ?

$\gamma\gamma \longrightarrow \pi\,\pi$

$\mathcal{F}(s)$ for each **I, J, λ**

$\mathcal{F}(s) \rightarrow \mathcal{B}(\mathbf{s})$ when $s \longrightarrow 0$

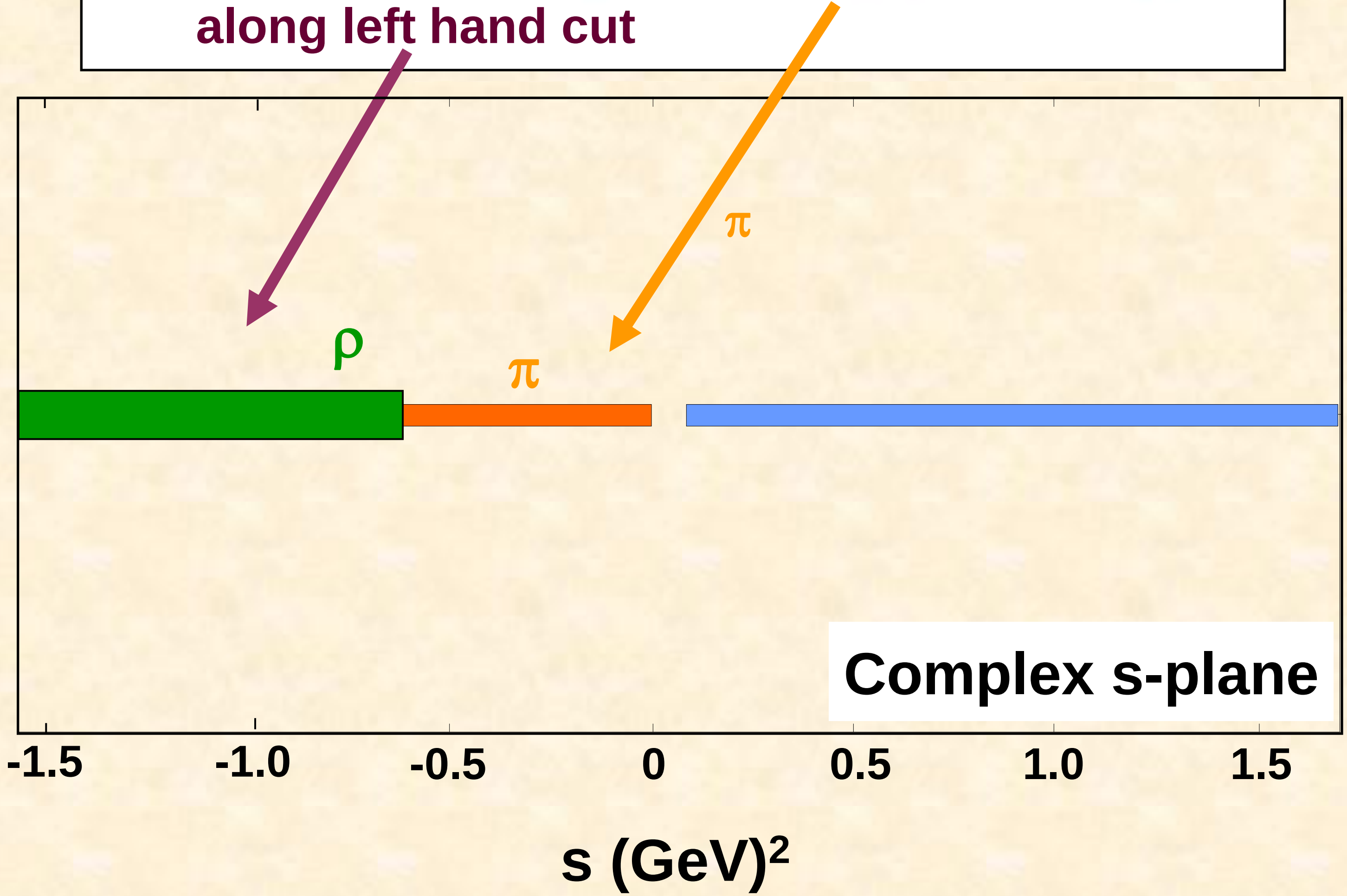


$\gamma\gamma \longrightarrow \pi\,\pi$

$\mathcal{F}(s)$ for each **I, J, λ**

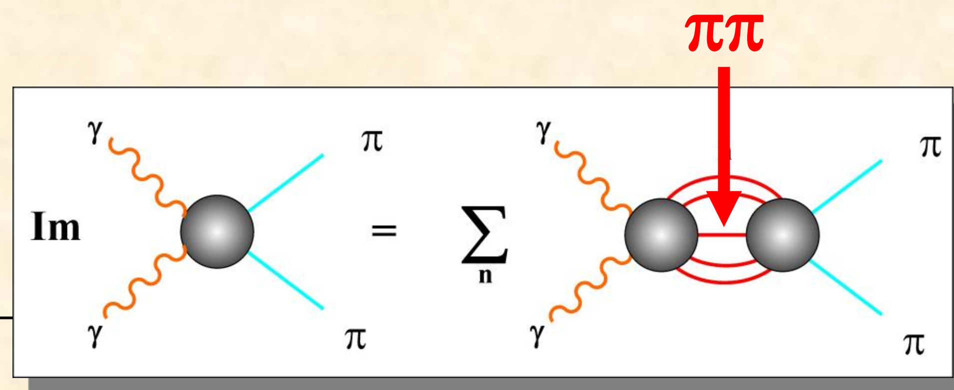
$$\mathcal{F}(s) \equiv \mathcal{H}(s) = \mathcal{B}(s) + \mathcal{L}(s)$$

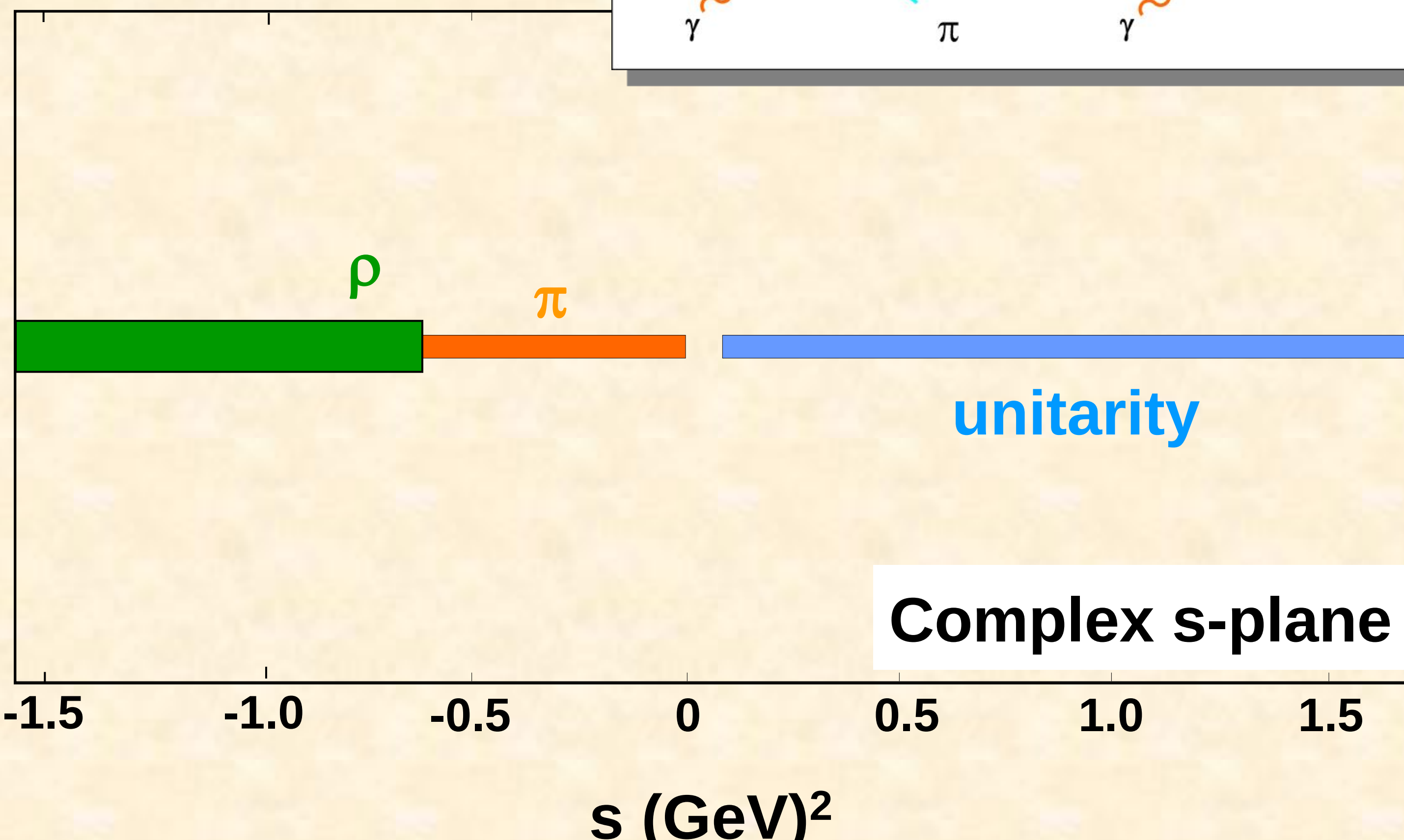
along left hand cut



$\gamma\gamma \longrightarrow \pi\,\pi$

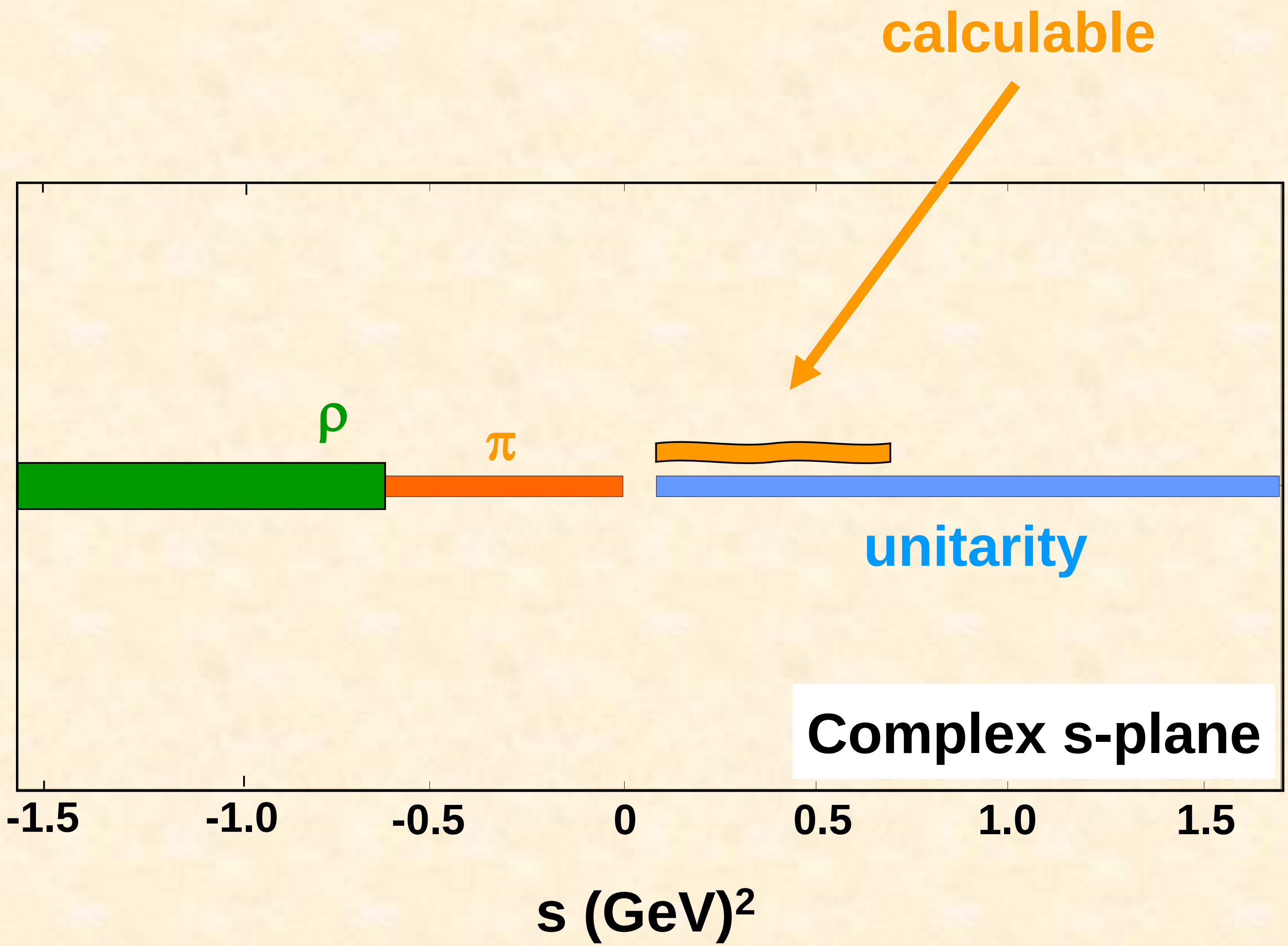
$\mathcal{F}(s)$ for each **I, J, λ**



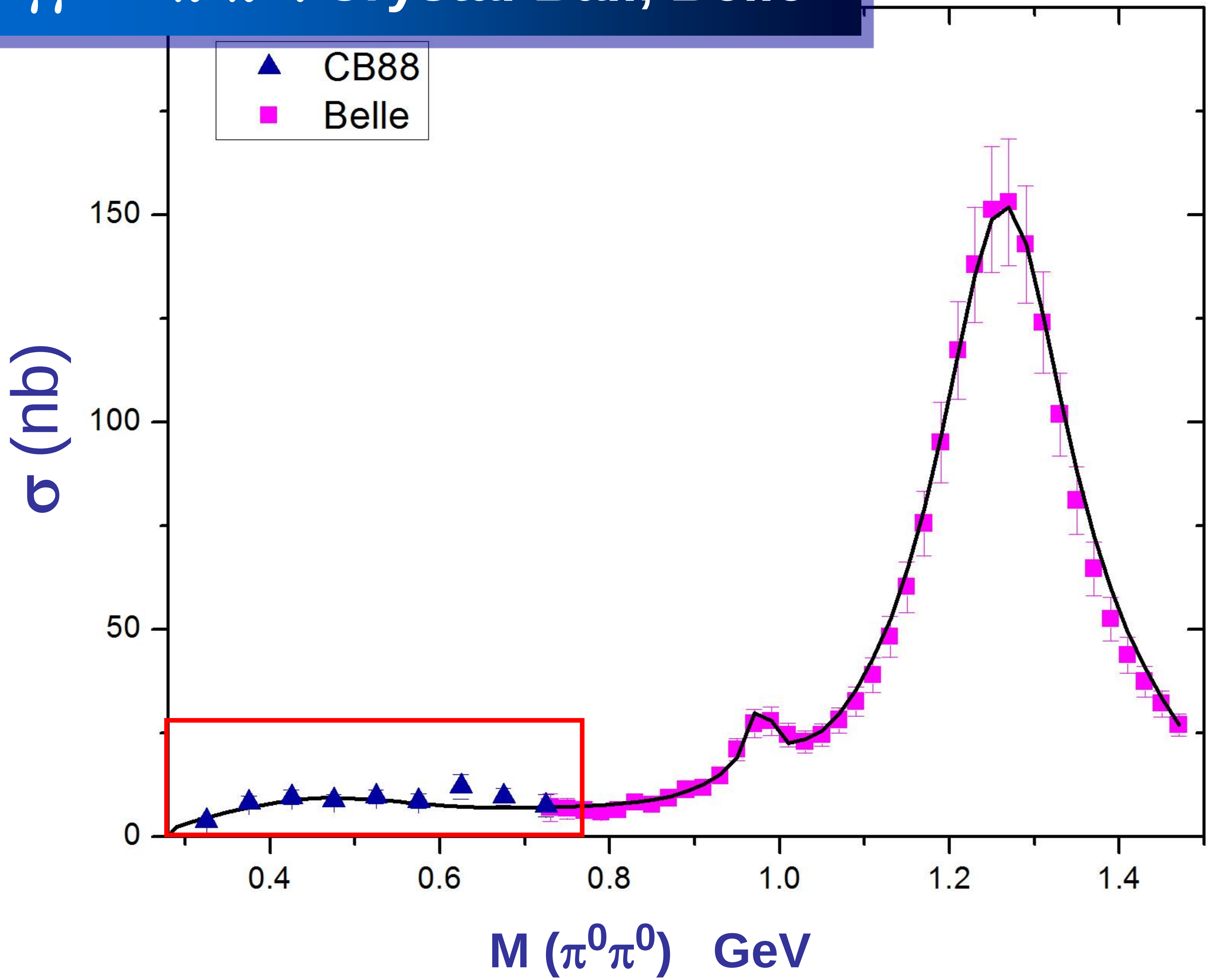


$\gamma\gamma \longrightarrow \pi\,\pi$

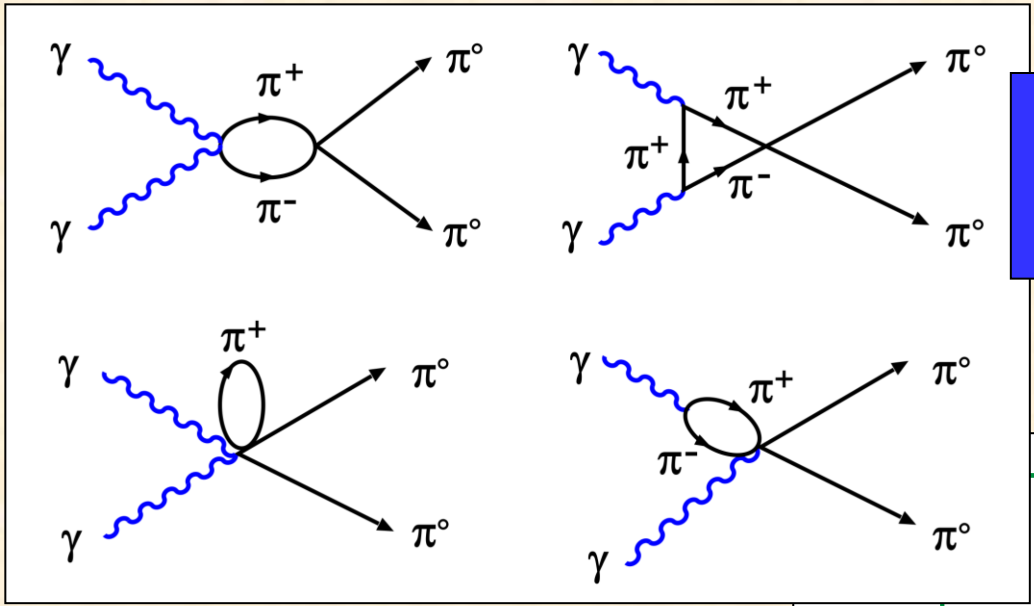
$\mathcal{F}(s)$ for each **I, J, λ**



$\gamma\gamma \longrightarrow \pi^0\pi^0$: Crystal Ball, Belle

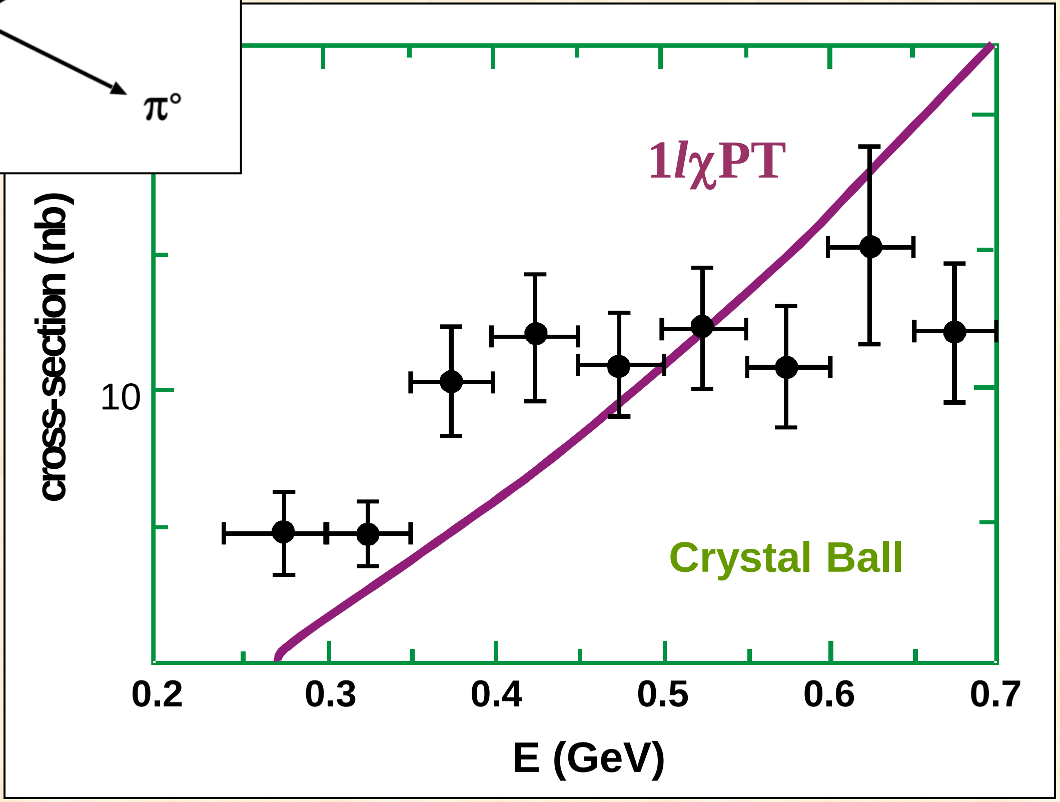


Calculating $\gamma\gamma \longrightarrow \pi^0\pi^0$

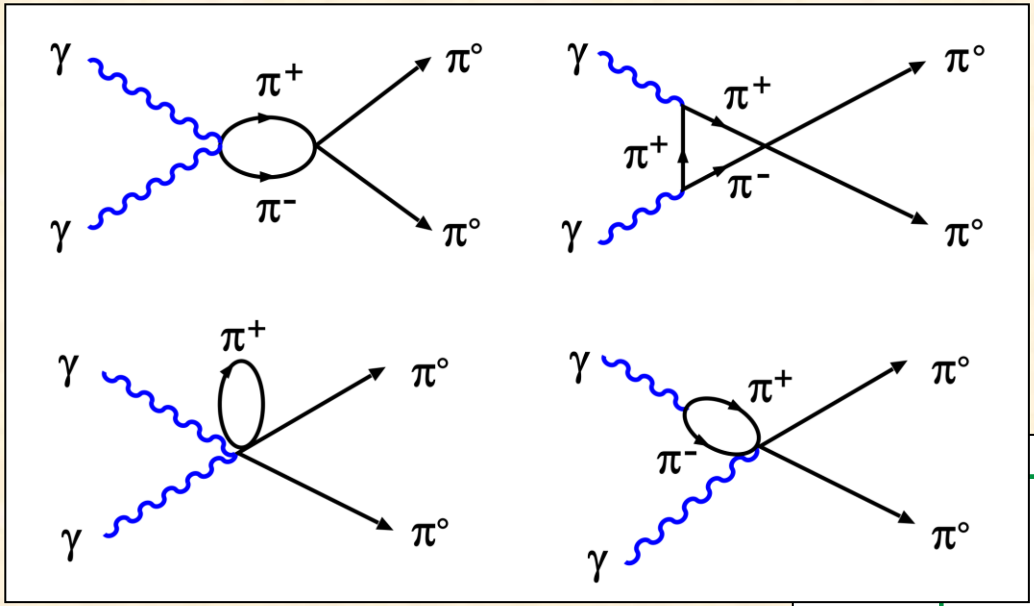


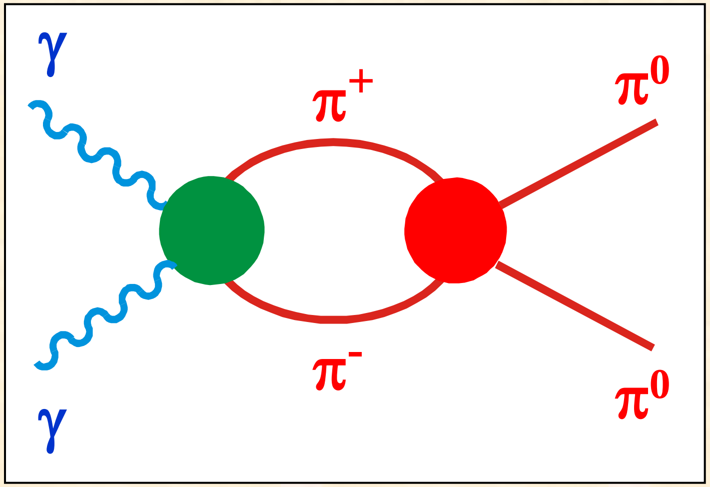
finite chiral loops

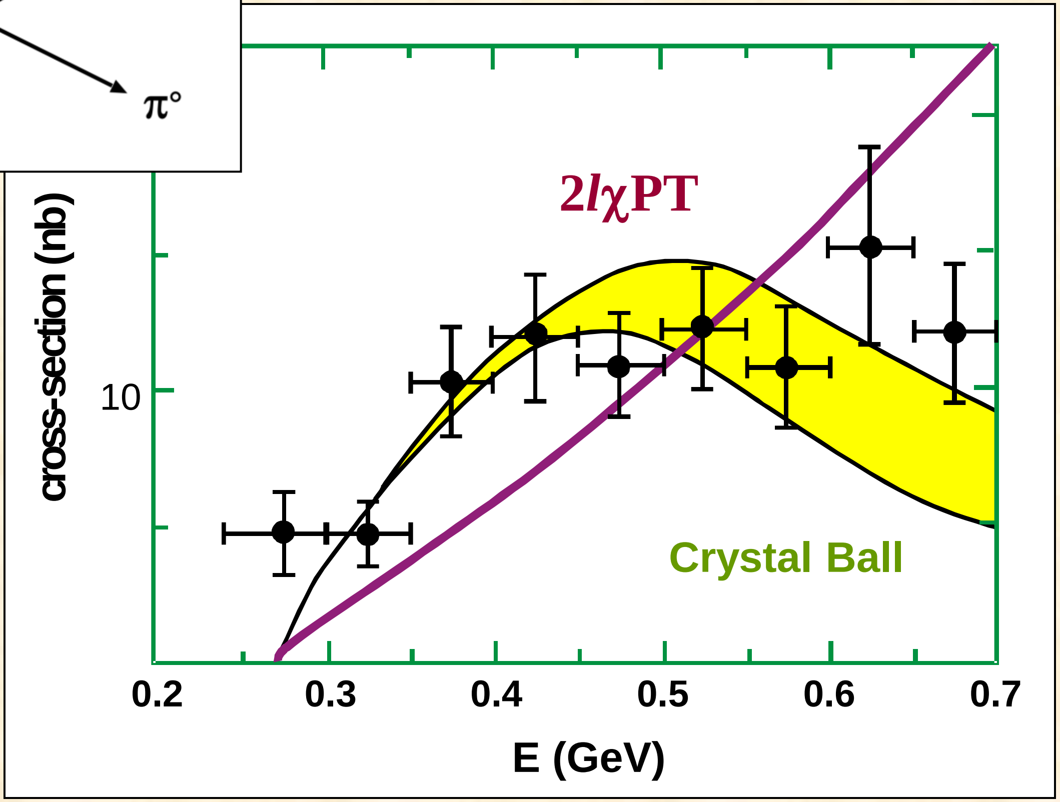
Maiani:
"gold plated test of χPT"



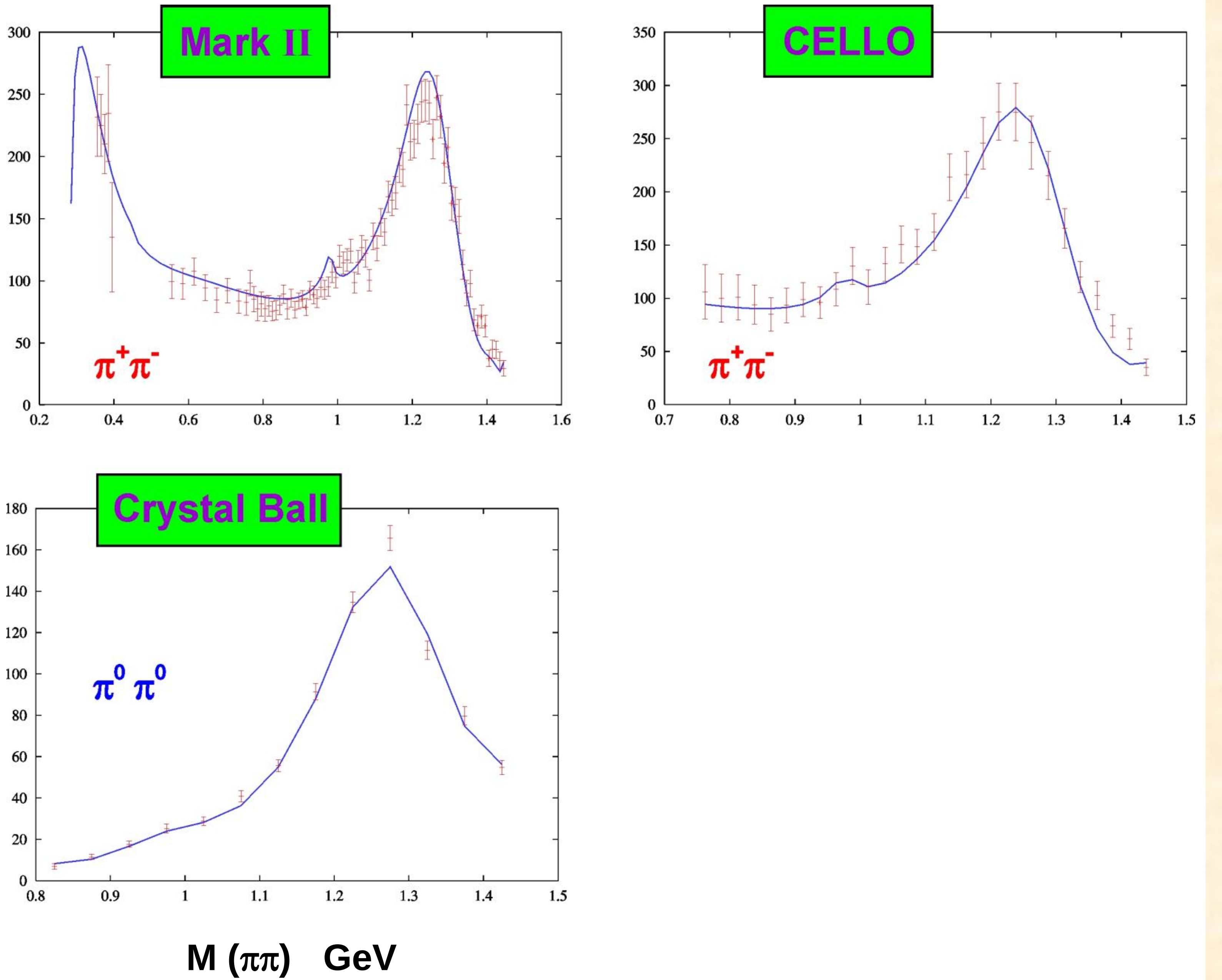
Calculating $\gamma\gamma \longrightarrow \pi^0\pi^0$

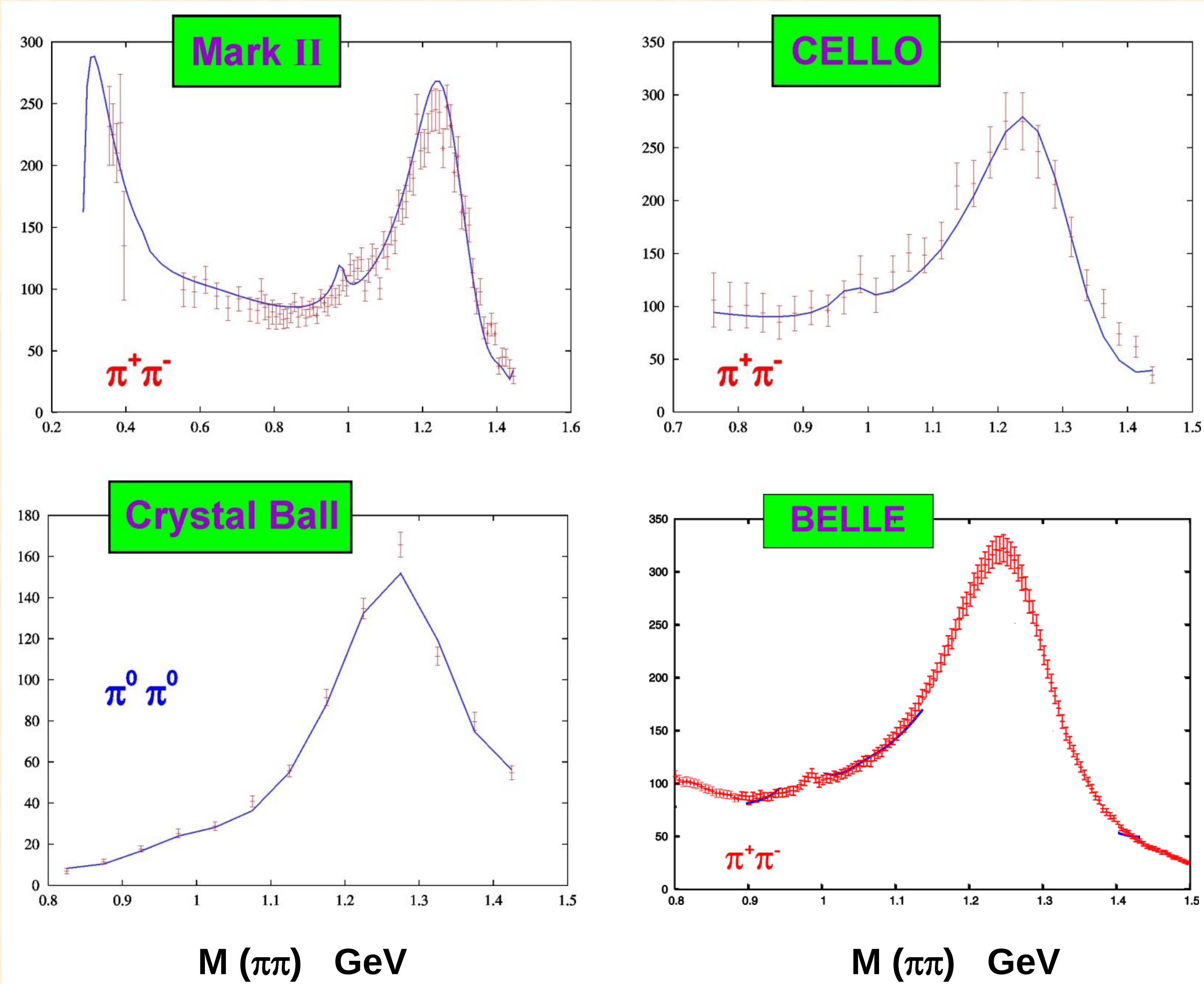




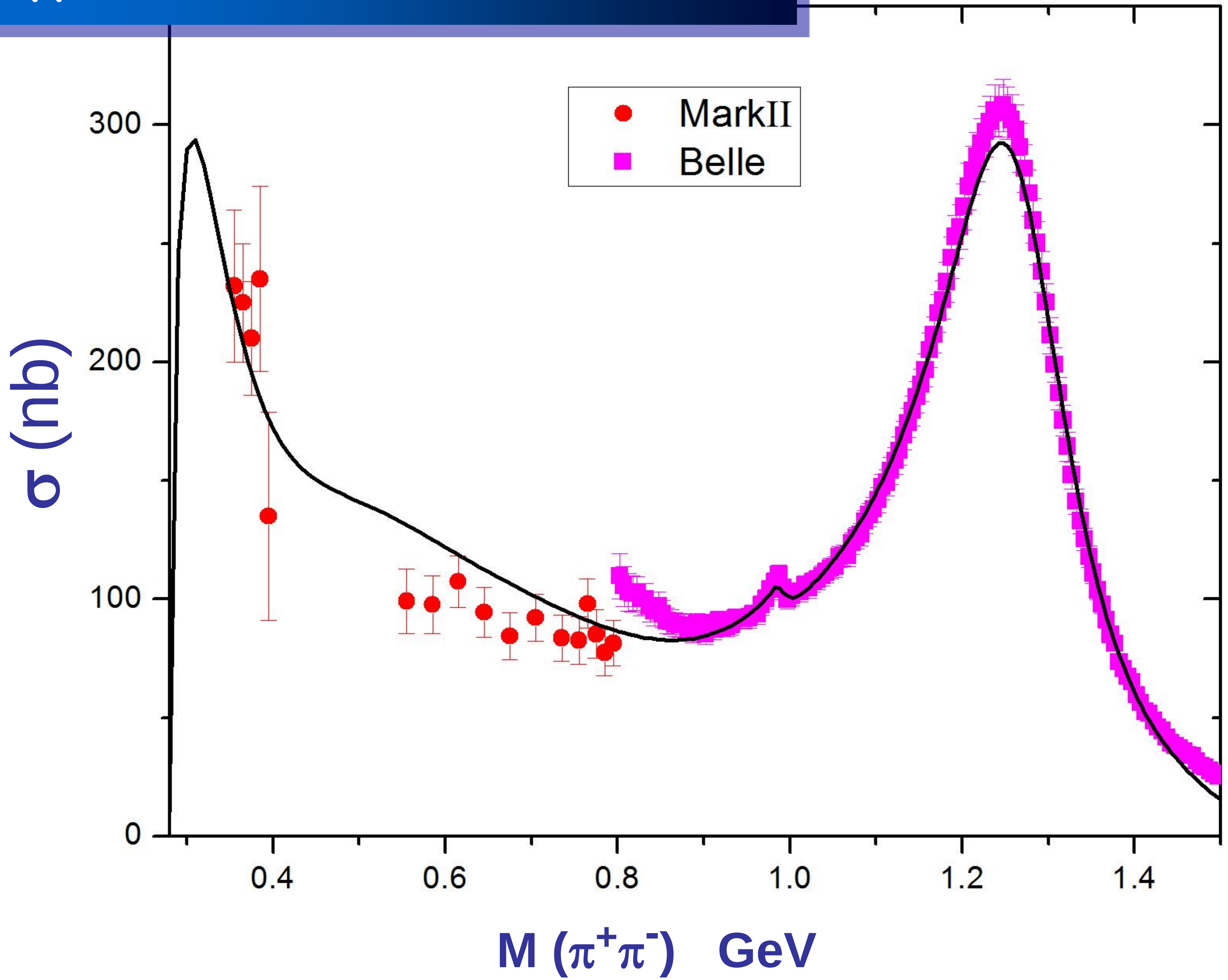


To perform a partial wave separation need
to know the partial waves at low energy accurately

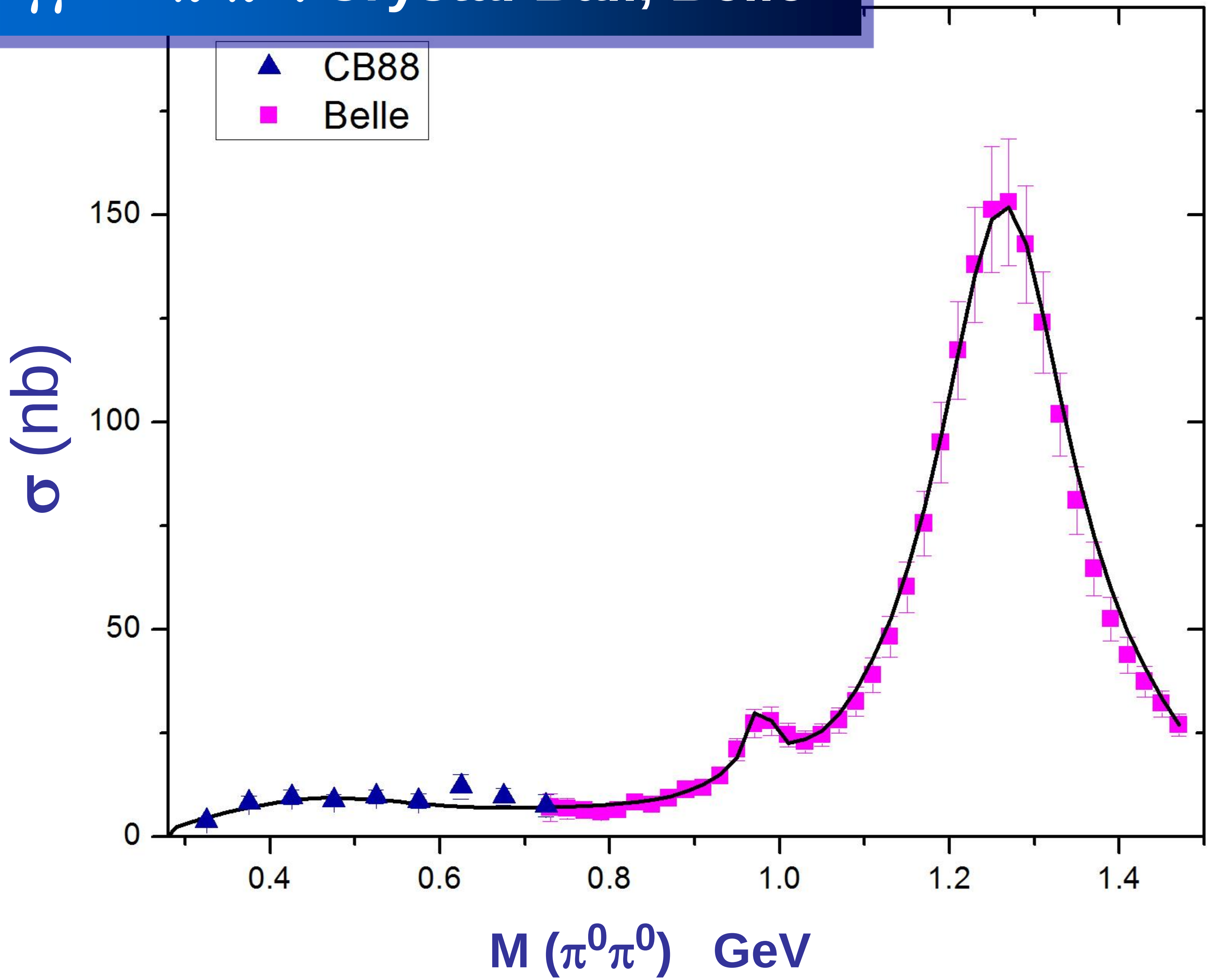

Mark II
$\pi^+\pi^-$
CELLO
$\pi^+\pi^-$
Crystal Ball
$\pi^0\,\pi^0$
M (ππ) GeV


Mark II
π+π-
CELLO
π+π-
Crystal Ball
π0 π0
BELLE
π+π-
M (ππ) GeV
M (ππ) GeV

γγ → $\pi^+\pi^-$: Mark II, Belle



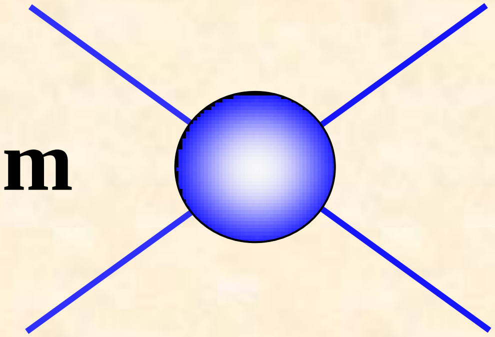
$\gamma\gamma \longrightarrow \pi^0\pi^0$: Crystal Ball, Belle



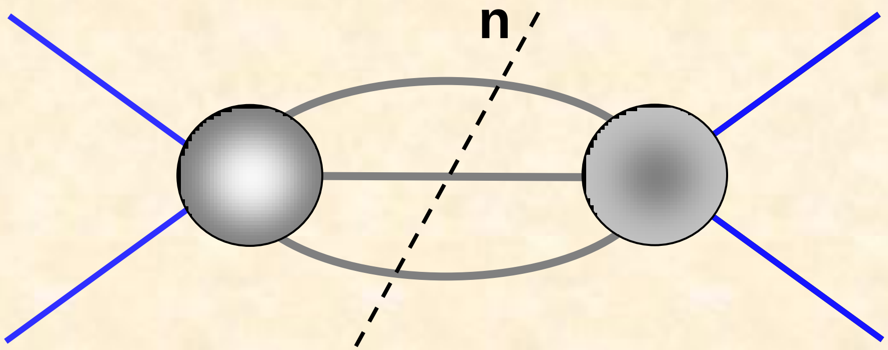
Unitarity

Amplitude with definite J^{PC}

$$\mathrm{Im}$$

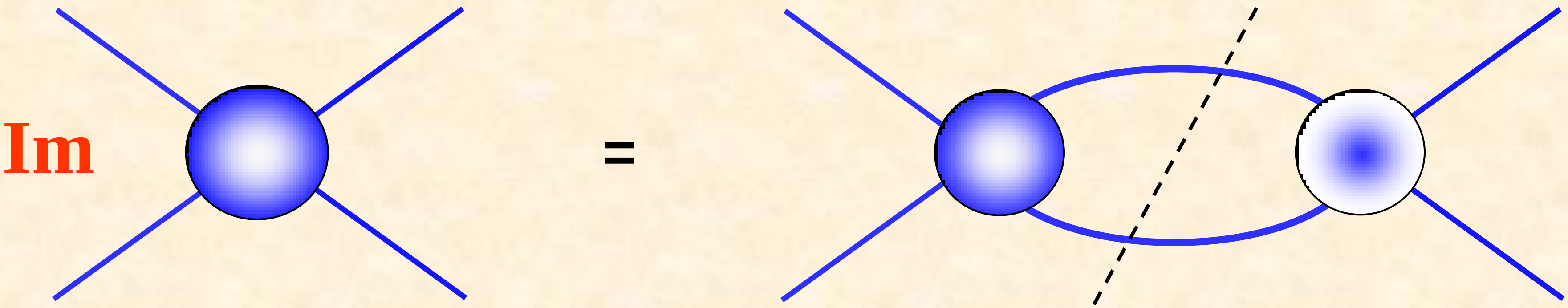


$$= \sum_n$$



Unitarity

Amplitude with definite J^{PC}



$1 = \pi\pi$

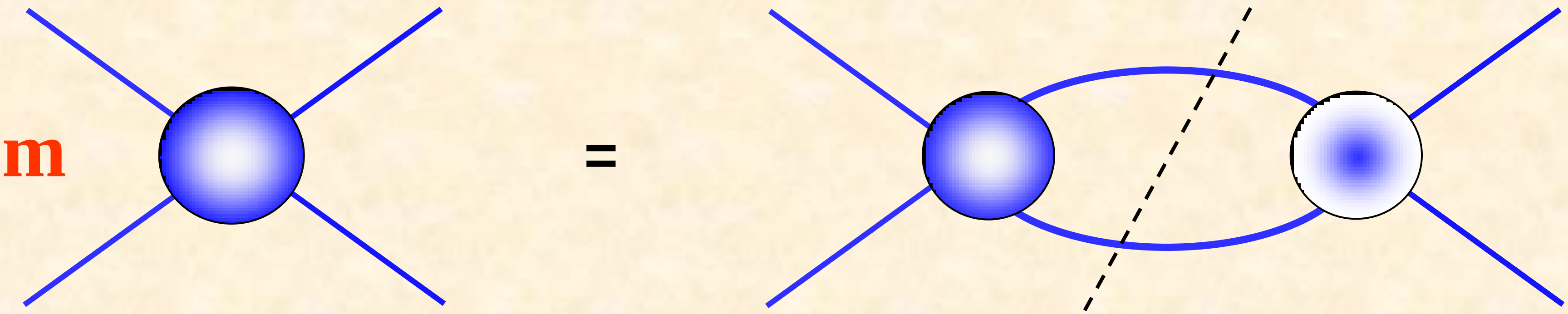
$$\mathrm{Im}\ T_{11}(s) = \rho_1(s)\ T_{11}^*(s)\, T_{11}(s)$$

$$\rho_1 = k_1/E$$
$$= \sqrt{1 - 4m_1^2/s}$$

Unitarity

Amplitude with definite J^{PC}

Im



$1 = \pi\pi$

$$\mathrm{Im}\ T_{11}(s) = \rho_1(s)\ T_{11}^*(s)\ T_{11}(s)$$

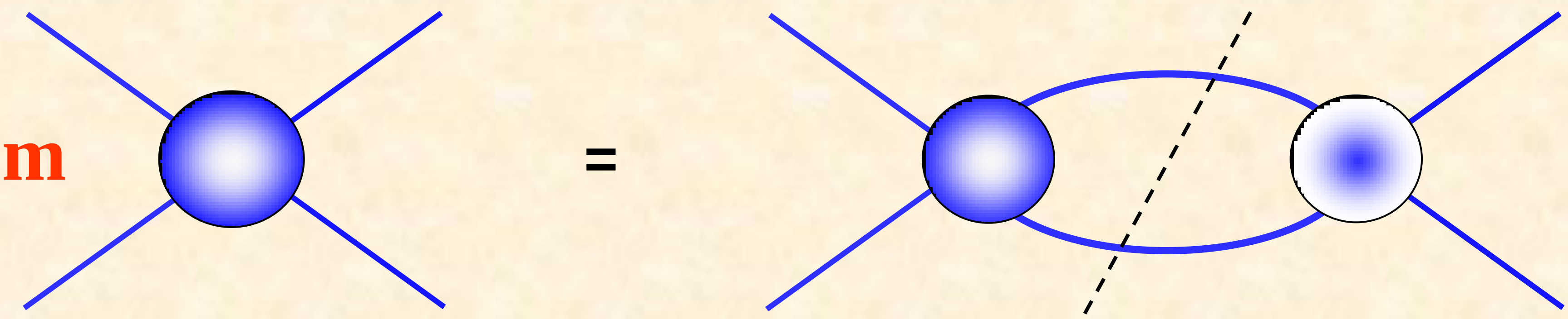
let $\quad T_{11} = \frac{1}{\rho_1}\,|T_{11}|\ e^{i\delta} \quad \Rightarrow \quad \sin\delta = |T_{11}|$

$\rho_1 = k_1/E$

$= \sqrt{1 - 4m_1^2/s}$

Unitarity

Amplitude with definite J^{PC}



$1 = \pi\pi$

$$\text{Im}\; T_{11}(s) = \rho_1(s)\; T_{11}^*(s)\, T_{11}(s)$$

let $T_{11} = \frac{1}{\rho_1} \,|\, T_{11} \,|\, e^{i\delta}$ ⟹ $\sin\delta = |\, T_{11} \,|$

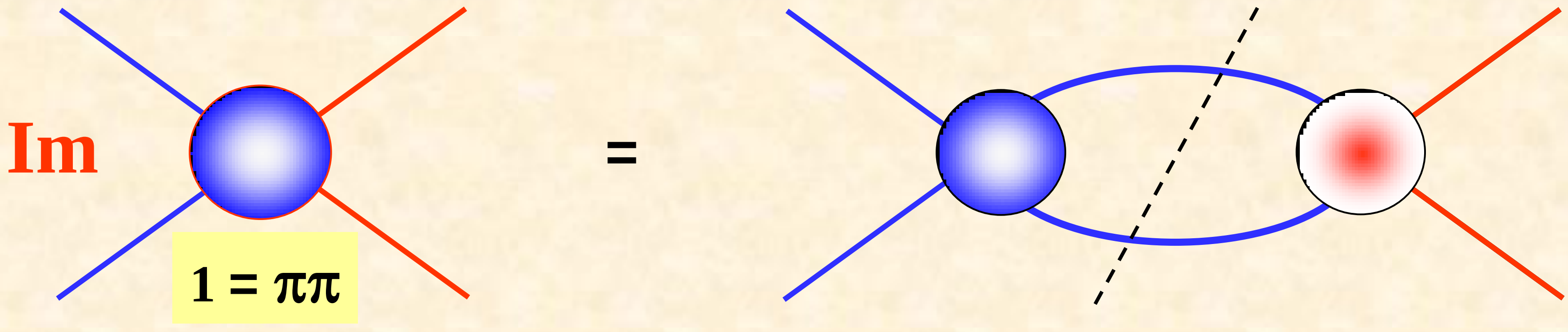
$\rho_1 = k_1/E$

$= \sqrt{1 - 4m_1^2/s}$

$$T_{11} = \frac{1}{\rho_1} \sin\delta\; e^{i\delta}$$

Unitarity

Amplitude with definite J^{PC}

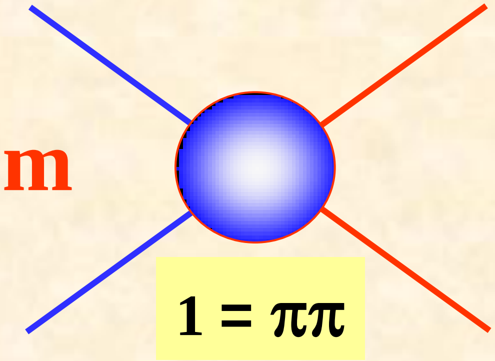


$$\mathrm{Im}\,\mathcal{F}_1(s) = \rho_1(s)\,\mathcal{F}_1^*(s)\,T_{11}(s) = \rho_1(s)\,\mathcal{F}_1(s)\,T_{11}^*(s)$$

Unitarity

Amplitude with definite J^{PC}

Im



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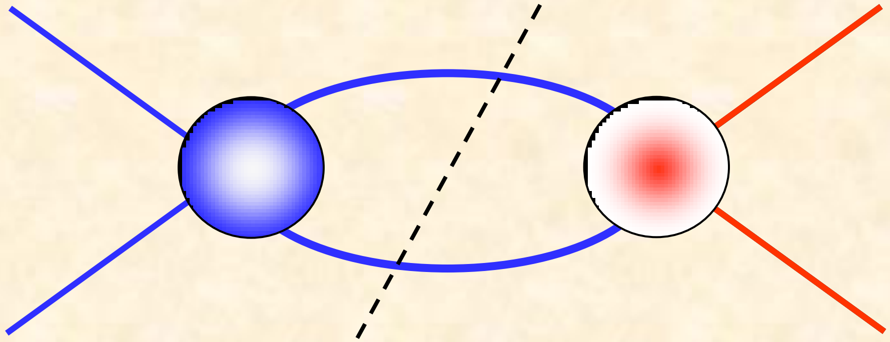


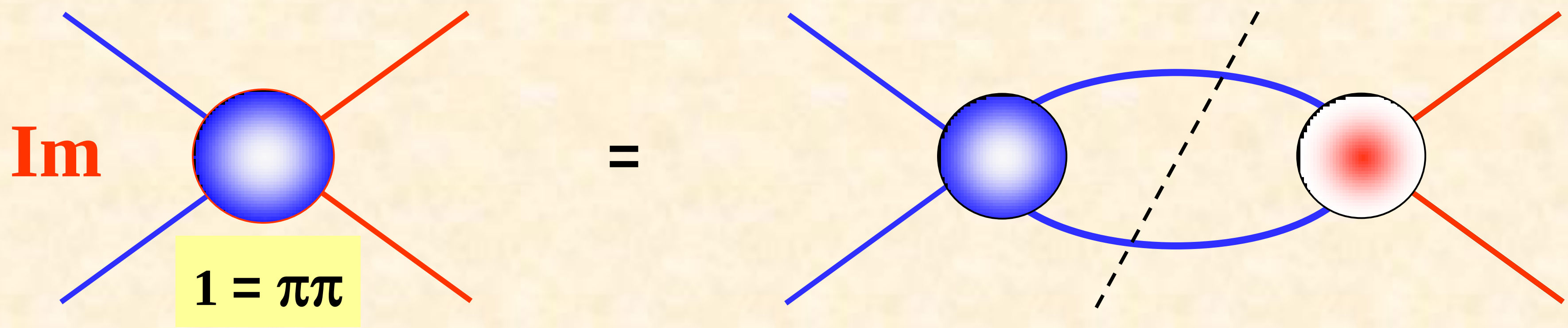
$$\text{Im}\,\mathcal{F}_1(s) = \rho_1(s)\,\mathcal{F}_1^*(s)\,T_{11}(s) = \rho_1(s)\,\mathcal{F}_1(s)\,T_{11}^*(s)$$

let $\mathcal{F}_1 = |\mathcal{F}_1|\ e^{i\varphi}$

recall $T_{11} = \dfrac{1}{\rho_1} \sin\delta\ e^{i\delta}$

Unitarity

Amplitude with definite J^{PC}



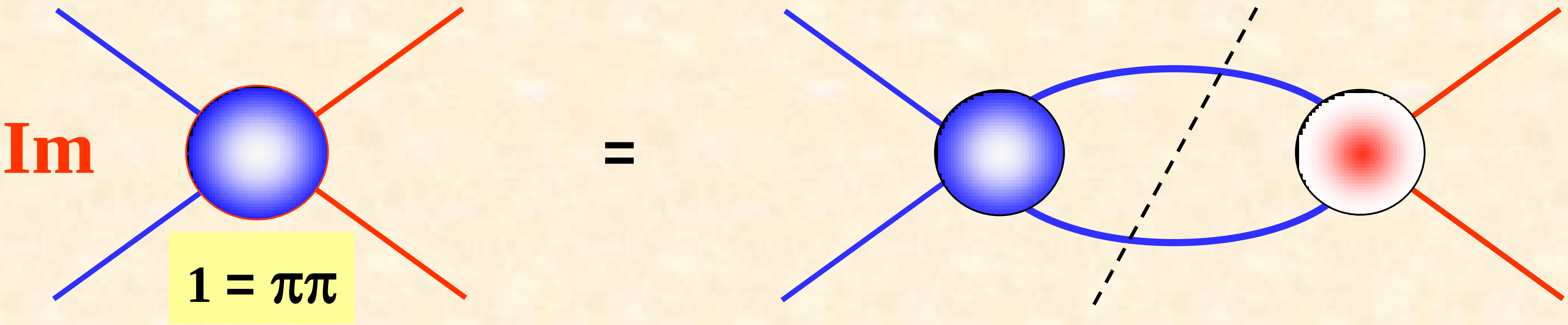
$$\text{Im}\,\mathcal{F}_1(s) = \rho_1(s)\,\mathcal{F}_1^*(s)\,T_{11}(s) = \rho_1(s)\,\mathcal{F}_1(s)\,T_{11}^*(s)$$

let $\mathcal{F}_1 = |\mathcal{F}_1|\, e^{i\varphi}$ recall $T_{11} = \frac{1}{\rho_1} \sin\delta\; e^{i\delta}$

$\Rightarrow$ $\sin\varphi = \sin\delta$

Unitarity

Amplitude with definite J^{PC}



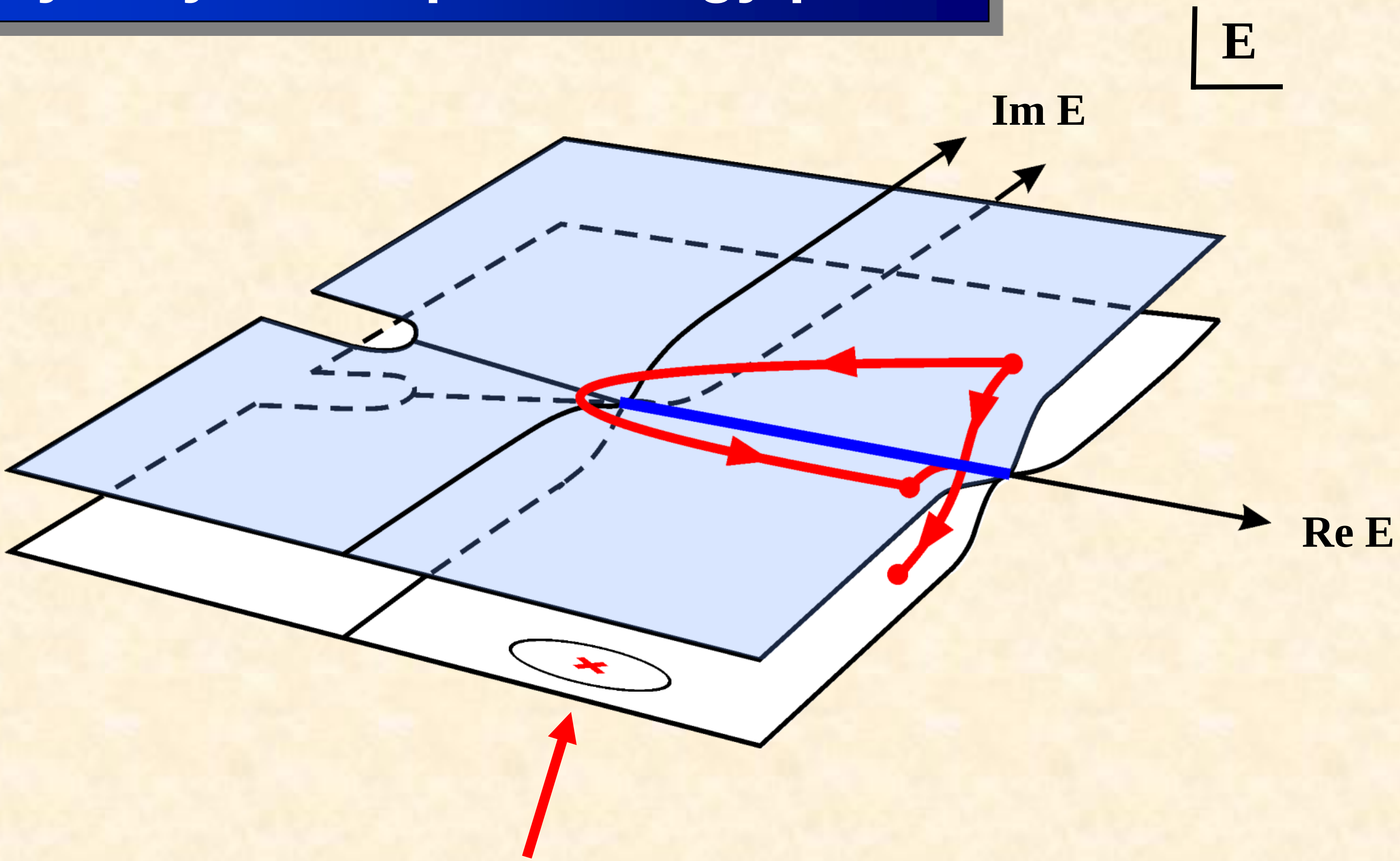
$$\text{Im}\,\mathcal{F}_1(s) = \rho_1(s)\,\mathcal{F}_1^*(s)\,T_{11}(s) = \rho_1(s)\,\mathcal{F}_1(s)\,T_{11}^*(s)$$

let $\mathcal{F}_1 = |\mathcal{F}_1|\ e^{i\varphi}$ recall $T_{11} = \dfrac{1}{\rho_1} \sin\delta\ e^{i\delta}$

⟹ **$\sin\varphi = \sin\delta$**

Watson's final state interaction theorem: $\varphi = \delta\ \ (+n\pi)$

analyticity & complex energy plane

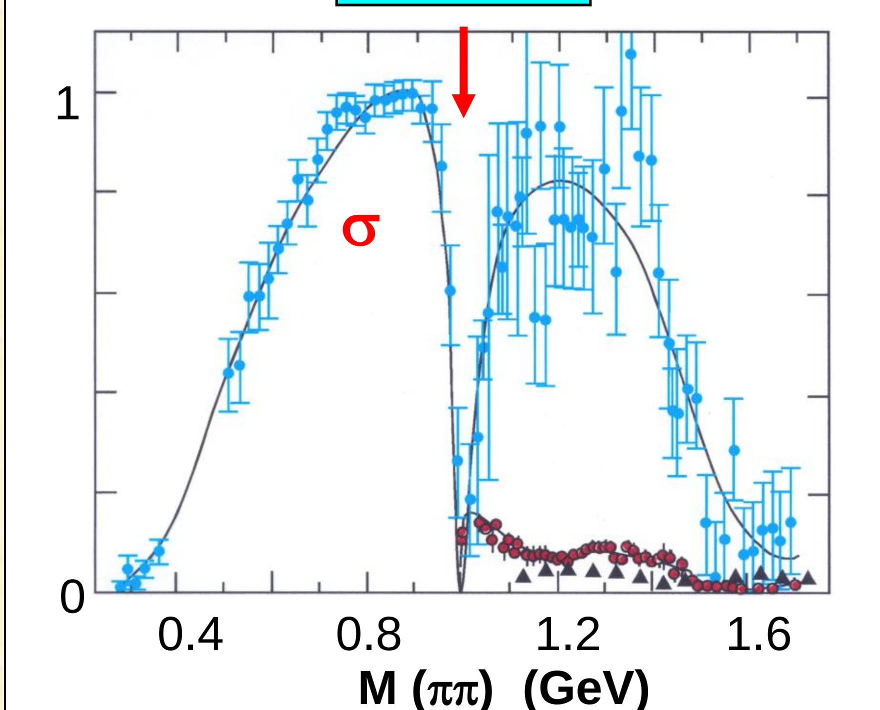


resonance pole

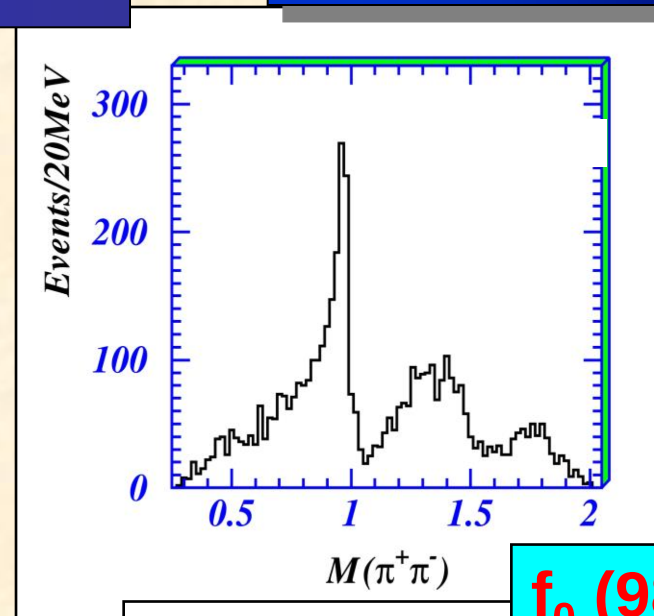
f_0 (980) in ππ and "$\bar{K}K$"

J/ψ → φ (MM)

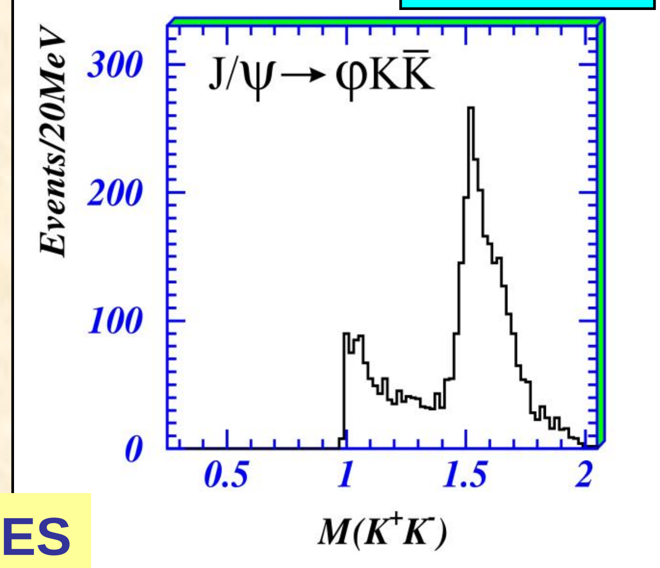
f₀ (980)



CERN-Munich, ANL, BNL



f₀ (980)

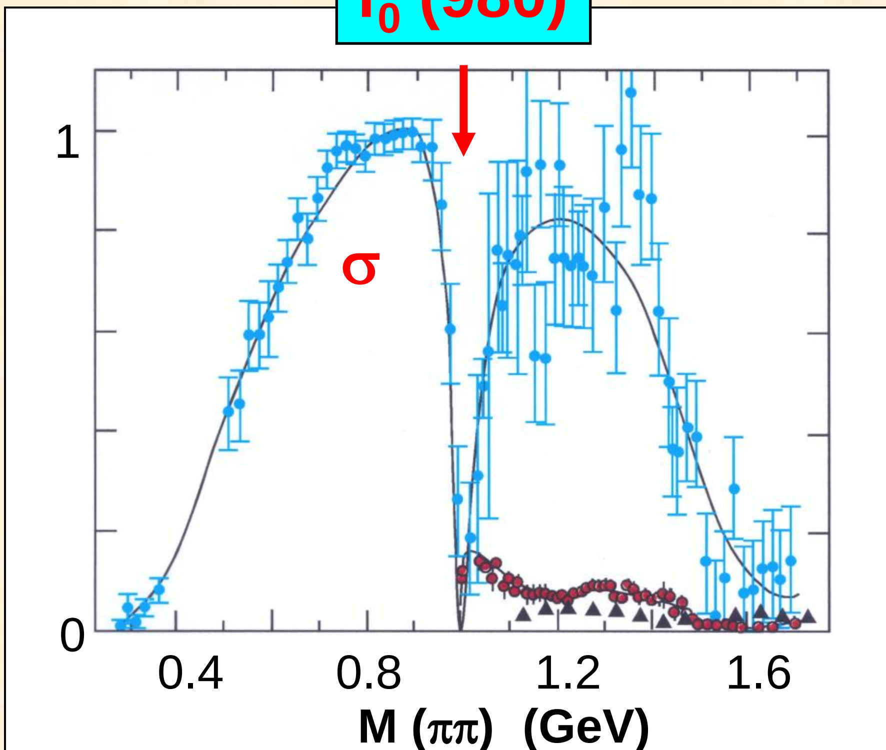


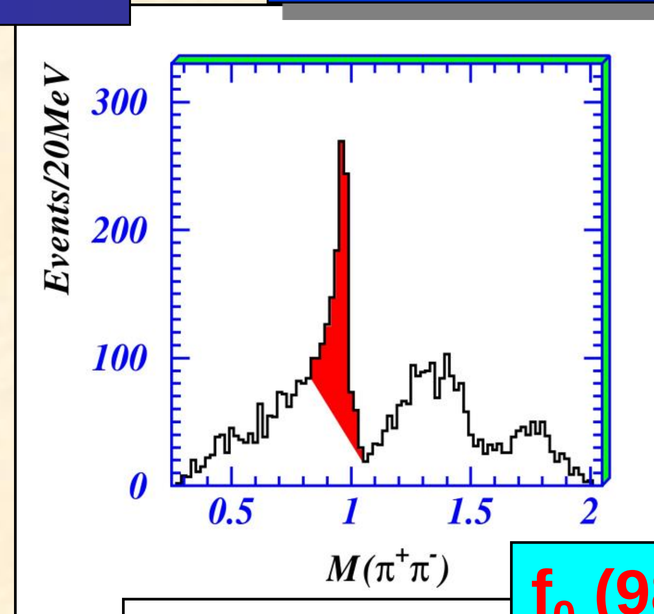
BES

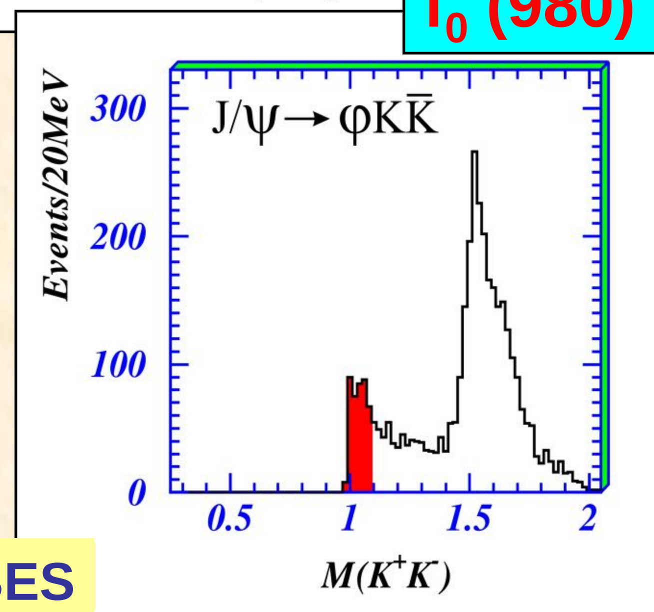
I = J = 0

f_0(980) in $\pi\pi$ and "$\bar{K}K$"

$J/\psi \rightarrow \varphi$ (MM)







CERN-Munich, ANL, BNL

BES

I = J = 0

$\mathcal{F}(s)$ for each **I, J, λ**

$$\mathcal{F}(\mathbf{s}) = |\mathcal{F}(\mathbf{s})| \exp[\mathbf{i}\varphi(\mathbf{s})]$$ for $s > s_{th}$

$\mathcal{F}(s)$ for each **I, J, λ**

$$\mathcal{F}(\mathbf{s}) = |\mathcal{F}(\mathbf{s})| \exp\left[\mathbf{i}\varphi(\mathbf{s})\right]$$ for $s > s_{th}$

define function

$$\mathbf{\Omega}(\mathbf{s}) = |\mathbf{\Omega}(\mathbf{s})| \exp\left[\mathbf{i}\varphi(\mathbf{s})\right]$$

with only a right hand cut

$\mathcal{F}(s)$ for each **I, J, λ**

$$\mathcal{F}(\mathbf{s}) = |\mathcal{F}(\mathbf{s})| \exp\left[\mathbf{i}\varphi(\mathbf{s})\right]$$ for $s > s_{th}$

define function $$\boldsymbol{\Omega}(\mathbf{s}) = |\boldsymbol{\Omega}(\mathbf{s})| \exp\left[\mathbf{i}\varphi(\mathbf{s})\right]$$

with only a right hand cut

then $$\boldsymbol{\Omega}(\mathbf{s}) = \exp\left[\frac{\mathbf{s}}{\pi} \int_{\mathbf{s}_{th}}^{\infty} \mathbf{ds}' \frac{\varphi(\mathbf{s}')}{\mathbf{s}'(\mathbf{s}' - \mathbf{s})}\right]$$

Omnes

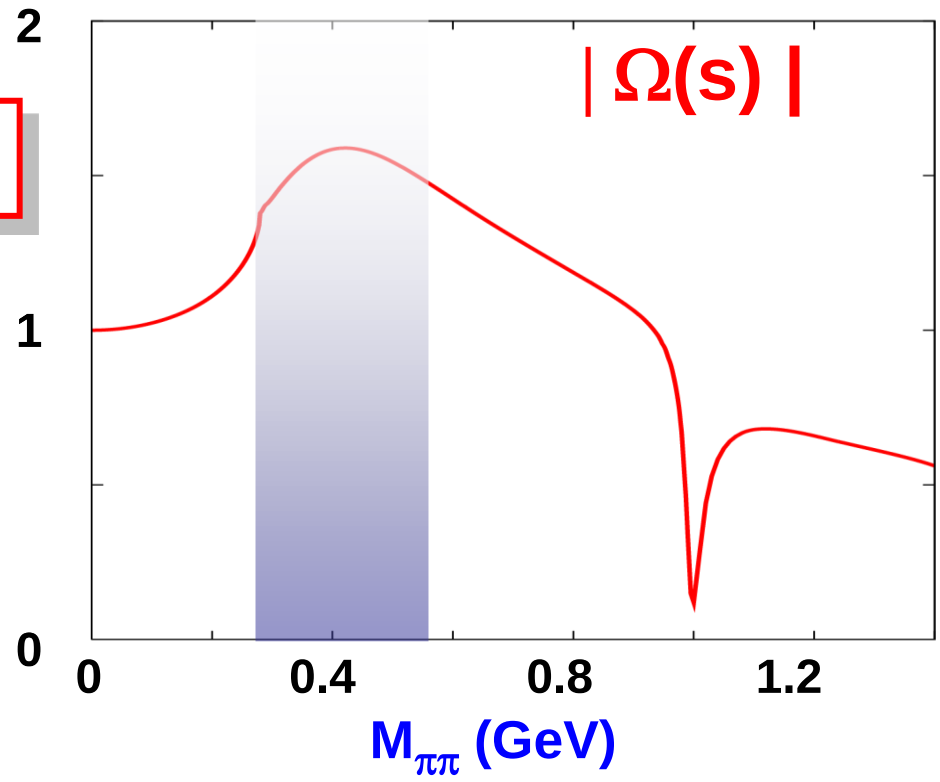
$$\Omega(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\varphi(s')}{s'(s'-s)}\right]$$

$$\varphi(s) \equiv \varphi(\gamma\gamma \to \pi\pi)$$

if →

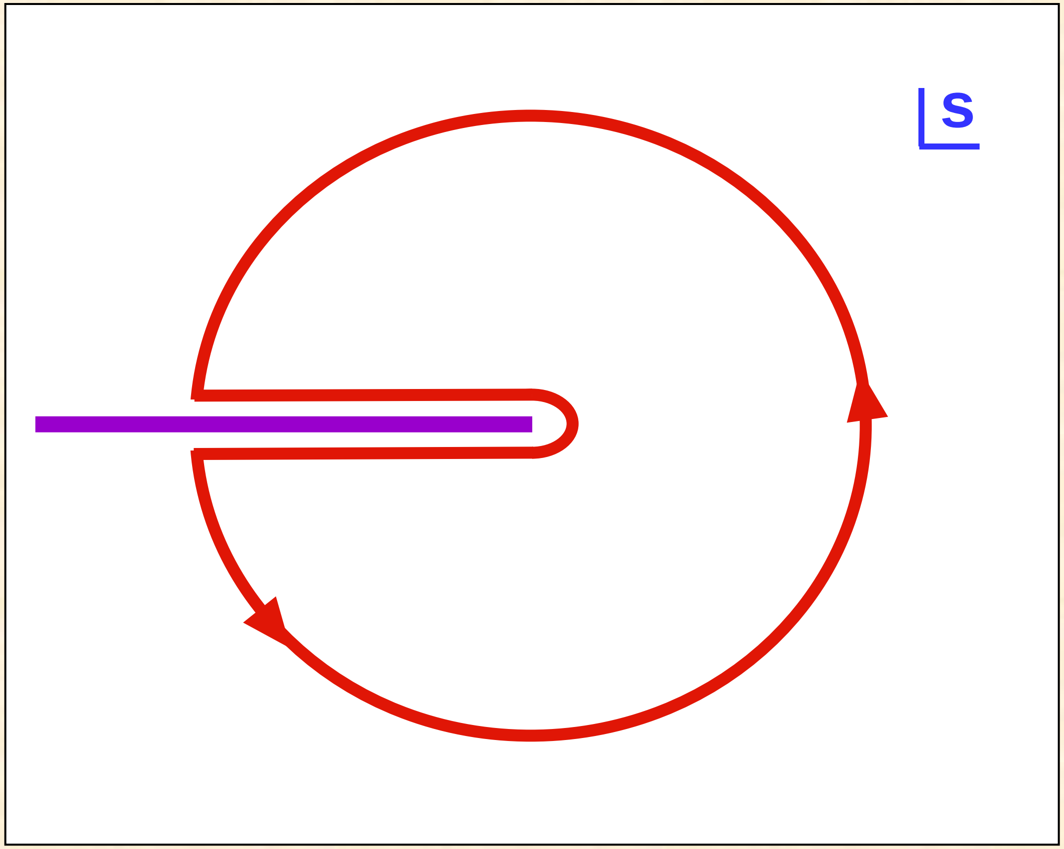
$$\varphi(s) = \varphi(\pi\pi \to \pi\pi)$$

I = J =0



construct $g_1(s) \equiv \mathcal{F}(s)\, \Omega^{-1}(s)$ with only a left hand cut

$$\mathcal{F}(s) = \frac{\Omega(s)}{\pi} \int_{-\infty}^{0} ds' \, \frac{\mathrm{Im}\mathcal{H}(s')\, \Omega^{-1}(s')}{s' - s}$$

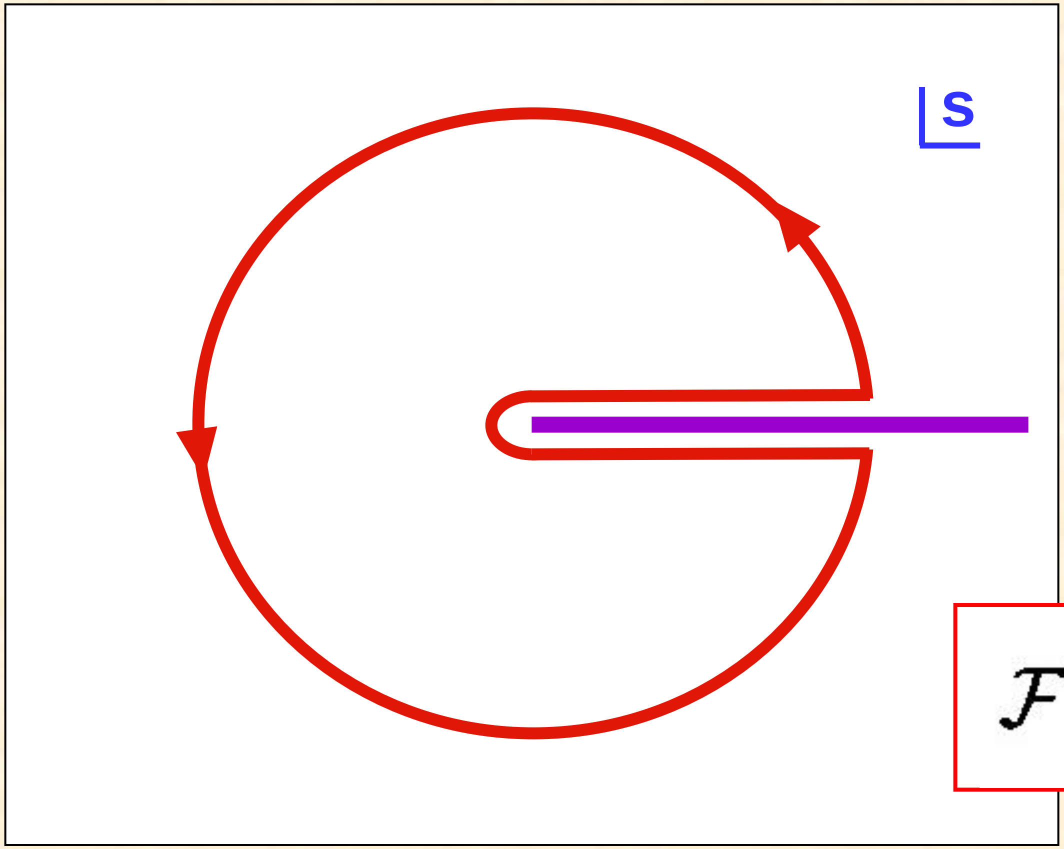


with no subtractions

construct $g_2(s) \equiv (\mathcal{F}(\mathbf{s}) - \mathcal{H}(\mathbf{s}))\,\boldsymbol{\Omega}^{-1}(\mathbf{s})$ then

$$\mathcal{F}(s) = \mathcal{H}(s) - \frac{\Omega(s)}{\pi} \int_{s_{th}}^{\infty} ds' \, \frac{\mathcal{H}(s')\, \mathrm{Im}\Omega^{-1}(s')}{s' - s}$$

with no subtractions



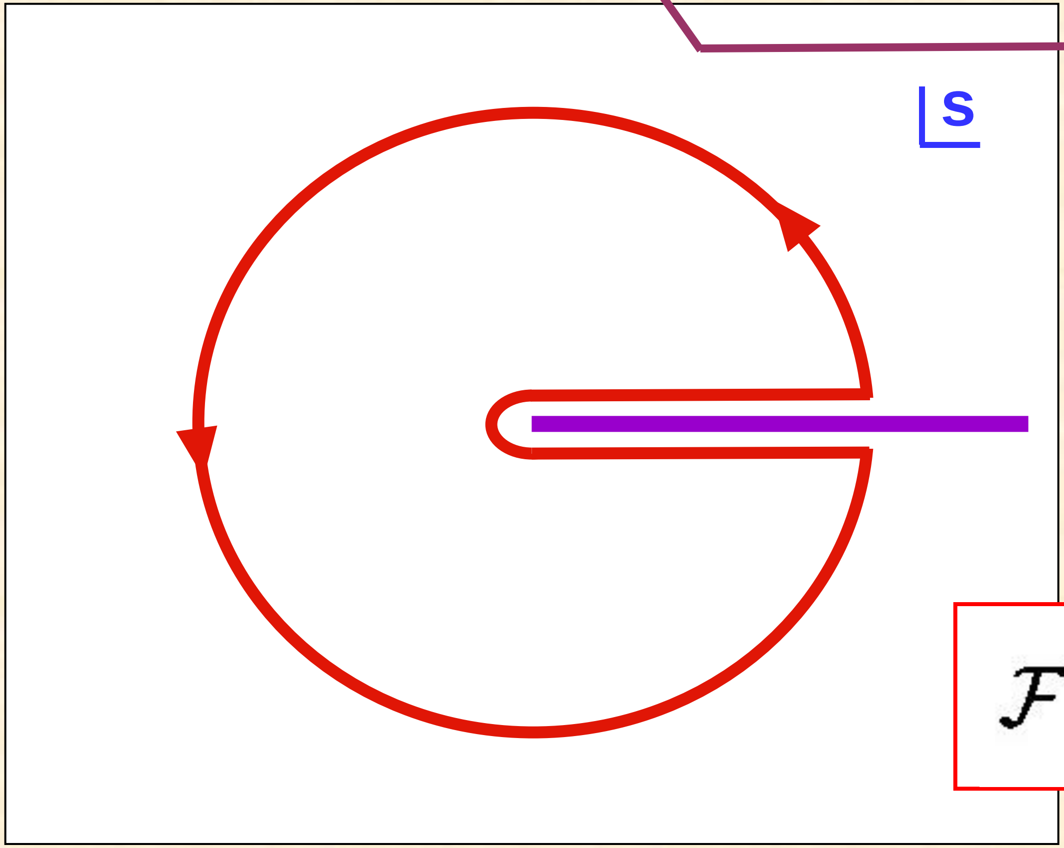
$\mathcal{F}(s)$ for each **I, J, λ**

construct $g_3(s) \equiv (\mathcal{F}(\mathrm{s}) - \mathcal{H}(\mathrm{s}))\, \mathbf{\Omega}^{-1}(\mathrm{s})/\mathrm{s}^2$ then

$$\mathcal{F}(s) = \mathcal{H}(s) + c\, s\, \Omega(s) - \frac{s^2\, \Omega(s)}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\mathcal{H}(s')\, \mathrm{Im}\Omega^{-1}(s')}{s'^2\, (s' - s)}$$

with subtraction constant *c*

fixed by χPT



$\mathcal{F}(s)$ for each **I, J, λ**

$$\mathcal{F}(s) \equiv \mathcal{H}(\mathbf{s}) = \mathcal{B}(\mathbf{s}) + \mathcal{L}(\mathbf{s})$$

along left hand cut

For $J = \lambda = 0$, consider $(\mathcal{F}(s) - \mathcal{B}(s))\ \Omega^{-1}(s)$ with $I = 0,2$

$$\mathcal{F}^I_{00}(s) = \mathcal{B}^I_{00}(s) + b^I s\, \Omega^I_{00}(s) + \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{00}(s')\right] \Omega^I_{00}(s')^{-1}}{s'^2 (s' - s)}$$

$$- \frac{s^2\, \Omega^I_{00}(s)}{\pi} \int_R ds' \frac{\mathcal{B}^I_{00}(s')\ \mathrm{Im}\left[\Omega^I_{00}(s')^{-1}\right]}{s'^2 (s' - s)}$$

with subtraction constants b^I

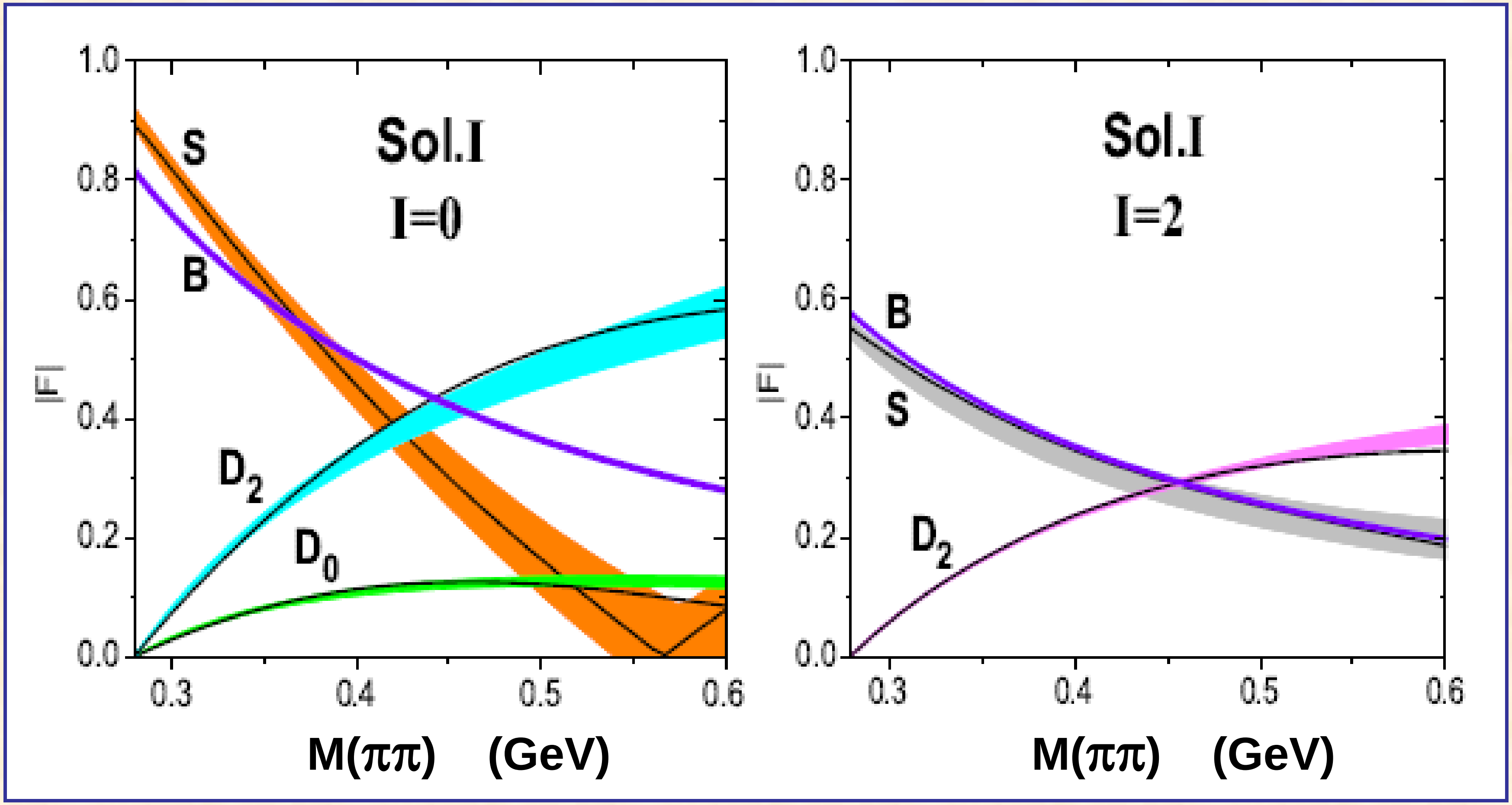
Consider $\left(\mathcal{F}(s) - \mathcal{B}(s) \right) \Omega^{-1}(s) \,/\, s^n (s - 4m_\pi^2)^{J/2}$

with $n = 2 - \lambda/2$, and $J > 0$, $\lambda = 0,2$, $I = 0,2$

$$\mathcal{F}^I_{J\lambda}(s) = \mathcal{B}^I_{J\lambda}(s) + \frac{s^n (s-4m_\pi^2)^{J/2}}{\pi} \Omega^I_{J\lambda}(s) \int_L ds' \frac{\mathrm{Im}\left[\mathcal{L}^I_{J\lambda}(s')\right] \Omega^I_{J\lambda}(s')^{-1}}{s'^n (s'-4m_\pi^2)^{J/2}(s'-s)}$$

$$- \frac{s^n (s-4m_\pi^2)^{J/2}}{\pi} \Omega^I_{J\lambda}(s) \int_R ds' \frac{B^I_{J\lambda}(s')\, \mathrm{Im}\left[\Omega^I_{J\lambda}(s')^{-1}\right]}{s'^n (s'-4m_\pi^2)^{J/2}(s'-s)}$$

Dispersive calculation of low energy partial waves



Unusual feature: large D-waves near threshold, I=2 as large as I=0

Born amplitude modified by final state interactions